Higher Surveying

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Module-6
Lecture - 17
Photogrammetry

Hello everyone. Welcome back on the course of Higher Surveying. And, we are in module 6 Photogrammetry. In the last lecture we discussed about the vertical photogrammetry, wherein we said that we are acquiring one vertical photograph. And then we realized that because of some error which is unintentional in nature it could be a little tilted also. And, then we defined 2 terms, which is vertical photograph that is an ideal situation and another one which is a real practical situation, we call the tilted photograph.

In case of vertical photograph we assumed that the optical axis of camera and the plumb line passing through a camera are coinciding or they are matching exactly. And, as a result we have derived some of the formulations for ideal case. Then, we have also seen that what could be the possibility in case of the tilted photograph. Moreover, we have also learnt, what is a relief distortion or relief displacement and again we have learnt, what is the tilt distortion. Also we talked about the scale of vertical photograph and tilted photograph.

And, what is the scale? In case of map we define scale as the ratio of the distance measured on the map to the distance measured on the field between the 2 points. Similarly, in case of photo scale we have defined scale of the photo equal to the distance measured on the photo, to the distance measured on the field. So, that is these are very simple terms, now you are able to understand very easily.

Today, we are going to talk about stereo photogrammetry. Let us go into the lecture today, this is the lecture 3 on the module photogrammetry. And, this lecture is about stereo photogrammetry ok. So, these are the books still I will say these first three books are still appropriate for this lecture.

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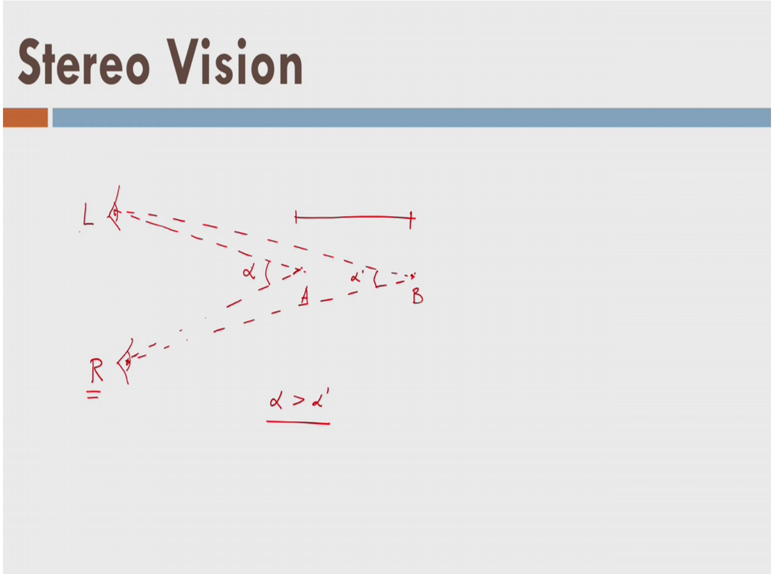

Now, let us understand, what is the stereo vision? Right, now let us say I am looking at a point these are my 2 eyes, and right I am looking at a point here which is a point called A. And, so my eyes are having this kind of rays intersecting at the and it this angle is formed.

Now, what if there is another point B and I am looking at this point B here and so, if there is a difference between H and B, that is very evident from this slide. Let us say I call this angle as alpha and I call this angle as alpha dash. Here, you can see that alpha is more than alpha dash. Why? Because the point B, it is far away from me compared to point A and that is the what we call stereo vision.

That means, I am having 2 eyes they are intersecting at a point let us say right now camera or I can say the point is camera, and there could be another point which is behind the camera. So, this point A is my camera and B is the point, which is behind the camera. So, I can easily say that, I am percepting the depth or I am measuring the depth or I am trying to understand or perceive the depth, because of the stereo vision ok. What is the contrary argument to that? Let us do one experiment and you might have done it many times before also. It is not a first time you are going to do, but let us repeat this thing.

So, let us say there is a point A in your room and try to stretch your arm like this, and make your finger this way and try to hide the point with your finger. Fine, after doing this thing what will you do now; you will close your one eye that is like this and try to see from the one eye that hided point. So, pointed hide at behind the finger and you are closing your eyes like this ok.

So, once it is done you open this eye and close this eye. You will find out that the point, which is hided behind a finger is shifted. And, this shift is called the parallax well, you we use this term also in astronomy to find out the parallax angle. And, the term parallax is coming from the similar concept; that means, on the successive exposure of the eye there is a shift of the point and this shift is called parallax well.

So, now if I have let us say my left vision here and this is my right vision here right eye and left eye. So, from left eye if I close my left eye and try to see point A right, you know which is behind my 1 finger and then I close the eye left eye and open my right eye, you will find out that there is an apparent shift of point A.

So, that is the idea here. And, this is an essential to create the depth perception. Even, I have 2 eyes and, but my eyes are incapable of doing the parallax creation; that means, I will not be able to see the depth of a point. Or I cannot compare 2 points, which are different in depth. So, that is called the depth perception using stereo vision. And, we are going to replicate the same phenomena in our photogrammetry. And, that is why we call it stereo photogrammetry. Well, let us go ahead.

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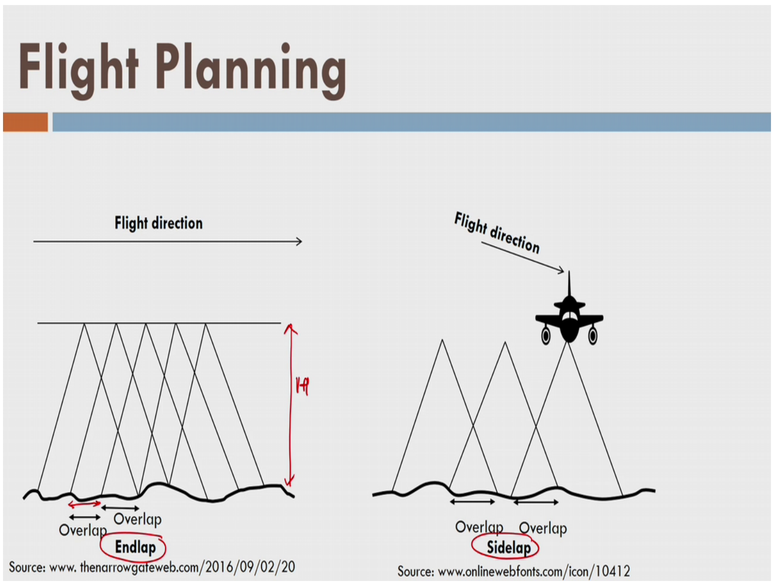

Here, before we start we have to understand one thing that let us see there is a flight and it is flight is mounted or the airplane is mounted with an camera. So, instead of creating 2 camera; that means, for stereo vision I need to have 2 eyes or maybe 2 cameras. So, what will I do now I will put one camera only one camera on a flight and then the airplane will move in certain direction.

So, this camera will keep on acquiring photographs and as a result I am trying to replicate the eyes or the multiple cameras this way in a line right. So, what is the understanding here? The understanding here is I need to first acquire the data, that is I need to acquire the many many photographs so, that I can create some kind of stereo vision.

Moreover, if you see in case of the example which I give you by finger, there is a point A or point B, but we have assumed that both points A or B or any point, it is visible from the each eye; that means, there is some kind of overlap or there is some kind of common area between the 2 eye. That is why, I can see one point common point, it could be point A or it could be point B or it could be any part, but there is a area where both eyes can see or both eyes are looking at same point on the same area, that is why I need to create the same replica or the same thing here with the aerial photogrammetry.

So, what I am doing I am trying to acquire the vertical photographs, which are like this, this, this, this, this and each and every photograph have some overlap with the adjacent photographs. So, that I have a common area where my both camera locations are looking at that point right. So, let us see my animation, what does it mean? Let us say this is my terrain and this is an aircraft mounted with camera. So, this is the field of view and it is acquiring photo now it has moved to this location, then this photograph, then this, then this, then this and so on. So, I have some this is my flight direction here.

So, this is the flying height I can write here, this is a flying height here, and if I take the average elevation I can say what is my value of h or even I can take. So, this is my overlap you see on the ground, this is my overlap which is marked in the photo it is image or slide also ok.

So, this kind of overlap, which is in the direction of flight, it is called Endlap here ok. What, if I see the flight coming towards me or going away from me like this. Let us say same terrain now aircraft is coming towards me. So, again this is my flight direction; that means, so, it is acquiring photograph which is perpendicular to the flight direction, this next position, this next position and so on. So, there is also an overlap and I call this overlap as the Sidelap.

And, Endlap and Sidelap both are important are required ok. Perhaps you might have understood, what is the concept of Endlap and Sidelap, that is overlap in 2 directions ok. Let us look into another animation, which will explain it again in a little better way.

