Higher Surveying
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Module – 04
Error, Accuracy, and Adjustments Computations
Lecture – 12
Condition Equation Method and Combined Method of adjustments

Hello everyone. Welcome back in the course on Higher Surveying. And we are in module-4- Error, Accuracy, and Adjustments Computations. In the last lecture, we have seen the observation equation method for adjustment computations. And then, we talked about how to find out the NPV, which is equal to the mean for given observations. And then, we have solve many examples, multi response example, single response example. And then, we have seen that how to construct the weight matrix.

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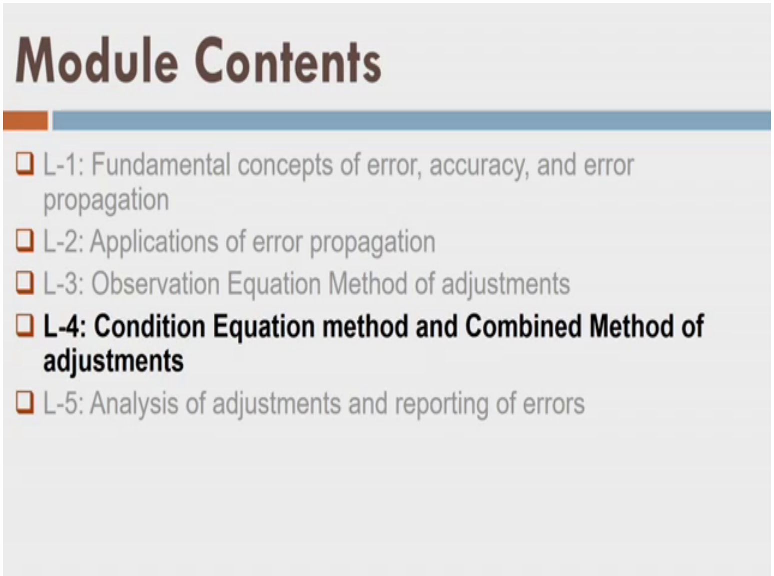

Today we are going to learn a non-linear example of observation equation method, but we are also going to consider how to construct the weight matrix, and what is the cofactor matrix. Secondly, we are going to see: what is the condition equation method, which is quite different from observation equation method. So, let us start here.

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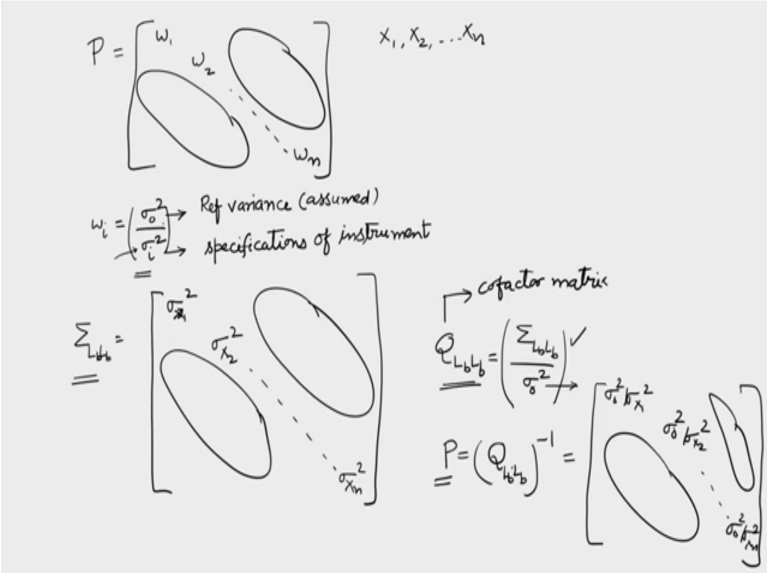

So, we have discussed that the weight matrix P that is written as w 1 w 2 and all the diagonal elements valid, and all other off diagonal elements are 0. So, I write big 0 here, and big here. Because, all these observations X 1 or maybe small x 1, x 2, x n are coming from field observation ok.

Then we have said that we have devised and a w i is equal to sigma 0 square divided by sigma i square, where sigma 0 square is my reference variance or variance of unit weight right. We have assumed this value, and also this value is coming from either experience or in general, it is coming from the a specifications of the instruments. So, I am using the specifications of instruments to estimate the sigma i square. So, it is general idea that we are meeting some instrument we are using some instrument for measurements. So, my instrument should give me this kind of quality in the observation, even if I take one observation.

After discussing this thing now if I consider the matrix sigma L b L b, which is nothing but sigma x 1 square, sigma x 2 square, and sigma x n square, it can be any variable, and just all the rest of the elements are here fine. So, can I derive the matrix P from this matrix, how can I derive. So, let us look into another matrix called Q L b L b that is called cofactor matrix and it is defined as sigma L b L b divided by sigma 0 square like this. So, I am dividing the each and every element of sigma L b L b by a constant value sigma 0 square here. And as a result, what are you get, I will get a P matrix as a inverse of Q L b L b, you can see here and so on, 0 and here off diagonal matrix elements are 0 here.

So, you can see here if I construct the Q L b L b in this from, and if I take the inverse of Q L b L b, which is my cofactor matrix here, I can find out the weight matrix of given observations, which I measured in the field. And, remember in case of observation equation method, we have a dependent variable, and we say that the Q L b L b or the sigma L b L b is explaining or describing the quality of my dependent variable. Moreover, my independent a variable is always error less, that is the assumption we make it in the observation equations method.

Now, here if you see carefully that by taking this ratio, I have elevated two problems. The first problem, which was the minor problem; the minor problem was the numerical error that means, if I take the observations in centimeter or meter, then the value of this sigma i will be in different different units. And as a result, if I take this ratio, it will happen that the ratio will be removing the factor of units. And so, even someone take centimeter or someone take the meters unit, does not matter, it is ratio. Secondly, I have derived a major advantage here. The major advantage is by taking this ratio, I need not to consider much about the sigma i square ok, because sigma 0 square is here, unit reference for unit weight. And as a result, if I take the ratio of these two, I can find out what is the weight of my observation.

For example, if I say let us say observation number 1 is better than 1.5 times, then my some other observation. So, I can say this observation, which is other observation, it is having unit rate 1, then this observation should have a weight of 1.5 or maybe some other weight, because this observation is 50 percent better than this observation or I can say it is 1.5 times better than this time. So, it is 1, then it is 1.5 width that is idea. And that I can establish with the help of taking this ratio ok.

