

Higher Surveying
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Lecture - 02
Coordinate System and reference frame

Hello everyone. Welcome back to the course of Higher Surveying. In the last module, that was a introduction to higher surveying, we have discussed one particular case, where we said that, let us say, we have forest of area 40 kilometers by 40 kilometers. In that forest, I want to measure the height of the tree, canopy size, vegetation volume and the diameter of the tree. Now, we discuss that whatever techniques we have can we use this what we learnt in basic surveying and the first technique.

For example, like theodolite total station compass, all the things we have used and we tried to find what are their positives and what are their negatives or what is the pros and cons of each technique. There we realized that, even we take techniques like total station or GPS. Although they are very accurate, but still they cannot deliver me more than one point at a given time and for that reason, we realize that, even if I take techniques which give me higher accuracy, but if they are not time efficient, I cannot use for larger areas.

Apart from that, if I look for easier techniques like plane table; which is a graphical technique and which is quick also. So, I can map the things, but still the accuracy has to be compromised more over all these techniques cannot consider the curvature of the earth or I need to take some special care for that aspect ok. So, we realize now that, whatever we can do many things with the classical surveying techniques, but still this techniques have lot of limitations. And hence, we discussed and we concluded that we need some better techniques and that is why, we develop that appropriate context of this course called higher surveying.

And now, in the same series of lectures, we are in module 2; where first we need to understand that for the sphere like earth or shape like sphere of earth. We need to have some kind of coordinate system and reference frame which should be quite different from the techniques that we have already learnt like plane table or may be total station or a GPS. Well, apart from that, we should first understand also for large areas, I need to have different requirements also, ok. So, let us dive into this module 2.

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Module Contents

- L-1: Understanding reference system, reference frame, and coordinate system for Earth
- L-2: Coordinate and datum transformations
- L-3: Projected coordinate system

Books

- *Geodesy*, by W. Torge, 3rd ed, Walter de Gruyter Berlin, New York, 2001.
- *Geodesy: Introduction to Geodetic Datum and Geodetic Systems*, by Zhiping Lu, Yunying Qu, and Shubo Qiao, Springer-Verlag, Berlin, 2014.

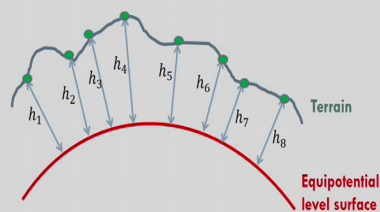
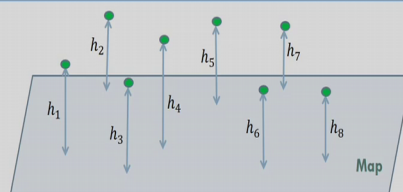
And the first lecture of this module is, the understanding reference system reference frame and coordinate system for earth.

Remember, that we are trying to define this reference frame, reference system and coordinate system for the earth and not for a small area of 100 meter by 100 meter 50 meter by 50 meter and so on, ok. So, it this module contains 3 lectures and will go one by one in a series and will connect them that how are these connected together. Well, let us start with the first lecture, ok.

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Introduction

- Classical survey technique
 - Planimetric survey (2D)
 - Vertical survey (1D)
 - Create 3D map by superimposing
 - Appropriate for small areas
 - Pseudo 3D survey for large areas
- Pseudo 3D survey (Burkholder, 1993)
 - Plane surface is not a reference surface
 - Reference surface (equipotential surface) is not plane
- Important factors for large areas
 - Gravity potential
 - 3D observations (simultaneously)



So, we have already criticized the classical survey technique, but in this classical survey technique there are some aspects that we should first understand in order to modify our concepts ok. So, what we did in case of classical survey? We did first the planimetric survey that is what we call 2 dimensional survey and we have created map showing the x y position of different points on the surface of earth. So, in this screen, if you see the terrain has been shown by a green colour line where few points are marked. So, let us understand that I am trying to measure these 3 all points.

So, basically they are 8 in numbers. So, let us say, I am trying to measure these 8 points on the surface of the earth and trying to draw a map. So, I might have used total station, I might have used theodolite or I might have used triangulation trilateration any technique. And finally, I could get the map that is showing the x y position of these 8 points, ok. So, we have projected these 8 points on the map plane ok, in the planimetric survey; that means, we are recording 2D coordinates of these 8 points on the surface of earth and then we are drawing on the map plane.

So, this is my map plane and we have projected these 8 points to indicate the horizontal position x y of these 8 points ok. Later, I did vertical survey that is a 1 dimensional survey and I collected the heights with respect to equipotential surface. Since I do not have any idea of equipotential service, so, that is why, I took the mean sea level by observing if the variation of sea level over a period of 19 years and then I found out what is the mean sea level average over the long period, ok.

Then, I called that as a 0 level and with respect to that level, I find out the level of other points. So, if a point is having its level that is reduced level, with respect to the 0 level or the mean sea level then, we call it benchmark, because benchmark is a point of known elevation using the benchmark at a given place. For example, in this campus of IIT Guwahati, I assume that there is a equipotential surface that is passing through the benchmark and I can measure the heights with respect to that equipotential surface. Well, that concept was for a small area and will see now why it is not appropriate for a long area or large area or long sites or long distances.

So, let us see what we have done. So, let us see, this is an equipotential level surface and for these 8 points, I have measured the vertical heights of the 8 points from the equipotential surface and so, these are the 8 heights we have measured h_1 to h_8 , right.

So, then I have created a 3D map by superimposing these heights on the map points so like this. So, I have given elevation to these 8 points from the map and then on the map, I call it is a 3D map.

3D map is showing me the $x y z$; that is, $x y$. I obtained from the some planimetric survey and the 3rd dimension that is the height, I obtained from the vertical survey by leveling exercise ok. Now, in this process, what is the problem? We have to first understand that and the first problem is, it is not appropriate for long areas as we already decided and why for the long areas expert of the field of survey, they call it that pseudo 3D survey, ok.

And if I word go to the word called pseudo, you will be realizing that pseudo is something that fulfills the characteristics, but still it is not the same ok. Just recall some movie where an actor as double role ok. So, the original actor or the original character comes into the movie and he is doing something and after few minutes of that, the duplicate or the pseudo person replaces him. Both are looking like same, but the pseudo person or the duplicate cannot replace exactly the original character.

And that is what we call the pseudoness; that means, it is again I repeat that the pseudo things, means it is fulfilling the characteristics. It is fulfilling the criterias, but still it is not the same or it is not an appropriate or it is not the exactly the same thing ok. So, it is kind of replacement.

So, what is there the pseudoness here and that is more important why experts says it is a pseudo survey. When I do 2 dimensional survey and I do perform the 1D survey by leveling and then, I super impose these 2D survey and 3D survey or I super impose the 1D survey on the 2D survey and I call it is a 3D survey.

So, here the expert says that, first of all, the problem is the plane surface, is not a reference surface plane surface means, the map surface. What we call here is, map. It is a plane surface because, we have projected point 8 points on a flat plane and the flat plane why did we assume it flat because, if you remember, we realize that for a long distance like 10 kilometers.

If I take the line of sight along the line of collimation with is horizontal and if I take the length of the sight along the surface of the earth, if I calculate the difference of the 2

length, the length difference is in millimeters and hence we said for a large distance like 10 kilometers. If differences are in millimeter; obviously, for 100 meter 200 meters or even 1 kilometer, they will be minimum.

And as a result, we said that, yes we can do the planimetric survey by assuming earth as a flat and that is why, we said that our map which is compiled by a 2D survey is a plane. So, a plane can always represent a map that will contain only 2 dimensional x and y coordinates of each and every point on the area of interest ok. But, we realize later that for a long distance like 10 kilometers or the difference of the vertical heights along the line of gravity will be in meters and we realize that, it is quite awkward if I use the concept of plane flat plane or the earth is horizontal.

