Higher Surveying

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Module – 04
Error, Accuracy, and Adjustments Computations
Lecture – 10
Applications of error propagation

Hello everyone, welcome back in the course of Higher Surveying. And we are in module 4 Error, Accuracy, and Adjustments Computation. We have already covered a one lecture on this module last time. In that lecture we have seen what is the error, what is accuracy and what is error propagation. Also we have seen that: what is the precision which is generally calculated by standard deviation of data. And then we have seen: what is the difference between the precision that is standard deviation and the accuracy.

Remember, in order to find out the accuracy we need to have some kind of condition. And in the last lecture we have given a example of a triangle, where we put a condition that the sum of the 3 angles of a plane triangle should be equal to 180 degree, and this condition was independent of the observation we made for 3 angles. And then we said that, since the data it creates or it we can calculate the standard deviation from the data, and that is why precision which is standard deviation is internal to the data.

On the other hand, since the accuracy is calculated by our condition equation which is independent of the data itself or the variable itself; so, we said that accuracy is always external to the data. Now we have developed the fundamental concept of error and accuracy. And in the last we have seen: what is error propagation. So, today we are in lecture 2, and we are going to look into the applications of error propagation.

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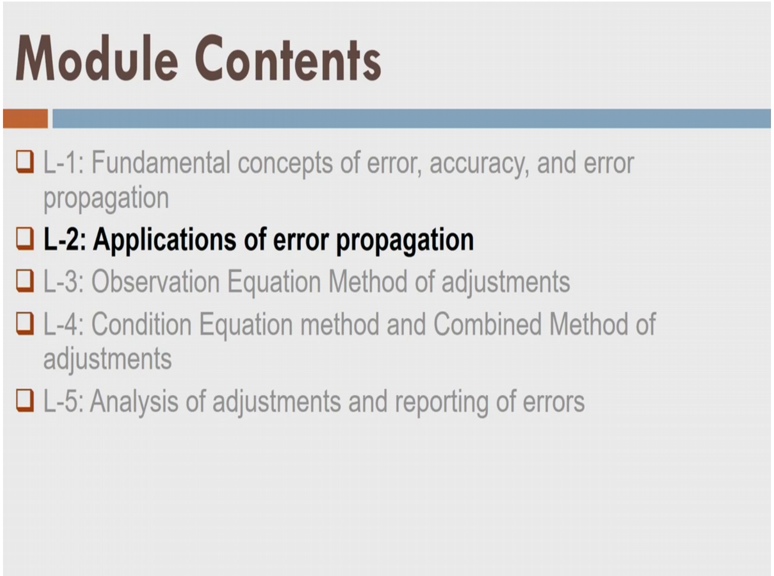

So, before we start in the detail, we will look some examples of error propagation by the logic or by the formula we have seen in the last lecture. So, let us start with that, ok. So, I started a simple example of area of a rectangle.

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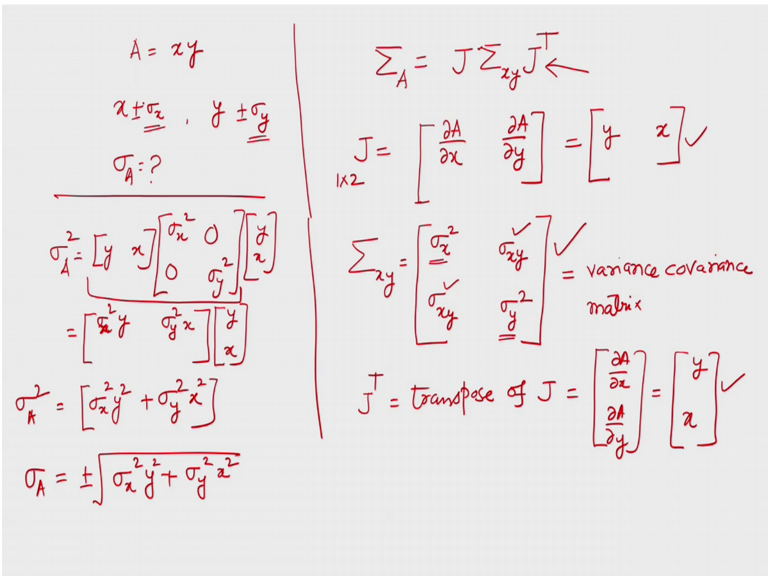

And area is given by x and y as a multiplication of x and y, ok. So, let us see that if x is measured, let us say the length of this room and width of this room x and y are measured with some standard deviation value, ok like this ok, now I want to find out what is the value of sigma A. And that is why I have given a simple formula there, that let us see if I want to find out the covariance variance matrix of output, then we will calculate it like this, ok.

Now, where we set the J is my Jacobian matrix and which is calculated like this. So, it is basically I can say 1 by 2 matrix, ok. What about the sigma x y? It is my covariance variance matrix of variable x and y. So, it is variance square of x, variance square of y, covariance of x y, covariance of x y here. So, these 2 elements here and here they are same. And these 2 elements it is variance of y, and here it is variance of x. So, this is my variance covariance matrix. And write J T is nothing but the transpose of J, which is nothing but dA by dx and dA by dy.

Now, if I write the value here, this value you can write easily, dA by dx is nothing but y and dA by d y is equal to x. So, my this matrix is nothing but y and x. Now, if I put all these values, this and this into this I can get it sigma A. So, a variable is a single variable. So, what is the covariance of the variable by variable itself, which is nothing right. So, we can have it is only one term that will be equal to the sigma A square, right.

The idea here is if there are 2 variables we have covariance, but there is only one variable, we do not have the covariance, because there are no 2 variables, or there not 3 variables, ok. So, fine what will I do here? Now, I will write sigma A square is y x sigma x square sigma y square. And I make it covariance as 0, why because; as I know if the observations are coming from the field the covariance’s are generally assumed as 0 between the 2 variables.

So, I am assuming that I am measuring the x and y using a tape. So, length and width of the room so, it is a field measurement. And now I can say that my covariance’s are 0, because both x and y are measured independently, and they do not influence each other or as far as their errors are not influencing each other. So, this is the total matrix here. Now, if I multiply then what will I get here. Let us say if I multiply first matrix to the second matrix I will get sigma x square y here, ok. And, then I will get sigma y square x here, ok then next y and x here. So, I multiplied these 2 mattresses first, ok. What next? Let us multiply these matrices, and I will get, I hope you should also get this thing, ok. So, that is my sigma A square.

You can also find out what is sigma A, that is precision, which is nothing but, ok. So, what is x and y? They are the measured values in the field or they are the measured value of room length and room width sigma x and sigma y they are the precision here. And you will look here very carefully that, sometimes I measure the length in general 1 time or 2 times. But I know with my experiences, what is my value of sigma x. So, I assume the sigma x and I assume the sigma y. Generally, we assume based on the least count of the instrument that is right now my tape, or if I am using a total station then I will use the least count of the total station or accuracy of the total station.

So, generally we assume sigma x sigma y and we do not measure it multiple times. So now what I am doing here is, I am trying to estimate the what will be the error that will be propagated to the area A if I measure the x and y in the field. So, again I am assuming here sigma x and sigma y, and I am trying to find out what could be the error with these assumed values.