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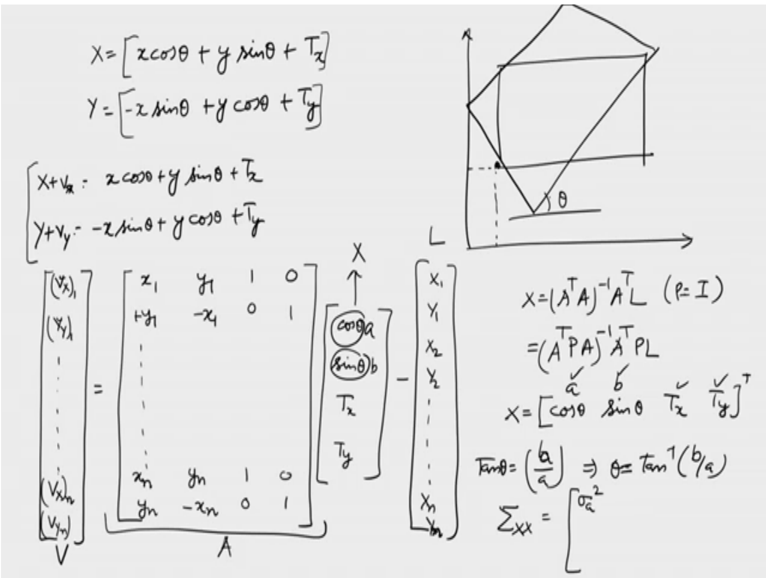

So, let us take an example here of observation equation method. One more example we will take here today ok. Let us see that I want to do the geo referencing. And the idea for the geo referencing was let us say I want to rotate an image to another image or I want to do geo referencing, so this was the case here x cos theta plus y sin theta plus let us say T x, that is my first equation. Second is minus x sin theta plus y cos theta plus T y. Here, T x and T y are the translation in the capital X and capital Y, such that I will get these two equations here like this; it is very simple equation of the translation and rotation that is conformal transformation about z-axis by theta angle, and then translation.

Now, it happens that suppose if I have map here or maybe I can say this is one image and I am trying to bring another image into that or I am trying to rotate this image by angle theta in the some coordinate system. Let us say this; I want to rotate this image by this angle like this. And this angle is let us say theta fine. And I want to shift this origin of this one, which is T x, T y right, so that is a equation I have written.

Now, as per the standard process what do we do, we introduce these error V X equal to x cos theta plus y sin theta plus T x. And similarly, Y plus V Y equal to x sin theta plus y cos theta plus T y. Now, if I try to find out V X and V Y for n number of observation, they will be like this v X 1 v Y 1 they are small v's and so on, I can write V X nth observation or V Y nth observation like that.

So, and then here what will happen, I can write my matrices as x 1 y 1 1 here, then I can write here let us say plus y 1 minus x 1 0, here 0, and here 1 well. Try to write yourself, I am not writing all the things x n y n 1 0 y n minus x n 0 1 and here my unknown parameters cos theta, sin theta, T x, T y. Try to check whether you are getting n number of equations, which are written by repeating this or not. So, this is my matrix arrangement.

Then finally, my L matrix, which is given by X 1, Y 1, X 2, Y 2, X n, Y n right. Now, I can call this as my L matrix here; this is my parameter matrix capital X; this is my A matrix here; and this is my V matrix here. And now, you can use X equals to A T A inverse A T L or I can also use using a weight matrix P, if I know or if I assume identity matrix. Then in this case, P is equal to my identity matrix. And here, it is not an identity matrix, so I am writing this thing, so that was the idea of observation equation.

But, if you observe now here X what is this X, this is my matrix cos theta sin theta T x and T y. So, was basically what I have done here, I have assume is cos theta and sin theta as my parameter a and b, similarly so I am writing here a and b. So, then I have four parameters 1 2 3 4. But, I have not determined theta, theta I have to determine again after finding out a b, then I will be writing like my tangent theta will be b upon a, and then I will be saying theta equals to tan inverse b upon a right.

So, what about the quality of the theta that means, if I find out sigma XX, so I will be finding out here sigma a square, and not the quality of theta, it will be quality of cos theta. And that is the problem with the kind of linear approach, when there are some non-linear functions like cos theta sin theta. So, let us look into that, how to deal with such situation.

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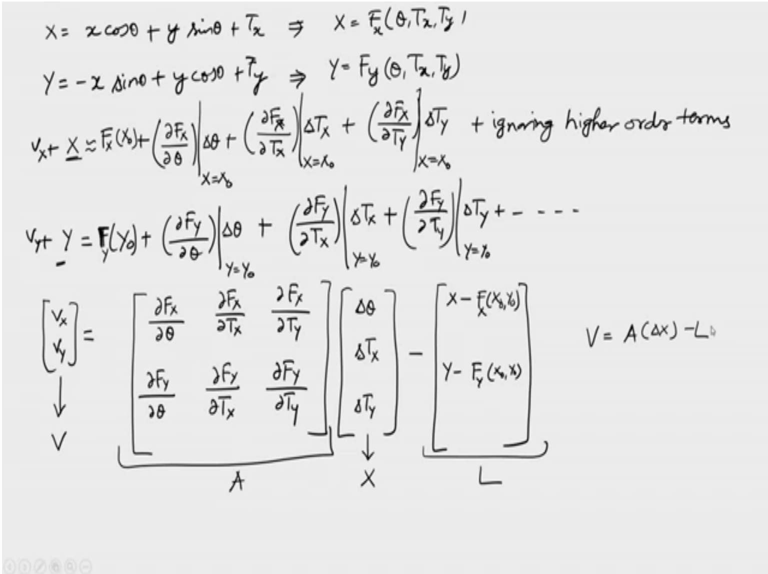

Now, once again write the same set of equations, and here I am writing let us say x cos theta plus Y sin theta plus T x; Y equal to minus x sin theta plus y cos theta plus T y ok. Now, my capital X is F of let us say theta, T x, and T y, let us say F x. And similarly, Y equal to F y is equal to function of theta, T x, T y.

Now, instead of using a linear approach, where I linearize my parameters, I will linearize my function F. What does it mean, so when I introduce the error V or I will say that use the Newton Raphson or the Taylor Series, and write it approximately as let us say X 0 value, which is a initial value here, then d F X divided by d theta into delta theta that is first order approximation of by Taylor Series F X divided by dou F by dou T x into delta T X, and here again I am writing dou F X by dou T Y delta T Y. So, what about these values, these values are evaluated at X equal to X 0 some known values of the parameters. This is what we do in a standard Taylor Series, we write about. And we are ignoring the other higher order term, right.