So, then we separated the 2 surveys. We say that for 2D survey I can assume earth as a flat plane, but for the 1D vertical survey I need to assume it to be a curvilinear and that is why, then we use the concept of equipotential surface and equipotential surface was replaced by mean sea level. And then, we realize let us say, stably some benchmarks and let us assume equipotential surface is passing through the benchmark and then we realize that how to calculate the distance in the direction of vertical line.

So, by leveling operation we have created the observations in the direction of vertical line or gravity line, that first we have done.

I think I have repeated it many times. Let us look into this thing so, the idea is the plane surface is not the equipotential surface because, equipotential surface is a curvilinear. Although it could be an arbitrary surface also, but still it is a curvilinear and hence that is the first flaw. Secondly, what happens is, the reference surface which is equipotential surface is not a plane. So, by assuming the height h_1 to h_8 super imposing on the flat map, I am indirectly committing 2 mistakes; first I am assuming that the equipotential level surface which is curved is a flat and secondly, I am saying that the reference surface which is not flat I am putting it 2 flat, ok.

Some kind of, you know cyclic problem, it is ok. So, let us say, what to do now? Especially, this approach is for a small area. It is like 100 meter by 100 meter 200 meter by 200 meter and I can say, let us say, you have an apartment or you want to built an apartment from the ground surface. So, apartment could be of size let say, 50 meter by 50 meter.

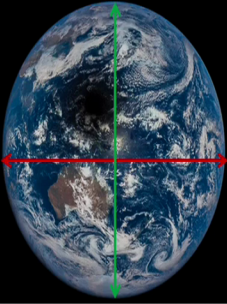
So, this kind of ideas are most appropriate for such small size or even if you want to develop let say, and library, library could be as long as 30 meter by 30 meter. Not more than that it is very used library. If it is 30 meter by 30 meter long, again for such a small area, this concept and of 2D survey and separate 1D survey is most appropriate and there applicable and there people have been doing it from last long many years.

But now for a large areas, we should consider 2 important factors. One is the gravity potential because on each and every point on this surface of earth, gravity varies. And remember that we are using the correction of gravity to measure our vertical heights, ok. And then, we should also take the 3D observation simultaneously. That means, I should not separate 2 dimensional survey and 1 dimensional survey and at any cost. So, if we acquire the 3D data simultaneously, that is, x y z where we also consider the gravity potential of each and every point, that will be the wonderful and best approach. So, let us look into this.

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Earth (Geodetic Characteristics)

- Shape of Earth (figure of Earth)
 - Real shape of Earth is very irregular and complex
 - Earth surface shape is close to 'oblate spheroid'
 - Earth shape changes frequently due to earthquakes, volcanoes, other natural hazards etc
- Earth movements
 - Earth spins at its own axis (diurnal motion)
 - Earth revolves around the Sun (annual motion)
 - Earth axis wobbles while spinning
 - Earth axis is tilted w.r.to Sun
- Position of points on Earth
 - All points on Earth moves with time
 - Position of all points remain mutually intact with each other (distance and angle between two points on Earth surface does not vary with time)



Shape of Earth (NASA)

So, now let us look into the shape of the earth. So, what are the earth characteristics because of that we want to develop our strategy or methodology in, so that, we can develop a 3D reference system in which we can measure our 3D coordinate simultaneously. So, this is the surface of the earth as visible from the satellites. And now, if we say that the real shape of the earth is very irregular and complex, as we can see

here, there are some sea, there are some lands, there are some hills, there are some big mountains and so on.

Well, if you see that earth surface is close to the oblate spheroid, what does it mean? I am showing you a red mark here. Red mark shows the diameter of the earth which is different from the diameter at the poles and the difference is approximately 50 kilometers. If I consider the diameter and if I consider the radius, it is approximately 25 kilometers. What does it mean? I cannot assume it to be a sphere. I should consider some other shape for that and appropriate shape is the oblate spheroid or ellipsoid.

So, next is earth shape also changes because of the earth quakes, volcanoes and any other natural hazard. So, how does it matter? Suppose, it is happening in some other country, how does it make a sense to my country? Nothing is that the way we are defining our reference system for the whole earth that will make a difference in the center of mass of the earth or what we call is geo center. So, geo center will be slightly shifted. So, I need to observe each and every such variation that will be created by earth quakes or may be volcanoes and so on. And other natural hazards which we cannot control, ok.

What about the other things? So, other things are earth movement ok. So, earth spins at its own axis; that is, we call it daily motion or diurnal motion. Secondly, we also have annual motion. So, earth revolves around the sun, what we call orbiting of the earth around the sun. Third thing is that earth's axis wobbles while spinning. What does it mean? This means that earth is slight, let us say, this is axis of the earth and it is wobbling and so.

We have the rotation like this; that means it is spinning every day. But at the same time, it is doing like that, but although the cycle of this rotation is very long approximately 26000 years. But still what is the problem, Why should we consider it? The reason is, since we are defining again the 3D reference frame and system for surface like earth and hence, I have to locate my 3D position with respect to time.

Why because, earth is spinning. So, if I want to locate a position, that is 3D in nature, I can say at time t_1 , it has $x_1 y_1 z_1$ position. At time t_2 , it will have $x_2 y_2 z_2$ position and because of that, I can always say with respect to time the position of the earth's surface is changing or may be my location itself changing with time.

So, I need to locate the 3D position with time and hence I need to consider all the motions of the earth ok. That is, problem is becoming very complicated, it is very complicated. Why because, earlier we realized that, in classical survey we are not bothering the time at all, we are just taking a reference arbitrary reference system and we started our work. But here, now just imagine that things are so complicated. Why because, earth has a single planet is has lot of different characteristics and we are trying to combine all together now.

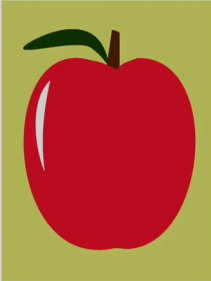
But, surprisingly, there are 2 things; the first thing is that, if I consider only 2 points on the surface of the earth and with respect to time, I know that these positions are going to change although, but they are relative difference in length and relative difference in direction. That is, azimuth is not going to change so does it give me a facility here. Yes in fact, I did not consider the time. If I calculate the difference of 2 positions on the surface of the earth, well, with this idea, let us try to understand that what we can do ok.

So, earth has a oblate spheroid as we have seen in this thing. So, let us assume that earth is looking like that an apple. In apple fruit, if I remove the root of the fruit which is shown in the figure and the leaf which is also shown here.

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Earth (Geodetic Characteristics)

- Shape of Earth (figure of Earth)
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- Earth movements
 - Earth spins at its own axis (diurnal motion)
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Earth as oblate spheroid:

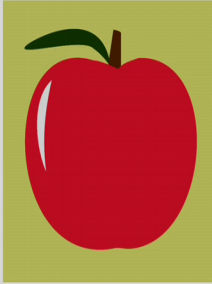
- flat at poles
- bulged at equator

So, if you can imagine that earth is somehow appearing flat or this apple is flat from the top and slightly bulging from the middle part and what we call as equator for the earth so by flat from the poles bulging out at the center part ok.

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Defining Position on Earth

- ❑ What is desired for positioning?
 - A basic and simplified 3D model of Earth for positioning
- ❑ Two approaches
 - Gravimetric approach: define a best fit geoid (equipotential surface of equal gravity values)
 - Geometric approach: define a best fit ellipsoid
- ❑ Problems



So, let us see how can we define the position on this planet which is bulging out and flat at the top. So, what is desired for positioning? I need to have as I said that, since we are considering the whole planet, all the complete surface of the planet as a 1 unit, I need to define a simplified and basic 3D model for the earth. And if I define that, I can write some mathematical equations. What will happen? I will have an advantage that let us say, if I write an equation of ellipsoid, I will have $x^2 + y^2 + z^2 = 1$.

