Higher Surveying
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Module - 8
Lecture - 29
RADAR (RADARgrammetry)

Hello everyone, welcome back in the course of Higher Serving. Today we are in the lecture 3 of module 8. In last two lectures of this module RADAR we learned that what are the geometric aspects of the RADAR, as well as radiometric aspects of RADAR, ok. In case of geometric aspects we saw that: what is the range resolution, what is the azimuth resolution, what is the slant range, what is the ground range and other aspects all the geometric aspects were discussed right.

In addition to that we saw that the resolution in the azimuth direction or the along the flight direction is very low or inferior. So, in order to improve that resolution we have also saw a concept called synthetic aperture RADAR, where we realise that azimuth resolution can be improved by observing a point on the surface of earth from many points, by the RADAR sensor on its trajectory, maybe airborne trajectory, a space borne trajectory. So, that was the concept of synthetic aperture RADAR ok and then we also realise that that azimuth resolution is improved by 2 types, ok.

Later in the next lecture we talked about the radiometric aspects. In case of radiometric aspects you first understand what was the brightness and how brightness is affected by the reflectivity of the material? As well as other circumstantial conditions, like bracket scattering or cardinal point and specular point right then we say that, if we acquire the data it is natural to have a noise and noise is created by many sources maybe hardware or maybe the terrain ok.

Then we say that, how can we remove the noise and we name it like speckle, ok. So, after remove speckle we have celebrated the image, the radiometric calibration and then we said that, now we have the image ready with us and image is giving me 2D coordinate plus brightness value ok. And we also said that brightness is decomposed or it is resolved in two parts one is imaginary part and one is real part. So, one we said quadrature and one is in face right.

So, now, we say that, after doing all this geometric corrections and radiometric corrections we have image available RADAR image available with us, and there we said that, a RADAR image gives me couple of pixels and each pixel gives 3 values; one is range second is incident angle and third is the brightness value. Now, in this lecture we want to understand that how can we use these values in order to do the 2D or 3D mapping and that is why we are using the word RADAR grammetry. Grammetry means to measure the 3D information or topographic information. So, we are viewing RADAR images today ok so, let us go ahead in this lecture.

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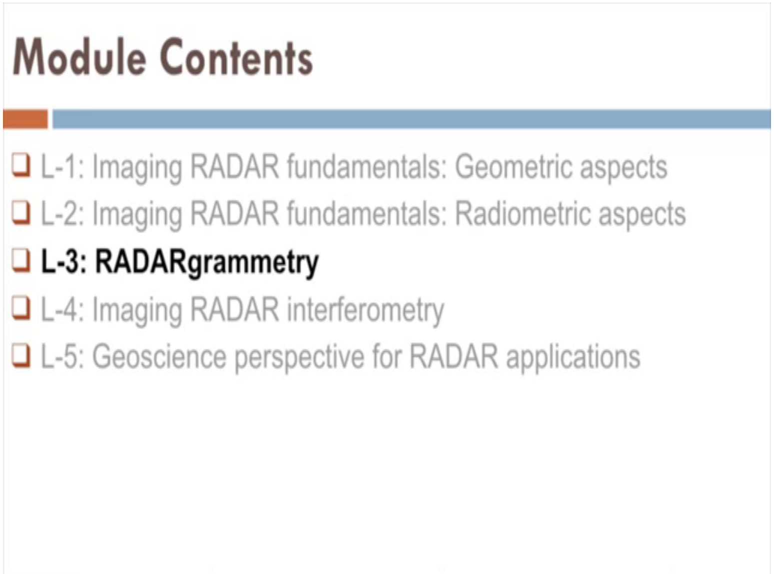

This is our 3rd lecture RADAR grammetry, now, these are the books. And what is the purpose of the RADAR grammetry?

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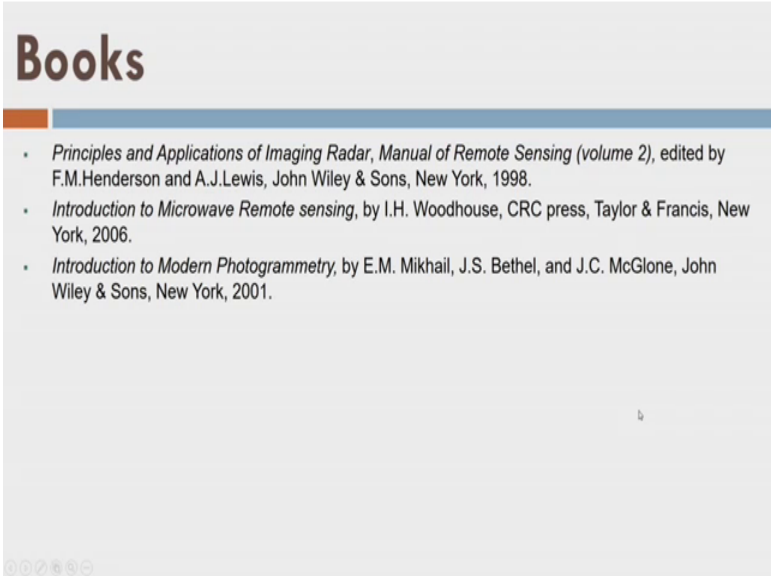

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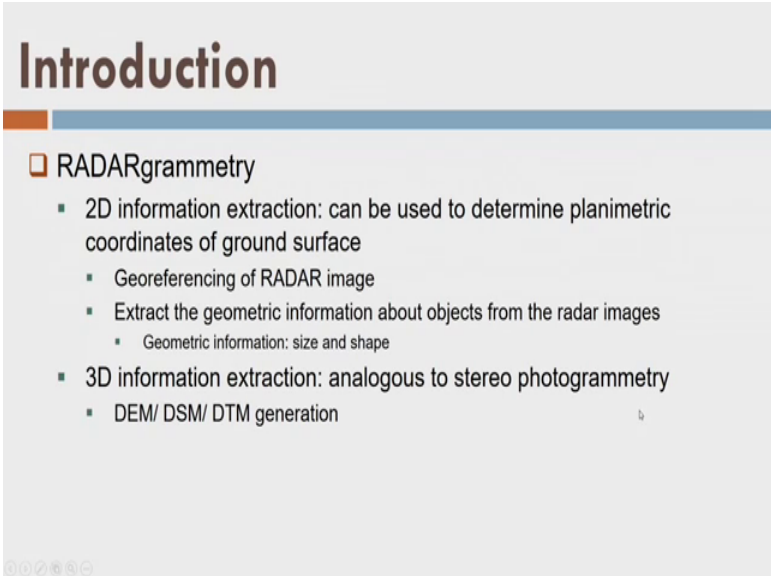

The first is the 2D information extraction for the purpose of 2D map. Now, in order to prepare 2D map I can perform georeferencing or I can expect some features from the RADAR image, what could be the features? They are the geometric features size and shape ok. On the other hand if I use the RADAR images in order to create the 3D that means, digital elevation model, digital terrain model or digital surface model right, I should be able to do it also fine.

So, what are the procedures that one should follow in order to first develop the RADAR image? Because, so far we have understood theoretically that how to do geometric correction or how to do radiometric correction, how to calibrate the image? However, we really do not know what my pixel is there and what my because if you remember correctly that pixel is decided with the help of range resolution and the azimuth resolution not be pixel size on the sensor. There is no system; there is no one to one relation between the pixel size on sensor another in case of RADAR pixel size on sensor does not exist at all. So, there is no correspondence between photogrammetry and the RADAR from that perfective right.

So, I hope that you got the idea in earlier lectures. Today will be talking little on the geometric part in a sense how my geometry is useful in order to acquire this 2D information or 3D information. So, we want to develop 3D information product that is DEM or 2D information product which is mapped right ok. So, let us once again look into the RADAR data acquisition.

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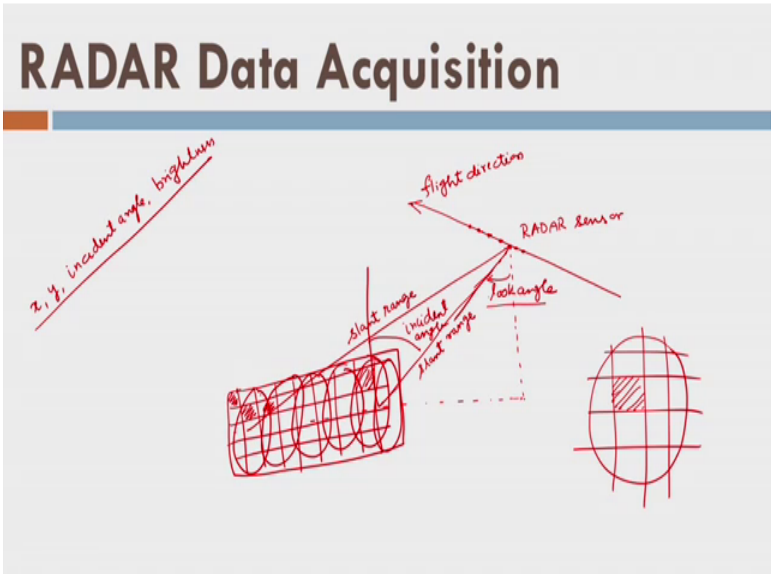

For example, let us see this is the flight track or maybe space one is spacecraft track and here my RADAR sensor is located at a given point. Now, this is my another line, I am not good at drawing so, this is not a line another here fine. We say that, in azimuth direction which is perpendicular to the flight direction this is your flight direction. We have acquired by side looking mechanism the first like this or the first footprint like this,

this is my footprint, then the last one is like this. So, this is another footprint here ok in between we have many footprints like this over prints.

