Higher Surveying
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Module-6
Lecture – 19
Photogrammetry

Hello everyone, welcome back to the course of Higher Surveying and we are in module 6 Photogrammetry. In last lecture we have developed some important relationship between the image space and object space and we call those relationship as collinearity and coplanatity equations. Apart from that we have also learnt some of the mathematical transformations and some systematic errors that a vertical photograph may have. Fine, so, today after learning the coplanarity and collinearity equations in last lecture today we are in the next lecture that is analytical photogrammetry 2.

And there we are going to utilize these equations so, that we can do some kind of space intersection and space resection, aerial triangulation and so, on.

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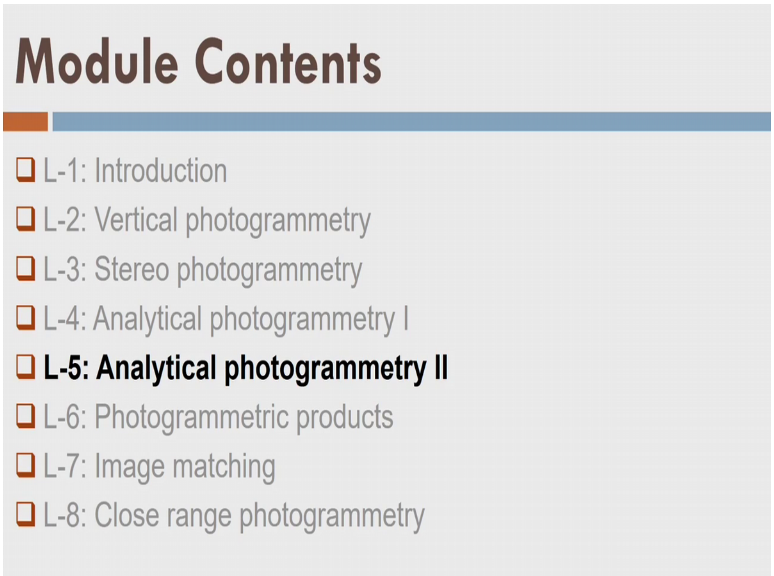

So if you remember or if you can recall the concept of resection and intersection from basic surveying we have done that resection and intersection in the basic surveying for the planimetric. Now, we are going to do it for the 3-D especially in the context of photogrammetry. So, now, let us try to regain the same knowledge again in this module.

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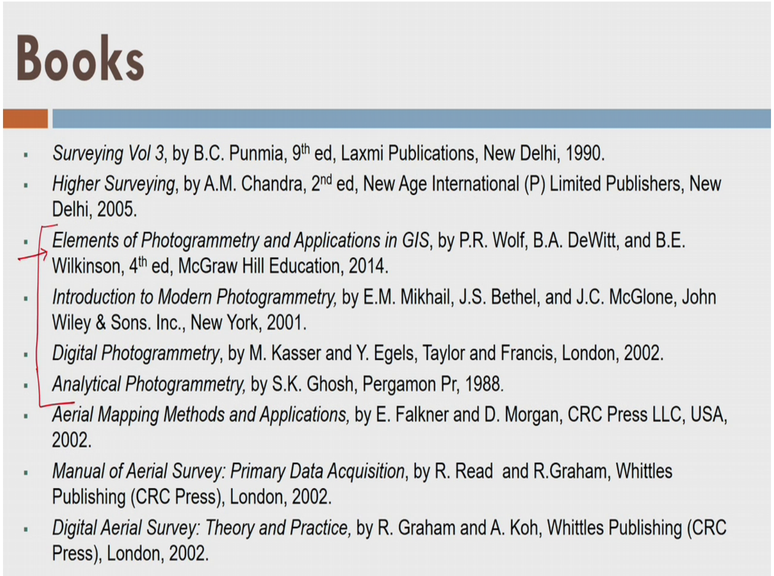

So, these are the books again and I say recommend you that these are the books we will need for this lecture also and this is again book available in India.

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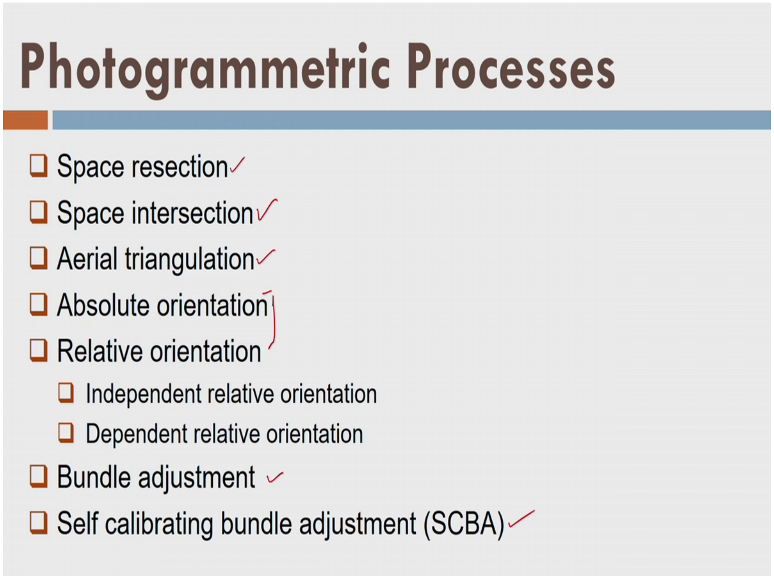

Now, in this lecture we are going to learn space resection, space intersection, aerial triangulation, orientation absolute and relative and then we are going to talk about the bundle adjustment and self calibrating bundle adjustment ok.

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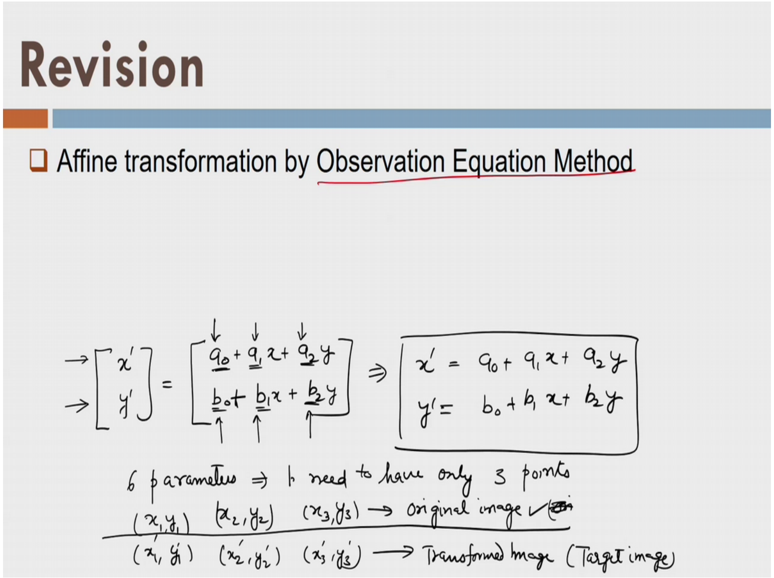

First of all let us do some revision because we want to use the computational photogrammetry or the analytical photogrammetry or the numerical photogrammetry. For that we need to understand some of the concepts like least square solution and for that purpose let us revisit the affine transformation as well as solving the affine transformation by least square solution. Especially we are going to use observation equation method.

So, if I write the affine transformation equation that we discussed yesterday it should be like that x dash y dash can be written in simple form a 0 plus a 1 x plus a 2 y b 0 plus b 1 x plus b 2 y the 2 equations we derived from this equation ok.

So, I can write a state that x dash equals to a 0 plus a 1 x plus a 2 y and y dash equal to b 0 plus b 1 x plus b 2 y. You will be surprised that where do we do it the affine transformation, where do we need it. Just I will give you a simple example let us say you have an image in your computer might be opened in some software could be windows photo manager or maybe MS paint wherever.

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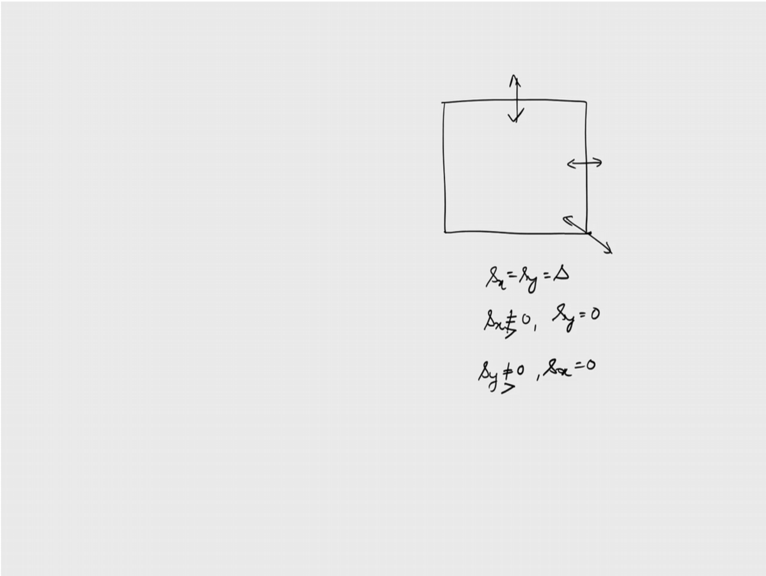

Now, what do you do? You take this image and sometimes you try to stretch it diagonally this way or sometimes in x direction or sometimes in y direction. So, what you will you do? When you stretch in diagonally you keep your scales same, but you make a bigger image.

