Higher Surveying
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Module - 04
Error, Accuracy, and Adjustments Computations
Lecture - 13
Analysis of adjustments and reporting of errors

Hello everyone. Welcome back. In the course of Higher Surveying we are in the module 4 and today we have last lecture: lecture 5. In the last lecture we have discussed about condition equation method, observation equation method and when to implement these 2 methods. And we also see that an example of chimney where we want to determine the centre of the chimney at certain height h, also we want to determine what is the radius of the chimney.

In that case we are realize that it is not possible to use either condition equation method or observation equation method, because both are not appropriate for that situation. So, let us look into this thing again with us while small review on the all 3 methods.

So, today we are in the lecture 5.

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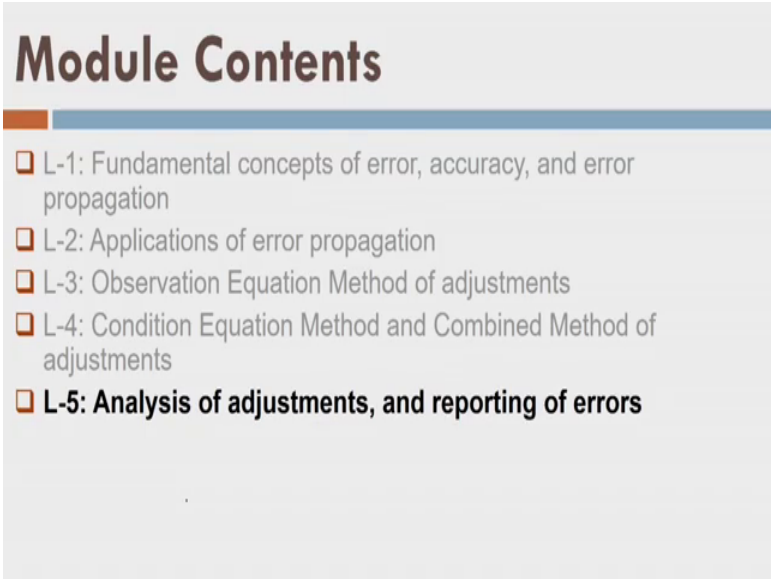

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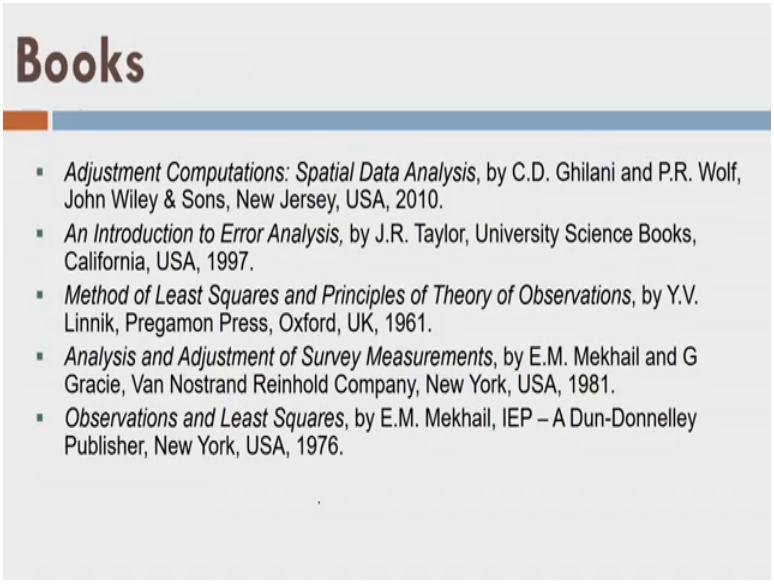

These are the books again fine.

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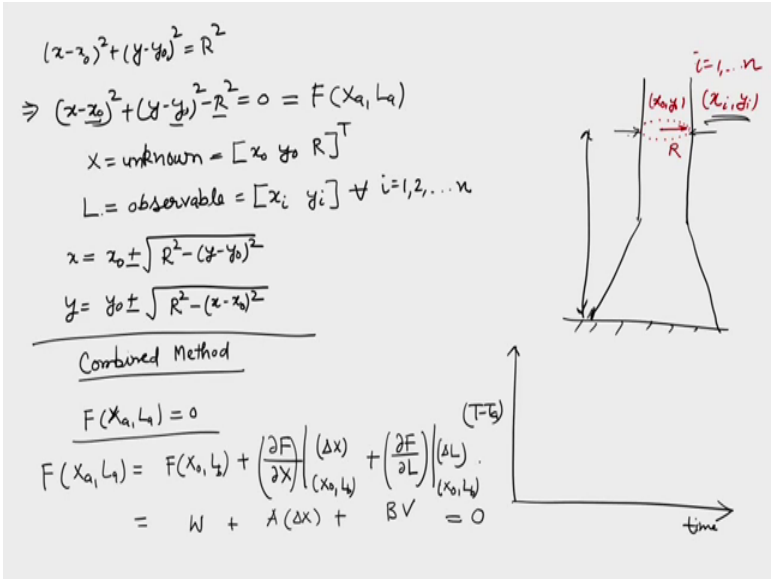

So, I am once again elaborating the whole situation. Let us see that this is ground surface here, and chimney has been installed or chimney has been constructed if it is concrete one. So, I want to find out the diameter as well as the centre point of the chimney at this level, at say at height some height. That is not important right now.

So, what we do? We acquire some points on the surface of the chimney using in the big total station of some other like this. So, this points I call him; I am using the x i y i where

i is equal to 1 ro n number of points right. This is what we have done. So, now I want to find out the centre let us say which is x 0 y 0 and the radius which is say R. So, I want to find out x 0 y 0 in R. So, these are my parameters. Now I can use the standard equation of the circle. So, I can write here x minus x 0 plus y minus y 0 square is equal to R square. Or I can also write it x minus x 0 square plus y minus y 0 square minus R square equals to 0. Or I can write it let us say this is my F x adjusted and some observed.

Why because; my these 3 parameters here and R, R nothing but x unknowns or I can say parameters. Similarly, what are my observables? Which are nothing but x and y. So, I right here x i y i for all i equals to 1 two n. So, this is situation.

Then we said can we use observation equation method. For the observation equation method, I need to express my dependent variable in terms of independent variable. So, I one can argue. I can write my x as dependent variable in terms of y like this. Someone can also argue that I can write y as a dependent variable I can write y in this form. So, mathematically these arguments are correct. However, if you look into physics of the problem, we realize that both x and y are independent of each other.

Remember, we give an example in case of observation equation method where we said that we are doing some experiment for the Newton's law of cooling where, we say that this is my temperature difference with ambient temperature and here this a time.

Now, here I know that I am observing the time and then based on the time I am taking some certain observation of temperature difference. So, I say that the temperature difference is a dependent variable. Because, I know time is not dependent on the temperature. Rather temperature is varying with the time. So, temperature difference is the dependent variable. I know from the physics of the problem; but here, if you look at x and y in this case right, y is not dependent on x or neither x dependent on the y. So, there is a problem I cannot use observation equation method to solve this kind of setup.

What about the condition equation method? In case of condition equation method, we need to observe all the variables. If you look at the all the variables there are 5 variables x 0 y 0 R and x i y i. So, if I observe all, then I can use condition equation method. And as a result I need to reject condition equation method also. Rather it is inappropriate, it is inefficient, it is cannot handle this situation. So, in such cases we use combined method, where we combined both condition equation and observation equation method.

And in this case, we write the fundamental equation like this. In this form we will always right. For example, what is my x a as I told these are my parameters. So, they are adjusted values of the parameters. So, after adjustment of observables as well as parameters I will get this equation ok.

Now, what is the meaning? Since, this is non-linear equation and because of that only I am not able to use the linear method treatment. The way I treat the linear equations, I need to linearize first. So, what can I do here? So, let us express this thing like this. So, I can write using my linearization scheme by Taylor's series like this. Let us see f x 0; that means some assumed values of the parameter.

I am trying to find out this function f as well as I am trying to find out dF by dx there is nothing x 0 here. X 0 is value of this one at given value of x 0 and L, delta x that is a change in the parameters. And then we have dF by dL delta L here. And evaluated at x 0 L. Remember that L R given or rather I can stay here it is my L b everywhere. Because L b are observed and L a are the adjusted values right, ok.

So, here if I write this as my W here this is my A. So, it becomes my delta x. And this matrix becomes B, and this becomes since it is delta L I can write it V. So, this is my form here.

