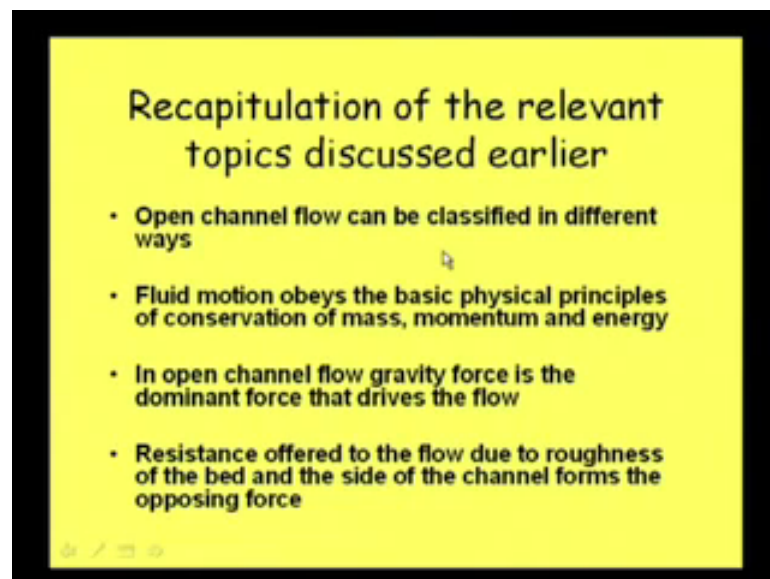


Hydraulics
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Lecture No. # 01
Uniform Flow

Friends today we shall be discussing one of the very important topic of hydraulic engineering, that is open channel flow; and then in open channel flow we will be discussing the uniform flow. Well, now, what is uniform flow? Some fundamentals of that, we have already discussed in our earlier classes, and some of the topic that we have discussed in our previous classes that will also come in this discussion. And so, let me recapitulate what we did in our earlier classes.

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We could see that open channel flow can be classified in different way, a different way means it is based on variation of flow parameter with respect to time, variation of flow parameter with respect to space, and then, of course, based on turbulence, then based on whether it is critical flow or not; so, like that we can classify each in different way And this uniform flow is coming under the classification that, we are making classification based on variation of flow parameter with respect to space, and that way we did classify

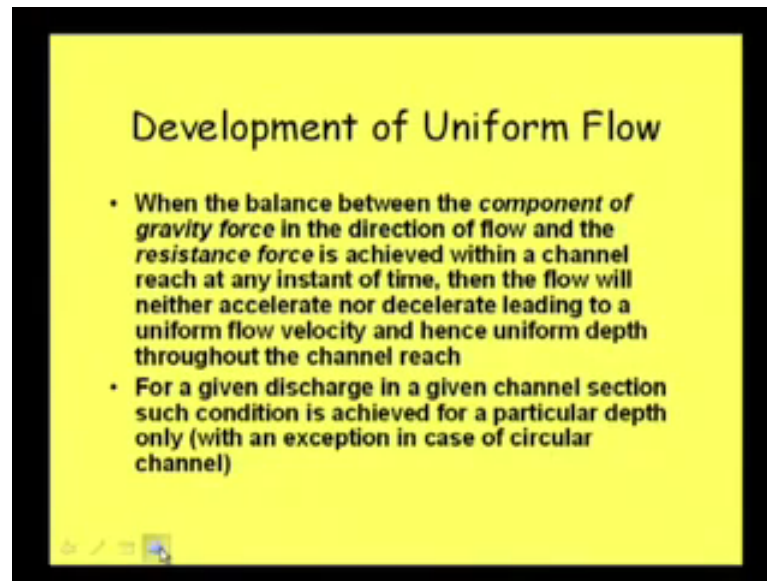
it as uniform flow and non uniform flow.

Now, in this particular class we will be going more detail into the uniform flow. Well, then another topic that we did discuss in our earlier classes is that, fluid motion obeys the basic physical principals of conservation of mass, then conservation of momentum, and conservation of energy.

Well, now, these very basic principals will always be using when we will be discussing this particular topic that is uniform flow. Well, then in open channel hydraulics we found that which force is dominating the flow that is the gravity flow. So, when the fluid is in motion, which is the force that causing the motion is the gravity flow, particularly when we are talking about open channel hydraulics; and then in a system when there is a force which is causing a motion then some forces will be opposing that motion, and that opposing force is coming due to fictional resistance offered by the bed and side of the channel.

So, that way these two forces depending on different abstraction, depending on the channel slope, and depending on the total volume of flow or volume of water that is coming; these driving force and opposing force keep on changing and that gets balance at some point of time, and that way **these** this makes the flow to sometime **become** uniform flow, sometime become non uniform flow; and I am saying that uniform flow, I am just considering that we understand that what we mean by uniform flow in a simple term. And because that, we have already discussed in our earlier class that when, that is **the flow parameter do not change** when the flow parameters do not change with respect to space that is the depth is not changing with respect to space, velocity is not changing with respect to space, then **we term** this sort of flow as uniform flow. Well, now, let me go a little bit deep into this particular topic, we need to know that how a uniform flow gets developed well.