And this is the reason to assume it. In fact, I should do let us say 100 measurement of length, and 100 measurement of width which is not possible. But I know what is the characteristics of the error of these measurements. And if the random errors are only present I know, what is the value that is possible for sigma x and sigma y. And so, by this assumption I go ahead. Now, you can take another example here.

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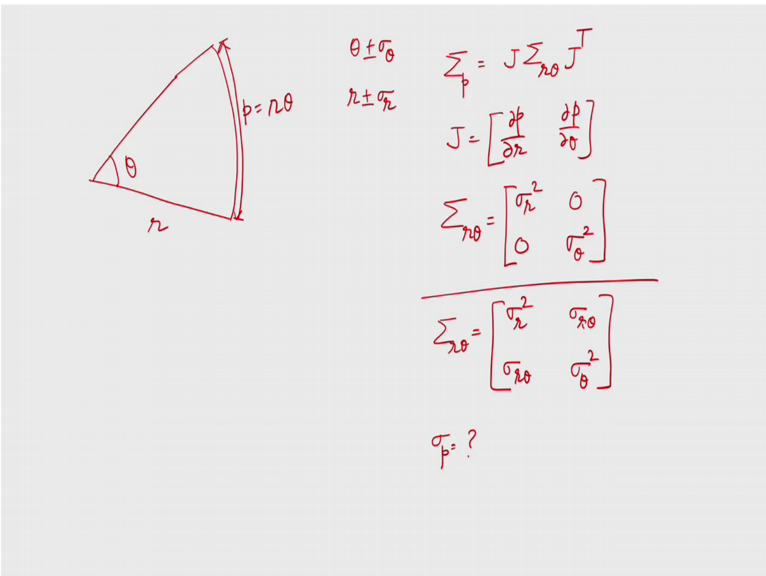

Where you will do? Let us see if there is a radius r and this angle is theta. Let us say there is a point, and now what is the length of this arc? Let us say it is p. So, p is equal to nothing but r theta, ok. Now if I measure the angle theta with theta plus minus sigma theta, this is the my variance sigma theta, theta is a value in the field. And r is the radius which I measure with sigma r, ok. So, what should be a value of sigma P by this formula? Sigma r theta J T, ok. What is the value of J? You can find out dp by dr, dp by d theta, that is it. And now I think you can go ahead with that.

Secondly you can also assume in sigma r theta, since I am measuring the observations directly in the field. So, I can assume 0 values of the covariance’s. I hope that is fine with you, and now you understand, ok. So, solve yourself this example I give it for you to solve it, and find out you will get the similar answer what we got in the last one. Also one more exercise is possible, if you assume that covariance’s are not 0. That means, this matrix is sigma square, sigma r theta, sigma theta square, sigma r theta in that case what will be your answer or how your answer changes? That means, you calculate sigma P and find out. You will get some term with sigma r theta also there, try to find out.

Now, so, we have seen 2 examples here, let us take multi response example. What is that multi response example, let us see we are measuring latitude and departure using some measurement.

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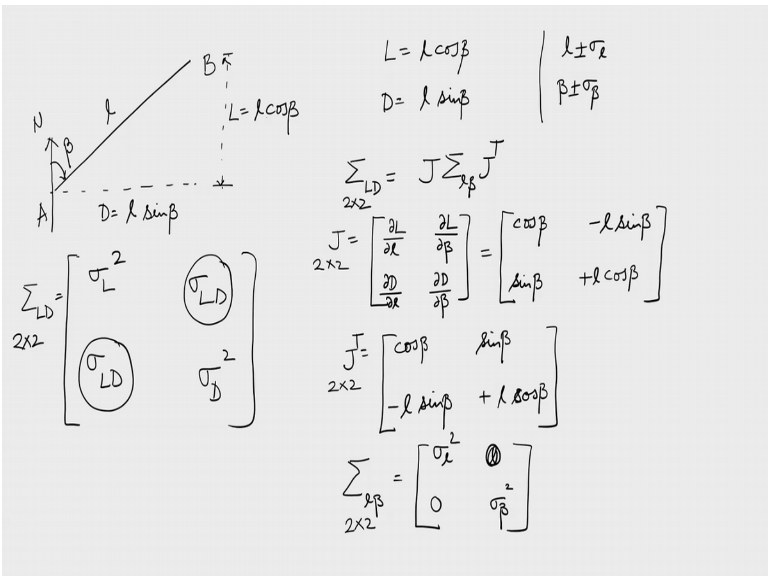

For example, let us say there is a line in the field fine. So, there is a line like this, ok. Let us call the line A B and the length is l, ok. And with respect to the north I measure the angle here or the bearing here as beta. So, what is my departure value here? And what is the latitude value in plane surveying? If you remember it, ok. So, l is nothing but equal to l sin beta or cos beta.

So, it is l sin beta departure, and it is l cos beta latitude fine. So, we write here that latitude equal to l cos beta departures equal to l sin beta in traversing remember, ok. So, let us see I am measuring the bearing with a compass, and I am measuring the length with a tape. So, both are independent first of all. Secondly, I can say here that sigma l is my precision of the length. So, I am writing the measured length l sigma l and the and bearing beta as sigma beta. Now you can see there are only 2 inputs as we have last time in the last example x and y and given the area. But here surprisingly we have 2 responses. One is latitude and one is departure. So, it is called multi response function, ok. In that case, how can I find out the propagated error, let us look into that.

Again now my output is sigma l d. And now J sigma l beta J T is my formula, ok. What is the J here? J will be slightly modified now, because there are 2 output variable. So, this is my dL by dl, d capital L by d beta. Second row will be d dou D by dou l and dou D by dou beta here, right. And if I calculate the value you can also do it yourself pretty simple. It is cos beta, here it will be minus sign beta into l, ok. What is the second row? Again it is sin beta, and here it will be plus l cos beta, ok. That is very clear to you, you can also develop what is your J T, that is transpose of J. So, that is my transpose matrix of J like this. So, that is it.

And what is sigma l beta? Which is variance covariance matrix of the input, again sigma l square sigma beta square, sigma l beta and making a 0 variance covariance, and now if you find out sigma l d, what should be the size of the sigma l d? Let us look into that, ok. What is size of J? J is the size of 2 by 2, J T is a size of again 2 by 2, it is always 2 by 2, since you multiply 3 matrix of 2 by 2 sigma l d will be 2 by 2. And if you try to calculate yourself you will get an answer like this. Sigma L D is equal to this way, and I have done it already for you, and I am getting few answers, I am getting some answers like this, here sigma l square which is, this slightly longer term. So, I am just writing it here sigma L square sigma D square sigma L D and sigma L D here. I have this is my 2 by 2 matrix here, ok.

And that is nothing but the covariance’s of the 2 variables L and D, remember. L and D are dependent variable or they are the response variable. And using the variances of basic measurement that is length and bearing, I have find out what could be the error propagated in the latitude and departure of a line, right. So, what should be the value of sigma L square? So, I got some values and I am sharing with you also try it yourself.