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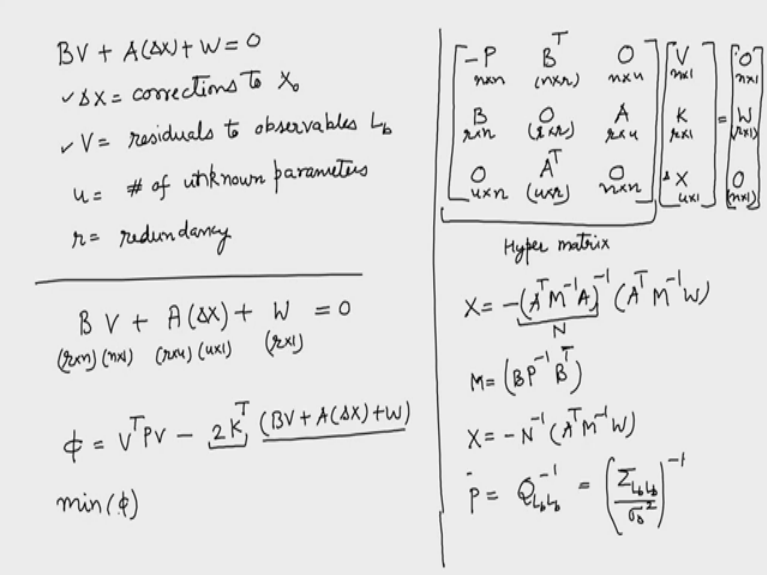

And I write it equal to 0 remember by Taylor series, because they are ignoring higher order terms here. This form here BV plus A delta x plus W equals to 0 ok. Delta x is corrections to assumed value of x 0 ok. V will be the corrections or residuals right, 2 observables. Now since we are doing the iterative process, because it is a non-linear problem and we have linearized it, now we need to converge it. When we do this convergence in that process we will keep on updating my delta the X as well as V fine ok.

So now let us proceed here with this idea. Here I can write that you are the number of unknowns or unknown parameters. And here I write R is nothing but the redundancy introduced by the set of equations. Now if we let us look at this equation again what is the size of each matrix, I can write here it is A delta X. Or A X 1 and the same thing they are symbolically written right. What is the size of B matrix here? R into n, because there are and then we have n into 1, because we have observed n values of the observable.

So, we have n number of residuals, right? Here we have R number conditions. So, we have R number of equations A, what about the A matrix? It is r into u delta x will be u into 1. W, W is nothing but misclosure matrix which is equal to the number of condition equations here right. So, this setup should very very clear to you.

So, let us move a lat us take this example; so now, if I take the V phi which was earlier VTPV. Now again this is equal to 0. So, if I make here 2 times KT, here this whole factor is scalar and BV plus A delta x plus W. That since this quantity is 0 even I added to this quantity, phi is not going to change. So, that is appropriate to right. Now if I do my minimize of phi with respect to my parameters and so on, then all the nodes remember my residuals are also unknowns.

So, if I minimize my phi with respect to both parameters as well as residuals; that means, all type of unknowns I will get some equations right. And then this will lead to A matrix is call hyper matrix; where this is my P which is of sin n by n, because there are n observations B T; which is size n by r here 0 and it is set of my 0 values n by u here.

Then we have B r cross r, 0 again here u cross n. I am sorry here R cross n here we, then we have 0 here which is r cross r. I A matrix here which is r cross u then 0 here n cross n. And then we have a T matrix here which is u, cross R. Then here V my V which is

unknown of n into 1, then we have k matrix here, which is LaGrange's multipliers for R condition equations, then we have x matrix which is unknown or delta x does not matter.

So, I am writing let us say delta x also is fine ok, what about equals to? Here if I write equals to here then it is 0 then n into 1. Then we have W matrix here which is disclosure matrix and equal to number of condition equation. So, r into 1 and then 0 we have n into 1. Ok and this matrix is called hyper matrix right. From this hyper matrix, I can say x is given as minus A at M inverse a inverse at M inverse W, where my n is equal to B P inverse B T right. So, I can write here? X equals to n inverse A T M inverse W right. So, what is my n here? This is my n here. So, I am writing n inverse right.

So, here if I use this minus sign here; now, what about the quality of the parameters? That will take later, but before that we should know that my P matrix is nothing but Q L b L b inverse, which is nothing but sigma L b L b divided by sigma 0 square inverse. And generally we take Q sigma L b L b as my diagonal matrix. So, no problem generally we get P matrix are very easily. So, let us look into the problem of this particular chimney example.

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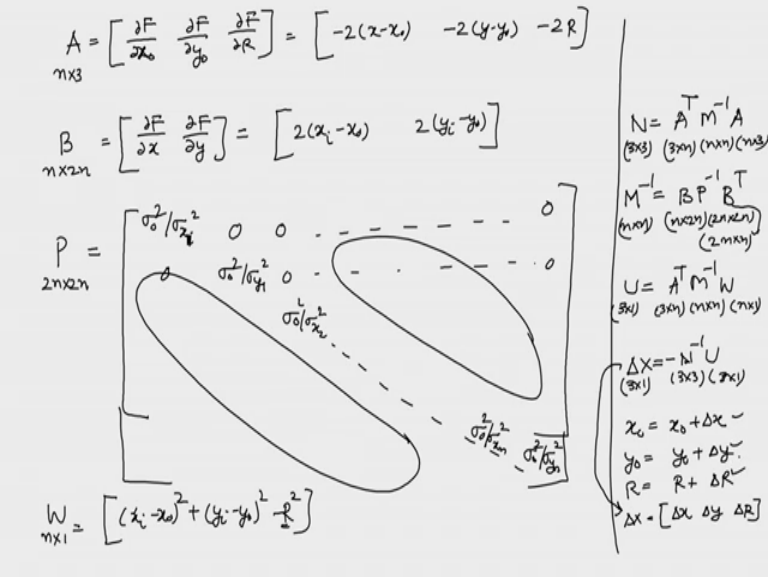

So, what is my A matrix here in this case? A matrix is nothing but differentiating dF by dx 0, sorry, dou dF by du y 0. Dou f by dou R. This is my A matrix ok. So, I can write it here all the elements here if I just differentiate it minus 2 R. So, these are the ok.

What about the B matrix? The size of this you can see n into 3. B matrix is n into 2 n size equals to equals to dF by dx. And it is remember it is the variable here and here dF by dy I can write it like this. 2 x i minus x 0 and 2 y i minus y 0 like this. Now what about the P matrix here? 2 n by 2 n, because we have x and y observations which are coming independently: so we have for a given observation one observation we have 2 data x and y. So, we need to consider wx i wy for all.

So, how can we do it? It will be diagonal matrix; where I need to write here let us say sigma 0 square by sigma x i square or let us say x 1 square here. Then we have all the 0's, then we have next element is sigma 0 square divided by sigma y 1 square. And all the elements are 0. Similarly, if I go to third element it will be sigma 0 square by sigma x 2 square and so on. I will keep on repeating this one, it will go up to like this, last element will be sigma 0 square divided by sigma y n square and here I will get sigma 0 square divided by sigma x n square.

. So, this is my 0 here. This is the 0 here that is my P matrix ok. What is W matrix? W matrix will be n into 1, and given by x i minus x 0 square plus y i minus y 0 square minus R square. Here these values are coming by some assumption or other I assume some value of the x 0 y 0 and R. So, that is why I can calculate for this one. So, that is I can have n number of observations. I can will have n number of such data in the matrix form. So, this is the sizes W ok.

Now I can calculate here my n matrix which is nothing but A T M inverse A, fine. And that will be finally, 3 by 3 matrix. Why because, this is 3 by n M will be n n by n by and a will be again n by 3. So, I will get this 3 by 3 matrix here M matrix and M inverse is equals to B P inverse B T. It is n into n by n. B is n by 2 n, T is 2 n by 2 n and then B T is will be 2 n by n. So, I will get ultimately n by n matrix here.

So, what about my unknown matrix; is u which is given by at M inverse W ok. It is 3 by 1, and it is 3 by n matrix here. We have n by n matrix here and W is nothing but n by 1 matrix here. So, I will get what about my delta x matrix? Delta x matrix is nothing but n inverse u right. I have written this matrix here like this so that I can write delta x very briefly. So, I get 3 by 3 here n inverse and here maybe minus n could be here 3 by 1 here ok. So, I will get ultimately the 3 by 1 unknowns that is my x 0 y 0 R right.

So now I need to update my x 0 as before we have done for non-linear examples; delta x and y 0. Again I will update with the y 0 into delta y and R equal to R plus delta R. So, basically my data x is nothing but delta x delta y and delta R. That I am determining from here. So, once I determine this by delta x I will update my 3 values, I will got another value and then I will again use them ok.

So, then I will keep on this thing iteratively. And finally, I will stop where delta x delta y and delta R becomes close to 0. There is there is no update in this thing, because of the process. And a result as I will say that yes they are the final values of the x 0, whatever I have the latest value of x 0 y 0 R. So, this is the way we do the adjustment processed by combined method.

Now, what about the quality?