Similarly, when you stretch in x direction what do you do? You S x non-zero, but S y is 0 and when we stretch in y direction S y is non-zero and S x is 0 or rather I can say this is more 0 or more than 1 right. And as a result now you get and another image which is stretched 1 right.

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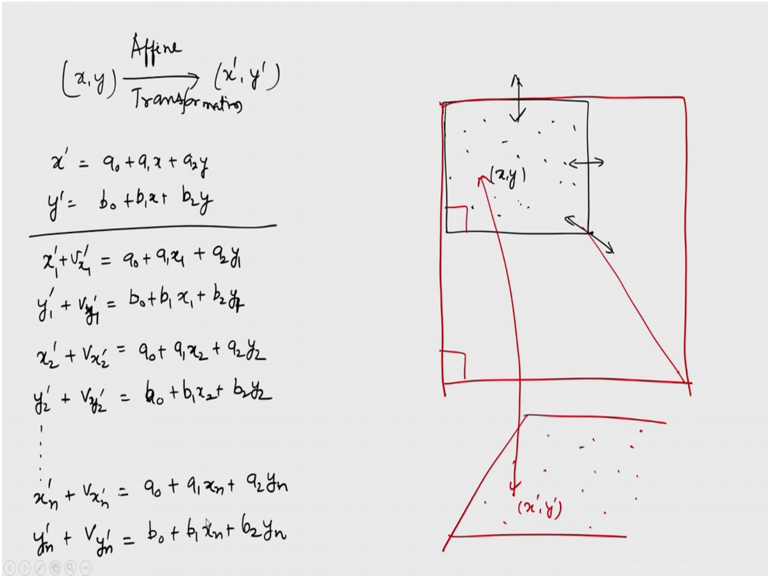

Like say this another image could be let us say this image bit it is stretched this image and you have made this image from here to here like this.

So, you get this image so, is not do not you think that it is an affine transformation where the scale S x and S y is different ok. So, we have changed the shape of the image, but since these angles are still maintain here and here I can say it is a conformal transformation. Because, S x and S y they are not different in their individual directions; however, if is makes some reshaping of this image and make it like this, this kind of image, the stretched image then it will be an affine transformation.

So, now, I can say that let us see in the stretched image a point p is having now x dash y dash coordinate and which was having originally here having x and y coordinates.

So, I have transformed my x y to x dash y dash by affine transformation or maybe other transformation. So, let us say right now we are talking about affine transformation ok. Now, fine let us see I have these are many many such point on this image and then I have measured same number of points on this image like this. And one to one correspondence is there the way these 2 are related I have same way all the n number of points have one to one correspondence between the 2 images ok.

So, now, let us do the least square solution; that means, I am trying to find out the parameters a 0, a 1, a 2, b 0, b 1, b 2 for given n number of points.

IF you see carefully here for these 2 equations and this 3 unknown parameter 6 unknown parameters this 3 and this 3 I need to have for 6 parameters I need to have only 3 points. What is the meaning? If I have let us say x 1 y 1 with corresponding x 1 dash y 1 dash, then second point is x 2 y 2 I have x 2 dash y 2 dash also x 3 y 3 I have x 3 dash y 3 dash. So, there are in the transformed image and there are in the original image on untransformed image right.

So, if I have this 3 values or 3 coordinates or 3 points in the original image as well as the transformed image I can say here let us say this is my target image and that is my Let us say the original image again same right. So, now, using this I can find out the unique values of a 0 a 1 and a 2, b 0 b 1 and b 2; however, what if I have more than 3 number of points? In that case I need to go for the least square solution that will minimize my errors and that will give me some solution right, right.

So, let us see that again if we write my equation x dash equal to a 0 plus a 1 x plus a 2 y y dash equal to b 0 plus b 1 x plus b 2 y let me introduce some errors if you remember.

Let us say x dash v x dash so, if I the first point then it will be a 0 plus a 1 x 1 plus a 2 y 1. Similarly, y 1 dash plus v y 1 dash equals to b 0 plus b 1 x 1 plus b 2 y 1. Then I can still right for the second point x 2 dash plus v x 2 dash a 0 plus a 1 x 2 plus a 2 y 2, y 2 dash plus v y 2 dash equals to b 0 plus b 1 x 2 plus b 2 y 2 and so, on. x n dash plus v x n dash equals to a 0 plus a 1 x n plus a 2 y n and y n dash plus v y n dash equals to b 0 plus b 1 x 1 plus b 2 y n right.

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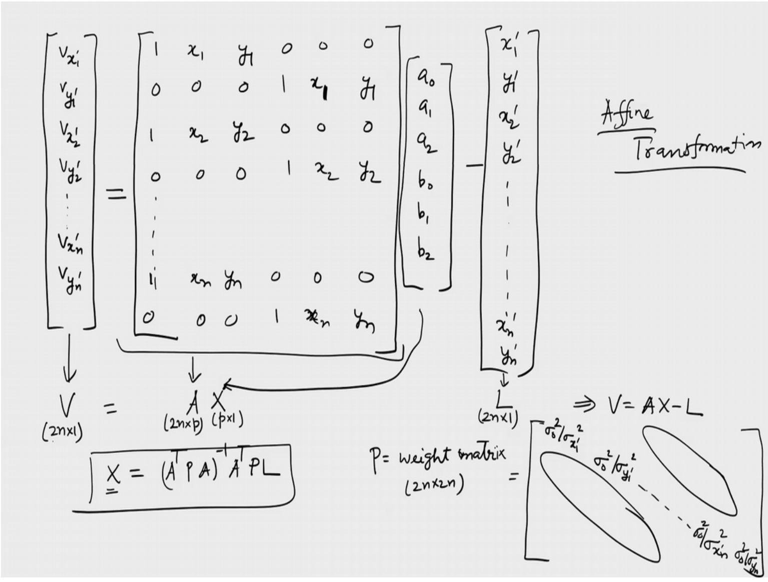

So, these are the equations I have written for n number of points now, I can straightaway write my V matrix here that is V x 1 dash V y 1 dash V x 2 dash V y 2 dash and so, on. So, here your V x n dash V y n dash and now I can make a complete matrix here called V matrix. I am just repeating the process I am demanding and now I can write here let us see try you try to write yourself x 1 y 1 0 0 0, then 0 0 0 1 x 1 y 1. Then here we have 1 x 2 y 2 0 0 0 0 0 0 1 x 2 y 2, then we have here 1 x n y n 0 0 0, then 0 0 0 1 x n y n. Then we have here all the parameters a 0 a 1 a 2 b 0 b 1 b 2 right.

And here I can write mu x 1 dash y 1 dash x 2 dash y 2 dash x n dash y n dash right so, ok. So, this is my V matrix equal to A matrix here, this is my X matrix here and then this is my L matrix here. So, this is the form called V is equal to AX minus L, where X is my parameters this matrix here, this is my A matrix here, V matrix here, L matrix here. I can write a solution called X equal to A T A inverse one more specifically A T P A inverse A T P L.

So, these are the values of my 6 parameters here and they are least squares values or rather these values will provide you the minimum deviation of the observed data to the fitted data or the simulated data. So, that was the idea and where my P matrix is the weight matrix and the weight matrix itself consists of size 2 n by 2 n, well regarding the size my X matrix has a size P into 1 this matrix has size 2 n into P, P is number of parameters, L is having 2 n 1 to 1 and V is having size 2 n 1.

Because, there are n number of points each point is providing 2 equations. So, we have 2 n number of equations and hence this is coming.

Now, what about the weight matrix? Weight matrix has 2 n by 2 n size; that means, it will be sigma 0 square by sigma x 1 dash square. Similarly, sigma 0 square by sigma y 1 dash square and so, on I can write up to here. Sigma x n dash square here and then sigma 0 square by sigma y n dash square like this and all these matrices are 0 fine. I hope you can recollect all those things back from the module 4 on the error accuracy and adjustments computation.

Well this was the idea that we have talked about the observation equation method ok. Now, we are going to implement the observation equation method for the photogrammetry and we have just seen the example of how to use observation equation method for the affine transformation. If I stretch an image from the original to the next one you can find out those values ok.