And if I now I want to observe a point let us say x, y, z or x_1, y_1, z_1 , I can find out whether this point lies exactly on this model; that is ellipsoid or it is lying below the ellipsoid surface or it is lying somewhere else, may be very close to the center of the earth.

Now, that is very good approach. I really like it, but there is a problem is how to fit such kind of ellipsoidal surface to this earth ok. So, there are 2 approaches not a fitting this ellipsoid, but to find out the overall reference system for the earth. So, that once we establish reference system we can establish some reference figure and you think those reference figure and reference system I can measure the 3D coordinate simultaneously. So, let us see there are 2 approaches.

So, first is gravimetric approach that says that define a best fit geoid that is equipotential surface of equal gravity values; that is first determine the gravity values on the surface of

the earth and let us fit a surface which is the best fitting surface. So, those gravity values good approach because, it is going to solve all the problems in a sense like I can if I measure the x y distance as well as 3rd the gravity direction distance, I can do the 3D surface the way I want.

But there is another approach called define a best fit ellipsoid that is called geometric approach; that means, forget the gravity values, but just look for geometric characteristics of the earth and accordingly, let us fix some kind of geometric shape to this planet let us see. So, this is my geometric approach so, this is let us say, my regional ellipsoid which says that there is some region on the earth and that if I observe that surface let say, with the help of some technique.

So, I can fit a surface like this to that region and because of that, I have a regional ellipsoid where the center of the ellipsoid is not the geo center or they have their difference which are quite long in hundreds of meters.

But does not matter because, that ellipsoid is serving the purpose of that regional area or I can say small continent. Now, let us see these are the A B C D. So, I can say A B are my 2 times semi measure axis and E F is my 2 times semi minor axis well. So, this is this the axis is of the ellipsoid which is not matching with the axis of the earth itself. The rotation axis, but yes, they are quite near because, the rotation axis of the earth will remain almost same even if I fit different ellipsoid. So, there will be slight difference between the regional ellipsoid and there will be a if at all there exist global ellipsoid.

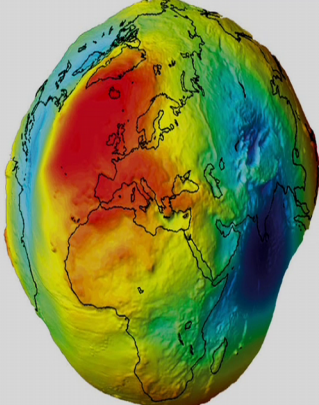
That we are going to know in coming slides well. So, what are the problems in geometric approach and the gravimetric approach. So, let us go by the gravimetric approach.

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Earth Shape Measurement

Gravimetric approach

- Geoid is irregular and complex surface
 - Analytical model not possible
- Geoid should be modelled by averaging of all ocean surfaces, but not possible.
- MSL is measured as local geoid measured by tide gauges in a region
- A local geoid is parallel to true Geoid but does not match.
- Due to changes in ambient conditions (ocean currents, winds, sea bed topography, tides, temperature etc), MSL deviates for each region in world.
- All regions in world have local geoids parallel to each other and true Geoid.



Shape of Geoid (NASA)

So, that is the shape of the geoid. If I observe the earth from a satellite, since we are observing the gravity values from the satellite, now we come to know recently last 30 years back that this is the shape of the earth, ok. And according to a this is the geoid shape; not the geometric shape. Geoid shape means, if I try to draw the potential values or the gravity potential values of each and every point, that is x y and I on top of that, I am putting the gravity value, I will get this kind of shape. And hence, I realize that it is not only difficult it is almost impossible to fit a geoid.

That is unique in nature and that can serve as a single surface for the whole planet. Very difficult because, you can see the 2 reasons; first is, first of all geoid itself is very irregular in nature. Secondly, if you look there are some sea surface there are some land so we can see that here on the right hand side on the screen, there is a blue patch which is a deeper that is a deep sea on the left hand side on this figure, there is a blue patch, but still it is a shallower.

So, now, I have realized that there are some deep valleys in the sea and there are some other seas they are shallow valleys or there is no valley at all. So, if I consider let us say that MSL issue that I want to establish the MSL for one country or unique MSL.

So, what are the problems possible? So, let us look into that. So, first is that, the analytical model or the mathematical equation to this surface is not possible because, in case of mathematical surface what I need to do? I need to put some kind of contour

problem or may be 3D contour or may be 3D surface and that surface should be of equipotential nature, quite difficult to fit across this particular shape ok.

Let us see, we have device a different approach, that is, we will average the ocean surfaces or the level of ocean surfaces. I will create level which will be uniquely defined.

So, what our earlier scientist did? They went to the sea surface and they tried to observe that, ok. What is the level of the variation of sea? So, they realized that, yes, over a 19 years of period, it varies the sea level varies and the observate. So, that particular country they have defined one MSL. Now, that country is having a deep valley near to the sea coast because of that, the effect of tides are little less.

Let us take another country where again I am trying to do the same exercise. Over a period of 19 years and there we realize that the effect of the sea tides are more because, there is no valley and eventually, it happened at 2 countries. We have different 2 levels and I am saying that this my mean sea level and my friend in another country staying. This is my mean sea level, ok.

Then we started our survey. So, we have assumed that yes there are some surfaces which are equipotential in nature and since their water surface which is uniquely different it always follow the gravity direction. It always indicate the value of gravity at each and every point; third thing, they indicate the same level of gravity value across the whole water surface.

So, one surface is passing through like this, another surface passing through like this and both surfaces are parallel to each other. But they are quite different in heights and that is why, they call the local geoid for one country and another local geoid for other country, ok.

So, this is the idea here. Now, as I said that there are various factors like ocean currents winds sea bed topography tides temperature etcetera, they keep on varying. And hence, these also decide the different levels of equipotential surface. So, one surface is like this curvilinear another is parallel to this and so we have local geoids and each and every country it is shown here in this (Refer Time: 26:47).

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Earth Shape Measurement

Gravimetric approach

- Geoid is irregular and complex surface
 - Analytical model not possible
- Geoid should be modelled by averaging of all ocean surfaces, but not possible.
- MSL is measured as local geoid measured by tide gauges in a region
- A local geoid is parallel to true Geoid but does not match.
- Due to changes in ambient conditions (ocean currents, winds, sea bed topography, tides, temperature etc), MSL deviates for each region in world.
- All regions in world have local geoids parallel to each other and true Geoid.

Shape of Geoid (NASA)

Let us say there is a train shown in the green color and if I fit the average surface through all the oceans, I will get the global geoid represented by G 0 with it is firm red line here.

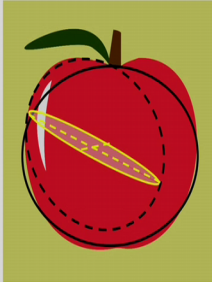
Now, if I have the 2 geoid which I has I said it is very irregular and in the last picture, I have already shown it that 2 geoid. So, let us say, 2 geoid is also there. Now, I fit in local geoid 2 country and it is coming out to be G 1 and this surface is almost parallel to the global geoid, but still it is quite different from the global geoid. Secondly, I have local geoid G 2 which is again for another country, but it is quite different from G 1 as well as G 0. So, hence we end up with the different level surfaces for different countries ok.

So, that was the problem with the gravimetric approach. Let us come to the geometric approach.

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Defining Position on Earth

- ❑ **Geometric approach**
 - Determine an ellipsoid
 - Earlier, ellipsoid is fitted through 2D triangulation data and 1D level data
 - (2D + 1D) data is regional, ellipsoid is regional
 - Regional ellipsoid fits best to a region
 - India: Everest (1830)
 - North America: North American Datum 1927 (NAD27)
 - Britain: Airy (1830)
 - Australia: Australian National (1966)
 - Egypt: Helmert (1906)
 - Russia (USSR): Krassovsky (1940)
 - South America: South American (1966)
 - Regional ellipsoid is not the best fit shape to Earth
- ❑ **Solution: Mean Earth Ellipsoid (MEE)**
 - Global ellipsoid (MEE); best fit ellipsoid to Earth surface that approximates geoid



Regional ellipsoid - - - -

Geometric approach says that let us fit an ellipsoid which is a geometric figure on the given surface of the earth ok. And we have already discussed that we have some regional ellipsoid which are already fulfilling the local purpose of a particular continent or particular country or particular region.

