Higher Surveying

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Module - 7
LiDAR (LiDARgrammetry)
Lecture - 25
Geolocation and Errors of Lidar Data

Hello everyone welcome back in the course of Higher Surveying. We are in lecture 3 today of module 7 that is LiDAR grammetry. In the last lecture will learn that how to acquire the air borne LiDAR data, terrestrial LiDAR data and mobile LiDAR data. So, they have given different names like airborne LiDAR terrestrial LiDAR and mobile LiDAR scanning ok, also we came to know in the last lecture that in case of airborne data acquisition in addition to the laser scanner we have to have the IMU as well as GPS.

So, that we can find out the 3D coordinates of the aircraft trajectory and using that trajectory, we can find out the location or the 3D coordinates of the points on the surface of earth where, laser tales is interacting ok. So, in this lecture today we are going to learn the process of Geo referencing, which means we bring the complete data into one reference frame. And that reference frame is ITRF or International Terrestrial Reference Frame right, the question is that how to do that process? So, the process of bringing the LiDAR data into the ITRF reference frame is called Geo location ok. So, let us go ahead in this lecture.

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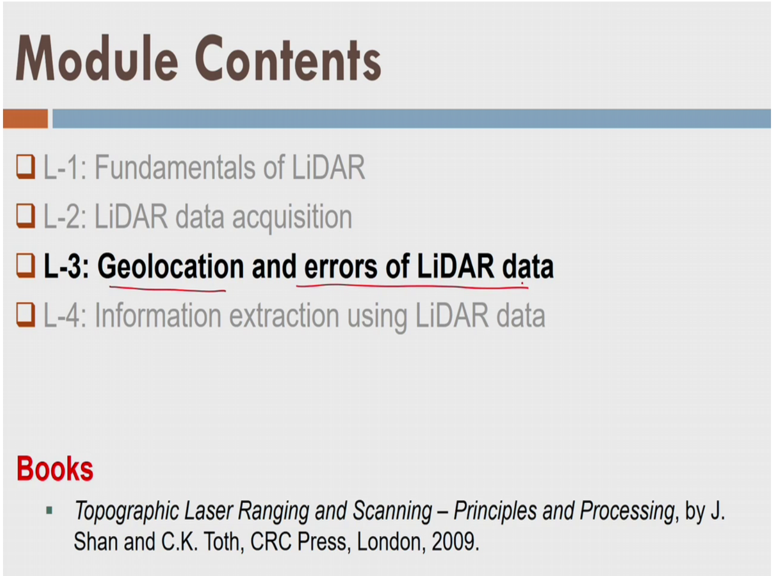

So, we are in lecture 3 that is Geo location and errors of LiDAR data right ok.

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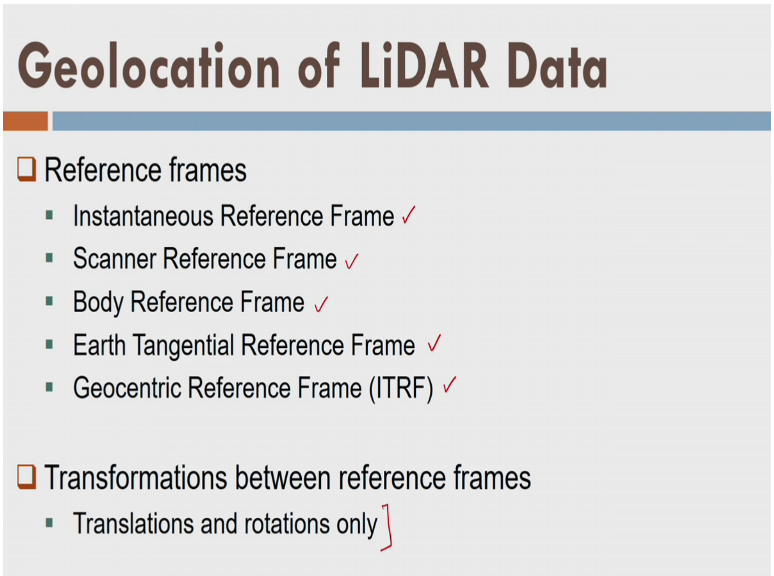

Now, in order to understand the geo location process, we need to define some reference frames ok. So, we have five reference frames here and that will connect our basic measurement of the laser pulse to the ITRF reference frame. So, the first is instantaneous reference frame, then we have scanner reference frame, then we have body reference frame, earth tangential reference frame and finally, geocentric reference frame or international terrestrial reference frame ok.

And we need to only do the translations as well as rotations among these reference frames in order to convert my laser data, the basic laser data or the that is the data acquired by the scanner into the ITRF fine. So, let us first understand what are these reference frame? The first of all my instantaneous reference frame is there.

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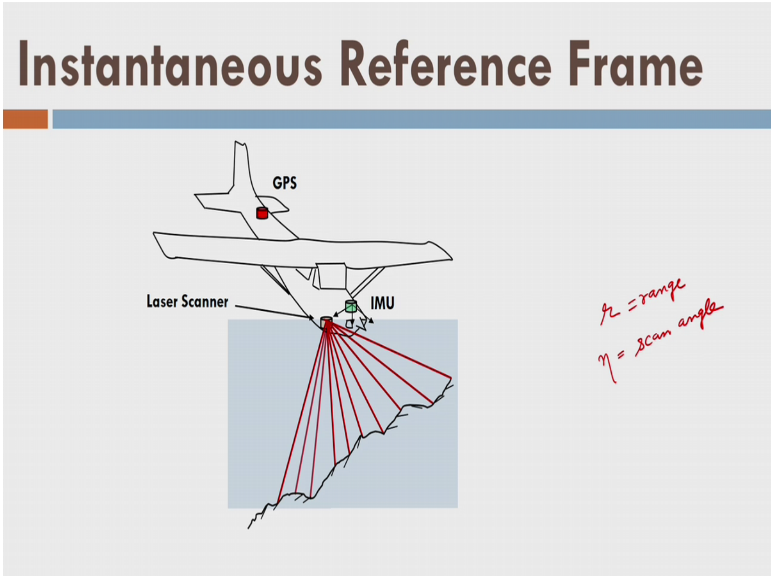

And you can see that there is an aircraft and aircraft is mounted with the laser scanner IMU and GPS fine ok and we have already discussed what is the role of IMU and GPS? GPS determines the trajectory of the aircraft ok. And it acquires the data at every one second between that 1 second we take help of the IMU which measures the role which and your with respect to the true north, true east and the gravity direction ok.

So, using that three angles we try to find out the aircraft trajectory coordinates during that 1 second where, GPS cloud not acquire the data fine. So, we can understand that we have the continuous data of the GPS trajectory ok. So, with this understanding now let us look into the animation and here we see that, there is a plane and there is a terrain and now laser is scanner is going to acquire the data like this fine ok.

Now, we can see here that these laser pulses are fired on the surface of ground and these are again reflected back to the scanner and the scanner has receive them determined the range are and the is scan angle eta, what is the is scan angle? Ok This scan angle is the angle of laser pulse with respect to the vertical.

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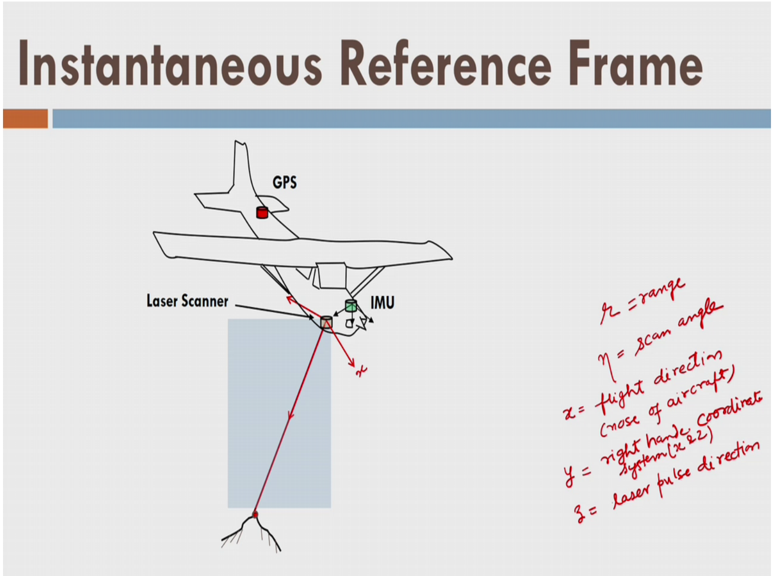

So, you can see here that let us considered one point on the surface of earth and this is the laser pulse. So, now, let us define the instantaneous reference frame. So, in case of instantaneous reference frame, this is my instantaneous reference frame here you can see that, the x axis is in the flight direction or the nose of aircraft ok, what about the y axis and what about the z axis.

