Higher Surveying
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Module – 6
Lecture - 16
Photogrammetry

Hello everyone welcome back on the course of Higher Surveying and we are in module 6 on Photogrammetry. In the last lecture of this module, we have discussed about lot of scattered information, which was very introductory in nature. And, then we realise that what are the types of platforms, what are the types of photographs? And many many terms like basic terms aperture stop, focal length, focal plane and so on. Now, in this lecture or rather from this lecture, we are going to be very very specific for a particular aspects of photogrammetry ok.

So, today we are going to talk about vertical photogrammetry. So, this is our lecture lecture 2 on vertical photogrammetry, if you remember or if you can recall the last lecture we talked about different types of photographs. First was vertical photograph, tilted photograph and oblique photograph. And, there in the vertical photograph we said that the nadir line is coinciding with the optical axis of camera.

So, that was our idea to categorise the photographs into 3 categories. So, if optical axis matches with nadir line or the gravity line or the plumb line of camera, then we say that it is a vertical photograph. If, it is tilted in a sense optical axis is tilted within 3 degrees from the nadir line we say the tilted photograph.

And, if it is more than 3 degree we say that oblique photograph. Well we will not go in to the bifurcation further, but we will try to understand the small nitigrities of these aspects today. Also, we are going to understand technically, what does it mean and how we can make the map using these photographs? Especially the vertical photograph and tilted photograph, because tilted photograph is a kind of tilt that is introduced unintentionally unknowingly, but we know what is a title? So, tilted photograph and vertical photographs are treated in this lecture.

So, let us go ahead. So, these are the books again well first 3 books are again still sufficient for this lecture.

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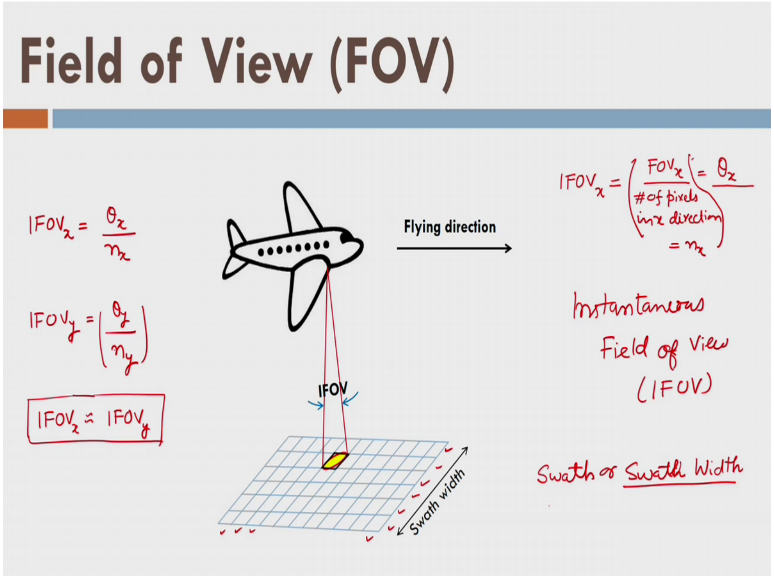

Ok. So, what is the field of view? If, you look carefully again in case of vertical photograph metric, where we say that a camera is mounted on an aircraft. Not in the other platforms. For example, terrestrial platform, we have then low altitude platform, we have aerial platform, and we have satellite platform.

So, in this case of vertical photogrammetry, we are saying that camera is mounted in aircraft or helicopter; that means, we are doing aerial photogrammetry.

Like this, this is my flying direction ok. So, this is what we call a pixel which is shown by a black colour over there right this is your pixel on the ground not on the sensor. Generally on the sensor we call 1 unit as pixel, but when it comes to the ground; that means, it acquire some area on the ground surface, then we call it the size of pixel as ground sampling distance or GSD right.

Secondly, I am not talking about digital cameras because they are the only popular one now; all the film based cameras are more or less obsolete. So, we are going to talk about now digital camera in the photogrammetry.

So, now we have a digital aerial camera, mounted in aircraft and that is acquiring one scene and this scene is explained by these pixels. So, in one scene it is acquiring it is a frame camera. So, it is making one exposure and acquiring this scene. So, in this scene I can say that so, many pixels are there and one pixel size on the ground is called GSD Ground Sampling Distance ok.

So, now, this angle which is along the direction of flight it is called theta y and putting it by a symbol and this angle is nothing, but the field of view along track and that is called a flying direction ok. Similarly, we have another field of view, which is called field of view across the track or across the flying direction ok.

Now, if you look at again the one GSD this pixel on the ground is going to subscribe some angle on the camera or on the sensor right. So, let us see this is the angle there. So, each of the pixel is subscribing some angle on the sensor, surface or a camera perspective centre. We will look into these further terms. So, just wait for few minutes. So, now the angle subscribed by the one GSD or one pixel on ground is called instantaneous field of view ok. If, I write it like this Instantaneous Field Of View or IFOV so, this is my IFOV ok.

So, this distance which is across the flying direction is called the Swath or Swath Width both names are popular we call it Swath or Swath Width both are the same ok. So, now, you can see that the instantaneous field of view in x direction can be calculated by the FOV in x direction or what we have written theta x divided by number of pixels in x direction alright fine. So, like this ok. So, I can write here let us say the number of pixels is equal to n x. So, then I can write that on this site the I F O V x equals to theta x divided by n x alright.

So, I can say here that these number of pixels are equal to from here to here there n x number. So, I can find out this. Similarly, if number of pixels here from 1 to let us say this number is n y. So, I can write I F O V y is equal to theta y or the F O V in y direction divided by number of pixels in y direction. One should note here, that in general c is the is the pixel size on the sensor is same for x and y direction; that means, we have square size of the pixel. That is why the F O V in x direction and number of pixels in certain direction x or y they are, basically decided or designed according to this criteria alright.

So, that we have we will have the square size of the pixel on sensor, as well as that square size of the pixel on sensor should create an, square size of grid, or square size of box or GSD a Ground Sampling Distance on the surface of earth alright. Let us come to the vertical photograph now the previous slide I assume, that camera optical axis and the nadir line or the gravity line are matching let us look into what does it mean.

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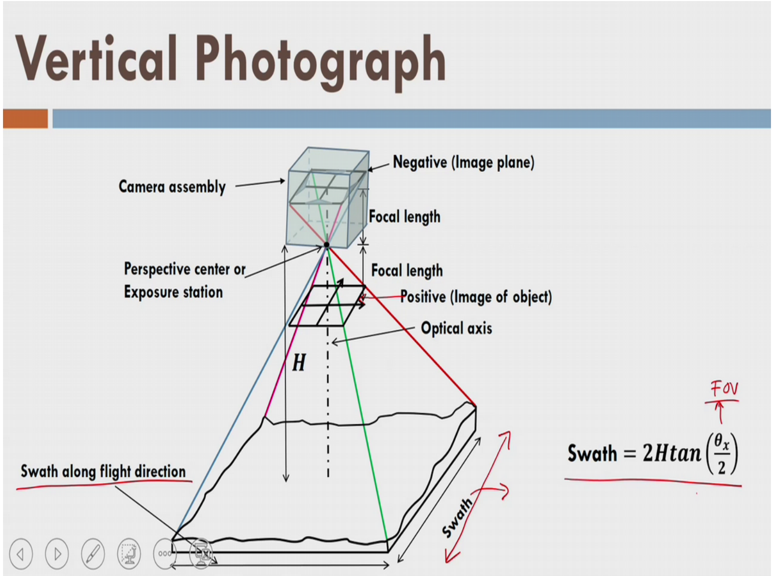

So, let us say there is a terrain where this distance is nothing, but a swath for example, ok.

So, let us say this is my camera assembly ok. So, this is my flying height H right. So, this is my negative image plane and this is my exposure centre, where all the rays from the terrain are going to meet and they are going to again divert towards the image plane. So, this is the optical axis here shown here fine.

So, this is the focal length as you know already from last lecture right. So, this is the way all the rays are converging to the perspective centre and again they are diverging to the image plane. So, and their creating an impression on the sensor or the image plane or we can say negative plane. Now, since negative and positives are no more a valid term, but still in the books these things are given and still we are going to use these in order to distinguish between a negative plane, that is a sensor plane or the image plane which is a positive plane which we really see.

Because, as we know that negative plane image is a inverted image and that we cannot work with and that is why we need a positive image. So, this is my positive image here right shown by here, and which is also located at a distance call focal length from perspective centre. So, that is overall idea you have and you should develop this idea first; that means, what is the mechanism of data acquisition for a given one exposure of the frame camera ok.

So, that is my Swath as already told well, what about this distance? This is also called the swath, but swath along the flight direction. So, this is the swath, what we call swath is my swath across the flight direction.

And, generally we do not write across the flight direction, we rather only write the swath. So, this is called swath the whole term. So, whenever you want to specify this swath which is along direction you have to specify like that ok. So, now, I can write that swath is equal to 2 H tangent of theta x by 2 because the across the flight we have theta x is my field of view, total field of view, it is very simple relationship you can find out yourself also.