Now, similarly I can write Y equals to Y 0 plus d F Y divided dou F Y by dou theta 0 plus dou F Y by dou T X into delta T X, again it is evaluated at some value of Y equal to Y 0, and X equal to X 0 right. And here d F by d F Y by dou F Y dou T Y into delta T Y yeah. And we are ignoring rest of the terms; again evaluated at Y equal to Y 0, and X equal to X 0 Y. Now, this is my linearized form ok. Now, I introduce the error V X and V Y at this stage. As a result, I can write this as my V X plus this one V Y plus this one is equal to this one, that I am that means, I am writing the adjusted value of the dependent variables now here.

And now, I can form the my V equal to A X minus L, where if I write this thing what will happen, V X V Y if I write small just for one observation, it will be what happens here, I can write here, it is my F Y here, it is my F X here right. So, let me write this form in this way, that it is nothing but I can write this way yet dou by dou theta by dou T X dou F Y by dou T Y into I can say delta theta, delta T X, delta T Y plus or minus, I can say whatever I want here.

Now, I can write here X minus F X 0 and Y minus X F Y Y 0 here like this. I can also write these values here, not only that, some X 0 Y 0, and similarly here, because there is a point X 0 Y 0 in the 2 d that I am considering here, no problem. So, they are basically same right ok. So, this is for one observation. Now, here this is my matrix A, this is my matrix unknown X, this is my matrix L, and this is my matrix V right. I hope I am conveying the correctly. So, this is the form V equal to A X minus L, rather I write instead of X, I have now delta X, which is the corrections to the assumed value X 0, Y 0 ok.

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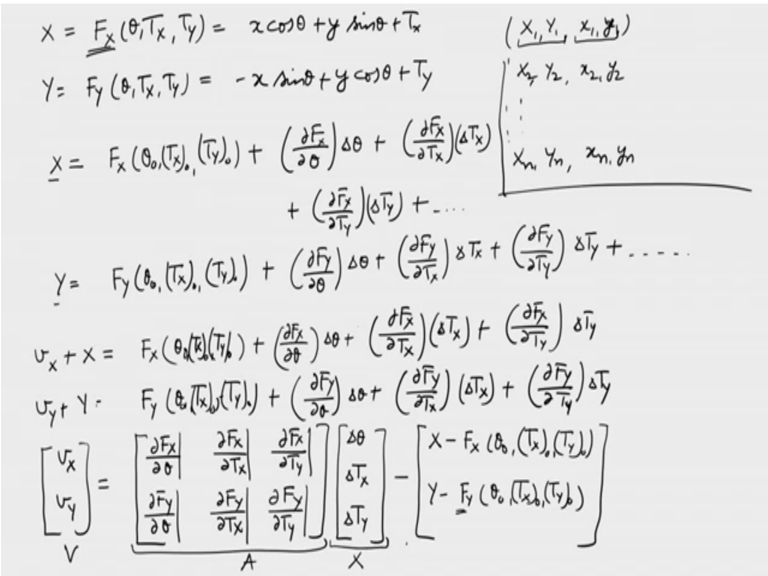

So, let us write this thing as X equals to my function F X of theta, T X, T Y or I can write here is let us see x cos theta plus y sin theta plus T X. Y equals to F Y of theta, T X, T Y, which is nothing but minus x sin theta plus y cos theta plus T Y ok, so that was the example, where I said that ok. Let us see an observing X 1, Y 1 corresponding to x 1 y 1, it could be (Refer Time: 18:35) transformation, it could be a two-dimensional transformation of the two images or two dataset anything. So, now, observing like this X 2, Y 2, which is corresponding to x 2, y 2 and so on, X n, Y n corresponding to x n, y n. And I want to bring this transformation, so my unknowns are theta T X and T Y.

Now, I can see that let us make X by Taylor series expansion that is F X, and now I am evaluating it at theta 0, T X 0, T Y 0 some known value plus d F by dou theta into delta theta plus dou F X by dou T X into delta T X plus dou F X by dou T Y into delta T Y that is my first equation. Same way, I can approximate Y by F Y theta 0, T X 0, T Y 0, so these are my initial values. Then again in the same fashion I will write, I hope you are trying to write it, and you can convince yourself here ok. Here, I would like to mention that we are ignoring the higher order terms in both F X and F Y, because we are taking first order approximation in Taylor series.

Now, let us introduce the error in my dependent variable X and Y. I am remember, I am observing both X, Y capital and both small x, y here. So, now introduce an error. So, lets say V X plus X equals to my this expression F X this, I am not writing the full form, and then d F X by d theta plus here some multiplication delta theta plus plus ok, here let us say theta 0, T X 0, T Y 0 right. Similarly, I can write V Y plus Y equals to F Y theta 0, T X 0 comma T Y 0 plus these three terms will come as it is like this, fine.

Now, let us construct our V equal to A X minus L, where let me show you what is the V matrix here, so this is my V matrix. And if I write the same thing here, how can I write in matrix form, which is nothing but d F X by dou F X by dou theta dou F X by dou T X dou F X by dou T Y dou F Y by dou theta dou F Y by dou T X dou F Y by dou T Y into delta theta, delta T X, delta T Y. Then, we write here X minus F X theta 0, T X 0, T Y 0; Y minus F X theta 0, T X 0, T Y 0 and so on. So, these are the values of function F here evaluated at theta 0, T X 0, T Y 0, which are initial approximation or initial estimate to start my calculations ok.

Now, similarly I have evaluated F Y (Refer Time: 23:47) Jacobian matrices are evaluated at the X theta theta 0, T X 0, and T Y 0, which are my initial approximation ok. Now, this is my V matrix, this is my A matrix, this is my unknown matrix parameters here, and this is my L matrix. And now, I can write for n number of observations like this.