Well so, this geo ellipsoids for fitted using 2 dimensional data and 1 dimensional vertical data. Well, again that is kind of compromise they did, but that was the only solution possible in those days, fine. And they fitted the regional ellipsoids and that is appropriate for a reason and for example, we are giving some examples here like for India in 1830, during the British period, some Everest ellipsoid was fitted and North America, North American datum, which is done in 1927.

Well, in Britain airy 1830 again and Australia, Australian national in 1966 Egypt Helmert, Russia, Kravssovsky I am sorry 1940 and South America, it is called South American which was done in 1966 well. So, regional ellipsoid is not the best fit to earth and this is the regional ellipsoid. So, what is desired here? I should fit a global ellipsoid and that earth should be that global ellipsoid should be called as mean earth ellipsoid and the idea is this geometric approach should be such a approach that, it should be representing the approximately at least that geoid surface of the earth.

And so, let us imagine something where I am saying that let us fit a surface around surface of earth that is a geometric shape, what we call as ellipsoid, but at the same time,

it is very close to the geoid. So that it can fulfill 2 surface 2 purposes; first purpose is; it is a geometric in shape so I can use it mathematically. Remember, I call it that if I take a point x y z, I can find out whether it is lying above the surface or below the surface or where it is exactly located with respect to the mathematical surface.

And second is that, since this it geometric shape will be very close to the geoid. I can always use it as a replacement to the geoid; that means, even if I can measure the height with respect to this surface, but still I can be confident enough that I am doing the measurement of vertical heights approximately in the direction of gravity and with this idea, the concept of mean earth ellipsoid came into picture.

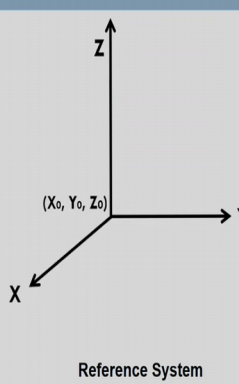
So, let us go ahead so let us see that E F is my rotation axes. So, I can say there is a global ellipsoid shown by black colour in firm line.

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Reference System

□ Concept

- Ordered set of three mutually orthogonal axes.
- Specific origin
- Each axis is indicated by a unit vector
- Cross product of two axes in order gives third axis ($\vec{X} \times \vec{Y} = \vec{Z}$)
- Position of origin and orientation of axes can be defined with time
- Example: International Terrestrial Reference System (ITRS)



Reference System

Now what about the reference system? So, we will now understand what do we mean by the reference system reference frame and coordinate systems that is required for this surface of the earth where, I say that there are 3 orthogonally oriented axeses at some origin. So, let us say origin is given by X 0 Y 0 Z 0 here and X Y Z is showing 3 orthogonal axeses. Well, it has a specific origin that I already shown you. Each axis is indicated by a unit vector, ok you already know what is the definition of unit vector.

So, if I take a vector of length L and divide it by magnitude, then I will get a unit vector for that vector L . Now, if I take the cross product of these 2 axes; that means, since they are orthogonally located; that means, they are oriented at 90 degree mutually to each other. So, if I take the X dot X cross Y cross product of 2 axes, I should get in order the 3rd one which means X cross Y equal to Z .

Z cross X is equal to Y and Y cross Z equal to X , you can just draw a cyclic order on a circle $X Y Z$ and you can do it. Well, this property is fulfilled. I call them as a 3 mutually orthogonal axes, well. So, position of origin and orientation of axes can be defined with time. So, now, since we have said that, we are considering 2 kind of systems where one is dependent on the time; that means, one is independent of time ok. There is nothing called independent and dependent of time, well. So, just try to understand, what do we mean by that ok.

The first thing is that, as I told earlier, that if I want to calculate the position of 2 points on the surface of the earth. I need not to consider the rotation of the earth or the movements of earth because, it is uniform for both the points and as a result, I can assume a reference system which is unique and same for the both the points. Because of that, now what will happen? Let us see that there is a reference system which is rotating with the earth and that is attached to the earth. So, because of that attachment it is also rotating with the earth the same speed the way other points are rotating.

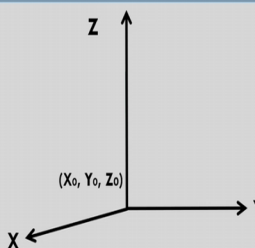
That means there are 2 points which are rotating with some speed because of the earth rotation. At the same speed, that reference system which is attached to the earth. It is also rotating and because of that such kind of system can be used in order to look at 2 points on the surface of the earth or look at many points, why not, well that is the idea here and such frames are or such reference systems are called non inertial reference system, ok.

What about the other type of reference system? The other type of reference system does not rotate with earth and such systems are called inertial reference system. We will talk about inertial reference system later in other modules. But here, we will work with the non-inertial reference system only, ok. So, one example of it is called International Terrestrial Reference System ITRS. Well, reference system is a conceptualization or is a concept its idealization, fine.

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Reference Frame (Datum)

- Concept
 - Realization of a reference system through geodetic observations
 - Determine all parameters (origin, direction of three axes)
 - Reference frame: realization of reference system
 - Example: a rigid body is a reference frame



Now, if I want to define something called reference frame or in other words datum, I need to understand that, if I try to realize the reference system, it is called reference frame. So, in other words, I can say the realization of the reference system is called reference frame.

So, what do you mean by realization ok. Let us say that, you are observing some points on the surface of the earth and using those observations you are trying to fit or you are trying to find out reference frame or a reference system. What does it mean? So, you are trying to find out where is the origin of the reference system and where is the 3 orthogonal axes are oriented. So, moment you realize this thing or you determine this thing, the definition what we got in mathematical terms that will be called reference frame, ok.

So, as I told that ITRS it is reference system that is a kind of conceptualization that is a kind of idealization of our concepts. But at the same time, if I call it ITRF, international terrestrial reference frame, it is a realization of ITRS. Well, I hope that you will understand this thing gradually as we go along this lecture or this module.

So, now you can imagine a rigid body. A rigid body is like may be this body this is the kind of remote control, it is rigid body because, with respect to time, it is not changing its shape. So, now, I can imagine a reference system which is non inertial that is, it is attached with that like this and now, if this body is moving in this way, for example, So,

my reference system is also moving like that. So, if I want to locate this point on the let us say one button on this mobile phone this remote control and another button.

So, I can always look it these 2 buttons with respect to this coordinate system or this reference system because, this reference system or reference frame itself is moving with the this device and this 2 points will remain consistent always. With respect to this, reference system provided this body is a rigid body.

So, here, we make an assumption that earth is a rigid body for some time; that means if there are some natural hazards or some volcanoes or earthquakes whatever is there, we take care of them. We consider them account for them and then we again find out what is a real shape, what is the chin shape, modified shape and based on that, we will change the definition of our origin of the reference system and coordinate axeses. So, this kind of arrangement you always consider and then we say that with this considerations let us assume earth is a rigid body, well.

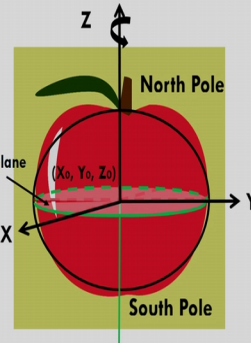
Now, let us see there is a reference system or a reference frame X Y Z with origin at X 0 Y 0 Z 0. So, let us fit reference ellipsoid to that surface where it is centered at the origin of reference frame, ok. So, I call it equatorial plane which is shown by a green circle here fine.

