Higher Surveying
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Module – 04
Error, Accuracy, and Adjustments Computations
Lecture – 11
Observation Equation Method of adjustments

Hello everyone, welcome back in the course of Higher Surveying and we are in module 4 Error, Accuracy, and Adjustments Computation. In last two lectures we have learnt many things, in first lecture we have differentiated between the precision and accuracy, we have seen the example of triangle and how to find out the accuracy of each angle or accuracy of the complete triangular system. Also, we saw that precision is indicated by standard deviation of a observation. So, let us say if I take n number of observations for angle alpha and if I find out mean and standard deviation. So, standard deviation of sample will represent the precision value.

But it is not an accuracy that is what we have learnt ok. Then we talked about the triangle example and triangle example what we have done, we have find out the adjusted values; that means, using a condition equation you put the observations alpha, beta and gamma, mean values of those three variables and then we try to put the condition that sum of the three angles in a plane triangle should be 1 equal to 180 degree. And using that 180 degree reference value on true value, we have find out the what should be the accuracy of triangle system or if E is the accuracy of the triangle system and we are measuring all three angles equally then E by 3 will be the accuracy of each angle.

That is what we have done in the first lecture. In the second lecture, we have learned how to use law of error propagation to find out the precision in the dependent variables, ok. Now, let us look again in the triangle example and as we said that it is an kind of adjustment process.

So, today we are going to talk about the adjustment process in detail, right. We are going to use law of error propagation also to find out the quality, quality of parameters, quality of variables and so on. So, let us go into the third lecture of this module which is observation equation method of adjustments, here, books are the here ok.

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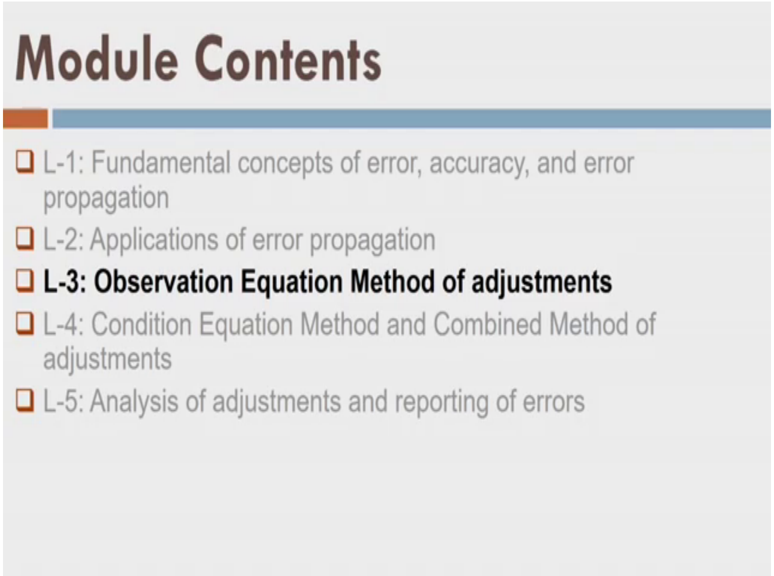

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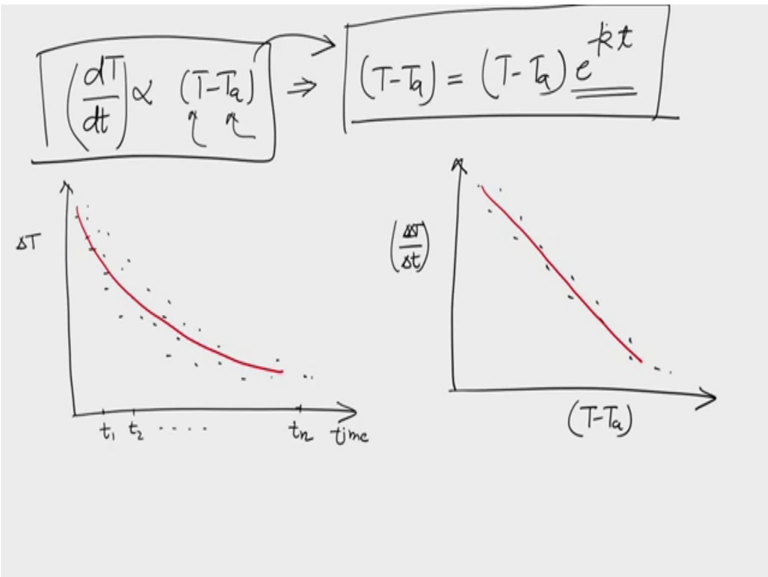

Now, you might have done the adjustments many a times, when you go into the bus and if there are two seats and you are three friends you try to adjust there, but when we talk about the adjustments and computations in higher surveying what does it mean, ok. Let me recall a simple example after the high school when we were in 11th standard, we conduct an experiment in physics and the experiment is the Newton's law of cooling and accordingly we write that d T by dt is proportional to d T or we can say T minus T a remember this was the thing there.

And what is meaning here that change in the temperature or the rate of change of temperature is proportional to the difference of the temperature with respect to ambient temperature. What is my ambient temperature? It is temperature of the surrounding or atmosphere. T is the temperature of the material from which radiation is occurring and heat is dissipated or energy dissipated, fine this is the rate of temperature change of the body that was the thing right and if remember the experiment we are drawn the curve also x and y axis here my x axis which is nothing, but time.

So, I mark time different times here let us say t 1 t 2 and so on let us say t n, ok. On the y axis we have delta T or d T fine and then we got some kind of this kind of data something like this. Perhaps you also might have gotten in this kind of data and maybe a little more, I can put it like this and then if I solve this equation analytically we know that I will get T a minus T a equals to T a minus T a into e to the power minus k t, ok. You are well aware of this equation, it is first order differential equation and by solving this first order differential equation I will get this analytic equation.

