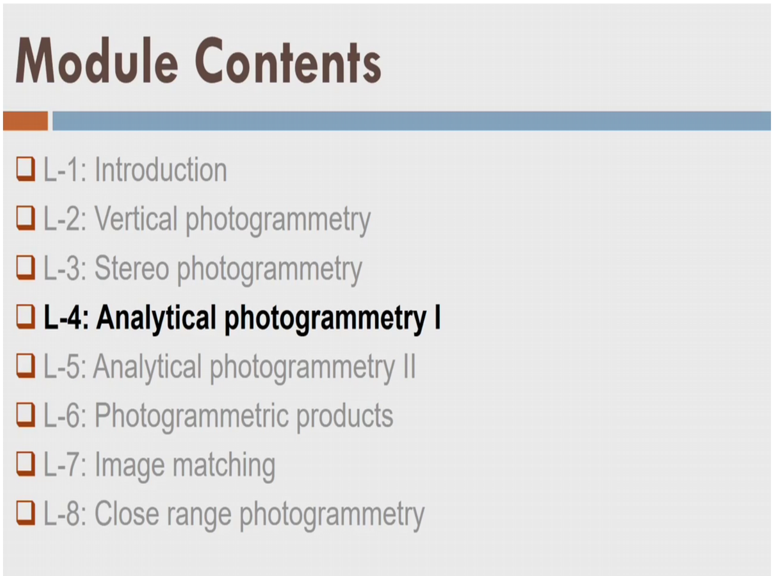
Higher Surveying
Dr. Ajay Dashora
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module - 6
Lecture - 18
Photogrammetry

Hello everyone, welcome back on the course of Higher Surveying and we are in the module 6 of Photogrammetry.

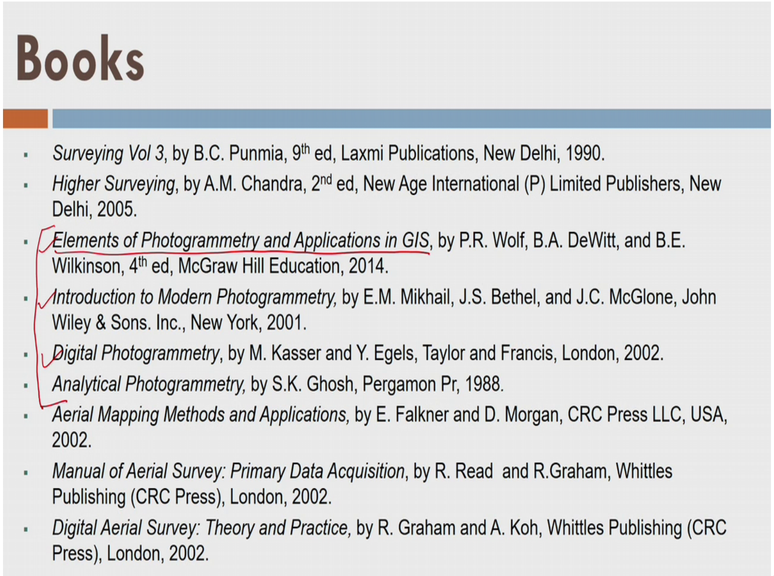
(Refer Slide Time: 00:37)



Today we are going to discuss about analytical photogrammetry and the next lecture we will also be on the analytical photogrammetry. So, we have named our two lectures as, Analytical photogrammetry 1 for today and for the next turn we have the Analytical photogrammetry 2 ok.

So, as we already discussed in the last lecture, that analytical photogrammetry is all about the mathematical treatment of the photogrammetric problems. So, in order to develop that kind of skill or that kind of attitude to understand a photogrammetric from mathematical perspective, let us, first develop the fundamentals of that. And so, we are saying that in the first lecture of Analytical photogrammetry we will go to the fundamentals and we will try to understand that how these fundamentals are made using simple mathematics. So, let us start with that.

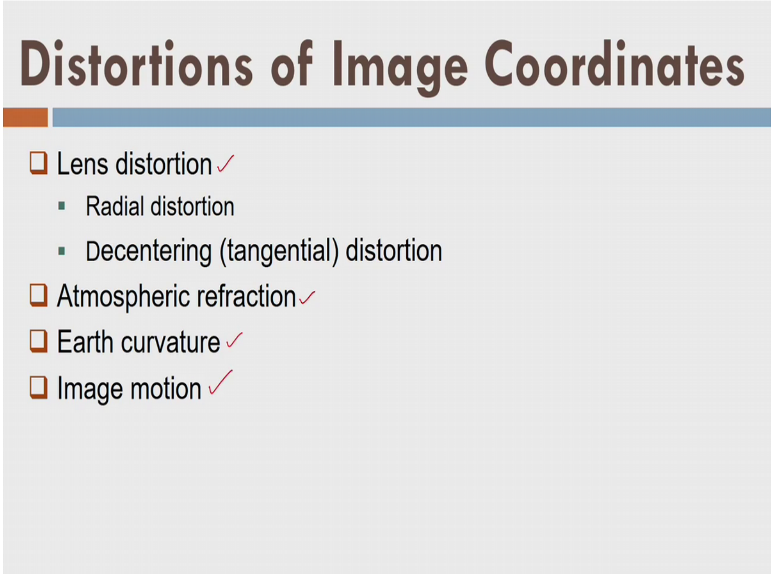
(Refer Slide Time: 01:35)



So, these are the books here and I would like to request you that here our first two books are not going to work rather we have to look for these books here. So, ,1, 2, 3 rights; in the first book in this three books, the first book that is Elements of Photogrammetry and Applications in GIS, by P.R. Wolf and Dewitt and Wilkinson. It is now available in India at low price. So, you can easily buy that book; however, for remaining two books you should join some library maybe a public library or institute library.

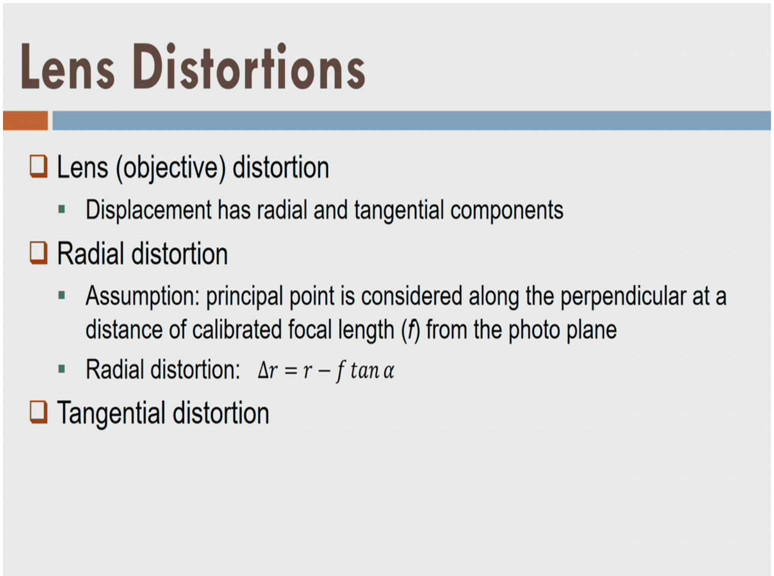
So, before we start a proper mathematical treatment to the photogrammetric problems let us, first, understand what are the systematic errors are there in the acquired photographs. In the last lecture we have learnt that how to acquire a stereoscopic photograph or stereo photos using airborne data acquisition and now, we are saying a still with our all cautions with our all preparation still there are some errors which are systematic in nature. And our images should be first corrected for those systematic errors.

(Refer Slide Time: 02:51)



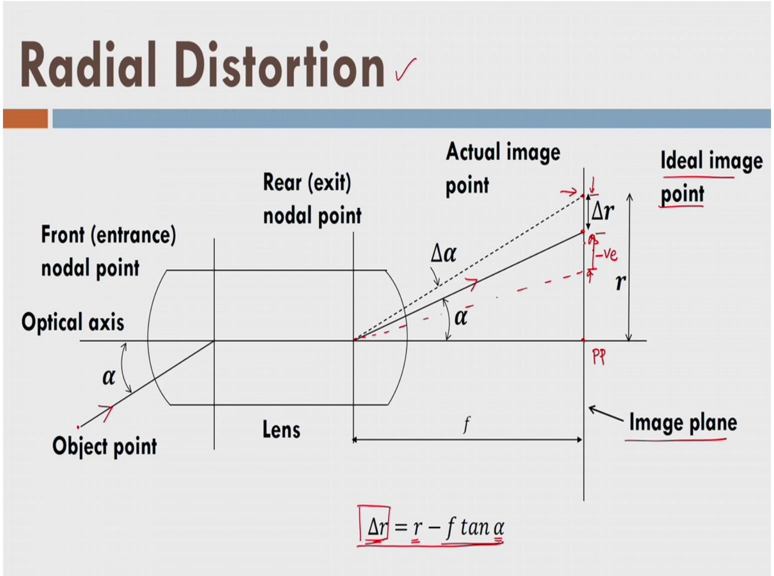
So, these systematic errors are Lens distortion, Atmospheric refraction, Earth curvature correction and the Image motion. We will go one by one and try to understand what are the importance of each error or ehat is the importance of each error, when to apply and how to apply and how to correct the image coordinates.

(Refer Slide Time: 03:13)



So, Lens distortion is also called the objective distortion and it has a Radial and Tangential component. The radial distortion as the name suggest it is Radial from the principal point ok, and the tangential is the tangent to the principal point so, let us look into one by one.