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Reference Frame (Datum)

□ Concept

- Realization of a reference system through geodetic observations
 - Determine all parameters (origin, direction of three axes)
- Reference frame: realization of reference system
- Example: a rigid body is a reference frame



Semi major axis = a
Semi minor axes = b

And this is my rotation axes and rotation axes is cutting at north pole and south pole and remember that if I fit the geocentric system; that means, the origin at geo center of the earth and the rotation axes is completely aligned with this thing, I can understand that it can be a global system. Let us look into detail of this thing

That is rotation axes and that is semi major axes a and b we have already defined it.

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Non-Inertial Reference Frames

- Geocentric terrestrial reference frame (X, Y, Z)
 - Origin: center of mass of Earth
 - Z axis: towards average value of Earth rotation axis
 - X axis: towards point of zero longitude (average value of Greenwich Meridian)
 - Y axis: forms right handed coordinate system with X and Z axis
 - Datum: geocentric ellipsoid
- Non-geocentric (geodetic) terrestrial reference frame (u, v, w)
 - Origin: center of reference ellipsoid
 - w axis: towards semi minor axis of reference ellipsoid
 - u axis: towards point of zero longitude and zero latitude on ellipsoid
 - v axis: forms right handed coordinate system with u and w axis
 - Datum: surface of a non-geocentric reference ellipsoid
- Local reference frame
 - Origin: user defined point (called topocentric if point is on Earth surface)
 - Axes: defined according to a global reference frame, ellipsoid, or gravity
 - Always Cartesian

Now, we have already talked about non-inertial reference frames. So, the non-inertial reference frame for the earth can be located at the geo center; that is the center of the mass of the earth or it can be non-geocentric or what we call as geodetic. Well, in case of geocentric system, we have the origin at the center mass of the earth geo center Z axes is towards the average value of earth rotation axes. Remember that, I called it is a wobbling and there is whatever motions are happening with the rotation axes of the earth, I am taking the average value with respect to sometime in past.

So, using that average value, today I define that average value is my mean rotation axes value and I will use for this moment. Let us say, after 1 year, this average will again change. So, I will update myself with the new information; that is, the idea here X axes towards the point of 0 longitude and which is very close to the Greenwich meridian, but not exactly at Greenwich meridian. Will see why is it so.

Well and Y axes is forming a right handed coordinate system with X and Z ok. Datum, I call since it is a geocentric system, I call it datum as geocentric datum ok. So, I can fit let us say, ellipsoid to this I can attach an ellipsoid to this X Y Z system and that will be a geocentric ellipsoid well coming to the geodetic terrestrial reference frame or geodetic or non-geocentric they are same meaning. So, origin is the center of reference ellipsoid. Remember, it is not a geo center or other. The center of ellipsoid does not match with the geo center of the earth. So, the w axes we call it u v w axes so, w axes towards the semi minor axes of reference ellipsoid.

U axes is towards the point of 0 longitude and 0 latitude on ellipsoid and not on the geocentric system remember it, fine, v axes again that forms the right handed coordinate system and datum is a surface of the non-geocentric ellipsoid. Well, that is fundamental difference between the geocentric and non-geocentric reference surfaces and terrestrial reference frames related to non-geocentric and related to geocentric again.

Let us see that, what may happen if I have on the surface of the earth some reference frame. I call it a local reference frame and the local reference frame. It is origin it is on the user defined point on the surface of the earth.

Then, we are access which are orthogonally located 3 orthogonal axeses and then we have defined a always a kind of cartesian system is preferable for local reference frame although other possibilities also there will look into this thing, ok. So, let us see this is my geocentric ellipsoid.

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Non-Inertial Reference Frames

- Geocentric terrestrial reference frame (X, Y, Z)
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 - Origin: user defined point (called topocentric if point is on Earth surface)
 - Axes: defined according to a global reference frame, ellipsoid, or gravity
 - Always Cartesian

Semi major axis = a
Semi minor axes = b
Geocentric Terrestrial Reference Frame (X, Y, Z)

There this is my equatorial plane. The rotation axis that is north pole south pole and this is the rotation axis direction or the direction of rotation of the earth. Well, this is the semi minor axis and semi major axes are shown. So, let us go ahead yes, here by mistake, we have written prime meridian and it is called a meridian; not the prime meridian let us make a correction here. And secondly, this is the parallel of latitude; that is, a circle or ellipse parallel to the equator. So, this is a kind fundamental terminology we always attach with an ellipsoid.

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Geodetic Terrestrial Reference Frame

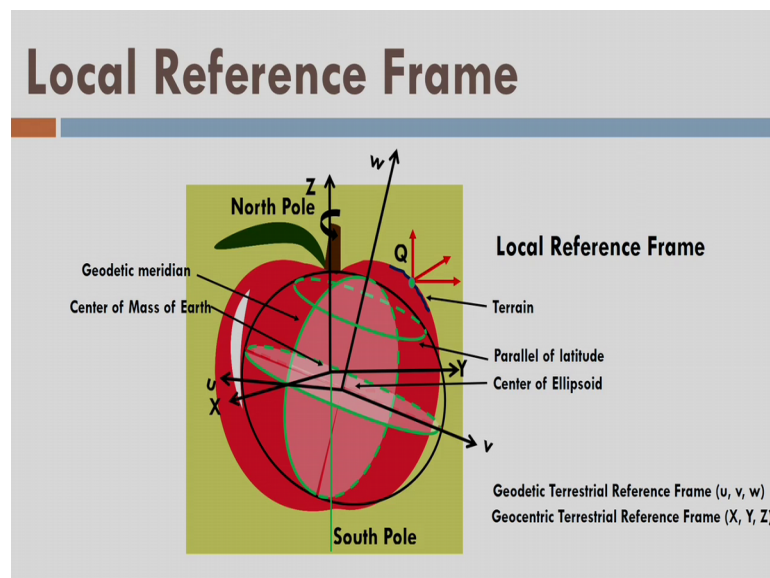
Geodetic Terrestrial Reference Frame (u, v, w)
Geocentric Terrestrial Reference Frame (X, Y, Z)
Center of Mass of Earth
Center of Ellipsoid
Geodetic Terrestrial Reference Frame

So, let us come to the geodetic terrestrial reference frame $u v w$ which is shown here and there the center of mass of the earth which is the geo center of the earth and at the geo center, we have a geocentric reference frame shown by $X Y$ and Z axis. Now, let us come to the geodetic one where $u v w$ axis are showing the 3 mutually orthogonal axes of the geodetic terrestrial reference frame where w is not exactly merging with the rotation axis of the earth, but still it is very close by and the difference between the 2 it is so minimum that we have very minor rotations between the 2.

But what about the geo center and the non geo center location of the ellipsoid? They are quite different; that means, the center of ellipsoid and center of mass of the earth are away from in order of 100 meters sometimes 150 meters 200 meters also, it depends in how the local reference frame or the geodetic terrestrial reference frame realized or fitted. Well, again we have seen that, we have parallel of latitude and we have a geodetic meridian. Remember, in the last, this slide we said there is a meridian and then it is called geodetic meridian in case of geodetic ellipsoid.

Well we have center of ellipsoid. We just away from the geo center and it is not the geo center and.

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So, we are showing here a geocentric terrestrial reference frame $X Y Z$ and geodetic terrestrial reference frame $u v w$ here, ok. What about the local reference frame? Well, again we are showing a geodetic reference frame and a geocentric reference frame in this

screen ok. And all the terminologies which are relevant to the 2 are also shown. Now, let us imagine that there is earth and I am showing the earth by the apple surface.

So, on the apple surface, let us see there is a terrain. That is a terrain surface which I am trying to manifest. So, point Q is there and on the point Q, if I define reference frame, in order to work on the surface of the earth, such a frame is called local reference frame.