So, I know that there is some exponential curve is possible between the delta T and time t as it is evident from this equation, but when we look into the data and if I try to fit or try to find out a curve, that is fitting for some point, it is very difficult for me to fit or to find a single curve. So, when I was 11th standard and I got this kind of data, I approached my physics teacher. I ask what could be solution here, how can I put a curve through this points? It is very difficult, it is impossible for me because if I add any this curve like this by some something like this, can I fit this kind of curve because these are the points here and for which if I fit a curve it should be like this ok.

But as I know that curve, ideal curve should be like, this because it is theoretically proven, and I am conducting an experiment ok, there. So, my physics teacher told me the real key here, he ask me to fit a curve or other he ask me pass a curve through this data such that the curve should be closest to most of the points. Or in other words I can say curve should be in a position such that it should have minimum deviation from most of the points or all of the points and that is the key here then what I did? I created some 7 8 such data set. Because I observed this data in the laboratory then I tried to do lot of manipulation and finally I could find a curve which I can say, which are something looking like this the curve was looking like something like this.

So, this was the curve I got it. So, now if you draw another curve in the same experiment, if you remember we have another curve also. So, if I draw the curve between the T a minus T minus T a on x axis and delta T by delta t; that means rate of change of the, this one so, I am trying to plot this curve. Now, so my curve, I got the data something like this, these are the data and if I fit a curve then it should be some kind of this straight line because here and again principal is same that the curve should pass through maximum number of points or the deviation of the points from the curve should be minimum.

So, we have done it the least square or the I can say the adjustment process or the curve fitting problem years back but perhaps we forgot it and many a times we have done this thing in our laboratory experiments, whenever we get some situation like this. So, today we will do this thing mathematically and then we will connect this process of curve fitting or maybe the line fitting or maybe some other thing to our adjustment computations what we are learning in the module; let us go ahead right.

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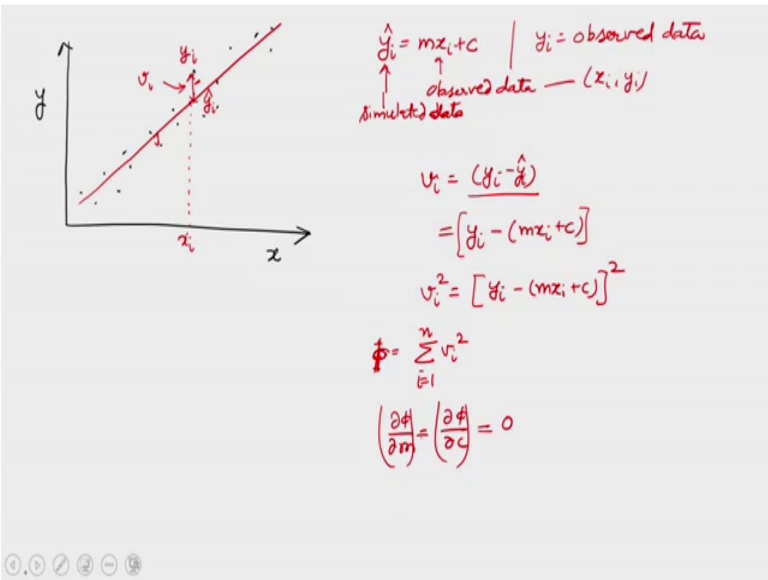

Here let us see that take the simple example of line fitting I have this thing here and some x axis x data here, some y data here and I put some kind of let us say data is something like this. I can see that they are this falling some kind of a linear pattern. Now, I fit a line through this, let us say this line, now let us calculate the deviation from any point i, what is the meaning i? So, if I drop here point here, this is my x i and corresponding point is y i, but the point which is lying on the line I call it y i hat, what is the meaning here. So, equation of line is y i hat is equal to m x i plus c this is the equation of line. Now I observed y i, this is my observed data and x i is also observed data.

So, if I make the combination here, this is my pair x i, y i and this y i hat is my simulated data by fitting a line, right. Now can I find out this deviation, I call this deviation as v i because it is a deviation from the observed and the simulated data. So, I write it v i equals to y i minus y i hat here or I can also write it y i which is observed data minus m x i plus c and like this, I can right it this way ok. Now you can see for this point this term will be negative and for this point it will be positive right.

So, I want to take care of both the things and as a result what I do, I will make the square of this term v i square it is nothing but v i minus m x i plus c whole square and then I make the summation for all the points; that means, it is a error or the summation of these squares of the error for all the points.

Let us say if I observe n number of points. So, I create this function I will call it function phi, well anyways I can right it phi this way. Now if you remember in case of classical surveying or the basic surveying course we have done using least (Refer Time: 11:24) principle and then we have find out what are the possible values of m and c by these two equations 1 and 2. We are going to implement the same thing here, but in a slightly different manner, why? Because I want to use computers because in higher surveying I have enormous data not like one point, two points, thousands of points, millions of points.

And so I need to use computers and for that I need slight ticking of the method keeping the essence of the methods same; that means, I am keeping the same method of least square solution, but I am deriving the solutions using matrices or computers. So, let us see; what is the method which is compatible with computers and which is very easy to implement. So now, let us see I am just explaining the things once again.