Similarly, you can imagine that if you are doing an affine transformation between the image and the let us say the map; they are both are planes. So, we are transforming one image plane to the map plane in that case also you can use affine transformation. I am not saying that it would be correct, but you can find out you can do it yourself right.

So let us go ahead and try to understand how to use this least squares method or observation equation method for the purpose of space resection and what is the specific resection first? Let us dive into that.

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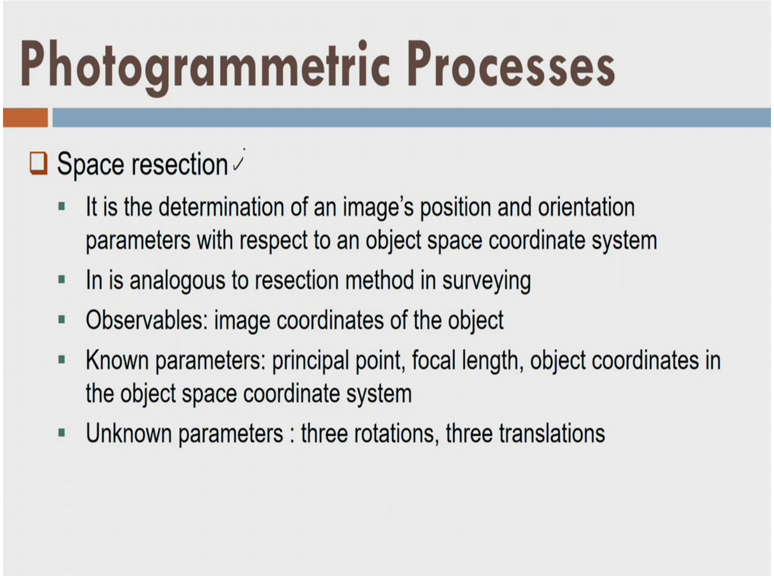

Now, so, it is one of the photogrammetric processes that we have already discussed and this is the space resection and let us go ahead.

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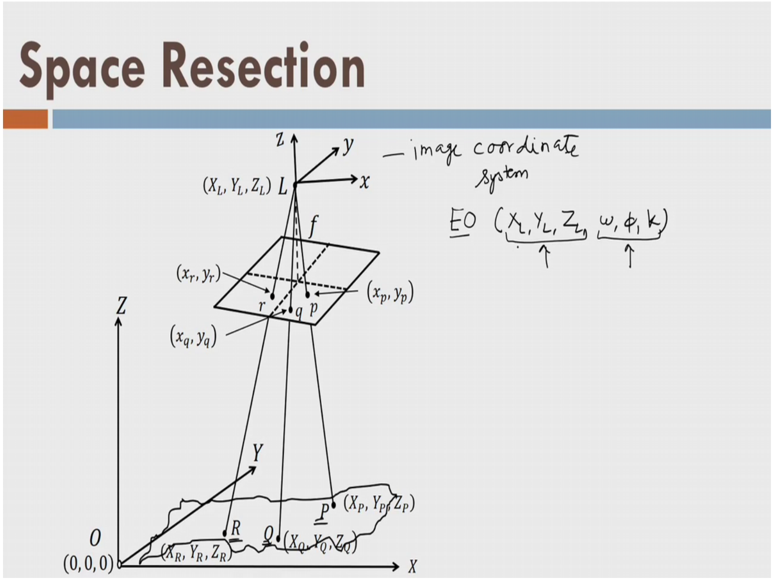

So, space resection let us see that there is a image, this is my exposure station given by L and it is my object coordinate system capital X, Y, Z. And now let us I define this my image coordinate system and both image coordinates system and space coordinate system or I can say object coordinate system are independent of each other. They are not dependent anyhow to each other right.

So, and the relationship between the image space and object space was developed by the collinearity equation; that means, the point or the vector in the image space system is or the some scaled version of the vector in the object coordinate system.

So, let us see this is a terrain and this is a point P well that is X P Y P Z P in the object coordinate system and that is been there are 3 points for example,. So, it is imaged here point P, X P Y P, point Q, X Q Y Q and point R. So, we have 3 images of point P, Q, R and then we have 3 points on the ground surface right.

Now, how can I use the collinearity equation there? I can write the collinearity equation for each of these point; that means, I can write for small p, small q and small r. So, I will have such 6 equations; however, here we need to understand; what is the space resection now.

Space resection is the same idea let us assume that I know the coordinates of point P, Q and R in the object coordinate system or ground coordinate system from there I want to find out the exterior orientation parameter. So, what are the values of exterior orientation parameter? I want to find out that.

So, can I use the collinearity equation? Space resection is to find out 6 parameters which I call EO parameters Exterior Orientation which is nothing, but X L, Y L, Z L and the omega, phi, kappa these are the called attitude of the image and they are called the altitude of the image or the exposure station.

So, it is altitude of exposure station that is 3-D position of exposure station and that is the attitude of image. Attitude means at which angle they are placed in the object coordinate system or the image is placed in the object coordinate system and it is going to change for each and every image, if you are taking couple of images together right.

So, now I want to find out these so, let us use the collinearity equation here ok.

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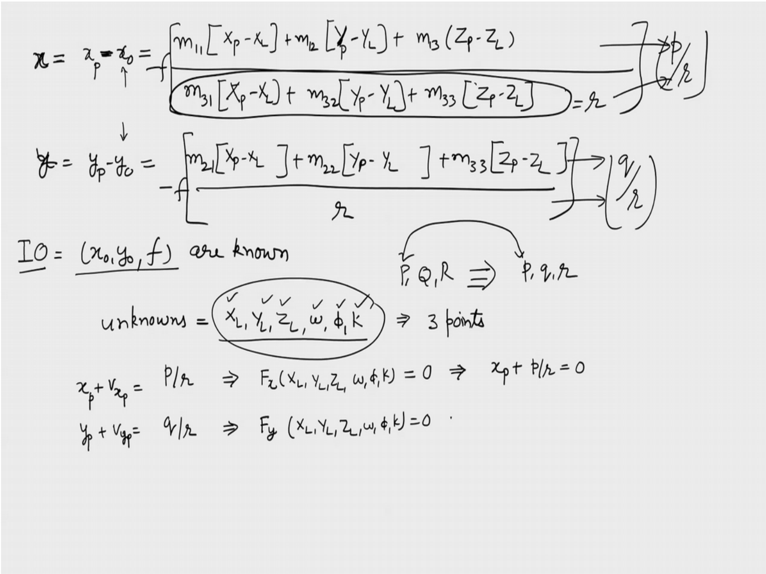

Let me write for any point the collinearity equation as x p minus x 0 here for point P equals to m 11, here I will write x p minus X L plus m 1 2 y p minus Y L plus m 1 3 z P minus Z L ok. Here m 3 1 x p minus X L plus m 3 2 y p minus Y L plus m 3 3 z P minus Z L.

Similarly, I can write y p minus y 0 x 0 is equal to m 2 1, same factor here m 2 2, same factor here m 3 3, same factor here divided by if I call this denominator this term the denominator as r, I can write r. Because it is same in the both equations ok I can also write equal to x here equal to y here right.

Now, for simplicity I assume that interior orientation parameters are known. So, here the factor minus f and here the factor minus f, they are known to me like this fine. And that makes my life little simple, why because if they are not if they are unknown no problem we can resolve it.

Secondly, there is a idea that I can also find them these interior orientation parameter by camera calibration process that we are going to discuss in the 7th lecture and 8th lecture, especially in the 8th lecture how to find out for the closing photogrammetry; however, we can also find out simultaneously here within resection itself that we are going to discuss today no problem.

So, for time being assume it they are known the interior parameters IO parameters are known here.

Ok, as a result I can place these values or simply I can assume that these values x 0 y 0 are 0, assuming x 0 y 0 equal to 0 makes my principal point coinciding with the image centre right. So, that assumption I am making here for making the calculation simple ok. Now, you observe let us say 3 points P, Q and R on ground surface and corresponding points n on the images are p q and r.

So, in order to find out 6 unknowns let us see X L, Y L, Z L omega phi and kappa I need how many points I need only 3 points. Why because, each point gives me 2 collinearity equation right; that means, pair P and p gives me 2 collinearity equations. Similarly, I will have 6 equation with the help of 3 points and as a result I can find out this unique values of 6 unknowns.

However, if I have more than 3 number of points I need to go for the least square solution. You think why should I go for least square solution when I can work only with 3 points. The idea here is you should be able to do it in a sense because later on once you do this least square solution after that whatever the calculated values are there you are going to use it for the whole image and any point.

So, if you use only 3 points these the solutions what you get by 3 points of these values they will not be reliable for the every point on the image; however, if you take more number of points, it will be more reliable for other points also we will see this thing.