(Refer Slide Time: 03:33)

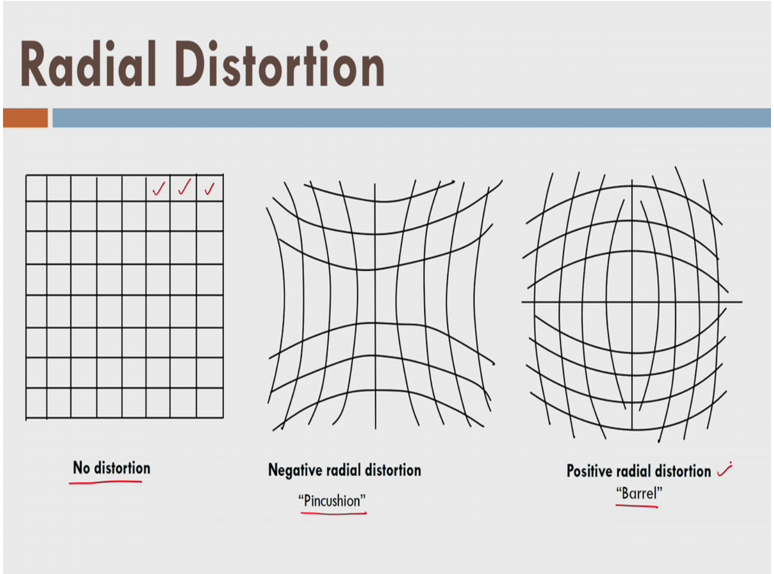


First, go for the Radial Distortion, what is meaning here? Let us say, some where we have object point in this direction and a ray is coming this way, and it is passing through the lens and ideally it should go to this point here. So, this is my Ideal image point; however, because of some defect in the Lens what will happen? The ray will go to this point here and that is my, delta alpha that is distortion and I call it the radial distortion. In fact, that is just an distortion in angle, but this is to, look at carefully, this is the distortion here; the point has shifted from ideal location to the this location here.

So, we call it the Actual image point here or rather I can measure this Actual image point in the image. So, this is the distance r, which is Radial distance from this principal point I can write it principal point here, imagine that this is your Image plane right, this is Lens and. So, you have this r as a Radial distance from principal point and this is your distortion called delta r and delta r is given by r minus f tangent alpha as it is evident from the simple mathematics of the figure ok.

So, that is my radial distortion. So, once I know the alpha that I can always calculate with the help of r, what will I do? I will measure the r and I will apply this correction f tangent alpha. So, I will get this delta r; that means, I need to correct my image coordinates by delta r. So, by applying delta r correction to the image coordinates what will happen? My image points will be corrected. So, I have remove the Radial distortion by this way right. It can be negative also, it can be positive also which means, this is right now, positive delta r negative means it could be like this. So, that will be a negative correction here, that is negative delta r right?

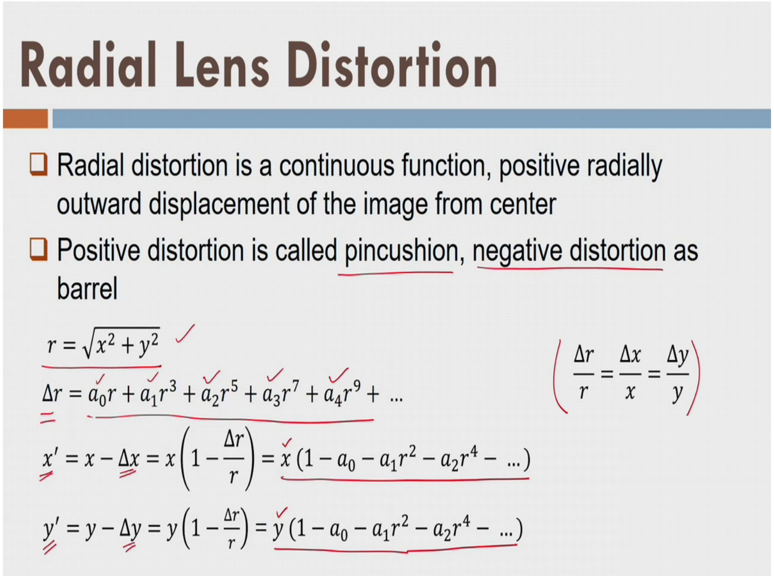
(Refer Slide Time: 05:47)



So, now, if there is no distortion in the image. So, I have so many pixels this one pixel second pixel and so on. So, these pixels are the ideal locations; however, because of the Negative radial distortion, I have these lines which are straight in the without distortion they are appearing as a curved line. You can imagine if you have a cushion at your home and you try to push that cushion you see, there is a pattern of line visible on the cushion. So, this kind of pattern only and that is why, we call it the pincushion as if, I am trying to pick or trying to prick a pin onto the cushion. So, some kind of pattern is generated and that is this pattern that is why it is called Pincushion or Negative radial distortion.

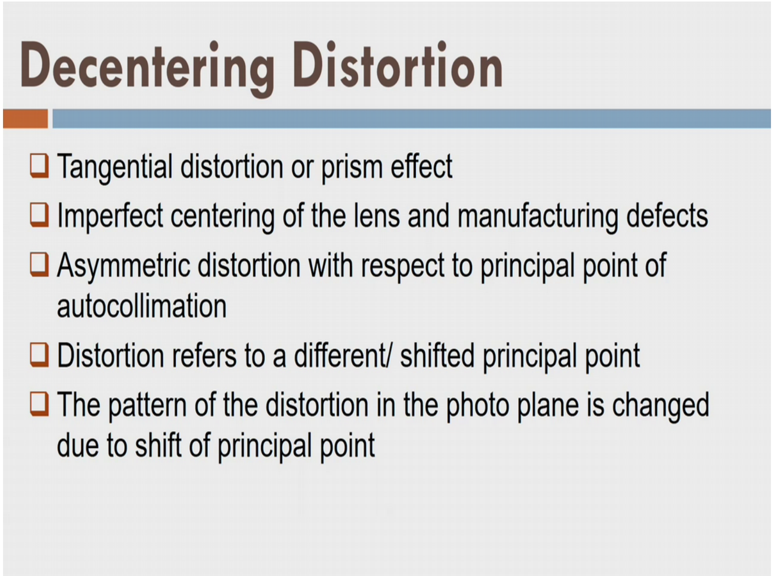
Similarly, what about the Positive one; because of the positive radial distortion what will happen? The straight lines will appear bulging out like a barrel and that is why we called it Barrel. So, this is my positive radial distortion here ok.

(Refer Slide Time: 06:49)



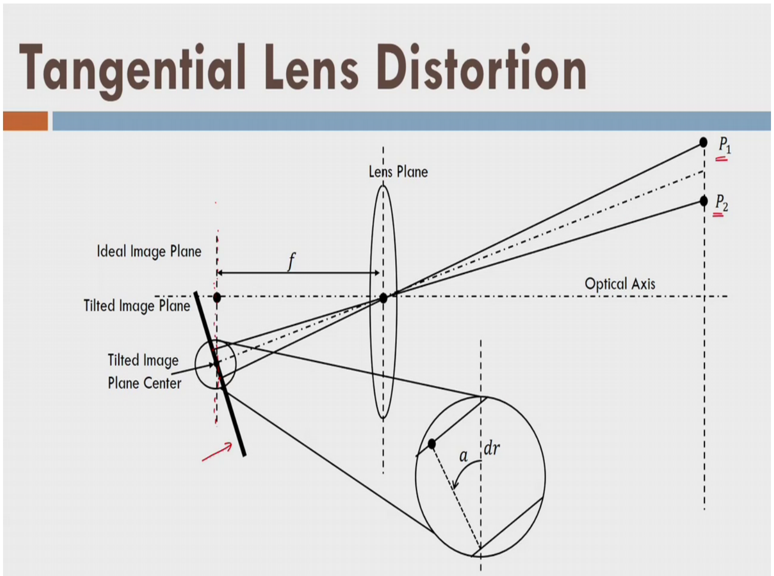
Now, the Radial distortion is a continuous function, positive, radially outward ok. So, it is positive distortion is called pincushion as we already discussed and negative distortion as barrel. This is my location of point where, I measure from the image and then the correction is given by this model where all these are some known or may be determined parameters ok. So, I this is my new coordinate or the corrected coordinates like this where these are connection and since I assumed this model and this is another condition I assume on this I can write this as my, corrected model; that means, this is the measured one, two values and if, I apply all this a 0 a 1 and the r is already measured. So, I can find out what is my x dash. So, that is a idea here.

(Refer Slide Time: 07:47)



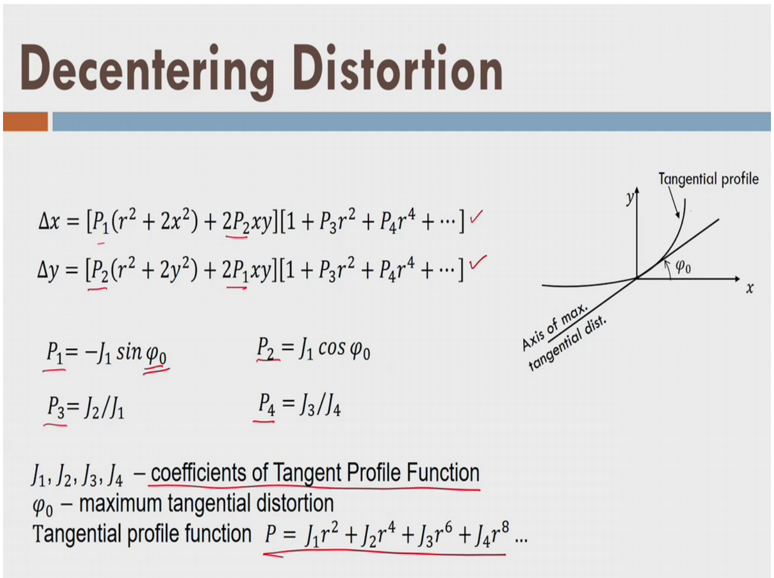
Go to the let us, go to the decentering distortion and it is also called the Tangential or prism effect. So, it is because of the imperfect centering of lens and manufacturing defects this correction is there and this correction is Asymmetric distortion with respect to the principal point; rather this distortion shift the principal point or if, principal point is already shifted this will further change it pattern of distortion. So, that is the decentering distortion. So, in spite of this theoretical description, look into the, what is the meaning here.