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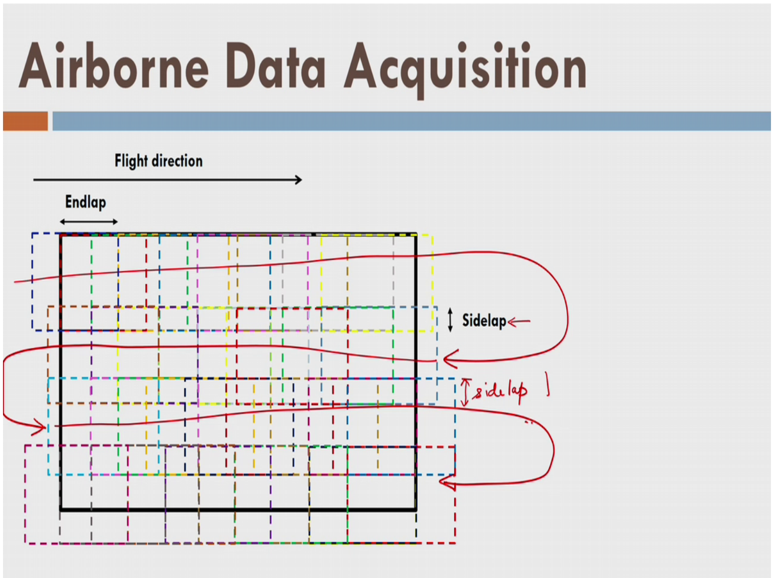

Let us say this is my flight direction and this is the area we want to ok. So, this is the 2 images in the direction of flight. So, this is my Endlap right and so, I can I am acquiring the photographs like this right. Further this is my Sidelap here you see this is my Sidelap here ok.

So, flight has travelled like this go on this way, coming this way, like this now it is going this way, this way, this way, this way ok. Then, it is come this all along this way, now going this way ok. Again, this is my Sidelap and again flight has come this way. And it will go this way and it is squared the complete photograph for complete area.

So, these many photographs have been acquired by the arrangement of one camera into a airplane. So, this is way in each photograph is a vertical photograph. So, now, I understand that this is called airborne data acquisition for aerial photography ok. Each photograph is vertical, because I want to analyse a theoretical case for that perception.

So, the idea here is how can I find out the depth or how can I make the map that is 3 D map using photogrammetry. So, let us look into that now, but before that we should understand that how many photos are required for an given area. And, before we go to the field we have this area in our beforehand, that this is the area I want to do a 3 D mapping. And, hence I want to acquire photographs for this area right.

So, in order to acquire the photograph I need to do a preliminary exercise, what we call as flight planning; that means, I plan my complete mission of data acquisition and then so, that with minimum cost I can acquire the data ok.

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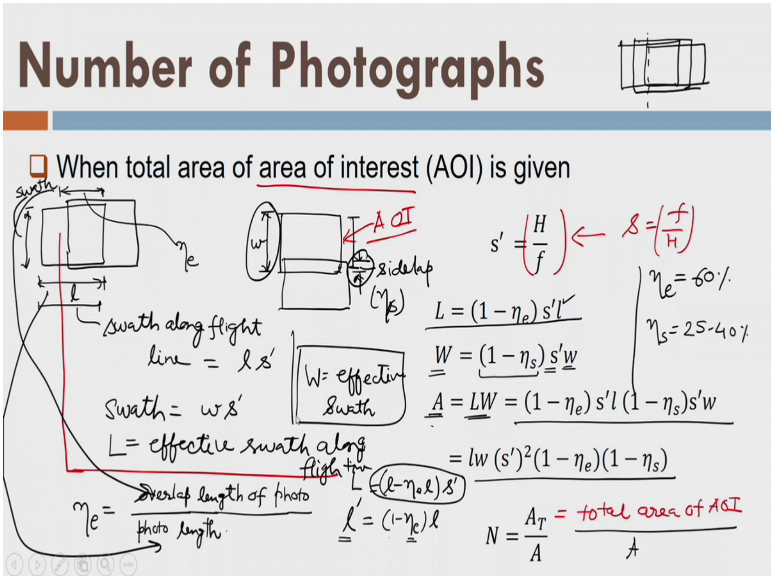

So, let us see there are 2 possibilities here and I want to find out what are the number of photos I need beforehand, before going to the field. So, let us assume that this is the area which is in rectangular in shape is given to me. Let us see the area total area is given by A T and I call it total area of area of interest or I call it AOI area of interest, the term I am writing here also ok.

So, let us look into that, that you know what is the scale, we define the scale in the vertical photograph, it was f by H. So, let us define something called s dash, which is inverse of the scale. And, now you see there is an Endlap let us see there is the first photograph is like this, second photograph is like this with some overlap and we call it Endlap here right. So, this is my Endlap and I write this Endlap as eta e, Endlap is nothing, but eta is the ratio of the photo length right, and here the overlap length of photo; that means, I am taking this ratio of this here to here 1 to the total this one length of the photo right. So, this is my denominator here and this is my numerator here, fine and I call it eta e.

Similarly, if I find out the overlap here between these 2 photographs, this overlap; what we call as Sidelap and I write as eta s. Similarly I will take the ratio of this overlap here and overlap to the this length. Total length is this width what we call here fine and that is what we say that ratio of this to ratio this is my Sidelap. Now, I can say that, what is the length covered by each photograph on the ground. And, there simple idea is there because I can say let us see this is the length of photograph.

So, that is an effective I will say let us say l dash it is my length of photo graph that is travel or that is effective coverage why because, this is overlap. So, in 2 photographs this area which is eta e into l, l is dimension length of this is my let us say l n length here ok. And, that is called let us say the small w width of the photograph. So, l into e is my overlap here. So, if I remove l minus eta e l, then I will get the effective coverage or the real coverage or the real area required by the image.

Now, if I multiply this by the scale or the inverse of the scale s dash, I will get the total length on the ground right ok. So, this is the total length covered by 1 photograph effectively, not in reality. In reality it is covering the complete length with overlap, but because of overlap I need to say that a on the ground we have moved by not the l distance, rather we have moved by l minus overlap.

So, this is nothing, but l minus overlap and I am multiplying with the s dash, which says that what is the length really covered on the ground, because these dimensions are for photo. So, I am multiplying the dimension of photo with the scale inverse I am getting the distance on the ground. So, this is the idea.

Similarly, I can say that this is the width of 1 photograph on the ground surface. So, what I do? I will 1 minus eta s, eta is my Sidelap and then again s dash multiplication and this is my width of the photograph. So, I marked here what is width and what is length l. So, what is the area covered by effectively that covered by 1 photograph. So, that is nothing, but A L into W right.

Now, I can write this thing here and this is nothing, but this you can do it yourself also this mathematics it is very simple ok. Now, if this is the total area of the AOI and if I divide by the area covered by 1 photograph or area effectively covered by 1 photograph; so, I will come to know: what is the number of photographs I need in order to cover the whole area A T.

So, this is what we call the number of photographs required in order to develop the stereo vision for the whole area A T area of interest. Moreover, I would like to say few of the things about the overlap. The first of all the overlap eta e, that is Endlap that is minimum 50 percent and in general it is kept 60 percent as minimum. Because, if I have 50 percent overlap what will happen you can see; that means, if I have this photograph here and if I exactly have the 50 percent overlap it will be like this. And, in order to avoid any chance that, I can you know because of some random error I lose it what we do we minimum keep 60 percent Endlap; that means, I am having this much of overlap like this fine.

Similarly, what about the Sidelap? Sidelap is kept in the range 25 to 40 percent and that gives me the continuity on the between the different lines of the flight lines or different lines of data acquisition the Sidelap. So, that is the idea here, moreover I would like to say here this is the swath, and this is the swath along the flight line right.

So, I am saying that this is not small l. So, here I am showing swath as on ground as L and swath along the flight line is nothing, but W and they are called the effective swath basically, both are effective swath because they are removing the overlap area right. So, I can say here that, what is my swath? So, it is my effective swath here L n this one. So, I should write here swath is nothing, but w into s dash and here what is this swath along the line length multiplied by s dash. So, what is my capital L on ground? I call it effective swath along flight line and similarly W is nothing, but effective swath right ok. So, let us move ahead.