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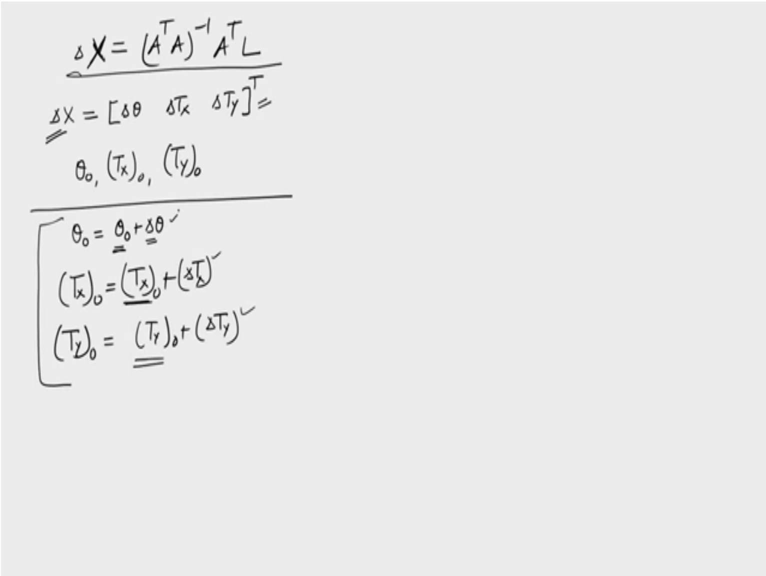

So, if I write for unknown number of observations V X 1, V Y 1, V X 2, V Y 2 and so on, V X n, V Y n. And finally, if I solve it, I will get X equals to A T A inverse A T L. And if I know the weight matrix P of my observations x and y capital X, capital Y, I can write weight matrix also here. Now, what is the message here? If you see what is your X or other delta X, here in the last slide, if you go here, this is your delta X call it delta X, because. So, now if you see your delta X equals to delta theta delta T X delta T Y transpose, which means I am finding out the corrections to the theta 0, T X 0 value, which is initially estimate, and T Y 0 here right.

Now, what is be the next? Next step is that this is my first iteration. Second iteration will be I will assume my theta 0; I will update my theta 0 as theta 0 plus delta theta that is my last value assumed. This is my correction, which I find out here in this step. And then, similarly I will write my new value of T X 0 as earlier value of T X 0 plus the update delta T X. Similarly, T Y 0 I will write here T Y 0 the previous value, and the update. So, by updating or by making these small corrections, I will find out new values. And these are the my previous values, which I assumed originally.

Now, I got the new set of values here. I will use this new set of values in order to again perform the same calculation, where I will use this formula that means, I will re construct my A matrix, I re construct my L matrix, and I find out my delta X again. Again it will give me some corrections, these corrections right. Then again I will update my second values, which is second time I find out in a previous iteration this values, this value, this value, I will put this one. Now, this process will be continuing till all this my corrections become very very small that means, there is no improvement in the corrections, there is no update in the theta 0 value. And as a result, whatever theta 0, T X 0, and T Y 0, I will get after n number of iterations, I will say that is my final answer. And this is the way we handle the non-linear problems by observation equation method.

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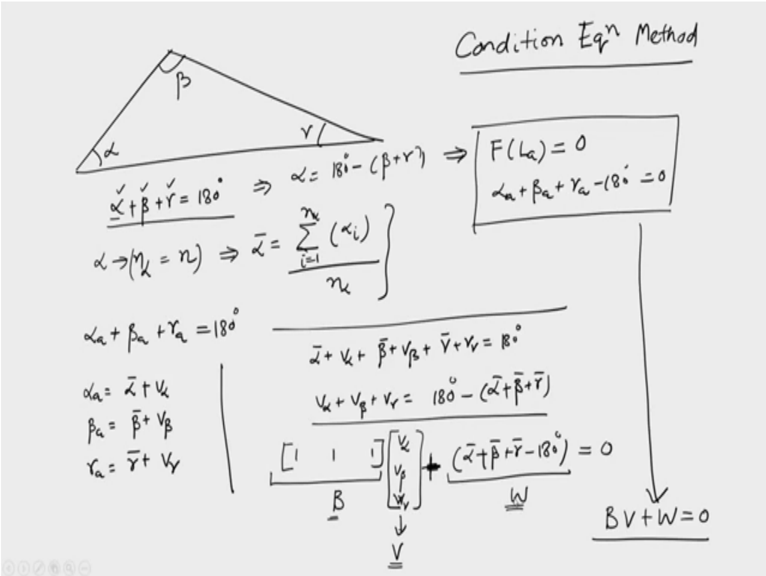

So, let us go ahead and see there are some situations, where I cannot use the observation equation method. And the first situation was if you remember our standard triangle example let us see this my angle alpha, this my beta, this is my gamma. Ideally, I can write alpha plus beta plus gamma, which are adjusted values equal to 180 degree. So, this is the condition equation.

However, we have also said that I am measuring alpha n alpha times or maybe n times whatever, and then I am finding out alpha bar as my mean right. And mean is nothing but I can say here summation i equals to 1 n times or maybe n alpha times alpha i divided by n alpha whatever. Same way, I find out beta bar and gamma bar. So, these are coming from my observations, and this is my idle relationship.

So, here if you see all the variables three variables are observed in the field, and there is no dependent variable or independent variable, because I cannot say that alpha is my dependent; and beta, gamma are my independent variables right. Although, mathematically I can write like this alpha equals to 180 minus beta plus gamma, but still there is no point of calling alpha or beta or gamma independent variable. And such a situation, when all the variables are measured, we can solve by condition equation method. So, let us introduce the condition equation method.

In case of condition equation method, we measure all the variables. Remember, there is no dependent variable and independent variable in some situations, but we are measuring all the variables. Remember, we have measured in observation equation method, we can recall that there is a dependent variable, which is a response of independent variable physically. And hence, we write that explicit equation between the dependent variable as a function of independent variable and parameters. This is not the situation here. All the variables involved in the physical phenomena that we are going to observe are measured ok.

Now, let us derive what is the condition equation method. I can write let us see that if I write these all the adjusted values let us see if I write alpha a adjusted beta a plus gamma a should be equal to 180 degree. What is the meaning here, if I say alpha a adjusted value is nothing but alpha bar plus V alpha, similarly beta adjusted value is beta bar plus V beta, and similarly the gamma adjusted value is gamma bar plus V gamma right.

So, now I can write this condition equation as here F of L a, L a is nothing but the adjusted values of the variables. Then I can write this kind of equation that means, I am writing alpha a plus beta a plus gamma a minus 180 degree equals to 0. And this is the form of condition equation method here right. Let us try to solve this one, using matrix method ok. So, I am solving it here the alpha a as alpha bar plus v alpha beta bar plus v beta plus gamma bar plus v gamma equals to or 180 may be whatever I can write here 180 degree.