So, z axis in the direction of pulse ok, what about y y is framing the right handed coordinate system with x and y right handed coordinate system with x and y with x and z right. So, now, this is my instantaneous reference frame let us go ahead these are laser pulse here.

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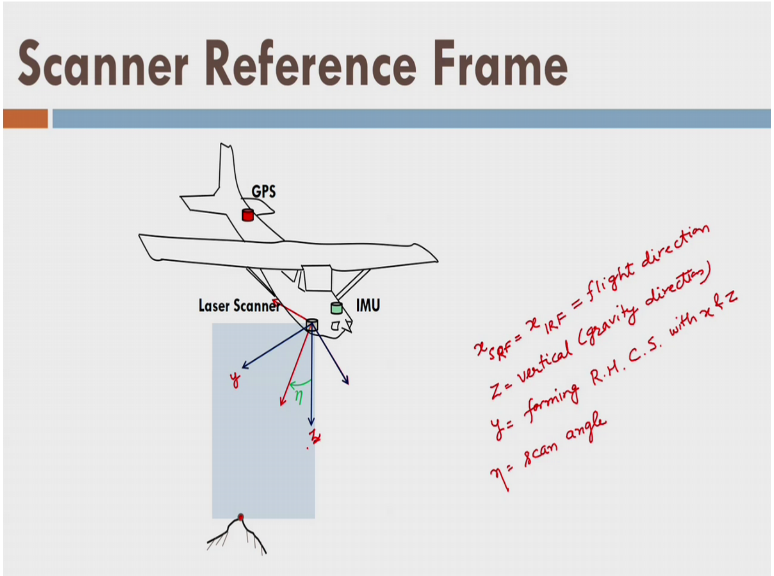

And so, we have defined the instantaneous reference frame like this ok. The now let us define the scanner reference frame which is like this where again the x of the scanner reference frame and x of the instantaneous reference frame are same in the flight direction right. Then we have this z direction of this scanner reference frame in the direction of vertical or gravity direction ok, then what about this y axis. So, y axis is again forming the right handed coordinate system, I am writing the short right handed coordinate system with x and z.

So, this is my scanner reference frame you can notice here the origin of the reference frame are at the laser scanner right, moreover I would like to say one thing here that we are using term reference frame, which means it is not a theoretical concepts, it is a realised reference system; that means, I know the origin of the reference frame as well as I know the orientation of the reference exercise fine.

So, that is why were using word reference frame let us go ahead. So, this is the angle the eta and what we call is this scan angle right. So, define this scan angle in the last side also. So, this is the angle of laser pulse with respect to the vertical so, this direction here gravity direction here right ok.

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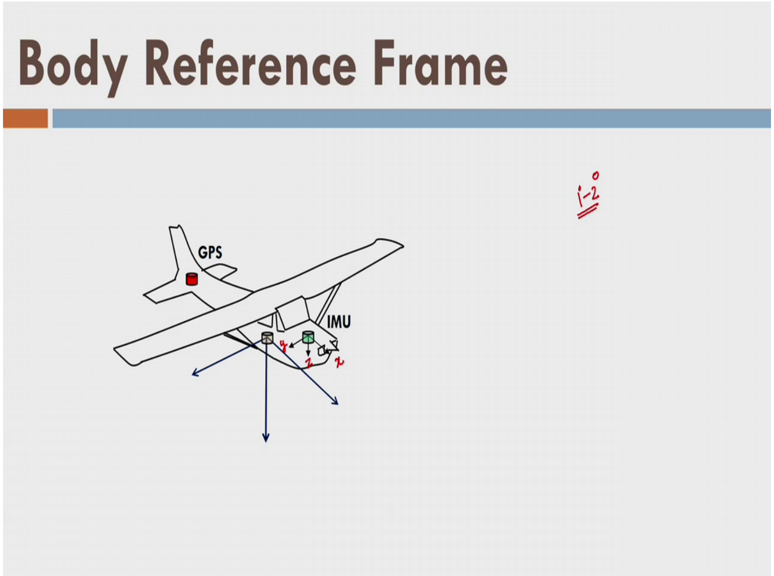

Now let us understand body reference frame, if you look here that we are showing the is scan reference frame in blue colour and now I show the body reference frame located at the IMU like this ok. So, in this case what happens is my direction of the IMU reference frame or the body reference frame at IMU and is scanner reference frame are moral less well aligned. That means, there orientation exercise are in the same direction, there could be slight difference and this differences is around 1 to 2 degrees only in each three axeses right. So, that is you should understand that they are moral less aligned ok so, that is called the body reference frame.

And now one thing about IMU you should understand that, IMU is initially when it is set up in the aircraft it is put in the switch off mode, what is switch off mode? It is turned off it is not turned on as a result when it is commissioned in the aircraft it is indicating in the x direction in the nose of aircraft and z direction here y direction here fine ok.

The moment it is turned on these three exercise will indicate the true north, true east and the gravity direction at a particular point; that means, when aircraft is flying on a certain path at that point of aircraft trajectory the switch on IMU will indicate you the true east, true north and the gravity direction. And now if you measure these three angles; that means, the IMU switch off position and IMU switch on position, the corresponding angle between the three axeses will be the roll pitch and yaw.

And so, they will help me to convert scanner reference frame or body reference frame into the other frame. So, that other frame will be the earth tangential reference frame so, now let us look into that.

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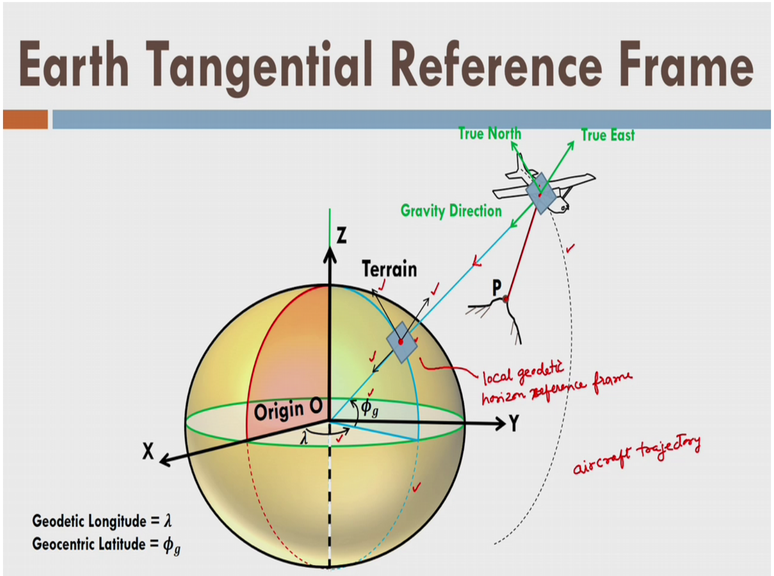

So, this is my earth tangential reference frame where we are showing one aircraft and that is following a trajectory like this ok. So, at this point where aircraft is right now, we are fired laser pulse towards the point P on surface of earth ok, let me tell you that this is the aircraft trajectory and this is the projection of this trajectory on the ellipsoid or WGS 84 ellipsoid fine ok. Now you can see here that I am defining a local geodetic horizon system here or let me call it reference frame ok.

So, now we can see there are two angle phi g which is the geodetic latitude of the aircraft position as well as lambda, which is geodetic longitude of the aircraft position here ok. So, let us defines this local geodetic horizon reference frame, such that I am also defining the same plane at this aircraft trajectory point here on the trajectory with this parallel to this plane ok. So, we know that this is the true north, this is the true east, this is the gravity direction here.

And so, we are defining the same reference frame or the same orientation of the reference frame at the aircraft trajectory like this. So, they are my true north, true east and the gravity direction, you can just see here that we have taken an assumption that this direction is my gravity direction. In fact, it is a gravity direction, but it is matching with the geocentic latitude in most of the areas this is very fair assumption for the earth surface, if you are using the W GS 84 coordinate system or ellipsoid right. And in certain areas if this assumption is not fair we should conduct some special study to find out the deviation between the gravity direction and the this direction, which is indicated by the geocentric latitude right. So, this is my earth tangential reference frame ok.