Now, what to do, how to use this collinearity equation with observation equation method? So, let us say that you have measured an image point x p and then you have introduce an error equal to p by r, where my numerator is p and this is my r numerator here. Similarly, I can write that y p plus V y p equal to q by r this numerator is q and this denominator is r. So, I can write q by r here.

So, I can also write this same thing here total let us say F x some parameter let us say all these unknowns I can say X L, Y L, Z L, omega, phi, kappa equal to 0, this equation as F y X L, Y L, Z L, omega phi kappa equals to 0 or in other words I am writing that x p plus this function called p by r equal to 0 here.

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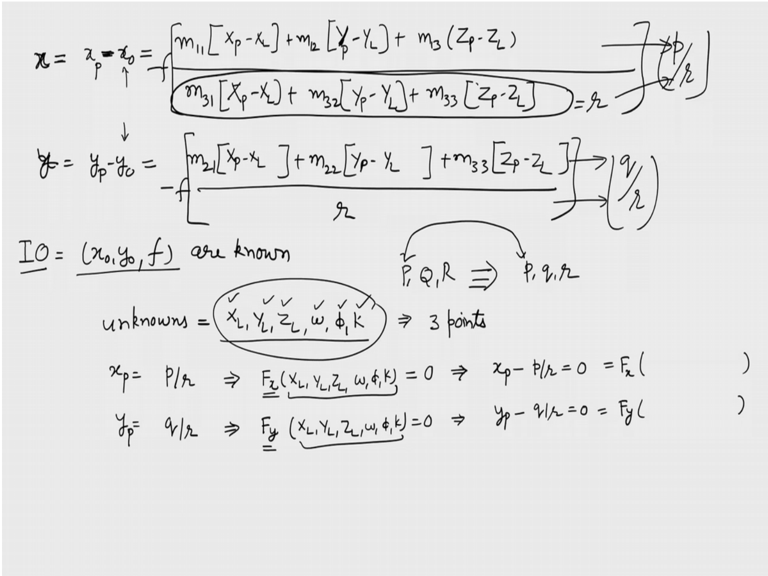

Let us write in a simple way like this let us do not introduce the error right now we will introduce it well ok. I can write here y p plus yeah minus here minus minus q by r equals to 0 and that is nothing, but my F x and here this equal to my F y right the same F x and F y. I am writing here parameters unknown parameters and here I am not writing parameters, but they are same. Now, let us use the Taylors series again to expand this F x because, F x is my non-linear part well what will happen here ok.

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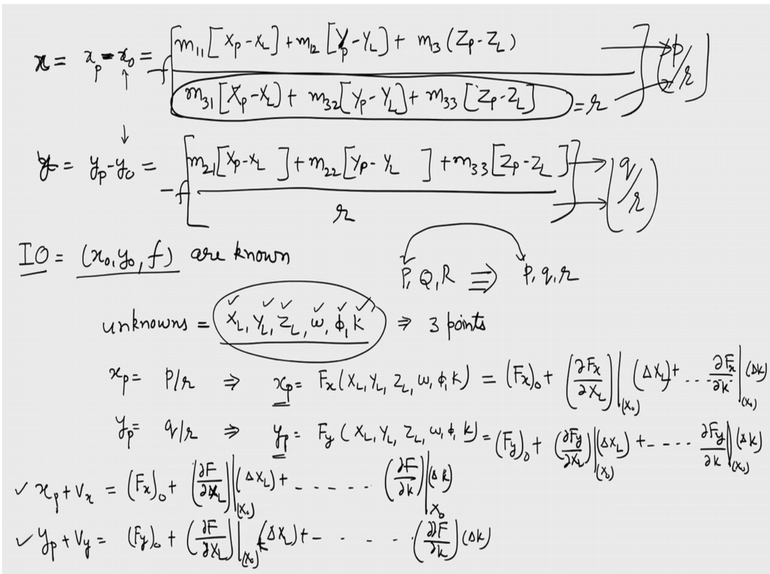

So, let us write this equations like this now as x p equals to function of x similarly, y p is equal to function F y kappa or I can write here that x p is function of all this unknown parameter, y p is function of unknown parameters.

Now, I can if I introduce an error V x and V y to x p and y p I can use the observation equation method. Because, my unknown parameters and my observe variables they are completely separate or I can I am writing this values as explicit values in terms of unknown parameters by functions F x and F y.

So, let us do like that and before that I would like to make one assumption here that I am using Taylor series in order to do the first order approximation. All my parameters will come here one by one. So, let us see the value at known place x 0 and delta X L plus all that first order term will come here. So, I will just write here the last term that is kappa at known position x 0 into delta kappa plus we ignore all the higher terms.

Similarly, I can write here F y 0 X L at some known location x 0 into delta X L plus all the first order term and the last term in the first order will be like this kappa term x 0, here also x 0 at some known place and then I am waiting here is delta kappa and we are ignoring in both equations the higher order terms ok.

Now, introduce some errors called x p plus let us say V x equals to 0 plus I can say here d X L into delta X L at known location X 0 about which we are expanding and. So, I have here d kappa into delta kappa right at known point X 0 fine.

Similarly, I can write y p plus V y equals to X L into delta X L here all 6 parameters delta kappa here I am writing delta kappa here ok. So, what are the intermediate terms or should we write it today itself right we will write on the next page in the matrix form.

So, now, I can write these 2 equations for the point q and r also and the moment I write this for 6 equations.

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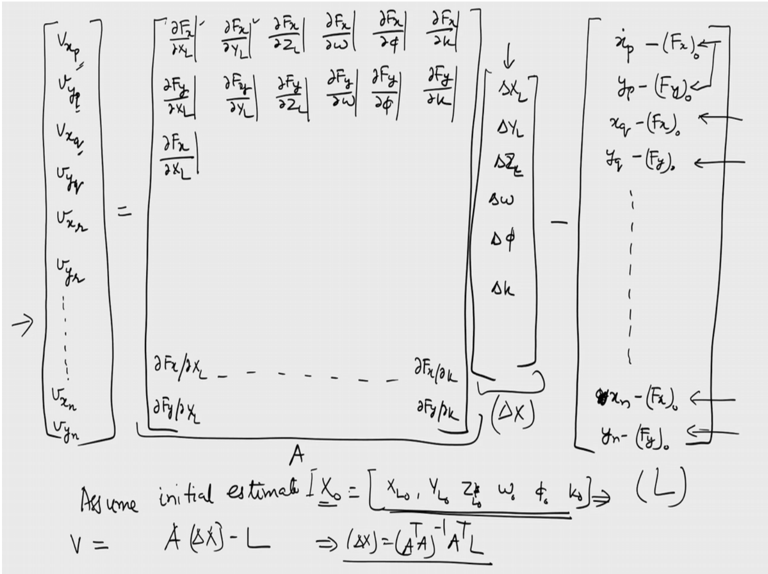

I can write them in the matrix from as let us see that V x p V y p V x q V y q let us say V x r V y r. If I have n number of points then it will be let us say V x n and V y n here equals to. One thing I would like to remind you here that I have written 2 things point P, Q, R and there also I have used denominator and in the collinearity equation small p and small r. Here let us make some clarification that let us write them as d I am sorry, call B divided by D and let us call by C divided by D, just to avoid any confusion among the points and this one.

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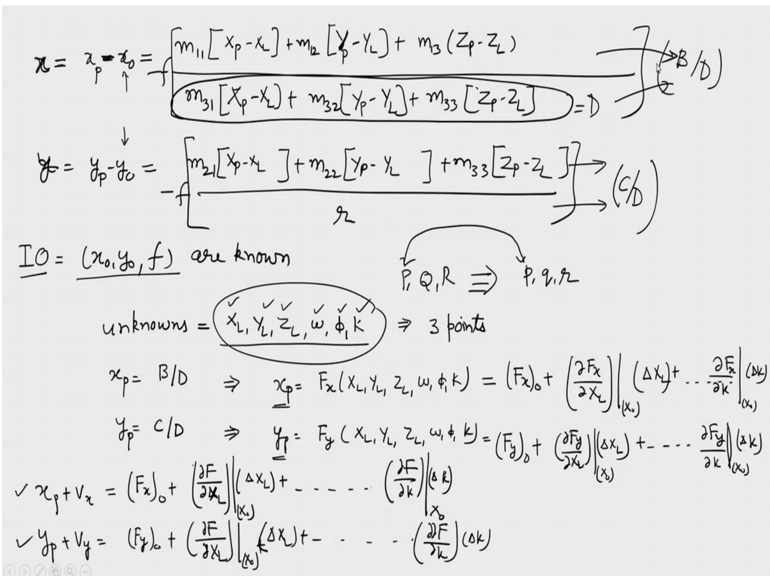

Now, I can write here just I am changing this thing here slightly which is nothing, but B divided by D and C divided by D here fine just avoid confusion among the point P, Q, R and this p r here. So, this is my D here right I hope you got it what is the meaning here ok.