And now what I want to do? I want to create an image like this, where each and every footprint is divided into multiple elements and each element will call as pixel fine so, these are my pixels here ok. So, let us say this pixel is there, this pixel is there, this pixel is there and so on. Now, you can see here very carefully that during this acquisition of from first footprint to last footprint that is my slant range for last footprint, it is slant range for my first footprint fine. From the first footprint to last footprint we can see that this v curve or the aircraft on the spacecraft is also moving from point to point here like this fine ok.

Also it has some kind of PRF Point Repetition Frequency that is number of pulses it fired in one set fine. So, that is a kind of dynamic motion that is happening and we want to measure the x y coordinate of each pixel like this ok. Here I would like to highlight one thing that we have said that it is my look angle, if you remember correctly and at any pixel I can say that is my incident angle here fine, remember all these terms fine right.

Now, this is very important thing here. Important thing is let us say that we have one footprint and we are dividing these footprint into many pixels and each pixel is giving me some value of slant range and incident angle in brightness that is what we have said. Just look here that if in case of airborne RADAR the look angle is equal to the incident angle. So, what basically data of RADAR gives me? It gives me x y pixel value and the incident angle value also we have the brightness value right. So, these are the data we can we get from the data. Now, what do you mean by x and y value? Ok, so let us look into the next slide ok.

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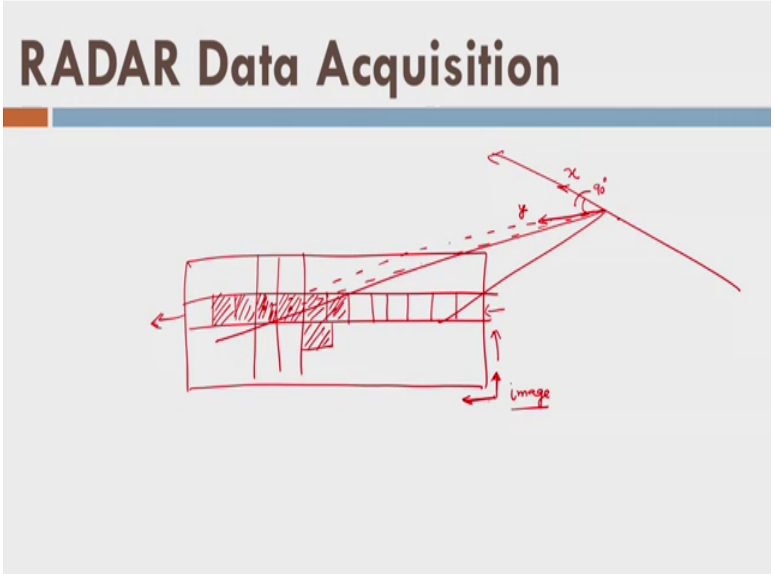

So, let us say that there is some image that has been acquired by the side looking mechanism. So, let us see this my flight track so, that was our first footprint, and this was our last footprint and so on right. Now, I would like to say let us say there is one pixel here in this line. So, I want to look at this pixel.

So, what will happen? So, this is my range here, it has been acquired from certain point ok fine. What about the any other pixels? That means, if I define some variable here like say for example, x and if I define my y along this direction; what will happen? This is 90 degree. So, this data will be having by value and it is nothing but this range of this pixel.

Similarly, for this pixel what will happen? My y range this range will change like this, but x will remain same and so if I have this pixel for example, so my x will also change here and range will also change here. So, now, I can see by looking at this aspect or looking at these values of x and y that if I try to map all these ranges of this pixels right they are monotonically increasing from which direction to this direction.

Similarly x is also increasing in the flight direction so, I can imagine here very clearly that I can develop an image where I can have a reference system like this image reference system what I call or image ordinate system right. And here this value are increasing y values of the range values, right and here it is my increasing x values.

Now, the question is how to calculate the values of x and y from the given range data and they look angle or the incident angle for a given pixel fine. So, in this lecture we are going to learn this thing. Moreover after that we will connect the small x y pixel coordinates to my ground coordinate system xyz using some kind of development of transformation right.

After that we will also learn the space resection in this the that means, given the pair of x y pixel and the xyz ground coordinate how can I find out the least square solution or how can I find out the trajectory point and it is orientation such that I can say that the whole image has been acquired from one point. So, for that purpose I need to use least square solution, I hope you are able to recall our photogrammetry concepts.

Similar concepts will be used, but similar not the same because RADAR is completely different from photogrammetry from RADAR and it needs a complete different treatment fine. So, let us go ahead into the lecture ok. One thing I would like to tell you here. Since this rangers are measured along this line for example, an increasing rangers are measured from this along this line, what will happen? Basically, because of the range we are saying that if we create a image using a range.

What will happen? Range is nothing but some diagonal value or you can say that I am representing the diagonal values or the radius values of a circle, right. So, based on this aspect we will compare RADAR and optical image as well as my map that is orthographic projection, right.

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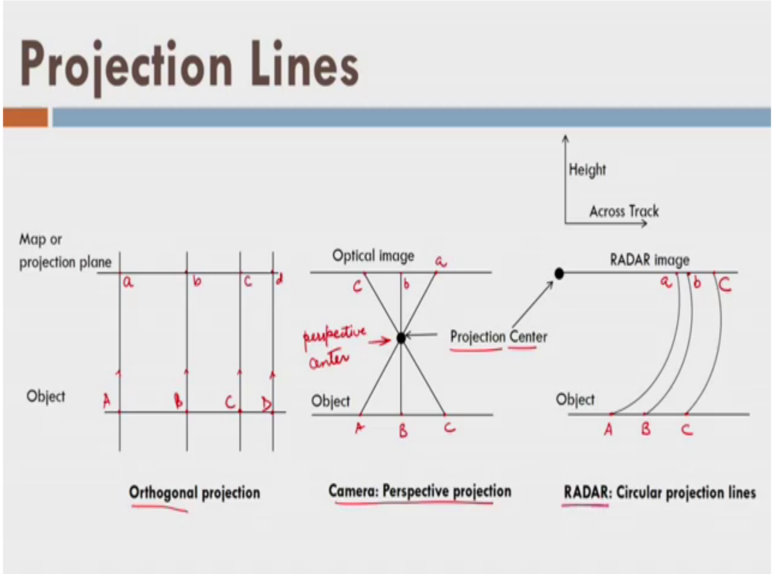

Now, see this is my orthogonal projection. So, these are the points here of the object surface and this points are basically projected here on the map. So, this are my point let us say if I call A B C and D so, these are my a b c and d, right ok. Now, this is my orthogonal projection or what we call a map. So, map is nothing but scale version of my true ground surface ok. Let us come to the perspective projection on the camera that optical camera point. So, there is a point A and point A is projected here at point a, point B is here and point C is projected here and this is my projection centre or I can said the perspective centre at the same time now, let me write it perspective centre also ok.

Now, if you look at the projection centre this is my protection centre and we have acquired by side looking mechanism. So, this point is A on the ground surface this is B and this is C. So, they will be predicted on the way front or what we call as the range are? So, this point is represented here as a, this point is represented here as b and this C point is represented here in c in my RADAR image, right. So, that is the difference between the RADAR image, optical image and the map.

Let us try to understand some of the definitions so that we can develop some relationship between the pixel coordinates and the ground coordinates, right that the ground coordinate is now my coordinates of the ground surface in certain object reference system or ground reference system. While pixel coordinates system is the coordinate of each pixel in a image coordinate system.

So, I want to develop a relationship here fine, so that we can conduct some kind of a space the section process. So, what you are trying to do here? We are trying to do some kind of development similar to the co-linearity equation for the RADAR. But it is not call co-linearity equation here, but I am saying that similar concepts we are going to develop, but the mathematical treatment of the problem is completely different. So, we should be very careful when we do this thing so, let us start this thing ok.

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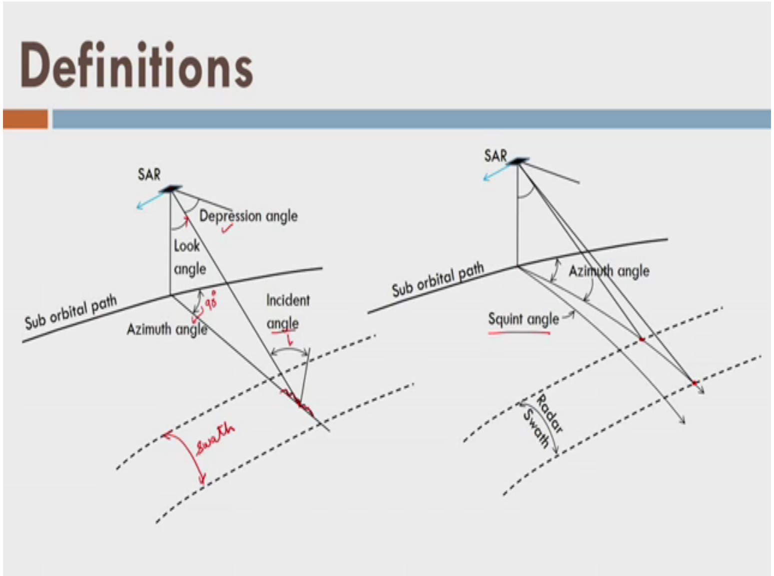

So, let us say there is a satellite on aircraft situated here and this is the look angle and this is my particular pulse that is transmitted and that is again reflected back. So, this is the incident angle here fine, and this is my terrain surface indicated here, so we know that this is my ground range and this is the azimuth angle which is generally 90 degree generally I will say ok.