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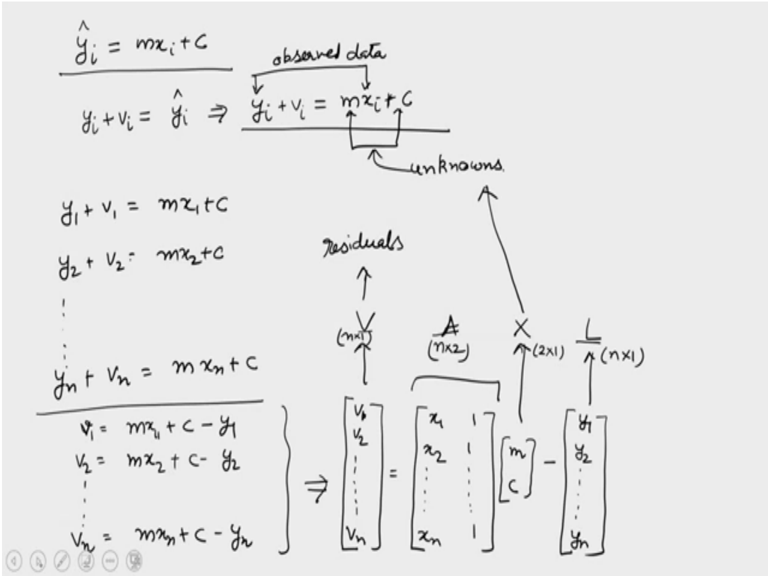

So, we have this equation let us say y i equals to m x i plus c and where we said it is y i hat ok, that is very clear to us now. So, let us write these equations one by one. So, what is the meaning here, this is my simulated one and let us if I observe y i and if I add this y i v i error that is residual then I will get y i hat or y i plus v i, I can write m x i plus c, ok. So, let me write this thing again and again. Remember these are my observed data and these are my unknowns that I want to find out for finding out the equation of line. So, if I write for n number of data this thing I will get this one and so on. If I write for n number of points I will get this, remember m and c are constant for a given line and data is varying ok.

So, let me write these matrices as let us form this matrix as v 1 equals to this is my small v here any how m x 1 plus c minus y 1 or v 2 equals to m x 2 plus c minus y 2 and so on. So, v n equals to m x n plus c minus y n, ok. I write it like this, let us see this is my v 1, I am writing in the matrix form, I can write the equation of matrices this way and slight picking and doing here x 1 1 x 2 1 x n 1 and here I am writing it this way m and c minus y 1 y 2 y n. So, I have written this n number of equations in the matrix form here.

So, this matrix I call V matrix, this matrix I call A matrix, this matrix is call capital X matrix and this matrix I call L matrix. So, if I write the size of these matrices it is n by 1 matrix it is I can say here n into 2 matrix x is nothing but 2 into 1 matrix and L if I just say n into 1 matrix. So, these are my unknowns here, ok A matrix is a kind of multiplication matrix or V is my residual matrix here. All the residuals are there and this is my observations y 1 to y n which are I m writing L here I am not differentiating right now observation x and observation y.

So, let us this is the situation we have come. Now, this is very easy to form, I can form A matrix easily, I can form L matrix easily, I can form V matrix easily and x also fine in this form there are standard solution of this form and let us see.

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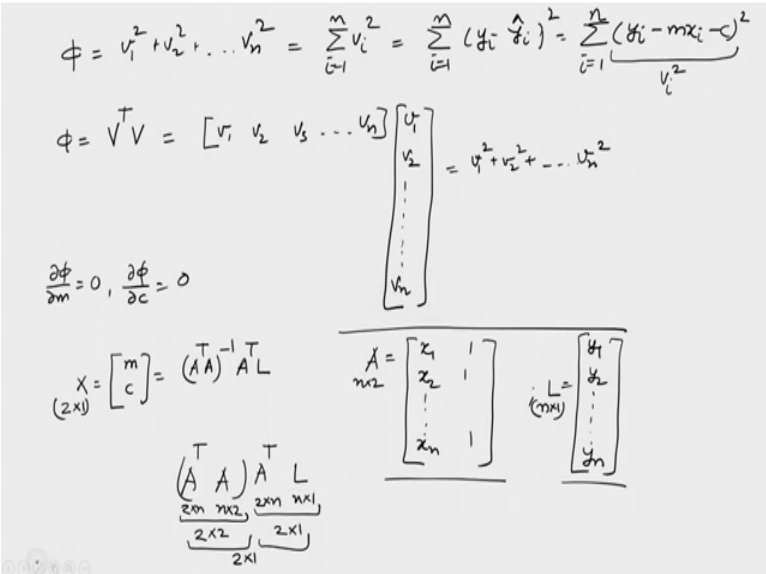

Now, I want to do my equation phi equal to that is nothing but V 1 square plus V 2 square plus V n square, if you remember because we have written summation v i square, i equal to 1 to n or we also write it like this, y equal to i equal to 1 to n, here y i minus y i hat square or I am just writing all the possible forms to avoid any confusion y i minus m x i minus c whole square right. That was we have already discussion first previous slides.

Now this is nothing but v i so v i square here so, I am writing it v 1 square v 2 square and so on up to V n square. Now i can write the same thing phi as in the form of matrix capital V T into V which is nothing, but v 1 v 2 v 3 up to v n and that is v 1 v 2 v n. So, if I multiply this thing I will get same thing here v 1 square plus v 2 square plus v n square. So, I am writing this thing and now using the same principle of least square what will learn the basic surveying, I can write what we have written this way and we find out equations.

And then we solve it here in the matrix form I can write the solution x which is nothing but m and c is given by A T A inverse A T L and for what is A T you can if you can remember otherwise I will make it from this is my a matrix x 1 1 x 2 1 x n 1. So, it is basically I said that it is n by 2 matrix here a t will be transpose of a L matrix is y 1 y 2 y n. So, now, I can find out the value of x matrix which is equal to m and c. So, what will I get just check what is the size of this one. So, if I multiply so, here I will get that 2 by n into n by 2 and then here again a t which is nothing but 2 by n here and l matrix which is n by 1 here if you remember the size.

Now, if you multiply these two matrices you will get 2 by 2 matrix if you multiply these two matrix you will get 2 by 1 matrix and then if you multiply these two matrices you will get 2 by 1 matrix and which is nothing, but x size of x is 2 by 1 right. So, we are very sure by dimensional analysis of matrices that we are going to get; some two dimensional or one column vector having two values. So, that is the idea here now you can easily formulate if you have the data x 1 y 1 x 2 y 2 and so on up to x n y n.