Here if I write the big matrix here then it will be d F by d X L X d F x by d Y L d F x by d Z L d F x by d omega d F x by d phi d F x by d kappa and here my unknown values delta X L, delta Y L, delta Z L, delta omega, delta phi, delta kappa ok. What about this values? You can write it yourself also well I will write it for you no problem.

But, before that let us fill this matrix d F y by dou X L dou F y by dou Y L dou F y by dou Z L dou F y by dou omega dou F y by dou phi dou F y by dou kappa. And then for the next point Q I am writing it dou F x by dou X L and then, but these point will be evaluated at known point called P or I can say x 0 at known point.

Similarly, all these points will be different different different right. So, here for point I can write here at point P ok. So, similarly I write here all this thing I am not writing all this in detail what do you mean. So, what do you do basically? You first assume initial estimate called 0 which is nothing, but some value of x L 0, Y L 0, Z L 0, omega 0, phi 0, kappa 0 and then you plug in this values X 0 here in this terms with this observed values of x p, y P and so, on right.

Then you find out all this matrices here let us say dou F x by dou X L and here dou F y by dou X L here and. So, on I can write it dou F x by dou kappa dou F y by dou kappa right. Now, you got all this terms these are your unknown or the corrections to these values all these are corrections estimated values assumptions, initial estimates. And what about here, what is the there?

So, you will have here x p minus F x 0, y p minus F y 0, x q minus F x 0, y q minus F y 0. So, on I can write here x n minus F x 0, y n minus F y 0 here.

Remember that these values are calculated by plugging in these values and the corresponding capital P coordinates; that means, these 2 equations are coming from collinearity equation of the P point. Similarly, these 2 equations are coming from the collinearity equation or the linearized collinearity equation for point Q and so, on.

So, this is the last point let us say nth point. So, these 2 equations are coming from the linearized collinearity equation for nth point. So, now, I have written this complete matrix and that is in the form V is equal to A delta X minus L. So, I can write a solution here delta X equals to A T A inverse A T L assuming the that the weights are units weights ok. So, this is my A matrix here, delta X matrix here and L matrix here. Just check whether L matrix is correct or not it could be wrong by minus sign, but correct yourself fine.

So, once you find out the solutions delta X now what will you do? You update your X 0 by adding those values to the initial estimates. So, you get the second estimate now you will repeat the same process of this thing and find out the next set of delta X again you will update your second estimate of the X 0 and you will get the third estimate and. So, you keep on repeating something this steps and you will go till this delta X value goes very negligible or close to 0. The moment you get this thing what will you do next, What will you do then?

So, that will be your final values of this one because, when they are 0 even you keep on updating these values the latest value of X 0 it will not be updated because, all the corrections delta X are there and there you say that I got my final value of X 0 and that is a corrected value. So, this is the way you find out the parameters of space resection.

Similarly, now you can do the space intersection also, the way you have done the intersection in the planimetric coordinates in basic surveying. So, let us look into that now.

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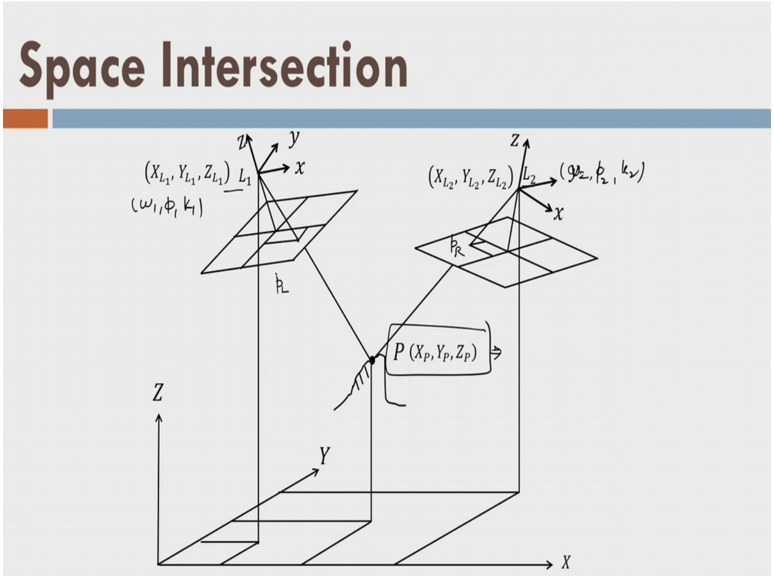

Yes, in case of space intersection what do I want to find out. I know; what are the interior orientation parameter for a camera, I know what is the exterior orientation parameter for a image like this. So, there are 2 images, I know they are exterior orientation parameter which means I also know omega 1, phi 1, kappa 1, I also know here omega 2, phi 2, kappa 2 now but I do not know coordinates of point P and I want to find out the coordinates of point P.

So, what will I do now? Situation is slightly different from space resection. In case of intersection I want to find out the coordinate of point P, knowing that I have the interior orientation as well as exterior orientation parameter of 2 images. And point P it is appearing in both images like this, say at this point call it maybe you can call it let us say p L left image and it is right image p R. So, what can you do again you can write for point p L and p R two equations, let us say that x left and y Left image you can write the collinearity equation and I am writing it here.

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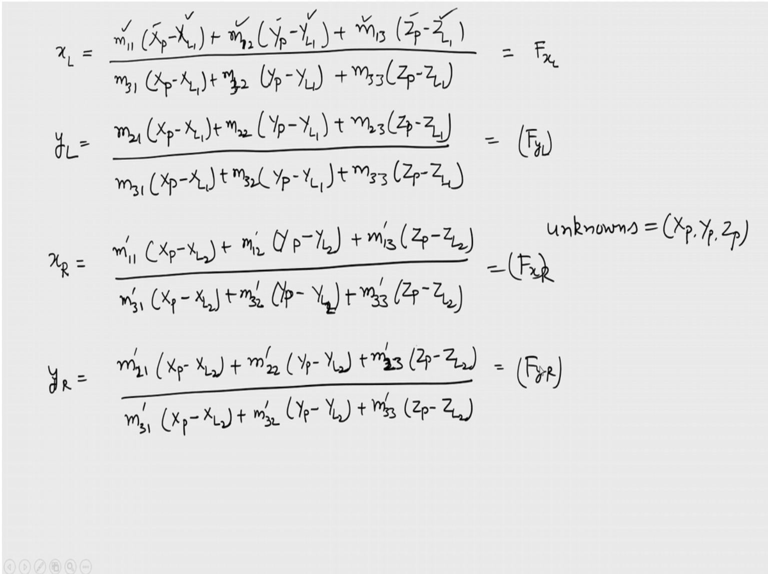

Let us see m 1 1 so, X P minus X L 1 plus m 1 2 Y P minus Y L 1 plus m 1 3 Z P minus Z L 1 divided by m 3 1 X P minus X L 1 plus m 3 2 into Y P minus Y L 1 plus m 3 3 Z P minus Z L 1.

Similarly, I can write for point m 2 1 X P minus X L 1 Y P minus Y L 1 m 2 3 Z P minus Z L 1 try to write yourself also m 3 1 this way ok. Now, for the right image I can write x R equals to here if you just observe these are known to you this thing and also the omega 1, phi 1 and kappa 1 which will come into this m 1 1, m 1 2 and m 1 3 are also known. So, I can say except X P, Y P and Z P everything is known in the right hand side.

Similarly, now I will write for right image and then I am writing it m dash why because they are different for 2 images. So, let us see X P minus X L 2 plus m 1 2 dash X I am sorry Y P minus Y L 2 m 1 3 dash Z P minus Z L 2 divided by m 3 1 dash X P minus X L 2 Y L 2. Both are Y P here, m 3 3 dash Z P minus Z L 2 here 2 ok, what about the y R? m 2 1 dash X P minus X L 2 now you can write it yourself very fast m 2 2 dash Y P minus Y L 2 plus m 3 m 2 3 dash Z P minus Z L 2 here right. So, these 4 equations you got.

Now, what are the unknowns here? X P, Y P, Z P and how can you find out using these 4 equations when these equations are non-linear? Secondly, here if you see that X P, Y P and Z P they are coming on the right hand side. Again I want to use the methods of least square or observation equation method. So, what will I do here F x L right similarly, F y

L here, F x R and are here F y R ok. I can write these 4 equations like that mathematically.