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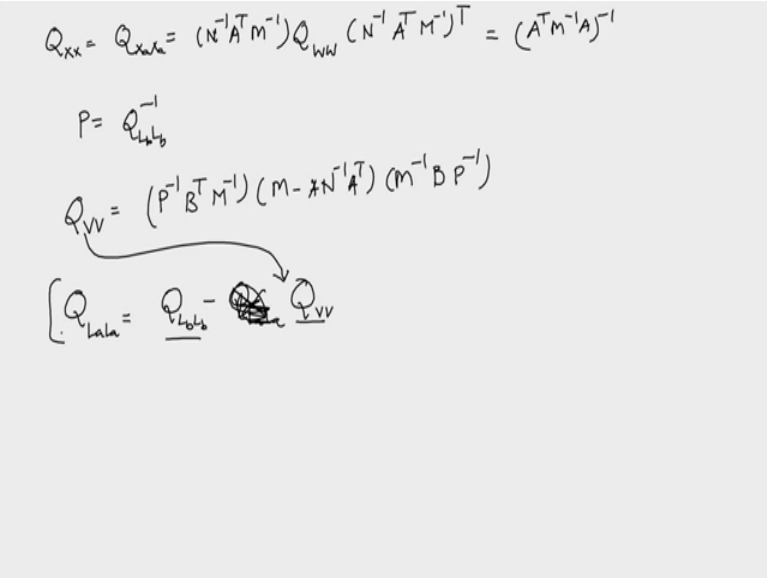

If I look on the quality; I can find out my Q x x as Q x a x a equals to N inverse at M inverse is Q w w here. Then we have n inverse A T M inverse transpose fine. So, whole equation can be written as A T M inverse a inverse right. Then again as we know P is equal to my Q L b L b inverse ok, W matrix you already know. What about the Q vv? That is P inverse B T M inverse n minus A N inverse at into M inverse B P inverse right. That is the quality of and finally, what is the quality of; again Q L b L b minus Q vv which is here, not this term.

Here you can see that I am reducing again Q vv from Q L b L b. So, the quality of my adjusted parameters and residuals are much better. So, adjustment processor brings the better treatment today; fitted parameters as well as observables. Now here we have to end our discussion about this 3 methods right. Now we will look into the again our adjustment process. So, we have done all 3 methods of adjustments.

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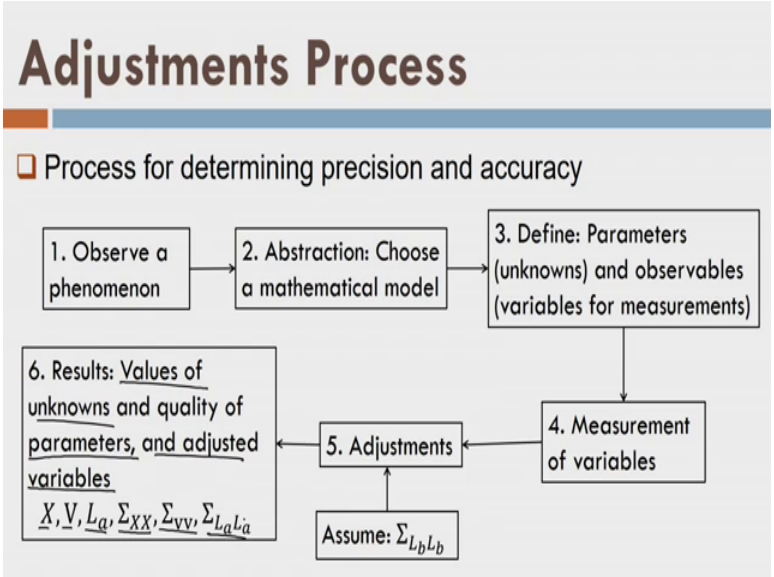

Let us discussed what we have done in the adjustment process. First of all, we have observe a phenomena right. Then we have created an abstraction. What is the abstraction? In case of Newton's law of cooling, we assume that there is only temperature difference and the time are the variables. We have ignore all rest of the variables and that was abstraction. When we observe these 2 things experimentally, we found that there could be a possibility curve which is following exponential track or exponential points. And so, we say that these 2 things are varying exponentially by a exponential curve.

So, that was the extraction because we have selected only 2 variables, it could be possible that in the reality some other variables in the real physics are influencing that law. Law of cooling of Newton's, but we have ignored other a rather when Newton postulated it he is ignored other things. So, that was an obstruction created by the Newton of this particular phenomena right. With after that obstruction what we do? We define our parameters; that means, we observe the data and we also define our

parameters which are unknown. That we want to determined with the help of adjustments computation.

Fourth processes we measure the variables. That means, once we decide; what is my parameter, what are my observables. So, in the field I will observe my observables with the help of some measuring instruments, total station tape whatever. Then we will do the adjustments here. In the adjustment process we will finally, calculate the values of unknowns and quality of parameters that is unknowns only and the adjusted variables; that means, all we calculate x this is unknown V is my residual. L a is my adjusted values of the observables. Sigma x 6 is the quality of the parameters. Sigma vv the quality of the residuals and sigma L a la is the quality of adjusted variables fine.

So, this is what we do. In the whole process do we make an assumption? Trying to think, remember, we have made an assumption about sigma L b L b here. We have created a P matrix, weight matrix. Either we have assume P equal to unit matrix or we have assigned weights some non unit values maybe 1.3, 1.5 to 2.5 whatever.

And there in order to simplify our work instead of considering sigma x 1 square x 2 square and all variances we have assumed one reference variance called sigma 0 square. And with the help of that we have taken the ratio of sigma x 0 square to particular reference, let us say to particular variance. Let us say sigma x i square and then we have created our P matrix.

So, let us look into this thing.

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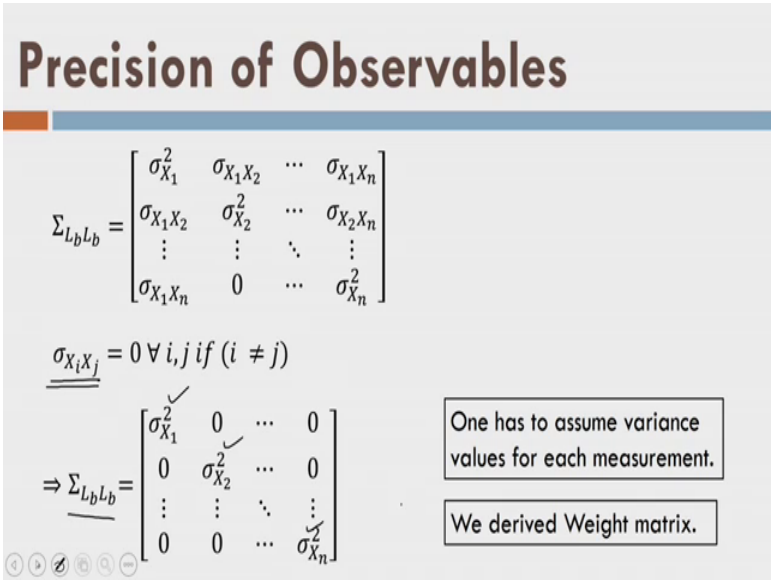

Here that was our sigma L b L b and from there we have seen that is I can put my covariance is equal to 0, because all these observations are coming from the field x 1 x 2 and so on. Now we have created sigma L b L b like this, and some there one has to assume variances values for each measurement. That means, you need to measure assume all these values. There from there we derive the weight matrix.

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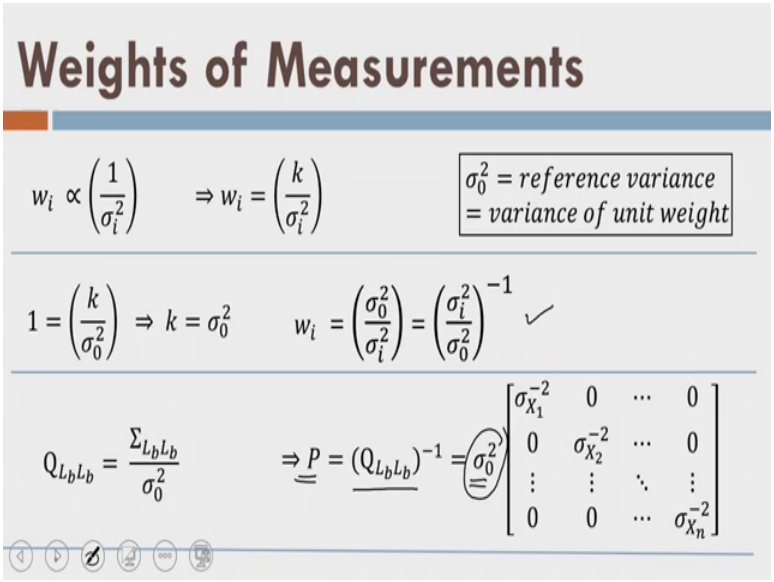

And this was the idea here. That means, we have done this process and we have derived our weight matrix like this all the individual weights. And this is was my P which is

nothing but inverse of Q L b L b and that is sigma 0 square into all these are inverse. And as a result I derived my P matrix here. So, in the whole process I have assumed many of the things.