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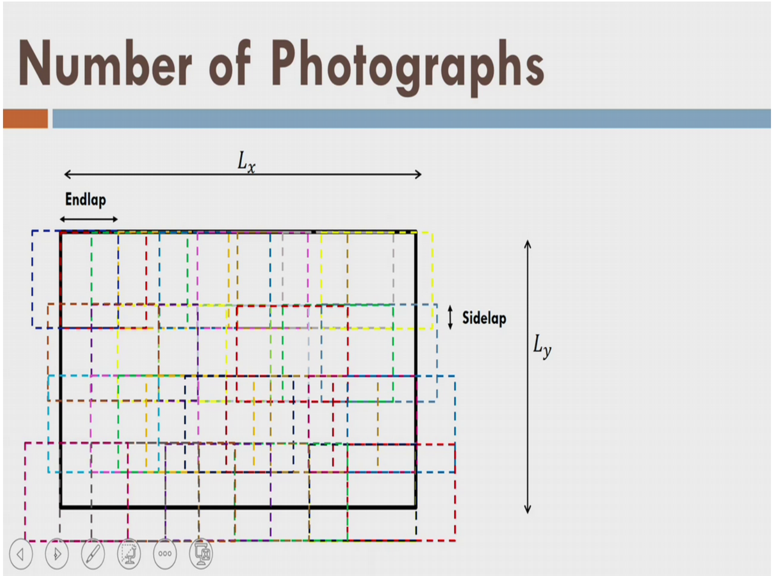

Number of photographs when situation is again slightly different here; let us say length is given to me instead of area. So, then this is the way we again we will cover the photographs yes. So, we have covered all the photographs.

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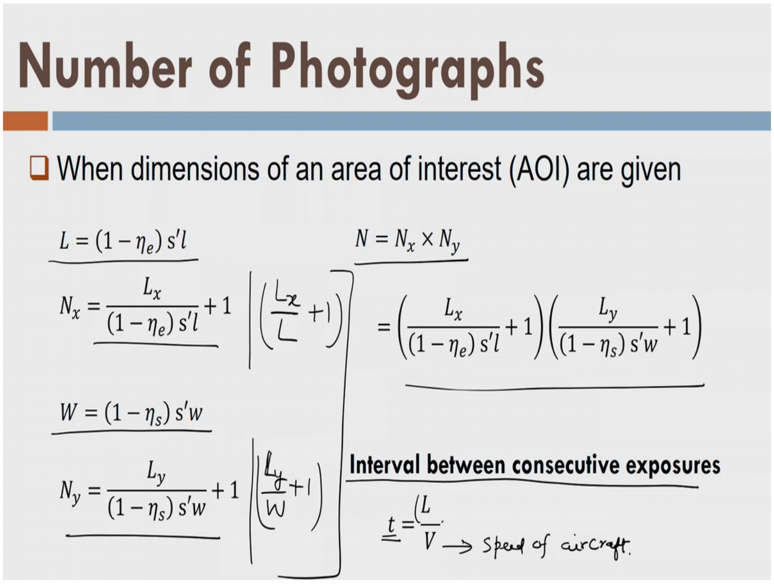

So, let us go to the next slide where I write, this is the effective length covered on the ground similarly, I can write effective width by covered by 1 photograph on the ground ok. So, what about the number of photograph along the x direction along the flight direction ok. Here, if I divide this L x by L you can argue like this if you divide L x by L. So, then in that case I should get number of photographs.

But, there is slight issue here. I need to cover the whole terrain or I need to derive my 3 D stereo view for the whole area of interest, and as a result what happens I need one additional photograph. So, that the last photograph, which will not be able to cover the area or which will just cover the area in 1 photograph. I want to cover that are in 2 photographs and as a result beyond the boundary of the area I need to have one more photograph that is the last photograph that will have Endlap.

Similarly, in case of Sidelap I need to have some additional photographs on the another edge on this side. And, that is the reason we write it the total number of photographs are L x by L plus 1, which is given here. Similarly, I have N y as L y divided by W plus 1. I need one additional photographs. Now, total number of photographs I can say N x into N y, again I will multiply here this thing.

So, what next? So, you can see now I can also calculate the interval between the consecutive exposure; that means, this is one photograph after sometime aircraft move to this it created another photograph, another photograph and so on. So, between the 2 consecutive photographs what is the time I need. So, what is the time exactly there in the field that is nothing, but if I divide this distance L by speed of the aircraft. So, you will get that is then interval between the consecutive exposures.

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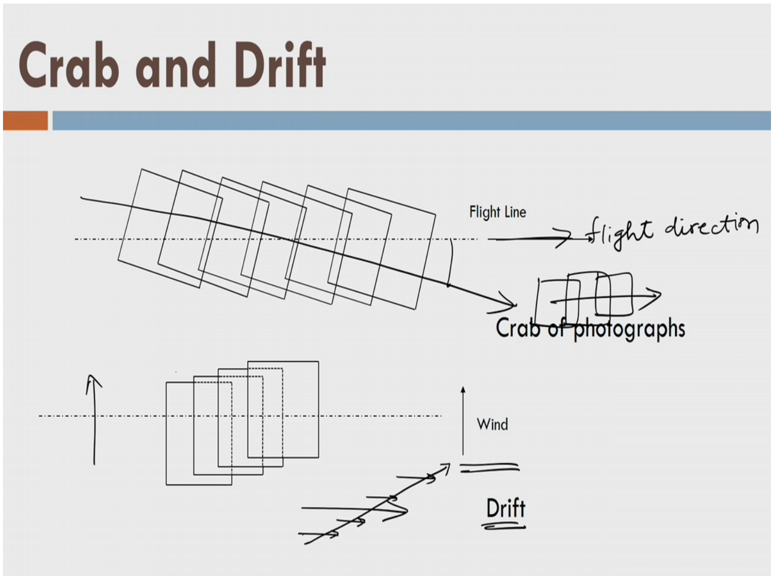

Let us go ahead ok. There are 2 phenomena's, while we acquired the photographs; one is crab and another is drift. So, let us look what is the crab first? Let us see there is a photograph like this and flight line they are acquired like this. So, this is my flight line in this direction or the flight direction.

And, the photographs are not aligned with the flight direction. Ideally, it should be like this the way we have said in the previous slides, but it is happening that way. So, the this line if I draw, and this line there is no flight direction, this angle is called the crab or this effect is called the crab effect. Similarly, what is a drift? So, drift is this let us say I am acquiring photographs like this and there is a wind. And, because of the wind what will happen, I am flying the flight is in this direction, but because of the wind every moment.

So, once I flight this one there is a wind flight will go like this, again it will go like this, it will go like this; that means, it is effectively going this way and because of that even the photographs are aligned with the flight direction, but they are having some shift in this direction. So, there are the shifts you can see here the photographs are shifted in up direction and it is called a Drift, Drift of the wind sometime or Wind Drift. So, because of the wind we have drift and because of the misalignment of the flight direction and the aerial photograph axis I have; so, that is the idea and these are the basically limitations in the aerial data acquisition fine.

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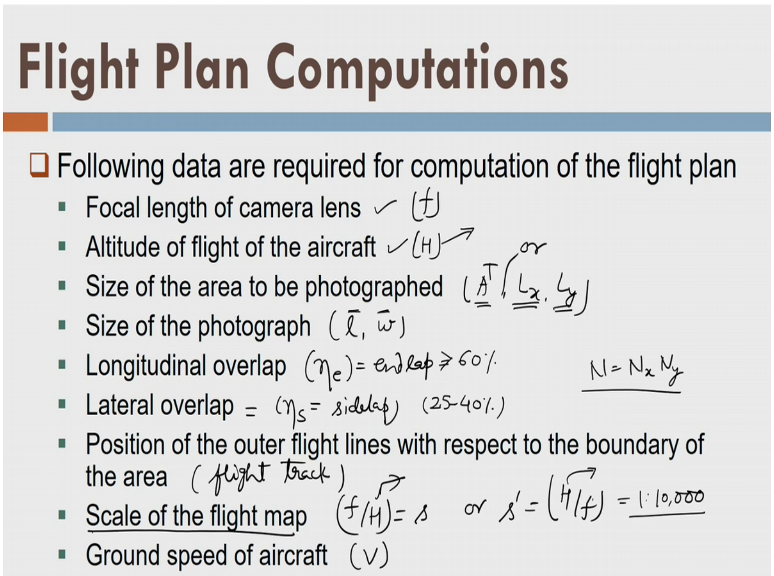

Now, I want to do the flight planning and for that I need to have some kind of computations. So, there I need to have some kind of idea, because we want to calculate the scale. So, I need to have focal length, I need to have H ok, size of the area of the photograph or that is nothing, but we call it area total or it could be like L x and L y whatever. So, I can write here or I need the dimension of the area of interest, size of the photograph; that means, I need lengths as well as width of photograph, longitudinal overlap, that is my Endlap I used to also called longitudinal overlap, lateral overlap, the Sidelap.

And, position of the outer light lines with respect to boundary of the area fine. So, what is a flight track basically? Scale of the flight map anyhow I will calculate f by H is equal to s or s dash as my H by f, that is basically the scale of the flight map the desired one. And, according to desired one, if I have the camera of focal length f I will decide my H first, that is flying height what should be flying height?

And, then I know that there is the limit of the capital H and hence I have a limit on the scale of the flight map. Then, we have ground speed of the aircraft that is speed V. So, if I have this all information I can plan my work before hand, before going to a field and that is what we call flight planning.