So, this is my depression angle and this is the sub orbital path or another line fine. So, this is the basically the swath here, this is nothing but a swath and since this swath is on the surface of earth. So, we are showing it by curvilinear line, or what we call is RADAR swath, right.

Now, imagine a mechanism where we do not have the azimuth angle equal to 90 degree rather it is less than 90 degree, ok. So, the difference between the azimuth angle an 90 degree is called squint angle here and it is shown here fine and basically such mechanisms are called either fore image acquisition or aft image equation, right. Fore means, it is something if there is sensor like that and we acquiring the point or we are throwing the transmitted pulse little ahead of the sensor it is called fore mechanism. And if it is the pulse or the transmitted pulse is towards backward from the nadir of these spacecraft or aircraft it is called aft mechanism.

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So, let us say it is either aft or fore whatever, there is an squint angle is formed here fine. Now, you can see this is my first point on the lowest look angle and it is the last point on the swath because of the highest look angle fine. So, this mechanism is very much clear to us ok.

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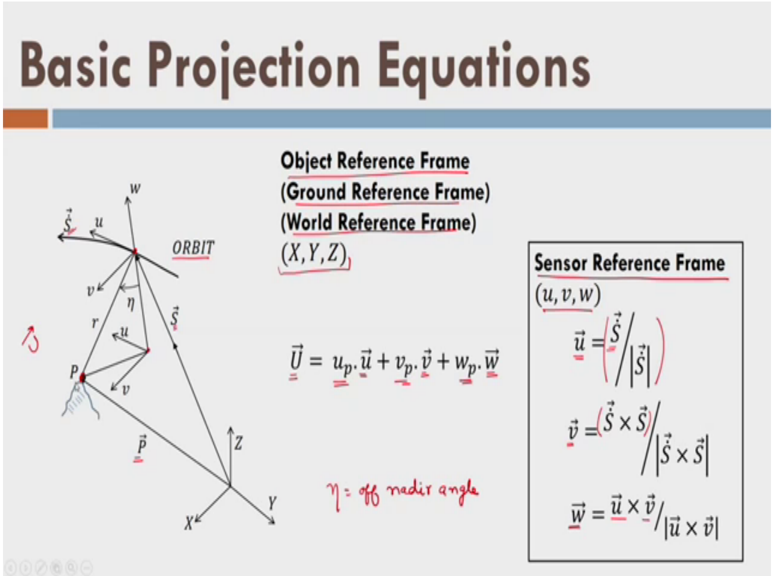

So, let us see that this is the orbit of the spacecraft or aircraft and this is the point where at present the RADAR sensor is located on this trajectory and this is the velocity vector of my RADAR sensor ok. This is the point that we want to acquire throw one of the pulse or throw one of the pixel and this is my object reference frame here fine, the ground reference and the world reference frame whatever name you give, but it has 3 orthogonal accesses capital X, Y, Z.

Now, first of all let me locate the position of the point P in the object reference frame or ground reference frame. So, this is the position of point P and it has a coordinate let us say X Y Z ok. Now, this is the position of the aircraft or spacecraft in the object reference frame and let us define a reference frame called u, v, w which is located at the RADAR sensor and I call it the sensor reference frame, right u, v, w. And where the unit vector u is defined along the u direction as tangential to the velocity at point here this point, right.

So, I am writing S dot the unit vector we are specifying here. Similarly the v is the unit vector which is obtained by cross product of the velocity and the position vector here fine and the w vector is perpendicular to the u and v. fine. So, this is my 3D orthogonal reference frame what we call sensor reference frame ok. In this sensor reference frame we measure the range are and let us say if I drop the w axis down then what will happen? This is my aft diagonal angle. Remember we are considering here the squint angle that is why we call that eta is equal to off nadir angle, fine.

Now, you understand that what is my eta and it is in some inclined plane ok. So, if you define this u and we axis on the ground surface here so, they are like this fine ok. Now, let us define the position of the point P in the u, v, w frame, how can I say that, let us define a vector u, right which is nothing but equal to the range r fine and so it is defining the U here I can say here like this.

So, U is nothing but u p, v p and w p that in this the coordinates of point p multiplied with the unit vector, unit vector and unit vector fine. So, that is my vector U and it is representing the vector form this point to point P fine. I hope that you agree with that and now using in some vector notations we would like to derive some information ok.

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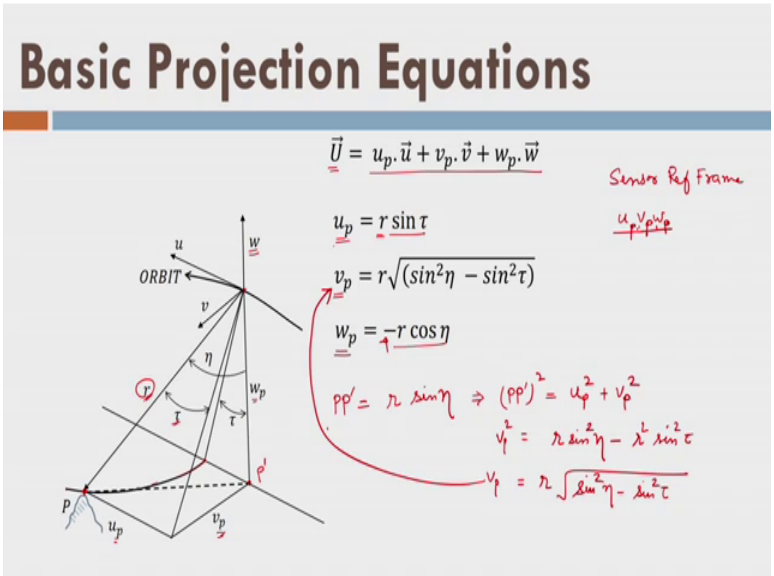

So, let us say the U vector U is there and we have already defined it like this ok. So, this is my range r now, you can see here this is the w p which is nadir point here on the ground surface ok. So, if I draw a line like this what will happen? This is the of diagonal angle eta, this is that means, peace if I am drawing a basically curvilinear path over there of equal range this range here. So, it will be making a circular path here and it will touch here. So, now, this angle if I call this is my tau or my sprint angle point. Similarly, if I project this line here like this ok so, this is also my squint angle I hope you agree with that, ok. What about my u p and v p? That is u and v coordinate of point p so, these are my u and v coordinate of point P.

So, now, you can easily see what is the relationship between the r, v p and u p ok? So, if I write up I can write up to r sin tau fine. Why because this is r, if I resolve thus r using this triangle I will get this equation u p is equal to r sin tau. Now, you can easily see that w p is equal to here r eta, and where minus sign here because, it is in negative direction compared to the positive direction of w ok. What about the v p? Here, in order to understand the v p you can easily understand let us calculate this dotted line length and it is nothing but I can say that this dotted line if I say call it let us say P dash. So, P P dash is nothing but we can write here r sin eta ok.

So, I want to calculate this v p it is nothing but I can write here that P P dash square is equal to u p square plus v p square, ok. So, v p square is P P square which is nothing but r sin square eta minus v p square is my r sin tau so, r sin square tau here or I can write here v p is equal to r times sin square eta minus sin square tau here and which is written exactly here this thing fine. So, now, I know that u, v and w coordinates of point P in my u, v, w reference frame ok. So, I have located point P in this reference frame or what we call is in sensor reference frame we have find out the coordinate of point p as u p, v p, w p, ok.

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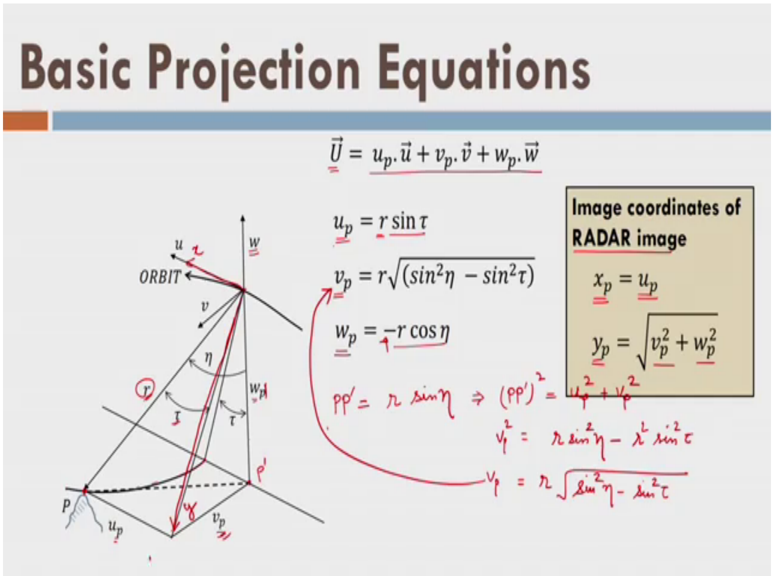

So, now, let us define an image plane such that my x p that means, the RADAR image coordinates are given by x p, y p which is u p here and it is y p here, y p is given by v p square plus w p square ok. If you look it very carefully, so this is the y p this range is y p and this coordinate is my up dash x direction and this is my y direction here, fine. So, you can imagine that this diagonal range or this diagonal distance what we call y here it is v p square plus w p square that means, v p square plus w p square here ok.