For examples that what is the reference variance or variance of unit weight; that means, this variance is going to give me is one weight 1, equal to 1. And as a result what will be the weight of other observations? So, that is why I take the ratio and so on, you know all the rest ok.

Yes, so that is my reference variance or variance of unit weight that I have assumed fine.

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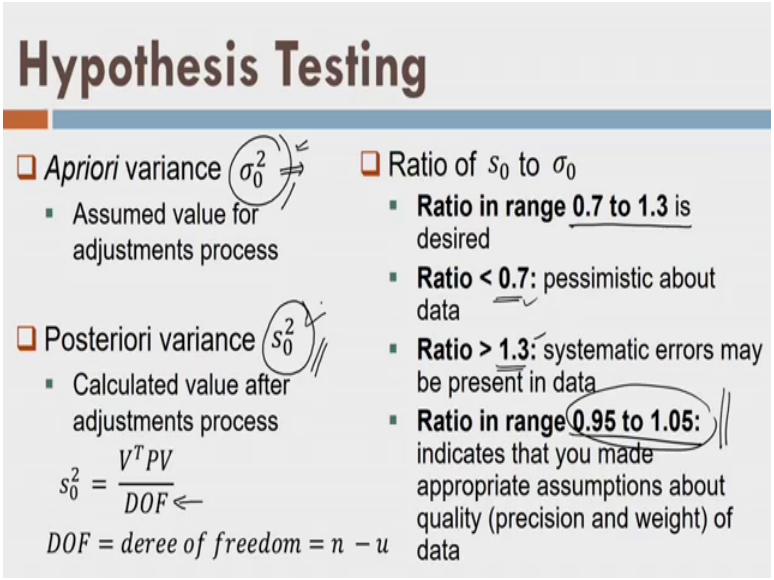

Now, I have to test whether whatever assumption I have made my reference variance or other variances are true or not, right. I can do one thing, I can check my reference variance, the movement it is correct all other variance will be automatically correct because they are just the ratios. Some multiples of sigma 0 only right. So, I have assumed the apriori variance, that is I have assumed sigma 0 square here right. And that is what I call a priority variance. That is the assumed value for the adjustment process.

Now I need to calculate my posteriori after adjustment process, again I need to calculate my sigma 0 that what I am getting and I call it s 0 square; that means, the calculated value. Remember, we have done in case of triangle example in lecture 3. There we have calculated; what is the after adjustment I am getting this value of h 0. There we coined

the term that this term is h 0 or h 0 square; which is nothing but the a reference variance or variance of unit weight.

Now, again I am verifying since I have assume this one, and now I have calculated after adjustment this one. They should be same more or less. So, if I take this ratio ok, before that I would like to give the formula. This is the formula where this is my degrees of freedom. We are going to discuss what is degree of freedom. So, here we calculate degree of freedom and then we calculate s is the ratio of s 0 to sigma 0.

Now, if this ratio is in this range, then it is desired one. If it is below 0.7; that means, you are very very pessimistic about data. What is the pessimism? Data is good quality, but you have assumed that it is a poor quality. And that is why you are given very high value of the sigma 0 ok. So, you can take this thing like one a simple example. Let us say you have an estimate of the fruit which is sold in the market say 100 rupees per kg. So, you took couple of bugs in your pocket to purchase maybe 2 kg 3 kg whatever or maybe 1 kg. And then you went to market in the market.

Even you go you realize that the fruits are 50 rupees per kg ok. What happens and your estimate of 100 rupees, you are very, very pessimistic about the prices. And that is why you carried little more with you, but in reality things are much better in the field. Similarly, situation could be contrary.

But first understand the first situation; that means, you have 100 rupees in your pocket, but the real price of the fruit is 50 rupees per kg. So, this is the situation here; that means, you are very, very pessimistic about the prices. Similarly, you are very pessimistic about the quality of the data here. That means, you have assumed very high value of sigma 0, but in reality when you calculate the s 0 it was much superior value. Similarly, now take the contrary situation, you went to market and there in the market when you inquired about the prices of the fruit.

That you are thinking as 100 rupees per kg, but now it was 200 rupees per kg. So, you are very, very optimistic, or rather there was lot of problem in your estimates. How could a think possibly be 100 rupees per kg, when in reality it is 200 rupees per kg. So, I should say that there was some kind of error in your estimates.

So, what we say here? There are some systematic error; that means, you have committed some kind of mistakes in looking at the information or whatever. And you have assumed very optimistic things right. So, in that case we say that when ratio is more than 1.3 in our data, this kind of ratio comes. Then we were some possibly a systematic errors is presented ok. So, what about the ratio in this range comes here? That is a most ideal situation; that means, whatever you have assumed about your data in fact, data is like that only. So, that is the most desirable situation. What happens if you do not get this situation or rather you face this or this situation.

In this case again you modify the value of sigma 0 perform the whole adjustment process again and again. Try to estimate s 0 again take the ratio, and to try to reach to the ideal situation. That is idea here of the hypothesis testing. So, the movement you reach here then you see that ok, whatever I we assumed about data is of course, of that quality. And so, our adjustment process which is just a numerical process; that means, you are giving some input some matrix P B A whatever A W, and it is giving some output to you. That numerical process is a valid process, because we have taken some quality constraints on our data.

And in fact, those quality constraints are real, because they are giving me the assumed equal to the calculated value of the especially the variance. So, here we assumed only one data here you see sigma 0 and we are trying to estimate it initially. Then we try to calculate it after adjustments process, then we take the ratio and. So, this process is called hypothesis testing and one should always do this thing.

Yes, there is a big question here. If we keep on you know doing this process, because I do not have any idea about sigma 0; so where I continue this process always. Here I can give you little. So, less or little sympathy that this whole process of adjustment computations is generally performed with the help of computers, because as we know computers can help handle the matrices very easily, efficiently, then a human and that is why you need not to worry about much. Only you need to modify your sigma 0 in your program; which you are writing python id or maybe some commercial packages like MATLAB.

Or maybe see which is completely free today. So, then in that case you need not to worry. You are writing only assuming sigma 0 you are changing sigma 0, and try to find out

adjusted values of parameters variables and so on. Then try to calculates s 0 and again check the ratio of the 2. So, the whole process has been performed by your program in computer, no worries at all ok.

. So, there now we have understood completely; what is my adjustment process. And there remember, that error propagation and adjustment processes are the tools for the higher surveying. They are essential tools rather without that you cannot calculate the quality of the data when data is very huge or enormous in number. So, that is the reason we have taken this 5 lecture module on the adjustment computation; where we have learned error propagation and adjustments process. Adjustment process is used to calculate the errors and accuracy, error propagation is used to calculate the propagated variances, given the variances of the independent variables. So, that is a difference between the 2, I hope you are already cleared by this time about the 2 things.

Secondly, adjustment computations gives me the quality or the accuracy of my unknowns as well as the original data. Because we have assumed here the value of sigma 0 then we calculate the values of s 0 after my completion of adjustment process, then I calculate the ratio. And it should be in this range. So, that is the whole idea here, ok.

Now, let us look into this thing that, how to represent the errors and one more thing I would like to say by error propagation we have realized that there is a n number of data and so on. So, my standard deviation is indication of precision by adjustment computation remember the example of one variable that we measure n number of times. And I have prove that mpv is equal to the mean.

And there we realize that error which is evaluated by the adjustment process is also equal to the sigma precision. So, if there is some random error we have proved again and again that sigma which is precision is somehow and indicator of the accuracy. And that is the main learning of this module. This is one of the most important learning; that precision may indicate the error or accuracy provided there are only random errors. And no systematic errors and no blenders involved in the data.

. So, let us assume that we have random errors only and then we are doing the adjustment process. And then we are getting all the results by minimizing of the random errors in our data and our adjustment process. Now we have talked about degree of freedom also here.

So, what is the degree of freedom? I will just give you briefly about this. So, degree of freedom I will give you simple example here.