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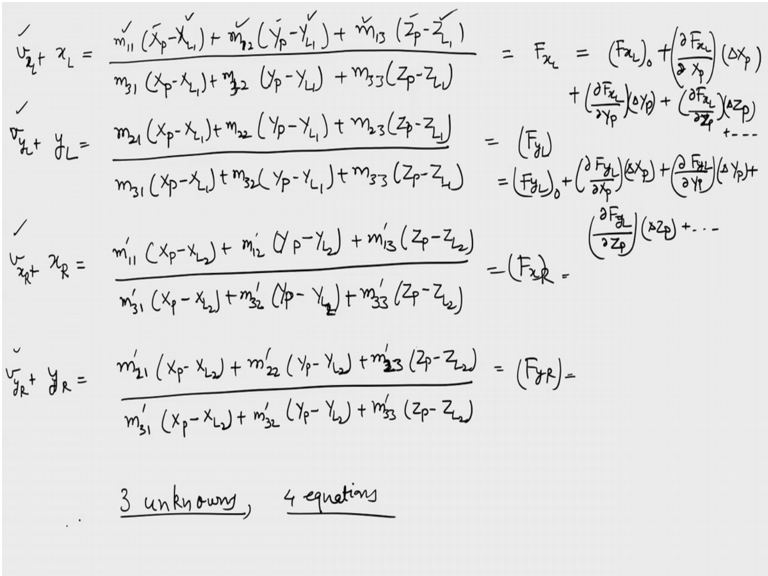

And now I can also write the linear expansion by Taylor series which is nothing, but F x L 0 plus d F x L divided by parameters here and parameters will be what? X P, Y P and Z P here. So, I can write here delta Y P plus d F X L dou by dou Z P here delta Z P right.

Similarly, I can write for F y L as equal to F y L at the known point 0 plus d F Y L by dx dou X P into delta X P plus dou F y Y L by dou Y P into delta Y P plus dou F Y L by dou Z P into delta Z P here and I ignore the rest of the term here right. So, I can write the same thing here also and so, here also ok. So, after writing this thing now let us introduce some errors here, V x L plus here, V y L plus this thing V x R plus V y R plus. So, these are my residuals because these equations are not going to be the exact because we have a lot of equations, they are not unique solutions.

Secondly, if you observe very carefully we are having 3 unknowns, but by just observing X P, Y P and in the 2 images I will have 4 equations and 3 unknowns. So, by default I have to go for the least square solution. Suppose, instead of that what if you have some more number of points for each point in the object coordinate system it could be P, it could be Q, it could be R, I have 2 set of collinearity equations in the image coordinate system 2 equations per image. So, total I have 4 coordinates like this, 4 equations for one point in this object space.

So, every time I can go for the least square solution and there well I will write it here.

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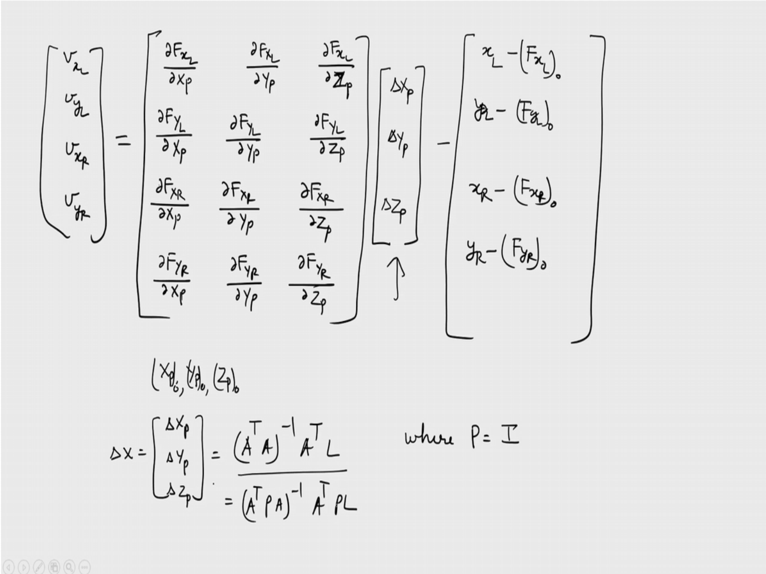

Let us say v x L, v y L, v x r and v x y r equals to I am using again the observation equation method here. dou F x L by dou X P then dou X L by dou Y P dou F x L by dou Z P. Here dou Y L by dou X P dou Y L by dou Y P dou Y L by dou Z P. Here dou x R by dou X P dou x R by dou Y P dou F x R by dou Z P by dou X P dou F y R by dou Y P and dou F y R by dou Z P.

So, this is my A matrix here into what are my unknowns here; delta X P, delta Y P, delta Z P minus now you can write from there that this will be once it is a minus and this is already available to us these values and this values. So, I can write here in the minus form that yes, X P minus F x L or this is my x L minus F x L initial value y L minus F y L 0.

Similarly, I have x R minus F x R naught like this ok. So, what is this? These are as corrections to the assumed value of X P, Y P, Z P. So, I call it that say X P 0, Y P 0, Z P 0 right. So, you make the corrections, calculate the corrections by again formula delta X equals to which is nothing, but delta X P, delta Y P and delta Z P equals to A T A inverse A T L, where we assume that P matrix weight matrix is identity matrix otherwise you can also assume the weights.

So, the weights will be again what size you can find out now it should be of what size try to find out yourself use the dimension here or you can also right here like this A T P A inverse A T P L right. So, that not to write I am not writing this form to you that is also right.

Now, you find out the corrections delta X, delta P and delta Z P.

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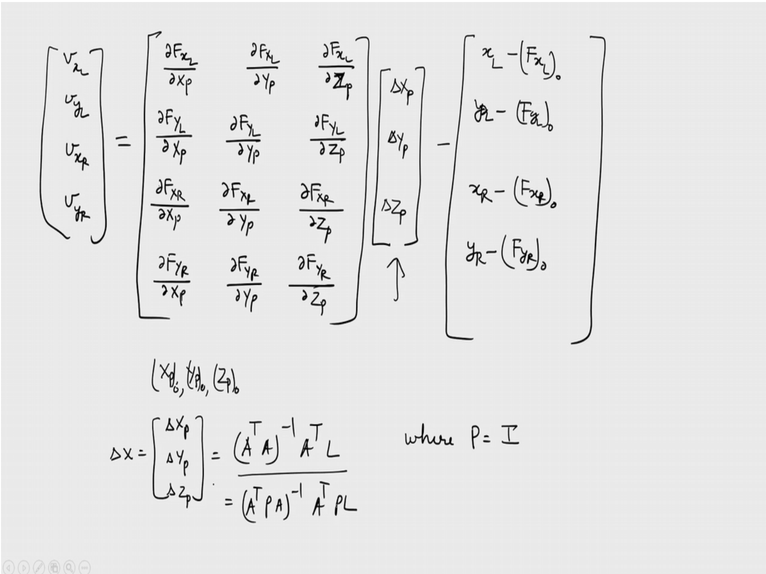

You will update your this thing like this; that means, new estimate of this one will be X P 0 plus. So, let us say I am updating it comma Y P 0 equal to the earlier vary of Y P 0 plus these values calculated comma Z P 0 equals to Z P 0 plus delta Z P. So, these are the updated values by adding these corrections.

So, we add this 3 corrections and you will get the updated values of the initial estimate. Again use the updated initial estimate in the same set up and try to find out the new corrections. Again add the new corrections and you repeat this process continuously every time. The moment you start getting these corrections very very small delta X values you will say that let us terminate the process and say that whatever values are the most updated values of the X P, Y P and Z P are you final values.

And this is the way you calculate the unknown point coordinates P, capital P in the object coordinate system. So, this is the process called space intersection ok.

Now, let us consider the aerial triangulation, what is aerial triangulation? If I combined the space intersection and space resection together in one module I call it aerial triangulation; that means, I will first estimate the exterior orientation parameter of all the images. Then I will find out the coordinates of unknown points by space intersection. So, this is the way we call it aerial triangulation.

So, now, aerial triangulation is also clear to you ok, here we have assumed that in the aerial triangulation, space intersection and space resection the interior orientation parameters are known to me. In fact, the interior orientation parameters can also be assumed unknown what will happen then; right.

Moreover, I would like to say some more things about the whole process and that are more important to understand now. We have understood the basic things like what is space resection, what is space intersection and how to use the least square solution or the observation equation method in order to find out the unknowns. In case of space resection, I have exterior orientation parameters as unknown. In case of space intersection the 3-D coordinates of point P or maybe other any other point as unknown in object coordinate system. So, I can use the 2 processes to find out any unknown value.

However, then I realize that aerial triangulation is a process where I can combine 2 things. So, that is our background now, let us imagine that you have a point P.

