Hydraulics

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Module No. # 01

Introduction to Open Channel Flow

Lecture No. # 06

Conservation Principle and IOIO Governing Equations

Friends, today we shall be discussing some of the fundamental principles of physics that we use for deriving the relationship useful for hydraulic engineering. In fact, these fundamental principles are same, almost same, but of course, some specialties are there definitely, and then, same means I am meaning that it is suppose when it is not a fluid in some other solid also we can apply this sort of principle, but there will be some specialty definitely. And so, today's discussion is basically on conservation principles and governing equation. Basically, these conservation principles lead to some governing equations of the flow.

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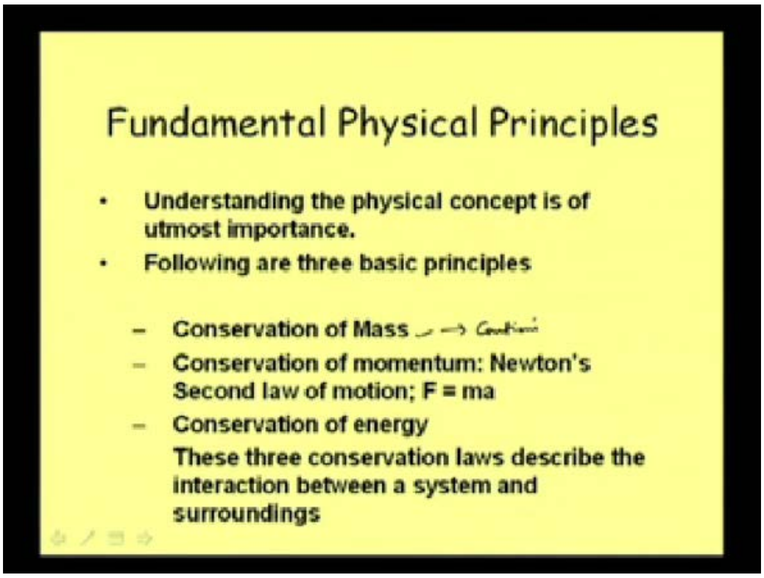

Well, what are the fundamental physical principles? Well, some governing equation will definitely be coming, but before that, understanding these physical principles are very

very important. And then, after understanding these physical principles, when we try to derive some mathematical relation from these physical principles, then, what are the assumptions we are making. This is also very very, these are also very very important and that way we will try to highlight some of those issue also in this particular lecture well. And, as we all know that what are the fundamental principle of physics, that is, the conservation of mass, you can concentrate to the slide, that is conservation of mass and we know that in fluid, particularly in say water, when we talk about water, then this we can lead, this lead us to continuity equation, continuity equation.

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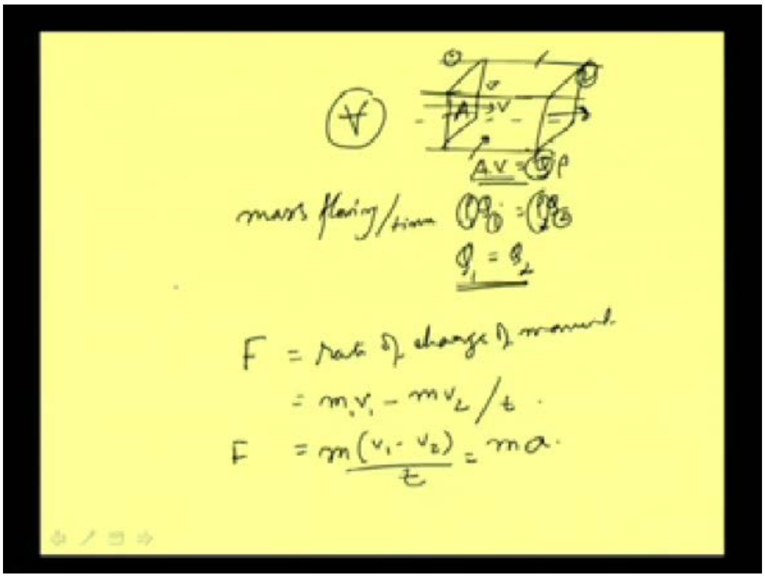

How? Say, conservation of mass means, mass is conserved, that is, say mass means if a volume of flow is say v and then it is that we can find that in a flowing fluid. Suppose it is flowing with a area A, it is flowing with a area A. Say this cross sectional area is a; let me draw a three dimensional view. Suppose, this cross sectional area is A, and then, it is flowing with a velocity v and then A into v, we can say A into v is equal to Q, area into velocity is equal to Q. That is all right, and then this Q multiplied by rho, Q multiplied by rho, rho, Q gives us basically the mass flowing say mass flowing per unit time, per unit time through this section, and then, when we say that conservation of mass, we talk that this mass flowing through this section and at a downstream, say mass flowing through this section will be same. What is, what mass we are having here, what mass we are getting here will be seen unless there is a lateral flow from this side or lateral flow from that side.

Well, that is what the conservation principle. So, rho Q is rho Q at section one. Suppose this is section two, then we can call in simple that is rho Q at section one is equal to row Q at section two. Now, in fluid, suppose if it is a compressible fluid, then it will be rho 1 and rho 2 will be different; rho 1 and rho 2 can be different, but in water, this rho 1 and rho 2 will be same. So, what we can have, that is, the Q is equal to ultimately Q 1 is equal to Q 2 and which we call as a continuity condition.