Now, I can write here that v alpha, which are errors v beta plus v gamma equals to 180 degree minus alpha bar plus beta bar plus gamma bar. I can write this equation in the form of matrix now as 1 1 1 v alpha v beta v gamma right is equal to right, this is nothing but I can also write this thing here minus alpha bar plus beta bar plus gamma bar minus 180 degree equal to 0 plus here sorry right. So, this becomes my B matrix here, this becomes my again the same matrix V here, and this becomes my W matrix here. So, I am writing the form BV plus W equal to 0 for the condition equation method here right.

What is the idea here, idea here is what is my unknown, and the unknown here is my V, because the moment I know the V, I will add it to the alpha bar beta bar and gamma bar, and I can find out the adjusted values that is it; this is what I want. So, here in case of condition equation method, my residuals are the unknown or I can say indirectly my variables each and every variable is unknown, but they are connected with the condition equation, and that is why I am using condition equation for my adjustment process right. This is my B matrix here, and this we call the mix closure or W matrix here.

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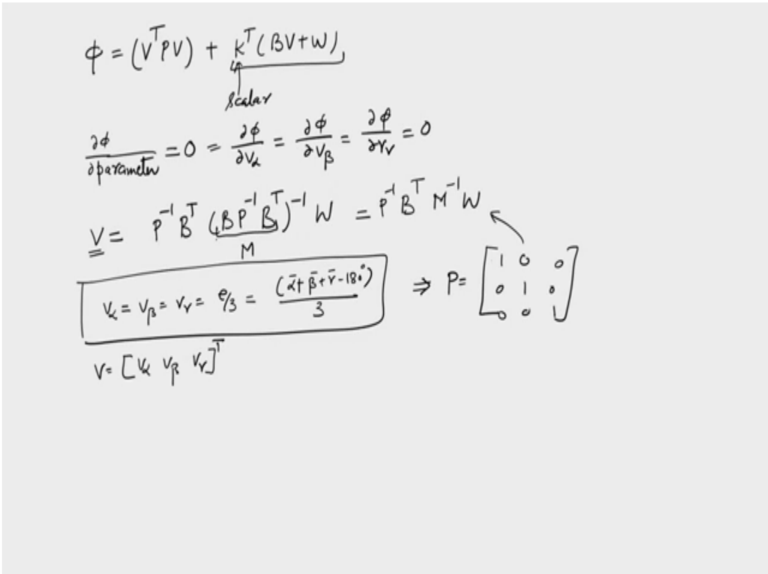

Now, before we write the solution, let us first go by phi fundamental principle, which we said that we want to minimize V T P V. Now, what if I add, since I know BV plus W is equal to 0. So, if I take a multiplayer K T according to dimension of BV matrix plus this one, this whole quantity is equal to 0, because this is a scalar quantity, what you call Lagrange multiplier, and this already 0. So, I am not adding anything to phi, I can write this thing like that ok.

Now, I will do the same process of least square solution, where I will differentiate my phi with respect to the parameter that is nothing but in this case V, and I will put it equal to 0, so which means here if I put d F by d V alpha d dou phi by dou V beta dou phi by dou V gamma equals to 0. I will get three equations; using those three equations, I will solve it and I will find my solutions right. Now, I am doing the same thing matrix method that means, using this only. We can write straight away the solution V is given by P inverse, if there is a weight matrix available B T B P inverse B T whole inverse W, which is mix closure matrix here. Now, it is also written here B T P inverse B T here M inverse W, where this matrix is also written here M, so that is my solution. So, we will find out the unknowns here.

Now, let us take the same example of three angles of a triangle. There can you use this formula to find out the solution, try yourself and find out that V alpha V beta V gamma will be equal to e by 3 what we have find out. What is e, e is nothing but alpha bar plus beta bar plus gamma bar minus 180 degree or you can write other way (Refer Time: 35:17) also. Why, because it is error could be positive, it could be negative also, no problem right.

So, you should get this answer, try to solve this V matrix. And what is my V matrix, which is nothing but V alpha V beta V gamma transpose or it is a column matrix. So, by putting the values here, and for simplicity you assume that your P matrix is identity, which is nothing but 1 1 1 0 0 0 0 0 0 ok; and try to put all the values here and you should get the correct value here right. So, let us homework or you can take as a slides challenge.

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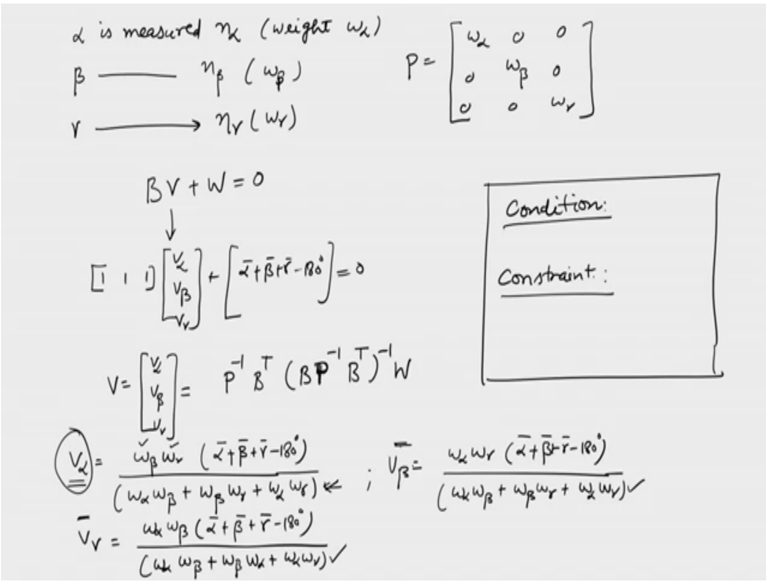

Now, remember we have said in our original derivation that let us assume that alpha bar is measured n alpha times, and hence it has a weight w alpha. Similarly, beta bar is measured and beta times, so it has a weight w beta. And gamma bar is measured or gamma I can say not gamma bar yeah gamma is measured n gamma times, so it has weight w gamma. What will happen, if I have weight that means, my weight matrix is like this, well I have solve it; and rest of the matrix will remain same that means, BV plus W form. In this BV plus W form my V matrix is nothing but V alpha V beta V gamma, and V matrices 1 1 1 plus W is we can say alpha bar plus beta bar plus gamma bar minus 180 degree is equal to 0 ok.