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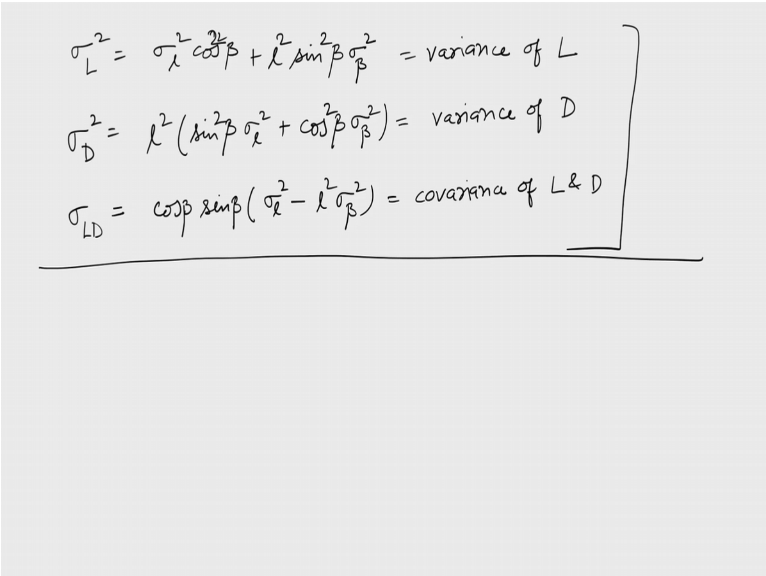

So, the sigma L square is given by sigma square cos square beta plus l square sin square beta sigma beta square. That is nothing but the various of latitude or L, ok. What about the variance of departure? Given by l square sin square beta, sigma l square plus cos square beta into sigma beta square. That is variance of departure or D.

Now, what is the covariance between sigma what sigma L D. So, covariance of l and d, ok and this is what I got is cos beta sin beta sigma l square minus l square sigma beta square. Now this is the covariance of L and D latitude and departure. And remember, that I have estimated these values. I have not ensured in the field, I have not measured in the field these value of sigma, ok. So, what is the idea here? Remember, we also do the error estimation by other logics by variation of the parameter, right for example, if I just draw here the variation of parameter logic here. For the area A x y, I do generally like this. Delta A by A equal to dou A by dou x into delta x plus dou A by dou y delta y, remember this formula, ok.

What is the idea here? What is the difference there between this error propagation and this error calculation? So, this is the based on as if I know what is my error delta x and delta y. So, I am trying to find out: what is error in the delta A, or what is the error delta

A. But here there is a assumption that I am measuring this I know what is my delta x and delta y. And so, if I know delta x and delta y, and if I can find out delta a what will I do before measurement itself I will correct my A; because I know what could be error in x and y.

However, in this case where we are using word sigma precision, I do not know what is the exact error is occurring there in the measurement. So, I am making an estimate of that error by sigma l square or sigma beta square or sigma x square, or sigma y square whatever. And using those as estimate, I am finding out what could be the estimate or in the error of area or in the latitude departure or whatever the dependent variable is. So, that is a idea here, right.

So, do not confuse yourself, what we are doing here is, we are trying to estimate based on some estimates. We are trying to estimate the error in the latitude and departure. Based on my estimate of error in the length and bearing value, right. So and why is it needed? Yes, it is needed in some of the applications. So, we will see the applications of error propagation today itself.

So, right now I just remove this one. Then you can verify yourself what is the answer you get here. This 3 answers you should verify yourself, and you have learnt the multi response function also, how to deal with the multi response function. And secondly, then you can do one thing, what will you do? Ok, before that giving little complicated exercise to you. Let us take simple more few more examples, ok.

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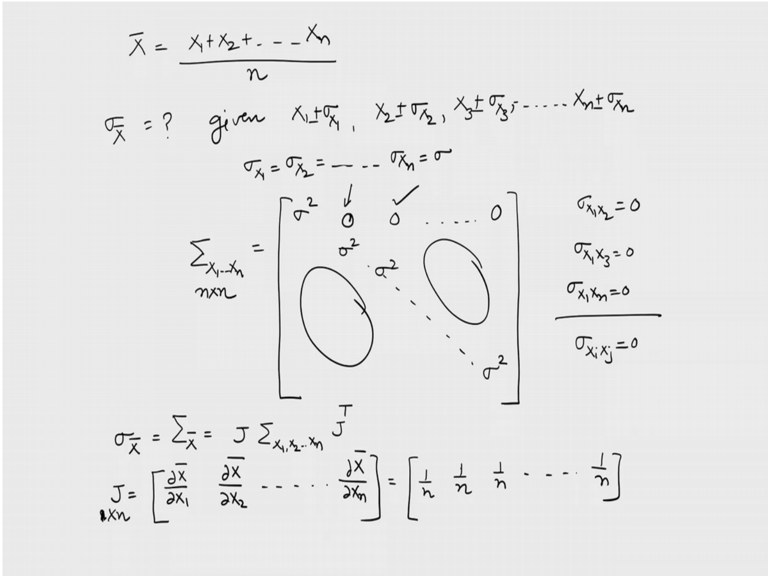

So, next example is, let us say I have variable X, and I am using it as X 1 X 2 and I am taking it is mean, and this is my mean. And I want to find out what is my sigma X bar here suppose it is X bar. So, what is my sigma X bar? Provided, given X 1 is coming from let us say sigma X 1 square or I am saying plus minus sigma X here, X 2 is coming from some distribution which is normal. But sigma X 2 is the variance value or a standard deviation.

Similarly, X 3 is coming from sigma X 3 here and so on. So, X n is coming with sigma X n; that means, they are these are the standard deviation of these values, ok. Let me make a simplification that all these values X 1 X 2 and X n are independent of each other. And secondly, all the values are coming from one distribution. So, I write here sigma X 1 equal to sigma X 2 equal to sigma X n equal to sigma; that means, they are coming from unique distribution, but although they are independent of each other; that means, they are not dependent

And as a result what will happen if I try to draw the covariance variance matrices it will be of size I am just writing it here X 1 to X n here. If I write it here sigma sigma square 0 because covariance variance matrix will be 0 between X 1 and X 2 0 X 1 and X 3 here and so on. So, then here sigma square and this is the leading diagonal here 0 like this. And all are I am sorry, it will be sigma square sigma square and all are sigma square here complete 0’s here complete 0s. And size of this matrix will be n by n.

Here I am saying that variance covariance this value is between sigma X 1 2. So, my sigma X 1 X 2 equals to 0; sigma X 1 sigma X 3 equal to 0 which is this value. And this value is 0 is nothing but sigma X 1 X n. Similarly, I can write all other values which are equal to 0. So, I can say in general that sigma X i sigma X j is equal to 0. All the covariance's between any variable is 0. Because all are independent of each other while we measure in the field, ok. Now I want to estimate what is my sigma X bar. So, use the logic again. Sigma X bar which is nothing but sigma this is the way we write J sigma this is my X 1 X 2 and X n into J T, ok. What is my J matrix? X 1 dou X bar by dou X 2 and dou X bar by dou X n, ok. The matrix size will be one row n column, ok.

And that is nothing but equal to if I take the this one what will happen? You will get 1 upon n plus sorry, not plus, all these n number n elements. You will get 1 upon n 1 upon n and such 1 upon n n times, like this. Check yourself are you getting it or not. Same way now I can construct the transpose of J which will be the column matrix.