And now as you know that if I acquire another point here in the same range, so we can easily see now, that as my ranges are increasing the y values are increasing fine and now I can say that I am can represent this data in the form of image where along the y axis or along the one axis my y values are increasing and along the another axis my x values are increasing. So, that is why these values of the RADAR images are nothing but the inclined ranges and the x the ranges along the flight direction, fine. I hope you got the concept why we can represent this data in the form of image ok.

So, we have defined our image now, how to acquire the image. So, image points are clear to us fine the question is now, how to connect these image points 2D object point or the point on the ground surface that means, I want to develop a one to one relationship between u, v, w reference frame and capital X Y Z reference frame. So, that is the next challenge we need to take now, fine. In order to do that let us do some kind of simple mathematics using a vector ok.

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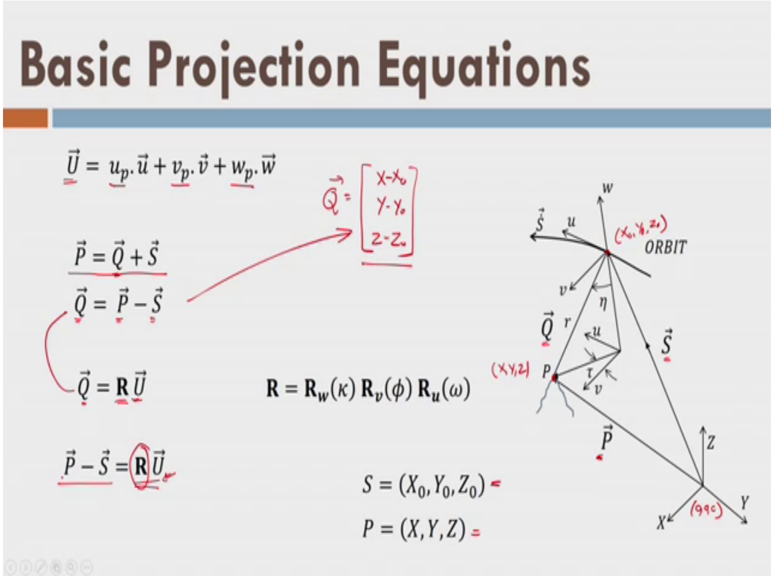

Let us call in the capital X Y Z reference frame, this is my P vector as we have written earlier, this is my S vector and this is my Q vector I am not saying u vector I am saying Q vector. Why because it is defined in this reference frame here, you may raise a question that, why are we defining different vector notation u and Q for the same range r. Now, I would like to say one thing that the reference frame meters here because, the Q vector is defined in the object reference frame ok, so the vector consists of two ends and we say that the coordinate fore end minus coordinate of the rear end and that is why these coordinates matter a lot.

And hence we are saying that in the object reference frame we are defining the vector Q. In some of the books you will find the same notation that means, Q in the object reference frame and the Q in u, v, w reference frame or sensor reference frame are written same. Well, if you can understand those notations is it is fine otherwise follow this notation, it is very easy to understand ok.

So, let us see that U was there the vector of point P defined in the sensor reference frame, but at the same term I am saying in the object reference frame I am defining the vector Q for the purpose of my understanding ok. So, I can write here the P vector here which is nothing but the point here is X Y Z which are written here, right and this point is my S vector and it is showing X 0, Y 0, Z 0 here which is written here ok. So, this is my vector P which is X Y Z minus 0 0 0, and vector S is there which is X 0 minus 0 Y 0 minus 0 Z 0 minus 0 0 ok.

What about this vector Q? So, Q we can write easily here in the matrix notation I hope you agree with that and if I can write this thing here clearly you can see here fine or I can also, right here Q equal to P minus S which is exactly the same here, right. But I know that vector U and vector Q are same, but if there is only one difference.

Now, we say that there exist a rotation between the two reference frame and the translation as well. So, translation is taken care here fine, but at the same time there is a rotation between the two reference frame so we say that if I rotate my u, v, w reference frame by some angle omega p Kappa around the individual accesses what will happen it will be oriented in the object reference frame and that is why we are writing this met this R here, I hope you got the idea.

Then now, by replacing this value here I can see that P minus S equal to RU. So, now, we can connect this using this rotation matrix the U coordinates u p, v p, w p, right here to the object reference frame. So, the important thing here is the rotation matrix now ok a similar job we have done in the photogrammetry, so let us repeat that thing again here.

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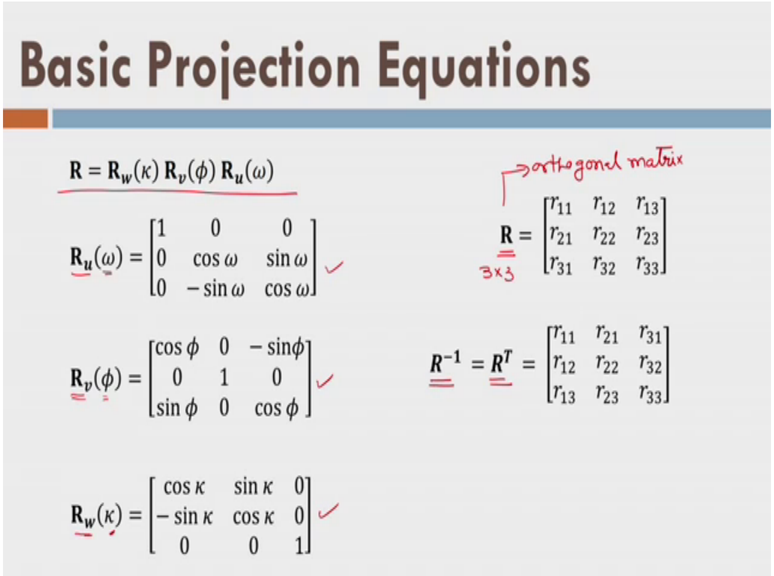

So, remember that was if I rotate the u axis by omega angle I can write this matrix similarly, if I rotate v axis with phi angle I can write matrix. And then if I rotate w with Kappa angle I can write this matrix or if I integrate all 3 matrices that is R equal to multiplication of 3 matrices. Then I can write also R in the 3 by 3 form that consists of this 9 elements and since it is an orthogonal matrix. So, R is my orthogonal matrix so, what happens here is if I take the transpose of R it will be R inverse and that is the beauty and that is the simplicity also ok.

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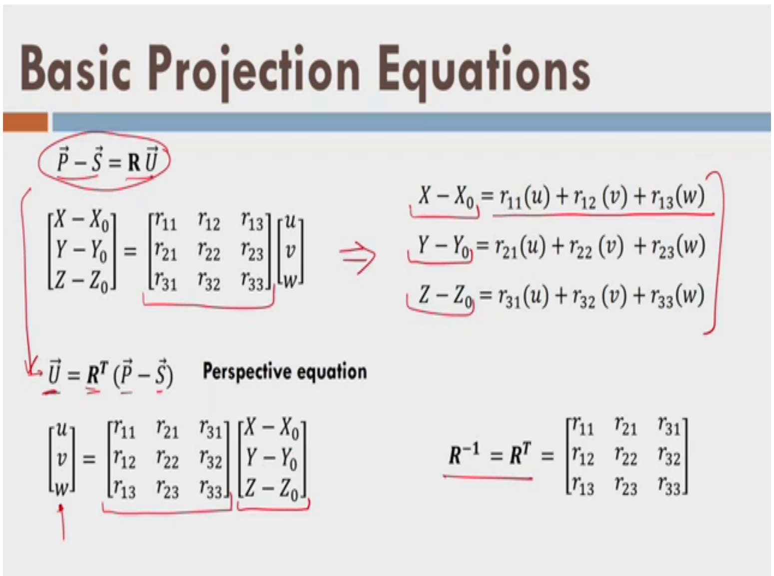

So, I hope that if you write this equation again here like this, I can convert using this matrix r the u, v, w into this system fine, I can write here this thing that U equal to that means, I am taking inverse of this equation here fine. Now, using this I can write X minus X 0 equal to this much and you can also try yourself this 3 equation we will get it from there ok.

Now, if you remember in case of co-linearity equation we said that we will convert first our image coordinates system to the object coordinate system, but ultimately we will express our image coordinates as a function of my object coordinates so same thing we are doing here. We are now presenting my U in terms of P and S using inversion of the matrix R.

So, I have written the same equation by inverting it. So, it is inverted equation, so I get my u, v, w in terms of the inverted matrix. So, it is inverse matrix here right and this is my translation vector between the point P and point S the RADAR sensor and the object point on the this is translation vector fine. So now, we got the u, v and w fine.