So, now I put the weight matrix here, then what will be the solution of V, which is equal to V alpha V beta V gamma is equal to P inverse B T B P inverse B T inverse I am again writing the same formula W here right. This is my P matrix here ok. You put this and try to find out. I have already found the solution, and what I am getting is I am just writing you directly here, I am getting V alpha equals to w beta w gamma divided by w alpha w beta plus w beta w gamma plus w alpha w gamma here. Here (Refer Time: 38:34) find out this denominators constant. And here we have this one right. And then, you get V beta equals to w alpha w gamma into again the summation of three weights multiply of the product of two weights each time like this.

Thirdly, what is the value of V gamma, which is nothing but w alpha w beta divided by again this summation. You can see here, the denominators in three terms is constant right. Now, assume that your beta and gamma are measured with higher confidence, so they have higher weights. What will happen, it means that alpha is measured with lower weight or alpha has more errors, because you have low confidence in that.

Now, it is reflected here. Why, because the V alpha which is error in the angle alpha, it is proportional to w beta and w gamma that means, if beta and gamma are measured with higher confidence, they will have higher weights, automatically this will contribute more error to this alpha angle; idea is very good here. Similarly, you can see with other angles like what is V beta and V gamma. Automatically if w alpha is less, it will contribute less error to V beta as well as V gamma that is the idea here. And now, you can understand that what is the magic of the weight, the moment you assign the weights to the observations, how your errors are going to change.

Secondly, you can also look into this thing that your observation equation method is not here, condition equation method here. And your adjustment computation process is working very very nicely here, they are adjusting the values, they are finding all the errors ok. I would like to introduce two terms here; one is called condition, you might have come across this time the condition equation, and another term is constraint.

Condition is a I can say mathematical equation or condition which has to be satisfied. However, under the errors the influence of random errors, it is not satisfied. And as a result, we use condition equation to find out the adjusted values, as well as to find out errors in adjustment computation process. But, constraints irrespective of any error whatever, has to be satisfied. If it is not satisfied, it is not a constant that is the difference between condition and the constant. So, let us go ahead and take few more examples of condition equation method.

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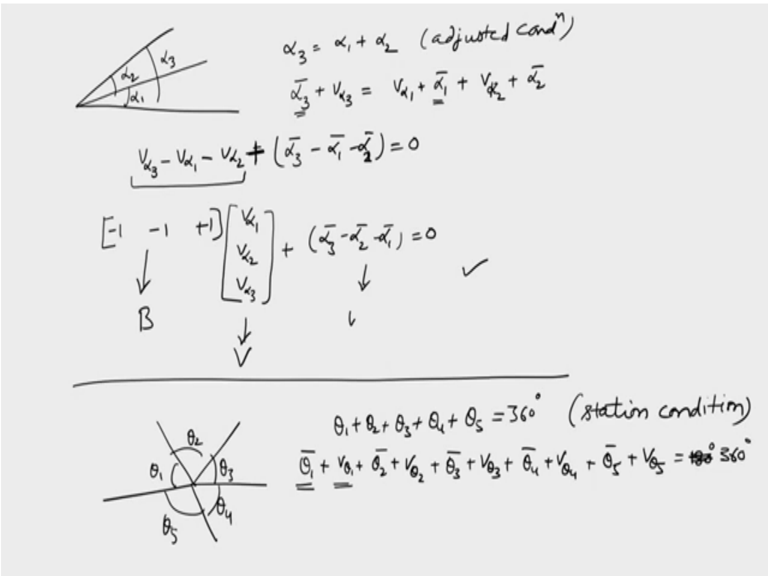

Now, I can give some homework here. For example, you have situation like that, where you measure angle alpha maybe n number of time alpha 2 and alpha 3. So, I have a condition equation here, alpha 3 equals to alpha 1 plus alpha 2; I can also write here alpha 3 bar that is my idle condition adjusted condition. And here if I write let us say V alpha 3 equals to V alpha 1 plus alpha 1 bar that is the measured value here, these values in the field, and then we have V alpha 2 plus alpha 2 bar here.

Can I construct my equation BV plus W here? That let us look into this thing. I can write here let us say V alpha 3 minus V alpha 1 minus V alpha 2 equals to I can write this thing or plus maybe here fine, what is my value here, alpha 3 bar minus alpha 1 bar minus alpha 2 bar equal to 0. So, this is my V; and I can write it here 1 minus 1 minus 1 plus 1

V alpha 1, V alpha 2, V alpha 3 plus this is my W matrix, which is alpha 3 bar minus alpha 2 bar minus alpha 1 bar equal to 0. So, this becomes my capital W; this is my V matrix; this is my B matrix. Again, we can find out the solution of B matrix, and then you can find out the rest of the values ok.

Let us take some other examples. For example, what we call here is station condition. What station condition? Let us say at a given station, I am measuring five angles theta 1, theta 2, theta 3, theta 4, theta 5. I can write the condition like this theta 1 plus theta 2 plus theta 3 plus theta 4 plus theta 5 equal to 360 degree, and what we call as, station condition.

Now, I can also (Refer Time: 44:10) follow the same process that means, I can write here theta 1 bar plus V theta 1 plus theta 2 bar plus V theta 2 plus theta 3 bar plus V theta 3 plus theta 4 bar plus V theta 4 plus theta 5 bar plus V theta 5 equals to 180 degree because, these are the measurements with some errors and so on. Now, you can again form this kind of situation here, find out what is B matrix, what is a V matrix, what is my W matrix, and then you are done. I am sorry, it is 360 degree.

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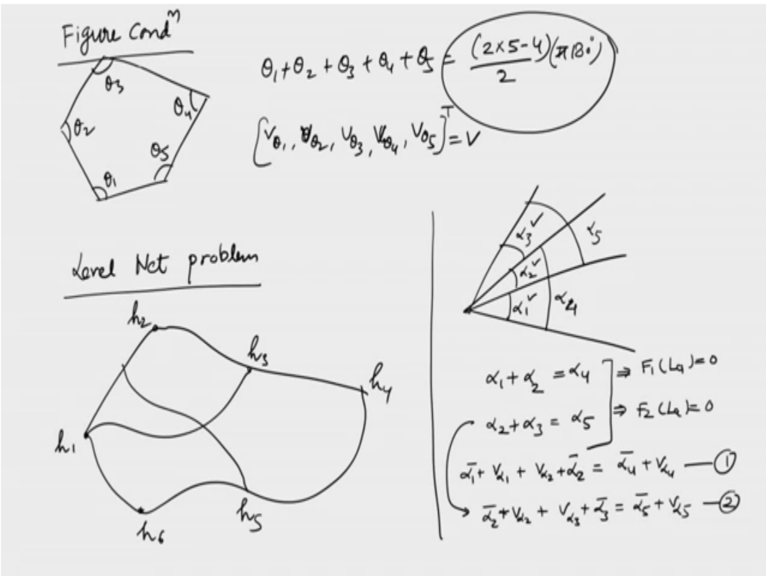