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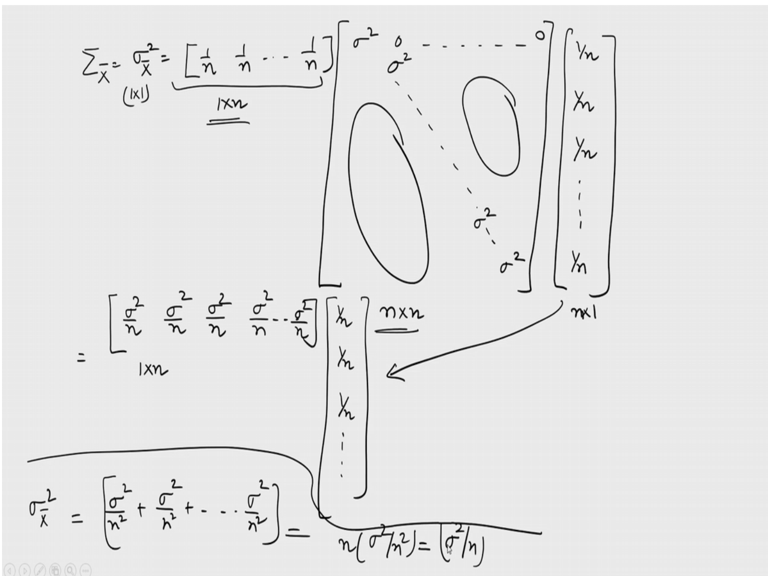

So, let us write this thing like this, what is my sigma X bar, which is equal to sigma X bar square, and J matrix if I write it will be 1 upon n. So, I am 1 upon n 1 upon n, 1 upon n n times here sigma matrix. So, sigma square 0 sorry sigma square here 0 0. Here it is up to sigma square every value is sigma square along the diagonal, and complete these values are 0, right.

Then J T, J T will be 1 upon n, 1 upon n, 1 upon n, 1 upon n it is equal to n by one matrix here. And this is your 1 by n matrix, here n by n matrix. So, if you multiply first 2 matrix you will get one into n size matrix, then if you multiply 1 into n into n into one size matrix, you will get 1 by 1. So, it is confirming the dimension of this value 1 by 1 here ok. So, let us multiply this thing you will get this kind of matrix here, right. And again the size of this matrix is, 1 by n and here we have this column matrix already this one. So, it is 1 by n 1 by n 1 by n and so on.

Now, if you multiply 2, what will you get? Ok, let us try, you will get sigma square by n square plus sigma square by n square n square. And these terms are n number of terms. So, I can write here my sigma X bar square equal to. So, this here difference is yeah n times sigma square by n square and you will get sigma square by n here, let us take next page.

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$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = n\left(\frac{\sigma^2}{n^2}\right)$$

$$\sigma_{\bar{X}} = \pm \frac{\sigma}{\sqrt{n}}$$

$$x_1 \pm \sigma_{x_1},\ x_2 \pm \sigma_{x_2},\ \ldots\ x_n \pm \sigma_{x_n}$$

$$w_1 = \frac{1}{\sigma_{x_1}^2},\ w_2 = \frac{1}{\sigma_{x_2}^2},\ \ldots\ w_n = \frac{1}{\sigma_{x_n}^2}$$

$$\bar{X} = \text{weighted average} = \frac{w_1 x_1 + w_2 x_2 + \ldots w_n x_n}{(w_1 + w_2 + \ldots w_n)}$$

$$\sigma_{\bar{X}} = ?$$

So, what is the value of my sigma X bar square? It is nothing but I got sigma square by n, or here I got the value by which is here, right. And there I can write here very easily, that my sigma X bar equals to plus minus root of this quantity so, sigma by root n.

So, that is the fundamental derivation we do, and we find out that the precision of the mean will be much better, or because it is much smaller. As we increase a number of observation n, I will get the factor of root n, and that will make my precision of the mean better. And that is prove somehow that yes, when we take the multiple measurements for a variable, the precision or the confidence is gaining up. Because precision is reducing there, by the factor of root n compared to a single observation. That is coming from sigma precision, ok. Let us make this example little complicated, and assume that observations are coming from different different distributions, which means X 1 is coming from sigma X 1 X 2 is coming from sigma X 2 and X n is coming with sigma X n.

And although each and every distribution is normally distributed; but all are different values of sigma. In that case, remember we called some kind of weights we assigned in basic surveying some weights and weights are assigned like this. Say, say W 1 is assigned something like this. W 2 is assigned like this for second observation and so on. So, W n is assigned like this. And in such cases we find out the weighted average; that means, what is my average value here. I call it the weighted average and which is equal to W 1 X 1 plus W 2 X 2 plus W n X n divided by W 1 plus W 2 and so on. That is summation of weights of all observations, this is what we say.

Now, find out what is the mean or the sigma X bar of the weighted average. That is my problem. Now I made problem little complicated. First I took a simple problem. Then, I make the problem little complicated; try to find out for this thing. And I will also solve for you here. So, what is my sigma X X here?

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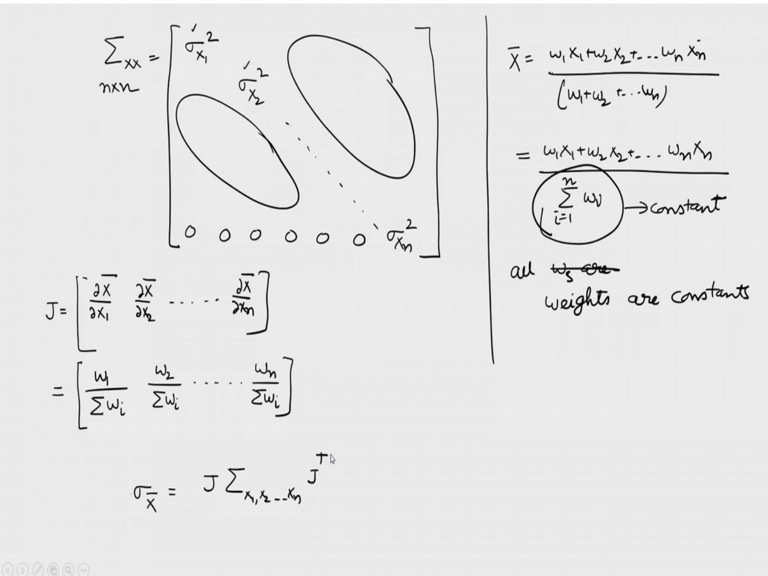

Sigma X X is sigma X 1 square sigma X 2 square sigma X n square, again remember, my observations are still coming from the field observation. So, I assume that all my variance covariance are equal to 0. And so, whole these matrix elements of diagonal are 0 in both cases. All this is a kind of diagonal matrix of size n by n.

But you can see these distributions are different. And hence all the values are different; sigma X 1 equal to sigma X 2 equal to sigma X 3 is not valid anymore. Here we are assuming that sigma X 1 sigma X 2 sigma X n and so on are coming from different different normal distributions, right. So, what is my J matrix now? In this case, when we write X bar as weighted average here. So, again for simplicity I right here W 1 X 1 plus W 2 X 2 W n X n divided by W 1 plus W 2 W n here, right.

In that case what is my J matrix? If I want to write here so, my J matrix will be dX bar by dX one dou X bar by dou X 2 and so on. Dou X bar by dou X n, right. You can differentiate yourself and you can find out that, you will get here W 1 divided by summation of all the weights. I am just writing it W i. Like this that is a first member here you will get W 2 divided by sigma W i and so on, you will get W n divided by sigma W i. Here I am not writing i equal to 1 to n, just it is understood here. Why because, you will see W 1 X 1 plus W 2 X 2 and so on W n X n here and divided by here W i i equals to 1 to n. And this quantity which is in the numerator it is constant. Because weights are constants we assume them. So, this is a constant quantity here, right

Similarly, because all weights are constant here, right, ok. So, I know my J matrix. You can also derive your J T matrix which is nothing but the transpose of J. Now we have to plug in all these values into the our basic formula, which is nothing but sigma X bar equals to J sigma X 1, X 2, X n into J T here, ok. So, let us put this value and try to find out what is the total answer here.