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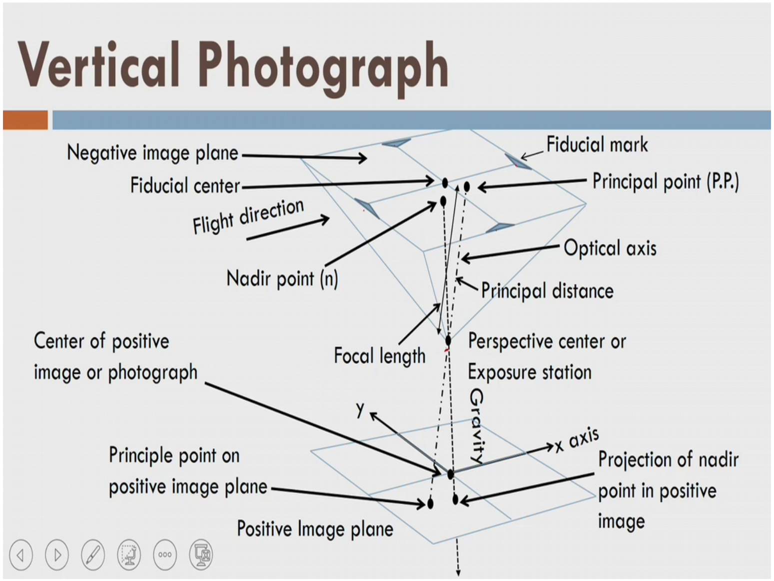

Ok. Let us look deeply categorically into the vertical photograph mechanism. So, that is my negative image plane, what I call and this is why this is my perspective centre or exposure station this point ok.

So, this point originally when we use to have the negative planes and negative images. So, once we develop those images then we have 4 marks. So, these 4 marks were called the Fiducial marks, which are at the centre of the edges of an rectangular image. So, if them connect them by straight lines they will intersect at point and that point is called the fiducial center of the image; that means, if I take 4 fiducial marks I should write all fiducial marks now. So, once I intersect if I line draw the line through the fiducial marks, which are opposite to each other then I will find intersection point and that intersection point will be called fudicial center let us go ahead.

So, this is my optical axis right and we can see here that fiducial mark and optical axis are not meeting at a common point. Why, because I am assuming that although it is vertical photograph, but there is some random error in the optical axis. And, as a result it is not matching with the fudicial center of the photograph. Yes. What about the random error, if you can recall the concepts of the adjustment computations and our error accuracy. It is an inevitable error that is random, it is out of beyond my control or I cannot measure it I do not know it is true value. So, still I am taking a chance and I am putting it slightly away to show that right. So, this is the point where the negative image plane and optical axis intersects this point is called principal point or it is also written sometimes P.

Next is this is the principal distance, that is the distance between the principal point and the exposure station along the optical axis and that is called the focal length, fine you agree with that ok. Now, that is my positive image plane, which is just shown here, fine. This is the against Fudicial center mark, Center of positive image or photograph or I can say the that is a Fudicial center on the a negative place same way impression of that or the same point I am marking on the photograph by taking the intersection of Fudicial marks, then I will get center of the positive image.

Then, if I extend the optical axis down towards the positive image it will interact somewhere here. So, this is my principal point on the positive image plane ok. So, let us draw some axis x and y like x axis and y axis to the photograph. So, that I can refer any point in the positive image like this is a Gravity direction. So, now, you can see even the Fudicial centre of the image is not matching with the nadir point the nadir point small and in the image is a reflection or is it the image of the real nadir point on the ground, where gravity direction is going to intersect with a ground fine.

So, that is a kind of mechanism of the vertical photograph that is we should understand before we go ahead. So, if you have any doubt just go through the video again to understand these things or one thing I would like to say here. So, we have realised by this photograph, that my optical axis that is need not to match with the gravity direction or there is a random error that is why we are not matching them. And, we know it that there is a random error although we cannot measure it.

Secondly, optical axis itself is not matching with the Fudicial center of the negative image right, but still optical axis gravity line or any other line they are passing through the exposure center right. And, exposure center is a place order point where all the rays from the object are coming and they are converging and then again they are diverging towards the image plane.

So, that is the idea here ok. So, this point let us say is the projection of nadir point in positive image.

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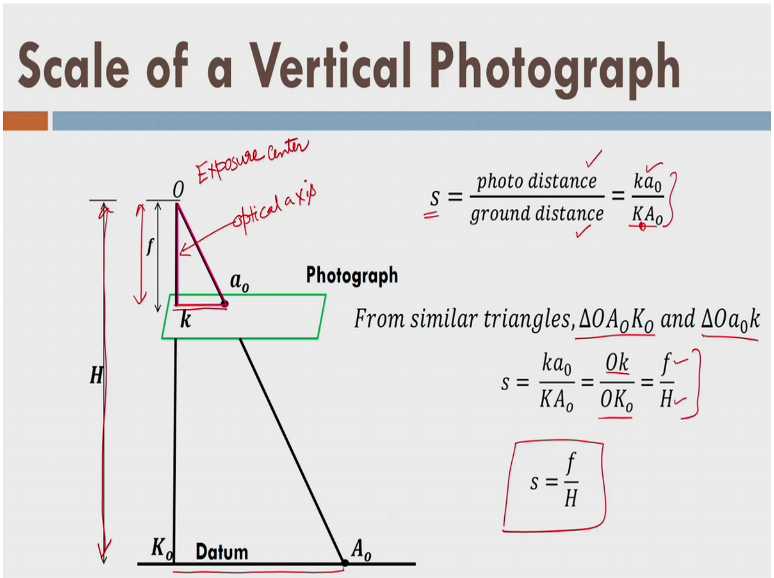

So, now, we can understand how are they different? So, let us talk about the vertical photograph and the scale; that means, if you remember the scale of the map from the basic survey, what is the scale? A scale we define as the distance on the map to the corresponding distance on the ground surface. So, same concept is there, but now we are going to do for the photographs. Moreover, we are taking the vertical photograph third thing we assuming that our optical axis and the nadir line or the greater gravity line they are matching exactly with each other.

That means we are talking about the theoretical derivation and there is no harm in doing this derivation, because it is going to simplify many aspects of my work and understanding. And, that is the reason we assume that there is no distortion in the optical axis or the principal point. So, principal point is matching with the Fudicial center of the image.

So, let us see there is a datum then we have A point a here call A 0, then we have optical center O here and this is the positive image and the small a 0 is a image of point capital A 0 in the my photograph. So, let us say this is k is nothing, but the optical axis O k this is my optical axis here. And, that is also matching with the gravity line or the plumb line passing through the exposure center O.

So, this is my exposure center or respective centre ok. So, now, this is the scale the distance on the photograph to the distance on the ground; that means, if I take 2 points on the photograph, then if I take the same points on the ground and, if I calculate 2 distances and if I take the ratio of the 2 distance is like this, this will be equal to the scale of the photograph. Because, I am assuming now that I am going to use my photograph to make the map. So, I am going to find out what is the scale of my photography so, that I can measure distances directly from the photograph so, that is idea k a 0 if you look at here k A 0 this distance this is divided by K A 0 that is it should be K 0 A 0 here.

So, now let us develop some simple mathematics of similar triangles first is this triangle and second is this triangle ok. So, the first bigger triangle is there shown by blue colour now and the second one is the small triangle shown by the red colour now right ok. So, what about s it is nothing, but O k divided by O K 0, why because that is my O k distance here, which is equal to focal length you can see here. And this is equal to the flying height of H. Why, because I can say that this distance, which is this and this distance, if I take the ratio of do like this, they are equal to this distance focal length and this flying height this distance and that is written here.

So, my scale of a vertical photograph is equal to the ratio of focal length to the flying height ok. So, now, we give a s is equal to f by H.

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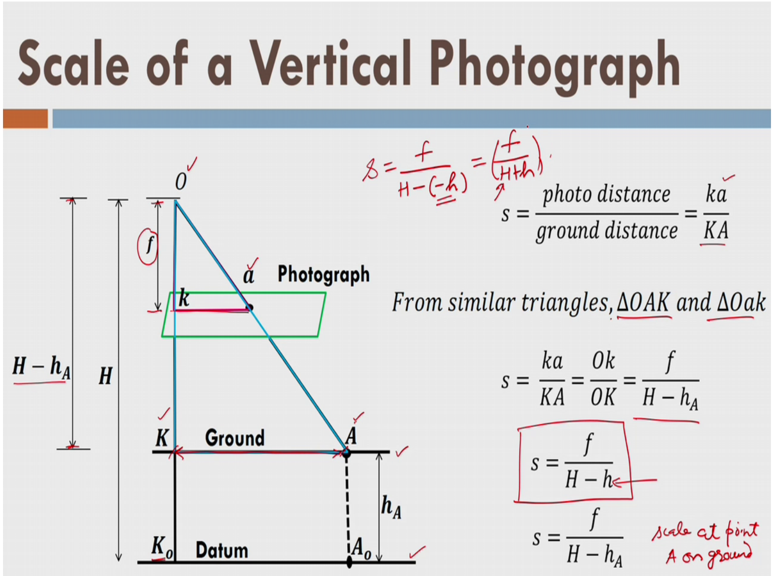

Ok. So, what again that scale of a vertical photograph once again we are talking about. Let us say this a ground surface like this and there is a datum like this; that means, I am measuring my flying height with respect to the datum. So, if you see that there is a point on the ground which is A and on the datum it is projection is A 0. So, if you look at here fine. So, this my point photograph is there this is the a point that is image of this point is this ok. Similarly, this is my optical axis that just I drawn and then if I say here that k point is my principal point or capital K point is the projection of the principal point and K 0 is projection of principal point on the datum.