This is the flight planning computations, that means I want to acquire the data in the field, before that I do some exercise where, I know which camera I am going to use. The moment I fix the camera I know what should be my flight height or the flying height for the given scale or to scale to be achieved in mapping or aerial photographic mapping. So, I know this is whatever scale for example, if I have 1 is to 10,000 scale, which is very high right.

So, far accordingly I will decide what should be the flying height H, because this flying height a pilot needs to fly the aircraft at. So, once it flies at flying height H the photographs will be of the scale f by H, because f is the focal length of camera.

Now, I already know this information the moment I fix the camera, what is the length of the one photograph and what is the width of one photograph. Thirdly, I know: what is the size of the area of interest, then, I fix my Endlap as minimum 60 percent or I can write it as more than 60 percent.

Similarly, it will be in the range of 25 to 40 percent the Sidelap. So, the moment I fix it, I can find out the number of photographs and which is N x and N y. So, what could be the possible cost in the field, if I acquire so, many photographs? So, beforehand I know what will be the projected cost or what the likely cost? So, should I go for this kind of mission, because what is the accuracy I am going to get by this data acquisition in my real map, does it really worth to go for this exercise or the data acquisition or to create such map by aerial photography. So, that kind of judicious decision I take by flight planning only and that is the importance of flight planning right.

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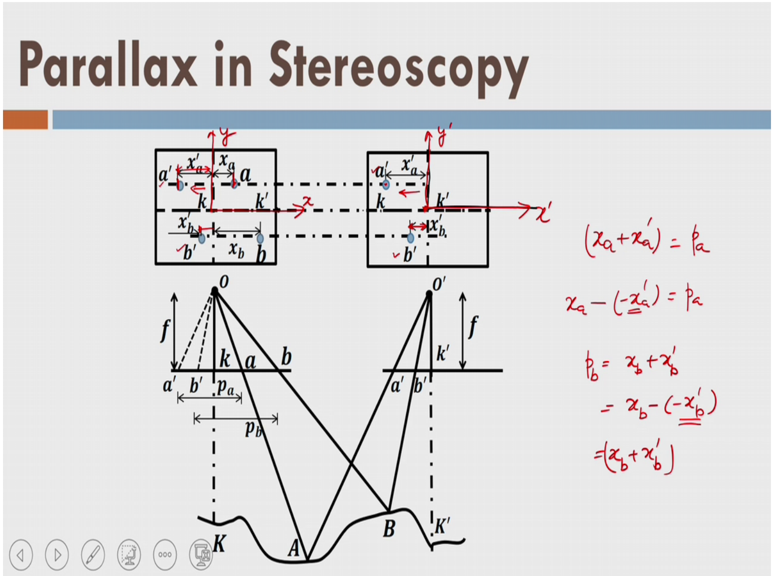

And so, let us see that we have acquired the photographs; that means, I want to now decided that yes by flight planning I have decided that yes, this machine is worth; that means, I am going to create the 3 D view of the terrain. And, that is going to have enough impact in a sense like this cost is justified. And so, let us go for the data acquisition.

So, once I acquire the data that series of vertical photographs having some Endlap and Sidelap. So, now, I want to create the 3 D using parallax ok. So, let us learn: what is the parallax again in terms of aerial photogrammetry. So, let us see there are 2 points A and B on terrain and then we have this point as my exposure station, then another exposure station O dash first image, second image. So, that is my principal point k. And, similarly I have focal length here and there is another principal point k dash for next image.

And, then we have focal length, because camera is same in 2 consecutive photograph or in the whole mission. So, I am writing the same focal length f. So, basically there is one camera only. Now, the point a is acquired like this. Similarly point a is again acquired in the another photograph next photograph so, called a dash location fine. Similarly, let us go for the point b, b and is like this. So, the image of point b in 2 photograph are b b b dash. So, let me call this as my left photograph and that is my right photograph and we are going from here to here by flight. So, these are my images like this is my central line. So, point k here, point k there, now k dash and k dash here you can imagine that both points are appearing in each image.

Because, we have 60 percent, in case we have 50 percent overlap k dash would have come at this point here, but now we have 60 percent that is why point k dash is appearing inside the image or photo first photo. Similarly, point k is appearing here and not there. So, this is the fine you can see there is a point a was here. So, this is my x a that is the x coordinate of point a in first image.

Similarly, I have point b here. So, I have x b here you see in the animation. So, try to look carefully in the animation I am going very slow here again point a dash. So, I have b dash. So, I have x a dash and x b dash. You can see here that x a and x b are positive in left image, but x a dash and x b dash are negative, because they are on the left hand side of the this. So, this is my coordinate axis here let us see x dash y dash and this is coordinate axis here is x and y. So, I am measuring x a and x b as positive in left photograph, but in case of x second photograph I measuring them as negative from this origin right.

Now, let us replicate this points a and b on the left image from here I am trying to replicate these 2 points on the left image. So, this is the point b dash and a dash like this. And so, here b dash and a dash you see I just place them here. So, nothing, but with respect to this origin I place this thing here. And, now I can write this is my x b dash this distance, which is equal to this distance.

Similarly, this distance is my x a dash, I hope you understand the simple idea ok. So, can I say this distance between point a and a dash. Basically, it is the shift in the location of point a on success of exposure, remember we have done the experiment we close our first eye then we close second eye. So, there is a shift in the point. So, this shift is equivalent to this x a and x a dash.

Because, at first exposure it was captured here or here, after sometimes there was second exposure. So, it was captured here. So, there is a shift here. And, since it is in the negative direction I am putting it into this negative direction here. So, that is the total shift. So, I can write here x a plus x a dash rather I can say numerically, it is equal to the parallax of point a I can write also here like x a minus minus x a dash, because it is in negative coordinate. So, I am basically taking the algebraic sum of the 2 locations with respect to their corresponding image centres like.

Similarly, I can write here p b is nothing, but x b plus x b dash which is nothing, but x b minus minus of x b dash here since it is again the negative coordinate and so in fact, it is it should be x b minus x b dash, but it is itself is negative and I get x b plus x b dash as my total parallax. So, that is the shift in the location of point on image, because of the 2 successive exposure of camera. I hope you got the idea of parallax on the photograph. Now, earlier we have done with our eyes understood the concept with eyes, now we have done with this camera and now we understood that on the photograph what will be the parallax or what is the shift in the point, because of these 2 successive exposures let us go ahead. So, here it is my p a and here it is my p b marked here, but I written here.

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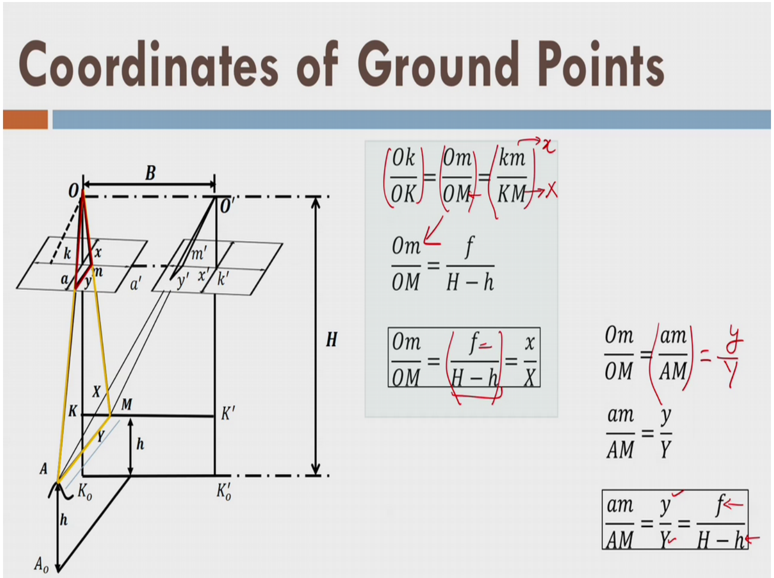

So, in order to find out the 3 D coordinates to find out x y and z coordinates. So, let us first find out the x and y coordinates. The moment I find out x and y coordinates, then I will find out z coordinate and then I can say that is ethyl photography is acquiring all the things together, I am developing a simultaneously measurement of x y and z system, but mathematically I am doing separately.

So, let us look into this thing. So, that is one triangle here and if I draw the similar triangle so, then I can write this thing divide by Ok capital OK equal to this ratio and equal to this ratio, you can do it yourself again fine again O m is nothing, but focal length and that is OM capital is nothing, but H minus h. So, thirdly if you look at in the similar triangle k m is nothing, but small x that is coordinate of point any point and here it is the point coordinates of same point in the on the ground. So, now, I can write it very easily. This is one way of expressing it.