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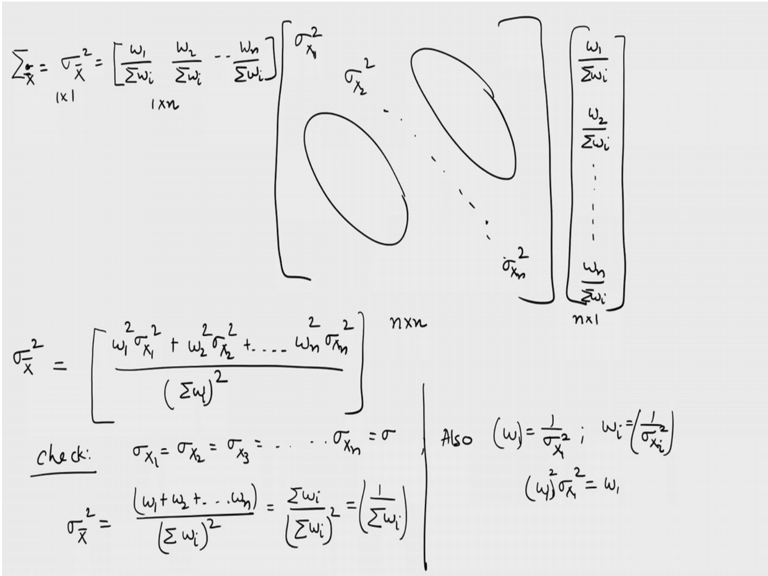

Well, if I put it, this one it will be something like this, W 1 upon sigma W i here, W 2 upon sigma W and so on up to W n. Then our diagonal matrix which is sigma X 1 square sigma X 2 square and so on up to sigma X n square, all the off diagonal elements are 0. And then we have this matrix which is transpose of J and so on. So, if we look the dimensionality it is my n into n matrix, it is my n into 1 matrix and it is my 1 into 1 matrix. So, if I multiply these 3 matrix I should get finally, one scalar number, one number. And that is the 1 by 1 and which is the size of this 1 by one or I can say here sigma X bar here. Yeah, X bar here, not here yeah.

So now, if I multiply what will I get here? You will (Refer Time: 34:46) to get you try yourself, and you should get the same answer what I got. I got finally, this answer, W 1 square sigma X 1 square plus W 2 square sigma X 2 square and W n square sigma X n square divided by the square of the total sigma W, right. That is why I am writing first in bracket sigma W and the square of the whole quantity. It is not W 1 square plus W 2 square, no. It is the first we make W 1 plus W 2 and so on the complete series and then the square of that number. So, it is called sigma W whole square right.

Now, so, you get the value of sigma X bar square. So, that is called the error of the weighted mean. Now, let us check whether this result is correct or not, how can you check it? Ok. So, check here is very easy, let us bring back to the same example, what we are solved with simplification, where we have assumed sigma X 1 equal to sigma X 2 equal to sigma X 3 equal to sigma X n and equal to sigma. Now you put this value here, ok. What will you get right. So, you will get sigma X bar square, in this case, you will get it if you put like this, then also I need to mention here, that we have W 1 is equal to 1 upon sigma X 1 square. And similarly, I have any W i equal to 1 upon sigma X i square like this.

So, here we will see that if I multiply these 2 i will get 1. So, basically what value get by multiplying? This with this equal to 1, or if I multiply here make the square of this term, I will get here W 1. You can see easily, right so, right, now if I put these 2 values here. So, what will I get here? W 1 plus W 2 and so on divided by sigma W i here also i square. And this is nothing but sigma W I divided by sigma W I square, or I will get 1 upon sigma W I, right. That is final answer I got it, ok.

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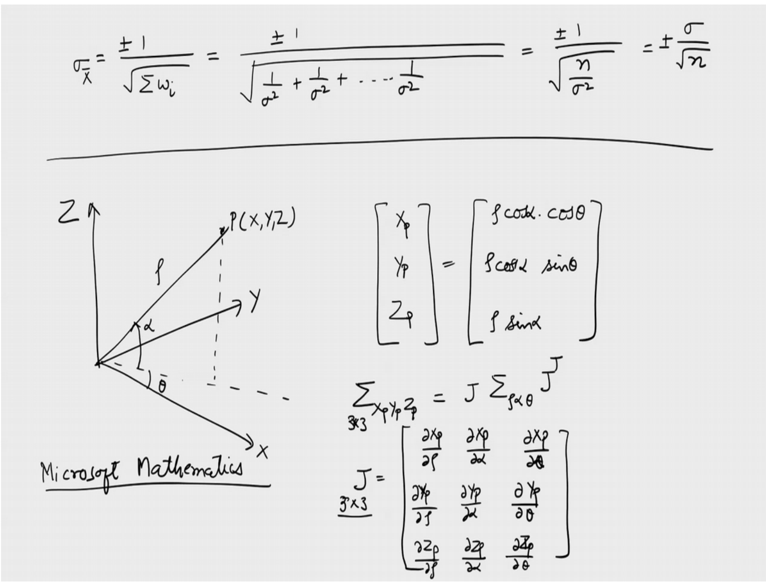

So, let us see: what is the interpretation here. Interpretation is pretty simple that I got my sigma X bar equals to 1 upon root of sigma W i, I got here 1 upon remember what is the value of so, 1 upon sigma square plus 1 upon sigma square plus 1 upon sigma square, equals to nothing but 1 upon under root of n time sigma square. Which is nothing but if I make it, I will get this sigma by root n. So, this is my check that my answers are correct. So, I have also find out what should be the error propagated in the weighted mean, right. And we have checked it also, that if we assume that weights are equal to same, I will get the same answer what I derived earlier. So, that is the idea here right here, right.

Now, if you look it very carefully that we have done many many examples, ok. Now I will ask you to do little complicated example, ok. And the example is, let us see you measure in a 3 dimensional coordinate system, using a total station you measure a distance called rho and you are measuring 2 angles. One is the theta, in the planimetric and one is the alpha angle, which is vertical angle, for point P and you are calculating the x y and z coordinates.

3D coordinates of point p, and this 3D coordinates are given by in a matrix form by this. If I am right, it is rho cos alpha, cos theta rho cos alpha into sin theta and rho sin alpha, right. This is a 3D distance rho here, alpha is my elevation angle with respect to xy plane, here this is my x, this is my y and this is my z, ok. Fine or I can write it also what is my coordinate of point P, X P, Y P, Z P, right, fine.

Now, what is idea here is you should do what is the value of sigma X P, Y P, Z P, right? By this formula sigma rho alpha theta J T; where Jacobian is given by, it is 3 by 3 matrix so, it will be given by this way. D X P by d rho d X P by d alpha d X P by am sorry, dou X P by dou l dou theta here,. So, it will be a 3 by 3 matrix here. Second row will be dy P by d rho d dou Y P by dou alpha dou Y P by dou theta. Third row will be dou Z P by dou rho dou Z P by dou alpha dou Z P by dou theta here. So, this is my 3 by 3 matrix.