You can formulate matrix a very easily you can formulate matrix L very easily and then you can write a solution just by multiplying and taking inverse and in most of the now the days commercial ideas or the free ideas, ideas means interactive development environment like python which is free. These inverse matrix multiplication and all these things are freely available or other they are in built modules. So now, you can use those ideas to find out a problem or solution of the problem ok. So, that is the idea of the least square solution using matrix method and this processes called the adjustments computation here ok.

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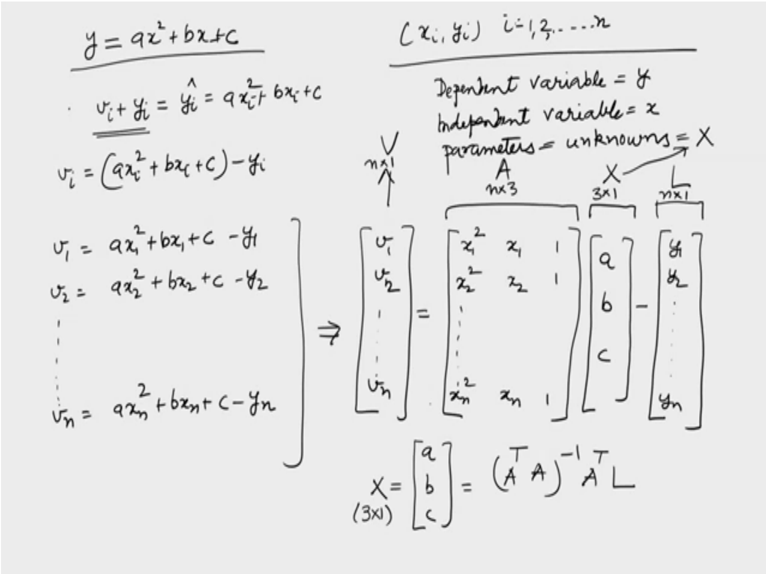

So, let us go ahead and try to do few more examples let us take example of second order. Let us say I want to fit a curve which is second order like this to my data where I observe this my data x i y i; i equals to 1 to n. Now remember what did we, like do last time in the linear example, we introduce some error v i to the real data such that it becomes y i hat or it becomes this way. So, again I can write here v i equals to a x i square plus b x i plus c minus y i. So, if for n number of data I can here v 1 equals to a x 1 square plus b 1 x 1 plus c minus y 1; v 2 equals to a x 2 square plus b x 2 plus c minus y 2 and so on for the nth data I can write a x n square plus b x n plus c minus y n.

I hope you agree with this things and using this let us say I am writing the matrix form v 1 v 2 v n equals to here if I write here let us say x 1 square x 1 1 x 2 square x 2 1 and so on x n square x n 1 and then here can write what is my three variables a b c they are my unknowns, no problem ok. Now here I can write y 1 y 2 y n so, again this is my V matrix this is my A matrix this is my matrix of unknowns I call it X and this is my matrix L. So, I write the solution A X equals to which is a b c here A T A inverse A T L.

Again let us look into the dimensions of the A T L everything A is nothing, but I can say n into 3 matrix here 3 into 1 matrix here n into 1 matrix and here V is nothing, but n into 1 matrix. So, you try to put it and find out whether there x matrix is 3 into 1 or not you will get it. So, that is the idea here we have done two examples now and you can now formulate any problem. So, let us try to find mathematically what we have done in a generic terms now ok. So, far we have done some examples there and now I will introduce you few terms.

What we have done basically here let me just use the same slide here in this slide I can say you this something called dependent variable and which is nothing but y because y is dependent on the x the moment. I am giving the input value x by some function f x I am getting y value which is function is this or may be y is equal to m x plus c in case of linear example or the line fitting example.

Ok what is my independent variable which is x here why because x is independent, or y is function of x moreover I always introduce error in the dependent variable y; that means, y plus v we wrote like this not in the x; x we assume that it is independent variable. So, it is coming without error in fact, nothing comes without error if you measuring the field, but still we assume in this method that my input variable or independent variable is coming without error and dependent variable has some error such that if I fit a curve or if I fit the adjustment process I will introduce some error into the dependent variable.

Now we have one more term called parameter or parameters which is nothing but unknowns. What are the unknowns here, these are my unknowns what I call here X this matrix all three are unknowns. So, now we are very clear on different different terminologies and different different limitations and different different assumptions here ok. Let us go ahead and try to find out what is the method. This method I call where I express explicitly y in terms of x it is called observation equation method.

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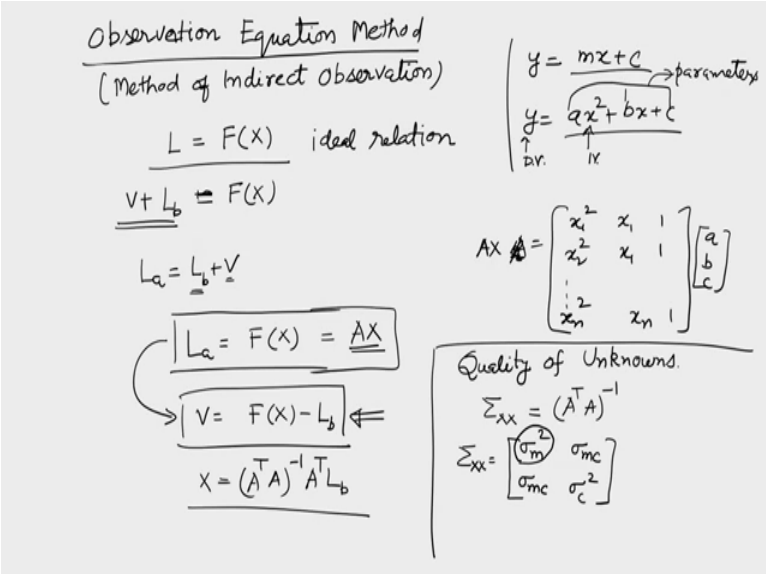

Remember I have taken two examples, the first example is y is equal to m x plus c and another one was y is equal to a x square plus b x plus c. So, this is my dependent variable, y x is my independent variable and a b c are my parameters. So, I write it, IV and it is my DV dependent variable and we always introduce error in the dependent variable.