So, on the similar line you can solve many such examples. Other examples could be let us say figure condition, I have let us say some kind of this figure of close polygon, where angle theta 1, theta 2, theta 3, theta 4, and theta 5, this is my pentagon. So, I have a condition here that theta 1 plus theta 2 plus theta 3 plus theta 4 plus theta 5 should be equal to 2 n minus 4 that is 5 minus 4 divided by 2 into pi or it is nothing but 180 degree. So, the sum of the internal angles should be equal to this value, for this pentagon ok. Now, again you can do it. Same condition equation method will go, and you will find out V theta 2, V theta 3, V theta 4, V theta 5 and so on. This matrix will be of V matrix.

Similarly, you can find out other examples. Let us see what are the other examples, level net problem here. This is the figure condition. What is the level net problem? I have some kind of level that is level net problem, where let us say this one point h 1, it has some height h 2, this point has height h 3, this is h 4, and let us say this is h 5 here, and we have h 6 here. Now, you write different different conditions here, which are independent, and then you do a level net problem analysis, there you will write multiple conditions.

For example, what do you mean by multiple condition, I will give you an another example here, simple one. Let us say there are some angles like this, first is alpha 1, second is alpha 2, third is alpha 3. Now, I measure the combined angle here alpha 4 here, and this angle as alpha 5 here. So, I observe all the five angles in the field, now I want to do the condition equation method to find the adjusted values of the these three fundamental variables alpha 1, alpha 2, alpha 3.

So, I can write here that alpha 1 plus alpha 4 should be equal to alpha 2 here yeah should be equal to alpha 4. And similarly, I can write another condition say alpha 2 plus alpha 3 should be equal to alpha 5. So, these are two independent conditions here, because all other conditions for example, I can write any other condition, that can be derived by these two equations right, and that is the idea here, what do you mean by independent conditions. Same way for this level net also, you will (Refer Time: 47:57) few independent conditions. And once you consider all the dependent conditions, introduce the errors in all the variables like h 1, h 2, h 3, h 4, h 5, h 6 like V 1, V 2 to V 6, and write BV plus W matrix and try to solve.

So, let us take this example, what does it mean by multi response or multiple function here. So, here I have two function, let us say F 1 L a equal to 0; similarly here I have F 2 L a equal to 0. So, what does it mean, if I write here alpha 1 bar plus error in alpha 1 plus alpha 2 bar plus an error here equal to alpha 4 bar plus error in alpha 4. Similarly, I can write another equation from here that is alpha 2 bar plus V alpha 2 here alpha 3 bar plus

V alpha 3 equal to alpha 5 bar plus V alpha 5. Now, I will have two equations there, how can I write it.

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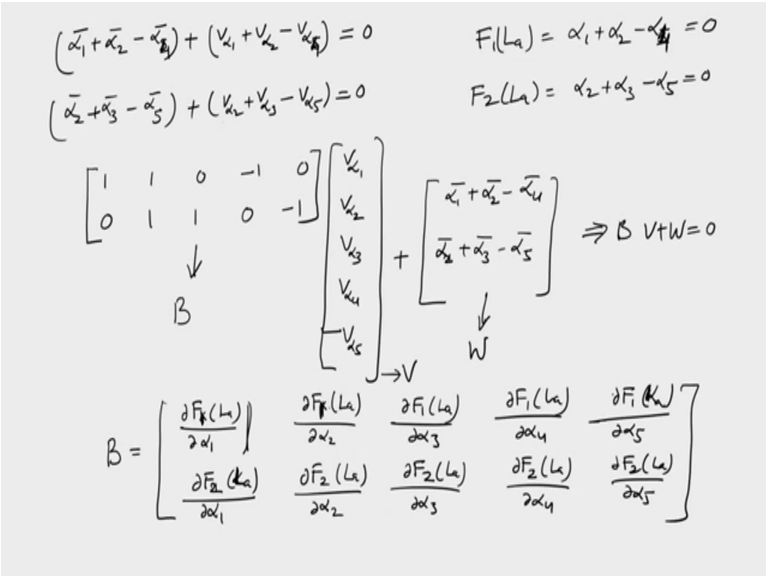

Let us see if I bring my all the things here in one side, then I can write here like this. Let us see first time alpha 1 plus alpha 2 minus bar bar minus alpha 5 bar, it is one thing plus I can write here V alpha 1 plus V alpha 2 minus V alpha 5 right equals to 0 here. Similarly, I can write alpha 2 plus alpha 3 bar minus alpha it should be 4 here, and it should be 4 here, it should be 5 here plus V alpha 2 plus V alpha 3 minus V alpha 5, and I can write it 0.

Now, I can write this thing in the matrix form. The first one I am writing 1 1, and then we do not gave the alpha 3 here, so I put 0 here, I do not have alpha 4 I have alpha 4 here, there no alpha 5, this is a matrix here. In this second equation, V alpha 2, no alpha 1 1 alpha 3 1, here 0 and minus 1; I can write this way V alpha 1, V alpha 2, V alpha 3, V alpha 4, and V alpha 5 plus I can write here alpha bar 1 bar plus alpha 2 bar minus alpha 4 bar, here alpha 2 bar plus alpha 3 bar minus alpha 5 bar. So, this is my W matrix; this is my V matrix; this is my B matrix. So, I have again got the form BV plus W equals to 0. So, this is what my condition equation method is.

Let us take the last one, how to solve the non-linear problem or we can take it in a simple way, where I can say that my B matrix is also given by here in this main equation, I can write this thing d F by let us say d alpha 1 in at fundamental equation I am writing it d alpha 2 dou F by dou F 1 at L a dou alpha 3 dou alpha 4 L a here and dou alpha 5. Here, I can write here dou F 2 L a dou alpha 1 dou F 2 L a by dou 2 here dou F 2 L a here dou alpha 3 dou F 2 L a over dou alpha four and the last factor is dou alpha 5. This is my B matrix given by like that right.

Here what is my F 1 L a if you remember, F 1 L a is nothing but alpha 1 plus alpha 2 minus alpha 5 equals to 0. And here F 2 L a is nothing but alpha 2 plus alpha 3 minus alpha 5 equal to 0, sorry; here it should be 4 like this. So, these are my two equations. I am writing my B matrix, so that is the way we handle the non-linear problems ok.