Similarly, if I write for y in the same photograph you see this animation this triangle and this triangle yellow triangle and red triangle are similar triangles. Using this I can also write like this or like this or like this, well again this, OM by here same and here you are calculating this one which is nothing, but y by Y. So, I can write this is my given the y coordinate and the image I can find out the capital Y coordinate on ground; provided I know the focal length as well as flying height and the height of the point right. So, that is nothing, but a scale at point O k.

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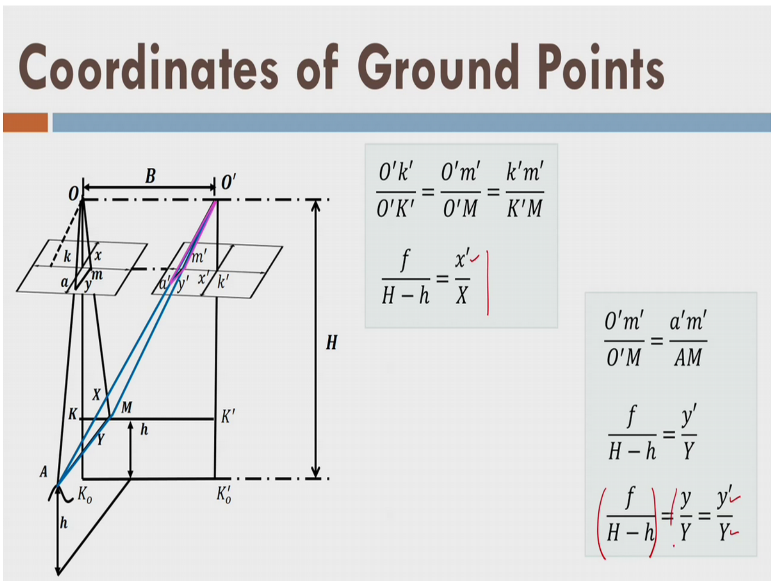

You see in the second photograph again this is triangle, this is similar triangle and so, I can write this thing ok, I am using the same time here and I am writing this thing. So, for the second photograph it is x dash.

Similarly, I can write with this triangle shown here you see this after this triangle and this triangle they are similar triangles. So, I can write the y dash also fine. So, that is the idea here that using y dash I am writing the Y knowing the scale photograph, also this is equal to this.

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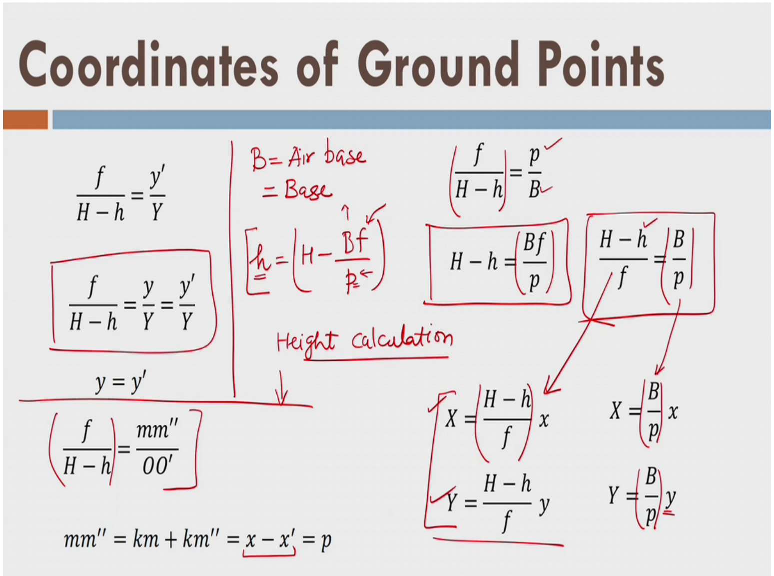

So, we have find out the ground coordinates in some way that is my this one that is also true. As, we proved in last slide so, y equal to Y dash like this and it is my parallax here, you see in the foot just look for the animation again. So, I am writing here p this term. So, f upon h is equal to p by B. Similarly, I can write that B into f I am multiplying here by p ok. What is this process if you just look at carefully? This process we start with the height calculation, here we have done the planimetric coordinate calculation that is x and y calculation right from here we start the height calculation. So, we are writing this way. So, I got that this expression; that means this is my X and Y planimetric coordinates and here I can say this is my height information h.

And I can also write this way, yes using this information now I am writing this you can see here this I am replicating here. So, B by p here, fine. Similarly I can write Y equal to B upon p into y. So, it is basically I am doing all the calculations together. Here, you can see here very carefully I can find out h as H minus B f upon p and p is nothing, but a parallax of any point. So, I am deriving a generic expression. So, this is the height of the point. So, once I know the height of the point knowingly that this is my parallax this is focal length and this is b.

B is nothing, but it is called air base, the distance between 2 successive exposure of camera it is called air base or a sometimes it is called the base also or distance this distance also in some of the books we will find out the distance of base distance base air base like that. So, now, you can see that I have done the calculation simultaneously for height as well as X and Y coordinate. And, here this is my X and Y coordinate and here it is my third dimension set the H height of a point you just re look into the animations again if you have any doubt. But, remember these are the very straight calculations fine. So, I am writing here is the scale of the photograph and I am deriving what is p B and everything ok.

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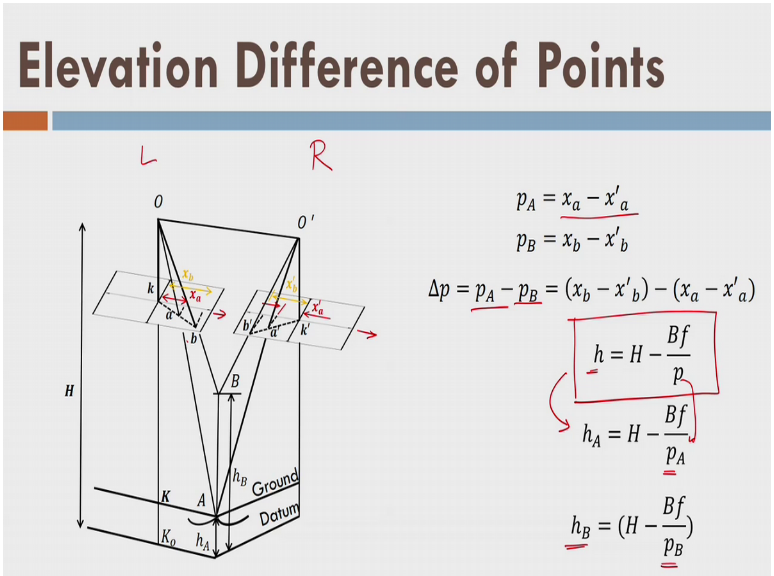

So, let us start again now I want to find out the elevation difference of 2 points. So, I have already shown you how to calculate elevation of a point. Now, I want to find out the elevation difference and there is a advantage that will see now. So, let us see there is a datum and there is a ground and there is a point A, which is height h above datum now point B is there which is exactly above the A and this has height h B.

So, O point is there and it is a exposure station and so, this is my image this another exposure station O dash and this another image, which is we call consecutively captured images right. This is the vertical line, principal point k, principal point k dash. Now, this is my flying height H. So, there is a point k ok. So, try to acquire point B. So, I have image of point b like that, in the left photograph call it a left and call it right. Then, I have image of point a as A fine, then I have a dash and b dash in the right photograph like this.

This is my base. So, this is my point b a dash and this is the point b dash on the image. By animation it is the purpose is to make you clear how that imaging has happened or the how images are acquired, but it happens in one single flash or single attempt like this it exposes the camera exposes and the light comes and it acquired. So, fast in microseconds milliseconds, but here the purpose of animation is to show that how things are developing or how we are thinking about the whole process right. So, now can we write the equation of parallax.

We can see here we are now talking about the relief displacement in some sense, you can see that point A or point B, because of the height difference let us say that it is a building and building height is represented by a and b. So, I can see what is the change of the location a, a has come to b because of the relief displacement of the point at a, because it has a height h B minus h A test concepts are there they are very very intermix concepts very nice concepts, we are derived about the relief displacement in last lecture.

So, now, we are using the basically the relief displacement in order to find out the third dimension or desired dimension, but using 2 photographs not different photograph just the idea here try to see. So, that is the x a coordinate. Similarly, I have x a dash coordinate right and you remember that we have calculated the parallax. So, this is my x b and this is my x b dash well.

I hope you agree with x b dash and x a dash. So, now, I can calculate: what is my p A it is this one. So, I am writing it in mathematical terms not in the numerical terms. So, it is like this. So, the moment I right algebraically in this particular case it will become x a plus x a dash, because x a dash is negative. If I take this direction as positive x it will become negative. So, if this is the positive x here right positive x here. So, it should be negative here right ok. So, I can write this parallax like this find. So, this is the kind of mathematical term here.