So, datum has certainly some height as you remember from basic surveying we measure the height from datum. So, now, I am saying that there is a plane datum and I am measuring the height of point A. So, this is the height h A of point A with respect to datum. And, as a result of that what happens in the scale just see now. So, I have the flying height H and if you see here this height above the ground of point A it is H minus h A right fine let us go ahead again. So, my distance now the k a is this distance and distance on ground is k a which is from this point to this point, this distance right.

So, again I will use the similar triangles, but there is small difference in the scale expression now, because of the point A is above the datum and with the height h A. So, let us look what is happening here? So, again use similar triangles, 2 triangles I am using so; that means, one is this triangle O K A, you can see O k and a. So, I can say the scale with the same expression as by H, now I am writing this ratio this f here which is nothing, but this distance from here to here and then this distance, which is from the exposure center to the point. So, this is the scale I get at point a. So, I can write in general for any point of having h elevation I will have the scale that is what we call the point scale, the scale at that particular point.

So, I hope you get the idea what is this so, at point A I have this scale, A on ground right. So, because h A is above the datum so, you will get better scale or higher scale on the photo. Similarly, what if the point A is below the datum? At, that case what will happen if I write in general a simple formula that is s is equal to f upon H minus of h. Because, this height h is measured below the datum that is why I am putting minus h and as a result I will get H plus h. So, if a point is below datum I will get inferior a scale or smaller a scale.

But, if a point is above the datum I will get the better scale or superior a scale. Remember, that we always needed the flying height with respect to the datum, because on the ground we do not know anything before going to the field, but from the map I know where is my datum? And, that is the reason we always take the flying height with respect to the datum. So, with respect to datum when we take the height H, then in that case this a scale is calculated ok.

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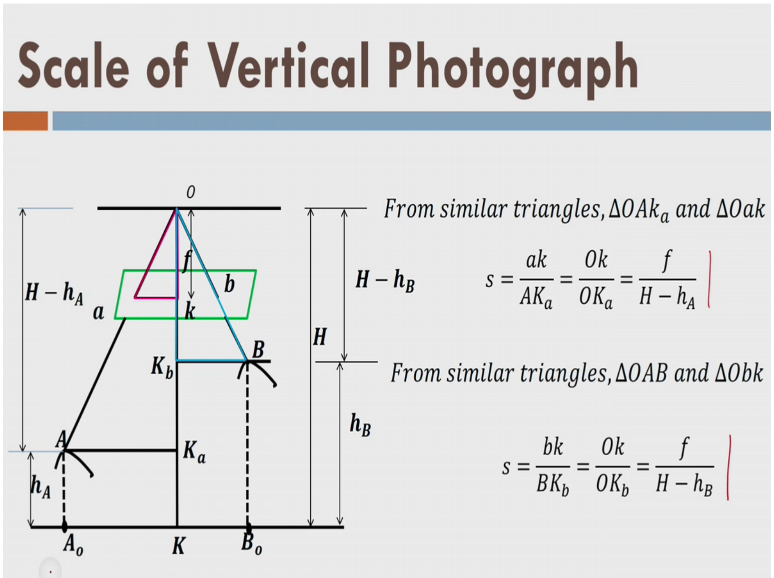

Let us go ahead. So, again we talk and these scale of the photograph here I am showing the 2 points you can see here this triangles. So, at point A, I have this is scale and at point B, I have this scale got it I hope you are very much familiar now with a scale idea ok.

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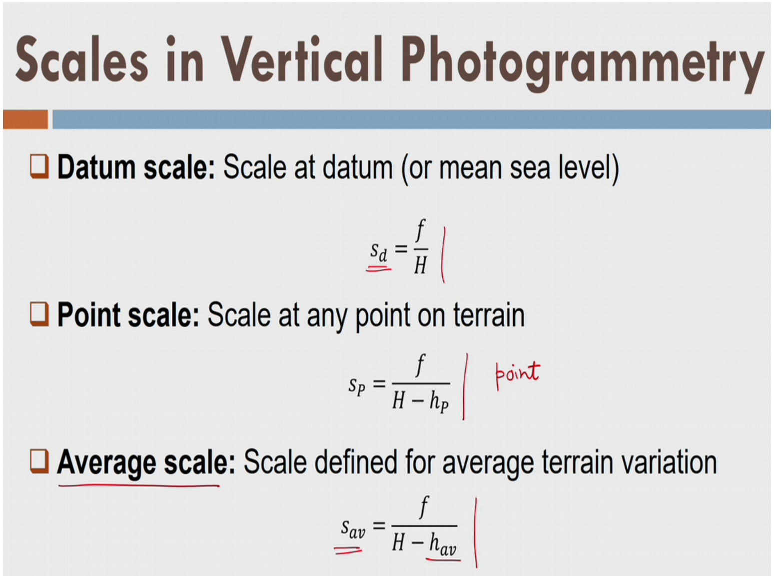

So, let me define some different type of scale. First of all I define a datum scale, which means a scale that is available at the datum; that means, if a point a is lying on the datum that will be the scale here. And, we call it S d that is a scale at datum ok. What is the point lies above the datum as we know this is going to a scale and that is call the point scale, because this varies from point to point ok. Now, as we know that terrain is always wearing like this or maybe some other differentiation is other variation is there in the terrain, because of that we will have different different elevation at each point. So, each point will have a different elevation, each point will have a different scale.

But, as you know that map is always having a uniform scale. So, I need to derive some uniform scale. So, what will I do? I will be derive a term called average scale where I will take the average value of the terrain elevation; that means, everything maybe I am averaging all the points. And, then I am taking the average elevation and so, I am defining the average scale. Remember, that average scale is not the final scale or it is not an absolute scale.

Definitely from each point scale will vary, but still in order to make a map I need to define an average scale and that is why I am taking average elevation h a v from most of the points. So, this is a preferable scale when the variation on the ground surface; that means, the height variation of different points on the ground surface are in same order right. And, accordingly it will side the scale. So, if the variations are very high for example, 200. 200 metres; in that case this average scale will not carry any meaning why because 1 point definitely having very high scale, which is on the top of a hill and there is another point which is on the ground surface, which is having very low scale.

So, if I take the average this average scale is not going to represent the real ground. And, that will give me the map created by such average scale will give me lot of errors. So, we should be very careful, while we use any scale ok. Let us go ahead now.

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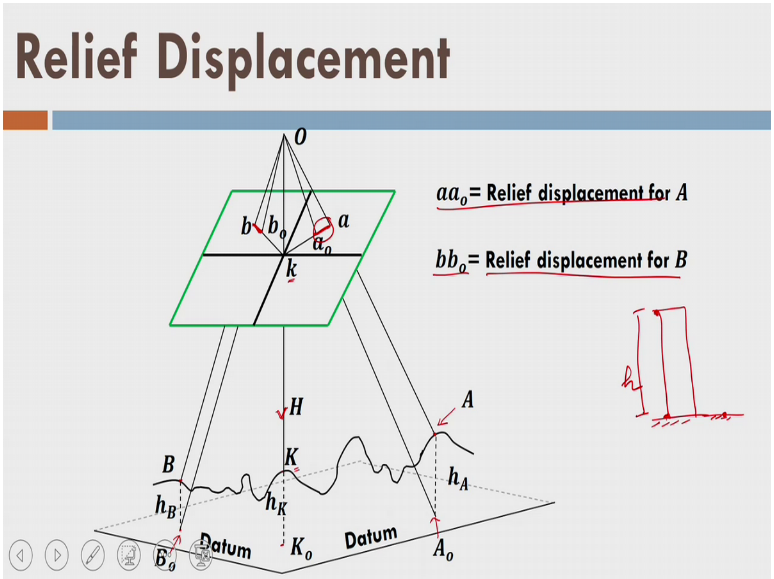

So, let us talk about the some kind of displacement that happens that are evident on the vertical photographs. So, first is the relief displacement. So, let us see there is a datum like this datum surface is this way fine. And, from datum surface if I measure the height h A and h B for 2 points A and B on the terrain so, I will have these points h A and h B. So, they are the elevation from the datum. Now, let us see the B 0 is this point here. And, similarly I have A 0, which is this point ok.

So, let us say there is an image and there is a exposure station and so, this is the optical axis, which is piercing here the datum here at K 0. And so, I have a principal point K here and the impression or the point principal point if it is touching the ground surface if

I extend the optical axis or the nadir line. So, in the gravity direction this is this point here fine ok.

So, the height of the point k with respect to datum is h K. Now, the point A here this point it is imaged here and that point I call a, similarly for point b, I call point b here right. Now, what if I make the image of point A 0, which is a datum maybe not may not be visible to me, but still it will create an impression of A 0 here. So, this is a point my this point is my A 0. Similarly this point is my B 0.