Now, here we should be careful I can write this kind of method observation equation method when I have a dependent variable explicitly given in the form of independent variable like this or maybe like this fine. So now, we also call this method as method of indirect observation or method of adjustment of indirect observations because y is my indirect observation and I am trying to adjust that y fine. So, how do I write mathematically, I write let us say L which is my dependent variable is function of some parameters X right ok. Since this is an ideal relationship like y is equal to m x plus c and so on or ideal function but the moment I have some error.

I call it L b should be equal to F X fine, but which is not equal. So, I introduce an error here V plus L b equal to and this V plus L b. I call the adjusted value of the observation; that means, if I add some V value to the L b observation, I will get the adjusted value which means that is going to follow my ideal mathematical adjustment process ok so, I write this thing here as if L a equals to F X finally. So, this is the main thing here of the observation equation method here and we write it like this fine we from here again I can write here that V is equal to nothing, but F X minus L b or I also explain the F X equal to A X; that means, there is a matrix which is multiplying to X parameters and I am getting some F X that is idea here and if you remember, ok.

Let me do like this if you remember A matrix and X matrix, X was let us say unknown a b c and if I write A matrix as let us see like this x 1 square x 1 1 x 2 square x 1 1 x n square x n 1 and if I multiply with the X matrix like this A X then it will be a b c. So, this is a meaning here I am writing it this way. So, there I give the solution X equals to A T A inverse A T L and more specifically it is L b right.

So, I need to reduce in this form the moment I bring this form here I can write straight away the solution of my unknowns what about the quality of unknowns; that means, that we are going to understand here. Right remember we use the error propagation in order to find out the quality of any variable sigma XX ok. I am not deriving the derivation here I can straight away write the solution here and that is given as A T A inverse that is the quality and nothing, but in case of linear example sigma XX is sigma m square sigma c square sigma mc sigma mc here.

That is the quality or the variance covariance of the m and c. Remember the quality of m will be sigma m or the precision of m well. So, before going into the quality aspects and all this thing let us take few more examples which are pretty simple to understand ok.

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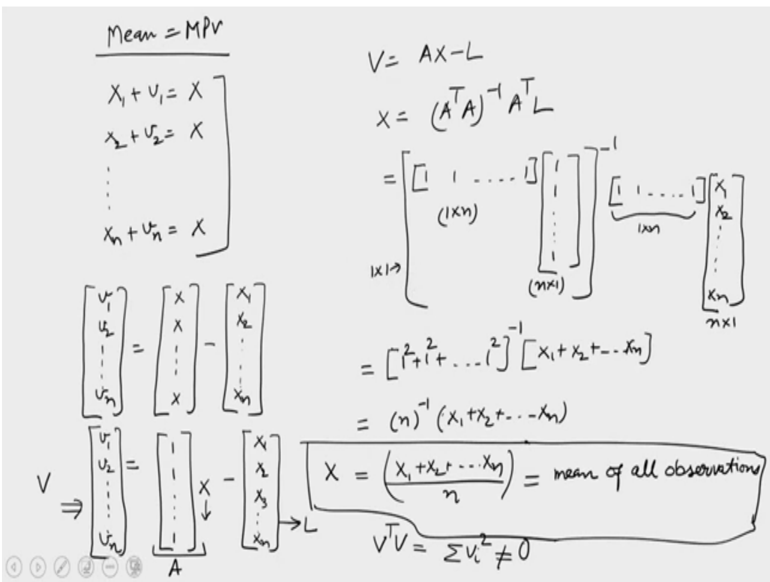

Let me take the basic example of mean we always say the mean is equal to most probable value let us prove this thing ok. So, let us see that your measured a variable X and you write the values like this X 1 plus v 1,you will get the true value X for example, fine.

Now, you take another option X 2, you add residual v 2 you will get true value and so on. Let us say you get X n plus v n we will get true value X even I can use lower alphabets also to write, but meaning is same ok. So, you are measuring basically the length and you are measuring L 1, L 2 or X 1 x 2 and so on. You are measuring one linear measurement or one non-linear could be whatever measurement your doing only one measurement fine. Now I can write my V matrix this way, v 1 v 2 v n equals to here I can write for example, x x x n times minus I can write x 1 x 2 x n fine.

I hope I am just transferring the things from here to here left to right and right to left ok. Here I can write this X matrix as 1 1 1 n times into unknown X minus x 1 x 2 x 3 and let us say x n same thing and here again v 1 v 2 v n. Now this is your V matrix, this is your A matrix, this is X matrix which is unknown single value and this is your L matrix. So, I have this form V equal to A X minus L. I can find out X given by A T A inverse A T L ok, very simple. Now let us put the value of A and everything it is very simple to put A T will be transpose of A so A 1 1 1 1 like this again A matrix 1 1 1 this way ok.

The whole inverse I will take again the A T L, A T is nothing but 1 1 1 and L matrix is nothing, but I can write here L matrix as x 1 x 2 x n right. Now try to multiply if I multiply these two matrices I will get one value here 1 square plus 1 square plus 1 square and inverse of the whole multiplication then if I write multiply these two matrices here I will get x 1 plus x 2 plus x n. Remember this is your 1 by n matrix, this is your n by 1 matrix. Similarly, here it is your 1 by n matrix and it is n by 1 matrix. So, you will get 1 cross 1 here of this size of the matrix is 1 cross 1 and here also you get 1 cross 1.