Now, at point Q as origin, I can take the measurement in the local reference frame and then I can connect those measurements with respect to may be geodetic system. Geodetic terrestrial reference frame which is non geocentric in nature or I can connect the observations of local reference frame to the geocentric reference frame that is centered at the center of the earth.

And by this idea, I can always do the measurements on the local reference frame and I can always connect those measurements to the some kind of global terrestrial reference frame which can be geodetic or which can be geocentric.

So, that is the idea here in order to define 3 kind of systems. Remember the idea of geodetic is somehow, it is with respect to the regional surfaces or the regional observations for country or for a region or for a continent, geocentric is about the whole planet earth as a single unit or I am most of the observations on this surface of the earth in all the places and then I am trying to find out reference system.

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Coordinate System

- **Concept**
 - Mechanism of defining a point in a reference frame
- **Types**
 - Cartesian coordinate system (X, Y, Z)
 - Polar (Spherical) coordinate system (ρ, θ, α)
 - Ellipsoidal (Geodetic) coordinate system* (λ, ϕ, h)
 - Astronomic coordinate system** (Λ, Φ, H)
 - Projected (Mapping) coordinate system* (E, N, h)

* An ellipsoidal surface as datum is required.
** A geoid surface as datum is required.

(X₀, Y₀, Z₀) Cartesian coordinate system

(X₀, Y₀, Z₀) Polar coordinate system

Now, coming to the coordinate system, a coordinate system is a mechanism of defining a point in a reference frame. Remember, we have already defined reference system and we define realization of a reference system is a reference frame. Let us see, I want to measure a point in 3D.

So, how can we measure let say there is a point P and there is a reference frame called X Y Z and it has a origin at point $X_0 Y_0 Z_0$. Now, I am showing you 3 measurements; z P that is along the z direction X P and then Y P and such a system, I call it cartesian coordinate system. Well, it is well known to you and we have been using such system cartesian system for long.

Now, understand that there is a point P is a same and I want to define the same point P with different type of coordinate system, because it is a mechanism to define a point. So, I am not changing my reference frame I am not changing the origin of the reference frame, but at the same time, I am just defining the point P in a different way.

So, let us look into next one and I call it let us say polar coordinate system. In the polar coordinate system, I have this distance row which is a direct distance. Distance from the origin and there is a point P dash which is again the perpendicular dropped on the point P on the plane X Y. And so, now, I measure the angle theta in the X Y plane and angle alpha in the vertical plane. Let us define the distance row that is the distance from origin to point P and there is a point P dash which is same here, both points are same.

And now, we are dropping the perpendicular from point P like this to the point P dash here, fine. Now, I am measuring the 2 angles in case of polar coordinate system and the angle is measured in the X Y plane from X axis like this. And another angle is alpha angle elevation angle which is measured perpendicular to the X Y plane like this. So, this is the rho theta and alpha which is 3D position again of point P.

So, this is the idea of locating point P in a 2 ways; one I am locating in the cartesian way and another way is polar or spherical coordinate system. Remember, the point is same reference frame is same, origin of the reference frame is also same.

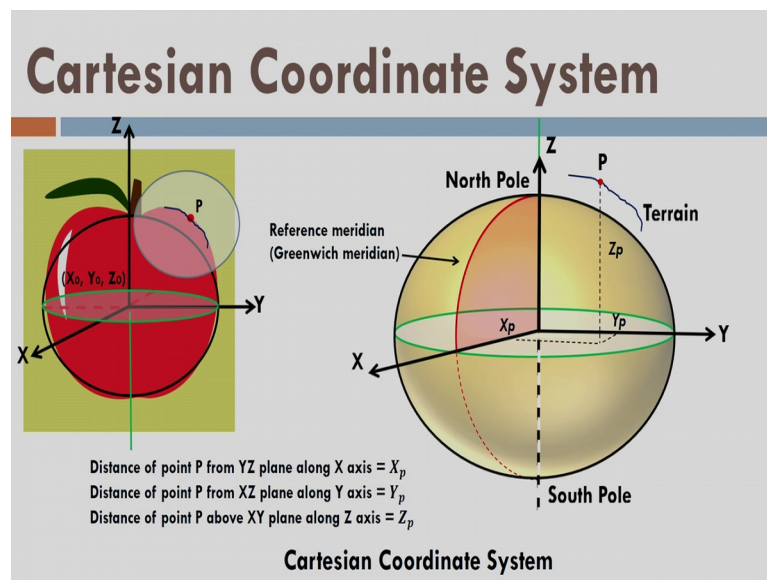
But it is a 2 way of defining a point right. So, we have other type of systems also which are very relevant for the earth; one is called ellipsoidal or geodetic coordinate system.

Remember, it is not a geodetic reference frame. It is a geodetic coordinate system and it is called lambda phi h where h lambda and phi are small alphabetical or the Greek letters.

Then, we have astronomic coordinate system where I am using capital lambda, capital phi and H and then we have projected mapping coordinate system which is nothing, but a 2D coordinate system.

What we have initially criticized in this lecture, but still it is useful for many of the reasons ok. So, in this lecture, in this coming few slides of this lecture, we are going to address all these reference frame which are very relevant and useful for the measurement on this surface of earth.

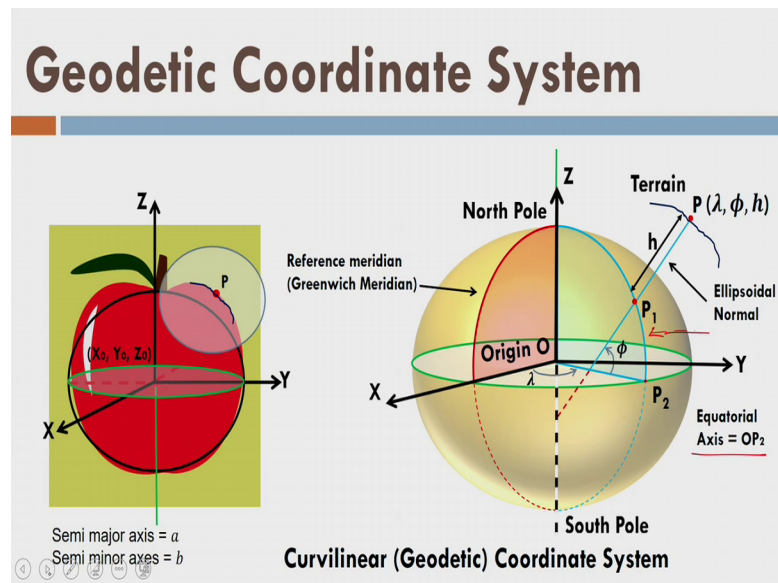
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This is my cartesian coordinate system for the earth. So, let us say point P which is zoomed here. So, now, for point P, I am drawing a Z P X P and Y P. So, this is my cartesian coordinate system and I am defining the cartesian coordinate system for the kind of geodetic or geocentric reference frame for the earth.

Well, you can read it yourself that the distance X P is the distance of point P from Y Z plane along X axis and so on. Let us go for the geodetic coordinate system, what is that. So, let us assume that there is a same point P and there is a geocentric system and the geocentric system has origin O, which is geocenter.

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Let us draw a perpendicular from point P to the ellipsoid that we are dropping a perpendicular from point P to the ellipsoidal surface. So, this is the perpendicular and this is the intersection point of ellipsoidal surface or ellipsoid with the perpendicular line from point P. Now, if I extend the line perpendicular line to the equator, it will cut here and now, if I further extend, it will cut the rotation axis of the earth.

So, this line is called ellipsoidal normal, the whole line from point P to the rotation axis and this line is perpendicular to the ellipsoid. Well, now through the point P₁ which is point P₁ is on the ellipsoid and if I connect the north pole, south pole and the point P₁, it is called the meridian of observer because, it is passing through the observer's perpendicular projection of point P on the ellipsoid.