Let me mark it b 0. So, you can see that because of the height of the point A or point B above the datum, you are getting the image of the point B at slightly shifted; that means, the point B if here at the b 0, then it should create an image here fine as you can see, but the point B is at height h B above the datum. So, it is creating image at B right.

So, either result I can see that the image of point B is shifted from b 0 to b because of the height of the b or height of the location b with respect to datum. And, I am still measuring the my flying height above the datum right. And, as a result now you can see that each and every point, which is above the datum, will shift in the photograph from it is bottom of a location.

So, you can see that if there is a building which is at ground surface. So, it is point on the ground surface will have different image than the top of the building on the image. And, that is what we call the relief or the relief displacement or I assume that building and this building does not have any height; that means, which is ground only right. This point will be imaged somewhere in the photograph, this point will be displaced because of this height let us say h.

So, this idea and this displacement image of the point, because of it is height above the datum is called relief displacement. What is a relief? Relief is some height and because of the height image of that point is displaced from it is bottom let us say like that. So, this is my flying height H. So, line k b 0 b I can say here.

So, this is my relief displacement here which is written here. Similarly, I have relief displacement bb 0 for point B this is this distance.

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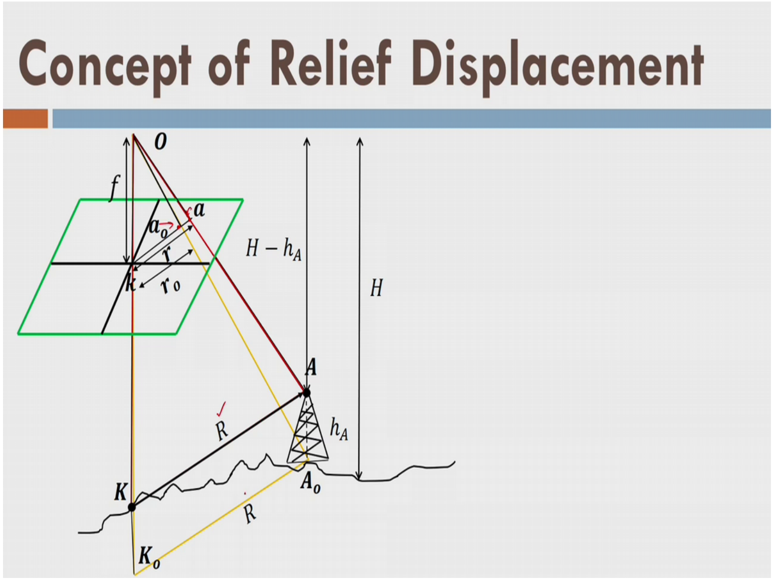

So, now, let us try to understand what is the concept again. So, let us say there is a tower and as a result I want to do some mathematic derivation. So, this is the one triangle. So, I can say that point r 0 it is showing the location of a 0 here right let us look into this thing. So, this distance is R and again this distance for point a 0 this is this point a 0 and this is point is a here. Now, I can see r and r 0 are the 2 distances radial distances from the principal point.

So, now if I draw the 2 similar triangles so, you can see here from the bottom also this distance is R and this distance also R right.

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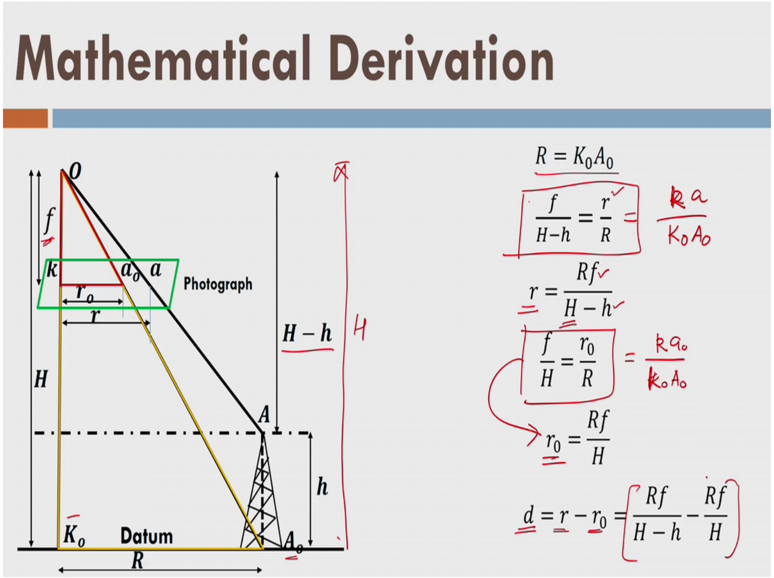

So, now, I want to drive the mathematically, what is the meaning of relief displacement or I want to derive some mathematical terms. So, that I can connect the height H of the building with respect to the relief displacement on the image, let us look into this thing ok. So, this is r 0 and r as we marked here fine. So, this distance is I can say here R equal to K 0 A 0. So, this is my K 0 here and this is my A 0 here, which is bottom of the building or the tower here. So, K 0 A 0 is my radial distance R as shown in the previous slide in a better way.

So, now if I take these 2 triangle this f, if I take the ratio of f to the flying height H minus h. This ratio to this is equal to the if you see here it will be equal to r by R, r is nothing, but k I can write small k to the a divided by K 0 A 0 right. And, you can see this relationship is very true ok. So, now, I can write that radial distance r of point A from the principal point is equal to this much R f into H minus h, you see the focal length I know I know the flying height. Let us say that I know the height of the point a above the datum also or above the ground surface also right.

So, above the ground surface means, I am fixing my datum at certain point on the ground surface that is the meaning here right. Similarly, I can write from other triangle this relationship f upon H; that means, the ratio of focal length to the complete height h is this height ok, equal to r 0; r 0 is nothing, but I can write here k a 0 divided by K 0 A 0 ok. And, as a result I can write from this equation that r 0 is equal to R f by H.

Now, what is the displacement let us look into this thing. So, this is the displacement of image r 0 to r. So, I can say that the displacement of the image point, because of it is height above the datum is d. And, which is given by this one right we are just doing some simple subtraction ok.

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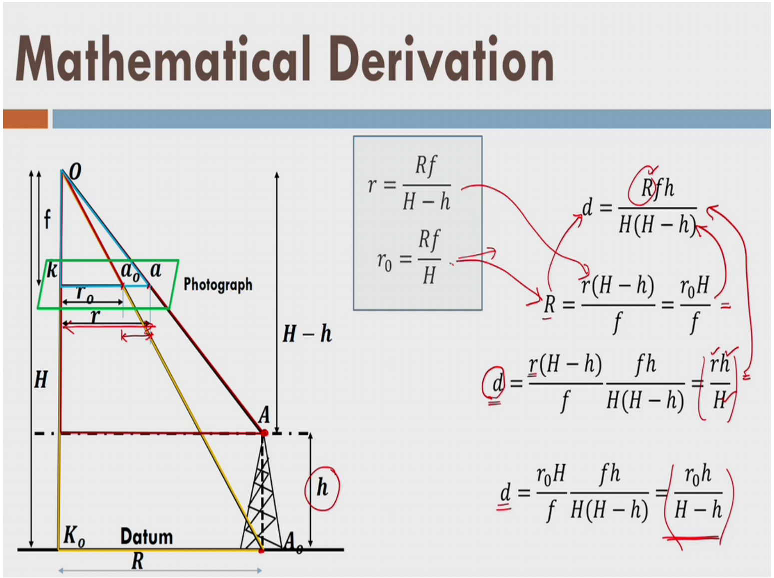

So, I can bring this thing here from there ok. Let me write this thing by just transferring I can write this expression, further I can say what is the value of R here. So, R is nothing, but I can write like this ok. And, then I can also write equal to this one. Why because from there I am writing this term, I hope you agree with that and similarly from this term I am writing this term and from here I am writing this term. So, that is equal to the r fine. And, as a result if I replace this thing here in the R from here, I can find out this distance this expression where d is equal to given by r into H minus h by f into this thing finally, this expression ok.

Now, if you can imagine that you can measure r on the image because you know if this is a tower. So, this is the bottom of the tower here and this is the top of the tower. So, I can once we see the image of the tower in the photograph. So, that is a point A and A 0. So, this is my r here from here to here and I can measure this think. Similarly, I know flying height and assume that I know the height of the point or that is height of the tower this, what is tower height above the ground surface. I can find out, what should be the replacement of or what is the shift in the image of the point?

So, similarly I can also write this thing, you can derive this thing instead of putting this expression this expression here. In this equation if you put this expression in this r you will get this expression and so, that is a idea here. So; that means, even you measure the r 0 which is nothing, but the distance of the image point A 0 from the principal point and A 0 is nothing, but the image of the bottom of the tower or bottom of a building and anything fine. So, still you can measure what should be the displacement of the image of a point a because of a it is height about it and that is what we call relief displacement on a vertical photography.