And now you can calculate it by your differential calculus logic. Moreover, now it is very difficult if the you know the problem is starts becoming more complicated. The equations becoming more complicated; so, the idea is very simple. You can use some automatic tool for this purpose. And the first tool you should use which is freely available on internet and that is Microsoft mathematics. It will help you to write your equations, right? You can write your equations like this, and then you can differentiate you can find out Jacobian matrix and so on.

You can also put some values if you have and you can test. What is the value of your sigma X P, Y P, Z P? And I can say here, the sigma X P, Y P, Z P will be 3 by 3 matrix here. And that is exercise you should do using Microsoft mathematics, it is pretty easy to learn and do it yourself. If you are feeling that is this is a too complicated example and you will be having lot of problem doing it manually, yes. So, that is the idea here, you should try it, ok.

Let us move to the further applications. So, first application we have seen of error propagation how to find out the errors that will be propagated in the dependent variable. Now we will look into some more interesting applications of the error propagation.

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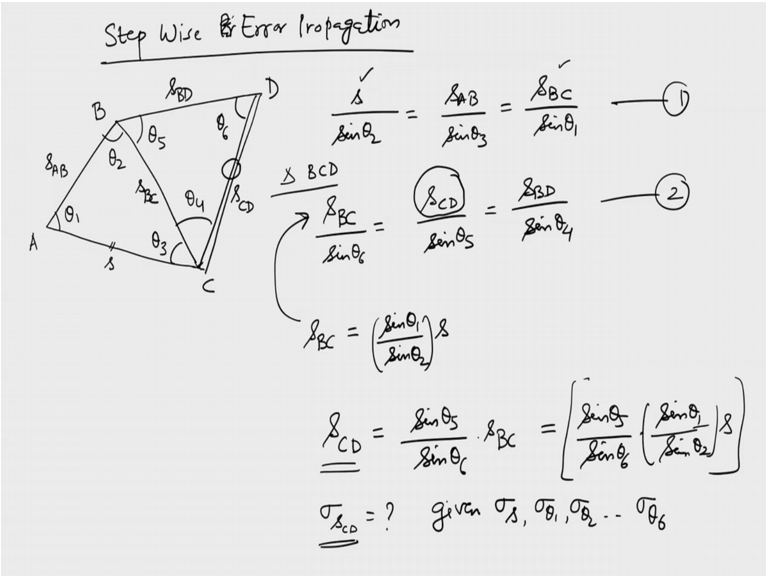

And the second application here is let us look into first a very simple example of related to the field study. One more thing I would like to say you here that is stepwise propagation here. So, let us look into stepwise propagation. What is my stepwise error propagation or call it, error propagation, what is the meaning here?

Let us look into simple example, let us say, I have a series of triangles with me, right. And by triangulation I measure the, this distance called S. And now let us see triangle, this network of triangle is A B C and D theta 1, theta 2, theta 3, theta 4, theta 5 and theta 6 here, ok. So, I have measured this distance S, and I want to find out what is this distance between C and D. So, I can use sin rule. So, first sin rule will give me S upon sin theta 2 equals to; let us say this distance is S A B, and this is my S B C, fine. So, I can write now S A B upon sin theta 3, and then S B C divided by sin theta 1.

Now, I know this distance measured in the field. I can calculate S B C, and again that is my equation number 1, and I can write equation number 2 as S B C divided by in this triangle in triangle B C D, I am writing this equation now, S B C divided by sin theta 6 equal to let us see S C D. And this is my S B D. So, S B C divided by sin theta 6, S C D divided by sin of theta 5 and S B D divided by sin of theta 4. And this is what we want to find out, in terms of S and theta angles. So now, using these 2 equations I can straight away write it, let us see first determine S B C. So, S B C is equal to sin theta 1 by sin theta 2 into S. Secondly, if I put this value here, and then I can find out S C D is equal to sin theta 5 upon sin theta 6 into S B C; which is nothing but sin theta 5 by sin theta 6 into sin theta 1 by sin theta 2 into S.

Now, if I keep on developing such network of triangle, these expressions as we propagates it will be very, very complicated and this keep on going. So, what is the remedy here? I want to find out what is my sigma S C D, what is this value? Given sigma x and all other value even let sigma theta 1, sigma theta 2 and so on. All the sigma thetas which are measured in the field are given. So, how can I find out this sigma S C D? One way is that let us try to make this as a function, and try to make the Jacobian matrix and you know the process. But there is some intelligent process also, that this call a stepwise error propagation which is like this

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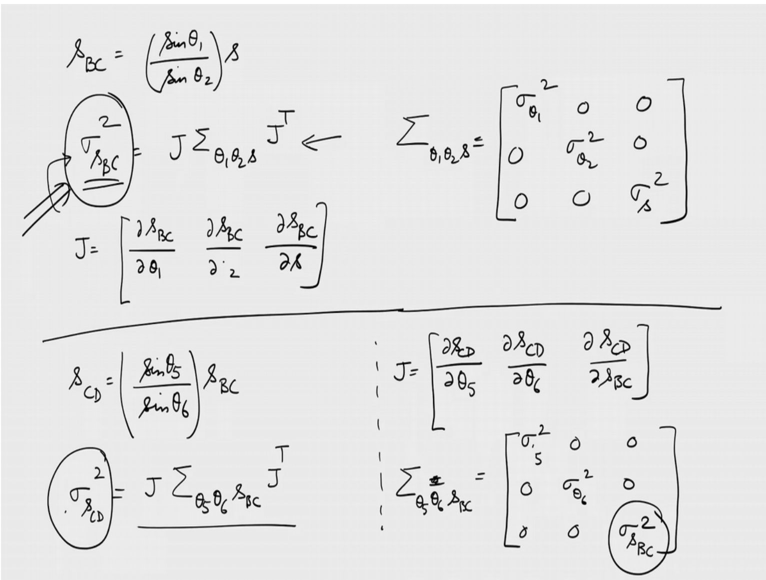

First I will do sigma S B C error I will find out, by a simple relationship, if you just see the last one here. This relationship I use that is a equation 3, and that is my equation 4. So, equation 3 I will use so, that is nothing but if I use this equation. So, S B C is equal to sin theta 1 by sin theta 2 into S, right. So, there are 3 variables and I want to find out sigma S B C. Here, I can find out sigma S B C S equal to J sigma theta 1 theta 2 S into J T. So, there will be my matrix J is given by ds B C dou S by B C by dou theta 1 dou S B

C by dou theta 2 and dou S B C by dou S. Well, you can calculate this matrix for you, ok. Then I can put this value and I can find out sigma S B C, right. I am sorry here, it should be square, right.

Now, I have calculated sigma S B C. Now we will use the equation 4, and equation 4 says that that S C D is equal to sin theta 5 by sin theta 6 into S B C. So, I write S C D equal to sin theta 5 by theta 6 into S B C, right this is my equation now. If I want to find out sigma S C D square, I can write that is a J sigma theta 5 theta 6 and S B C right into J T. So, what is the value here of my J matrix? J matrix says that is Jacobian dou S C D by dou theta 5 dou S C D by dou theta 6 dou S C D by dou S B C.