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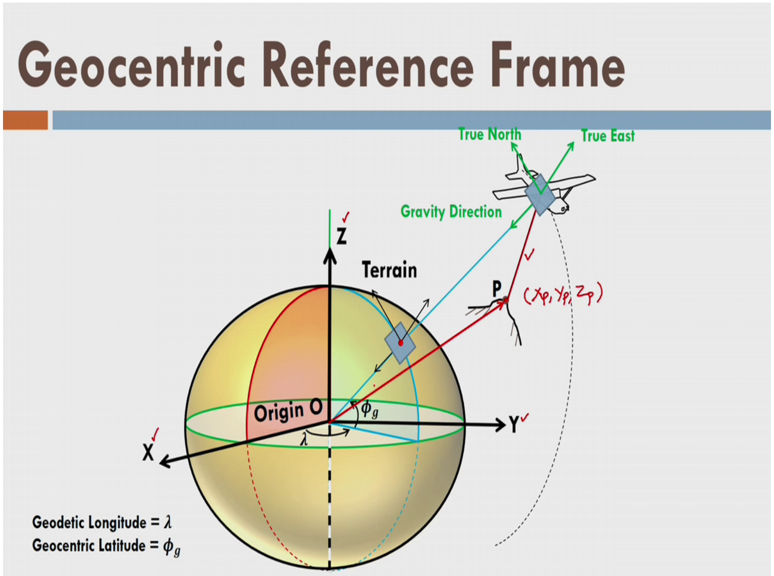

Now we have defined it here ok, now define the geocentric reference frame, in case of geocentric reference frame we have the ITRF or international terrestrial reference frame here, which can see here this is my Z that is rotation axis this is a X axis and this is a Y axis and this is my international terrestrial reference frame ok. So, ultimately we want to find out the coordinate point of P which is my XP YP ZP in this reference frame using information obtain by the laser pulse.

So, that is the introduction about the reference frames and now we are going to convert the reference frames into another reference frame by simple processes of translation and rotations, but that process has some linkages to try to see those linkages.

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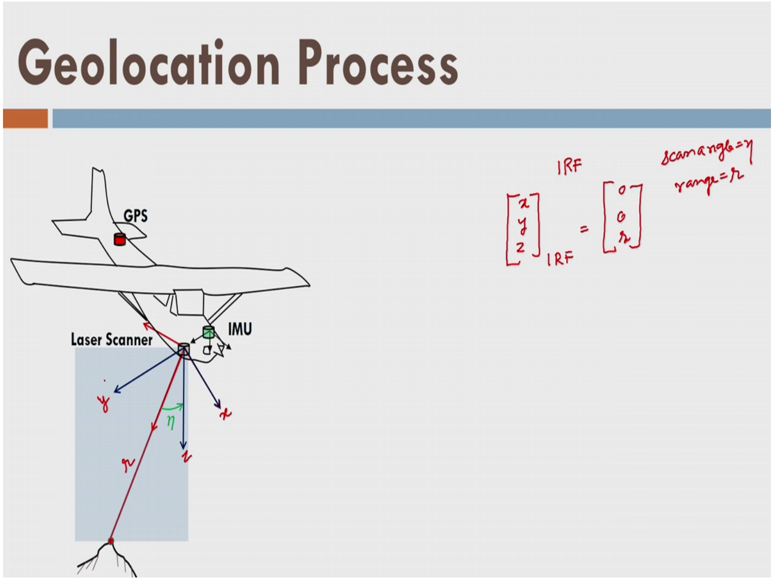

Now, the first step of the geo location process is to convert the instantaneous reference frame located at the scanner, which is acquiring the laser pulses in to the scanner reference frame which is also located at the laser scanner ok.

So, you see this is a laser pulse and so in the instantaneous reference frame I have the coordinate of a laser pulse as xyz is equal to 00 r, what is r? R is nothing but the range right, you can see it verified moreover, as we already told that laser scanner measures the range and is scan angle right and range is r ok. So, using the range and scan angle I would like to convert this range into the scanner reference frame.

So, how can I do it? So, let us define this is my is scanner reference frame where it is x it is z and it is y and the angle between the two reference frame is eta angle; that means, this is my eta angle that is scan angle for laser pulse right. So, now, if I rotate my instantaneous reference frame into the scanner reference frame like this.

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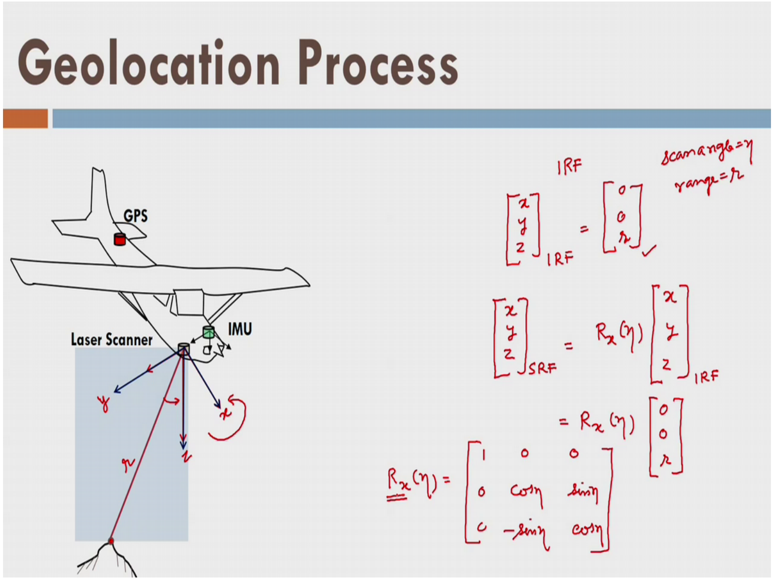

It will convert this coordinate of if instantaneous reference frame into the scanner reference frame. So, I can write now that my xyz of the scanner reference frame is equal to rotation about x axis by angle eta and since it is anticlockwise rotation here you can see this rotation and so, I am giving positive value here. And I can write here the x y z of the IRF instantaneous reference frame or I can write here very clearly Rx eta and here 0 0 r. So, that is my xyz in the scanner reference frame ok. What about the Rx eta? You already know from our module two that it is 1 0 0 00 cos of eta sin of eta minus sin of eta and cos of eta.

So, this is my Rx eta, you can here that, we have rotated this instantaneous reference frame in anticlockwise manner like this about x axis. So, this rotations is happening here this rotation is converting my instantaneous reference frame into the scanner reference frame ok. Let us go ahead and try to now convert this scanner reference frame into the body reference frame right, you can see here as I already told that as such there is no difference between the orientation of a scanner reference frame and the body reference frame right.

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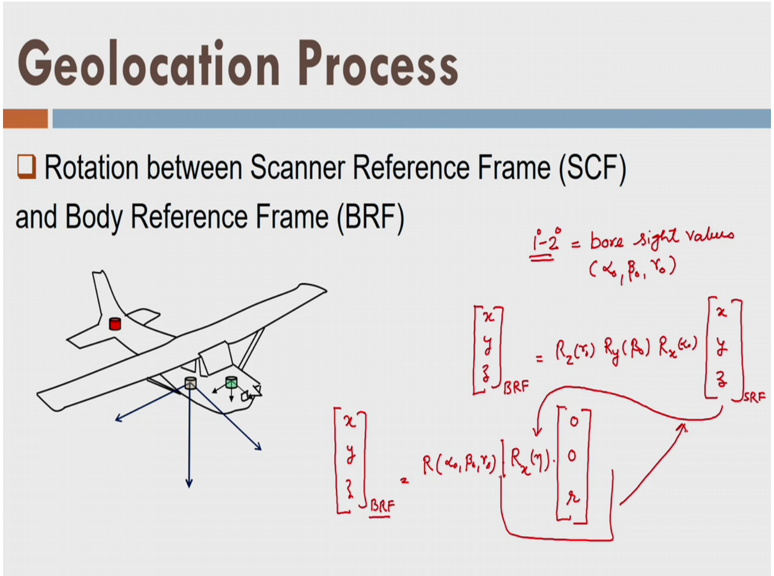

But, there could be slight difference of may be 1 degree or 2 degrees something like this. Now the origin is different so, what do we do? We will orient our scanner reference frame into the body reference frame, what is the reason here? The reason is very simple that when IMU will be switched on it will be indicate of the direction earth tangential reference frame. And as a result if my scanner reference frame is aligned with the body reference frame I will be able to convert the scanner reference frame to the earth tangential reference frame so, let us the purpose.

So, let us align the scanner reference frame into the body reference frame by 1 or 2 degree and we are indicating by some vibration here in animation ok. So, I can see there this values are called the bore sight values. And this bore sight values are determined before the we start the airborne machine and there also determined after the airborne machine.

That means, we take the average value of the two set up observations, why because, it quite possible that the orientation between the body reference frame and scanner reference frame may change because of the aircraft motion during the data acquisition and as the result we observe it after words as well as we observe the both site values before also before the airborne data acquisition.

And as a result we take average value of the 2 and then we say that let us see we have alpha 0, beta 0 and gamma 0 and my both sight values about x axis y axis and z axis respectively ok. So, what can I do here? So, my x y z in the body reference frame is equal to let us say R z gamma 0 Ry beta 0 and Rx alpha 0. So, xyz in my scanner reference frame right.

So, now you can express it like this I am writing it in a combined form the complete single matrix alpha 0 beta 0 and gamma 0 ok and then I am expressing Rx eta in to 0 0 r. So, this is the coordinate of the laser pulse in the xyz body reference frame right, we can just to put this value here this complete value so, this is nothing, but scanner reference frame coordinates ok. So, now, we have converted the coordinates of point p into the body reference frame right.