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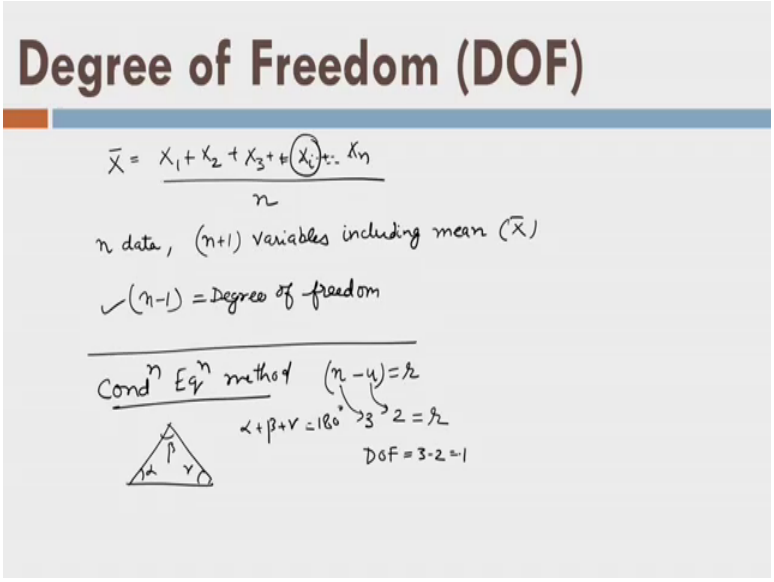

Let us say you know the mean that is observed from the n number of data like this right. What is the degree of freedom? Now there are total n plus 1 there n data. And if I include my mean then n plus 1 variables I can say here right, ok. So, I am including mean here, including mean, that is x bar.

So, what if I do not know let us say some x i data, it is not known to me. But I know the mean say for example, I can find out the x i value by this equation, because I know all the n number of data. It could be mean also; that means, I know that particular data is missing somewhere, but I know the mean or let us assumed that you have written all the n values in your notebook. And you are calculated mean also, after sometime you realized that a particular value got erase from a data. And as a result because you know the mean you have evaluated that value.

So, at a time we always know n number of data here. Even we calculate mean that means, we know n number of data before that. So, we say here that I have n minus 1 degree of freedom. You got the idea here? I hope. That means, I can fix my one variable with the help of n values or with the help of n variables I can fix the value of 1. And as a result I can do n minus 1 possible variation here, and degree of freedom. I have freedom of n minus 1 choices. What does it mean? Out of these values, I can choose any n minus

1 as per my choice, but at a time only one. So, there are n numbers n minus 1 possibilities are there to choose any of this data. And that is what my degree of freedom.

Similarly, in case of condition equation method, when we go for observation equation method, this is my n minus 1 degree of freedom. For condition equation method I have R equations that is my n minus u here which is R. That is number of condition questions here right. So, here the number of variables are 3. So, I have 3 and what is u? That is number of unknowns. Remember, if I know 2 any 2 angles, I can find out the third angle.

So, I have a choice on the one angle whichever angle I want to select. And that is my degree of freedom. So, my degree of freedom is becomes equal to number of condition equations here that is 3 minus 2 equal to 1. So, that is my dof equals to 1.

Similarly, I can find out degree of freedom as and when in a given situation I need. Remember, the degree of freedom is the kind of freedom or kind of flexibility I have to select certain variable, still I can fix my phenomena. So, that is a I can say that this is a determinant system. There I need minimum number of observations to find out minimum number of variables, but the moment it is over determine; that means, I have in axis of observation in then the required observation I find out, what is the possible flexibility I have. And that is what we called degree of freedom.

So, let us go ahead and look into how to report the errors using precision, because we have already said that precision is somehow and indicator of the error or accuracy. So, today we will before finishing this lecture. That is last lecture of this module. We will try to see whether different-different expressions of the precision to express accuracy for one dimensional case, 2 dimensional and 3 dimensional case.

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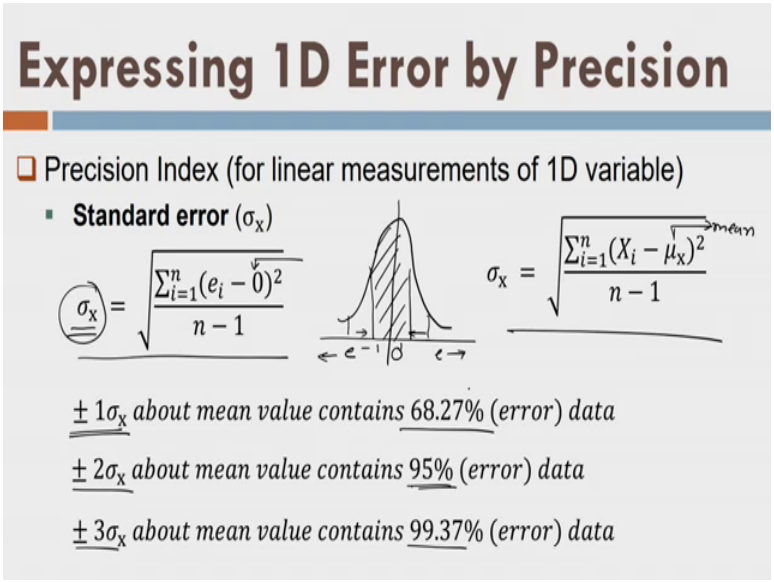

Let us go it ok.

First of all, we know that standard error what we call sigma x and we calculate by this formula. And that is sigma x for the x, data where this is my mean or the sample mean instead of population mean I am taking sample mean. In case of errors we know that errors come from the normal distribution; where this central values my 0. And that is why we put here the 0 here. And this is my error which could be negative. On this side which could be positive on this side. So, this standard deviation is called standard error. Remember, it is also are indicator of the precision, but now we are saying that we are trying to evaluate the accuracy assuming that there are only random errors.

So, precision is an indicator of accuracy. So, this is my standard error sigma x. And we know that if I include plus minus 1 sigma x, around 0 I will have this much of errors in the data. That means, this is my one sigma here one sigma here plus and minus. So, this range includes 68.27 percent of the error data because it is a error. So, we are seeing error data again and again and again ok.

Now, what if I make this range plus minus 2 x? So, that will contain 95 percent data plus minus 3 x that will contain almost 100 percent almost 100 percent. So, that is the idea, I can see that this could be a possible 100 percent accuracy if I consider plus minus 3 sigma. I calculate sigma x, then I say that ok, this is my mean 0 minus 3 sigma plus 3 sigma. So, this total range around 0 is going to give me the total 100 percent error data.

And as a result I can say that this could be possible maximum error range. This is 95 percent error range. This is almost 66 percent or 68 percent error range or 2 third of the error range, right. And then we say that this is the accuracy here different-different ways here clear, 3 sigma accuracy, 1 sigma accuracy like that.

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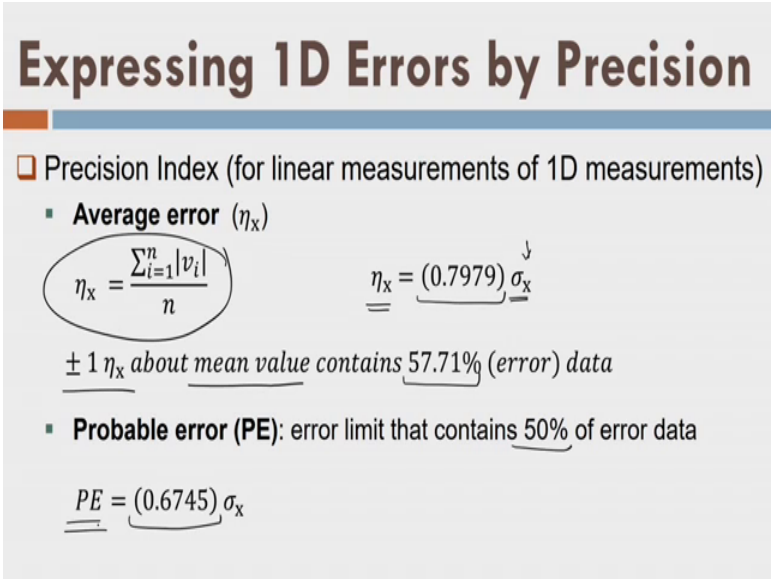

What about the average error? We defined average error as the mode of my residuals like this; that means, and then theoretically I can say that if I calculate my standard deviation and if I multiplied this value I will get this value. And that is the idea here; that even if I calculate only my sigma x, I can find out my average error by this formula. Similarly, here if you look that plus minus n x about mean value contains this much of data, almost 60 percent we can say or 55 percent better.

Now, we define another error called probable error, right, that error which contain exactly 50 percent data. Here we say 55 percent or 57 percent. Now if I multiply sigma x by 0.6745 factor. I will get probable error such that, plus minus 1 probable error we will contain 50 percent of the data, the error data.

So, I can if you want to have report the accuracy 50 percent accuracy, or the accuracy reported a such that 50 percent of the accuracy is are within that limit. So, you multiply sigma x with the help of point this factor, and then you can report here probable error. That means, if you try to verify your accuracy in the field, you will get let us say you taken n number of observations around the point and you are trying to find out what is

the error that my point, you will find out that 50 percent of the error will be lying in the range of this range, around that point, that is the meaning here ok.