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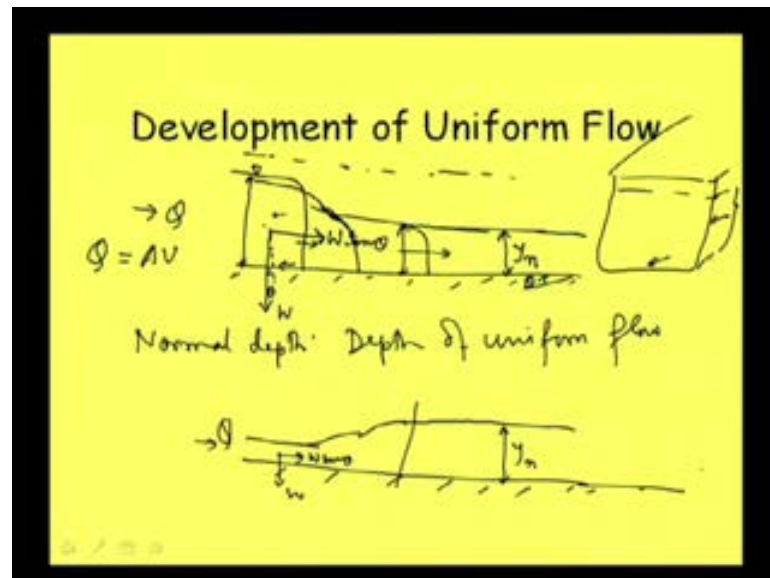
So, let me go to the development of uniform flow. Well, we can say that, when the balance between the component of gravity force and depth again; when we say component of gravity force we are talking in the direction of flow component of gravity force in the direction flow, and the resistance force, that is resistance force means the resistance offered by the frictional resistance bed inside; when these two force get balance, and when we achieve this balance within a channel reach within a channel reach at any instant of time, then the flow will neither accelerate nor decelerate.

We know that when the force are getting balance then I mean there will not be any acceleration or deceleration of the flow, and that lead to the formation of uniform flow velocity, and when we are getting uniform flow velocity without any acceleration and deceleration within the channel reach ultimately we are getting uniform depth at every point, and that flow we refer as uniform flow.

Well, now, one important fact is there, for a given discharge, when a given amount of discharge is flowing through a given channel means our channel section is fixed; suppose, in that channel section a given amount of discharge is flowing then this balance, that is the balance between driving force, there is a gravity force and the resistance force can be achieved or is achieved for a particular depth of flow, this can be achieved for particular depth of flow, of course, there is exception, that is the exception; in case of circular channel in case of circular channel, of course, there is exception, in that case that

will become later, but right now we can consider it like that, this balance is achieved for a particular depth and that depth, in fact, we call as uniform flow depth or there is another name for this uniform flow depth that we call as normal depth, because well that normal depth means normally we get without any disturbance this sort of flow, so we call it as normal depth.

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Now, let me just give you one example or let me explain this with some diagram, you can just concentrate on this slides, say, there is a channel of this slope well, and we have put some amount discharge Q here, some amount of discharge Q is been allowed to flow through this channel, and say by some means we are putting this amount of discharge with a depth of this much, suppose this is the depth, well, this is the depth we are putting, that means, the flow is in this part we are not showing right. Now, **and** this is depth we are putting right now.

Now, for this depth of flow, now what this amount of water will be having some weight in this direction, that is say weight W , **and then this will be...**, if this is an angle θ this is an angle θ , then this is θ , and then this will have a component of this force W in this direction, that we can write as $W \sin \theta$, and is **(())** that this is this channel is suppose like this, and then water is flowing like that, and we are putting some amount of water here like that, so this is like that.

And then from the side of the channel, suppose this is the channel boundary top of the

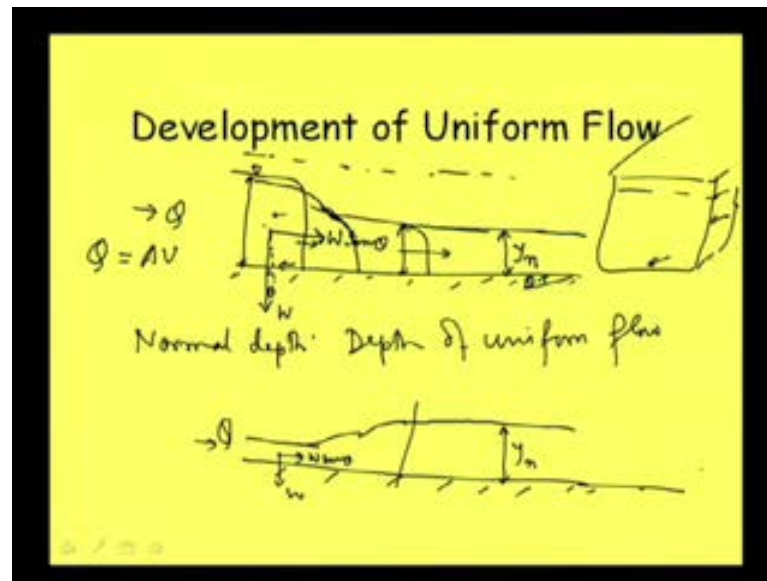
channel I am writing I am drawing here; now, this force is trying to move the water in this direction, then what force is opposing that side of the channel and from the base of the channel, so all these from side of the channel is emerging from the side of the channel, means, I am talking about this one base of the channel, means, I am talking about this one; so, that way from the side of the channel and from the base of the channel there will be opposing force.