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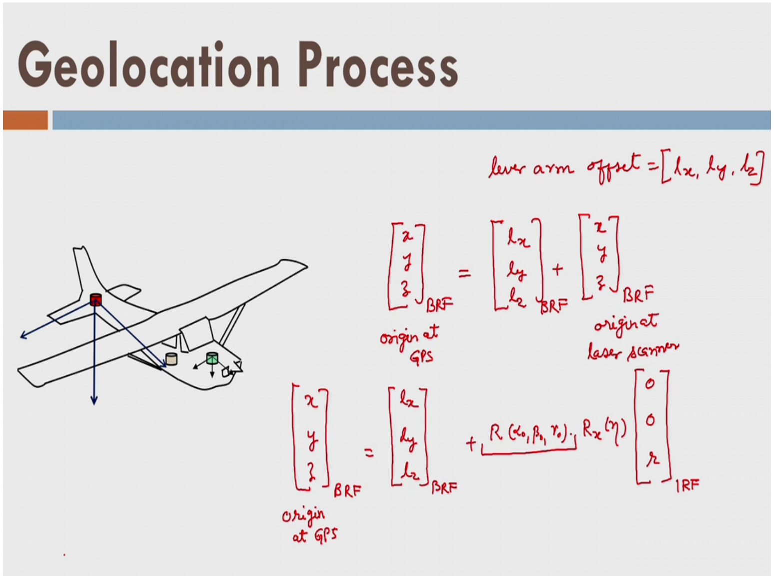

Let us go ahead and now ok so, before sitting on the IMU we would like to convert or we would like to translate our body reference frame, which is located at the scanner to the GPS because, GPS is measuring the aircraft trajectory right.

So, let us do some translation here first and then remember that the body reference frame is a aligned with the IMU right. So, an IMU is the switch off position at turn off position right now so, let us do this translation. So, now, we have brought our body reference frame into the body reference frame only, but we have translated by the lever arm. So, this distance between the GPS and the scanner is called the lever arm or lever arm offset and it is given by in three directions as lx ly lz and remember this lx and ly lz they are measured in the body reference frame.

So, now I can write here that xyz of body reference frame and I am writing here that origin at GPS is equal to lx ly lz plus xyz body reference frame origin at scanner, I hope you agree with that ok. Now, if I write it in elaborated form I can write like this body reference frame is equal to I am just writing it here BRF because they are measured in the body reference frame BRF plus now I have rotation alpha 0 beta 0 gamma 0 by bore sight values, then we have rotation about x eta and then we have this thing 0 0 r.

And this was measured in the instantaneous reference frame. So, this is my measurement in the instantaneous reference frame here right ok and this the values are indicating the connection between the scanner reference frame and the body reference frame fine. So, this is what we have done now ok. So, here let me write here that origin at GPS so, we have body reference frame look it at the aircraft trajectory now ok.

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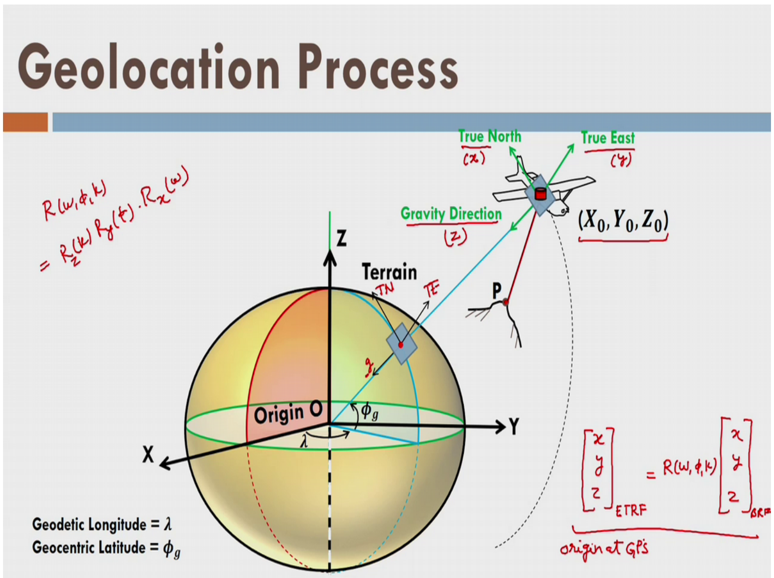

So, let us see that GPS is measuring X 0, Y 0 and Z 0 coordinate of the aircraft trajectory here, shown by the red colour there right ok, Now let us define the earth tangential reference frame, which is we have already defined this is my local geodetic horizon and I have defined again here.

And this three directions indicating the gravity here true north and true east here all the geodetic north, geodetic east and the gravity direction here. And remember we have made an assumption that this direction is gravity and it is a fair assumption ok. So, this is my true north, true east and gravity direction at the GPS, now you can see here that I want to bring the body reference frame into the earth tangential reference frame.

And for that purpose if we turn on the IMU it will indicate these three directions true north, true east and gravity direction and surprisingly x direction is this after turning on this is your y direction and this is your z direction of the IMU right, or what we call as earth tangential reference frame.

So, now you can see here if I want to bring my body reference frame into the earth tangential reference frame, I can write here xyz into the ETRF is equal to nothing, but I have to rotate by omega phi Kappa xyz of my body reference frame ok. So, now, you can also write here what is my R omega phi Kappa it will be R z Kappa Ry phi and Rx omega. So, right now we are writing this thing here ok. So, now, we have brought the our coordinates of a point P into the ETRF or earth reference frame and origin at GPS right, fine where again this is my ETRF frame.

So, true north, true east and east ok so, this is a 90 degree angle as you can see here.

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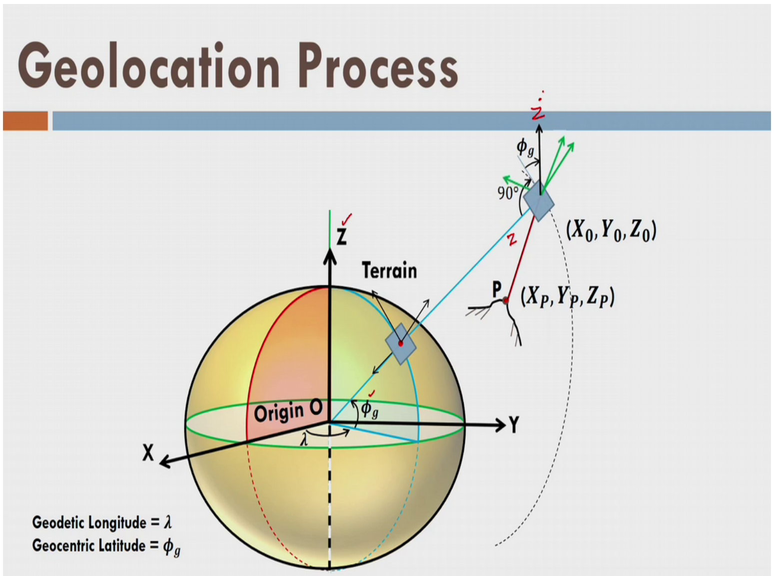

And now I want to bring this earth tangential reference frame into the geocentric reference frame. So, now, you see this is your Z direction here is Z direction and what is the angle here this is your phi g angle which is written here ok. So, total rotation between the Z axis this Z axis of ETRF and this Z axis of the ITRF is equal to 90 degree plus phi g ok. So, let us do the rotation here like this ninety degree and then this 90 degree plus phi g. So, that is my Z axis is aligned now ok.

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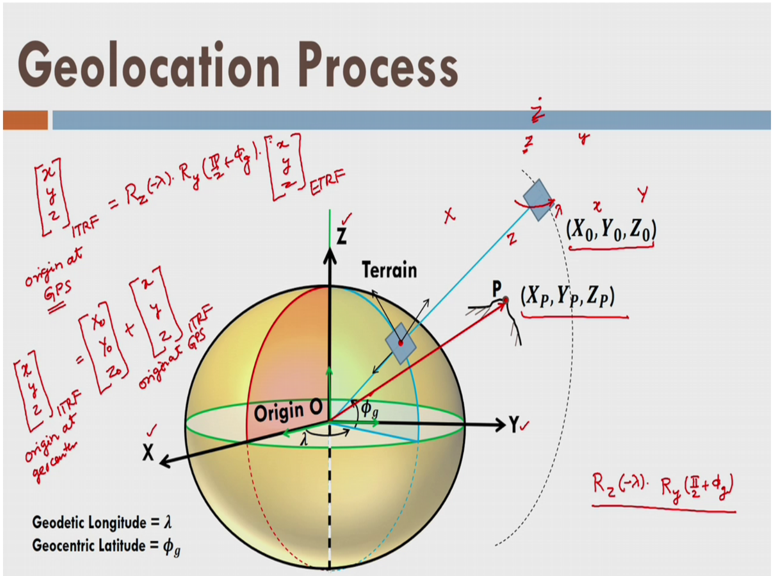

So, I can write here the first rotation is about here right we have rotated about the Y axis ok. Now we will rotate about the Z axis and such that this X axis will be aligned this is your x axis this also y axis before axis is after rotation and this is z axis right this is x axis here capital X here this x axis is same and now Y axis here and now we can see that this angle here is nothing, but lambda fine ok.