(Refer Slide Time: 08:23)



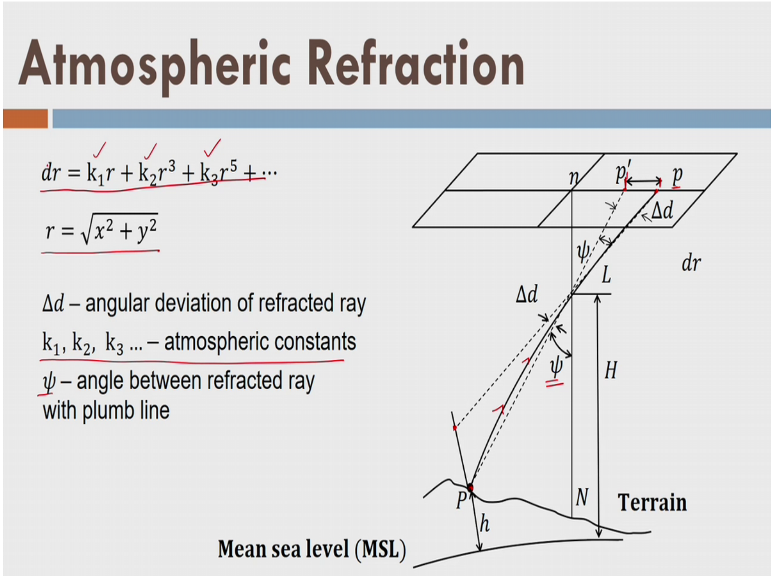
See, this is the ideal image Plane which is kind of vertical here it should be vertical here like this dotted line and now this is the Tilted Image Plane because of tilt your principal point has shifted because principal point is already having some error and it has shifted. And as a result I can see if, P 1 is a point here or P 2 is a point here, it will be image somewhere at different location here or here. So, what is the meaning here? Let us say, rays are coming this way they should be intercepted here; however, if image plane is like this, they will disturbed they will reach somewhere else like this and that is what we call Tangential Lens Distortion.

(Refer Slide Time: 09:05)



And this distortion is given by this formula delta x and delta y where, P 1 they are all coefficients P 2, P 1 here P 2 here and they are given by this one. So, if you see J 1, J 2, J 3 and J 4 they are coefficients of the Tangential Profile Function and this Profile Function is given by this formula and here this Tangential Profile Function is drawn here. So, now, if I draw a line to that; so, it is a kind of Tangent here. So, and this is angle phi 0 which we have here. So, we are doing some kind of approximation here and we are applying this correction. Well, this approximation is fair enough for our requirement.

(Refer Slide Time: 09:47)

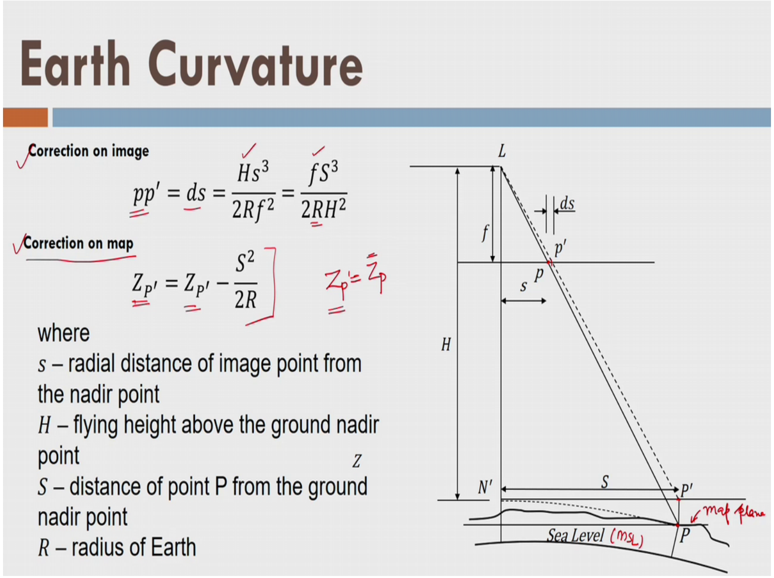


Now, coming to the Atmospheric Refraction if, you look and we have already talked in basic surveying that atmospheric refraction happens for a large distances; that means, when an aircraft flies carrying a camera and then it is acquiring one photograph like this what will happen if there is a point P here shown here that will be you know because of the refraction the rays will go like this and it will create an impression here or the image of point P will be here at P.

Now, you see ideally this point should follow this dotted trajectory and it should come on the P dash here. As a result I call that this is the distortion here from this point to this point shown by the arrow here ok. Now, what is the effect of that? The effect is there if, I draw a tangent line over there on this curved line which is my line of sight what will happen this will touch somewhere here. So, the point that should be here, it is appearing here that is a effect of that; that means, in the height the elevation of point P is increased because of the atmospheric refraction and that is somehow not desired.

So, what to do? Let us see, this r distance on the image of a point and. So, this is a Mean sea level. So, I am measuring the height with respect to that. So, this correction is given by this value k 1 r, k 2 r 3 plus k 3 r 5 and so on. We can increase the order they are called atmospheric k 1, k 2 constants and phi is the angle between the refracted ray with plumb line this angle you see here. So, we can now, calculate if I know the values of k 1, k 2 and k 3. So, values of d r and I can apply to each and every point fine.

(Refer Slide Time: 11:41)



So, let us look into the Earth Curvature; yes, Earth what is the Earth Curvature Correction? If you remember if you are using the map in the module-2 lecture 1, we say what we used to do or for surveyors before when we do not have the GPS with us. We used to do a kind of pseudo 3D survey, where we used to say that we are measuring the heights with respect to the Mean sea level, which is a kind of curved surface or equipotential surface and these heights are superimposed on a plane in order to indicate the height of a point. And this is what is done here and as a result let us see, the point P is located on Terrain here and this is your sea Level and what I calls as MSL.

So, let us see, this another point here and because of the Earth Curvature what will happen and if, we are using a map then it will happen basically now if you are using a DGPS or GPS or any system which is taking care of earth curvature this correction need not to apply, but; however, if you are still using a map remember that map is a plane over which we have superimposed the height which are perpendicular to the mean sea level not the map.

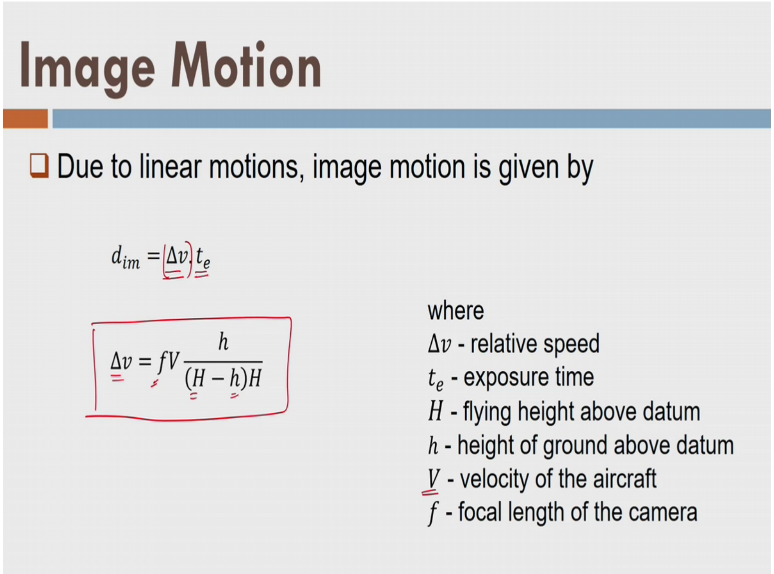
So, what happens here point p should be imaged here at p here ok, but if you suggest look at this is my flying height and point p will be projected here above the plane surface this is my map surface here map plane ok. So, the point P which is measured with respect to Sea Level is now projected to point P dash ok. So, what is the consequence? So, this is my map plane. So, this is the point p dash. Now, remember is it really happening or we are doing some kind of manipulation here just look into this thing carefully.

We have actually in the field we have acquired this point p only, but now, we are using the map in order to correct our image or in order to develop a 3D model and that is why instead of point p, we are saying that point p is p dash in the map and because of that if I use the map what will happen I will use point p dash instead of point p and as a result, I will commit some error over there which is equal to this and this is the error here ds and this ds error is called pp dash on image and it is given by this, this is a flying height focal length, S is this distance from N dash to p dash and R is my radius of Earth ok. So, that is the correction, we have to calculate and we have to correct our image.

So, on the other hand, we have another way, why can't we correct our map itself? Because, my p point is image correctly there is no problem with a point p as far as Earth Curvature is concerned, but only I am assigning p dash point instead of p point when I correct my image with the help of a map. So, let us correct our map or let us correct the elevation of point p dash and that is correction is given by S square by 2R. So, basically this Z p dash is nothing, but Z p only because we have measured in the field Z p and now, we are assigning to the point p dash or rather p point because we have assigned the p point above the map and it becomes p dash.

So, this is the correction we need to apply here. So, that I will correct the elevation of point p dash itself and then. So, either I will do this or do this either of the two, I can do and I will correct my image or maybe I can correct my map itself.

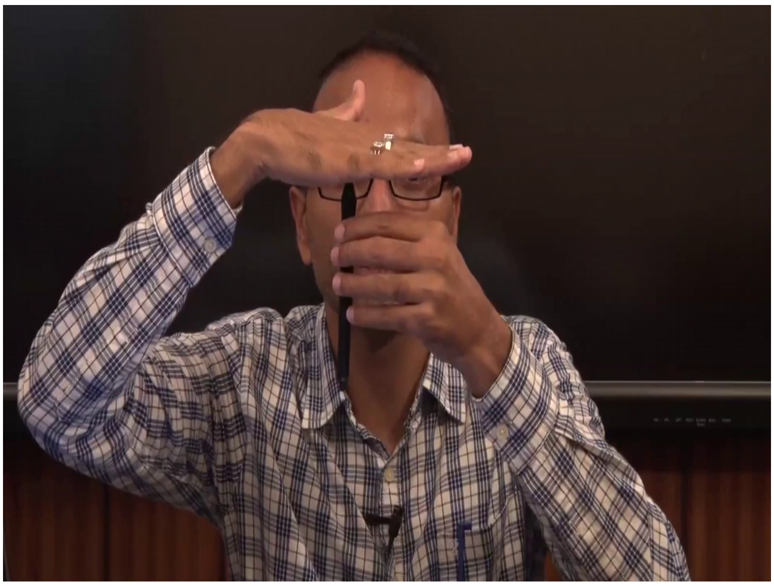
(Refer Slide Time: 15:43)



So, these are the basically, three correction we have talked about. So, it was the Earth Curvature. Now, let us imagine the another situation where, the aircraft is flying having a camera. So, camera exposes the scene or rather it clicks and the moment it clicks the scene is acquired fine ok. During this time which is in microsecond for which camera opens the shutter that is called the exposure time t e here.