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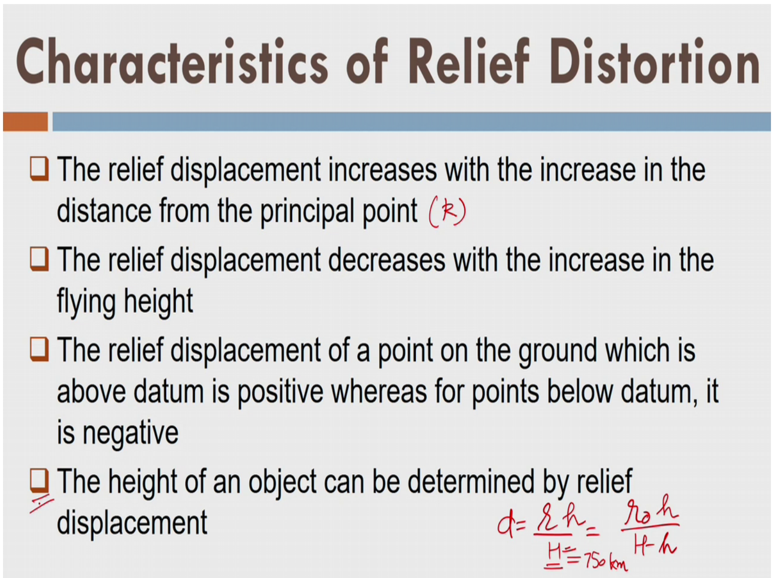

So, if you look at the characteristics you can see that relief displacement increases with the increase in the distance from principal point. The point principal point is nothing, but small k on the image the relief displacement in decreases with the increase in the flying height, because if you just see back this is the expression we have h upon H minus h here.

So, if you can here that relief replacement of point on the ground, which is above datum is positive as I already explained and below datum is negative. And, relief displacement decreases with increasing in the flying height if I increase the flying height here. The relief displacement will be lower right and that is a key if you do not want relief displacement.

So, better to increase the flying height, what will happen in that mission all the buildings will appear almost very very small, but in other sense relief displacement also give me some sense of height. So, choice is yours, whether you want to suppress the heights or whether you want to increase the heights represented in the photo choice is yours. Secondly, if you can imagine now satellite, satellite at the height of 750 kilometres. So, if the aerial photography define the relief displacement with this help height H. In case of satellite I will put this H is equal to 750 kilometres or 900 kilometre whatever the height.

And, as a result you will not find any relief displacement in case of satellite images and that is best source; that means, the satellite images are best source for the planning metric mapping; that means, I want to do the x y mapping, not the 3 D mapping right and that is the idea here. What do we prefer the aerial photography or aerial photogrammetry compared to the satellite photogrammetry? The satellite photogrammetry will give me minimum vertical acceleration or the vertical distance measurement possible, and as a result I cannot create a reliable 3D information maps using satellite photogrammetry.

But, in case of aerial photogrammetry I will have relief displacement and buy some way if I measure the height of the buildings or height of the terrain points with respect to datum I can prepare the 3 D map ok. The, height of an object let us look into this can be determined by relief displacement and let us see how?

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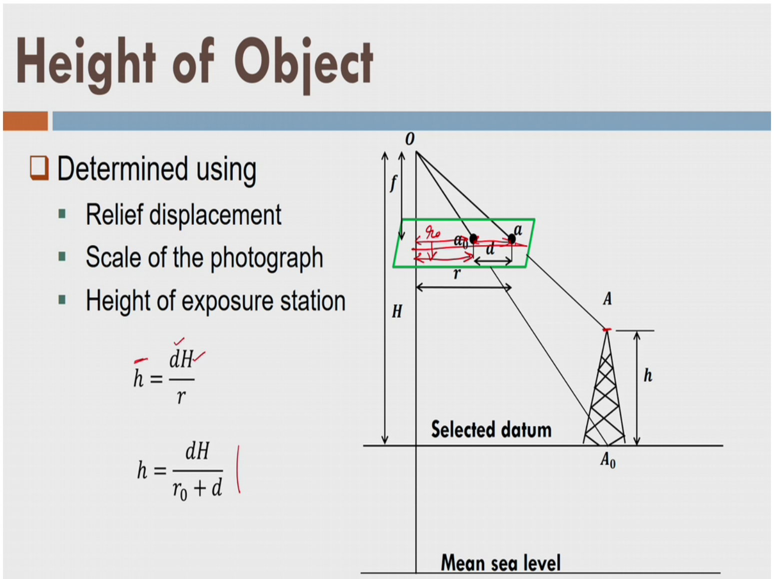

Ok. So, height of object here you see let us a mean sea level or the my datum selected datum and I want to find out point height of this point A this height h. So, let us see this is my image I am making it again without going to detail, because we already seen this thing this is a d right.

So, I can write h is equal to the height of the tower or building I will measure this d which is nothing, but this distance ok. I know the flying height and I know the r that is the image of the top of the point from the principal point here k. So, this distance if I measure is my r I can find out or I can also find out if I measure r 0, that thing, but this is my r 0 from here to here small distance for I can write like this is my r 0 here. So, I can still find out from this formula what is the height of the point above the ground surface?

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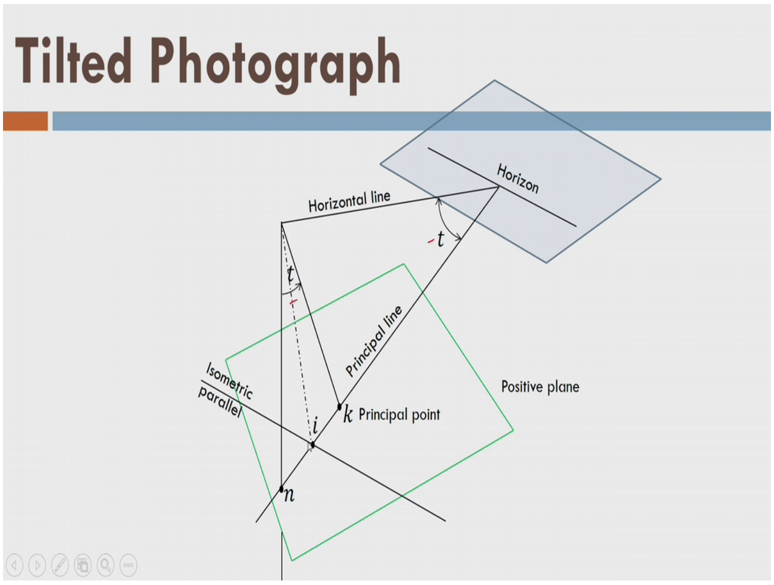

And, that is a advantage the vertical photography, that is advantage vertical photogrammetry especially aerial photogrammetry gives me compared to the satellite photogrammetry.

Now, let us take the tilted photographs. What is the meaning of tilted photograph? As, I told that when we say that there is a vertical photograph, which means the gravity line is matching with the optical axis, theoretically it is correct no problem, but we again say that because of some random errors. We can assume that optical axis not vertical or it is not matching with the gravity direction like this. Rather, it is slightly shifted in the photograph and it creates the principal point away from the Fuducial centre. Now, imagine that there is a tilt; that means, optical axis is tilted with respect to the nadir or the gravity direction.

What is the meaning here? Again, it is unintentional tilt, but it still there is a tilt and I know it there is a tilt; that means, it is no gone a random error it is kind of systematic error that I know. So, tilted photograph I am saying I know what is the tilt? Although, it is unintentional it is undesired, I am not doing it deliberately it is happening, but still I measure the, what is the amount of tilt I know it beforehand.

So, let us look into that that how my scale will going to change? Because, we have seen that if because of the vertical photograph where optical axis and the plumb line are exactly matching, in that case what is a vertical scale of the vertical photograph?

Now, we will see what is the scale of the tilted photograph? So, let us see the image plane this positive image plane. So, this is a point n, which is the nadir point on the image I can say here right. So, this is my optical axis and it is creating principle point k. So, this line which joins the small end to k it is called principle line. This is angle is tilt fine and if I draw a horizontal line, in horizontal plane, like this. Then, you can understand that this is angle is tilt here t they are same this t and this t, but they are in different planes they are appearing different in this animation.

So, I am saying I am showing that this is my tilt here with the horizontal respect to horizontal and there I am showing this is the tilt here between the this my nadir line or a plumb line and this is a optical axis I am showing like this. So, this is my optical axis by finger and the pen is showing the plumb line. So, this is my tilt angle here you can see here tilt angle. So, that is shown in the animation. Now, if I bisect the tilt angle by a line as shown in the animation it will intersect at point i and this point i is call the ISO centre of the image.

So, now we are trying to find out i also right and there is a reason for that we will see how?