So, let us rotate about z axis by minus lambda like this ok. So, now, we have the rotation about z axis minus by minus lambda ok. So, you have seen this thing very clearly that I can convert my xyz here, it is in the now I can say ITRF, but origin is at aircraft trajectory or GPS not with respect to centre of earth. So, I have rotated this by R z minus lambda and R y pi by 2 plus phi g and then we have x y z here I can write here ETRF ok.

So, here origin is still the GPS that is it aircraft trajectory it, now since both reference frames are parallel to each other, but origin is GPS what will I do I will do the translation now like this is my translation ok. So, because of translation I am translating by this value X0, Y0, Z0 here right.

So, I can write here that xyz in the ITRF an origin at the earth geo centre is equal to X 0 Y 0 Z 0 right and plus I can write here x y z ITRF origin at GPS right. So, now, we can elaborate all the terms let us look into the next slide. So, this is the Xp Yp Zp I got here it is nothing, but xp yp zp these coordinates ok. So, let us write the equation of the translation and rotation between the instantaneous reference frame to the ITRF into elaborated form or the complete form so, let us write it.

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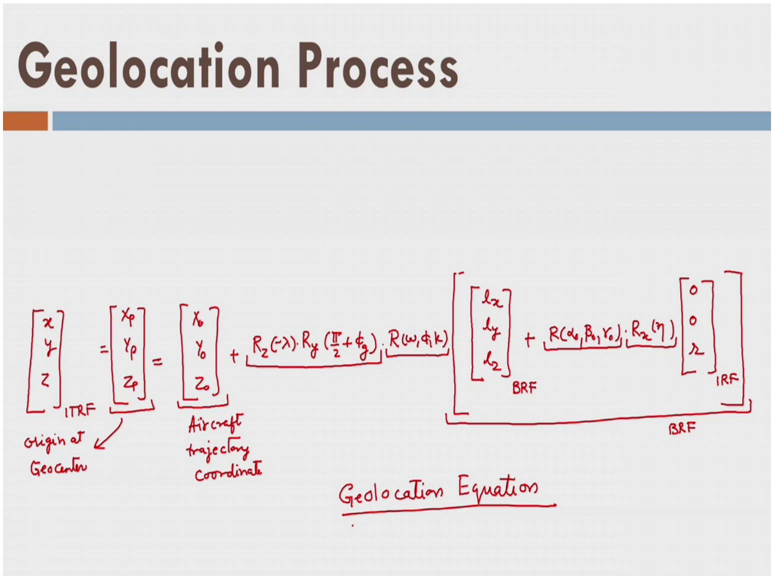

So, the x y z in ITRF where origin at geo centre or I can write here xp yp zp equals to x 0 y 0 z 0 plus R z minus lambda Ry pi by 2 plus phi g ok, what next then we have rotation by omega phi Kappa and then we have the observations in body reference frame.

So, lx ly lz plus R alpha 0, beta 0, gamma 0 into Rx eta into 0 0 r. That is the total equation we have here I would like to indicate that this the measurement was in the instantaneous reference frame ok. This is the rotation between the instantaneous reference frame to scanner reference frame and this is the rotation between the scanner reference frame to the body reference frame and then we measured this lever arm offset in the body reference frame and finally, this whole quantity was in the body reference frame.

And then we converted this quantity by rotation to the earth tangential reference frame and finally, the earth tangential reference frame was made parallel to the ITRF by this rotations ok. And then we have added the GPS aircraft trajectory coordinate and then we brought this one and which is nothing but the coordinates of the point p with respect to the geo centre of the earth right. So, this equation is called the geo location equation or sometimes geo location LiDAR equation right ok. So, let us see that how to use this equation in order to determine the errors of the LiDAR data.

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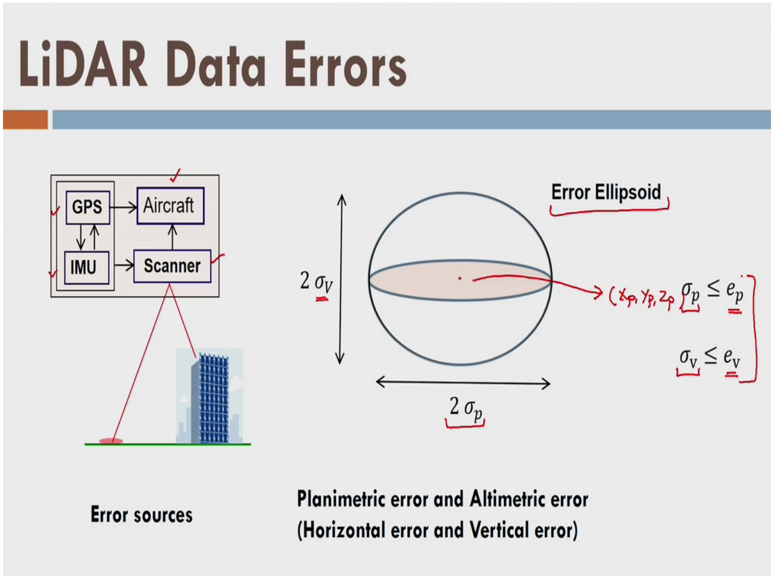

Now LiDAR Data Error evaluation is going on where, we know that there is a GPS this IMU this is an aircraft and this is a scanner and all four elements of the air borne data acquisition are contributing errors into the laser point data right. And we want to estimate the what is the error in the laser point right so, what is the error in xp yp zp fine? So, now, you see that this is the error ellipsoid and we want to estimate this 2 times of sigma p that is planimetric error and 2 times of sigma v that is vertical error and here the point is Xp, Yp and Zp located.

So, I want to find out what is my error ellipsoid here ok. So, there is a process to that right finally, the purpose is that the user generally define maximum allowable error ep and ev in the planimetical and vertical direction. So, these errors sigma p and sigma v should be less than these values right; that means, I should acquire the data in such a way that my sigma p and sigma v should be less than the maximum allowable errors demanded by the user. So, let us look into the process of errors determination.

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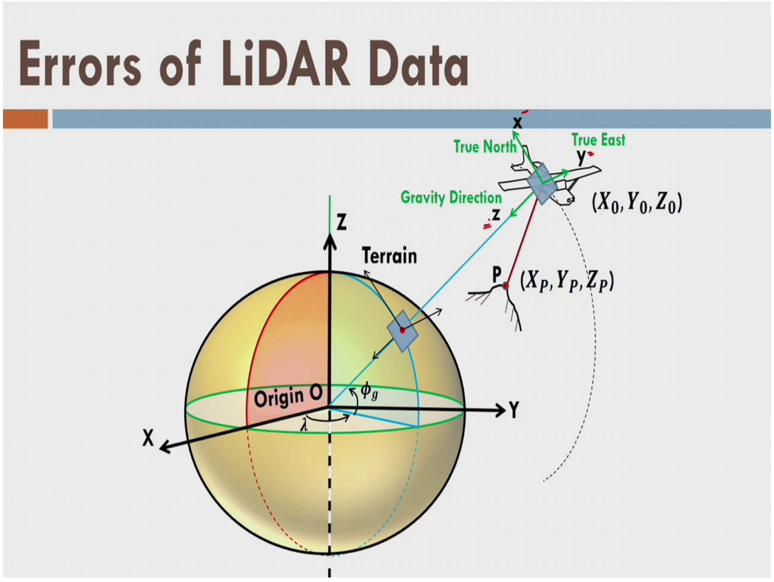

Now, you see we have defined earth tangential reference frame, where this was my X axis here this was my Y axis here and this was Z axis in the gravity direction.