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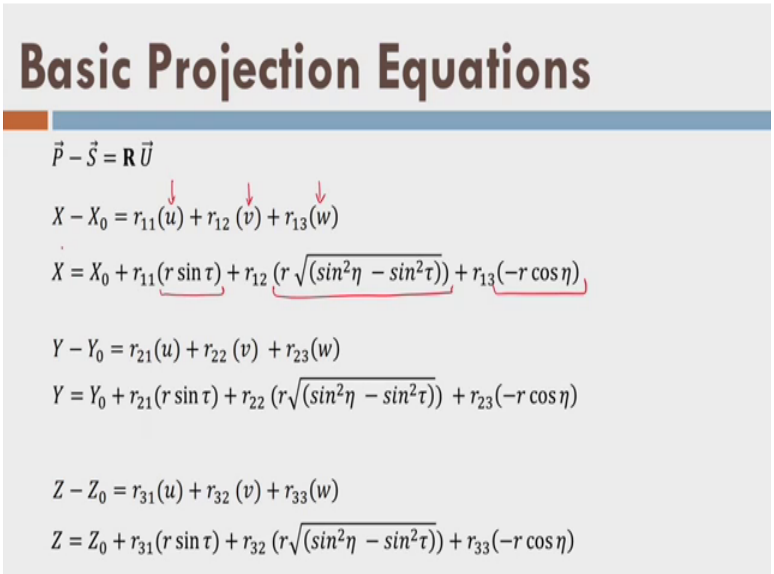

So, if you replace the value of u, v and w well, we will get this values here inducted, right and then we can write this equations also ok.

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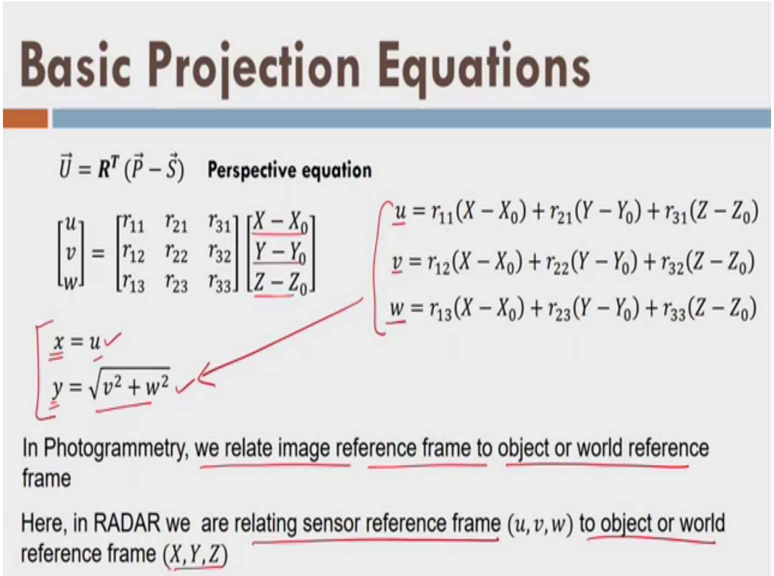

So, here we have seen that we got some u, v, w in terms of X minus X 0, Y minus Y 0, Z minus Z 0 and we know that x the image coordinate x is given by u and image coordinate y is given by this one. Now, we can see that indirectly we are connecting the image coordinates x, y with the object coordinate capital X, capital Y and capital Z. Also we can see here that X 0, Y 0, Z 0 and omega phi kappa are also participating variables here, fine.

So, now we can see that in photogrammetry we relate the image reference frame to object reference frame, here we are relating this sensor reference frame to the object or world reference frame here, right. Now, using the indirect method we are relating my sensor reference frame to the image reference frame or we can say that we are indirectly relating our image coordinates of RADAR image to the object coordinate or the ground coordinates fine. Now, we can write this equations here u, v, w and then you can use this value here in order to find out x and y, all right fine.

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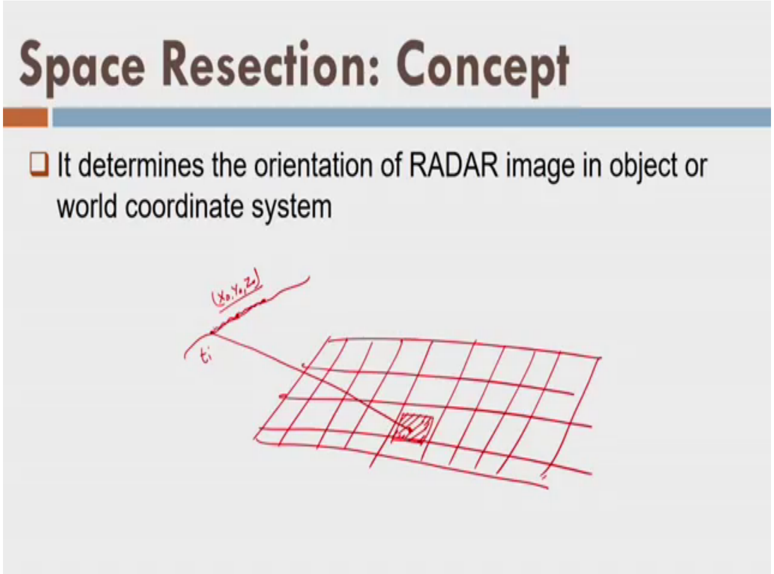

Now, we are developed some kind of relationship between my image coordinate x and y to the object coordinate system X, Y, Z. So, now we have developed the relation between the image reference frame or image coordinate to the object reference frame that means, x y is some function of the capital X, capital Y, capital Z. So, various variables like omega phi Kappa X 0, Y 0, Z 0, I hope that it is very much visible from the previous slide ok.

Now, what is the space resection? Let us understand the fundamental concept here ok. Let us imagine that air craft, a space craft is moving on certain trajectory like this and at this point it is timestamp t1, it throw some pearls and try to acquire some kind of data on this pixel. Remember pixel is a part of the footprint and now we are converting the one footprint into many pixels so, this is one of the pixel ok.

Let us imagine that it is moving like this on the spacecraft trajectory that is vehicles moving on this trajectory and after sometime it acquires another one and so on. So, while it acquired the whole image like this from here to here by side looking mechanism, what happens here is it also move from t to t another point here a space, right ok. Now, what I want to do in case of space resection? Ultimately, when I have the whole image here, so I want to find out that this image here should be acquired from one point as if it is acquired from a frame camera, well that is the idea here.

So, that I can use the idea of mapping, 3D mapping, 2D mapping whatever. Now, what I need to do here I need to identify a point let us say X 0, Y 0, Z 0 and the orientation omega phi Kappa of the my spacecraft sensor or my air bond sensor such that I can say that it has acquired a single image RADAR image from that point, I know that it is moving all the time.

However, we say that that during the whole movement from this point to this point I am trying to find out a single point between the movement such that from that point it appears to me that I am acquired an frame image. And that frame image consists of many pixels and each pixel is representing some range and each pixel is representing some x value and y value. I hope you got the idea here now, what we want to do.

So, what can we do here? We have to go for the some kind of least square solution, right and that is what we call these space resection where we are trying to find out this point X 0, Y 0, Z 0 from where we are acquiring an frame image. Also we are trying to find out omega phi Kappa that is orientation of my sensor, given that x y z coordinate of a point on ground surface and the corresponding point on the image x y.

We have already learnt that how to find out x y and we can acquire ground control point from the real field data fine from the field data ok. So, let us look into the process of space resection now, fine. So, these are the equations, we have ones again writing here, we are writing once again and this is my equation here ok.

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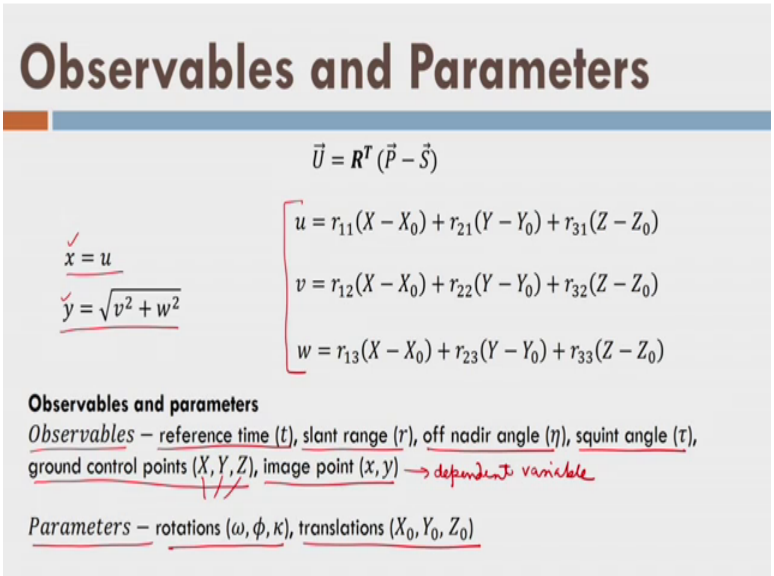

So, in order to make our process very simple what I need to do for least square solution, I will be first finding out the errors in the x and y image pixels and I will try to minimise hose by least square method. And then I will say that in the whole process of least squares I will determine X 0, Y 0, Z 0 omega phi Kappa ok. You can see here that treatment is completely different, instead of making the things complicated let us put the things in a simple form, right.

So, what are the observables here? First of all I know what are the ground control points and what is my corresponding image point? So, image point is my dependent variable here and in case of space resection X, Y Z are my independent variable ok. We also know what is a reference time t, what is slant range r of a pixel, what is of nadir angle eta and squint angle tau, right, so using that we have calculated this x and y, fine.

So, we limit our discussion to x y u, v, w and capital X and capital Y, capital Z, X 0, Y 0, Z 0 well and the omega phi Kappa, right. So, what are the parameters here? The rotations omega phi Kappa and translations X 0, Y 0, Z 0 or my perspective centre coordinates ok.