So, this lead to continuity equation. Of course, this is just in very simple term I am trying to explain, and then, when we talk about say conservation of momentum, so you know that mass into velocity gives us the momentum. Well, by conservation of momentum, what we mean that, suppose that momentum in a upstream section we are calculating, momentum in a upstream section, and then momentum at a downstream section, then whether these two momentum will be equal or by conservation of momentum what we mean basically?

As we know that from the Newton's law, that is, the change of momentum, rate of change of momentum is equal to that applied force. So, momentum at upstream and momentum at downstream, these two section upstream and downstream momentum between these two section may be different, but the change of this momentum per unit time, rate of change of momentum per unit time will be equal to the external force. So, that is what we mean by conservation of momentum.

Similarly, when we talk about, say that is why this formula that we use for F is equal to mass into acceleration. This is a well known relation. Basically, what we are meaning by this formula, that is, the force, as we are saying force is equal to rate of change of momentum, say rate of change of momentum, change of momentum. So, change of momentum per unit time, say in unit time if we have some mass into velocity here and v 1 at a particular section and then we have say mass into velocity at another section.

Then if we talk about per unit time, then we can divide it by time, and then, say when mass is constant for a particular fluid, we are talking about say for this thing, then what we can write this is equal to v 1 minus v 2 by t and this is nothing but change in velocity per unit time. So, you can write this as mass into acceleration. So, that way we can ultimately come to this relation, that is, the force is equal to ma. That is well known Newton's second law of motion.

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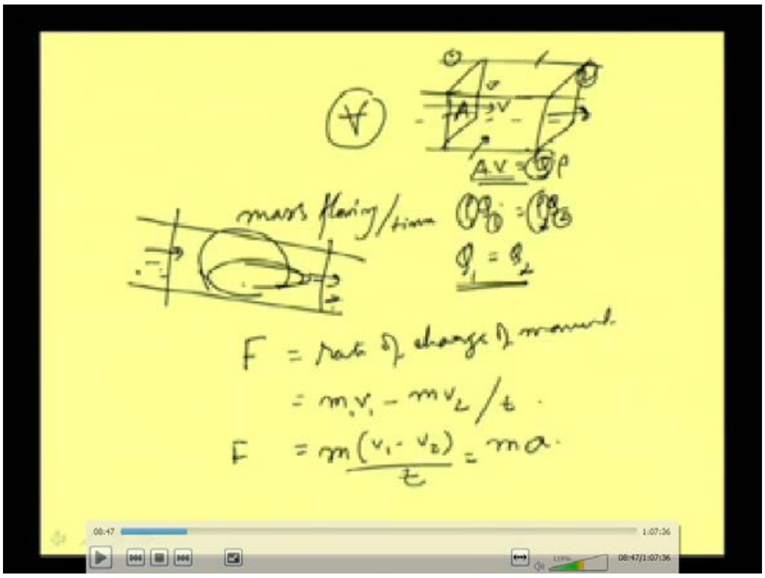

Then, the third basic principle is conservation of energy well. Now, conservation of energy again here also suppose if you consider energy at upstream section and energy at a downstream section, well where the energy will be more definitely. When a flow is moving from upstream to downstream, when a flow is moving from upstream to downstream, then energy level at upstream must be moved in downstream. Then only, that is why the flow is moving from this in this direction.

Now, so, by energy conservation, what we mean that whatever energy we are having here, whatever energy we are having here it is not that this should be identical, but some energy can be lost here. So, energy at this point plus loss of energy during this flow if we aid, then we get, suppose if we deduct the loss of energy from this one, then we are getting the energy at this point then. So, energy is getting conserved, and it can be transferred to some other form or it can be use. Suppose somewhere lot of turbulence is getting created, some energy is getting lost here.

So, energy at upstream and the energy at downstream will be definitely different, but some energy is being used in the creation of this turbulence and all. So, that way we have basically three principles, and from these three principles, we can have three equation and these three conservation laws, as you can refer to the slide that these three conservation laws describe the interaction between a system and the surroundings.

So, when we talk about a fluid, when we talk about a flow, then we for deriving the equation we will define a system, we will define a system. By a system what we mean that we will define a boundary and within that boundary interaction or we will observe the things within that boundary, but always the things what is going on within the boundary will be having some relationship with the things existing outside the boundary and that is we call that environment system environment.

So, there will be some interaction between system environment and the system and these three principles we use for explaining that and then we get some equation. Out of that, we derive some equation. Well, now, with these equations, we can do lot. Once we get the equation, then mathematically we getting, we are getting the equation. Then we can just solve some of the problem associated with the flow, well, but one interesting point is that many author uses these equations; that means, say three equation we are having many author uses these equation and you will be finding that whether we find that, sometimes the equation, suppose continuity equation is looking different and then another author is using his continuity equation is looking different; momentum equation is looking different.

So, when we talk about different, we talk from different point of view, then we, we, derive the equation from different point of view or equation take different shapes, and then many a time we get confused that which is actually correct form or which form we should use for what purpose, and in those aspect, we should know that when we are getting a different form means we are using a different model of the flow, we are using a different model of the flow and that way we are getting different forms of the equation, and then, we are making different assumption in deriving that equations. That is why we are getting different form of the equation.