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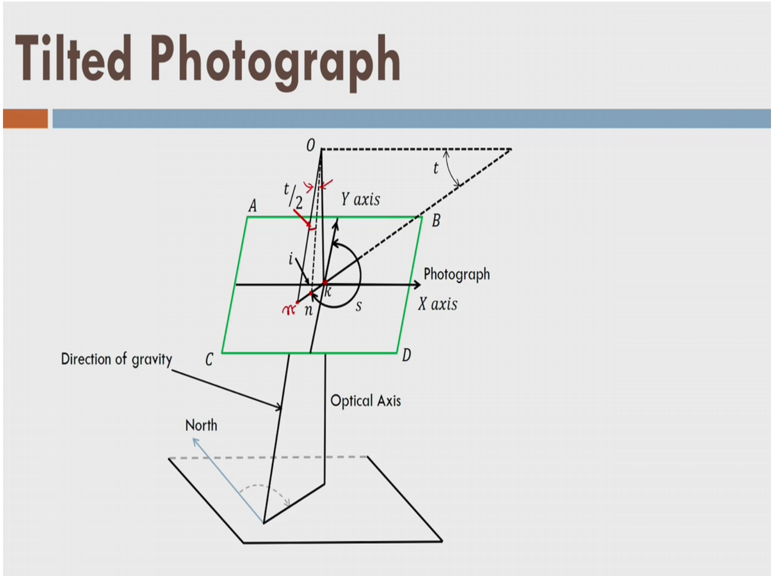

So, this Isocenter is there. Now, we have understood what is a tilted photograph? So, in the book you will find this kind of photograph right. Where, this is my n point here, this is n point and this is my k point. And, so this is a tilt angle total tilt angle is this. So, as I said this point is my Isocentre which is the bisector or if I bisect that tilt angle I will get the point i Isocenter on the image.

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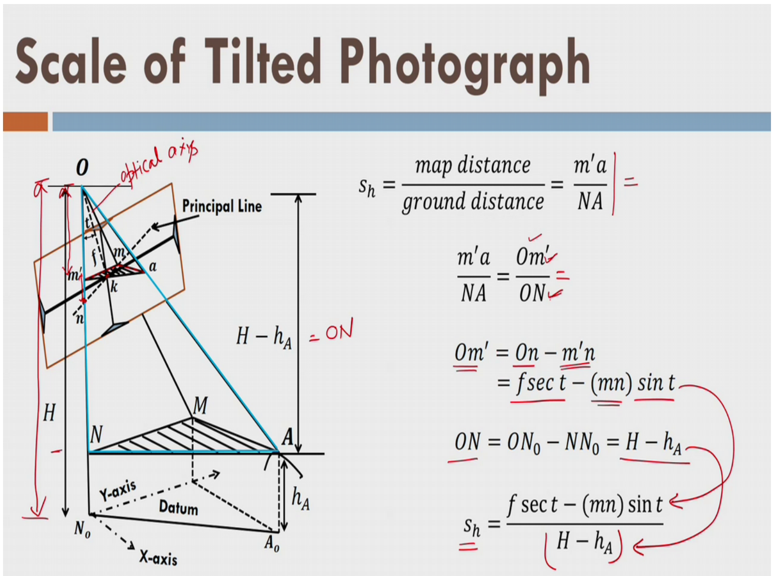

Now, let us measure the scale of the tilted photograph. So, let us say this is a point A on the terrain it is a there is a datum. So, point a has height h A ok. So, let us say there is the image. So, there is a point n and there is a point a here which is imaged.

Now, this is my focal length f, because this is my optical axis ok. So, this point line joining n let us say this is a line n here, point n here, and the point k here. So, this is my principal line ok. So, if I take point a and draw the perpendicular to the principal line. I will get my point m and similarly if I draw the line from point m to the vertical line or the gravity line I will get m dash right. Because of the tilt; that means, my photograph is not like that and like this and I know that my nadir point which was like this. Now, has shifted from here to here it is passing like that. So, optical axis is like that fine. So, this is my focal length.

This is a nadir line here right. So, this kind of arrangement is there. And, as a result what happens is because this is focal length which is perpendicular to the image tilted image and now this one. So, I know that this distance, which is little smaller than this distance right has to be taken for the scale calculation. So, let us see this distance if I take here up to m dash, then this distance divided by the flying height which is nothing, but this distance this is my scale in case of tilted photograph ok. So, let us draw an angle triangle, which is in the horizon plane.

So, now we are also trying the same triangle in the ground surface also, which is shown by N and A ok. So, this is my flying height H this is the point H minus h A this triangle; that means, O M dash a in the image and this triangle. Capital O, capital N and capital A they are similar triangles. So, I can say m dash a divided by N A is my a scale of the tilted photograph. And, not the vertical photograph I am saying this m dash a. And, as I told that m dash a is not measured along the nadir line here, but this distance is from this point to this horizontal plane like this distance fine.

So, this distance I am trying to measure and I am saying that ratio of this distance to the ratio of flying height, which is H minus h A is going to be the scale for point A, if point A is on the tilted photograph ok. So, now, m dash a divided by N A, it is nothing, but O m dash again I am taking the same ratio right by O n. So, O n is nothing, but this distance which is nothing, but equal to this is my O N distance H minus h A. Now, let us calculate the O m dash which is slightly complicated because of it is involved mathematics. So, let us do this thing.

Now, O m dash it is nothing, but O m dash I am writing it On, which is O n you can see minus m dash n right you can see this thing from the this simple photograph. So, O n is equal to the f sect t where t is my tilt angle and m dash n is equal to a m n into sin t. So, we can use this triangle shown here fine let us go ahead. So, now, coming to the O N as I already told O N is equal to H minus h A. So, I can write that s h the scale of the tilted photograph is equal to this ratio to this ratio or this distance to this distance. So, this is my this distance and this is my this distance here, this is the scale of my tilted photograph.

Still there is something to say here let us wait.

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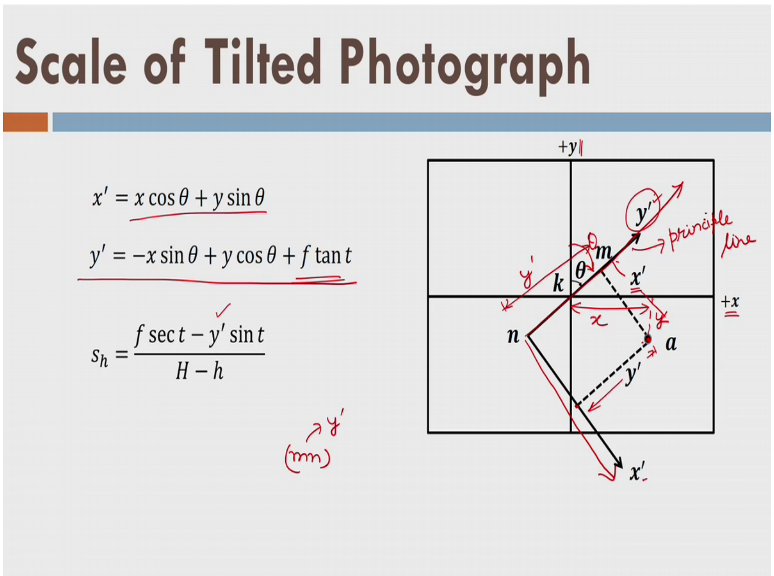

Again, now what happens is if you go back to the photograph here. We have assumed that, we are measuring these distances along the principal line. In fact, on a photograph we do not measure anything on the principle line. It is just a concept we measure with respect to our Fudicial centre only and the axis is which are parallel to the edges of the photograph. And, as a result let us draw this thing here. Let us say point a and that is n k and m. So, this angle is theta right. So, this is y dash and now x dash axis.

So, what I have doing I am saying that, I am measuring this point a here in the image coordinate system what I call x y, but I know that I should measure this in the x dash y dash system, which is at angle theta because I know if this is the principal line shown like this it is angle is theta here with respect to the y axis.

So, I can write here that these are the desired coordinates; that means, distance from a to m here right and this distances are required from a to this line. So, I can say this is my y dash and I can say it is x dash, because x dash is measured along this axis x dash like there and y dash is measured along the y dash axis here and this is my principal line if you see carefully fine. So, now, I have to correlate my image measurement, which is x and y. So, what is my x and y this is my y for this point and this is my x right for point same point a.

And, this is my x dash from here to here and this is my y dash ok. So, let us write x dash equal to I can write by simple transformation as you already know beforehand from the module 2. Similarly, I can write y dash equal to this much plus f of this term is also come because plane is tilted it is not in horizon plane image plane is tilted plane here ok. So, now, we can see that in the last slide if you go and there was a term called m n which is nothing, but equal to the y dash here if you see m n this term was this distance was there and which is nothing, but y dash.

So, I am putting the y dash here instead of m n and my y dash is given by this. Now, you can measure the scale on the tilted photograph also try it yourself try to do some experiments with your mobile camera put it like that, create some object try to put some kind of tilted image like this and capture it and try to calculate the tilted photograph scale. Also try to calculate something related to the relief distortion; that means, if there is a point like this. Let us say there is a some building like this you have. So, put it on your table like this you can put it. And, then try to acquire the photograph like this and try to see how this point the bottom of the pen and it top of the point has shifted in the image.

Because of the height of the pen you will find out, that there is a this point has shifted in the image outwardly from the principal point fine that is the idea here try to do this experiments yourself this is very simple, because you have the mobile camera with you assume that mobile camera is mounted in aircraft and your hand is an aircraft.

So, now, you are acquiring the photo of a certain terrain where this is a building. So, now, if it take the photograph in which this is appearing. So, you will find out the bottom and the top of the pen are in different appearing at different different positions. And, the difference from the centre which is an assuming that centre of the photograph is the principal point.