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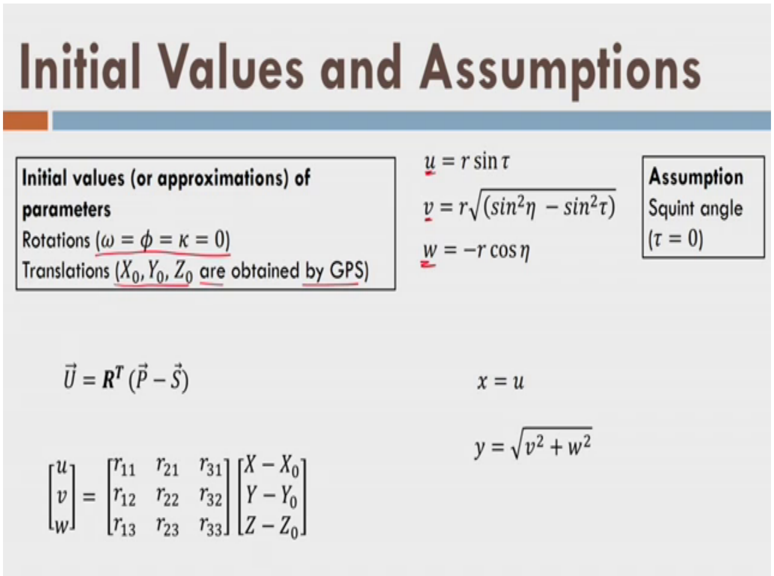

So, what are the initial values that I take for the parameters? So, I will take omega phi Kappa equal to 0 and I will take that this translation X 0, Y 0, Z 0 are obtained by my GPS or any position sensor that is mounted with the RADAR sensor and the aircraft or spacecraft, fine all right. So, these are my relationship also, here well they are no more required and then we have this relationship here.

So, let us see in this equation what I need to find out if I differentiate my x and y in order to find out the error terms dx dy. What can I do here? dx equal to du and dy equal to this term the standard differentiation term. So, I need to find out now, du dv and dw and that we will find out from the equations which connect u, v, w to x y z, fine.

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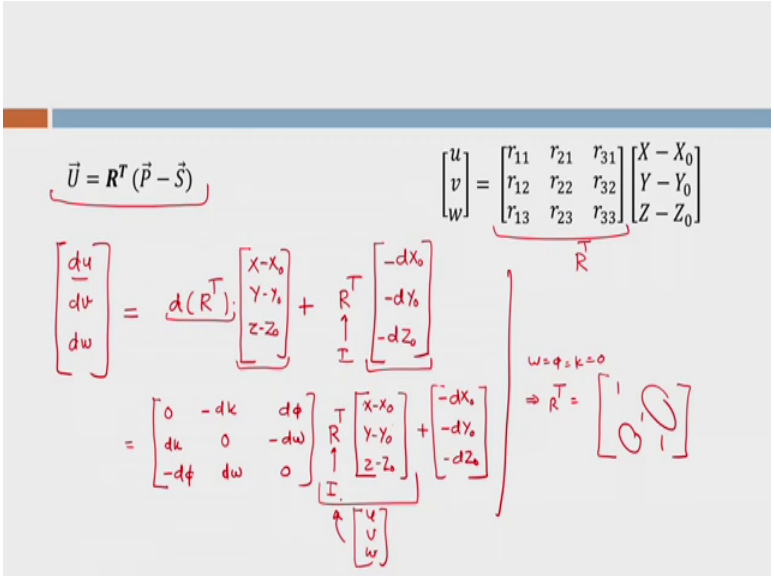

Now, I need to linearize them using the Taylor series because, they are now, linear in form in order to find out the differentiation ok so, let us do the differentiation here ok. How can I do the differentiation? It is very easy let us do the differentiation. So, I can write here in a matrix form only du, dv and dw is equal to I can say here the R matrix this one or R inverse whatever you say ok.

So, let us call the R T, fine so, I can say here that differentiation of R T into this matrix. I am doing the simple differentiation plus R T into differentiation of this term which is nothing but dx 0, dy 0 and d Z 0, I hope you agree with that from the standard notation of the differentiation for matrices ok. So, what can I write here, here if you see surprisingly if I take omega phi Kappa equal to 0, then the matrix R T becomes identity that means, 1 1 1 and all 0s here, right.

And so if I calculate the differentiation of these terms what will I get here? I will get these terms 0 minus d Kappa, d phi, d Kappa, 0, minus d omega, minus d phi, d omega and 0 and R T multiplication. Well, we are assuming that these angles are very small and that is why we are writing this terms here with this assumption we are working, fine. So, that is R T and R T we already know this is my identity matrix, here so, it is identity matrix.

Now, I need to write this term which is multiplicative term here, fine and then what about this R T? It is my identity matrix, ok, so plus I write this terms here. So, this my dx 0 minus dy 0 minus dz 0, right alright ok. So, I got the values of du, dv and dw if I multiply this and which is very simple, fine.

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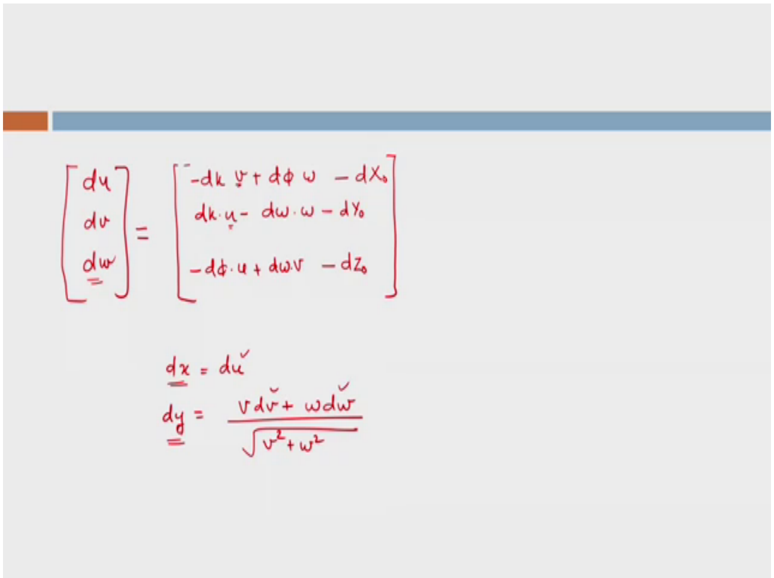

So, we get these terms liked du, dv, dw equal to minus d Kappa into v plus d phi omega minus dx 0. Next is d Kappa into u minus d omega into omega minus dy 0. Further you got here minus d phi u plus d omega v minus d z 0, right ok. You will be surprised how do we get this terms u v and w here again, if you just look at back there we know that these terms are R T into P minus S here.

This complete term here, it is nothing but my u, v, w fine so, these terms are u, v, w only originally from this equation you see here. So, I write the site here same thing here so, I get this term here u, v, w also again and then this simplified terms are like this ok. We already know that my dx equal to du, and dy equal to v dv plus w dw and it is under root of v square plus w square. So, if I put this value of du, dv and dw from this here I can get the form what is the dx and dy here, right.

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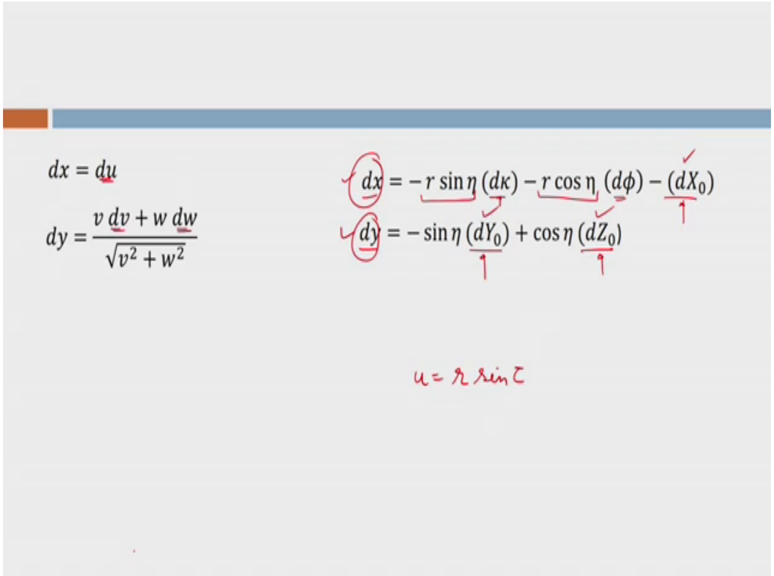

Ultimately, if I do what will I get here, we get these terms and there if I put the values of du, dv and dw, right, I will get this terms dx equal to dy equal to finally, this one I am putting it again the values of u v also. So, finally, we get this one you can see here that there is no value of d omega it is only d Kappa and d phi ok. So, we have replaced the values of u v also in terms of for example, u was given by if you remember r sin tau. So, we are put here these values and we got this values finally, right so, you can just take it yourself ok.

Now, what is the speciality of this equations? You can see there is no presence of d omega that means, omega does not affect my space resection process ok. What next? Here, you can see dX 0 is there, d phi there, d Kappa is there, dY 0 is there, d Z 0 is there, but at the same time now, or we can say that omega phi and Kappa basically they are derived from the position vector of the space bound for the RADAR sensor ok. And because of that what happens is my omega phi Kappa are no more unknowns, they are known values because they are derived from the positions fine ok.