Basically the physical principles are same - conservation of mass, conservation of momentum and conservation of equation energy, but equations form may be different because of considering different assumption there. Well, an another point I want to just say here that say in flow, when in open channel flow, when we do study, then for any section, we are interested to know what is the flow velocity, what is the depth of flow, and then, say what is the discharge.

Now, already we have seen that if we know the depth of flow and if we know the sectional area; that means, sectional shape if we know the sectional shape, then knowing the depth we will be able to know what is the sectional area is, and if we know the velocity, then say the product of this velocity and the sectional area will be getting the discharge. So, our interest is generally to know the flow velocity, the depth of flow and the discharge, but out of these things basically if we can get two, then we can get the third one. If we suppose know Q, then if we know the velocity, we can calculate the y, that is, the depth, or if you know the depth, if you know the velocity, we can calculate Q. So, like that.

So, two unknown generally remain in our analysis, and then from very basics of our mathematics that, if we have two unknown, then to solve for these two unknown, we required two equation. Well, then fortunately we have three equation - one is that conservation of mass. From that principle, we get continuity equation. Then, again from conservation of momentum, we get one equation. That is called as momentum equation. Then again from conservation of energy, we get one equation. That is called energy equation.

And basically these momentum and energy equation that we can call as a equation of motion also. Now these two equations are more or less same but with some differences. That will be coming later and say out of these three equations for solving our problem where we have two unknown variable, only we have two unknown variable. For these two unknown variable for solving that problem, we need two equations. So, what we can do? We can choose two equations say we are choosing mass, equation of mass conservation of mass and the equation of conservation of momentum. So, continuity equation and say momentum equation.

Again sometimes we can select, we will, we can take continuity equation and energy equation. Now, which two we will be taking and that two in fact we call as a say continuity momentum couple or continuity energy couple like that also we say. Now, which two will be selecting that depends upon the, upon the, problem. Say for example, we have a situation where flow is moving like this, where let me draw here. Suppose we have a situation here; flow is somehow coming like this and then this is moving like this lot of turbulence in this portion and then it is moving like this.

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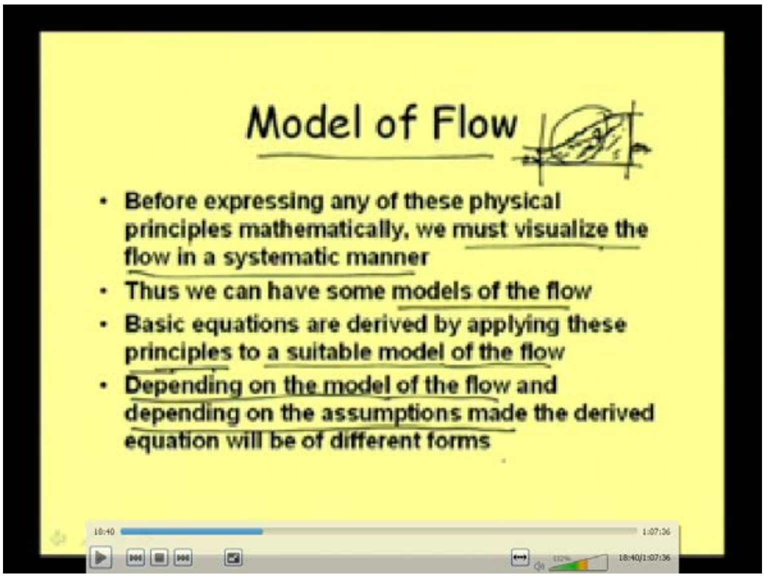

So, now, we want to relate what is going on here and what is going on here and then we want to derive some of the relationship between these two values to this section. Now, if you go for energy equation here, what will happen? Basically, in this portion because of turbulence, lot of energy loss is there and we may not be in a position to have full understanding of how much energy is getting loss.

So, if we try to equate the energy at here, energy at here, this will be wrong; we will have to do energy at here minus energy at here; energy loss is equal to energy here. So, then energy equation application may be difficult, but if we talk about the momentum, then if we consider this as the fluid to tell our, consider flow field under consideration, then we see what are the forces acting here, what are the forces acting here. Then we can see what are the external forces acting here. Then we know the situation here we know what is the momentum here; then we know what is the momentum here.

So, in that case, we can relate the change of momentum or rate of change of momentum to the external force without bringing in the complexity of energy loss in between. In that case probably momentum equation will be better. So, like that considering or looking into the situation we need to select our equation. Well, and now, let me explain some of the point regarding the model flow. Why we are coming to this point? At very initial stage, we may not require these things, but I feel that if we discuss these point, then we will be knowing why the different equation or why the same equation, same equation means equation derived from the same principle takes different shape and which shape we are using that should be clear.

So, say before expressing, you can concentrate on the slide that, before expressing, any of these principle mathematically we must visualize the flow in a systematic manner, we must visualize the flow in a systematic manner, and that is what actually we how we visualize, that is what we call as a model of flow. Thus we can have some model of flow, and basic equations are derived by applying these principles. These principles means mass conservation, momentum conservation and energy conservation to a suitable model of flow, to a suitable model of flow, and as I have already said the depending on the model, what model we are choosing, a model of flow, and depending on the assumption that we are making, the derived equation will be of different form, derived equation will be of different form.