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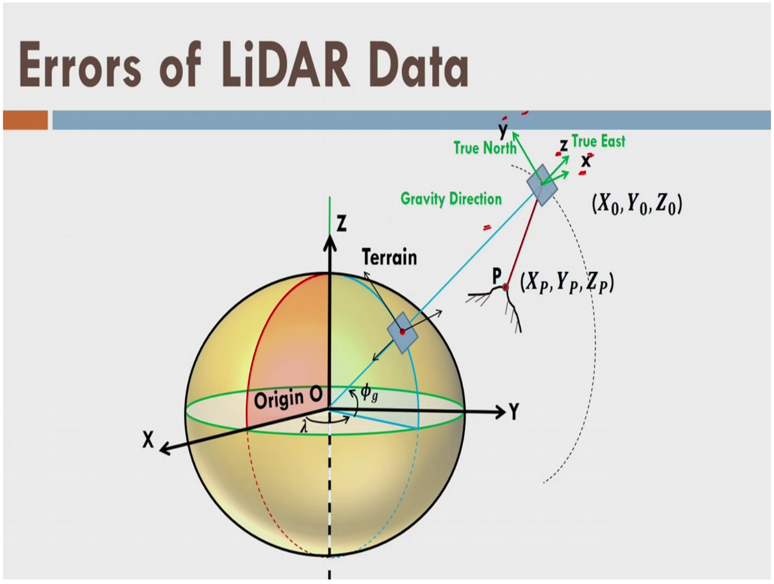

Now we flip the Z axis like this and X and Y axis; that means, my Y axis come to the north axis X into the true east and Z in the opposite to gravity direction. So, it is an orthogonal reference frame located at the aircraft trajectory ok, now let us translate it like this.

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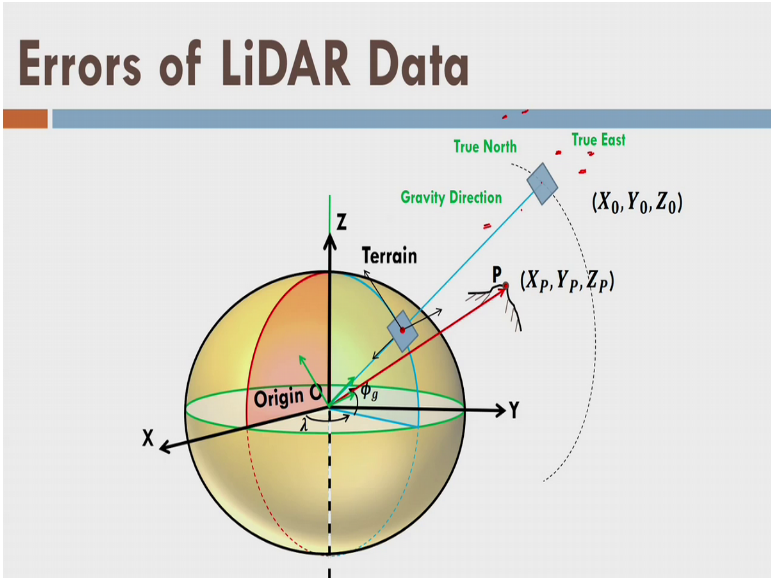

So, I got the coordinates of point P with respect to this reference frame which is green, but located at the geo centre and what is the purpose of this thing. We want to analyse the error in the green reference frame, but located at the geo centre for each and every point this reference frame will be changing because, for each point we have different trajectory points and each trajectory points as different direction of true north, true east and gravity ok.

So, by that I can analyse the data in a true manner or what does it mean by true manner? True manner means at a point P or at a trajectory point, whatever the gravity direction is there what is true north and what is true east an accordingly I am getting the coordinate of point P.

So, in that frame only I want to analyse each and every point. So, I will have for each and every point a different reference frame at the centre of the earth so, let us do it for one point. So, I can write this translation as it is we have done before, but the reflection of the accesses.

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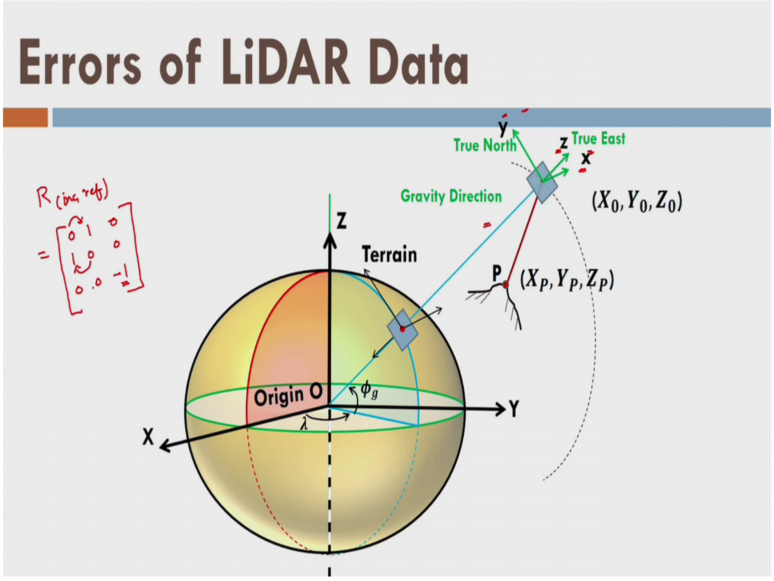

This reflection as well as transformation I can write by this matrix, let us say R inclination and reflection is indicated by this 0 1 0, 1 0 0, 0 0 minus 1 here you can see that I am changing my x to y and y to x here and as well as we are flipping over the z axis by minus 1 right. So, and then we are doing the translation by the X0, Y0, Z0 like this. So, let us write the equation which you have modified.

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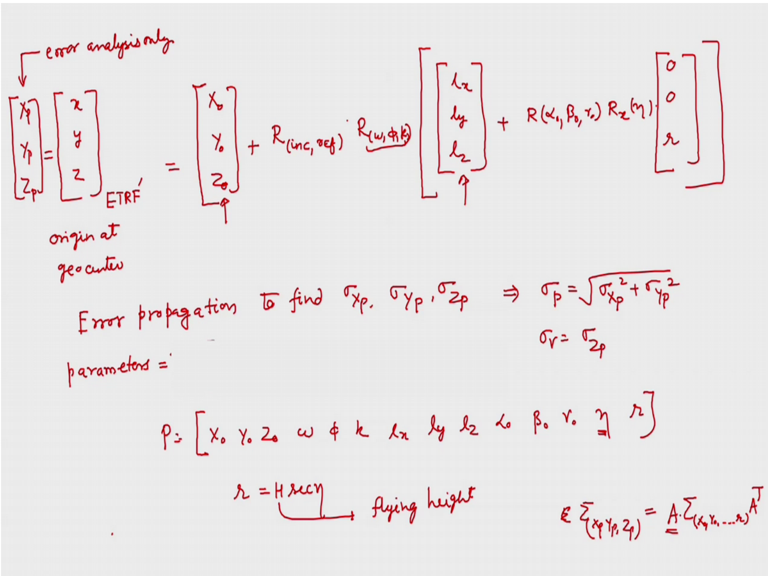

So, I am writing x y z and let us call it ETRF only or not ETRF we say ETRF z opposite, so, ETRF dash and the origin at geo centre and is equal to X0 Y0 Z0 plus R this matrix into we will write the up to ETRFS ETRF we are not change anything fine. So, we have this R matrix omega phi Kappa and then we have lx ly lz plus gamma 0 eta 0 0 r I hope you agree with this equation ok. In the above equation we can see that we are doing the analysis of each and every point individually and now I want to use the law of error propagation to find out what is my sigma Xp sigma Yp sigma Zp.

So, my sigma p will be equal to sigma Xp square plus sigma Yp square and sigma v equal to sigma Zp here right, there is a idea here ok. Now we can see by this equations now I have here Xp Yp Zp and this is derived for the purpose of error propagation or the error analysis only, you are writing this xp yp zp like that ok, what are the parameters here or what are the variables that we measure? Call this matrix P may be P matrix like this ok.

So, first is aircraft trajectory co ordinates then we have omega phi Kappa these three variables variables this lever arm offset and then we have alpha 0 beta 0 gamma 0 ok, then eta angle and the range r where, range is given by r h into sec of eta, eta is nothing, but my scan angle this and H is my flying height right. So, what do we have to do is we have to use the law error propagation like this is my c error or I can say sigma Xp, Yp, Zp is equal to A times sigma all this is my 14 parameters starting from X0, Y0 to upto r into A T. So, what is my A matrix? A is nothing, but the Jacobin matrix.

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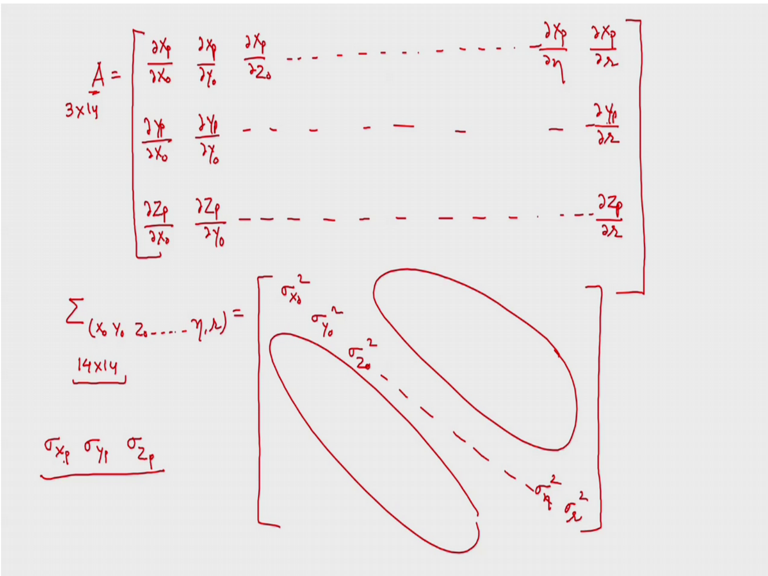

So, A is given by delta XP dou XP by dou X0 dou XP by dou Y0 dou XP by dou Z0 and now I write this terms with respect to all 14 parameters. So, ultimately I will get dou Xp by dou theta dou XP by dou r here similarly, I will get here, similarly dou Zp by dou X0 dou Zp by dou Y0.