So, now you are having this much. So, if you add 1 square 1 square n times you will get n inverse and here summation x 1 plus x 2 plus x n and I can write it x 1 plus x 2 plus x n divided by n. So, that is nothing but the mean of all observations right and here if I write capital X here which is a one variable. Now I have proved it using I have just meant computation principal that my mean is the most probable value; why because if you just calculate now the V T V right which is nothing but sigma V i square right that is not equal to 0. In case of true value or the population mean it should be equal to 0 but in case of sample mean it is never equal to 0 and hence we say that we have find out the best possible value from the given observation but it is not equal to true value and so we call it, it is a most probable value or other it has the value which has maximum probability of occurrence. So, that is a concept of N P V and now we have proven it that why mean should be taken as N P V not any other value and the idea here is the least square solution we have done the least square solution of the given situation here given data here ok.

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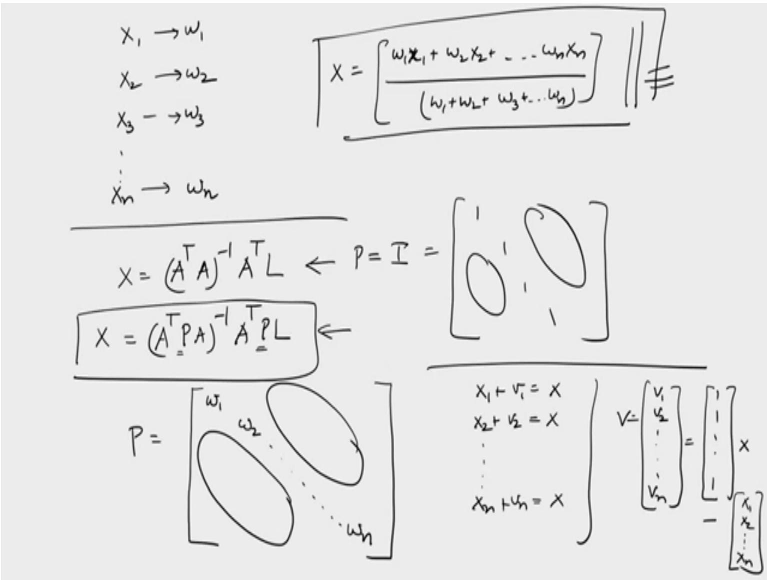

What if you have some weights what about you should get the weighted average try yourself which means, let us say that data is coming with different, different means; that means, the X 1 has some weight w 1 and so you are writing X 2 w 2; that means, an X 3 is coming from w 3 weight and so on X n coming with w n weight ok. So, you should get the answer X equal to w 1 X 1 sorry, capital X 1, w 2, X 2 w n X n divided by w 1 plus w 2 plus w 3 after w n should get this answer here.

I would like to modify some of the solution here when we write X equal to A T A inverse A T L here we need to modify if we consider the weight of the observations. So, if you consider the weight than let us say there is a weight matrix P. So, I write A T P A inverse A T P L. So, what is weight matrix here? So, in this case we have assume that my weight matrix is identity matrix or 1 1 1 1 and all the 0s here diagonal matrix I assumed here.

So, now, when we consider different weights then what will be the weight matrix here w 1, w 2, w n.

And since observations are coming from field and their independents so, I am assuming that they are independent and there is no covariance variance between that time. So, only the variance of the X 1 is deciding w 1. So, I write P matrix like this well try to write this kind of P matrix and try to solve this one and try to find out whether you get this answer or not try to get it you will get it again. Just develop V matrix how will you develop V matrix again, V matrix will be same; that means, you will get X 1 if you add some value v 1 you will get the true value if you take X 2 if you add v 2, you will get the true value and so on.

So, in the nth observation if you add some residual v n you will get the true value right. So, now you can construct V matrix v 1, v 2, v n in the form of A matrix which is nothing but 1 1 1 1 t 1 here X here and minus let us see X 1 X 2 X n. So, you think the same thing but now you will consider the P matrix here also and here also. So, check whether you get this answer or not you should get it do it yourself take a homework should we take few more examples ok. Let us take a multivariate example ok.

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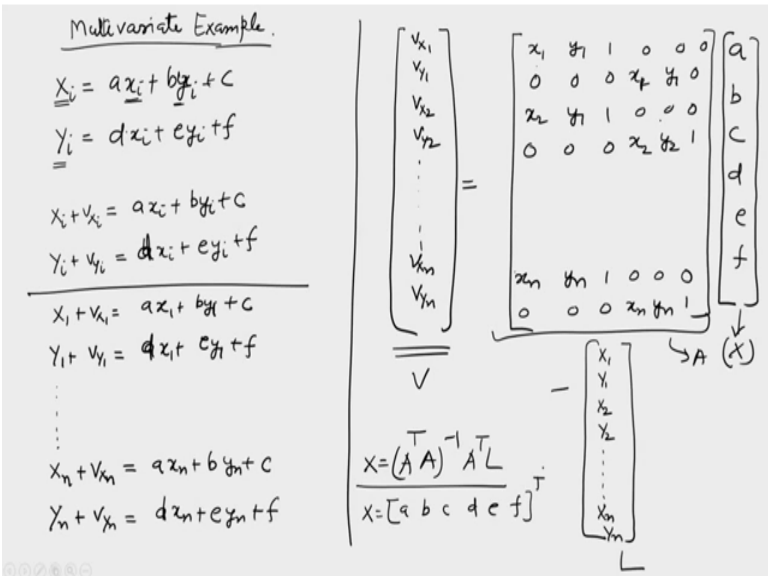

So, what is the meaning of multivariate? So, far we have taken single output variable multivariate example ok. Let us see that I want to have an image here and I want to geo reference or refer this image to a map. Map is slightly you can see bigger one. So, what I do in case of map I observe capital X and capital Y and in case of image I observe small x and small y. So, I know the pixel number or maybe I can find out x and y. So, let us see these are the points I observe in the image let us say n number of points corresponding; that means, it is my small x i y i and here in the in maps I have some points like this same corresponding 1 to 1 connection.