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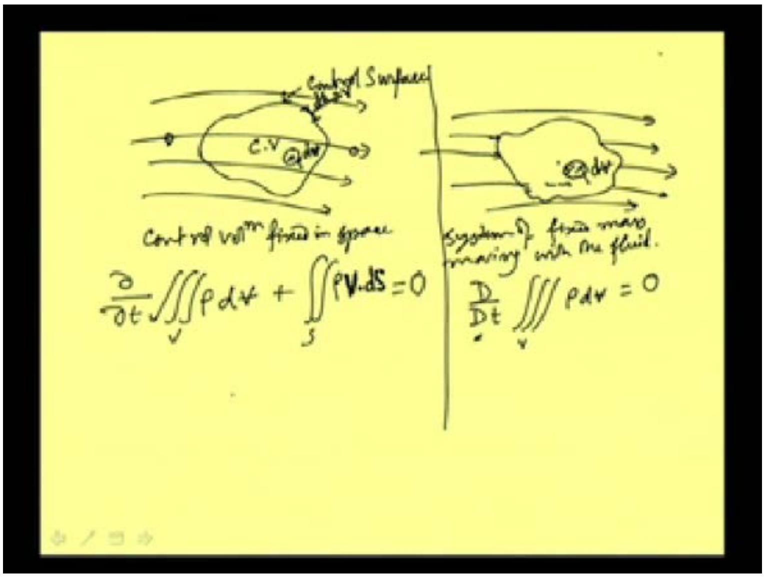

Now, say model of flow one we can, one very popular term here in hydraulics we use is control volume. Say this is one control volume. You can just concentrate into the slide this we use. We are using the, this is a, this is a control volume; we are naming this is a control volume and this surface of that volume we are saying as control surface, control surface.

Well, now, here, if we just consider a small area, if you consider a small area, then there will be, the, suppose this volume is small volume. So, let me write it as, say you can write it as dv small volume, and then, what the mass is we can write rho into d v this one. Anyway, that we will be coming later again, but first let me talk about the flow model. Say in one of the model, what are we interested that we see what is happening inside this control volume, what is happening inside this control volume, and the flow is say moving like that, flow is moving like that. Flow is passing this way.

And what we mean that fluid mass, suppose a fluid element is here it is flowing through this and it is reaching this point. Our interest is not that what is happening to this fluid mass. Say it is carrying some a, something and it is going, and what is happening to this particular fluid mass? We are not interested. Our interest is to see what is going on within this boundary, what is going on within this boundary. So, this sort of system we call that control volume, control volume, fixed in space, control volume fixed in space.

Again in another model, what we can consider that, say the, this is the volume and it is not standing. In this space, it not fixed; it is moving, it is moving. Say it is moving with the flow it is moving. So, at this moment, it is there, and next moment, it can move, and what is that mass within that control volume? This also sometimes referred as control volume and or we can call this as a mass and these things itself we can call as a system and this system is say moving, this system is moving and. So, we can call that a system of, system of, fixed mass system of fixed mass moving with the fluid moving with the fluid Well.

So, this is another way of just making the model of the flow before analyzing it now. So, well, these two system when we consider, then our derive equation will be of different type. Suppose we say that this mass is moving, then it, it, can be compressed, it can be expand also like that when it is getting compressed or expand; that means, this volume can keep on changing, but this mass is not changing, its mass is not changing.

So, if we consider a small volume, small elementary volume and then suppose this small elementary volume is a say dv, then the mass of this small elementary volume v that I am using, and then, it will be, say mass will be we can write rho into dv, and then, say we can write D D t that is the change of this mass. How much will be the total mass? First, we need to say how much that will be the total mass. If we integrate it over the volume, if we integrate it over the volume, that is the volume integral. This will be say rho dv and then the change of this mass with respect to time is equal to 0. That way also we can have our equation, that way also we can have our equation.

This we are writing as say D D t, capital D D t because we are talking about a substantial derivative here as the fluid mass is moving. So, there can be change due to its movement as well as there can be change due to time also. So, that way we are using this term. That we will be discussed in more detail of course later, but here, to give you introduction, I am writing this expression. But now, when we talk about this system, when we talk about this system, here also we can have a small volume say this is dv and then we can d of volume and then we can have that rho is the density.

So, this is the mass. Now, here, what will happen that total change when we talk about, suppose total change of this control volume, then we can say that the change in the, I mean mass of this control volume if we say, then it will be that the, what change here it is occurring. If we talk about decrease or increase, suppose the change in decrease in this entire control volume is equal to what is going out.

If I consider a, if I consider a, surface say small surface ds and say using velocity in vector form is. Suppose I am writing v because now we need to consider direction also. From here, if I consider, it is going out; from here if I consider in the opposite direction, then it is coming in. So, if I just talk about the entire surface when we integrate it, it will mean what is coming in and what is going out everything. So, what is coming in, what is going out and that will indicate what the change is. So, that we can write it as suppose what is the small volume that we can write rho into dv; this is indicating rho into d v. This is indicating the small mass, and that if we integrate over the entire volume, entire control volume, then we are getting the, this volume and change of this mass we can have say del del t and this plus.