And finally, I get the last term as dou ZP by dou r. So, the size of this matrix would be 3 rows 14 columns ok, what about the sigma X0 Y0 Z0 eta and r that will be 14 by 14 and we believe that all this parameters are uncorrelated. So, we have only diagonal elements like this on rest of the elements are 0 here fine.

So, now using the law of error propagation I can find out the values of my errors; that means, we have determined sigma XP sigma YP sigma ZP you have find this process very complicated if you are doing it using the manual calculations. So, use some kind of symbolic mathematics use Microsoft mathematics and try to write the equations there then differentiate it find out the A matrix use symbolic multiplication and find out the complete expression of your errors. So, you can find out here ok, what about the values of this variables ok. So, I am going to share you the typical values what are these values in general ok.

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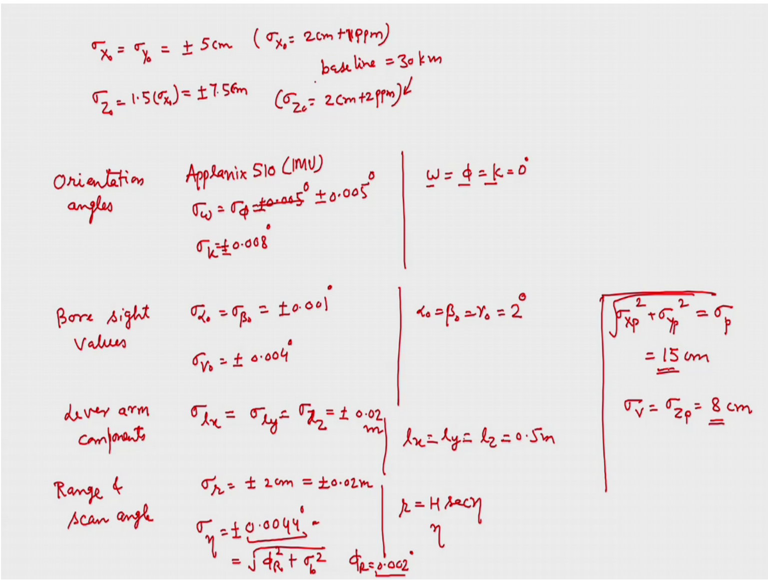

So, the GPS trajectory coordinates are generally having 5 centimetre errors because, the sigma is 0 is given as 2 centimetre plus 1 ppm and we have the baseline equal to maximum 30 kilometres. So, if you calculate you will get this value, what about the sigma Z0 it is 1.5 times sigma X0 Y0. So, it is becomes 7.5 centimetres right and now you can see sigma Z0 is generally given by 2 centimetre plus 2 ppm.

So, now you can also calculate this value ok, what about the errors of the omega phi Kappa right. So, if I take the some IMU model let us say I have taken Applanix 510 IMU. So, it has values sigma omega sigma phi equals to 0.005 degree and sigma Kappa is 0.008 degree fine ok, what about the values of omega phi Kappa.

Generally we take omega phi Kappa equals to 0 right ok, then and we can take original values also if you have, but for time being we have assumed you assume 0 degree. Then we have the bore sight values, these values are generally 1 to 2 degree so, you take 2 degree for worst case and what about the sigma alpha 0.

So, these are the typical values right ok, what about the lever arm components. So, the typical values of the lever arm is 0.5 metre ok, what about this sigma values 0.02 metre or 2 centimetres ok, what about the range and scan angle ok, range you can measure as H sec eta where, eta is my scan angle. And sigma r is again 2 centimetres or 0.02 metre sigma eta is we have taken this value right ok and this is the reason for that we have taken we have considered the resolution of the scanner as well as we have considered the footprint angular value. So, by this we have calculated this one generally phi R is 0.002 degrees right. So, we have find out this value considering the height and everything.

So, if you put all these values you will find out that your sigma X P square plus sigma Y P square that is sigma P generally comes out around 15 centimetre and sigma v it is sigma Zp comes out around 8 centimetre 7.5 to 8 centimetre right. So, you can see here that the error in the vertical direction of LiDAR is much lower or 50 recent of the error in the planimetric direction and that is the beauty of the LiDAR compared to the photogrammetry or compare to the GPS.

In case of photogrammetry in GPS you might of observe the vertical error are twice of the horizontal error is or planimetric error is in case of LiDAR it is completely reverse or I can say that LiDAR is more accurate in vertical direction right. So, that is the idea here. So, now, you can do this kind of study also ok.

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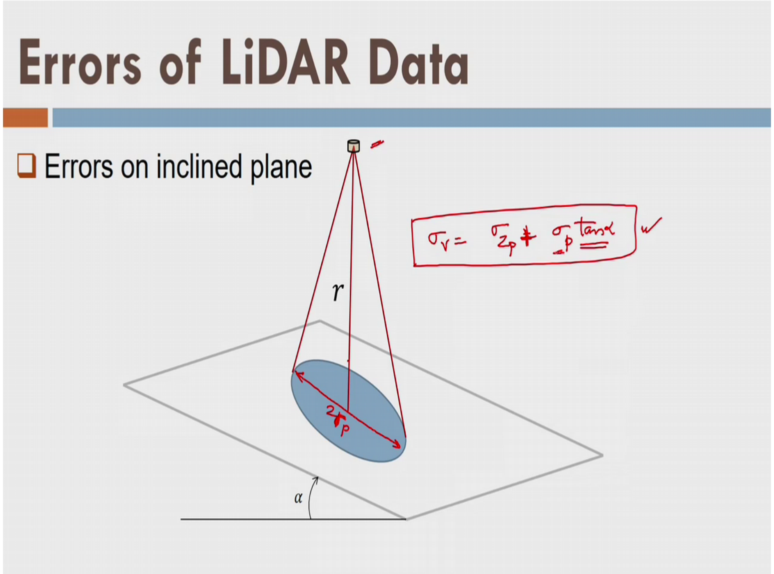

What about the errors on the inclined planes? So, far we have considered the plane as horizontal plane ok. So, now, let us see this is my horizontal plane and there is a laser scanner what will happen now, this is the centre point or the range r here like this ok. So, this is my beam divergence either result it will create an error ellipsoid right. And which is also inflated or the size of the ellipse will be increasing and this is my now 2 sigma P right fine. So, what about the sigma z you can see the sigma as z has increased right and sigma z is given by sigma Z p.

So, sigma v the vertical it is given by sigma Zp that you determined from the error propagation logic sigma p again from error propagation logic and this tangent of alpha. So, my sigma v has increased you see in case of inclined plane. So, you should be careful if you have some heliterrain where the hills are highly inclined; that means, alpha values are very high right maybe more than 45 degrees also or maybe if it is still it is around 20 degrees then you have to consider the error in the vertical direction and that will increase because of the tangent alpha factor as well as sigma P here right ok.

Now, you have determined the error so, you have estimated the error, but you want to verify you error also in the real field. So, there are two ways to verify the error.

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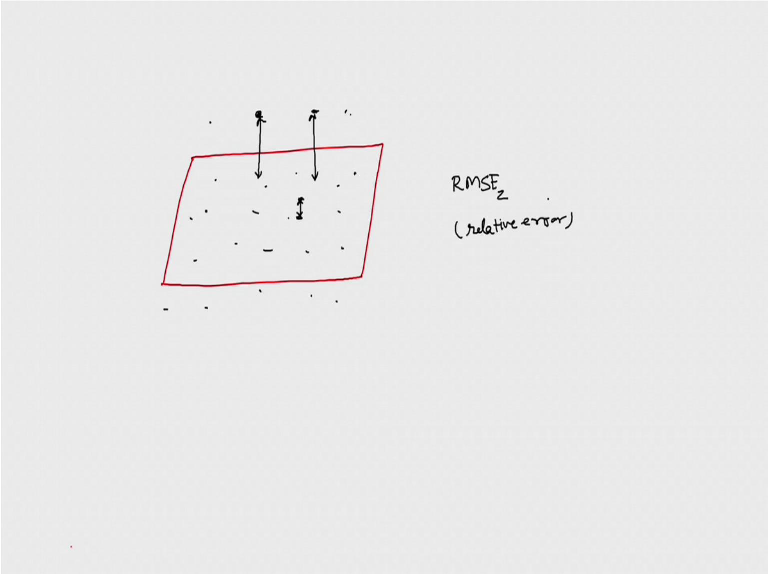

The first way is the relative error, in case of relative error what do we do we acquire the data of one swath and we try to fit a plane into the particular swath data right, what will happen now. So, I have taken let us say I have fitted plane fine for the given data which is like this few of the data in the plane and few may be lower than the plane and few may be the above than the plane right. So, I have a fitted a plane now I will determine this error for this point this error for this point let us say this error for this plane.