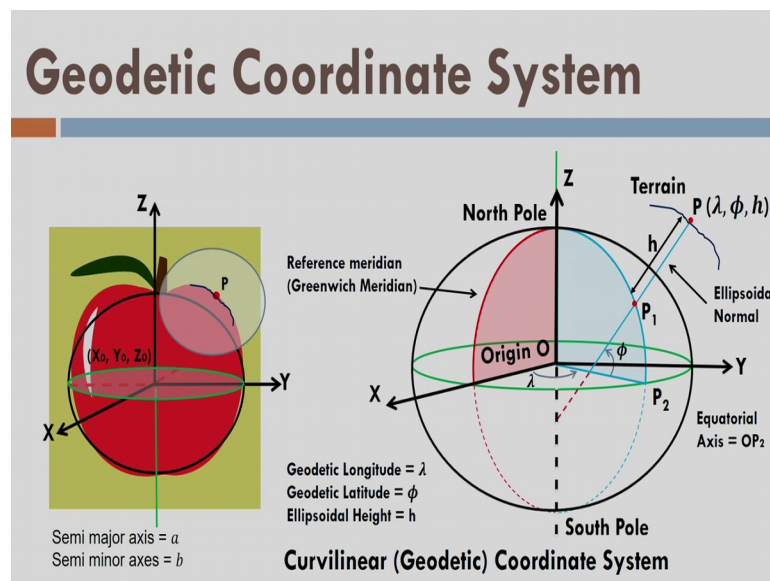
So, remember observer's meridian is the circle or ellipse passing through the north pole, south pole and the projection of observer's point on the ellipsoid. Well so this is called the terrestrial meridian, but remember all these meridians reference meridian which are defined on the surface of the earth or observer's meridian or any other meridian. They are called terrestrial meridian.

So, the OP₂ if you look at, it is called equatorial axis. Well, now this angle which is I have just shown by animation which is the angle between the reference meridian and the observer's meridian and this angle is lambda. Once again, I repeat this thing.

Let us see, this is the angle between the reference meridian and the observers meridian and this angle is called lambda or longitude. Remember, it is called terrestrial longitude of place P with respect to some reference meridian on the surface of earth ok. What about the latitude again or terrestrial latitude? This angle which is between equator and the line joining that is ellipsoidal normal. If I calculate this angle, that is called phi angle or latitude or terrestrial latitude of point P 1.

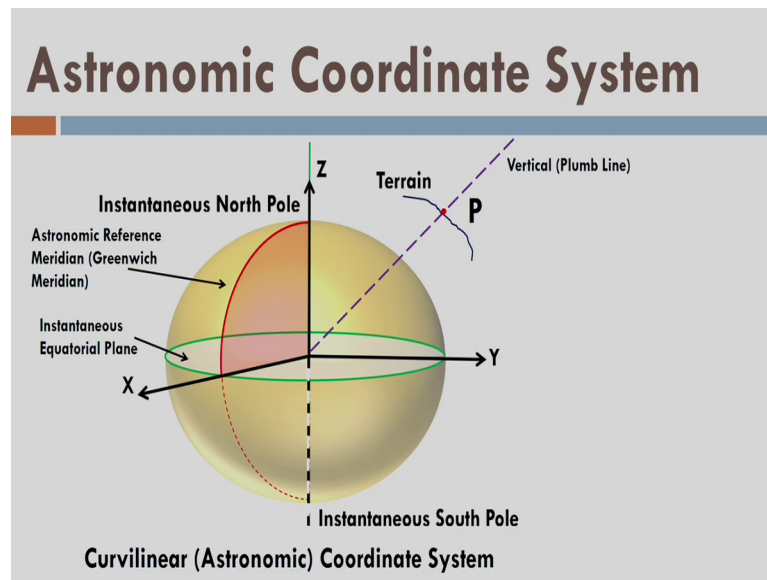
Well, I have fixed point P 1 now in 2 dimension; that means, point P 1 or P; they are still same because, both have lambda and phi same value. So, now, I define this distance which I am going to fix with respect to the ellipsoidal surface the distance from the ellipsoid along the ellipsoidal normal to point P is called ellipsoidal height. So, this is my ellipsoidal height h. Well, now I can fix the point P because h is also fixed. So, point P is uniquely defined now in 3 dimensional way and such a system is called ellipsoidal coordinate system, also called geodetic coordinate system or sometimes called curvilinear geodetic coordinate system.

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So, I have a geodetic longitude here lambda geodetic latitude phi and ellipsoidal height h; well this kind of figures are available in the book.

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Let us go to the astronomic coordinate system. Here in case of astronomic coordinate system, again point P is same, but let us define the plumb line or the vertical line, that is called vertical or sometimes called plumb line in literature, that is the line of gravity passing through point P. So, it is going to intersect the geo center of the earth and that is why, we are straight away drawing it to the geo center.

Well, let us see there is a point here and let us see this is a kind of terrestrial meridian that we call astronomic meridian of the observer ok. What about this angle which is between the reference meridian and the astronomic meridian of the observer and this angle is called capital lambda or astronomic longitude. Again, this angle, it is called astronomical latitude which is between the equator and the vertical line passing through the point P that is called astronomic latitude. Again both are terrestrial in nature.

Secondly, in case of astronomic observation when I perform some astronomic observation I will do some observation at certain instant and at what instant are at that instant whatever direction of axes of the earth is there I will adopt that and as a result we call it instantaneous north pole and instantaneous south pole that has framed by the instantaneous rotation axes of the earth and that becomes the Z axis.

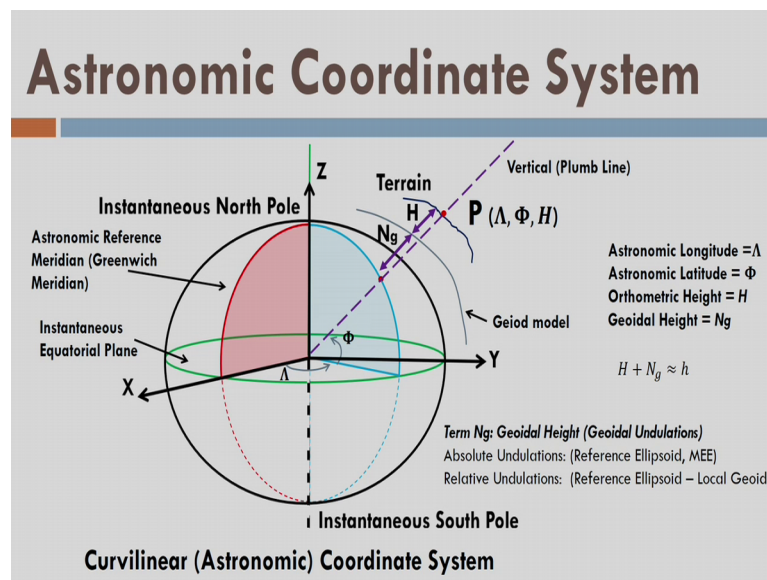
Secondly, Greenwich meridian or the reference meridian is again instantaneous because, at that instant, wherever I find the 0 longitude, I will say that particular meridian is my reference meridian or I can say the Greenwich meridian. But remember, it is an

astronomic reference meridian. Same way, I have instantaneous equatorial plane why because, I am considering the instantaneous axis of the earth and perpendicular to perpendicular to that axis, I have instantaneous equatorial plane.

Well so, this is called the astronomic coordinate system by yet still it is a 2 dimensional system I need to fix the point P and for that, what I do? I will take geoid model. Remember, the geoid model; it could be local geoid and it could be global geoid. Let us assume it could be global, but remember it is decided by the mean sea level variation ok. Say, from geoid, I measure this height of point P and I call it capital H and this is height is nothing, but the orthometric height.