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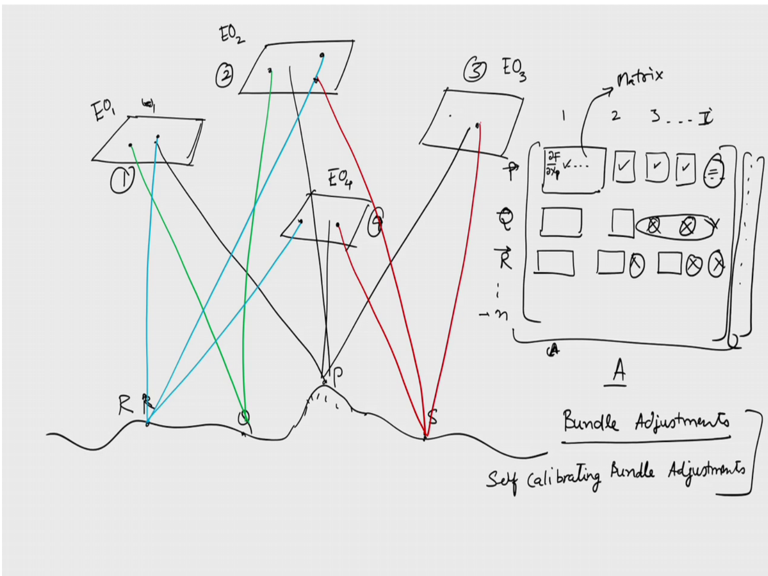

Which is like this given the terrain right, it is imaged in many many images not 2 images let us say like this let say there it is imaged in 4 images and with the same camera like this, like this, like this and like this. So, let us say image number 1, image number 2, image number 3, image number 4.

Similarly, if on the terrain if I have many many points let us say point Q here, let us say point R here and let us say point Z here or point S here. And in this image if I can say this point S is imaged here, it is imaged here, it is imaged here ok. Let us see that this point Q it is imaged in this image, in this image only and 2 images only point R is imaged in let us see this image, this image and let us say this image also ok.

Is it very complicated situation? No, this is not very complicated it is only the is appearing complicated, but now what will you do for point R here you can write the collinearity equation for this point, for this point and for this point. So, total you have 6 collinearity equation for point R.

Similarly, for point Q you have 2 collinearity equation from here and 4 here; that means, you have total 4 collinearity equation for point Q, for point P. Now, it is imaged in all 4 images. So, you have total 8 equations for point P and then you have for point S you have 1 and 2 and 3, for this point, for this point and this point. Each point gives you 2 equations so, you have total 6 equations I hope you got the idea.

Now, again you can build up your least square solution, if you find out let us say space resection in that case you will say this is my EO parameters of image 1, EO parameters of image 2, EO parameters of image 3 and EO parameters for image 4. So, now, you can see let us say there are n number points like P, Q, R, S, I have instead of 4 points n number of points we have. And as the result I have will use those n number of points in different images to develop my whole system of equations. In that thing you will find out that for this image for example, if I take the point P, it is appearing in all 4 images that means.

If I draw the big matrix in the least square sense then the matrix A will be very very big and it will indicate one portion for let us say if this my image is 1, 2, 3 and let us say I number of images I have and here I have number of points say 1, 2, 3 and n number point here.

So, I am writing this thing so, what will happen here let us say point p which is here it is appearing image 1, image 2, image 3 and that is it let say image 4, but all 0. So, what will happen? Here my all this differential coefficients will come here. Let us say like this or maybe I can say d X 1 and so, on right. Similarly, it will also come here it will also come here it will also come the complete matrix in case of space resection similarly intersection also now.

So, we have this thing, but here every matrix will become 0 let us say point O what will happen? Point Q is appearing only in 2 images so, 1 and 2. So, for point Q here one set of matrix will come here will come, but complete this are 0s nothing here ok. Therefore point R, I can write it is appearing in point 3 images 1, 2 and 4. So, it will come here 2 here not there and there and not anywhere else.

So, such a set of matrices we will have, this is itself is a complete matrix and this is B A matrix which is complete this matrix is very very big matrix here. And now we can see that there are matrices locations where it is 0, here it is 0 0, complete matrix is 0 right.

So, in such a case I would like to save my memory instead storing with the values 0 and such a process where we are try to become memory efficient by considering this 0s. So, what can I do here? We will do some kind of matrix partitioning and then we solve the whole system where we find out all the unknowns together the list of unknowns will be very high; that means, if I consider all the 4 images. So, each image has exterior orientation parameter 6 parameters. So, maybe 6 into 4 24 in that case of space resection I am finding out all the 24 parameters together here right.

And that is why it is very very important to understand that the process we follow here a numerical process that partitions the matrices and finally, it calculates all the parameters in case of space resection or in case of space intersection or in case of aerial triangulation; that means, aerial triangulation is combination of space resection and intersection together. So, such a process where we are becoming numerically efficient process it is call bundle adjustment.

Where it can consider any number of images and any number of points in those images where it is not necessary that each and every point is appearing in every image. No, each and every point will appear in couple of images. And finally, we use that and generally this approaches bundle adjustment is used in the software writing. So, whichever software you are using whether it could be free software of photogrammetry or it is a paid software or commercial software of photogrammetry they are using bundle adjustment.

Now, at this stage we have assumed that we know the interior orientation parameters. Let us think that we do not know the interior orientation parameters. In that case they also become in unknowns and such a system where we use the bundle adjustment where we include interior orientation parameters as unknowns. We call it, self calibrating bundle adjustment.

I hope you got the concept of bundle adjustment and self calibrating bundle adjustment. They are not very important if you are not writing a software or if you are not a software engineer in the photogrammetric world. But still, they are good concept to understand you will find this in the books in detail.

Now, let us consider there are few more things are there.

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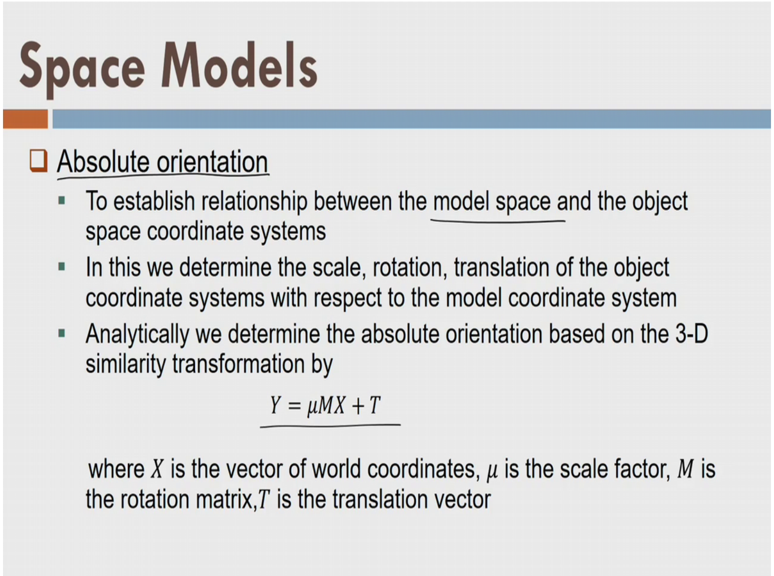

One is absolute orientation; that means, using bundle adjustment or using space resection and intersection. We have calculated the unknown coordinate parameters; that is X P, Y P, Z P in object coordinate system. And now you are ready with the lot of 3-D coordinates you can develop a 3-D model.

So, let us see that you have a 3-D model using 2 images only and another set of image is there; which is give you another model 3-D model and you want to orient these 2 models together.

So, what will you do? And that is called the absolute orientation. That establish the relationship between the model space and object space coordinate systems different different systems we have. So, if you bring everything all the models together 3-D models together and then orient themselves together each other; then that is called the absolute orientation right.

So, generally we use a kind of 7 parameter transformation here that means one 3-D model, another 3-D model and bring them into the same coordinate system object coordinate system together right.

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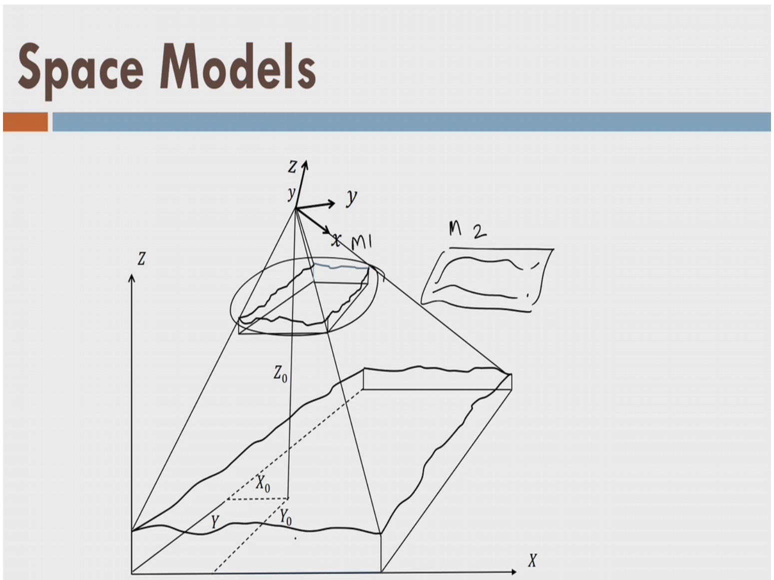

So, it is specified here like this, this is a 3D model here shown here and I am bringing into this coordinate system there could be another 3D model here like this and I am bringing model 2 and model 1 both into this coordinate system capital X, Y, Z. So, that is what we call absolute orientation.