Now again this is the 3 variable problem. Here, I have already find out this value and I got this value, ok. So, what was the value here? My sigma value in this case if I let me just draw the 2 cases differently, sigma theta 1 theta and S. This was my variance covariance matrix 0, 0, 0 sigma theta 2 square and 0, 0, 0 sigma S square. So, find out my value here of sigma S square S B C here, my sigma matrix again, now I am doing the stepwise.

So, I find in first system I find out sigma S square S B C. Using this value of sigma square S B C in the second step, I will find out what is the value of this using this logic. Where my sigma theta 5 theta 6 and S B C will be there, and that is nothing but that C is given by sigma theta 5 square 0, 0, 0 sigma theta 6 square 0 0 sigma sigma S B C square. And which is this value is nothing but this value. So now, I am doing instead of a complicated big job I am doing the job in stepwise manner. So, that is pretty easy to understand all the intermediate steps. Try yourself; try to solve this example, ok.

So, we have learnt: what is error propagation we have seen few examples, then we learnt: what is the stepwise error propagation. Now, we will see: what are the real applications which are really required in the field in a project.

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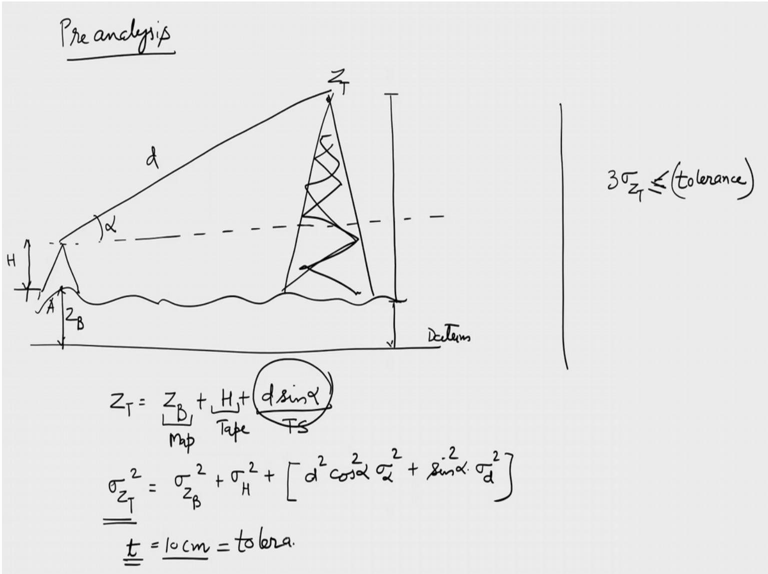

And the first application is the pre analysis that I am going to take today. So, what is the pre analysis? You have already seen in the basic surveying. So, I am allocating it little. Let us see you want this terrane and this is a tower of A, let us say any mobile tower I can say like this, something like this or maybe you can make some criss cross like, ok. You want to find out what is the height of the tower; from some benchmark value called this is my datum. So, what is the height of tower, or given the point a here with the height Z A. What is this J T? Top of the tower, you want to find out.

Z B is coming from say map a contour map, now you put a total station over there this point at a right, and then you establish a horizontal line of site, then you measure this height, this distance let us say d and you measure angle alpha. Elevation angle and distance, that we do always with the help of total station. Now if you also measure the height of the instrument of this point which is let us say H, I call it capital H here, ok. So, let me write the Z T is equal to let us say Z B plus H plus d sin alpha. That is the z coordinate of (Refer Time: 53:26). Now if I say let us determine: what is the Z T you can determine it by putting these values, but what if I ask you what is the sigma square Z T. So, what is if you do this kind of measurement what is there.

Now, you see that Z B is coming from the map contour map. So, which is completely independent of the height of instrument because, you have measure by height of instrument with a tape, and you have measured this one from the T S. This is you measured with a tape, and that you have coming from the map, ok. So, both all 3 are the sources are independent. So, I can now write easily, because they are not correlated to each other. I can write here sigma Z B square plus sigma H square plus, now I need to do error propagation, because there are 2 variables available. And since if I assume that these variables are independent to each other, alpha and d which is generally we assume in the field I can write here directly that if I do error propagation for this only, only this quantity with respect to d and alpha.

So, I will get a value like this d square cos alpha sigma alpha square plus sin square alpha into sigma d square here, right, B here, right. Now you see that there are 3 sources and 3 sources are contributing to the error propagating to height of the tower with respect to some datum. Now someone says that I want to measure the height, or client comes to you and say that I want to measure the height of the tower with respect our datum, but this height error should be within some limit. Let us see a threshold of t. Yeah let us say it is 10 centimeter or 5 centimeter whatever I am just giving you.

So, what will you do now? Because you know that is sigma Z T you can calculate it your error propagation that I have already calculated it for you. But now in the field you will ensure that what will you do? You limit the 3 times sigma Z T equals should be less than equal to t tolerance; which is given by the client, ok. I am just saying tolerance here, and it is tolerance here. And try to go to basic surveying also for a detailed one, if you really want, ok. Let us take the next one which is compatible measurement again, ok.

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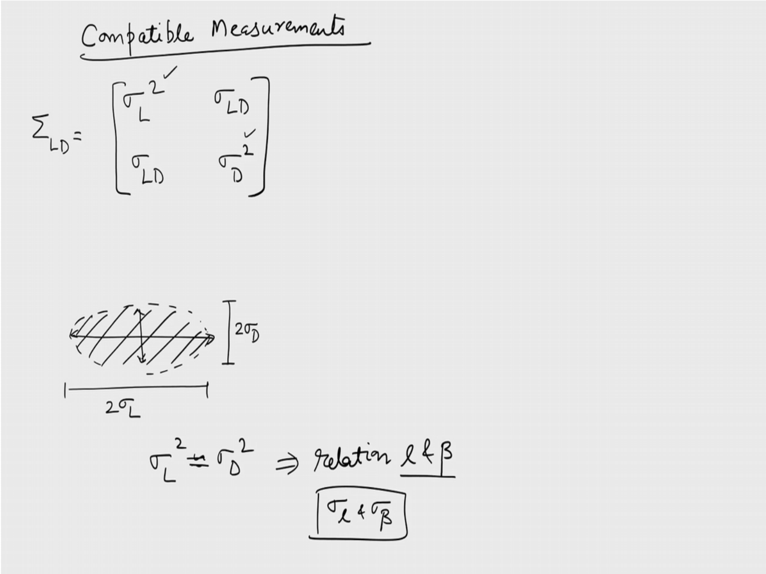

If you remember we have find out the matrix sigma L square sigma L D sigma D square in case of latitude departure example, this one right, as we said it is my sigma L D. Here if you see that if my length measurement is very, very accurate or bearing measurement is very, very accurate and as a result of that I am getting some value of sigma L and some value of sigma D, which are quite different, right.

You see the expression and try to fit some inappropriate values and try to see what do you get sigma L square or sigma D square. So, it could be possible this is my value of sigma L right, like this and this is my sigma D which is quite small, right. And that is why we say that if this is an error of the location, right where this is my let us see 2 sigma D I can say. And this is my 2 sigma L. So, this is a kind of error I am expecting in the location of a point which is given by L and D.

Now, what will happen you see I am wasting either a resource or I am trying to match 2 inappropriate sources; that means, either I am doing length measurement. So, accurately that it is not compatible with the bearing measurement. Or I am doing my bearing measurement. So, accurately that it is not compatible with my length measurement. Fundamental measurements are length and bearing if you remember in latitude departure calculation. So, I am doing something wrong there. So, how can I do the proper selection of the bearing instrument?