So, what are the unknowns here? There are only 3 unknowns that is dX 0, d Z 0 and dY 0 or I can say X 0, Y 0, Z 0 and we are trying to find out the corrections to X 0, Y 0, Z 0 by least square solution by minimising these errors in the image coordinates x and y. Remember the least square solution ok, but here there is a small point here that we should always understand. Because of the movement of the spacecraft or aircraft what happens is during that period of movement it also acquires the data, right and which has to consider the aircraft trajectory or the spacecraft trajectory. So, generally we assume that spacecraft or aircraft is moving any in the second order trajectory or the trajectory has a second order equation.

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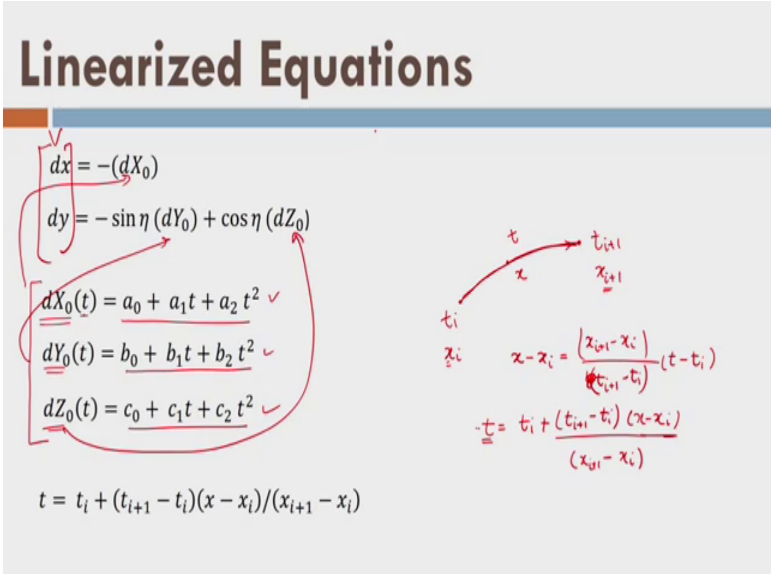

Now, so how can I write it? So, this is the way we are writing and we are writing that these my corrections or the function of time and it is nothing but they are second order curves, fine. And now we can say one thing here that we have 9 unknowns because, as we said before that there are 3 unknowns and we are replacing the unknown by this equations, where t is my time and how do we calculate the time that is more important here ok.

So, let us see if there is a timestamp t i and this is timestamp t i plus 1, fine. And this is my some image coordinate x which is acquired at this point x t i and at this point some other x i plus 1 will be acquired here. This is my image coordinate x and this is along track path I am saying now, fine ok.

What happens here is let us say this is time t, at this time t some point x is acquired or the image coordinate x is acquired ok. How can I write this thing using the linear relationship? Fine, we are writing it like this. Remember the equation of the line it is nothing but x minus x i is given by is equal to x i plus 1 minus x i divided by t i plus 1 minus t i all right into t minus t i ok. Using this thing I can write t is equal to t i plus t i plus 1 minus t i into x i minus x i x minus x i divided by x i plus 1 minus x i. So, for any given coordinate x you can find out what is the time t, that is at that time thus image pixel x was acquired fine.

So now, you will put this time t in this equation and you will have finally, these equations where I am putting this value say here, right, so dY 0 here and d Z 0 here in terms of time. Now, you can see there are only 9 unknowns and we have 2 equations. So, how many points do I need? Remember we need to have minimum number of points that means, minimum number of points are 5.

So, if I have 5 number of points what will happen? I can find out the unique solution of X 0, Y 0, Z 0. Rather I have 9 unknowns here as a result I will have still with 5 point some least square solution, but what if I have more than 5 number of points. I have to go for the least square solution that and the size of the matrix a in case of v equal to a x minus l will be increasing.

So, this is my V vector here fine ok. And if I put the values here I will get X vector, where X vector unknown vector will be a 0 to c 3, 9 variables and there will be a matrix you can write it very easily. Now, fine that is the kind of development we have to do. Now, if we develop the a matrix as we take more data for more number of pixels that means, I have x y pixel and ground control x y z for this couple what will I have, I will keep on increasing my a matrix and I will find out a better least square solution if I have more number of points. So, that is my process of space resection where I have determined X 0, Y 0, Z 0.

Remember one thing that d omega d phi and d Kappa, d omega was not present at all, d phi and d Kappa they are also replaced in terms of my x the sensor trajectory coordinate capital X 0, Y 0, Z 0 because, we want to estimate d phi and the Kappa as a first derivative of the sensor position in the trajectory. So, there also we say that that will also participate fine. So, doing by these 3 job I have determined my basically the X 0, Y 0, Z 0 and the point where I am acquired an frame image. So, that is my space intersection process, fine. Now, after that we want to learn that how to create the 3D if I know the X 0, Y 0, Z 0 that is my perspective centre coordinate fine ok. So, let us look into the space intersection here ok.

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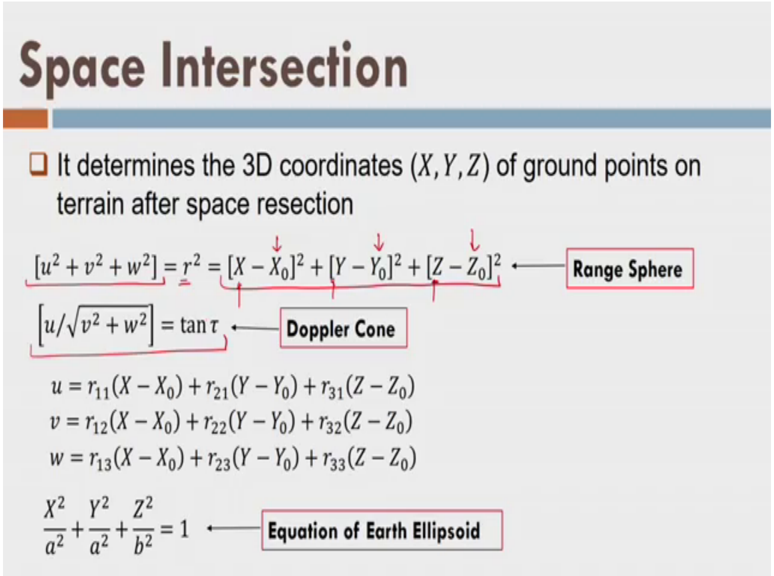

So, in case of space intersection what do we do? We use some simple process here, first of all if you remember that u square plus v square plus w square is equal to r square and it is also equal to this that is my range fine. We already know this by space resection process and now I want to find out these x y and z given the image pixel and the X 0, Y 0, Z 0 that is the trajectory coordinate of my sensor RADAR sensor and using that information I want to find out what is the ground coordinate that is that corresponds to the image pixel x y. So, that is my space intersection process.

So, what will I do now? So, I am using the term space intersection and space resection in order to correlate with the photogrammetry fine that is the purpose here. I hope that you remember those definitions till now ok no problem. One more thing I would like to highlight here that is my Doppler cone. What is the Doppler cone? At the air craft trajectory you can imagine there is a cone like this fine, where this and this centre point let us say this point this is representing the range.

So, now, you can imagine that if that Doppler has, Doppler cone is having some kind of rangers and it is creating a cone here getting it ok. So, now what will happen? These rangers will be intersecting with the ground surface fine ok. So, now, we know that if I assume that there is an ellipsoid well and there is a range sphere of this and there is a Doppler cone. So, all 3 are intersecting at one point P and P has coordinates x, y and z. So, now, we use this logic here in case of space intersection, fine.

So, let us see that if this is the equations I will replace by this one so, I have some omega phi Kappa which are known to me and this X Y Z they are unknown, but they are knowns, right so, they are knowns, fine. So, I will replace this here in this equation and that is called the Doppler cone I can also determine this triangle and that is I can calculate also that can be given also. Now, this is my equation of the ellipsoid or the earth ellipsoid.

So, we know that point P is situated at 3 surfaces one is the ellipsoidal surface that is ground surface, it is also situated on the range sphere of my RADAR sensor and it is also situated on the Doppler cone. So, now I have 3 set of equations and I have 3 unknowns is very easy to do the job in order to calculate capital X, capital Y, capital Z which are 3 unknowns and rest of the information is completely known to me.

So, now, we can do this space intersection very easily, but only problem here is we need to linearize this equations because, these equations are not the explicit as well as they are not the direct equation. So, that we can find out capital X, capital Y capital Z for the given x y pixel coordinates and the rest of the parameters, fine.

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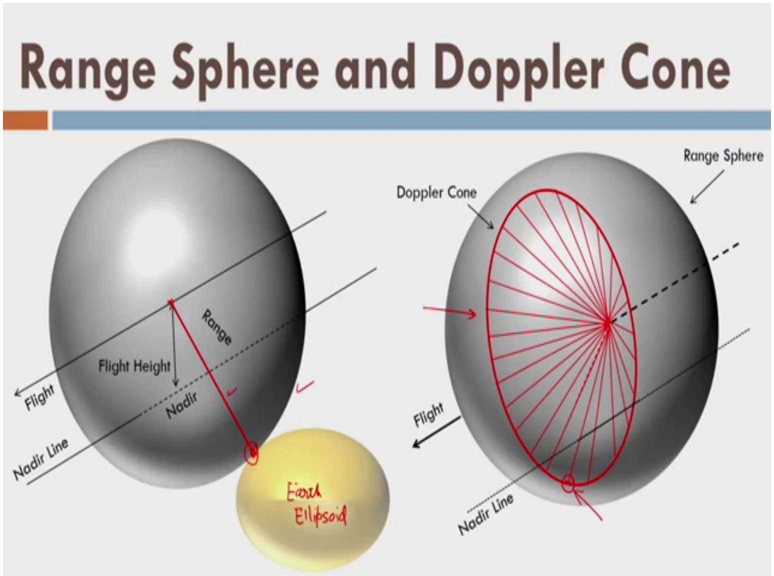

So, here this concept is shown physically here. Let us say this is the RADAR sensor located here and this is the range so, this is my range sphere fine, in both cases these two are range sphere ok. So, this is my range here so, this is a point on the surface of earth so, this is my ellipsoid or earth ellipsoid I can say ok.