So, let us say so, all this errors I will determine I will report the RMSE z as my relative error and this relative error as per the mapping agency should fulfil some criteria ok. What about the absolute error in case of absolute error I will post translate first I will transform my complete LiDAR data into the geocentric reference frame ITRF and then I will do some analysis. So, what kind of analysis I should do there?

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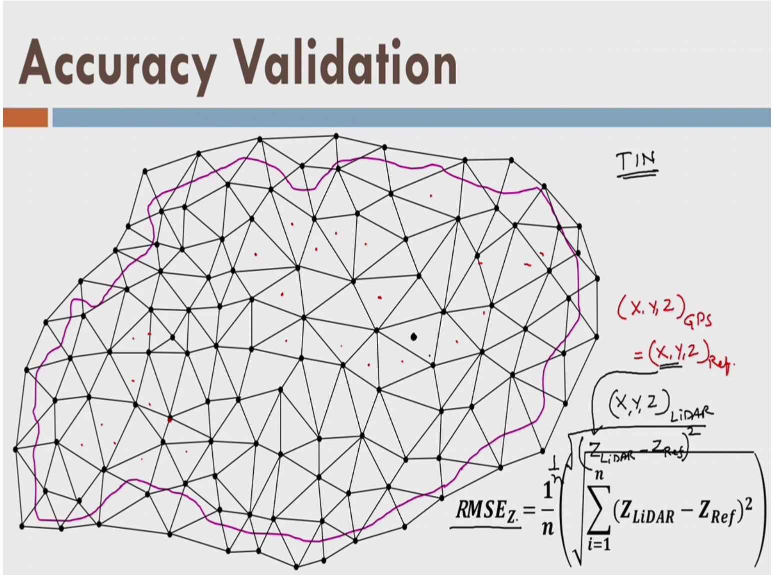

So, let us look into this thing, using the data points I will first create the triangular irregular network what we call as tin right and then I will go to the field and I will measure the few of the points using GPS or maybe which is a system let us say total session which is more accurate than the LiDAR data.

Let us say I have assumed randomly these points somewhere ok, let us say I have measured all these data points maybe like this ok. Now what will I do, I will find out the this is my X, Y, Z let us say from GPS, I call it reference data because, it is more accurate on the way hand I have the data of the LiDAR and let me write it black colour here. So, that is my X, Y, Z by the LiDAR ok, now I will use the XY of the reference and then if will put it here ok, then using this triangulation tin I will find out since it is my tetrahedron or 3D tin.

So, what will I do? I will find out for this XY value, what is the Z value interpolated by this triangle right ok. So, let me call this as my Z LiDAR that I determined by this XY value ok and then I will use my Z reference of this point because I already observed this point by GPS and then I will make the estimate of the RMSC. So, that is my RMC Z. Now I have to report this error also how to report this so, I have determined RMSE Z, but in case of LiDAR you have some different characteristics of different land types and now considering those land types we have defined three types of accuracies like this.

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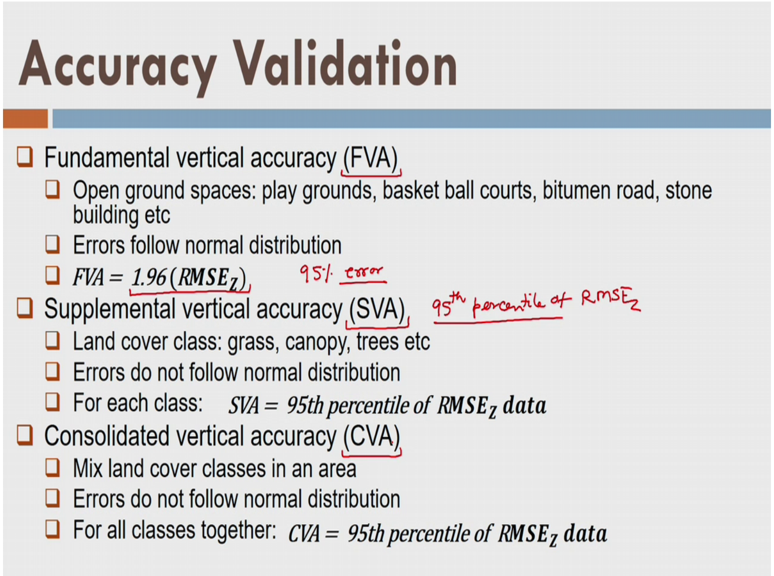

The first idea here is the fundamental vertical accuracies, fundamental vertical accuracies are determined on the surfaces which are horizontal as well as which are flat like concrete surface, rooftops, stone surface, plane ground basketball courts and so on. Now what happens is the error on such surfaces follow the normal distribution.

So, what will I do now I have taken such areas, I have collected my GPS points and I find out my RMSE Z. Now using that RMSE Z, I will report my fundamental vertical accuracy by this logic; that means; I am reporting the 95 percent error or accuracy and reporting ok. Now let us consider some other areas some other areas could be where errors are not normally distributed, such areas are vegetation trees, grass ground or grass lands and so on.

So, in such area we defined supplemental vertical accuracy using 95th percentile of my RMSE Z, we will see later, but before looking into how to find out my 95th percentile, what we will do we will first define one more error accuracy criteria call consolidated vertical accuracy what is the CVA? Let us integrate all the areas which follow normal distribution of error or non normal distribution of error.

Let us see we have complete football ground, basketball court, concrete tops, concrete roof tops, stone surfaces as well as vegetation grass land, shrubs like that.

Now, you collect some n number of points using GPS on the or may be the total session on this complete area fine and then we defined your RMSE Z using that RMSE Z which is on combined the area, you will find out the 95^{th} percentile of that errors and then you will say that 95th percentile will be my consolidated vertical accuracy. So, this are the three criteria as we defined, now come into the how to define the 95^{th} percentile that we have to see now ok.

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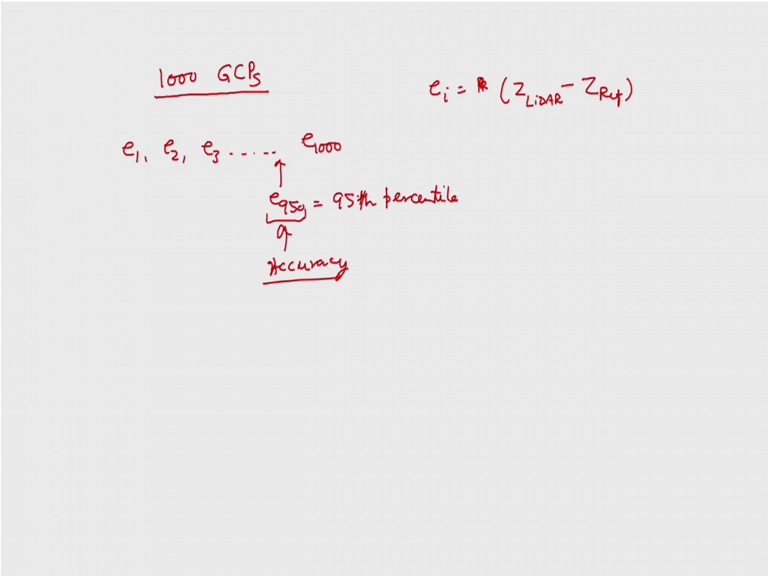

Let us say we have 1000 GCPS taken by total session or GPS ok, for this 1000 GCPS you have determined the errors and now you are writing those errors, let us see in the descending order in the ascending order I am sorry ok, you are writing these errors in the ascending orders or the So, I call it let say e 1, e 2, e 3, e 1000 where e is nothing, but Z LiDAR minus Z reference are you may be taking the mod off that whatever fine.

So, you have determined these errors and then you will find out where your e 950 that is nothing, but the 95th percentiles. And then you will report this e 950 as your accuracy or the error of your LiDAR data right and that is the idea about the 95th percentile error reporting. So, that is your accuracy here, so, this is the way we determine the accuracies ok. Now we will finish this lecture here itself and in the next lecture we will try to find out that how can we extract the information is in the LiDAR data which is already available in a reference frame let us say ITRF right.

Thank you very much.