That is, bearing measurement instrument, that is a my compass and the length measurement instrument that is my tape or total station idea is very simple here. Let us equate or make them equal, approximately equal let us theoretically make them equal, and try to find out the relationship between l and beta. And then you can fix your a specifications of compass which will measure beta angle. And we can fix the compass specification of your tape which will measure the L.

So, will you end up with some relationship between sigma l and sigma beta, right. So, try to find out this relationship yourself, and then according to this compatibility measurement. You can fix the sigma l and sigma beta, which is nothing but the precision or the quality or the we can say accuracy of the your instruments tape and bearing measurement which is compass. That is compatible measurement idea; let us look into the next one.

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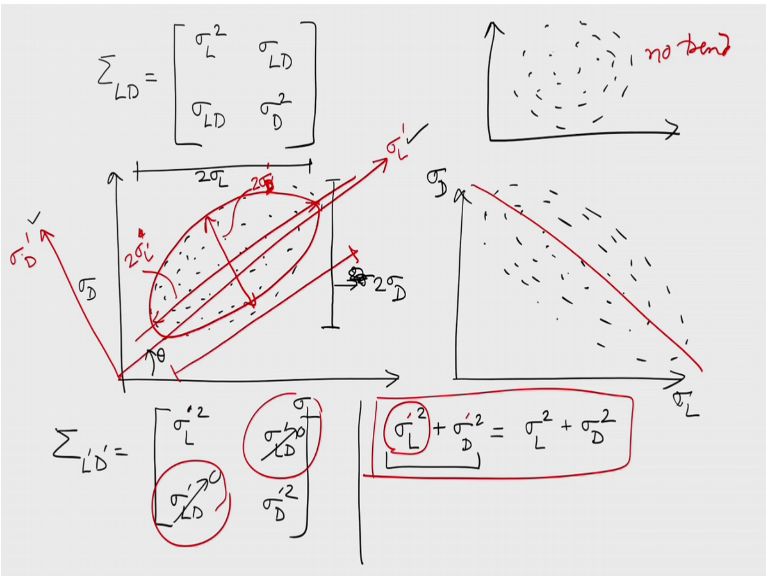

What is meaning here? The next is: what is the maximum error here, again take the example of the latitude departure measurement only here we have sigma L square, sigma D square sigma LD and sigma LD here, right.

Now, if I take couple of samples of length and bearing, then I try to find out latitude and departure for each line, different different lines. And let us assume that each line is representing me a sample. So, I have length and I have a bearing of that line. So, I find out for each line this one. Now if I try to plot those errors on let us say sigma l here, and sigma D here. So, since I have n number of line. So, I am trying to plot, this plot could come like this. I do not know it could come, these errors I am trying to plot. Different different n number of samples something like this, they are randomly distributed. I am just seeing one possibility. There could be another possibilities also like this; where they could be like this. Something like this n number of measurement.

This is another possibility; let us say this is my sigma l here and sigma d there, ok. One more possibility is there like this, where n number of observations are like that, ok. Here we can see here there is a clear linear trend I can draw here, like this; I can see here there is a linear trend between the 2 variables. Here I can see linear trend here, but there is no trend here, right. So, in these 2 cases where I can observe the trend, I can say one thing that there could be a relationship between the 2 number one between the L and D; that means, their errors are correlated what I am saying is right, ok. So, I can find out correlation between the 2 that is the coefficient of correlation. Again, you can just look into the basic surveying or you can look into the any fundamental book of statistics what is correlation.

Apart from that, I have one more query: what is the maximum error, because it is elliptical shape. And I can see here that this error is maximum here. And here I can see that this error is minimum which is across this (Refer Time: 62:18) error moreover I can see, I want to make errors such a way that these errors which are appearing, now correlated sigma L and sigma D. They should become uncorrelated to me. So, what can I do now?

Simple logic here is let us rotate, this axes in such a way that this becomes my new axes like this. And I called up let us say sigma L dash and sigma D dash, like this. And if in the original data, if I say like this is my 2 times sigma L and this is my spread along D is 2 times sigma D, then what should be the value of sigma L dash and sigma LD such that sigma L dash and sigma D dash are uncorrelated?

And that is where we come to the principal component analysis, where the principal component analysis says that let us preserve the variances. So, what does it mean? So, if I do some kind of rotation theta here to the my axes, sigma L and sigma D such that they become uncorrelated. So, what is the meaning here? If I write now after this L dash and D dash, it will be sigma L dash square, sigma L dash D dash here sigma L dash D dash here sigma D dash square, I am just rotating that by matrix. So, I can see here that if this my covariance's becomes 0, then I say that they are uncorrelated.

Hence, with this idea we got mathematically first of all. If you do like that, in that case, first of all my variances are preserved. What is the meaning hare? So, if I am adding sigma L dash square sigma D dash square is equal to sigma L square plus sigma D square; that means, I have rotated my axes theta in such a way that the variances remains, but covariance's becomes 0. So, what is the idea here is mathematically if you see I have shown here. But after rotation, if I see, this is my and this my new axes where this is my 2 sigma l dash.

And this is my 2 sigma B dash. I am sorry, not here square I am sorry, here one should write like this not this way I have change the variances value not the variables. So, do not do this mistake here, let me just rewrite this thing again here, yes, this is my sigma dash and sigma dash L D. Remember, I have not changed my variables, I have changed only the variances value.

Yeah, so, my covariance's are now equal to 0, why 0? Because we can see that these 2 yeah sigma L dash here and sigma dash D are uncorrelated to each other, because they are along the major axes of the ellipse and the minor axes of this ellipse, this data ellipse, or the array data ellipse, right. And they are so they are uncorrelated, ok. So, in this case if I ensure this thing; that means, if I rotate by angle theta, such that this is there I ensure this property, in that case, I can prove it mathematically that my variances are preserved.

At the same time, I have find out the maximum error here, you see, the maximum amount of error which is nothing but. So, this is the maximum value of the ellipse here, or the I can say major axis which is equal to 2 sigma L dash. So, that is the idea here, I have for really find out the principal component what we call. So, we can also find out what is the formula of the principal component, and this is written like this.

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$$\sigma_L'^{2} = \left(\frac{\sigma_L^2 + \sigma_D^2}{2}\right) + \left[\frac{\sigma_L^2 - \sigma_D^2}{4} + \left(\sigma_{LD}\right)^2\right]^{0.5}$$

$$\sigma_D'^{2} = \left(\frac{\sigma_L^2 + \sigma_D^2}{2}\right) - \left[\qquad\qquad\right]$$

$$\tan 2\theta = \left(\frac{2\sigma_{LD}}{\sigma_L^2 - \sigma_D^2}\right)$$

I am writing here, this is given by sigma dash is equal to sigma L square plus sigma D square for 2D variable I am writing it. And you can look more into the principal component analysis if you want, to the power 0.5, ok. Similarly, here writing it here, minus and the same quantity here and try to find out yourself whether it is correct or not in the books, ok. So, where we give the formula of theta as like this; so, this is the principal component analysis. And here we stop and we will meet in the next lecture. With this idea we have finished our error propagation, and the applications of the error propagation.

Thank you.