Remember, we have decided in case of plane survey or in case of classical survey one surface call equipotential surface and with respect to that equipotential surface. We have measured some height and similarly, this height h is also the height which is measured in the direction of gravity. And this height is called orthometric height and orthometric heights are defined with respect to the geoid, ok.

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What about the role of ellipsoid here? So, far we did not find any role of ellipsoid. Although, we are showing it here for the purpose and the purpose is the height between the geoid model and the ellipsoid is called geoidal undulations and represented by N_g . Well, and earlier we assumed for a regional ellipsoid that, the regional ellipsoid the line vertical line and the ellipsoidal normals are parallel or they are basically the same and

there was some region for that, rather it was a limitation to do that. Why because, in those days, when this concept came of ellipsoid and so on and those days, there was no observations possible with respect to an ellipsoid.

So, what scientists use to do? They used to fit an ellipsoid or defined parameter; that means they are already fixed the ellipsoid. They decided for that particular reason we are going to fit that particular ellipsoid and for that, how do they do? It they used to observe triangulation networks. They used to determine some 2 dimensional points. They used to determine the vertical heights and this vertical heights are the orthometric heights H .

Using this one dimensional survey for vertical and 2 dimensional survey, by triangulation for planimetric survey, they used to combine this data and they used to fit the ellipsoid through those 3D points and that was the idea. And since there was a problem that we did not have any global reference, what they used to do? They used to fix some stations called Laplace stations. On those Laplace stations, they used to perform some astronomic observations and using those astronomic observations. They used to bring all the network of points into the global reference frame.

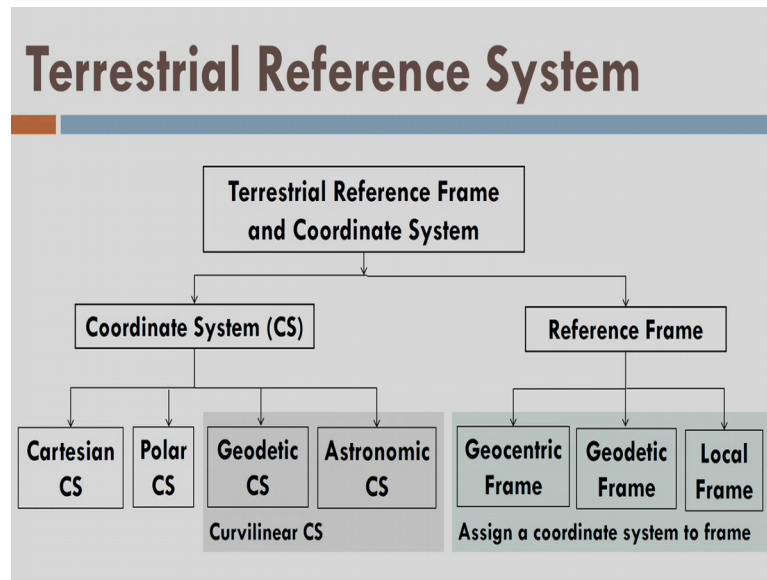
They used to assume that the ellipsoid at one place on that region is matching with the geoid; that means the surface of the earth and ellipsoid is matching at certain point and with this idea, they used to work around. Then, they used to say that now, we are measuring the heights or the heights which are orthometric in nature although, but they are being measured with respect to the ellipsoid and that was the kind of manipulation. It was in the place before the space geoid is started.

So, astronomic longitude is called λ , capital λ , astronomic latitude is capital ϕ and orthometric height is H and geoidal height N_g or geoidal undulations. So with the assumptions that I said few minutes before H plus N_g is approximate equal to the ellipsoidal height because, we assume that ellipsoid and the ellipsoidal normal and the orthometric height or the vertical line are approximately close to each other. And with this assumption, I can find out the N_g value of N_g or value of H . If I know the N_g and since there was no N_g available to us, earlier scientist they used to assume N_g equal to 0 at one point on the surface of the earth.

I hope that I just I am repeating the same idea again. So, the term N_g is called geoidal height or geoidal undulation. So, it is an it is called absolute undulation. If I take the

difference between the reference ellipsoid and mean earth ellipsoid or it is called relative undulations. If I take a difference of reference ellipsoid to the local geoid so that is the idea of the astronomic coordinate system.

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So, let us conclude all the terrestrial reference system. So, we have terrestrial reference frame and coordinate systems like this we have coordinate system, cartesian coordinate system, polar coordinate system, geodetic coordinate system and astronomic coordinate system and we have explained it that, how do they measure a point. The point is same, but coordinate system is a way to measure the point and all these cartesian polar geodetic and astronomic coordinate system are 3D in nature. At the same time, we have reference frame, we have geocentric reference frame, we have geodetic reference frame, we have local reference frame.

Remember geocentric reference frame is located at the origin or the origin of the geocentric frame is located at the geocenter or center of mass of the earth. If reference frame, it is in global in nature, but if it is not located at the geocenter of the earth or it is slightly away from that; it is called geodetic reference frame and remember we have defined various ellipsoids also and various reference frame like Everest, Clerk, NID 27. All these are geodetic reference frame because they are particular to a continent or particular to a region not for the complete one.

Then, we have local reference frame. Local frame we need to define in order to take the observations on the surface of the earth. Well, because geodetic and geocentric reference frames, they are good for conceptualization. They are good to work around, they are good to imagine or good to calculate the values of 3D coordinates of point in a global reference frames for large areas, but still we need to collect observations on the surface of the earth and for that, we need to define a local reference frame.

So, these are the basically importance of different concepts of a reference frame and coordinate system. In the next lecture, we are going to perform the transformations between the 2 coordinate system, between the 2 reference frame and how can we do change of the geocentric reference frame. All these things we are going to learn in the next lecture. Before concluding, I can say this is my curvilinear coordinate systems, that is geodetic and astronomic and we have to assign a coordinate system to a frame. So, that we can measure the coordinates in a given reference frame ok.

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Some Variants of RF and CS

<ul style="list-style-type: none"> □ Reference frames <ul style="list-style-type: none"> ▪ Global/Terrestrial frames <ul style="list-style-type: none"> ▪ Geo-centric (Earth centered) ▪ Geodetic (Non-Geocentric) ▪ Local frames <ul style="list-style-type: none"> ▪ Topocentric ▪ Non-Topocentric 	<ul style="list-style-type: none"> □ Coordinate systems <ul style="list-style-type: none"> ▪ Cartesian coordinate system ▪ Polar coordinate system ▪ Curvilinear coordinate system <ul style="list-style-type: none"> ▪ Geodetic coordinate system ▪ Astronomic coordinate system
<ul style="list-style-type: none"> □ Variants of Local reference frames and coordinate systems <ul style="list-style-type: none"> ▪ <i>Local terrestrial frame</i> ▪ <i>Local geodetic frame</i> ▪ <i>Local astronomic frame</i> 	<ul style="list-style-type: none"> □ Variants of Global reference frames and coordinate systems <ul style="list-style-type: none"> ▪ <i>Global cartesian coordinate system</i> ▪ <i>Geodetic cartesian coordinate system</i>

Now, finally, we can see there are some variants of reference frame and coordinate systems. For example, we have reference frame global or terrestrial reference frame, then we have local frames in case of global or terrestrial frames, we call it geocentric or geodetic in case of local reference frame, it is called topocentric and non topocentric. Remember, if the origin lies on the surface of the earth of a local reference frame, it is

called topocentric reference frame and if origin lies somewhere above or below or somewhere else, we call it non topocentric.

So, then we have variants of local reference frame like local terrestrial frame, local geodetic frame, local astronomic frame, then we have coordinate systems. We have already explained them in detail. Then, we can have the variants of global reference frames and coordinate systems; that means I am combining different coordinate system with different frames. So, I can say global cartesian coordinate system. I can also say geodetic cartesian coordinate system and so on. Well, let us conclude this lecture and this is the reference I used and thank you very much for being attentive.

Thank you very much.