What happens is because of the movement of the aircraft the point which should come here like this.

(Refer Slide Time: 16:17)



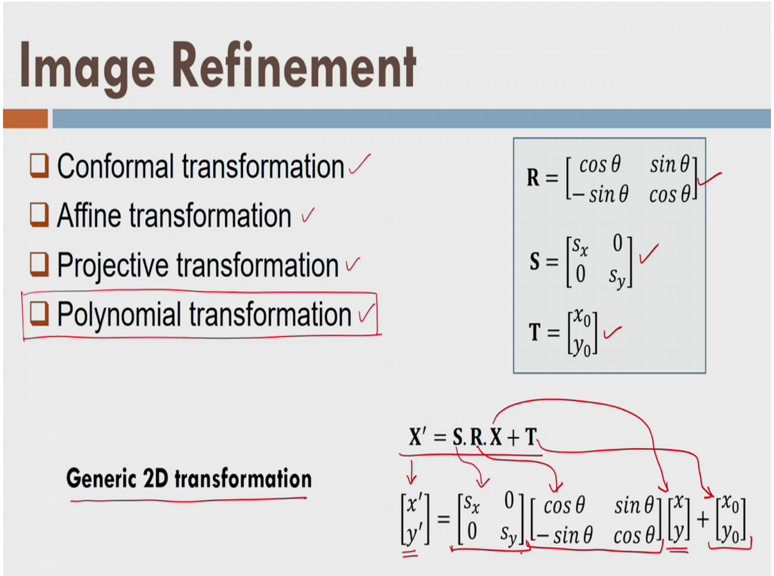
This is my image it should come here it will be shifted why because camera is moving this direction and this point which should come here it will come here again, imagine that point if it is aircraft is stationary it will come here. However, the point is moving or rather the image is moving like this and as a result this point by the time rays are reaching they will reach somewhere else.

So, there is a distortion opposite to the direction of flight and this distortion is called Image Motion or rather the image of a point is moving, it is in the motion and this is given by d m an where delta v is the motion or the motion of a point image point and it is given by this formula.

Where, f is focal length, V is a speed of the aircraft, H is my flying height and h is the height of a point which we are trying to image on the ground surface right ok. So, now, you can see if, I multiply this delta v with exposure time. I can find out what is the during that exposure time what is the motion of a point which is moving with this speed delta v and that is nothing, but the image motion.

Now, the days camera technology especially for digital aerial cameras is. So, excellent it is. So, self sustaining. So, self taking care that it corrects automatically this thing or even in the post processing when we acquire the image after that it will automatically correct this image motion. So, now, the days image motion is not that relevant from the user perspective. Yes, if you are in company which is working for acquisition of data you should consider this one or rather you should think of whether Image Motion is correct or not well.

(Refer Slide Time: 18:05)

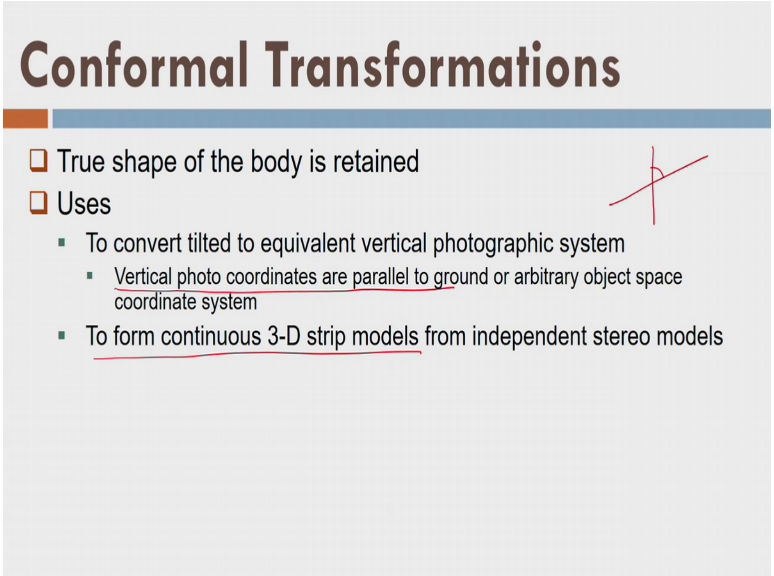


So, these are called the four systematic errors. After that let us, look into some mathematical error or let us, look into some mathematical models of Image Refinement. What is the meaning here? Systematic error I know that there are some errors which follow some pattern or which follow some mathematical rule. So, we have corrected it; however, there are some errors which always follow some pattern and those pattern are very much known to us and according to those pattern we try to correct our images if you look for such patterns first. So, if you find some pattern then we will immediately use a particular type of transformation and what we call as mathematical transformation.

So, these transformations are conformal transformation, affine transformation, projective transformation and polynomial transformation. So, there are some situations under which we use this thing ok, especially the polynomial transformation, something different; it is not used for the image refinement, but still, we will look into this thing fine ok. So, in order to learn this thing let us look into the generic 2D transformation which is written like this where, these are my corrected image coordinates this s is my scale R is my rotation, matrix T is my translation and X is my measured or the observed image coordinates.

That means I want to rectify my x and y image coordinates and I want to calculate x dash, y dash. So, I am using this scale it is very Generic one. Then, there is a rotation matrix here and this is a translation matrix. So, written here translation scale and rotation and it is happening in 2 dimension only. So, let us look into the Conformal first.

(Refer Slide Time: 19:57)

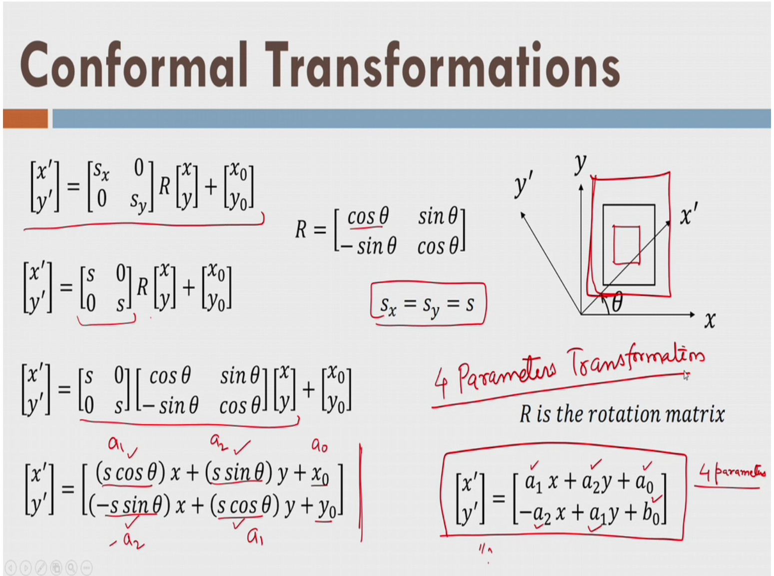


What is the Conformal Transformation if, you remember it from the our earlier, modules on this course. It is a shaped retaining there we talked about a mapped projection. So, Conformal Transformations of Conformal projections are reprojections which remain in the shape which you know preserve the shape; that means, the angle between the 2 lines like this and this is the angle say this angle will be preserved before and after transformation ok.

So, now let us say you have a tilted photograph and you want to bring it to the vertical photograph, yesterday we have seen some corrections. So, we can also do by Conformal Transformation because in fact, if you use analyse those equation carefully, they will appear to us as a conformal transformation only right Conformal is very easy to do in mathematically using a computational technique right.

We assume here the vertical photo coordinates are parallel to the ground or you can assume that they are parallel to this; that means, x y plane of vertical coordinate system is parallel to the ground surface. Now, we will also used conformal for this purpose that, we will look into the next lecture, for time being let us first, understand how to do the conformal transformation ok.

(Refer Slide Time: 21:07)

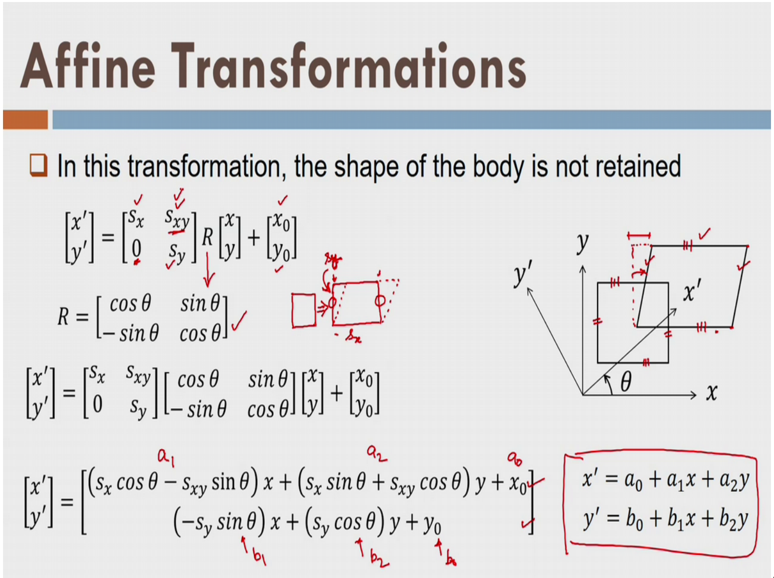


So, let us look into the basic equation again here and there I put s x equal to s y equal to s. So, what is the meaning here? The scale in the generic transformation when made equal in two directions it is called Conformal Transformation which means, rotation will definitely happen here by theta angle, but scale in two direction x and y will remain same; that means, this rectangle will rotate we see here that scale is same and as a result this rectangle can be enlarged like this, after transformation or it can be strength like this, but still the scale in the two direction is same; that means, enlargement factor.

If, you remember the aspect ratio lock aspect ratio and some of the softwares they right, which means the scale is same in both direction. So, this what is there same scale ok. Now, if I just put this value here and then if I put all this value over there what will I get here multiplication as, s cos theta into x, s sin theta x 0 s sin theta s cos theta and here. You can try this equation write yourself you will find the same equation you see here, s cos theta is there and this is also there.