Now, depending on roughness of these sides or roughness of these base, some amount of resistant force will be developed, and that will also depend of depends on **how much area** how much surface area is actually in action, that is how much surface area is in contact, and that way will be getting a total amount of resistance force; now, if that resistance force is less than this $W \sin \theta$, that is force from this direction, then what will happen that the flow will accelerate flow will accelerate in this direction, that is velocity will be increasing, and then what will happen when the velocity is increasing that flow depth as total amount of Q is remaining same, and from the vary continuity equation we know that simple continuity equation we are talking about it is area into velocity.

So, when our Q is remaining same flow is accelerating velocity is accelerating the flow depth will be reducing. So, when it is moving downward, then it will be coming like this, it will be coming like this; and then what is that, this again the gravity force we are having this amount of water will be having a flow force in this direction, and again depending on the contact area of this particular flow with the side and bid resistance force is developing; if it is again less suppose this $W \sin \theta$ is more again the flow depth will be coming down and it is moving like this.

So, that way after sometime and some distance what will happen **will be arriving at the depth** will be arriving at the depth, say, this Y depth for which we are finding that the component of gravity force in this direction and the resistance force offered by the side and base are exactly balanced; and in that case when the force will get balanced, this flow there will not be no more acceleration or deceleration, and then the flow is moving with the same depth in this direction; and then what we are getting is call uniform flow, and this particular depth of uniform flow we refer as normal depth.

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So, normal depth means the depth of uniform flow, that time we will be always using that is call **normal depth** normal depth, and normal depth means depth of uniform flow. Now, here I am just saying about depth of uniform flow. Now, here I am talking about, suppose, the flow is accelerating and from higher depth it is coming down, and we are having a uniform flow the things can be just reverse also; for example, suppose we are putting some small amount of discharge, say this is again base, and **we are putting small...**, we are we are putting amount of discharge Q , but with a very small depth we are forcing this depth, say there can be some abstraction, and we are allowing the water only to flow through this part.

So, what is happening that, this volume or the mass of fluid will be having some weight, and again this $W \sin \theta$ component will be having here; and then opposing force is, suppose, much more here, it is more opposing force is increasing, then what will happen that **flow will retarded** flow will be retarded, and then when the **flow velocity is decreasing** flow velocity is decreasing, then your depth is increasing. So, in the next moment that depth will be increasing like that, and that way the depth is increasing, and it will ultimately reach a depth where again you will be finding that the force driving the flow and the force opposing the flow is equal.

So, that way in a channel for developing a uniform flow we always need some initial portion, I mean, in nature, of course, the things will be coming as it is, and we can have

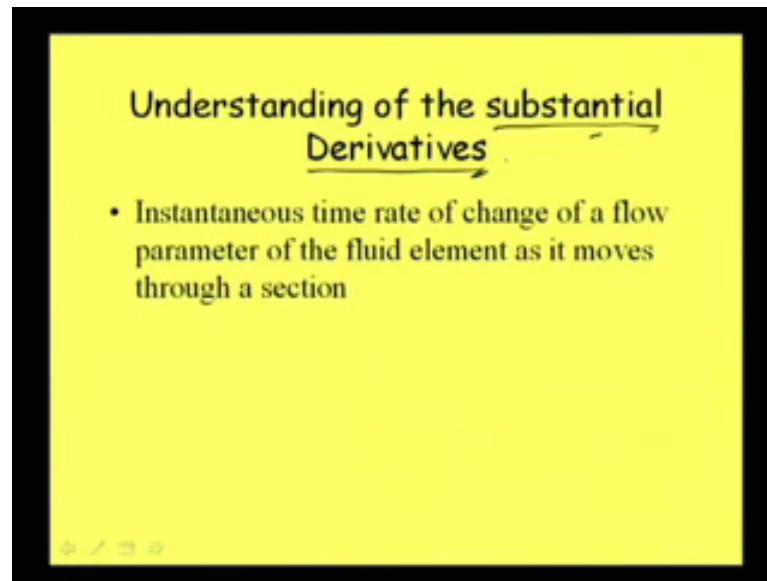
say, so it is coming with a uniform flow depth y_n ; then when there is a change when there is a change, then the flow will be coming like this in this point, the detail will be discuss in later, of course, say the flow is coming like this; and here of course, there will be critical flow that aspect we will be discuss in later, but again then the flow is coming down, and this part of flow this part of flow is not uniform flow, because here as the slope is more velocity is increasing V_1 , and here suppose sorry if I consider V_1 here, it is suppose V_2 , this V_2 will be moved and V_1 ; that means, there is acceleration and the flow is increase a flow is changing like that.

And then after some time based on the resistance of this part and based on the slope of this particular reach, again it will be gradually moving to a uniform condition like that; and then here also we are getting a uniform flow y_n normal depth, this is also normal depth; and then again, suppose, here we are going there will be retardation, so flow will be rising like that flow will be rising like that; again, we may get another we will be getting another uniform flow here, of course, if there is drop like this then there will be fall like that.

Of course, based on how much is the depth here, and how it is entering, this particular channel reach depending on all these fact, there can be different phenomenon, and I am just talking about a simple situation and drawing one typical keys where it can be; now, what we could see that, for a given discharge for a given discharge uniform flow or normal depth in a channel we can have here we are having this depth here we are having this depth both are normal depth, but here it is more, here it is less. So, what it means that, normal depth depend on slope S_b , base slope S_b , if your base slope is more your depth will be less, that is uniform flow will be moving lesser depth; if slope base less means it is moving with higher depth like that.