Now, see here that this is my range sphere and this is my Doppler cone fine. Now, you can see that any point here this point here and this point same so, in order to we did not merge the 3 because it will very clumsy ok. So, now, this point is intersected by this range or the range sphere, it is intercepted by the Doppler cone here at this point and it is intersected by the ellipsoid of the earth at this point. So, 3 point are merging so, now, we can write 3 questions for this point, right that was a physical concept that I showed in the previous slide ok.

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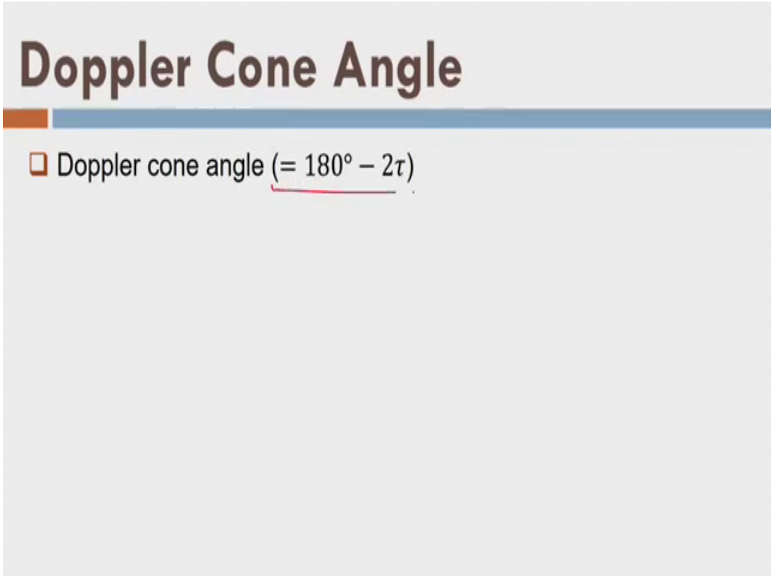

Now, the Doppler cone angle we should know that the Doppler cone angle, the Doppler cone angle which is at the RADAR sensor point it is having an apex angle 180 degree minus 2 tau where tau is my squint angle, remember this thing, fine.

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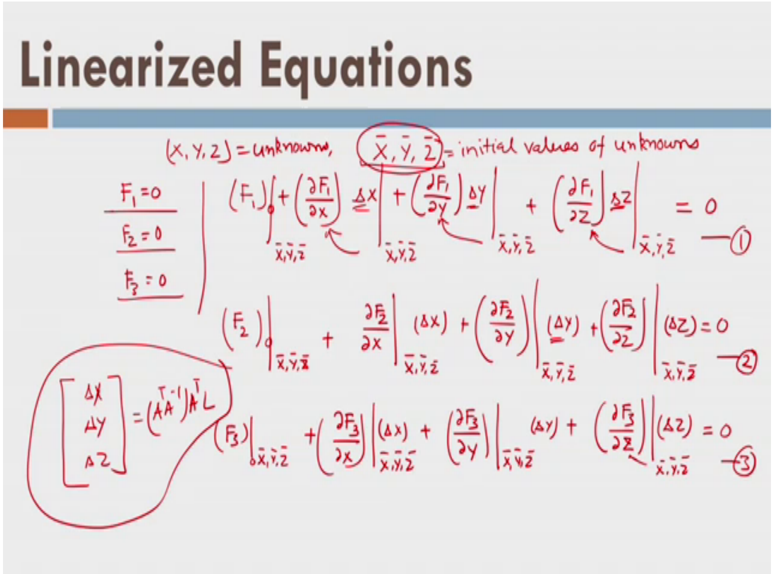

Now, using the linearized process, so what will I do? I will linearize the 3 functions here F 1, F 2 and F 3 equal to 0, full 0 equal to 0 and I want to find out my unknown as X, Y, and Z. So, let us say I have some estimate of this one as my X bar, Y bar and Z bar, fine. So, these are my initial values of unknowns, fine so, what will I do? Now, I will write this 3 functions using Taylor series expansion and up to the first order Taylor series.

So, this is my nothing but F 1 0 plus I can write here d F 1 by dX into delta X plus d F 1 by dY delta Y. So, I can determine here that these are determined here point X bar, Y bar, Z bar; X bar, Y bar, Z bar plus d F 1 by dZ delta z determined at X bar Y bar Z bar so basically we are determining this value of derivatives at this point fine. So, I may be writing little different terms, but again it is equal to 0 fine, the first order expansion of Taylor series. Similarly, I can write F 2 determined at 0 point or I can write here X bar, Y bar, Z bar similarly X bar, Y bar, Z bar plus d F 2 by dX.

So, I can determine it let us say X bar, Y bar, Z bar into we say that delta X plus d F 2 by dY determined at the same point. Again I am writing it by shortly delta Y plus d F 2 by dZ determined at X bar, Y bar, Z bar into delta z equal to 0, equation number 2, equation number 1. And one more equation I need to write F 3 determined at X bar, Y bar, Z bar where I call it 0 here plus d F 3 by dX determine at X bar, Y bar, Z bar into delta x plus X bar, Y bar, Z bar into delta Y plus d F 3 divided by dZ. So, dou F 3 by dou z into delta Z here find determine that this value determined at X bar, Y bar, Z bar equal to 0, equation number 3.

Now, you see that these are the corrections in my this value of initial approximation. So, these 3 corrections are there. So, now, we are 3 linearized equation ok so, if we put the values of this initial values I will get these linearized equations in form of delta X, delta Y, delta Z solve it, and find out the using the matrix method what is the vector delta X, delta Y and delta Z, fine. If you remember the use logic is A T A inverse A T L, where now we can construct this A matrix form these terms. Once you find out the values of delta X, delta Y, delta Z find out the new values of X bar like this, Y bar like this, and Z bar like this.

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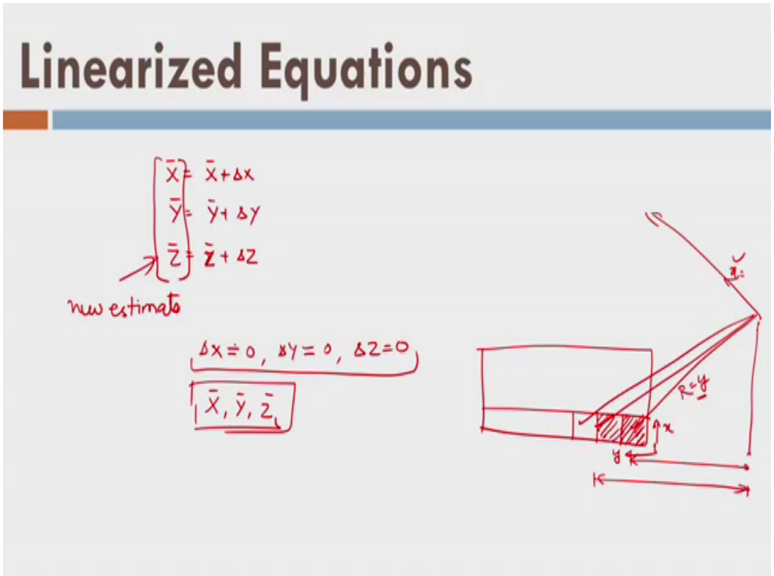

Now, use these values on the left hand side as my new estimate or the initial approximation and then find out the new set of delta X, delta Y, delta Z and repeating repeat this process, unless till you get delta X is almost equal to 0, delta Y is almost equal to 0 and delta z is almost equal to 0. So, that is a point where the solution will be converging.

So, the final point ones you get delta x equal to 0 the whatever the final point is there X bar, Y bar, Z bar what you get that will be your coordinate of the ground control point. Here, I would like to say now, that if you remember that the images are inclined images, right where we are saying that this is the range remember and range is representing the pixels or pixel is representing one range y value, here this is y value. So, if I indicate on the screen on the screen I know that my image side looking image. So, one pixel here, next pixel here, it is indicating let us say tract here one y value, the next y value, next y value and y values are increasing.

Now, what happens is I know that this is my ground range from here to here fine, for this pixel this pixel I have this ground range. So, what will I do? I will convert this y value range values into the ground range values and using the resolution the ground resolution I will convert this range resolution into number of pixels or the distances. So, then I will say that this is the distance the ground distance or the ground range this pixel is situated. Similarly, this is the distance I will see like that, right.

So, this is the way we say that I have inclined images like this and, but I have make it horizontal parallel to the ground surface and that is done in the radiometric processing. And finally, we use those images for the processing purpose, processing in a sense for calculating the digital elevation model or 2D maps.

So, here we would like to finish this lecture. And in the next later we will talk about the interferometry, that how can be used the RADAR interferometry or the RADAR phase values for of the brightness in order to create the 3D models.

Thank you.