Similarly, s sin theta is there and it is minus s sin theta. So, we know that, if I write here let us say a 1 and a 2 and let us say a 0 here. Then I can write here minus a 2 and a 1 and this is what we are writing here, and this is called the conformal transformation or you may find this equation in the books also saying that it is a Conformal Transformation. So, that is the source here, of this equation moreover what are the parameters a 1, a 2; a 1 and a 2 two parameters and a 0, b 0 which are translation parameter. So, total we have four parameters here. So, it is also called the 4 parameter transformation, right ok.

(Refer Slide Time: 23:33)



So, now, let us understand, what is the Affine Transformation? Ok. In this transformation as written on the screen the shape of the body is not retained. So, what does it mean? So, here you can see that, there is a rotation matrix here which is rotating the body uniformly by angle theta; that means, all the points of this rectangle shown on the right hand side will be rotating by angle theta alright. So, rectangle will still remain rectangle; however, if you look at here there is a scale factor s x and s y; that means, I have different scales in x and y direction what will happen here? A square may become a rectangle like this that means. So, x direction scale is s x and in y direction scale is s y here alright, this way.

So, now, you can see that in y direction the dimension has changed and the x direction also dimensions are changed. Now, because of this square has turned into the rectangle alright, one more factor you can see here the shear factor and what is that? Because of the shear it will happen that, some rotation can also happen or rather a particular length here we can see from this point to this point, it can deform or rather this lengths can be deform alright and because of that it may take a shape like this; that means, a rectangle can further become a parallelogram ok. So, that is what we understand by the Affine Transformation here we see that translation is also there ok.

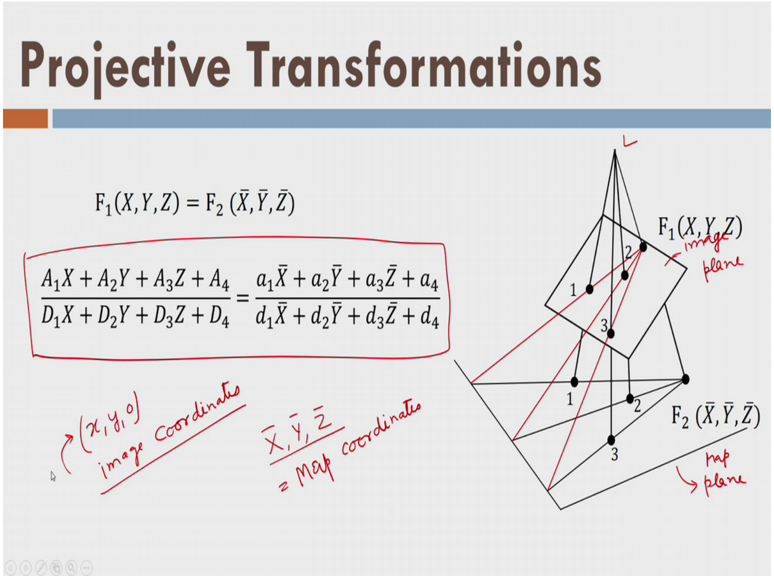
So, if I write this equation in this form and if I try to expand a matrix I will get these equations. So, x dash equal to complete this term here written like this and y dash equal to this term here and now, you can see here that this factor I call it a this factor, I call it let us say a 1 I call it a 2 and a 0.

So, this factor becomes the b 0 this becomes b 2 and this becomes b 1 here alright and. So, I can write this equation, in order to represent the Affine Transformation ok, what happens now? Let us see, this is my new axis x dash y dash like ok. So, this is the angle theta which is represented by rotation matrix theta here ok. So, that is still I will say that only theta rotation here and then this rectangle has become a parallelogram alright.

So, for changing this thing I can see here that that s x y has played a big role here right; because, now there is one more factor 0 here which means that s x y shear is happening only along one direction not in the other direction; that means, we can say that one of the base is fixed and other base is moving alright. So, if this is a base this point to this point is fixed all though scale is changing s x and s y is also changing, but here is whole happening only in the x direction and as a result, this two points are shifting in the x direction. And as a result, we are having some kind of shear here ok.

So, this is the amount of shear here from here to here ok, and this is angle between a either shear angle ok. So, you can see here that the beauty of Affine Transformation is parallel lines like this they still remain parallel ok, on the other hand this parallel lines I write three marks here, three marks here and they also parallel here ok, and that is the beauty of Affine Transformation and that is why, we have the zero shear here, but shear in one direction here right. So, that is the role of the s x y here ok.

(Refer Slide Time: 27:36)



Now, let us look slightly better thing here which is called Projective Transformation. So, what is the meaning here? Let us say, there are three points on an image plane; let us say 1, 2 and 3. Now say, there is a point exist here that connects all the three points by straight lines ok, this anther plane here. So, we are drawing three lines in this plane let us say, image plane this is my image plane here and let us say, this is my map plane fine.

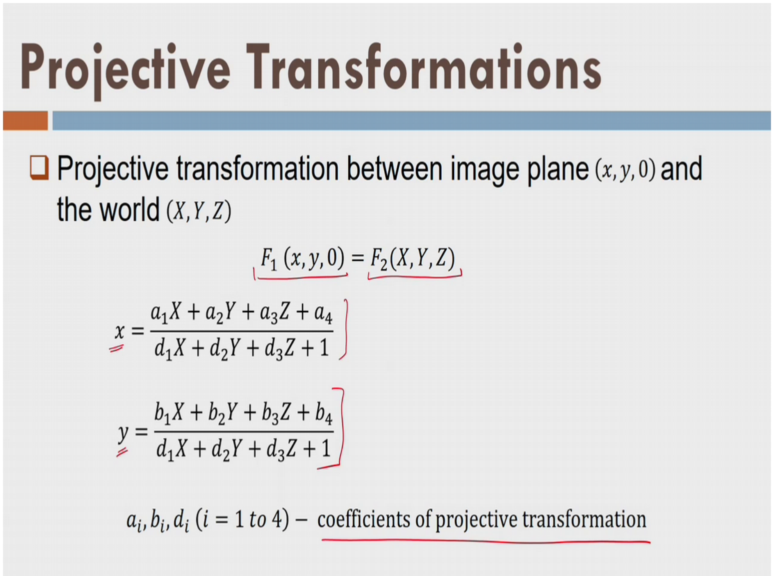
So, in case of image plane I have three points image on the point this one and then such that they are coming from one point here like this. So, now, these three points in the map plane, then they are connected to this point like this and these two points are such that, that let us say, there is one more point exist that is perspective centre and that is connecting these two points like this. All the three points are connected; that means, they are present in both the planes. So, one plane is image plane another plane is a map plane.

So, in the map you can see all the three points in the same, of the same area if I acquired the image three points are there. Now, I can do the Projective Transformation or the this is my perspective centre here, I will there ok, and there is one more relationship happens because this perspective centre will also connect to this point like this and that is the called the mathematics of projective transformation.

Now, I can write that, if there is function f 1 for image plane and there is a function f 2 for this one and if, I try to develop some Projective equations in 3 dimension I can write like this and there I can write like this. So, there is a one plane here. So, I am writing image plane fine. So, if it these are just generic equation, for a project transformation because generally we write for image plane like this x y 0, my image coordinates ok, what about capital X bar Y bar Z bar here or may be capital X Y Z they are called object or Map coordinates. So, let us see that you are trying to develop your image you are trying to row correlate your image with a map.

So, you find some common points which are having this coordinates and your image has this coordinates fine. So, this is the kind of Projective Transformation why because Image is one plane map is another plane.

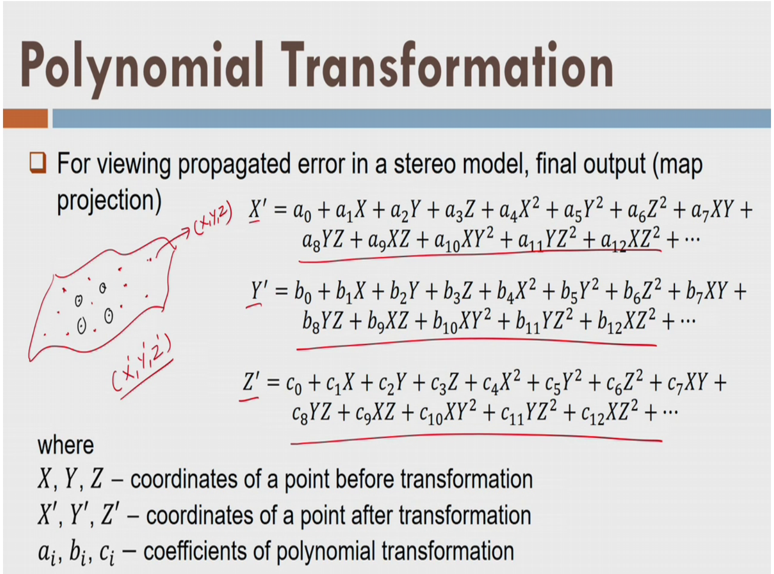
(Refer Slide Time: 30:25)



So, now, I can use this equation and I can write my Image coordinates like this. So, F 1 as instead of x y z I am writing x y 0 and my Map coordinates are X Y Z. So, I can write x equal to y equal to like this, this equations I can write easily; you can understand it, very simply. These are my all coefficients a 1 to all d; a, b, c, d all right. And they are called coefficients of the projective transformation.

So, in the next lecture, we will try to know how to find out this one if, I have common map points or I am having some image points and corresponding to those image points I have map points also right. So, how can I find out all the transformation or all the parameters of any transformation right? So, we will look into tomorrow, the simple process is there and we are going to use the method of least squares, that we already discussed in the last second to last module which was m 4 and that is the reason we have discussed all this thing before.