Suppose over the surface something is going out. So, what is going out through this small surface, that we can have say in unit time, how much is going we are, because we are talking about per unit time. So, what is going on out in unit time is the, the, surface area into the velocity will be the volume of flow that is going in unit time, and if I multiply it by rho, so v rho into v means vector velocity and then it is also in vector notation ds we are getting.

So, this much is the things that is going not from this part, and if I integrate it over the surface, if I integrate it over the surface, then I am getting the total mass that is flowing out from this, and that way this total summation if we say that it is not changing, it is not changing, that this total summation should be equal to 0.

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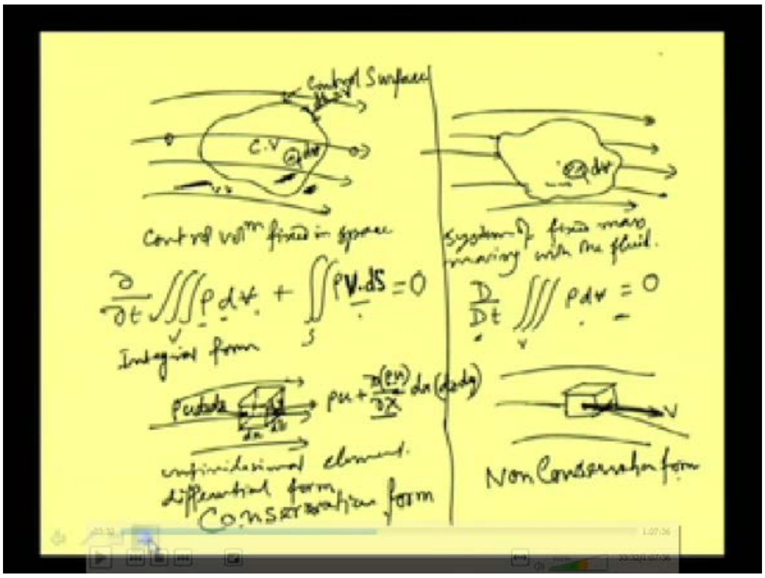

So, we are getting two different form, we are getting two different form, well. Then rather than considering these entire volume, we can again sometime consider as a infinitesimal small volume like that; we can consider small volume like that of definite shape and here also we can consider that flow is moving through it in this case and say we are considering another small shape here and then we are saying that it is moving with the flow, it is moving with the flow.

So, it will be having a velocity in this direction. At the next moment, it can move in this direction also, but at this point, it has a velocity in this direction and then flow is moving. So, here this element is moving, but the concept is same. But here, we are considering say infinitesimal small element, infinitesimal small element we are considering. Now, when we are considering a definite shape, then we can write say this as dx. Then, this we can write as say dy this height and this we can write as dz.

Now, again if we derive our relation based on these understanding, then it will be taking a different shape, it will be talking a different shape. For example, just this small point I want to note here. Suppose the flow velocity let me just draw it like that. The flow velocity at this point is suppose u the flow velocity at this point is u. Then the flow velocity u and rho, the, u, and then how much total volume is moving through this part, how much total volume is moving through this part that we can write as u and the sectional area. Sectional area will be dz dy, d z into d y, and then multiplied by rho, we are getting what the flow is here, and then, on the other side, on the other side, what we can do that here the flow velocity is u, then or say when the flow velocity is u per unit area, the mass flowing is rho u.

So, this rho u is definitely here. Then plus if there is a rate of change of mass flow, if there is a rate of change of mass flow in this direction with respect to x; that means, if we write say del rho u del x, suppose there is a rate of change of mass flow in this direction, then after a distance dx, after a distance dx, it will be having a total change of this mass. So, rate of change of mass flow at this phase can be written as this one, and again then if this is the rate of change of mass flow per unit area, then total change will be this multiplied by again we can put dz into dy this cross sectional area, so, dz into dy, well.

Now, that is, just I am deleting this part and that perform this side again. We can do something and some other equation. I am not going for deriving the entire equation, but what I am meaning? Here, we are using a term del rho u del x that is the mass flow, rate of mass flow we are considering that it is differentiable, it is differentiable. Now, when we can consider this as differentiable, when this change is continuous, say the velocity vector is continuous, value of the velocity is continuous and these are continuous then only we can use this differential term; otherwise, it will be, it will not be possible.

So, the, the, flow the velocity or the depth these things will have to be considered as continuous in this sort of analysis, but when we are talking about this control volume and we are talking about this integral form, here we are getting it in differential form. Of course, I am not writing the final equation, but this is the differentiation term we are using. Here we are not using those term in this part and that is why here that sort of assumption is not required means in this part, well.

So, this equation we will be calling as integral form, integral form, and this we can call as say differential form, differential form. Of course, I am not writing the, I am repeating, I am not writing the entire equation here, and then, here also it is integral form; here also it is differential form. But when we go by this concept, when our flow model is this one, then we call this one means the first one that I am considering. You can concentrate on slide. When my flow model is this one, then we call this as the equation that we get from this starting from this point. Then we call this as this equation to be in conservation form. We call this in conservation form, conservation form, and this we call if we go by this concept, then the what the ultimate equation we get, that we call as in non conservation form, non conservation form. Of course, it does not mean that it is not related to conservation of mass or conservation of momentum, but the name we give as non conservation. Form here, it is conservation form.