Similarly, I can write p B like this and again that difference of the parallax. Now, I am calculating delta p, which is difference of parallax A and parallax B given by this ok. So, now, I also know from the last previous slide, this is a reality or this is the formulation we have done before ok. So, now, I am writing the same expression for point A; that means, I am putting p A here and that is it similarly h B, I can write like this. So, if I measuring the parallax of point B I can write h B, if I measuring the parallax of point A I can write h A ok. But, I want to find out the height difference of A and B, that is the height of the building because it is a vertical height of a building right. So, let us look into that now.

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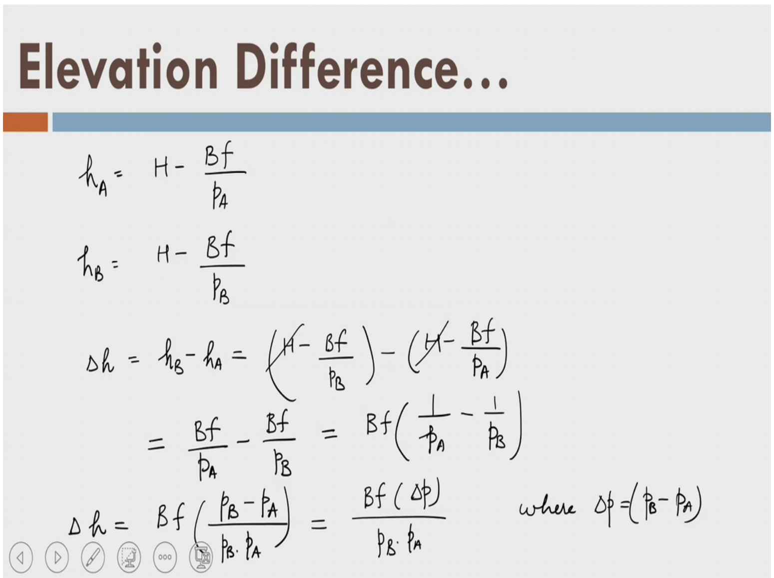

So, let us do some derivation by hand. So, h A is nothing, but h A minus H capital S minus B into f divided by p A. Similarly I wrote h B is equal to H minus B into f by p B. Now, I can say delta h, which is difference of 2 points h B, because h B is higher and h A is smaller than h B. So, I can right here H minus B f upon p B minus H upon B f upon p A, I hope you agree with that.

Further, if I calculate this term it will be actually cancel with H, because of the minus sign and I will have B f upon p A minus B f upon p B or B f into 1 minus p A minus 1 upon p B, I hope you agree with this term at least. So, we can do it yourself try to practice with me. So, at least you will also have a command on the derivation fine. Now, what next? I can write delta h this term as if I do the simple mathematics again b f into p B minus p A divided by p B into p A like this. I hope you are doing with me try to do yourself also right.

Let us call this term as delta B or the parallax difference not the height difference. So, delta p is my parallax difference p B into p A. So, I can write here where delta p is nothing, but p B minus p A right ok. So, I got a formula.

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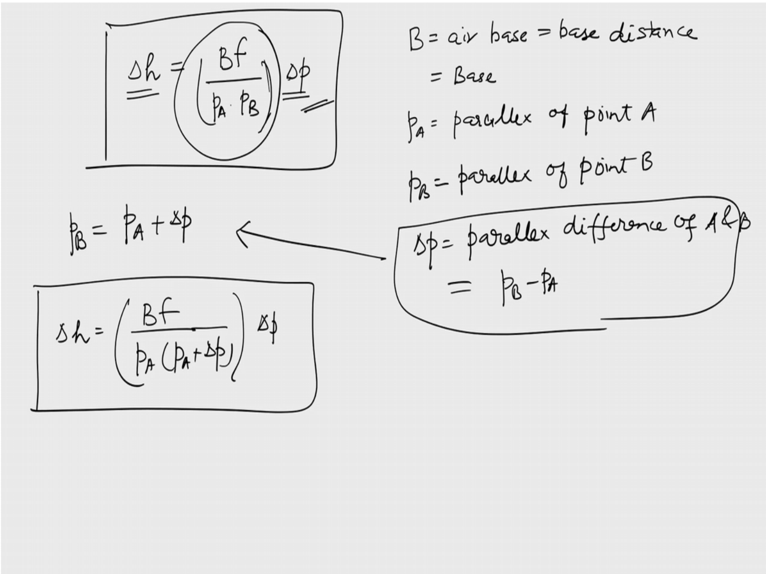

Where I write delta h in terms of parallax difference and parallax values I am writing it as del delta p. So, you see that how the delta h is a function of delta p; that is the height difference is function of parallax difference or I can see if I multiply the parallax difference by this term here I can get the height difference.

Where B is my air base again I repeat or the base height or base distance or the base also called base right p A is the parallax of point A, p B is my parallax of point B. Then, we have delta p parallax difference of A and B, which is nothing, but p B minus p A. So, let us look into this thing that is one expression I got it for my height calculation. So, I am using delta p from the image and now I am calculating the delta h ok. So, let us write p B equal to p A plus delta p from here I am writing it this thing this term.

So, I can replace the p B, because I want to have minimum number of variables. So, I can write here delta h; that means, I am replacing p B with p A plus delta p into delta p right. So, that is a expression where I have minimising number of variables p A is there p B was there I removed p B in terms of p A. So, I have delta p because I measured from the image 2 images I took and measured it by some way and then I have p A, which I already measured from the image. So, like this ok.

So, I can write this thing here that, this is the expression I have fine. So, let us look into another form of this and what is written in the books call alternative form, if you are reading the books suggested here.

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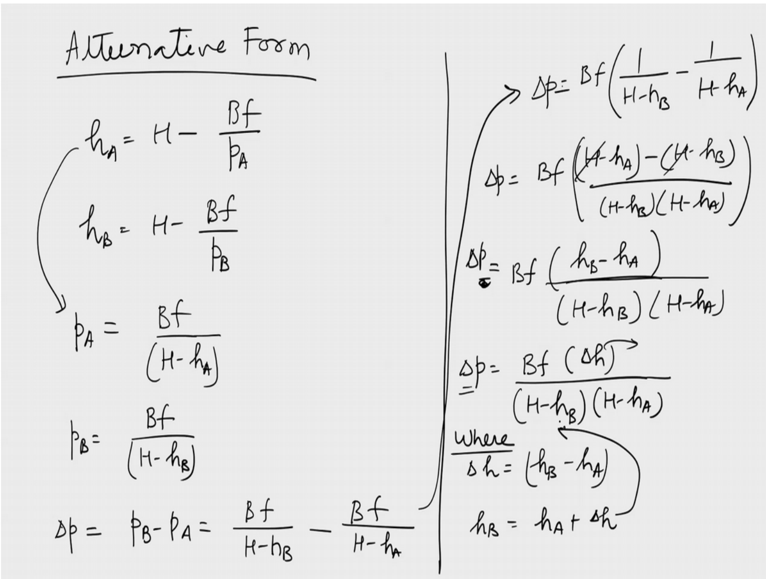

So let us look into the so, called alternative form it is nothing, but alternative in the books it is written alternative form, but no problem is another expression I can say. Again write h A equals to H minus B f upon p A and h B equals to H minus B f upon p B right.

Now, I want to do the same exercise, but in terms of not in terms of delta h in terms of delta p; that means, I want to calculate, what is the change in the parallax if that is change in the height? So, let us do like that. So, calculate p A from here I can write here see that, how to write it B f upon can I write this way H minus h A.

Similarly, can write p B B into f divided by H minus h B right; so, let us calculate the delta p it is nothing, but p B minus p A which is I can say here B into f upon H minus h B minus H minus h A. So, let us divide the slide and here from here again I have writing here, that delta p is equal to B f into one upon H minus h B minus 1 upon H minus h A or delta p is equal to I am doing simple mathematics nothing special.

H minus h A minus H minus h B divided by H minus h B into H minus h A. Again equal to B into f into h will cancel out here and then we have h B minus h A, then we have here in denominator H minus h B into H minus h A. I can write here delta p is equal to if you remember B into f into nothing, but delta h like this, where my delta h is difference of height that we already defined before also right.

Now, I calculated the parallax difference or the difference in parallax. Now, it is better to write again the h B and h A here. So, we can write here h B as h A plus delta h and again will put it here so, that I can minimise my number of variables.