(Refer Slide Time: 31:31)



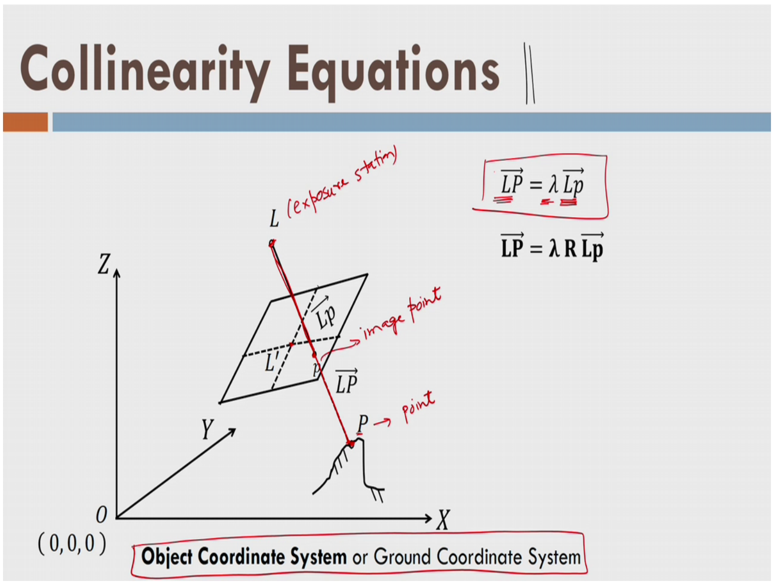
Now, Polynomial Transformation this transformation is no for it is not available for the Image rectification, but it is easily available or rather it is a 3D transformation between the two coordinate system which are both are in 3D. So, let us imagine that, you have developed some kind of a stereo model using stereo images and you want to bring that stereo model into the object coordinate system or some other coordinate system which is of object type. You know you want to bring your stereo model the local coordinate of the stereo model in an object coordinate system. So, what will you do now?

The simple idea is, we assume that any in the, x y z model of stereo I can develop some kind of polynomial model like this kind of surface, like this right, and there I will measure some points here in the stereo model, which is my say these points what I call xyz here, each point is x y z. Now, after transforming those stereo models into the object coordinate system; I will get transformed coordinates as x dash y dash z dash and. So, I am saying that, all this points I am fitting up Polynomial 3D Polynomial through this and these are written that, this Polynomials 1, 2 and 3. So, once I fit this three Polynomials for each coordinate differently. Then I can find out any new point.

For example, let us say, I want to find out these new points here, here, here, here. Which I do not know in the field, but now, after fitting these, Polynomial what will I do? I will do the interpellation and I will find out the coordinates of the new these new four points or may be multiple points. So, that is idea and it is again remember that polynomial transformation is not for the image refinement. So, Image refinement is generally done by the projective system, projective transformation, conformal transformation and affine transformation ok.

So, these are the four mathematical transformations we have learnt now, for Image rectification as well as for the model development or the model transformation ok. But, there is some physical equation also or there is some physical understanding also we can develop about our imaging process. For each and everywhere or each and every point on the ground surface that is image in the photograph. And now, let us try to look that from the perspective of a mathematician or from an observer perspective right.

(Refer Slide Time: 34:17)



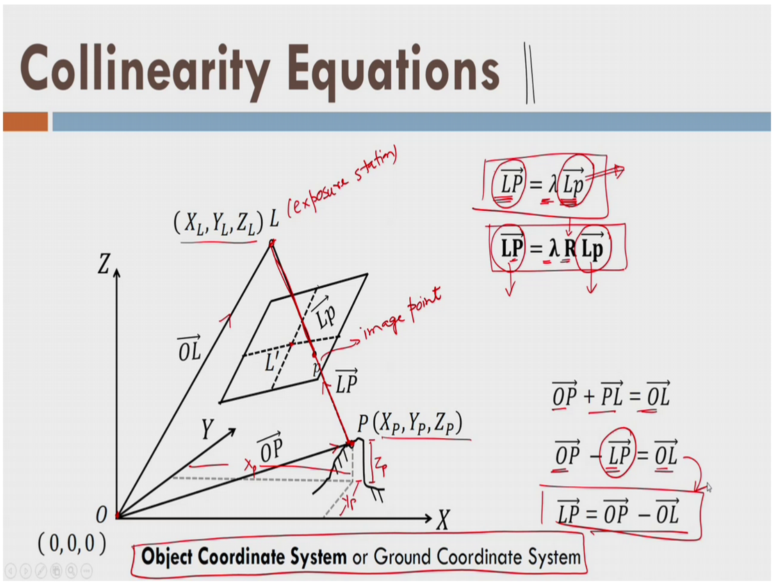
So, let us say, there is an exposure station L here. So, we are trying to understand now the Collinearity Equation here ok. So, there is a exposure station l here and there is a point p on the ground surface here, Now, there is a image and the L dash is the projection of l onto the image or rather L dash is nothing, but the principal point. So, assume that, we have acquired the image ideal image and we have corrected it for all the systematic error or maybe some non systematic error that that are mathematical errors by transformation.

So, now my I have an ideal image; that means, it does not have any distortion. So, as a result, I am saying that there should be some kind of mathematical relationship I can develop. So, let us see this point is image at point P like this as shown in the animation ok. So, this vector is my L P, this vector from here this point to this point this vector ok, what about the other vector? I have another vector from here this point L to point P this vector fine.

So, I can see here that three points this is my exposure station, point P here that is called image point, and this is the point on the ground or I can say point they are in one line ok. I hope you appreciate that concept it is very simple or I can write here that the vector L P which is larger in size is some scaled factor of the L P. So, if I multiply this L P this one, by some scale lambda I can get capital L P. So, that is the idea and that is what we called the collinearity equation; that means, if three points are collinear I should be able to write this relationship very easily there is no doubt in that.

Now, let us see there is an object coordinate system. And what we call as, object coordinate system here, or it is also return as Ground Coordinate System in the books. Well, these two names are very, very common. So, if you come across any of the term do not get confused they are same object coordinate system or ground coordinate system ok.

(Refer Slide Time: 36:53)



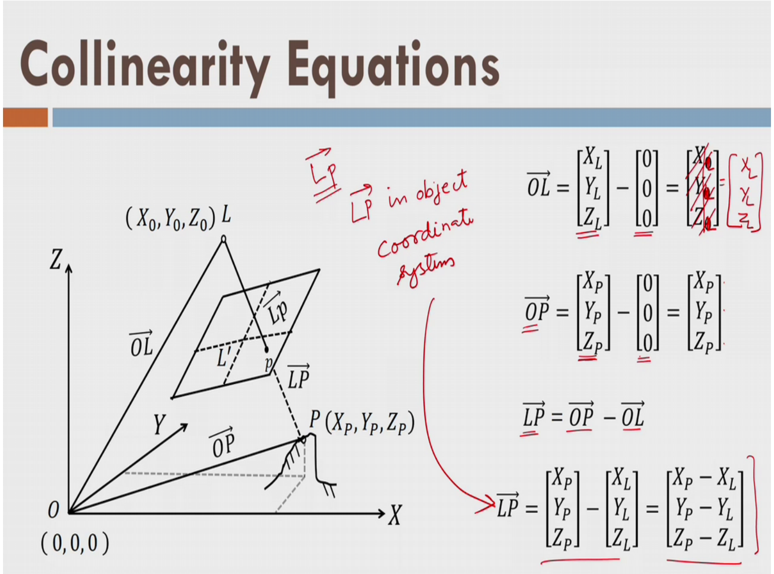
Now, you can see that this vector L P and this L P may not be, in the same Coordinate System. So, if I bring my L P this Coordinate System or this vector into my object coordinate system then, I need to have some kind of rotation what we call as Conformal Transformation that we have already done before in module 2.

So, if I write like that my scale is still there and I need to rotate my L P by R. So, R is my rotation matrix basically right. So, I can write like that. So, it still it is representing the collinearity equation only where R is the rotation matrix. So, that both this L P and this L P are in the same Coordinate System right, ok. So, let us PL see if, point P is given by X P Y P, Z P Now. So, I want to find out what is my L P what is my L P both ok. So, let us try to find out what is my L P first this one.

So, let us see, this is my coordinates like X P, Y P, Z P shown here. So, this is my Y P here, this is my X P here and this is my Z P here ok. So, similarly for this point L, I have three coordinates X L, Y L, Z L in the object coordinate system remember, I have this vector OP like this I have vector OL like this and. So, I can write OP plus PL equal to OL. You see O here O to P this vector is equal to the plus PL this vector is equal to this vector I can easily write this thing that is simple vector mathematics or I can also write OP because, I want to find out this LP which is here. So, I am writing minus LP equal to OL.

So, now let us, look into this what is OP and OL because this is the way, I want to find out. So, I need to first find out what is my OL and OP also I can write here LP is equal to OP minus OL. You can see here you can do yourself also. So, this is the way, I am writing it the same equation here ok.

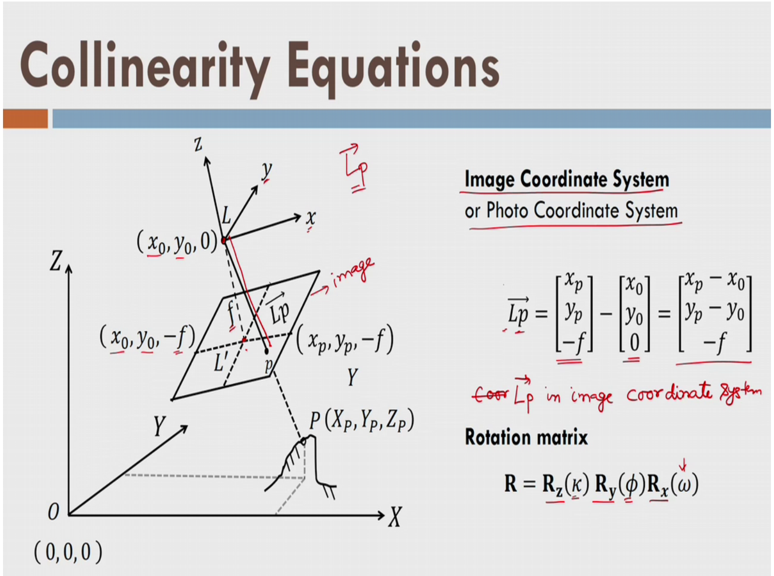
(Refer Slide Time: 39:21)



Now, let us look into the OL what is my OL if, you look at here, this is my OL here, OL is nothing but the coordinate of L minus coordinate of origin that is my O. So, I get X L, Y L, Z L right, I just remove this one and I am writing it here instead of X 0 similarly. Now, I have vector OP where, OP means the coordinate of P minus coordinate of origin. So, I can write here X P Y P Z P.

As a result now, I can write LP which is op minus OL here like this. So, I hope you agree with this. So, I have expressed this vector LP, in object coordinate or the Ground Coordinate System by this quantity here. Now, I need to find out this LP in object coordinate system, but I do not know this LP in the object coordinate system. So, first I develop an image coordinate system right. So, let us develop the image coordinate system to measure this LP, this LP from here to here.