So, that way you could see that, when our flow model is different, then our equation ultimately that we get is different. Again flow model means with this, again we get different, but one point is that, we can always after deriving a equation, we can go from one form to other form; from integral form to the differential form we can go, and from again from non conservation form to the conservation form. We can go from conservation form to non conservation form you can go, and this way our energy equation or momentum equation or the continuity equation can take different shape.

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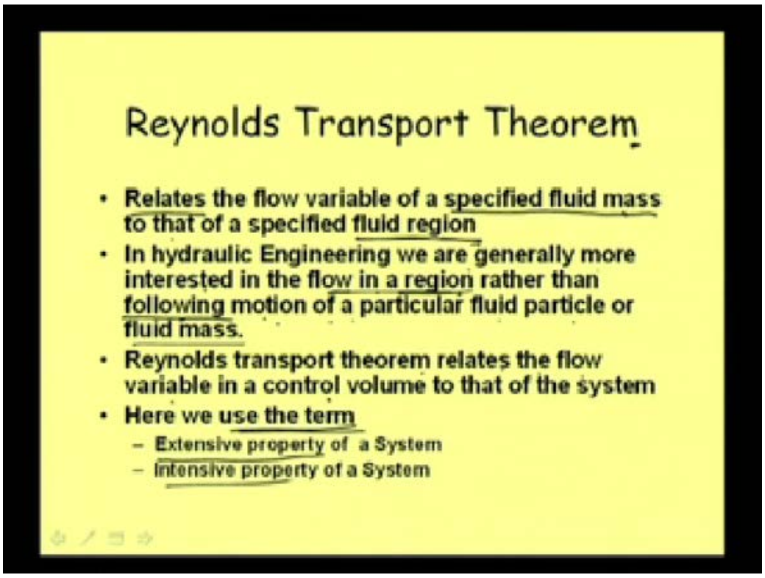

Well, we are not going into much detail of this part, but still I am just considering, I am just trying to explain some of the very fundamental. Well, then you can concentrate again on the slide that Reynold's transform theorem, Reynold's transform theorem, this is also one of the important theorem that we need to know before deriving those equations. Well what it means that Reynold transport theorem relates the flow variable of a specified fluid mass; that means, suppose we know the flow variable, flow variable, of a specified fluid mass, then we can relate this property, that is, flow variable of a specified fluid mass to that of a specified fluid region.

By fluid region, we mean say a control volume, and then, say if we know the property of the fluid mass, then we can relate to the property of the entire fluid region. This is for the Reynold’s transport state, and of course in hydraulic engineering, we are generally more interested in the flow, in a region, in the flow in a region. Say normally when we say when we tried to understand a flow, we see that what is the happening to a particular region.

Suppose if we need to talk about a channel, we will be interested to know that in a channel reach what is happening. How the flows are changing or what is happening. Of course, how a individual fluid particle is moving or say fluid mass, a small fluid mass is moving from one point to other end. What is happening to that particular fluid mass that can be of course, of our interest, but in hydraulic engineering, generally we are more interested in this part, that is, in flow in a region rather than flowing motion of a, rather than flowing motion of a particular fluid particle or fluid mass.

Well and Reynold’s transport theorem relates the flow variable in a control volume to that of the system. So, that is what is Reynold transport theorem, and for the, for just explaining that part, we use two term - we use the term extensive property of a system and intensive property. Now, what is extensive property and what is intensive property that we need to discuss before writing some of the equation. Well, what is extensive property and intensive property?

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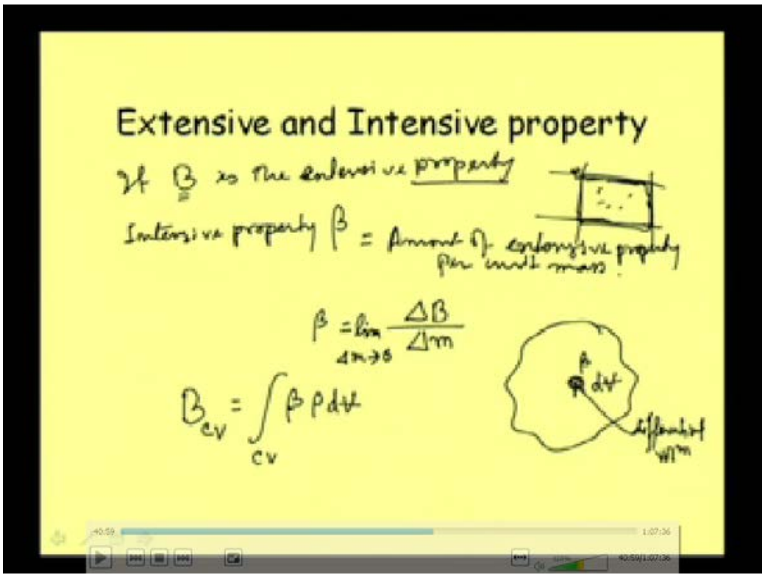

Well, suppose B, let me write suppose B, if say B is the extensive property, extensive property. Well by property what we are meaning, this can be suppose we are considering a system, we are considering a system, it may be like that. Say a channel, let me talk about a channel and then say this is the control volume that we are talking about and we are considering this system. Suppose the fluid what is flowing here and we are concerned about this particular things.