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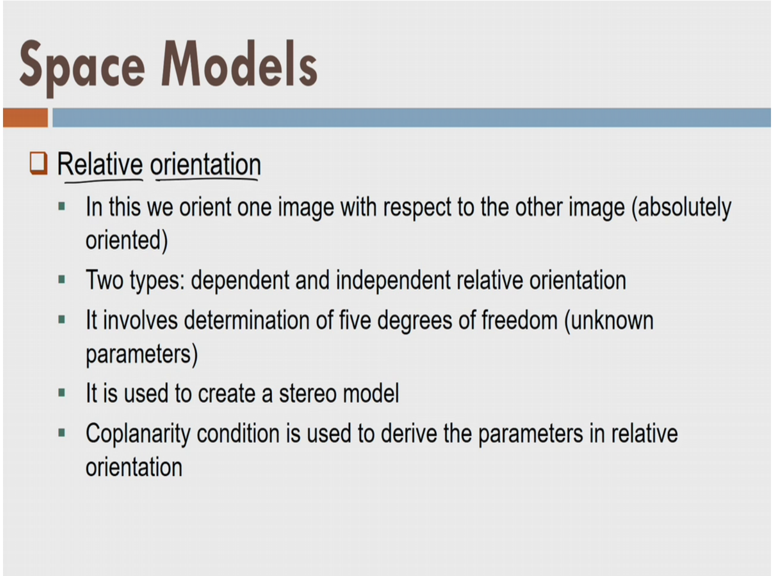

Now, let us think of relative orientation now just imagine that you have 2 models like this.

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You have 2 images only, in 2 images 2 points are coming or let us say 1 point is coming and you want to bring them together, you want to orient them together.

So, what will you do? You will take one image and you will try to orient another image like this, making this is as a reference and you are orienting it relatively. So, let us see you are oriented like this and such that it is started creating you the model 3-D model, now we can see it so, that is called the relative orientation. Relative orientation can be done 2 types.

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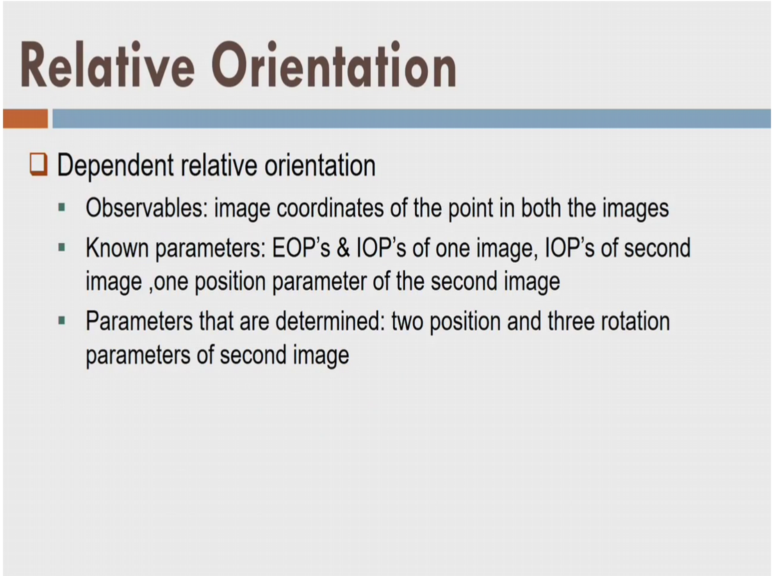

First is dependent relative orientation, means all the one is becomes my reference and another becomes my target or slave and that is now moving like this. So, that is called dependent relative orientation, where we have aligned one image with respect to another image and then we have created the 3-D model.

Apart from that let us imagine another situation that we are bringing all 2 images both 2 images into one system object coordinate system some like this try to see. So, the moment I bring it I am moving all 2, the moment I bring it them together it becomes my independent relative orientation, I am moving them independently. So, that is shown here.

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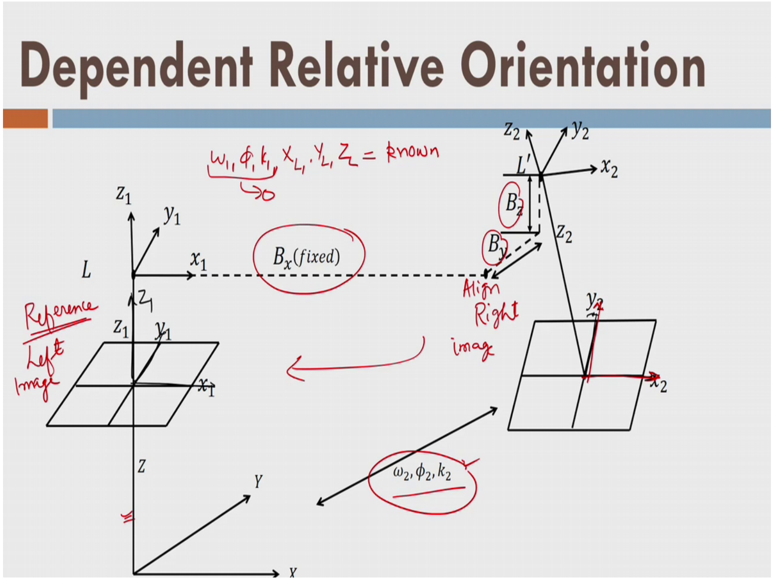

Let us see one system here dependent relative orientation. So, I have assumed that this is my known. So, let us say this is my x 1, y 1, z 1 that is my y 1, this is x 1 here and this is my z 1, here right ok. Similarly, I have x 2, y 2 here for another image like this and these are my exposure stations.

Now, so, I am writing like this, and let us see there is a base which into which is fixed and which is known to me because I am saying that the reference system for this which is aligned with the first image or the left image is known to me or rather I am saying that the left image itself is my reference right.

And so, align the right image ok. So, what will happen I know that only omega 2, phi 2, kappa 2 are my unknowns. Why because one more thing this B x and B y they are also unknown to me. And I have reduced the number of unknowns why because we assume that this is known to me. Moreover, whatever omega 1 phi 1 kappa 1 and other parameters like say X L 1, Y L and Z L are all known or here this say rotation parameters are 0.

So, now, we need to find out these 3 relatively. So, the one moment I find out these relatively I can orient my 2 images and I can create the 3D view and that is what we call dependent relative orientation.

Similarly, if I do the assume that there is the left image itself is not reference image, we have some reference coordinate system which is slightly away or different then I can bring all the 2 images together in that coordinate system like this. We are moving them together and finally, I can do the relative orientation independently also. So, that is called independent relative orientation.

So, here in this lecture we have learned how to use the computational photogrammetry to do the 3-D development right and especially we learn space resection, space intersection, aerial triangulation. And then we said that if we do some kind of real numerical process we are serious for that we should try to save our memory in a computer and then we realized; what is the bundle adjustment. Well, bundle adjustment is very important for the software writes especially. Or if you are planning to write your own software you should always think for bundle adjustment. It is very clearly explained in the books right.

Then we said that if I have interior orientation parameters as unknown which we assume as known in case of resection, intersection and aerial triangulation as well as in the bundle adjustment as well, but if I feel that these are also unknown I should go for the self calibrating bundle adjustment. Remember, in case of space resection and intersection also I can assume then unknown also right, but that is not self calibrating bundle adjustment. Self calibrating bundle adjustment is a numerical process which is slightly better than the bundle adjustment or slightly advanced right. Again these are things.

Now, then we realized what is my relative orientation; that means, I do not want to do bring my object coordinate system into place. I just want to do some kind of only a relative orientation or the assuming that one is image is a reference another is a target or slave and I am trying to orient to create my 3-D model like this.

So, we defined a term called relative orientation fine, then we also define the absolute orientation, there where we want to bring 3D models which are individual in nature which are prepared from couple of images this model this model this model and so, on. So, I am bringing every model in one object coordinate system then we call it absolute orientation.

So, we have learnt all the computational processes so far that is very very popular in the photogrammetry industry. So, here we stop and in the next lecture that will be on the photogrammetric products, we will talk about the how to generate digital elevation model because, in the space intersection process we have created the 3-D points.

Now, we want to develop some kind of maps and some kind of 3-D maps using the 3-D coordinates that we calculate in the space intersection or may in the bundle adjustment or may in self calibrating bundle adjustment. So, assume here that you have developed the 3-D coordinates. In the next lecture we will talk about all the products which are possible using photogrammetry. So, thank you till then bye.

Thank you very much.