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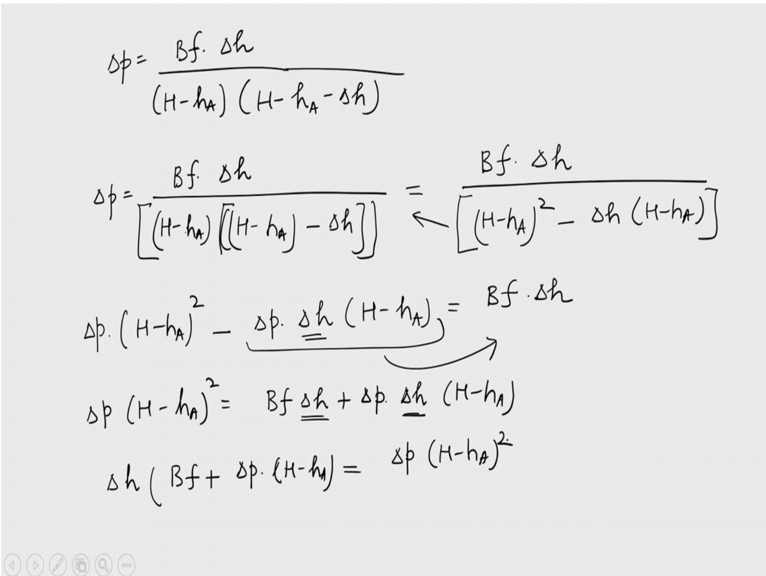

I right here now delta p is equals to B into f into delta h, which is here H minus h A into H minus h B I write h A minus delta h here fine ok. So, there I can calculate delta p is equal to B into f into delta h into and now multiplying it inside, H into h A, that is I am writing it like this H minus h A inside term minus delta h like this.

And, I like this B into f into delta h divided by H minus h A whole square minus delta h into H minus h A right ok. Now, do some kind of transformation shifting. So, I can write delta p into H minus h A whole square so; that means, I am multiplying it up here minus delta p into delta h into H minus h A equal to B into f into delta h right.

So, you have this term now. So, there I can easily right now that I want to calculate delta h basically in terms of delta p by this formula. So, let us shift this term on the side. So, once you shift it will get h A I take this term common then I can write here delta h into B f plus delta p into H minus h A equals to delta p into H minus h A whole square.

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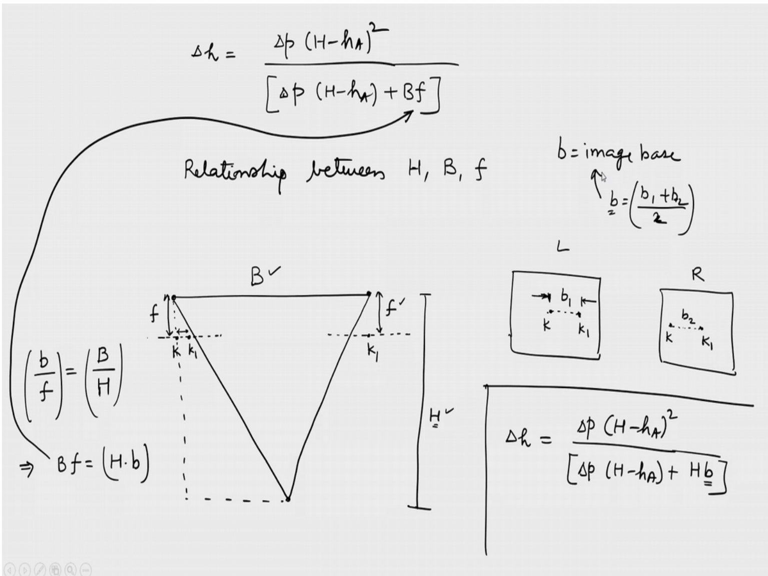

So, we expressed the height difference between the 2 points delta h as delta p, H minus h A, square divided by delta p, which is difference of the parallax of the 2 images for the 2 points like this ok.

So, can we replace the air base B with the help of flying height or in terms of flying height so, that we can reduce 1 variable in this equation. So, let us see what is the relationship between flying height H air base B, and focal length f ok? So, let us imagine that there is a air base B and there are 2 exposure stations of perspective centre of 2 images alright. So, from these 2 places point in both images are acquired a point is acquired in 2 images this is the left image here and that is the right image here.

So, this is my focal length from here to here and this is the flying height h here ok. Now, you can see very easily here that let us say that this is the principle point k of first image and this is the principle point k 1 of the right image. Now, says the 2 images have overlap the common overlap of 60 percent. So, what will happen if I draw these 2 images left and right? So, the left image will have point k 1 around centre. And, similarly it will have point k 1 along the flight direction.

Now, this point will this image will have point k here and at the centre it will having k 1. So, if we align in the flight direction, which is this and this here what will happen the distance between the 2 points here in this one image is my image base b ok. So, this image base b is just an representation of my air base B. Why? Because, that the real train or the 3 D condition or the 3 D topography of the train, it is you know image in an image it has been photographed in an image with the help of focal length. So, my focal length is basically representing the flying height H alright.

So, from here you can develop some kind of triangles and then using this triangle logic I can say that the air base B and the image base B are basically connected, but how are they connected? Now, if it take the ratio of B air base to flying height, I can see the same ratio exist between the focal length and the image base alright, because this point k 1 let us say is here right. So, this is my image base B from here to here and now I can write it very easily using the similar triangles that b upon f is equal to capital B by H.

Because, they are in the same ratio why because they image geometry which is using the focal length f, it is recreating the same scenario at some small scale and that is why we can write this ratio easily here. And, you can also prove it by the similar triangles. And, as a result now, I can write that B into f is equal to H into b here. And, now I can replace this term here in this thing and I can reduce 1 variable. So, what is a final result here? That delta h is equal to delta p, into H minus h A square divided by delta p into H minus h A plus H into b here fine.

So, how to find out this image base b? We call this image base this term b here and how to find out? Well we can take in practical how do we do it with take the 2 points or at first we try to find out the location of the principal point k 1, which is the principal point of the another image. So, we find out at 2 locations k and k 1 in 1 image and then be we measure the b for the left image. Similarly, we also measure the b for the right image. And, then we take the average of the let us say b 2 and b 1 and then we define b equal to b 1 plus b 2 divided by 2. So, in the real practical case we find out b like this and we call it the image base here.

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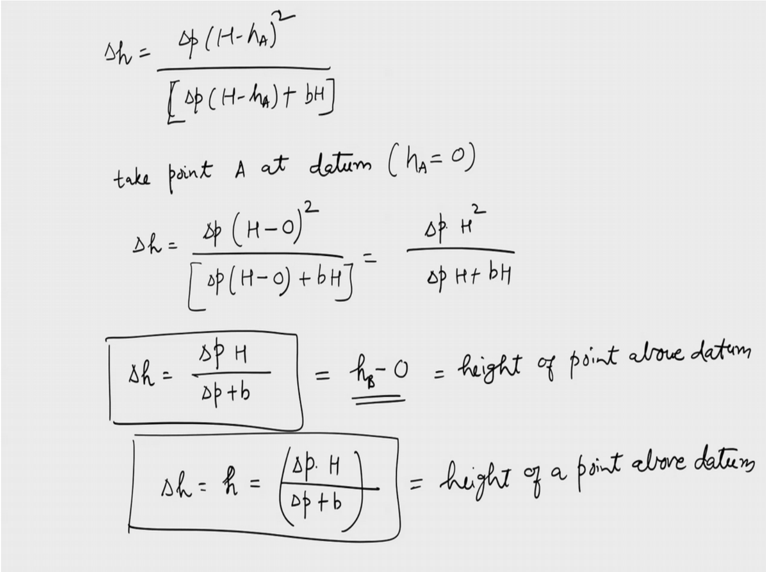

Here I am writing here, this way delta h equals to finally, what we got here delta p H minus h A square divided by I have here delta p into H minus h A, we have derived it today this term into b H. Now, what if I take point A; take point A at datum right. So, I will it will give me the height of point b with respect to datum or point A because point A is on datum. So, I will get the height of point b with respect to datum. As a result h A becomes 0, because we are put it on data and we are measuring every height from datum, where the height of datum itself is 0 into.

So, now, I can write here delta h equals to let us say delta p into H minus h A 0 whole square divided by delta p H minus again 0 plus b into H. Or this nothing, but delta p H square into delta p H plus b H, or I can write delta h equals to delta p now you can find out very simple term we are cancelling h with h on so, I have get this term. So, this is nothing, but h B minus h A. So, h A is 0. So, this is the height of point B above datum.

So, I can write any term like that the height of any height point let us see I call it any point h equals to delta p into H divided by delta p plus b. I hope you agree with this term. So, what we are getting here? So, this is a basic very simple formula finally, we have devised. So, it is nothing, but the height of up point above datum right ok.

So, what is the interpretation here? Interpretation is very simple. And, interpretation is that if you assume any point at datum; that means datum is passing through point A and point A itself is on the surface of earth.

That is a building bottom of the building you assume that the datum is passing through the bottom of the building. And, as a result you can find out, what is the height of the point with respect to the bottom of building. So, even you can calculate the height of a tree with respect to that bottom of building. And, what is the height of a top point of the building? Because, you are putting your datum at point A which is at the bottom of the building so, that is the idea here and that is the interpretation of this formula. Now, what about the accuracy of your height measurement; so, let us look into this thing.