So, channel cross section will influence and base slope influence, and for the same base slope and same channel cross section for a given Q we can have single uniform flow depth normal depth, of course, with the change we can get another uniform flow depth. Well, so, briefly saying about the development of uniform flow in nature we can have like that.

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And let me go to another term or little mathematically expression of these uniform flow, and of course, not some of the term we use that is call say substantial derivatives ,this is not directly related to uniform flow phenomenon only, but before explaining some of the term in uniform flow or in non uniform flow even or even in unsteady flow, we need to have some understanding of these term, that is **substance** substantial derivative; why it is important, because we are talking about say acceleration, say when there is no acceleration not when there is no deceleration, then we are getting uniform flow right.

But now acceleration of what **if it is solid body** if it is solid body it is moving with a velocity v suppose and then we can very well understand this solid body is moving fine and then it has different velocity with time what the change of velocity time rate of change of velocity then we can call this as a acceleration; but in case of fluid there can be changes, because **changes** the changes are not that simple, and we need to have some better understanding of some of these term.

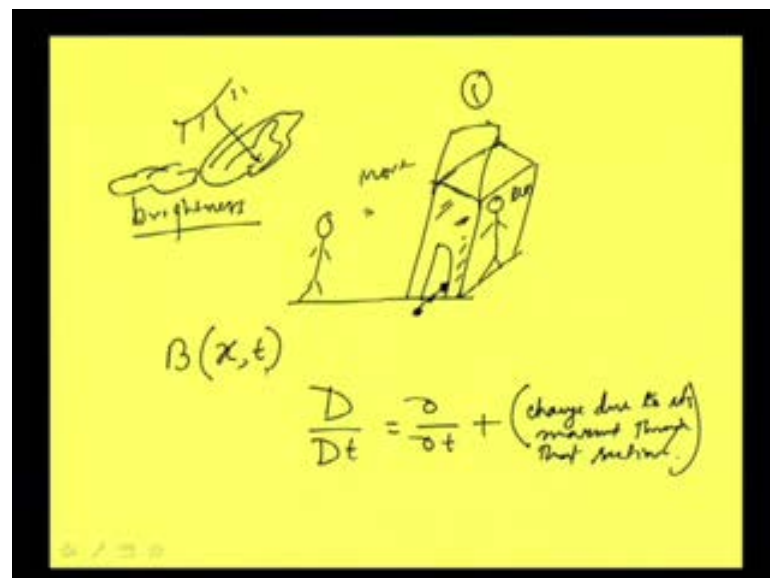
And so, **we refer**, suppose, when we say that change of velocity or acceleration we are talking about that, that means, time rate of change of velocity, then we need to talk it in this term what we mean by substantial derivative, of course, we use the term to tell derivative also; well, but let me first explain this in physical term, and let me read this definition of substantial derivative first. Instantaneous time rate of change of a flow parameter, so it can be any flow parameter, of course, when we talk about acceleration

then only we talk about velocity, but change of velocity, but this substantial derivative that term can be coming for any flow parameter, say, if we talk about density, suppose row density of the fluid, and if this density of the fluid is changing from point to point time to time, then we can talk about the time rate of change of that particular parameter also. So, **any parameter** any flow parameter we can talk about and then we are talking about **instantaneous time rate of change** instantaneous time rate of change.

Well, what we mean by instantaneous time rate of change; instantaneous means, at that instant of time what sort of **time rate of change** rate of change of depth flow parameter with respect to time we are getting again; again at a different time we can have different rate of change of that parameter with respect to time. Well, so, by substantial derivatives we talk about instantaneous time rate of change of a flow parameter of fluid element as it moves to a section as it moves to a section.

Well, let me give another example to explain what we mean by substantial derivative; well, say, any parameter let me not talk about in fluid, let me take a different situation where it may be very conveniently explained what is substantial derivative, say, let me I am not good in drawing, but let me try.

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Suppose, a person is moving in this point, and he is trying to enter a building, say he is trying to enter a building like this; and then, well, now let me talk about what are the changes this he can experience well, suppose there is a sun light here, and let me talk

about the sunlight is falling on this person, and then when he is entering through this, **let me talk about one parameter say brightness**, let me talk about the parameter brightness, so how the brightness of this person is changing. So, when he is moving from this point to that point through this point he is entering. So, our interest is at this particular section we are observing at this particular section, we are observing where there is a door, and when he is just entering from here to here, then when he is inside, then say **inside the house** inside the house the brightness will be definitely less, here we have more brightness, and here **we have** inside the house we have less brightness.

So, when the person is just moving from this point to that point, at that instant of time when we are observing, because of his movement from this point to that point his brightness will change, because of his movement of from this point to that point when he is crossing through this section, my section is this one through my section is this one section I am talking about; so, at this particular section when he is moving from this to that point then his brightness is changing, so there is change in his brightness **because of his movement** because of his movement.

Well, so, suppose brightness B that is a function of space, suppose space been I am writing as x and also a function of time also a function of time, so when I am talking that he is just moving from here to here, at that instant actually time change is not coming right now, say, as he is moving from here to here something is changing; but now again you imagine that when he is just moving from this point to that point, at that instant of time some cloud has come, some cloud were here, and that cloud has come here at that instant of time; now, because the cloud is coming for that also there is a change in his brightness, because the cloud is coming, so brightness is changing, and at that instant of time because of that his brightness is changing.