So, if you take the radial distance from the centre to the bottom image or the image of the bottom point and image of the top point on the photograph, we will find out we are lying in a 1 line approximately assuming that there are no distortions in the optical axis and the gravity line.

So, radilly they are outward. Similarly, you can do another experiment assumed that, this point building is like that or there is a some valley where you are saying that this is at top point now. This is your bottom point and this is the point below the bottom or below the datum. So, you can find out what is the negative relief or the effect of the negative elevation on the relief you will find out, that it is inward in the radial line. So, that is the idea here about relief displacement. Similarly, we just talked about the scale of the tilted photograph, now if you go ahead right.

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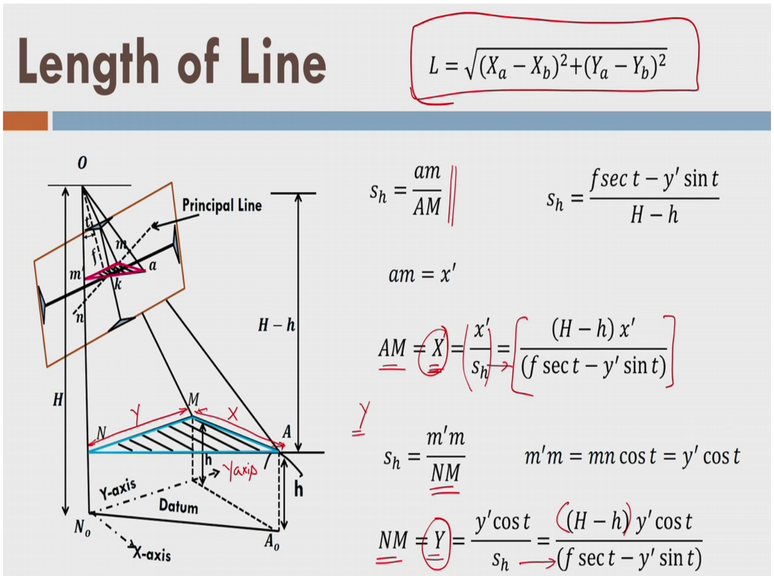

So, length of a line, so let us measure the length of a line on a tilted photograph.

So, what does it mean? We are trying to measure the length of the line on the ground surface using tilted photograph; that means, I am going to measure the measurement on the tilted photograph of the image points, I know the scale of tilted photograph that I already determined right. So, then I am finding out what could be the distance between the 2 points, which I do not want to measure in the field rather I am measuring from the photograph itself let us look into this thing.

So, this is my tilted photograph and I can see the red triangle here shown by red colour M, M dash A is my in the horizontal plane and tilted photograph is this. So, I have a plane like this and this is a tilted photograph like this.

So, in this horizontal plane I am defining my triangle M dash and A as we did it in the previous slide also ok. So, let us see s h is equal to am by AM, you can see now the defining we are defining the scale of the tilted photograph here right. So, am is this distance in the right triangle. Now, am is equal to x dash x dash perpendicular to the principal line which is nothing, but widest line in the photograph ok.

So, I can write here that am which is x coordinate here in this x and y axis system this is my y axis. So, AM is nothing, but this measurement this point to this point this measurement capital X, I am writing it here.

And, that is x dash divided by Sh and given by this way you can refer the previous slide for the final expression ok. Now, again I can write further in order to find out what is the y distance? But, now this time I am using m dash m, which is measured parallel to the principal line right. So, I can write this thing here similarly m dash m is equal to this and NM is nothing, but my y distance which is here and that is this distance, NM I am sorry this distance yeah.

So, this is my y distance and measuring along the y axis and so, it is y dash cos t divided by s h and then I can write the values of s h and here H minus sign putting the value of scale only. So, because of the inversion it is coming this one right and the scale is nothing, but same this is my scale S h I am putting here also I am putting here also.

Now, I can say that what is the distance here for 2 points, let us say you measure X this distance for 1 point and y also for 1 point A. Similarly, you also measure for point B and then you can find out the length on the ground surface is equal to this distance that is the idea here.

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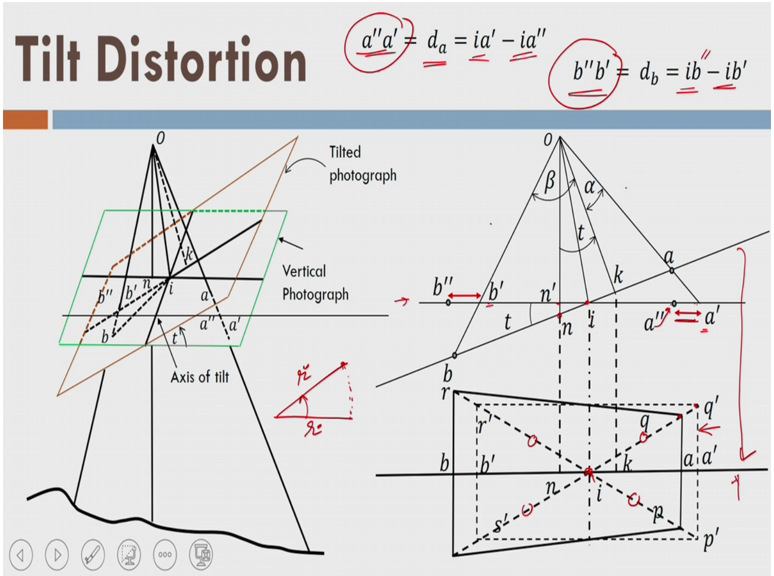

Let us look into the tilt distortion ok. Because of the tilt distortion what will happen my images appearing like that not the rectangle 1 fine. So, this is my tilted photograph. So, let us say this is a plane of tilt ok. So, this plane of tilt is this which is shown here, on the image planimetric image right. Now, O is my exposure centre and this is nadir point. So, this is the image of the nadir point here ok, k there is a k point and this is my tilt angle. Now, let us see there are 2 points a and b on the tilted image ok.

They are appearing like this. At, the extreme of the image I am deliberately taking it and we will see why ok. So, this is my Iso center I which is nothing, but here this is my Iso center I here this point ok. If, I draw now the vertical photograph, which is should be correct one. So, this is my vertical photograph like this. So, this is the vertical photograph here.

So, it is a plane of the vertical photograph indicated by this line. Now, this vertical photograph will have n dash as the, if it is vertical than this optical axis sorry this is my optical axis here. And, this is the plumb line from the exposure station right. So, plumb line is intersecting the vertical photograph at n dash.

But, it is intersecting at n for the tilted photograph as you can see here. So, this is my again tilt angle. Now, I am connecting the exposure station with the point a and b on the tilted plane. So, these are the point a dash and b dash, which are the image if we have taken the vertical photograph. In fact, they are trying to derive what is the distortion? That means, if photograph was vertical I could have get a dash b dash, but now real points are a b on the tilted photograph and as a result I want to find out what is the distortion from point a dash and b dash because photograph is tilted. So, let us see. So, this is my a dash and b dash ok.

So, let us see these angles are alpha and beta right ones again we look into animation. So, I want to tilt my tilted photograph such that it will meet with the vertical photograph. And, that will give me an idea that if this is point a and it is going to make point a double dash I already know what is point a dash. So, this point is going to make a double dash. So, this will be the my distortion end point a.

Similarly, I can find out distortion endpoint b also on the image. So, let us rotate the tilted plane and make it equal to the vertical plane, match with the vertical plane or the match the tilted photograph with the vertical photograph like this.

So, because of the rotation what happens is the distance from this point i to b is equal to i to b dash. So, this distance I can write here a dash a is equal to d a that is the distortion tilt distortion in point a, this is this distortion you can see here. And, it is it may equal the i a dash i to a dash right minus I double dash a dash right. This is point is a double dash here and this is your a dash.

Now, I know that i a dash is similarly I can write for point b right. So, b dash b is my distortion in the point b the tilt distortion and I can write it i b minus i b double dash i b dash here. Well you are agreeing with that ok, yes here I have written i b dash and i b straight away right. You will see I can also write here instead of i b I can write i b double dash also in the way I have written here.

Why because, because of the rotation the these distance i b is equal to i b double dash we are not projected the point b on to the vertical photographer rather we have rotated it. So, it is kind of this effect I can say here let us click. So, if I have a radius r here. If, I rotate by some angle, then it will again hey go here and this point will be go here in a circular arc. So, this distance is r and this also r. So, the same thing I am doing here.

So, now, I can see that if I join these points different rays. So, this is my final connection between the tilted photograph and the. So, this lines you can see they are intersecting also at I and their intersecting here as well as here and so on. So, that is a concept of the tilt distortion these 2 amounts are may tilt distortions.