(Refer Slide Time: 40:43)



So, why do I need Image Coordinate System because, this length is basically expressed in the image coordinates only because this, point P is imaged in the image not in the ground. So, I cannot have the ground coordinate system for point P and as a result first I will measure this LP thus this LP, vector in the image coordinate system and from there I will transform by rotation to the object coordinate system and then I will put the condition of the lambda scaling or I will scale that transformed LP by lambda and I will equate to the capital LP here. So, that is the idea here.

So, let us look into the here, let us see L dash there is a focal length f and now I have an image coordinate system which is x and y plane small x y z where, x y plane is parallel to the image, but centred at the exposure station. So, I have x 0, y 0, 0. Since, I also do not know where is my principal point it is unknown to me now. So, this is what I call my image coordinate system or Photo Coordinate System.

So, they are two names again do not get confused in the books. So, this point L dash if, you just look at it will have x 0 y 0 and minus f because, it is vertically exactly down with respect to the point L. And since, this x and y plane is parallel to this image plane. So, f automatically becomes the perpendicular distance from the image plane and so, I have the coordinate of L dash as x 0 y 0 minus f.

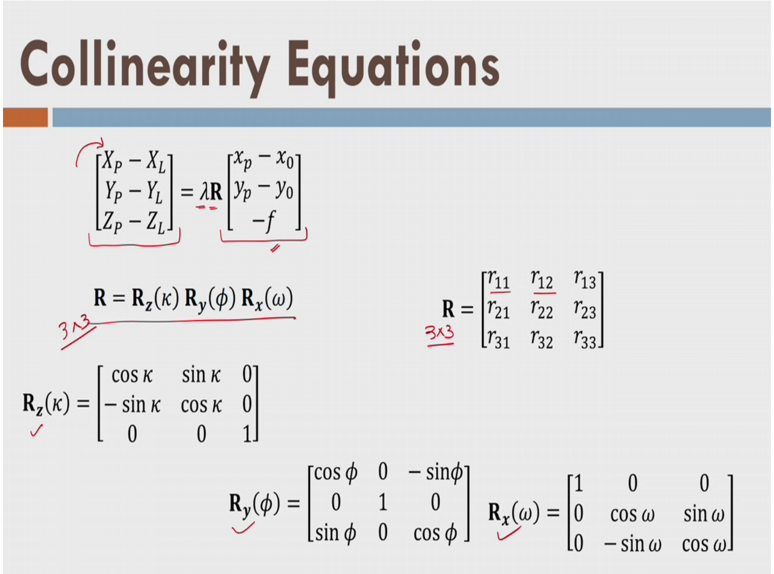
Now, what about the coordinate of point p which is lying in the image plane and for the point p it will be x p minus p y p minus f I hope you got it. So, we are measuring the things with respect to principal point as 00, but I do not know where my principal point is it could be somewhere near here, here. It could be around that L dash, but L dash is the centre point of the image. So, I have established an image coordinate system at point l that is it is exposure station, where I say that it is exactly above the image perpendicular above the image and we know that focal length is a distance between exposure station and image. So, this is the f here. So, I developed these all coordinates.

Now, how can I find out the LP ok, LP is nothing, but again you can straight away write the coordinate of P minus coordinate of L. So, coordinate of this P and coordinate of L is this right, this vector between L to P and I get it this one. So, that is the Image Coordinate of point P in Image Coordinate System. So, that is nothing, but LP in, image coordinate system it is still not in the Object Coordinate System I need to bring it. So, what will I do now? I need to rotate it.

So, I need to apply a rotation matrix and this rotation matrix is given by r x, r y and r z rotations. Such that these rotation in consecutiveness or in the sequence will bring you the image coordinate system parallel to the object coordinate system that is the idea here. So, I do not comment anything on omega phi kappa right now, but you have to when you really do something you have to imagine those values first and also you can change the, order of rotation if you find it is easier for you to apply some other angle instead of omega, if you want to apply phi do it no problem right or instead of if you feel that it is easy to apply kappa first. So, that I can bring kappa phi omega this kind of sequence of rotation will bring my image and I have some imagination over there use that no problem.

Let us say, that you brought your LP from image coordinate system to object coordinate system; that means, both having the same coordinate system now. So, I can easily write the collinearity equation.

(Refer Slide Time: 45:07)

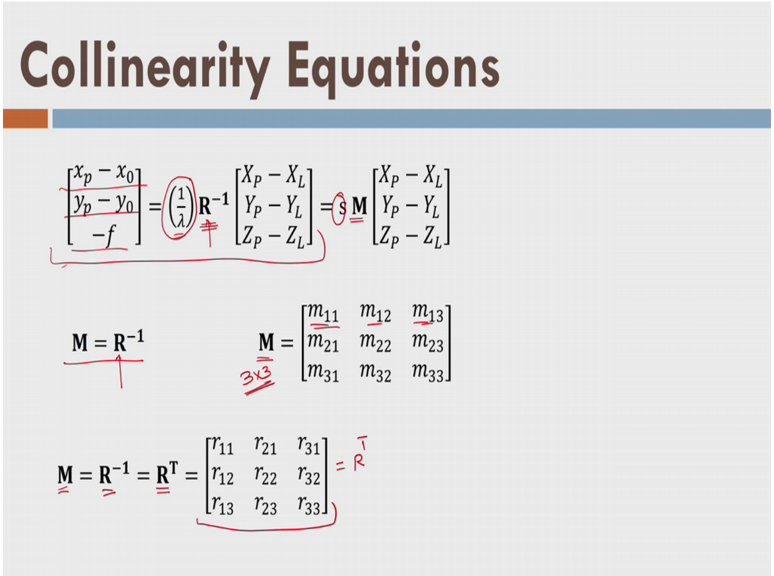


Now, or the collinearity will be like this; that means, my this is function of lambda times rotation matrix and this. Where, R is given by this and if, I write this is by 3 by 3 matrix R. So, if I write like this R 11, R 12 all 3 by 3 elements here where R z is this, R phi is this, and R x is this. I am not going into detail of these matrices now because, it is well known now we have already learnt in module 2 while we doing the coordinate transformation.

So, we have now established what is what we call collinearity equations. So, called a collinearity relationship we have established, but there is small clutch here, there is small problem here. In fact, we measure the Image coordinates and not the ground coordinates they are ground coordinates or the coordinates of point P or point L in the object coordinate system.

In fact, I am not interested I am interested to do measurement on the image right. So, what will I do? I will know, express this coordinates as my output as my derived coordinates or as, my you know calculated positions on the image

(Refer Slide Time: 46:26)



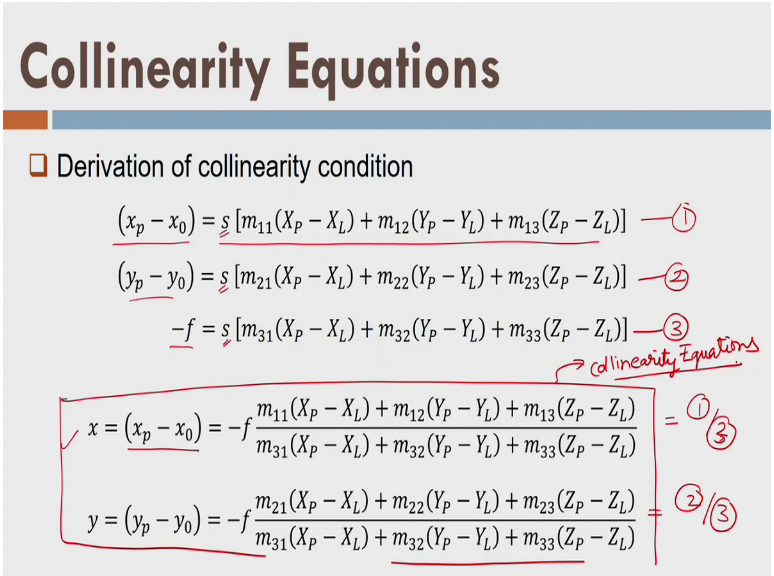
So, what will I do I will do like that I will take an inverse I will write 1 upon lambda here and I will take R inverse matrix and I will write in this form. And now, I will call my M matrix which is R universe here and this 1 upon lambda is nothing, but S here. So, 1 upon lambda is inverse scale I can say let us fine.

Now so, it is written here ok. So, whatever element I get they are called M 11, M 12, M 13 and so on. So, it is again 3 by 3, but fortunately this R matrix is Orthogonal matrix which means if, I take the transform of the R, I will get it is inverse like this. So, as a result, I am writing the transform R T here. So, my R matrix in the last slide was whatever it was it is a transform of R. So, it is very easy if you know the R matrix and remember one thing it is very, very crucial because we imagine omega phi kappa as the conversion from image coordinate system to object coordinate system we have our imagination that way only.

So, my these matrices is developed using those imagination use imag[ine]- using omega those values of omega phi kappa from image to object coordinate system not other way round. So, do not imagine the do not you known confuse yourself that we have taken the inverse of the matrix R; that means, our rotations are also negative no it is not like that we started from there because it is easy for us to understand from developing the rotating an image into the object coordinate system it is very easy to imagine because we have.

So, many images and we have to rotate each and everyone in one object coordinate system. So, that we can develop our 3D model in one object coordinate system right.