Now, that part of changes brightness has nothing to do with his movement from this point to that point; so, that means, when he is entering into the room, at the same instant of time a cloud cover is covering the sun, so his total brightness from here to here is changing; and it has two distinct part, one part is that at that instant something has happened with respect to time and because of that his brightness is changing, and again as he is moving from this point to that point, so his brightness is changing.

So, similarly, when we talk about a change of any parameter, suppose, in the fluid field

we talk about the change of velocity with respect to time, it is the time rate of change; but again that has to do here we need to think about that when a fluid is moving from one point to another point, and during that instant of movement what is the change occurring due to which movement one point to another point; again, **another** we need to observe that at that instant of time with respect to time what it is changing, so that way **we get two distinct component** we get two distinct component, and that two distinct component we just covering these two distinct component **we call this as a substantial derivative**, we call this as a substantial derivative.

We can write this in this form that the notation use is say D/Dt , D/Dt is the notation, and this D/Dt what we write that this is the change with respect to that is change partial change with respect to this time plus there will be another component, that is change due to the variation, say, if we are talking about a particular component, then I am writing in literal meaning here I will be coming to mathematical expression later; so, this is the change of that parameter with respect to time, and then change of that change due to which movement **through that section** through that section. So, that way we can write.

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Expanding in a Taylor series about a point y $V = iu + jv + kw$

fluid density = ρ
 $\rho(t, x, y, z)$
 density of point 1 - ρ_1 time t_1
 " " " " 2 - ρ_2 time t_2

$$\rho_2 = \rho_1 + \left(\frac{\partial \rho}{\partial x}\right)_1 (x_2 - x_1) + \left(\frac{\partial \rho}{\partial y}\right)_1 (y_2 - y_1) + \left(\frac{\partial \rho}{\partial z}\right)_1 (z_2 - z_1) + \frac{\partial \rho}{\partial t} (t_2 - t_1) + \text{higher order terms}$$

neglecting the higher order term and dividing by $t_2 - t_1$

Well, now, let me just see how we can express this substantial derivative, how can I express this substantial derivative well; let me just take one example, say, let me take Cartesian coordinate, suppose, we have a Cartesian coordinate, and in three-dimensional space we are talking about, and then we are considering a small fluid element **very small**,

and then this is a position 1, and that is in time say t_1 , and it has a velocity say V ; now, when we write the velocity in Cartesian coordinate we are writing, that means, this velocity will be having in space we are writing. So, this velocity will be having its component in all these different components, say, this component we are writing as suppose this is the x direction, this is the y direction, and this is the z direction; and in vector notation how we can express this velocity, this is basically **we are because** when we say velocity we give magnitude and direction and, so it is a vector of quantity basically, and this vector, this velocity vector **we can** how we can express, say, **you need vector is...** I then you need vector in this direction is j , and you need vector in this direction is say k .

Then we can write depth this velocity vector v is equal to say the value of this velocity in the x direction, and y direction, and z direction, we have some value, suppose this you u this is v , and this is z ; then what we can write that, we can write that $i v + j u$ **sorry** $i u$ plus j into v plus k into w , so, that way these things will give this expression is giving us the velocity of this point; now, **this flow is moving from this to...**, suppose it is at moving like this, and tangentially I am drawing this direction, and it is moving to another point here; here it has it may have another velocity V_2 , say, it is suppose V_1 , this is V_2 , and this is the position 2, this is the position 2, this is position 1, and say this point it is reaching **after a time t_2** after a time t_2 ; and now, within this fluid element, this is the small fluid element, within this small fluid element which can be consider to be a single point only and this is fluid element.

Let me talk about one property, say, density fluid density let me talk about say fluid density; well, the fluid density ρ , and let us first generalize, let us consider that the things are moving and **everything can changed here** everything can changed here well. So, this fluid density ρ **it is basically depending on...**, it is depending on the time t , because at this time it can be something at this time, it can be something, and also **it is depending on its position** it is depending on its position; so, position means, here say its density is something, and when the fluid we are considering the density at this point it may be something.

So, this is depending on say x , that is **the** in confusion code I am talking about, this is in x , what is happening then in y and then z . So, this density is a function of these depending on all these different ranges locate position and time well. So, we can call the

density at this point at the point 1, so density say density at point 1 is, say, ρ_1 and density at point 2, and that when we are talking about point 2 means, well, when we are talking about point 2 means, this is a time t_1 , and this is a time t_2 , this is a time t_1 , and that is a time t_2 well, so this is say ρ_1 and ρ_2 .

Now, from the Taylor series expansion, now if we just talk mathematically the Taylor series expansion a parameter is dependent on all these value, and then the Taylor series expansion how we can write that ρ_2 , ρ_2 , this is ρ , ρ_2 is equal to ρ_1 plus, we can write that the change of say $\frac{\partial \rho}{\partial x}$ partial derivative of this particular variable with respect to x at time at the point one; that means, I am writing one means if time again time 1, but the point 1 I am talking about the same thing, this into change in that direction that is we can say that x_2 minus x_1 , say, this is (Δx) x_2 minus x_1 plus we can write the change of these $\frac{\partial \rho}{\partial y}$ again at the point one, and what is the change in this particular direction that is y_2 minus y_1 , then plus say $\frac{\partial \rho}{\partial z}$, and what is the change in this direction that is it will be z_2 minus z_1 , again there is another part that is $\frac{\partial \rho}{\partial t}$ with respect to time and then we are getting t_2 minus t_1 .