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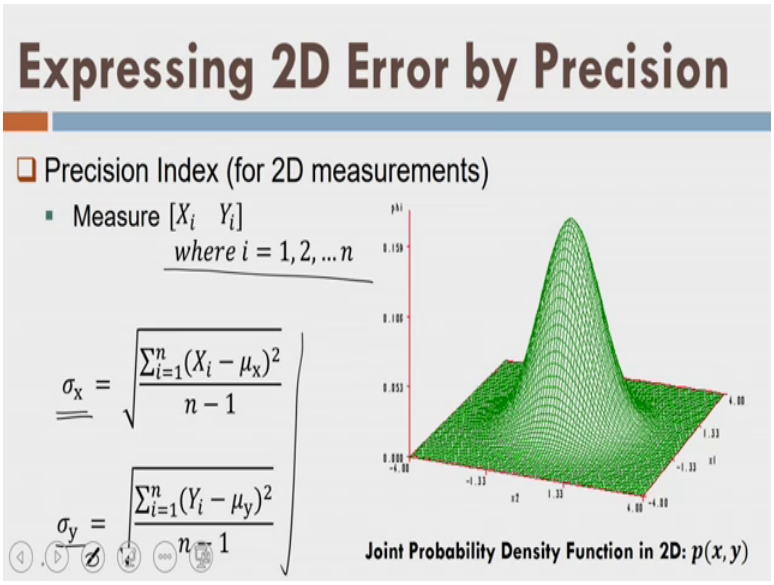

Now, let us see we have 2 dimensional measurement. That is x and y coming as a couple. Or sometime we have 3D measurement xyz coming is a duple. So, right now we have 2 d measurements say. So, we are assuming that they are basically independent. In fact, they are not in reality. And we should not assumed it, but now we have devised some formula so that even we assume them independent I can find out their joint contribution. Let us look into this thing. Let us say you are find out assuming that they are independent and you are find out sigma x and sigma y by this formula which is nothing but standard deviation ok.

Now, the reality this is my joint probability distribution. I have assumed that x and y are varying as a univariate variable, which is a single variable normal distribution. In fact, in case of 2 d variation or 2 d data they vary like that, especially the errors. Right now we are seeing by data, but I will let us talk about the errors. So, errors they do they are connected in 2 dimension to each other. And that is the joint probability distribution. Not the they are not univariate any more because that 2D data we are observing; however, I cannot deal this it is it is very, very mathematically complicated case, if I try to write this pxy here and you justice with that.

Therefore, our earlier scientist they have already devised some approximate methods; so that we can find out my correct errors or the sigma values for 2D distribution using this values only. I will use these values, but I will find out the errors in the 2D case. So, let us look into those formulas.

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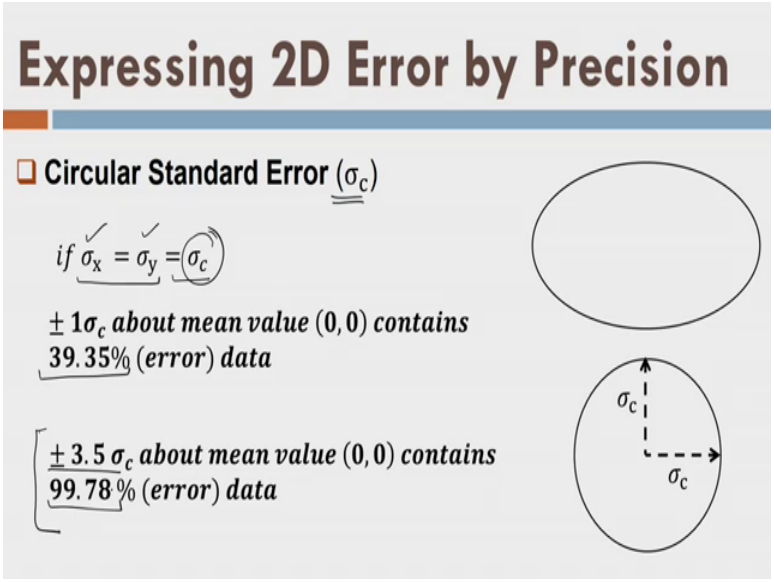

First is circular a standard error written by sigma c, and which could be an elliptical error, it could be a circular error. So, if I say let us see sigma x and sigma y equal to sigma c that is both are equal. So, I can find out that plus minus 1 sigma c it contains almost 40 percent of the data for 2D distribution remember. This is for the 2D distribution. And therefore, the 1D distribution assuming that x and y are coming from one d one d distribution. And so, this is my 2 d distribution and that is giving my sigma c value; that means, on the joint probability distribution I am trying to find out what is the value of sigma c where those many of data is within that circle or within that ellipse.

So, here this is another figure given. If I take 3.5 times of sigma c, it will contains almost 100 percent data.

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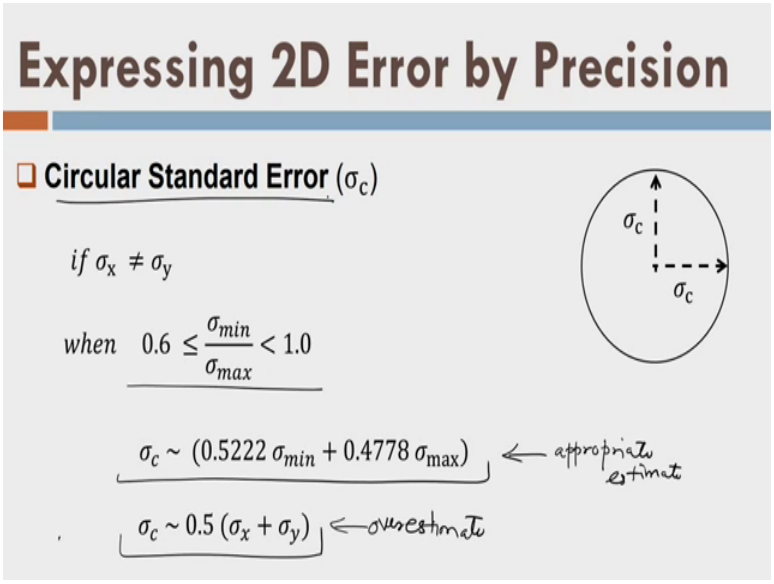

So, let us look into the; what is sigma c how to calculate sigma c, given sigma x and sigma y. Or we calculate sigma x and y for the couple of xy data right. So, when this range is there, that is since I know they are not equal ok. In that case we say that one is minimum one is maximum. So, if I take this ratio, and this ratio is in the range of 0.6 to 1, I can calculate approximately my sigma c. Or my sigma c is like this that is, half of the sigma x plus sigma y. One is minimum, one is maximum ok.

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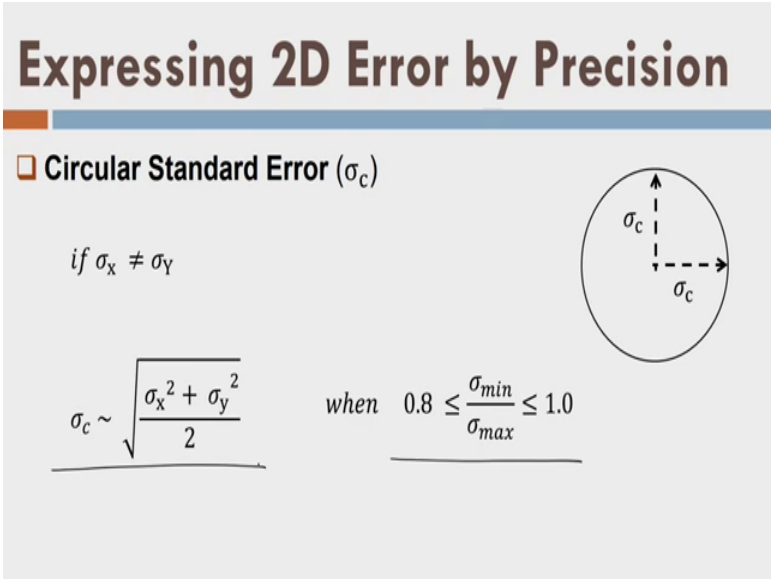

Then we define, again circular standard error, if they are in this ratio minimum and maximum sigma values. I can define again approximately like this.

If you go back, here this is an over estimate I will say. But still overestimate in terms of error is better, because you are trying to report your error accuracy. So, accuracy slightly increase better then reporting very, very optimistic value. Here it is appropriate estimate, overestimate of the sigma c. Now that is also done here ok.