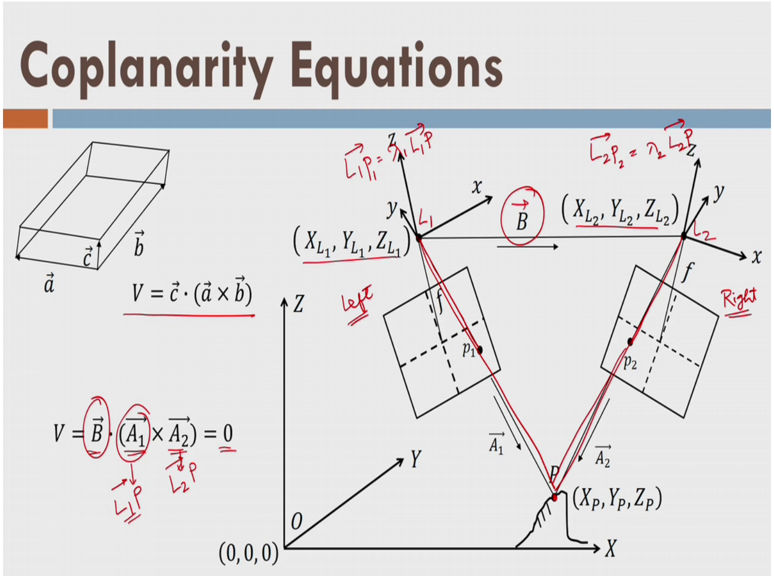
(Refer Slide Time: 48:25)



So, now, if I write these equations one by one, three equation 1, 2, 3. I can write like this X p minus X 0 equal to, something like this similarly this and this you can try it yourself ok. Now, this scale factor is still unknown to me I do not know what is the value here even because I was not knowing what is lambda. So, I would like to remove it. So, what will I do? I will make the ratio of X P and f; that means, I am dividing this equation 1, 2 by 3 both 1 and 2 by 3. So, I will get this equation by as the ratio of equation one to equation 3.

Similarly, this is the, ration of equation 2 to equation 3. So, I can get this one where I have removed the scale factor and these two equations are known as, collinearity equations. And that is a fundamental governing equations in the Photogrammetry ok.

(Refer Slide Time: 49:27)



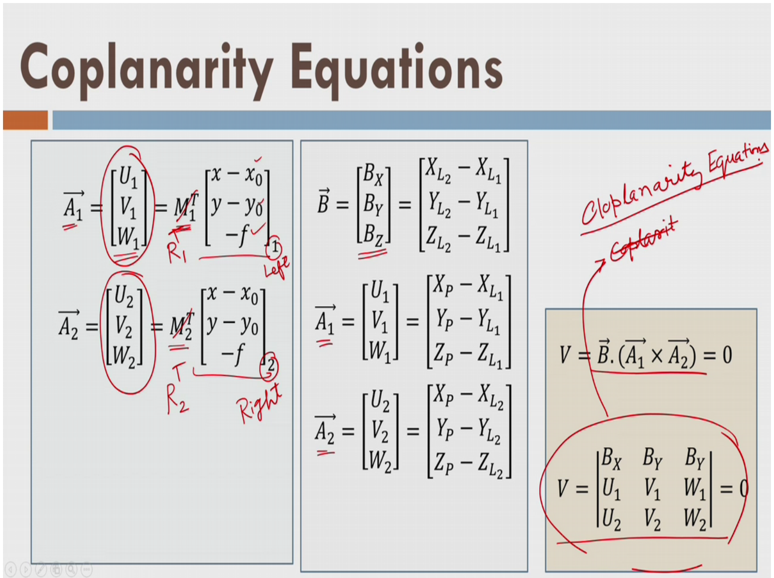
Let us go ahead. So, one more condition is possible now the coplanarity equation. So, let us, imagine that we have acquired 1, 2 stereo images; that means, S stereo pair ok. So, this is the image and this is the image let us say left image here, and this my right image here, fine and this is the point P here on the ground surface and it is being imaged at point P 1 and P 2; that means, point P is coming in both images it is imaged in both images ok.

So, what is the distance between the two exposure station? Let us, call L 2 and this is my L 1 station. So, these are the coordinates here ok. So, I can now write the collinearity equation for this like this LP 1 is the like this L 1 P is let us say, lambda one times l one capita L P. Similarly I can write collinearity equation here. So, I can write L 2 P 2 here. So, lambda two times L 2 capital P like this. At the same time if now, if I see very carefully if point L 1, L 2 and p are imagine in one plane there is nothing wrong in that because 3 points are always connected with a triangle right.

So, what will happen? If, I put or if I calculate by these three vectors b vector this vector L 1 P vector, L 2 P vector here. So, all these, are creating one triangle and what about the volume of a triangle which is generally, we calculate volume by three vectors like this. So, I can write this volume and volume of a plane is always equal to zero right and so, this is my B vector here, this is my A 1 vector here, what I am writing as, L 1 P and here it is my L 2 P.

Now using the collinearity equation that we learnt this previous slide I can write my L 1 P in terms of image coordinates of first left image. Similarly, I can write my L 2 P in terms of right image coordinates like this ok. So, let us there is a coordinate system here ok.

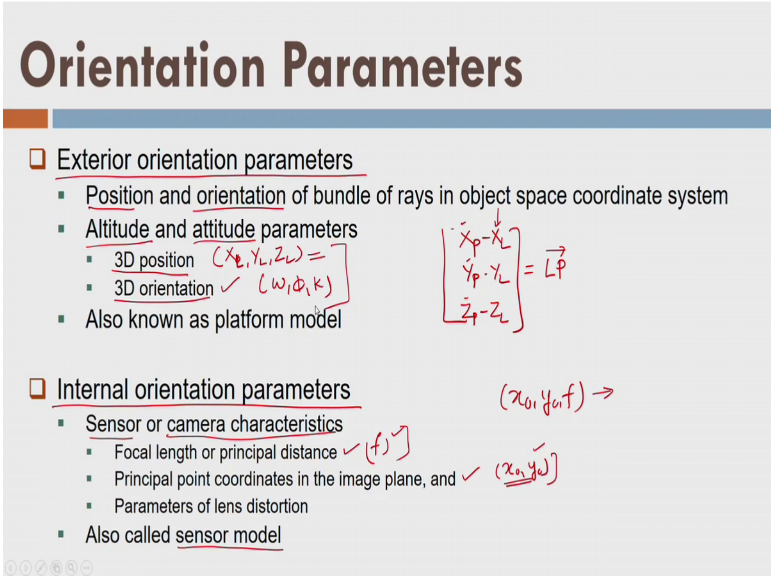
(Refer Slide Time: 51:47)



So, I am writing A 1 as U 1, V 1, W 1 using some transformation that is m matrix, I am writing first image coordinate here and. Secondly, it is the second image coordinate here right.

So, I should write it let us say, with our convention R 2 T and here, R 1 T because I am converting like that now I can write let us see I got this U 1, V 1, W 1 and U 2, V 2, W 2 that is the object coordinates of these points. So, it is my first image or what you call is Left image, this my second image what you call as Right image ok. So, I got this b matrix as B X, B Y, B Z like this and A 1 matrix A 2 matrix vector ok. So, if I put like this I will get this kind of determinant equal to zero, just refresh your twelfth standard matrix knowledge you will get this one. And this is called the Standard Coplanarity, Equation. So, because it will lead finally, two three equations, because it equations. So, they are called Coplanarity Equations.

(Refer Slide Time: 53:17)



Now, we understood what is the Coplanarity and Collinearity apart from this we also talked about the systematic errors in the images and then we also talked about the image refinement and we have also discussed about the polynomial transformation today ok. So, let us stop here, for time being and try to imagine what we have done in case of Collinearity in Coplanarity and how are they different and how are how can I say that they are physical models ok.

If you remember carefully we have done some rotations and some translations. So, something I call an exterior orientation parameters if you just look and then we have interior orientation parameters and it is nothing, but let us, call the interior orientation parameter first. So, if I know the focal length or the principal distance principal point coordinates if you remember X 0, Y 0 focal length is f here and parameters of lens distortion.

So, I can find out what is the relationship of the image in the Image Coordinate System. that is what is my focal length how image is placed ok, what is Orientation of this one right. So, I can say that, all image or the all bundle of rays in one image they are creating one image and as a result all bundle of rays will have these common parameters focal length as well as principal point.

Secondly, once I know these measurely parameters then I can reconstruct my image or I can reconstruct my camera model and that is why, it is called the sensor or camera characteristics, right they are also called this sensor model these parameters are called sensor model. So, moment I say that what is the sensor model; that means, I am asking about this three parameters, x 0, y 0 f for a particular image which is acquired with the particular camera. So, these will remain fixed if camera is fixed and that is why they are called sensor model or they are called camera characteristics. So, once I fix these my camera is also fixed or camera is known to me.

Now, we have done some image transformation or we have rotated our image into the object coordinate system by some angle omega phi kappa we call them 3D Orientation; that means, we are rotating each and every bundle of ray or each and ray in the bundle of rays which are creating the image. So, I am rotating by angle omega phi and kappa since all the rays are creating one image and as a result I say that I am rotating my image fine.

So, now what happens here these rotations are omega phi kappa; that means, I have rotated my image by this angle omega phi kappa not only that I have done some translation here by X L, Y L and Z L remember in this matrix X P minus X L, Y P minus Y L, Z P minus Z L. So, I have rotated or rather I have translated my X P Y P and Z P by this. So, this becomes my vector L P right. So, these are nothing, but the coordinates of point l or they are the translation of point p with respect to point l. So, that is why these are also the 3 D position they are called positional parameters or they are called altitude parameters Orientation parameters are called attitude parameters and they are basically developing the relationship between image and the object coordinate system.

So, what is the conclusion here this position and orientation if these are known to me I can orient any image into my object coordinate system and that is why they are called exterior orientation parameter. So, they are quite different from interior orientation parameter because interior orientation parameter fixes the camera and image relationship; that means how a camera will acquire an image. So, in what position it will be there what is the position of principal point what is the focal length right on the other hand my exterior orientation parameters they are going to develop a relationship between image and the object coordinate system.

So, if I have n number of images and each will have different, different exterior orientation parameter. So, if I bring them each image into one coordinate system what will happen I will need this, six parameters for each image and that is why they are called Exterior Orientation Parameter and they are no more dependent on the camera. So, let us imagine that you have one camera and you are acquiring some photo like this like this like this. So, each image that is acquired from your camera will have different, different exterior orientation parameter according to your position and according to the orientation of the camera itself or the image right.

At the same time your interior orientation parameters are remain same for all 3 images because your camera is same for all this turn let us say you have mobile phone. So, you are acquiring n number of images for mobile phone. So, all n images will have same sensor model or they have same, camera characteristics at the same time each image will have different exterior orientation parameter. So, here we stop and we conclude that we have learnt the coplanarity equation and collinearity equation which are the physical relationship between the imaging system and object coordinate system.

Secondly, we have also talked about the transformations which are mathematical in nature and then we have also learnt about the systematic error on the vertical images or the vertical photographs. So, with this we stop here and we will meet in next lecture, with the mathematical treatment on this equations; so, till then wait and bye.

Thank you very much.