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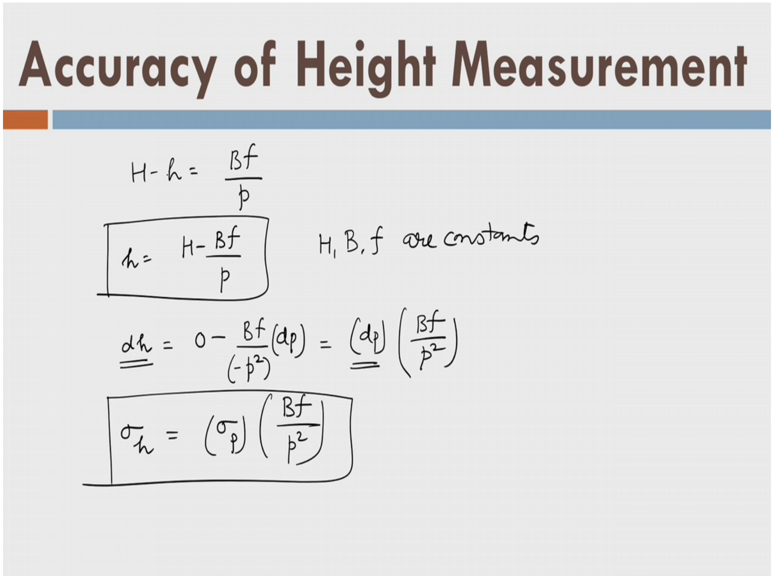

So, it was like this kind of term we have derived B f upon p or I can write h is equal to H minus B f upon p. From here I can write the d h equals to I am differentiating it 0 minus B into f upon p square and I can say here minus into d p right. So, I will get d p into B f upon p square. So, that is the error in the parallax, that we give me this kind of error in height measurement that is the idea ok.

Similarly, I can write here if you remember the law of error propagation you use it on this formula, assume B and f are constants; then you can prove that sigma h is nothing, but equal to sigma p and B f 1 p square will come try it yourself I leave it to you. So, one more expression is there of the height measurement.

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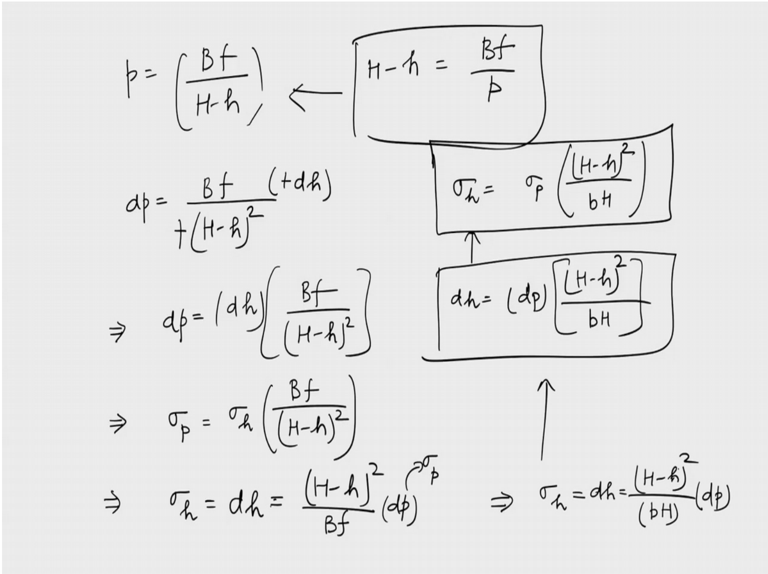

That is nothing, but now write p is equal to B f upon H minus h from the formula, we know that H minus h is nothing, but equal to B f upon p. So, I am writing from here this formula. So, there I can also say what is my d p? If, I differentiate it B constant f constant, but H is not constant, so I write H minus h whole square minus and minus b h right.

There I will get it d p is equal to again minus and minus will go and then I have d h into B upon f into H minus h whole square. So, I can write here also that sigma p is nothing, but sigma h into B f upon H minus h square. And, there I can write it sigma h or d h is equal to H minus h whole square upon B f into d p, or I can write sigma p also right. There finally, I write sigma h is equal to or d h is equal to minus H minus h whole square divided by b into H. This is nothing, but B f into d p. So, finally I write as d h is also given by d p into H minus h square divided by b into H here.

And therefore, I write here sigma h is equal to I can also write like sigma p into H minus h whole square divided by b H like this, and this 2 formulas we have. So, there I have 2 expressions and in the books it is written as unit height or unit change method. In fact, it is nothing, but unit change I am doing try to find out what if I make a unit change in the parallax. So, what will be the change in the height, that is if and that is why they name it unit change method. But in fact, we are just doing a differentiation or we are using law of error propagation to find out the error in the height measurement given the error in the parallax measurement ok. So, let us look into the how do we do this thing using some instrument?

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So, this is the instrument what we call as stereoscope. We have 2 type of stereoscope: one is called mirror stereoscope, that you put 2 images and remember that 2 images are aligned in a way that flight lines; that means, if I go back to slide last slide. That is blank slide.

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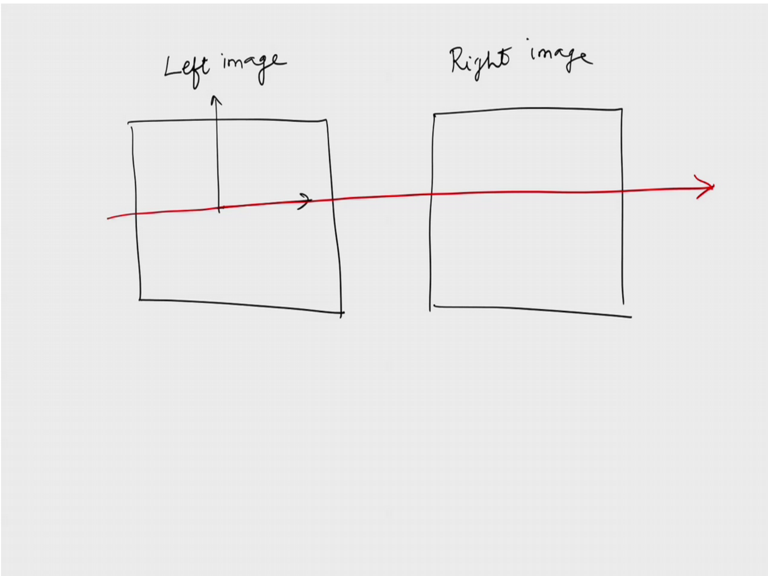

So, that 2 images 1 is left image call it left image and the right image. Here, in the all derivations we have assumed this is my flight line like this is my x and y axis or this is the flight line from here like this; that means, here I assume that I have already aligned the flight line of the 2 photographs, which are consecutive acquired in a same flight.

So, by that I say that this once you align the flight lines of 2 photographs and you keep it like this, then you can do this operation using stereoscope. So, there we do what do we do here? We put these 2 photographs such that their flight lines are aligned correct. And, then once you align it you can see the 3 D view using the mirror stereoscope. In the field mirror stereoscope is difficult to carry, although sometime to will carry it if work is so intense. But, in a field you want to have kind of a relief idea how terrain is varying in 3 D.

So, you use pocket stereoscope, which is of very few grams 50 grams even less than that, mirror stereoscope is little heavy around 1 kg or so. So, you carry the pocket stereoscope in the field you take 2 photographs and you tried to see how the terrain is varying and you take some decisions in the field. And, that is the instrument used for stereoscopy. Remember that you have to align 2 photographs in such a way that the flight lines are along the straight line. And, that comes by trial and error by experience, I hope if you have some instruments in your college or university you should try it.

And, by for taking the photograph; now you can use any camera like mobile camera and try to develop in your room put 2 objects, which are slightly away from each other having some kind of depth difference and then try to acquire the photograph for 2 places, one from here one from here. So, this is left and right photograph and take the printout of those photographs on a simple paper, put on the mirror stereoscope try to align the flight line, which is from here to here this is my flight line and try to see, whether you can see the depth of the 2 points or not.

And, you can also find out the relative height of 1 point assuming that another point is at 0, try to do yourself is very interesting exercise. Stereoscopy can also be done with the help of computer and what we call computational photogrammetry. And, that computational photogrammetry is also called analytical photogrammetry. And, that we have to learn it. So, in the coming next 2 lectures l 4 and l 5, we will learn the analytical photogrammetry, that how computer does these same operations, but it will use some kind of mathematics matrices. Because, computer does not understand, what we say or what we understand. It understands language of mathematics. So, in the coming two lectures we will talk about analytical photogrammetry.

So, till then thank you, bye.