Let us say I have marked 7 points here mark 7 points here. So, let us this is 7 points and here I can say let us say this is my capital X i Y i. So, if I say this is my capital X 1 Y 1. I have corresponding small x 1 y 1 here in the image that is a meaning here; that means, I have measured one to one correspondence between the points. Now I want to do some transformation between the image and map so, that I can refer the image into the map or I can do the geo referencing of the image. So, what do I do, I write let us see these two equations; that means, my X i equal to a x i plus b y i plus c is one equation.

And Y i i explained as plus e y i plus f. So, there are two such equations or other it is a multivariate in input we have x i y i and output, I have map coordinates capital X capital Y. So, let us assume that if I introduce the error X i i plus V X i I will get a x i plus b y i plus c. Because I am transforming my image into the map so, I am writing this equation. So, let us see y if I introduce Y i here I will get d x i plus e y i plus f. So, if I write for n number of points I can write here such equations. Let us see X 1 plus V X 1 equals to a x 1 plus b y 1 plus c; Y 1 plus V Y 1 equals to a, I am sorry here d x 1 plus e y 1 plus f and so on.

I can write for n number of points X n plus V X n equal to a x n plus b y n c; Y n plus V Y n equals to d x n plus e y n plus f. So, how can I write the V matrix and all this thing so, let us write here itself. So, here I write V X 1, V Y 1, V X 2, V Y 2 and so on. I can write V X n V Y n here then I am transferring this on the right-hand side so, I can write here that is see this my x 1 y 1 1 here then I am writing here 0 0 0. Here I am writing here a b c d e f minus X 1 Y 1 X 2 Y 2 X n Y n like this. So, now I can fill these values like x 1, y 1, 0 here x 2, y 2 1 0 0 0.

Here I can write here x 2 y 2 1 and here 0 0 0 similarly 0 0 0, similarly if I write here x n y n 1 0 0 0 0 0 0 x n y n 1. So, now this is my V matrix, this is my A matrix, this is my X matrix unknown and this is my L matrix. So, now I can write X is equal to A T A inverse A T L. So, I will get X as a b c d e f transpose there is no space. So, I am writing as a transpose you will get a column matrix here. So, now we have seen multi variant example single variant example or multi response example, also it is a multi response example multivariate; that means, you are getting more than one response as if you give the input variable you will getting multiple outputs X and Y and so on.

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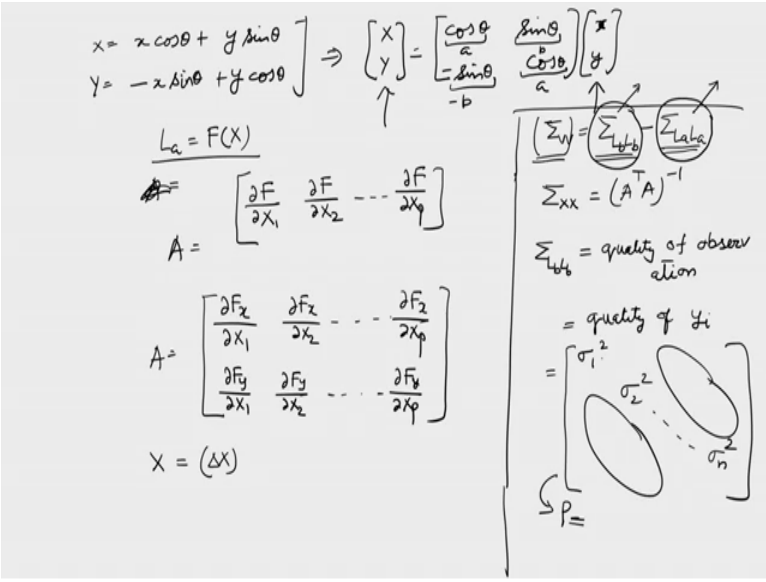

So, this is the idea here, let us take one more now which will be a non-linear example. What is a meaning here by non-linear example ok. So, non-linear example could be let us say I want to rotate. So, what I do I write basically X equal to x cos theta plus y sin theta right and Y equals to minus x sin theta plus y cos theta here, if you remember famous equation of rotation X Y is equals to cos theta sin theta minus sin theta cos theta and here I am writing x and y. So, again take example of image and map I want to rotate my image into map coordinates system but I do not know theta angle.

So, what can I do here? I can solve this problem linearly as well as nonlinearly, but choice will be mine because there is a cos theta. If I assume that a this is cos theta, I will say a sin theta is equal to b and this is my minus b. So, I can write linearly but I am telling you, you can solve the same thing the using non-linear also and where the matrix A will be given by in the L a equal to F X form. This is my basic (Refer Time: 49:00) form where I can write my A is also equal to d F by d parameter. Let us the first parameter here 1 d F by d X 2 and d F by let us say if I have p parameters then I can write it this way this matrix.

In case of multi response I can write my A matrix equal to let us see d F x the multi response X 1 X y first parameter, second parameter, last parameter pth parameter and here d F y divide by d X 1 d F y by divided by d X 2 dou X 2 and we are finding out the Jacobin matrix. Basically, y divide by dou F y by dou X p. Here I can this my A matrix here what about the X matrix, X matrix will be some variation in the delta X or the variation in the parameter fine. At this stage we need to understand that the observation equation method can be used when function or the output variable or the dependent variable is given explicitly in terms of independent variable or input variable right.

In the next lecture we will look into the problem of non-linear cases and we will also talk about the quality of the parameters which we said here sigma XX which is nothing but A T A inverse. Moreover, I would like to say here we know sigma L b L b because we are observing the observation and that is why we know the quality of observation here. So, what is the meaning we are talking about the quality of y i measurements; that means, we write it here. So, sigma 1 square sigma 2 square sigma n square all these are 0. Remember using this only we have constructed our weight matrix P ok.