Well, and then again, of course, when we are talking about Taylor series expansion there will be some higher order term, **there will be some higher order term**, higher order terms will be there; and of course, when these say small distances and we are talking about very small distances, and we are expressing a difference, and this higher order term become negligible, and that **we can neglect this term** we can neglect this term, and then neglecting this term what we can write, so neglecting the **higher order term** higher order term **and dividing by...**, because our interest is to find how the things are varying with respect to time, so and dividing by say t_2 minus t_1 dividing by t_2 minus t_1 then what we can give well.

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Handwritten mathematical derivation on a yellow background:

$$\frac{\rho_2 - \rho_1}{t_2 - t_1} = \left(\frac{\partial \rho}{\partial x} \right)_1 \frac{x_2 - x_1}{t_2 - t_1} + \left(\frac{\partial \rho}{\partial y} \right)_1 \frac{y_2 - y_1}{t_2 - t_1} + \left(\frac{\partial \rho}{\partial z} \right)_1 \frac{z_2 - z_1}{t_2 - t_1} + \left(\frac{\partial \rho}{\partial t} \right)_1 \frac{t_2 - t_1}{t_2 - t_1}$$

When $t_2 \rightarrow t_1$

$$\frac{\rho_2 - \rho_1}{t_2 - t_1} = \frac{D\rho}{Dt} = \frac{\partial \rho}{\partial x} u + \frac{\partial \rho}{\partial y} v + \frac{\partial \rho}{\partial z} w + \frac{\partial \rho}{\partial t}$$

$$\left(\frac{D}{Dt} \right) = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

In Cartesian Coordinate vector operator

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$$

$$\mathbf{V} = i u + j v + k w$$

we can write this like that that is rho 2 minus rho 1 divided by t 2 minus t 1 bringing the rho rho 1 is this side, and this is equal to..., here we can write say del rho del x and we have to say x 2 minus x 1, and this point is of course 1 divided by t 2 minus t 1 plus say del rho by del y into this is of course in point one y 2 minus y 1 by t 2 minus t 1 plus say del rho by del z this is one again z 2 minus z 1 divided by t 2 minus t 1 and then plus del rho by del t here again of course we can write t 2 minus t 1 divided by again t 2 minus t 1, so this part is not becoming one.

Well, this we can have; now, when we write this particular expression, this rho 2 minus rho 1 divided by t 2 minus t 1, and when we talk about say t 2 minus t 1 when it tends to 0 or say we can t 1 approaches t 2, that means, we are talking about instantaneous time when this time change is very in insignificant, insignificant means in a sense that very small instantaneous; so, when t 2 approaches t 1, then for this condition when t 2 approaches t 1, then say rho 2 minus rho 1 divided by t 2 minus t 1, this particular expression we write as total derivative t sorry substantial derivative of d D Dt of rho D Dt of rho, that is what basically we call as this one.

And this now on the right hand side the same thing, if x 2 minus x 1 again here divided by say x 2 minus x 1 divided by t 2 minus t 1; if t 2 approaches t 1, that also can be expressed as del rho del x; well, so, on the right hand side we can write say del rho del x and sorry this change in x with respect to time, this is what the we know that..., what

you mean by velocity, that is the change of distance with respect to time. So, here also we are talking about change of distance say x_2 minus x_1 , this is change in system with respect to time.

And we are talking about this change that component of change, suppose, there is the total change, then in the x direction what is the change, and that particular change with respect to time we have already given the name that this is equal to u in our vector, we have send this velocity, this is basically velocity in the direction of x , so we can write it like this plus say $\frac{\partial \rho}{\partial y}$ into this will be v plus $\frac{\partial \rho}{\partial z}$ into say W that we were writing as W and then plus say $\frac{\partial \rho}{\partial t}$ $\frac{\partial \rho}{\partial t}$.

Well, so, this is what the substantial derivative; and when we write this term say for any variable, suppose now we were talking about the change of density and the variable we did consider the as the density, but say we can talk about change of any variable; that way the substantial derivative we can write that expression suppose $\frac{D\rho}{Dt}$ is representing the change basically, and this we can write as say $\frac{\partial \rho}{\partial t}$, I am writing the time part first, and then I am writing this part, this is time part plus $u \frac{\partial \rho}{\partial x}$ plus $v \frac{\partial \rho}{\partial y}$ plus $W \frac{\partial \rho}{\partial z}$.