Then this B may be, capital B by capital B we are meaning extensive property and this extensive property or by say simply by property what we meaning, say mass can be one property; then say momentum can be one property. So, that way different things, by property we are meaning different things. If B is the extensive property, then say if we use the symbol beta for intensive property, then say intensive property, intensive property if we use the simple beta, that intensive property means that amount of extensive property.

So, when we talk about mass, suppose amount of mass, it can be amount of mass also; it can be amount of momentum also. So, it can be amount of any property. So, amount of extensive property per unit mass. That is what the intensive property we call. So, intensive property means amount of extensive property, extensive property, per unit mass, amount of extensive property per unit mass, per unit mass, and generally, we talk about a very small mass, and then we see that we try to see that, if this mass tends to 0, then what is happening to that? So, beta is basically beta we can write as say what is the small extensive property that has in small amount of mass and we write that limit delta m tends to 0. So, what is small element? We talk about this property; we call as a intensive property.

Well, now, let me again draw this diagram, let me again draw this diagram. Say this is a control volume and we have a small a volume dv v cut I am writing, and then say intensive property of this is suppose beta; intensive property of this is small beta, and this small volume will generally refer as we call as differential volume, we call as differential volume, differential volume, well. Now, so this differential volume has some intensive property beta.

Now, knowing this intensive property beta, we can find out what will be the extensive property of the entire control volume, but I mean corresponding extensive property. Suppose if we talk about mass here, then what is the mass of the entire control volume. That we can find out say b of the control volume is equal to we can integrate it over the control volume. I am writing the volume, and then, it is say beta.

Beta is the intensive property; that means, it is the, that particular property per unit mass. So, what will be the mass here? If the volume is dv, we need to multiply it by rho. So, that we are getting mass. So, that means in this mass, this is the intensive property. So, if we integrate it over the control volume, we are getting the total extensive property of the entire system.

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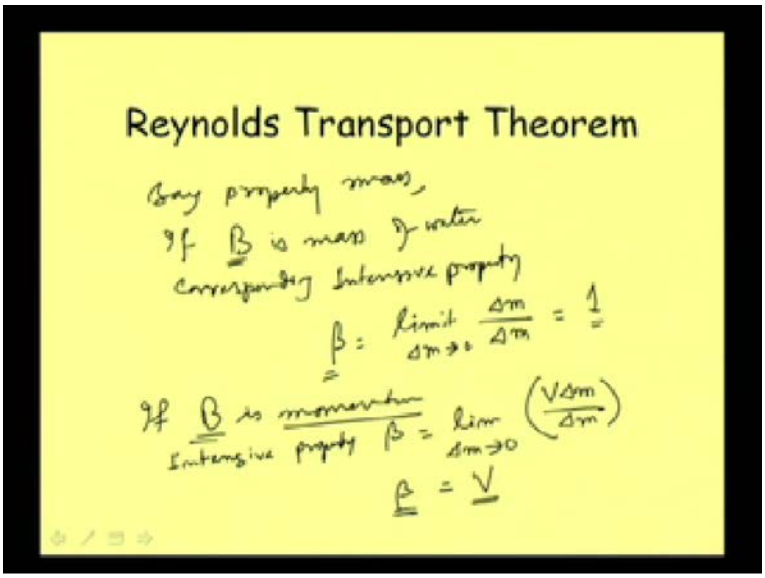

So, this is how we relate, and for example, let me give an example. Say for example, if we talk about, say our let me take our property is say the mass, property is mass, property is mass. We can talk about momentum also. This can be mass momentum anything, and then if say B is mass of water, if B is mass of water, then corresponding intensive property, corresponding intensive property, will be; that means, corresponding intensive property means we are talking about again mass only that we can write that limit delta m tends to 0.

Now, we are talking about mass. So, our property itself is say mass. Small property, a small mass in in that small area and that, that, way it will be this mass and then per unit mass. So, this is equal to 1. So, what we can write that, if our mass is the property, then the beta is, if capital B extensive property is mass, then intensive property become one, and then, if our suppose extensive property, if capital B, that is the extensive property is momentum suppose.

If extensive property is momentum, then our intensive property will be what? Then our intensive property will be intensive property beta. We can write as intensive property beta we can write as say limit again delta m tends to 0. You can write say momentum of this small mass. Mass we can write as d v into delta m. This is the velocity into this and per unit mass, per unit mass.

So, when we write like that, ultimately this become when delta m tends to 0 automatic it; gets cancelled and beta is equal to v; that means this is the velocity. So, when our extensive property is momentum, corresponding extensive property is momentum, then corresponding intensive property is the velocity. So, that way we can have this sort of relation between extensive property and intensive property, and then, already we have explain what this Reynold transport theorem state that it relates the flow variable of a specific fluid mass to that of the specific, to that of a specified fluid region.