So, now what about the quality of the adjusted values; that means, if I find out the quality of the residual sigma VV I write a relationship which can be proven sigma L b L b minus sigma L a L a. So, what is a meaning here the quality of given or observed values a sigma L b L b which is inferior than the quality of the adjusted values of the same variable. So, I am trying to adjust the variable y and I can see here confidently that y is the variable which I am trying to adjust. So, after adjustment whatever quality it has here it is superior to the whatever quality I had after observation or by the what is the quality of observation I have right why because sigma we will definitely will have variance terms and that will be positive and I can say that in order to make this matrix positive I need to have this matrix higher values than this matrix.

And I can say that the error in this L a L a that is adjusted values will be less compared to the what I observe sigma L b L b, that is a meaning here ok. Then in the next lecture also we can take it briefly here now that how can we construct the P matrix using the sigma values of the observation remember these sigma values are for the y 1 y 2 ends y n because x we assumed that there is no error in the x. We will take only 5 minutes now to understand that how to develop this P matrix weight matrix and what is a problem. So, I have not taken the non-linear example here but before that I would like to explain how to develop the P matrix or the weight matrix ok.

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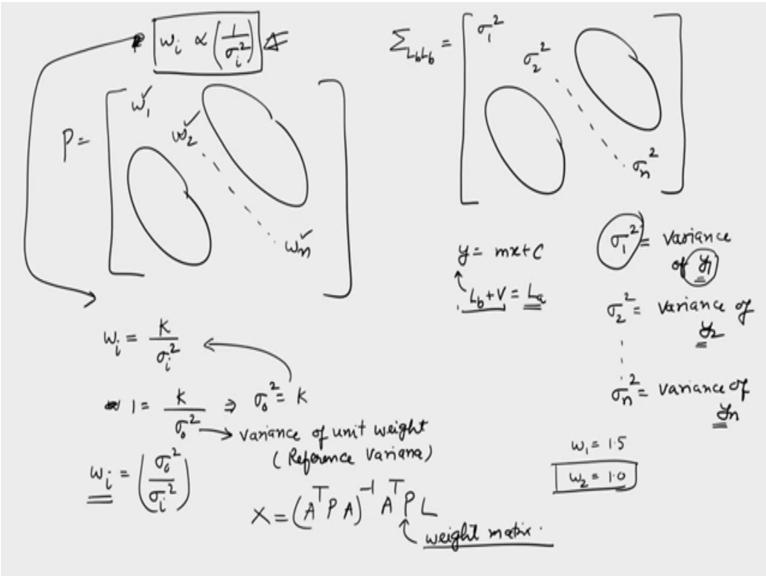

Let us see we know that weight of observation which is given by let us say w i of any observation is proportional to 1 upon sigma i square right and as a result I know my sigma L b L b it is nothing but sigma 1 square sigma 2 square and sigma n square here because they are all coming from the field observation. So, I say that they are un correlated and here remember in case of y is equal to m x plus c my this is L b because if I adjust this value by V I will get the adjusted value of a; that means, what is adjusted value here if I add the error in to the observed value for sigma 1 square is for variance of y 1 sigma 2 square is the variance of y 2 and sigma n square is variance of y nth.

All we assume; why would you assume, because we do not have any estimate of this ok. Now how to assume? There are three ways first if I am using some instrument I know what could be probable error there, say suppose total expression I know the characteristics or the specification of total expression, I can also assume these values with my experience. So, if I have a lot of experience, my experience I assume then I will use their specification of instruments or if you do not have experience.

So, even you can estimate (Refer Time: 55:37) no problem, but you should try to develop some experience there. So, it can be arbitrary because it is assumed value although it should have some logic to assume, but it can be arbitrary also.

Since it is an assumption now let us see that you have assumed or based on your experience or based on your specification you have estimated these values by somewhere right and we call it sigma L b L b; this whole matrix sigma L b L b. Now there is a little problem if I use this logic to find out the weight matrix P which is nothing but here w 1 w 2 and w n all 0s here. There is a big problem here that first of all if I take the sigma square equal to 1 upon sigma square equal to w, there will be problem of the units which is a minor problem. Let us say someone observing a centimeter someone is observing a meter. So, I will have different units of the sigma i square and accordingly weights will be affected.

Higher the weight of could be lower weight something, but there is the minor problem that can be handled numerically, but there is a major problem and the major problem is, first of all I do not know the value here. So, what can I do but I know one information I know that whether this observation y 2 is superior to y 1 or y n was superior to some other observation based on that I can say, yes I am confident about y 1 that it is at least 50 percent better than y 2. So, I can say the weight of y 1 is 1.5 times while weight of y 2 is 1.

Because I am feeling that y 1 observation is 50 percent better or something like this. I know how many times something is better than other. So, I have some idea about weights that way there I use this information to construct the weight matrix but I also use the variance information.

So, how to do it let us say there is a weight is given by K upon sigma square which is nothing but this principle only. Now I write w i equal to k by sigma square, ok. Let me tell that let us see assume some observation having one weight. So, if I write here 1 equal to K times and if I write sigma 0 square which is also I am assuming so, I can say sigma 0 square equal to k and if I replace this value over here I can write w i equal to sigma 0 square upon sigma i square.

So, here first of all two things are happening I call this sigma 0 as variance of unit weight or I also call it the reference variance. So, reference variance of unit weight or I call it the reference variance. Again, I am assuming this value so, it elevating two problem to me first the unit problem is gone here because I am taking the ratios of sigma 0 and sigma i no problem with the units.

Secondly even if I do not know my sigma 0, sigma i still I can construct the weight by assuming sigma 0 sigma i. So, that way I will have the weight matrix in terms of some numerical numbers which are ratios and so, I will have this w 1 which is independent of unit, but they are still representing the quality or the mutual quality of the all the observations together. So, this is the way we construct the weight matrix and now in the logic or in the formula you can use it A T P A inverse A T P L the P as your weight matrix.

Here after doing this thing we will stop here and we will meet in the next lecture with some new method and with what we can understand from this adjustments computations we will try to explore more on that. Thank you for having patience and try to practice what we have done and try to do your homework also and that will help you to understand this method more rigorously.

Thank you very much.