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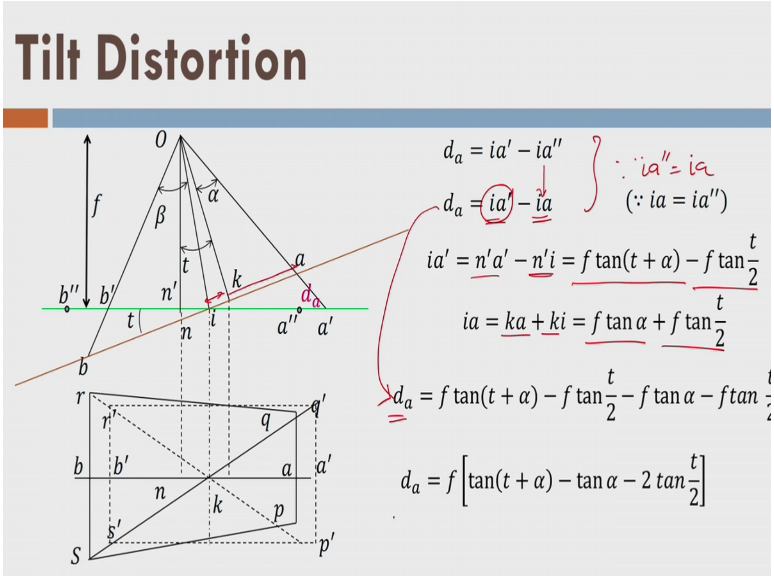

Now, let us try to find out mathematically. What is the amount of tilt distortion? So, this is my focal length in the tilted plane and this is also equal to the focal length in the vertical photograph, because I have finding out what is the equivalent vertical photograph of the tilted photograph right. Now, I can say this is my tilt distortion d a in the photograph in the animation. So, I can write it like this I can also write it like this you can see here that i a dash is equal to this one ok.

Similarly, we can see here now i a dash here is equal to n dash a dash right and minus n dash i. And, I can write it easily f tangent t plus alpha and my f tangent t by 2 so, that is i determined is quantity no I want to find out this quantity i a.

So, i a is nothing, but equal to k a plus k i; that means, this is k a and plus k i these 2 distances shown by the dark colour here and it is equal to f tangent alpha and f tangent t by 2. So, you can use these triangles and find out what is my f here ok. Now, I can straight away derive this term here, what is the tilt distortion power point a, which is lying below or above the tilted photograph you can imagine right. And, similarly I can write this expression.

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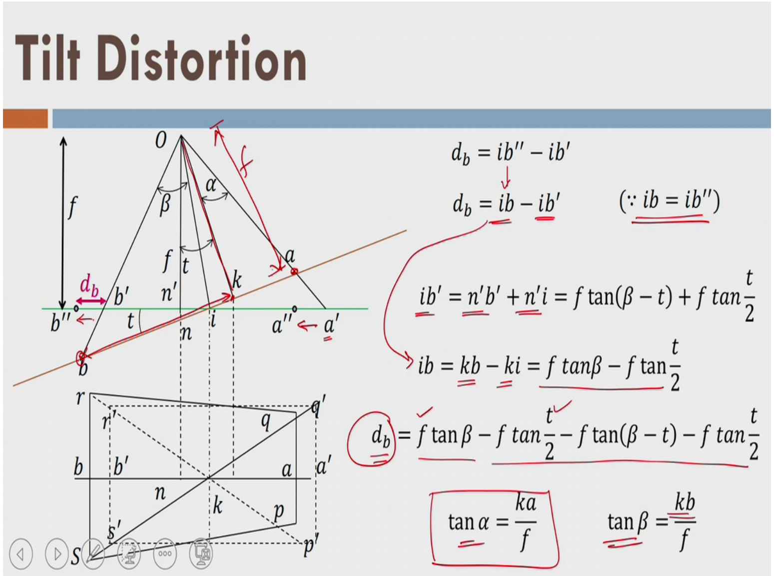

So, what about point b again this is my focal length f. So, let is my distortion db you can see here that because of the tilt distortion point a is shifting inside like this, but point b is shifting outside right.

So, I can write this thing here and they are equal basically right (Refer Time: 64:13) because they have rotated the image tilted image into the vertical image right. So, I can write here d b is equal to nothing, but i b minus i b dash, and i b dash itself is equal to n dash b dash plus n dash I from this simple relationship of the vertical photograph ok. So, now, I can see that i b is equal to k b here I am writing i b here, k b minus k i and equal to f tangent beta minus f tangent t by 2. Now, similarly I can write that d b is equal to f tangent beta minus this distance; that means, I am just doing some algebraic summations here. Fine, and where this tangent alpha is nothing, but k a by f here this distance if you see alpha is k a divided by f fine.

Similarly, tangent beta is nothing, but kb divided by f. So, my k b I can say here kb this distance k b divided by f this is vertical this is the f distance here focal length. So, I can find out tangent beta also tangent alpha also focal length is known to me tilt is known to me I can find out what is a d b or the tilt distortion of point b. So, you can find out for any point and that is why we have taken 2 points; one is below the photograph and one is above the photograph; that means, one is below the vertical photograph point a for example, this point b is below the photograph vertical photograph, and this point a is above the vertical photograph in the tilted plane.

And, we find out the expression of tilted foot scale.

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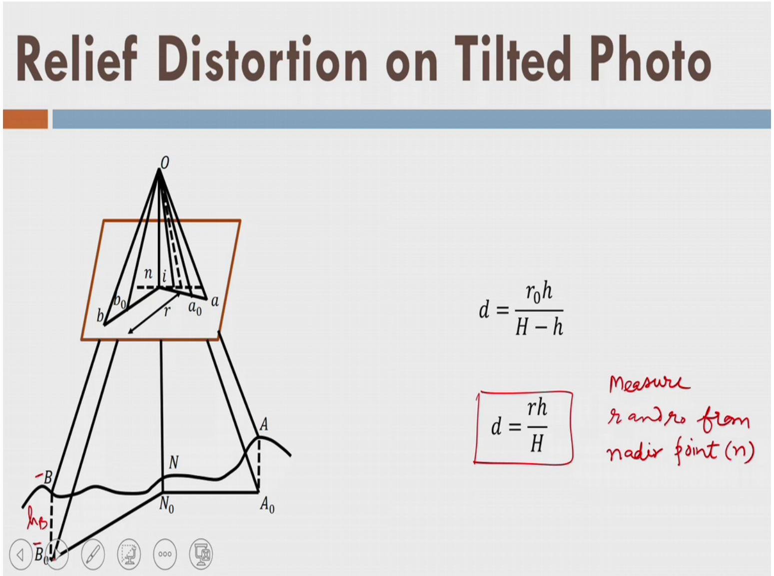

So, let us look into the relief distortion on the tilted photograph. So, this point a here this nadir point here a 0 I can say ok. So, let us see this thing a 0 here now this thing I am drawing some animation you can see with carefully there very easy to understand Iso center, now this is a situation here right. Now, that is a r 0 distance from Iso center to the B 0; that means, this is the B 0 here and this is the B point and this is the height h B here. Now, this distance is r and I can see here that this equation is still applicable if I measure the r and r 0 from nadir point and not the principal point.

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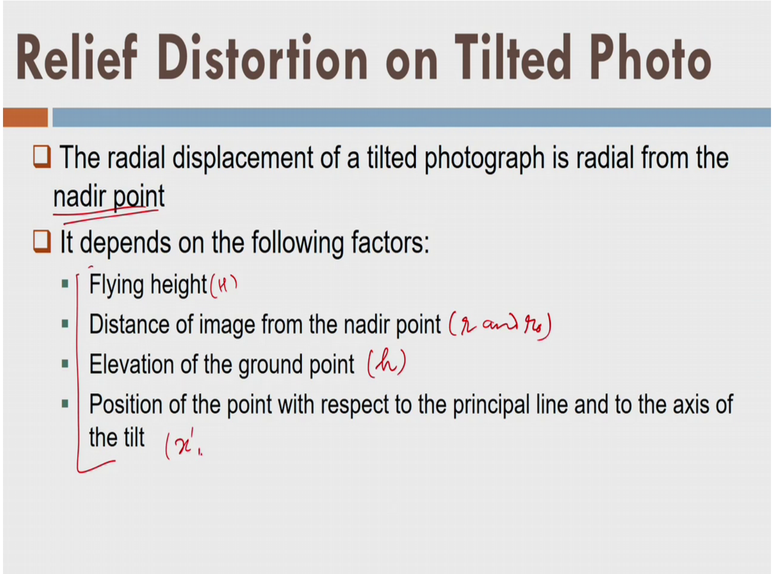

So, I can write this expression also. So, finally, the radial displacement of a tilted photography is radial from the nadir point and not the principal point that you should remember in case of tilted photograph. And, this depends on these factors. As, we see in the expression we have all this in all sectors right r n r 0 here, h small h this could be principal line and the so, that is nothing, but my x dash and y dash. So, here we finish our this lecture on the vertical photogrammetry and that is all about the vertical photogrammetry.

In the next lecture, we will talk about the stereophotogrammetry that is a principle of creating 3 D from multiple images not the one images. Today we have seen that we have find out the height from a tilted photograph or from the relief displacement of a 1 single photograph. And, photograph could be vertical photograph or tilted photograph, but tomorrow in the next lecture we are going to see how can we used to stereoscopic photograph and the concept of stereoscopy to find out the 3 D points or the 3 D elevation of various point with planimetric position also. So, till then.

Thank you.