Well, so, this way we are writing the expression; and then again this can be express in factorial notation also, as we know that one symbol is used normally in confusion code as vector operator; so, in say confusion code unit we use as vector operator, the symbol is used normally to represent this particular vector operator, that is sometimes rather than making in bold some used a dash here, and this is equal to say $i \frac{\partial}{\partial x}$ plus $j \frac{\partial}{\partial y}$ plus $k \frac{\partial}{\partial z}$; and we know that v already we wrote that v is equal to say $i u$ plus $j v$ plus $k w$, so this two vector we can just combine here, and we can write an expression for this substantial derivate as say D/Dt that is substance here derivative is equal to $\frac{\partial}{\partial t}$ plus $\text{this velocity vector not the...}$

Well, so, these two component distinct two component, we are getting this component two distinct component, and this same expression $\text{this same expression}$ we can also get from the very basic of calculus; and let me just explain that part say using chain rule of calculus also, we can write if a particular variable v again we are talking about suppose here I will be talking about the velocity vector, because as we started we came to all these from the point that we need to know $\text{what is acceleration}$ what is acceleration. And

so, when we talk about acceleration basically we are talking about change of velocity with respect to time. So, we will be talking **our parameter** our flow parameter which is important here is the velocity V we are talking about.

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Using Chain Rule of Calculus

$$V(x, y, z, t) \quad V = u + v + w$$

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

Acceleration $\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \left(\frac{dx}{dt}\right) + \frac{\partial V}{\partial y} \left(\frac{dy}{dt}\right) + \frac{\partial V}{\partial z} \left(\frac{dz}{dt}\right)$

$$\frac{dV}{dt} = \left(\frac{\partial V}{\partial t}\right) + \underbrace{\left(u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y} + w \frac{\partial V}{\partial z}\right)}$$

So, let me talk about this velocity V , and we have already written this, this is a function of x, y, z , and t - x, y, z , and t . And so, by using chain rule of calculus say **change in the velocity V** change in the velocity V we can write that as partial change of velocity with respect to time into that how much time has already allows.

Suppose, a small time we are talking about dt , then this is what is the physical meaning of that, that total change in the velocity is equal to partial change of the velocity with respect to time, and then this is the this is giving you the rate, and then multiplied by the actually how much time change has occurred plus partial change of the velocity with respect to x , and how much actual change in the x has occurred; then partial change in the velocity V with respect to y , and how much change in the y ; similarly, we can write about z also that $\frac{\partial V}{\partial z} dz$.

Well, this is very well know rule; and then say acceleration $\frac{dV}{dt}$ if we divide it write $\frac{dV}{dt}$, then what we can have, that will be $\frac{\partial V}{\partial t}$ plus $\frac{\partial V}{\partial x} \frac{dx}{dt}$ plus $\frac{\partial V}{\partial y} \frac{dy}{dt}$ plus $\frac{\partial V}{\partial z} \frac{dz}{dt}$; now, what we mean by $\frac{dx}{dt}$, well, this is very common that $\frac{dx}{dt}$ means the velocity, basically **the change of velocity with respect to** change of distance with respect to time, that is what the velocity $\frac{dx}{dt}$

$\frac{d}{dt}$; and this velocity we are talking about is the velocity component in the direction of x , and we already wrote that the velocity vector $\mathbf{v}$ is nothing but that i into u plus j into v plus k into w , means, it has three components u , v , and w , and in a **say** confusion code unit in the direction unit vector i in the x direction, j in the y direction, k in the z direction.

Well, so, what we can write this **$\frac{d}{dx}$** $\frac{d}{dx} \frac{d}{dt}$ or $\frac{d}{dy} \frac{d}{dt}$ $\frac{d}{dz} \frac{d}{dt}$ that can be written as u $\frac{d}{dx}$ and w ; well, so, we can write this is equal to $\frac{d}{dt} \frac{d}{dx}$ is equal to $\frac{d}{dt} \frac{d}{dx}$ plus for this I am writing as $u \frac{d}{dx}$ plus for this I am writing as $v \frac{d}{dy}$ plus $w \frac{d}{dz}$; well, so, this acceleration term we have two distinct component - one is say I can break it from here to here, and then another term is this one.

Now, all these term when we talk about the change of velocity with respect to time at any instant of time, then all these component will be existing subject to some initial understanding or subject to some condition, that means, what sort of condition basically we are talking about that if the flow is steady; now again we need to call back what was steady flow, what was unsteady flow that we did discussed earlier; steady means, when the flow parameters are not changing with time that we call as steady; and when the flow parameters are changing with time then it is unsteady.

Now, when the flow parameters are not changing with time, suppose, it is a steady case then what will be the rate of change of any flow parameter with respect to time, it will be 0, because it is not changing at all, so, $\frac{d}{dt}$ term will become 0 in that case; but if it is unsteady case, when they are exist a change of flow parameter with respect to time then only this term that is the first term $\frac{d}{dt}$ will be there; and then when there is change of parameter with respect to distance, then let me come to uniform and non-uniform, this part, this is the second part, second part is basically representing the change of that flow parameter with respect to space x , y , z , means, we are talking about change of this parameter with respect to space.

And when some flow parameter changes with respect to space, then that type of flow if you remember if you recall our earlier discussion then that type of flow we call as non-uniform flow; if the flow parameter whether it is velocity or depth, when the flow parameter do not change with respect to space, then we call that as a uniform flow.

Well now, that means, all these component or all these value of the second term will be existing in case of non-uniform flow. Now, we need to know that our say we need to

appreciate this point that why discussing the uniform flow we have started with all this deduction and we have come to a situation that, in case of uniform flow the second term will also not be there, because there is no change of flow parameter with respect to space, and if it steady case there is no change of flow parameter with respect to time, but we need to understand what we mean by that, that is not there means, how it is not there that we need to know; again say we did discuss in the classification of flow we did discussed about one-dimensional two-dimensional and three-dimensional flow.