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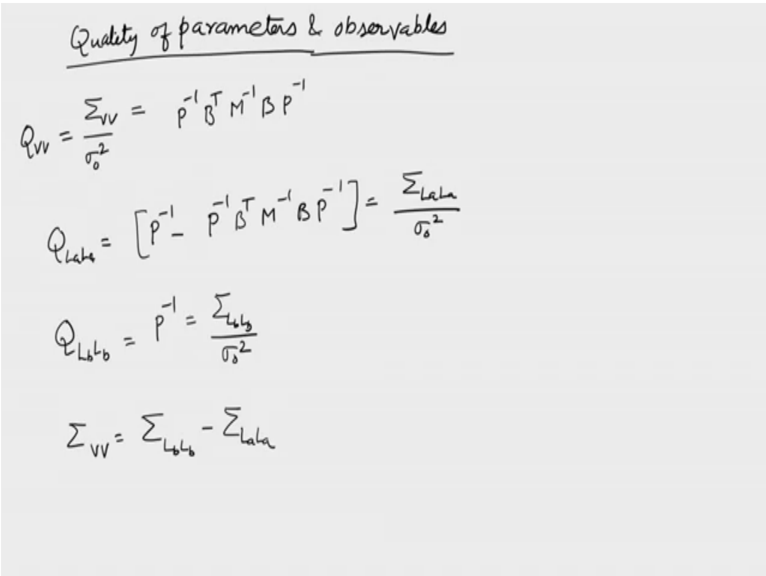

Now, let us take the quality of my parameters in condition equation method and observables what I say, not other things, observables ok. So, how to give the quality here, my first of all sigma VV is given by since we know that weight matrix is coming with the help of so, I have Q VV here, which is given like this. P inverse weight matrix B T M inverse B P inverse, it is given like that ok. What about the Q L a L a that is may the quality of the adjusted parameters is given by P inverse minus P inverse B T M inverse B P inverse is nothing but equal to if you remember, sigma L a L a divided by sigma 0 square.

What about Q L b L b? We already observe it in the field, so I know it, which is equal to let us say P inverse equal to sigma L b L b divided by sigma 0 square ok. Again I can prove here that sigma VV is equal to sigma L b L b minus sigma L a La that means, my adjusted parameter will have higher quality or compared to the observed value of variables in the field ok.

Here, we can finish the condition equation method, and let me tell you little more about that. There are some situations, where we cannot use condition equation neither we can use observation equation method fully, what could be personality here. Let us take some example before you finish this lecture ok.

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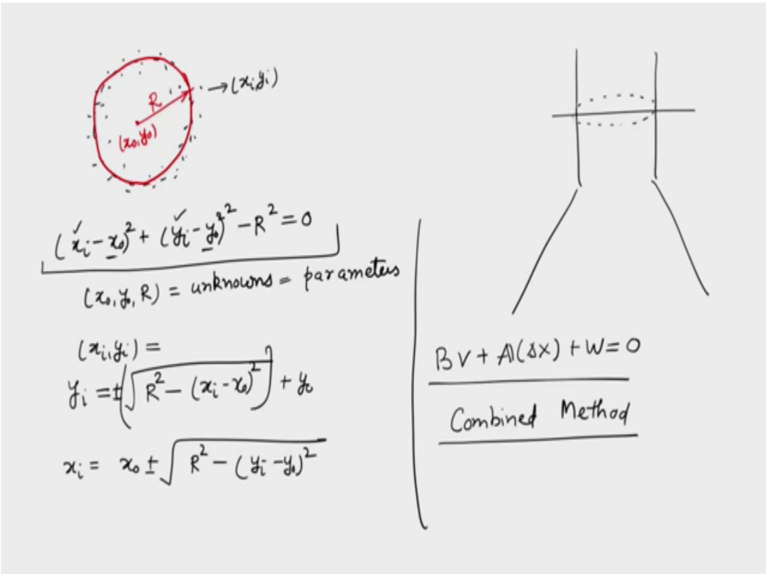

The idea here is let us say that you observe a chimney, which is something like this. And here on this level, you observe some points on the surface of the chimney like this. And then, you observe these points, and then you know that these points should fall on the circle, but you know because of the error in the measurement that you performed using total station maybe, it will never be a good circle. So, what will you do, you will find out some points like this like that. And now, what will you do, you will try to fit a circle through these points like this, to find out what is the exact diameter of chimney.

So, now let us see this diameter, which is determined by circle will be R, and this centre is my x 0, y 0 offside circle ok. Now, these are the observed points, each point I can say here let us see x i, y i. And so, I can write an equation of circle here x i minus x 0 square plus y i minus y 0 whole square minus R square equals to 0 here. Now, in this situation, you see very carefully. Can you use condition equation method? No, because I am observing only x and y like this, not this one. So, I do not have a condition, where I am observing each and every variable. No, there are unknowns, which are x 0, y 0, and R, they are my unknowns or I can say parameters.

On the other hand, can you use observation equation method? Ok, in observation equation method, remember I should mention the dependent variable explicitly in the form of independent variable, but here if I am observing x i and y i, and both I can say which is the my independent, and which is dependent, you can argue that. I can write y i is equal to plus minus I can write this way.

But, one of you may argue that why cannot I write x i equals to x 0 plus minus R square minus y i minus y 0 square. What I am saying here is, it is very difficult to say who is dependent and who is independent among x and y, which I observed in the field. Moreover, when I observe x and y using total station or any other device, they are coming as a couple right. And they are coming by one device, and I cannot say which is dependent and which is independent.

In such a situation, I need to combined both observation equation method and condition equation method in order to solve such problems. And in that case, my generic form will be BV plus A delta X or maybe X plus W equal to 0 will come, so that is what we call here. In such a problem, where I cannot use explicitly my observation equation method or condition equation method, because some situations are very (Refer Time: 58:52) different, and both are not applicable rather; in that case, I use the combined method.

So, in the next lecture, we are first going to discuss about the combined method, and then we will see how to conclude all this, our knowledge that we have gained in this four lectures so far. Thank you very much for patiently listening. And I request you to practice some of the problems that I shared with you on paper. So, once you learn all this thing, you yourself will become more confident. And if you can learn the python, which is free IDE, then you can also do your small, small work on the python IDE solving the complicated problem, where you have enormous data that you want to handle. Thank you once again.

Thank you very much.