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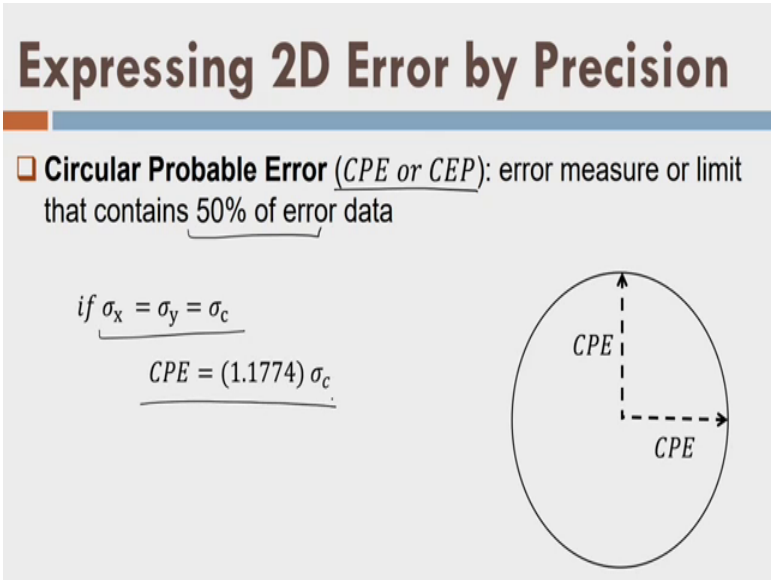

Then we define circular probable error CPE or CEP. Remember if you on the interface of the geotrance this was reported. Again check it in module 2 when we talk about geotrance, for the map protections and return transformations and the coordinate transformation as well.

There this is given on the interface itself. You might have not noticed it ok. So, this is an error measure that contains 50 percent of the error data in 2 dimension, not in one dimension right. And so, again we say that if this is the situation, then CPE is given by this value right.

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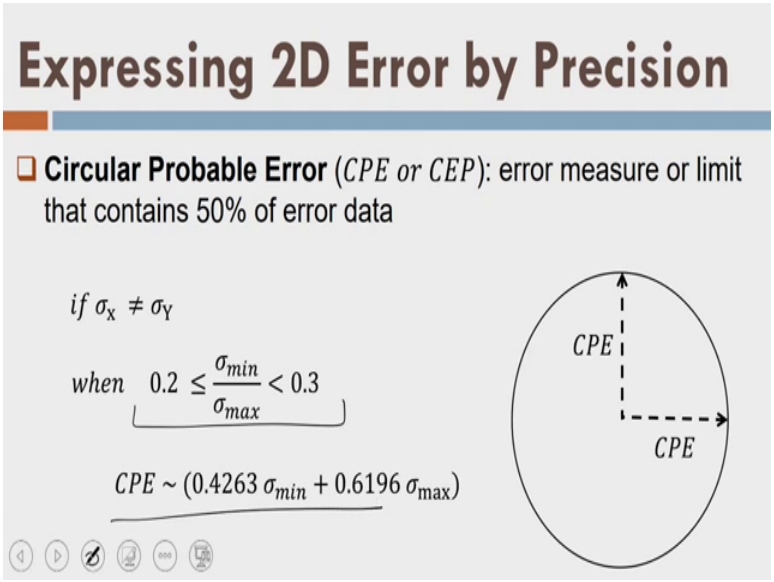

Next what if they are not equal in that case if this is the situation, we calculate CPE by this formula or if this situation is there we calculate by this again.

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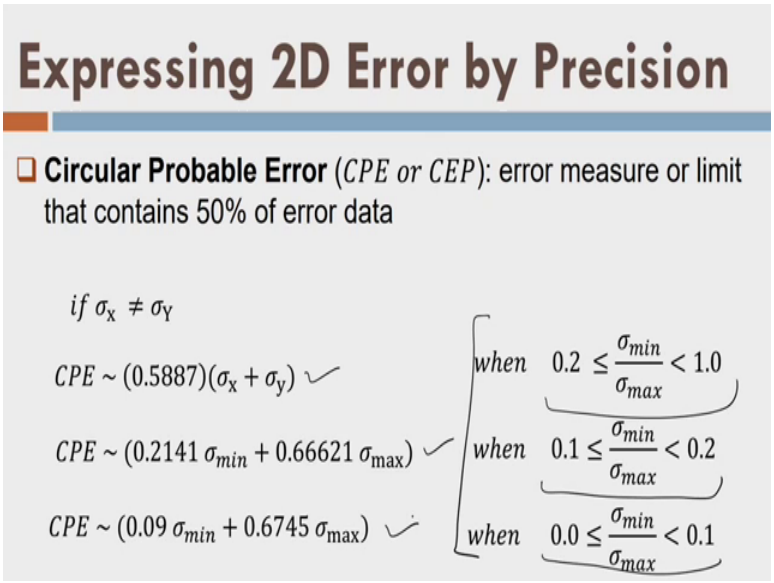

So, I just have this few more formulas here different-different situations; that means, in this range if there which is quite wide range, that is an appropriate formula, if below 0.2 to up to 0.1, we have this formula and below 0.1 we have this formula. And they are all approximations, but still they are good approximations. Remember, we are trying to

evaluate estimate the 2D error joint probability distribution or the joint error using the 1D values.

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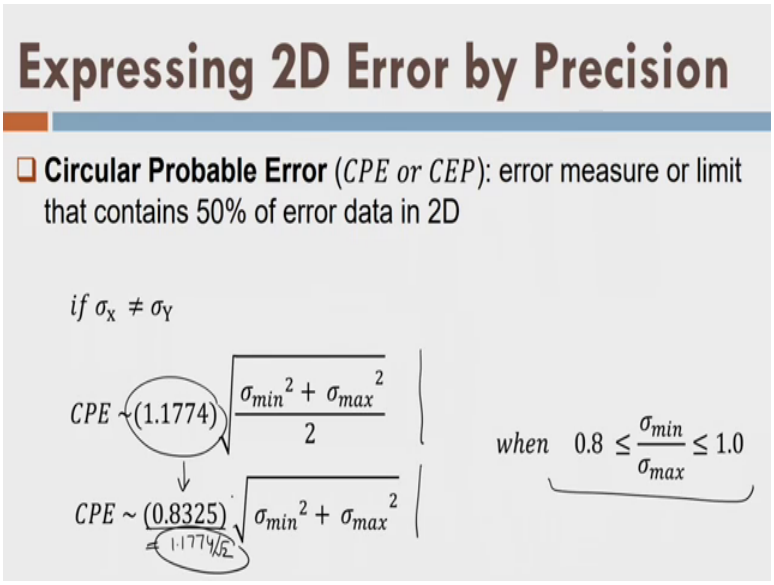

Then again CP is still given by this formula. So, if you are in this situation, these are the better estimates; that means, when sigma min and sigma max are almost equal they are equal to almost a circle. And that is why we are having this assumption here, right. Now they are basically equal if you put this root 2 outside. So, this divided by root 2 we will get this thing. So, here it is nothing but this is equal to which is coming from here right.

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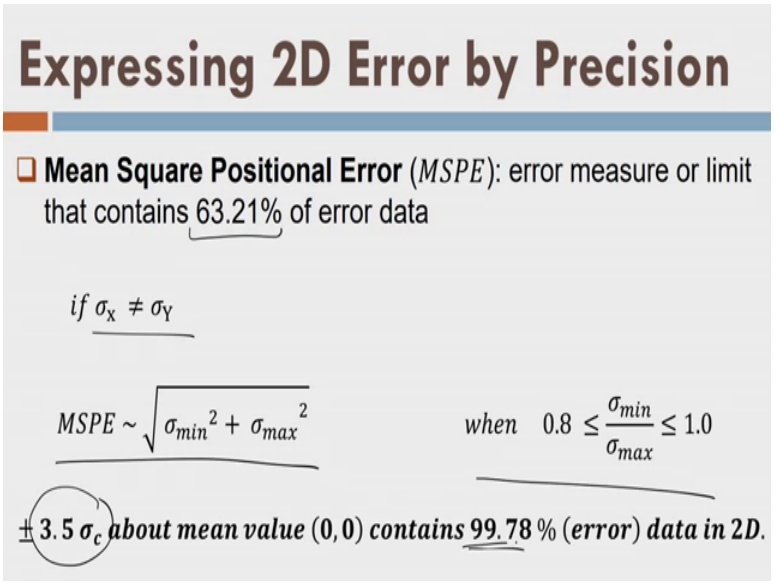

Next mean square positional error that is called MSPE and that contains this much of data in 2D; so if they are not equal, we define in this situation like this. And we know that after all this measures we know that if I take this much of 3.5 times sigma c, I will have almost 100 percent data in 2D right, ok.