And here as we are writing this velocity vector as say in the component of u, v, and W **u v and w**; that means, we are considering three distinct directions, it has component in three distinct directions, so we are writing this like that; and that means we are writing this for three-dimensional condition, but for many flow situation in nature we consider this as one-dimensional, and then when make one-dimensional some of the components will not be there.

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One dimensional, velocity in the x direction = V

$$\frac{dV}{dt} = \left(\frac{\partial V}{\partial t} \right) + \left(V \frac{\partial V}{\partial x} \right)$$

↓ ↓ ↓

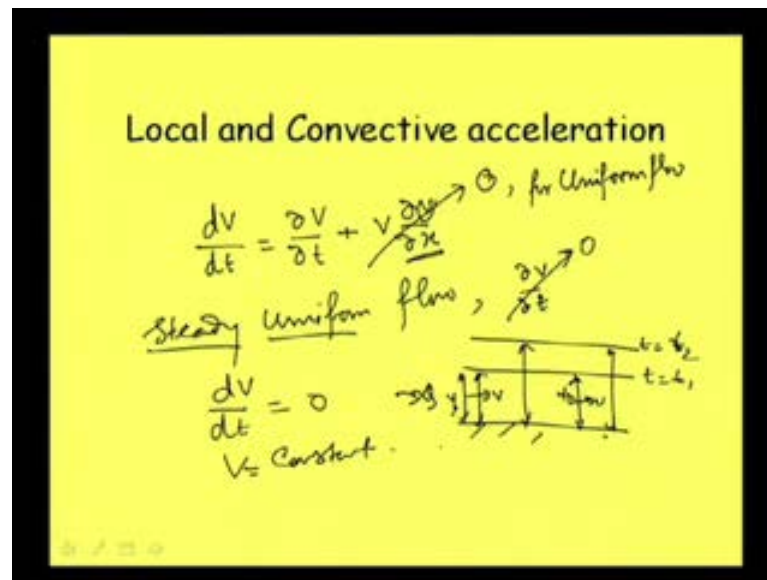
Total Accⁿ = Local Accⁿ + Convective accⁿ

Now, say let me write this expression dV/dt , and say our case is one-dimensional **one-dimensional one-dimensional**, that means, the velocity only we are talking that suppose x is the only direction and **velocity we are talking that** velocity in the x direction is equal to say V; well, **so**, now, what we can write that dV/dt is nothing but equal to $\partial V/\partial t$ plus $V \partial V/\partial x$, this is in one-dimensional case.

And this particular component that is **in** when we talk about **total acceleration** total

acceleration, then we have basically two component, this component and that component; and this component we call as local acceleration; and this component we call as convective acceleration **convective acceleration** - local acceleration and convective acceleration.

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Well, so, out of these two distinct component local acceleration and convective acceleration, for say uniform flow case $\frac{dv}{dt}$ if it is unsteady then only this $\frac{\partial v}{\partial t}$ component will be there, and this $v \frac{\partial v}{\partial x}$ this **convective acceleration part will be 0** convective acceleration part will be 0 for uniform flow; the reason is that, in fact, when we are talking about uniform flow, there is no change in the direction, there is no change in the velocity with respect to x , so, this component will become 0 for uniform flow.

And then if we say that it is a steady uniform flow, **it is a steady uniform flow, it is a steady uniform flow**, then what will happen that this $\frac{\partial v}{\partial t}$ component as we have already discussed **this will also become 0**, this will also become 0. So, what we are getting that total acceleration $\frac{dv}{dt}$ is equal to 0 $\frac{dv}{dt}$ is equal to 0; well, then when $\frac{dv}{dt}$ is equal to 0 we are meaning this as steady uniform flow, of course, if **we mean the $\frac{\partial v}{\partial x}$ equal to 0** when we say that only $\frac{\partial v}{\partial x}$ equal to 0, that mean, that also mean that it is uniform flow, but it may not be steady, but when we talk about steady uniform flow then $\frac{dv}{dt}$ equal to 0.

Well, but in our earlier discussion also we did mention that in reality, it is very difficult

to get a unsteady uniform flow say in any moment of time, suppose our flow is here, t equal to t_1 , then next moment of time, if t equal t_2 we are having again a uniform flow, so uniform flow means everywhere it is same, and everywhere it is same like that.

So, this sort of changes is not possible; so, that way when we say dV/dt is equal to 0, it means that **V is equal to constant** V is equal to constant; and then when we say total V is constant, total variation of velocity with respect to time is constant means we are also meaning that dV/dx is also equal to 0; and V is equal to constant means if is same discharge is coming, here also V, here also V, then our depth is also constant, say, this is y , this is also equal to y , and that is what we call as uniform flow where our velocity is constant all through, and it is not changing, and when our depth is also not changing, well, so y and y this depth of uniform flow is call normal depth.

Now, with discussion on uniform flow we could understand that by uniform flow what we really mean; and then now as we did discussed that, it is basically we need to balance the component of gravity force and the opposing force that resistance force offered; now resistance for offered to calculate it analytically it is very difficult, so mangier time we use empirical relation for expressing the uniform flow condition; these are the fundamental understanding, but we need to take recast to the observation, we need to take some of the value empirically, and that will be discussing in our next class that how we express these relation of uniform flow balancing the component of gravity force and the resistance force.

So, thank you very much.