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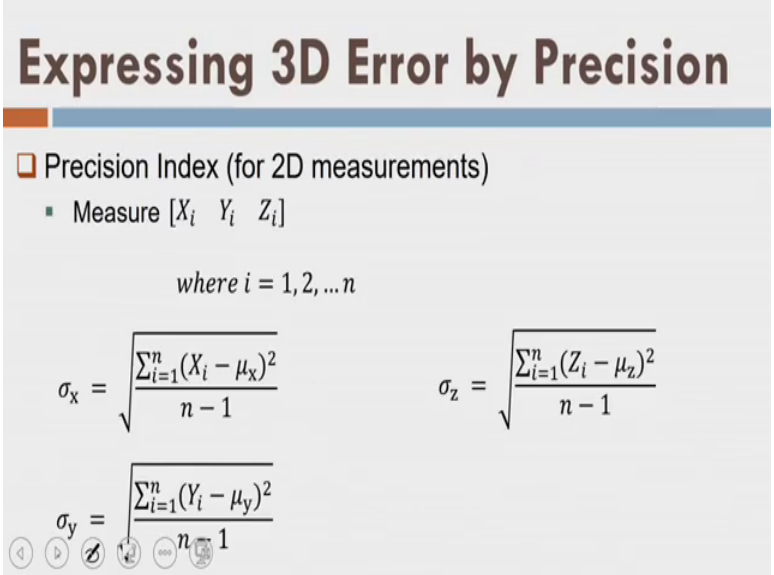

What about the 3D error? Now I need to consider that distribution of error in 3D the joint distribution in 3D, because the data is coming in tuple not couple or not single data. So, we have calculated sigma x sigma y and sigma z this way. And then we are saying instead of this P x y z we will do some approximation ok.

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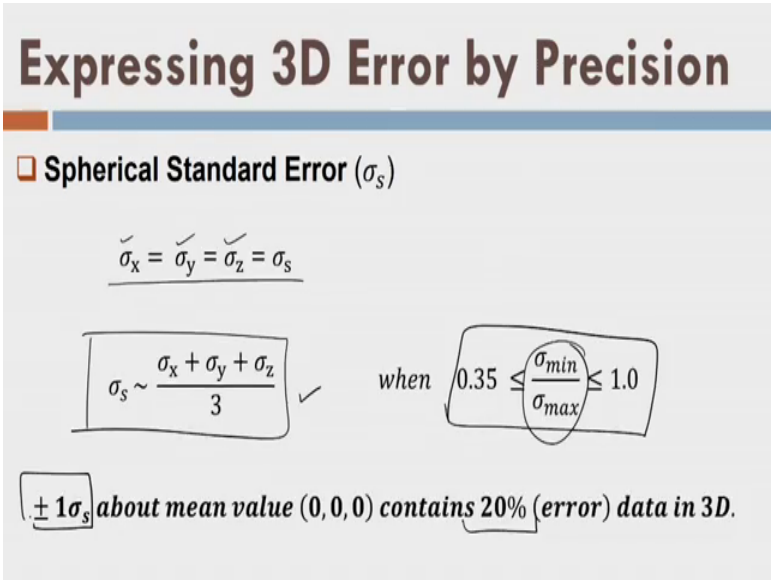

So, here assume that let us see they are all equal; that means, they are close to a sphere. What will happen? I can assume this as my sigma s that is my a spherical standard error. And if situation is prevailing like this; that means, I have 3 data here sigma x y and z. I assume one is minimum, one is middle and one is maximum. So, if I take the ratio of 2 here, minimum and maximum and this ratio comes into this range, I will use this formula in order to find out my 3D joint distribution error or the error in 3D considering the joint distribution of the 3 errors.

So, it almost in 3D it contains 20 percent error if I use plus minus one sigma s.

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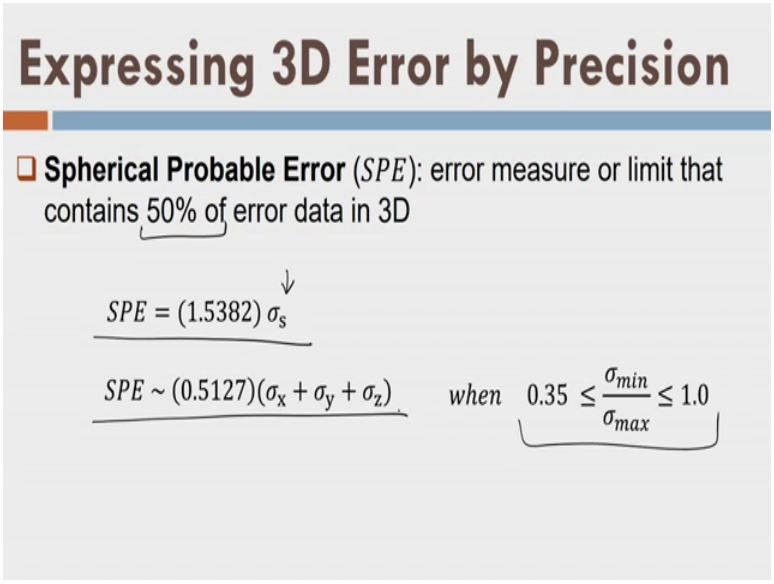

Spherical probable error again we define another one, such that we have 50 percent of the data. So, it is given by this formula, if I estimate sigma s by the previous slide, then if in this situation it can be given by this approximately.

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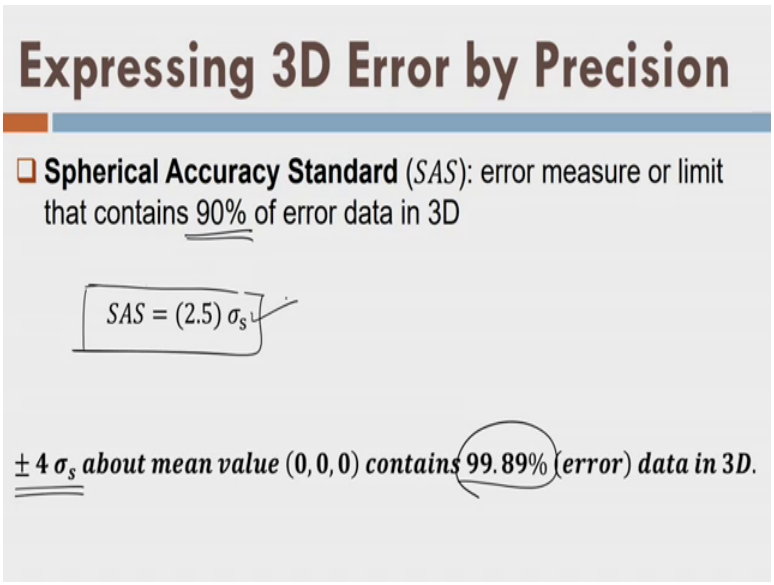

Then we have spherical accuracy standard SAS which is error measure or limit that contains 90 percent of the error data. Remember, we are trying to do 50 percent of the error data sometimes 68 percent.

Now, if I contains 90 percent of the error data which is called spherical accuracy standard. And that is given by this. So, you estimate sigma s first, and then try to estimate SAS. Moreover, in case of 3D data, if I estimate this quantity, I will have almost 100 percent data, error data (Refer Time: 52:31). That means, if I calculate sigma s, and after that I want to report what could be the maximum error possible here; that means, all 100 percent error data will come within that circle or within that sphere. So, what is the radius of that? And that is this radius 4 times sigma s. So, that is the idea here for reporting of error in ideal cases. So, they are working approximately well these criterias, because we for a large number of data, we know that errors are distributed normally.

But when data is not one dimensional, it is multidimensional let us say 2 dimensional or 3 dimensional, in that case we should consider the joint probability function of the 2 dimensional error or maybe the 3 dimensional errors. Since, it is very very complicated mathematically. So, we have derived some approximations. So, this was this whole story here. So now, we have equipped ourselves with 3 things. One is error reporting recently we have done.

The second thing adjustment computation process, that is one of the fundamental tool I need for higher surveying. Third error propagation, because error propagation gives me the value of propagated variances. Using these propagated variances, I can find out what is the precision that is propagating in 2 dependent variable. Adjustment process will tell me; what is the accuracy I should expect from my adjustment of the data. I can report the errors now.

Third thing I also realized that in the whole game of adjustment computation and error propagation, I concluded one thing here again and again and once again I am finally, concluding before finishing this lecture. That in the presence of random errors precision is somehow and indicator of the accuracy. Although, accuracy and precisions are completely different concept, precision is the quality of the data; that is, closeness among the observations right. And that is why we represent it by standard deviation. What is accuracy? Accuracy is the closeness of the data to the true value, reference value and that we cannot calculate, because true value is not known.

So, that is why we use condition equation for example, for triangle, sum of the 3 angles equal to 180 degree. So, my 180 degree becomes the reference value or the true value we

know it universally true, right. So now, we are realized that how to calculate accuracy and precision. Again, precision may indicate accuracy if there are only random errors.

So, we have proved all the things in this module. And with this now we are equipped with the very strong tool a very fundamental tool a very very important tool. That is called adjustment computations and error propagation and error reporting. And now we will keep on using this tool again and again in coming module. So, next module will be on the GPS, till then bye-bye.

Thank you very much.