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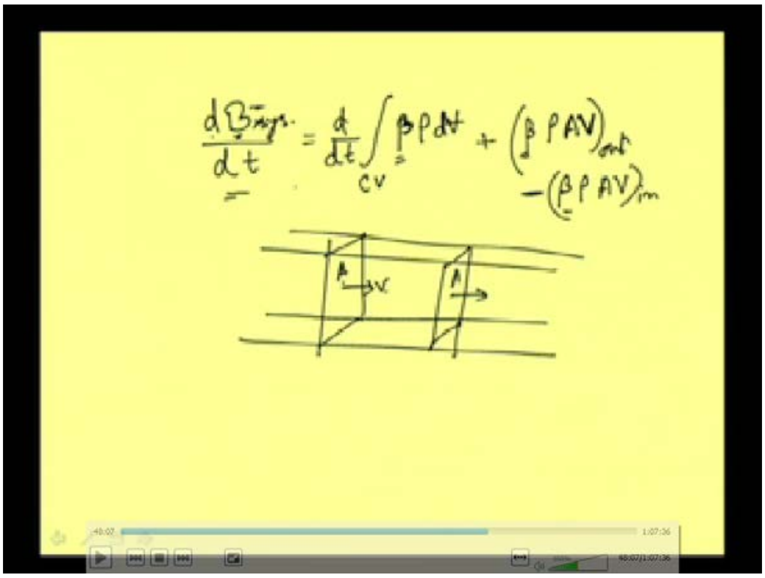

So, from that if we come here, from that if we come here, then we can of course write B. Suppose let for a particular system we have extensive property of the B system is say B and then change of that extensive property with respect to time if we write. Now, let me see that what corresponding, suppose this B is the extensive property. Corresponding intensive property is say beta, corresponding intensive property is beta, and if we talk about a small elementary volume dv, then per unit mass it will be rho dv.

So, per unit mass is B that property, and so, B multiplied by the small mass of that small differential volume dv. Then we are getting the intensive property and that we can integrate over the control volume, and then, when we are talking about d,t that is a total differentiation of the extensive property, then we can call the differentiation of the intensive property. Then plus, plus, what is that, what is coming in to the system and what is going out from the system.

If we talk say decrease, then we can say that plus, what is it if it is say decreasing that thing, suppose we are talking about momentum or mass then if it is decreasing, then we can say what is going out and what is coming, what is coming in that also we can write. So, this will be we can write say beta, and then, if the sectional area is a, if the sectional area is a, let us consider, we are just writing this for one-dimensional system.

If the is sectional area a, say this sectional area is a, if this sectional area is a, and then, this is suppose B this sectional area is B, then, sorry, it is a 1 and this is suppose a 2 or let me write it as a itself in the control volume. Then, what we can have this is equal to beta into if the velocity here is V, then A into V is the volume coming in and multiplied by rho; it will be the mass coming in. Then intensive property of that mass if we say that, this is going out, and then, again we can write. This out itself indicating whether it is in the one section or in two section, that is, say out. Then can have using the sign itself negative here. We can write beta rho AV in. If we are talking about decrease, then what is coming in, we will have to deduct, and then, what is going out, we are just heading up and that way we are finding the changes.

So, this way we can find out the a relationship between the extensive property and the intensive property and this sort of relation we can use for any of the intensive property, any of the property of fluid and we can go for deriving some equation, and from these fundamental relation, we start and we go for deriving what is momentum equation or what is energy equation that you can do.

Well, these are some of the very fundamentals of the fluid. Some of the very fundamentals of the theories that are used for deriving some of our relation. Of course, just to start, just to start, without going for this sort of without, without, going for application of this sort of theories for a very simple situation. Simple situation means when we consider our flow to be steady, it is a very simple, and then, for when we consider this to be one-dimensional, then again it is very simple. So, for those simple situation, we can derive some of the relation in a very convenient way.

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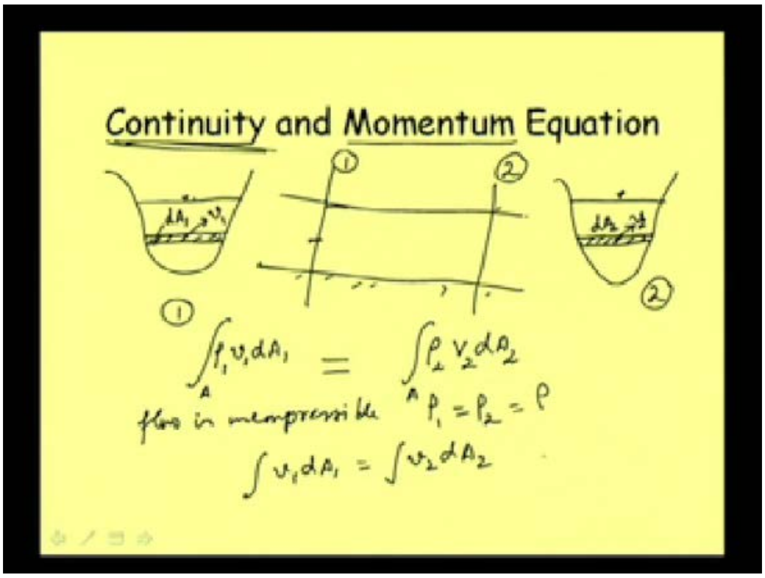

So, let us see for a simple situation. First, how we can derive this continuity equation and momentum equation? Now, please concentrate on to the slide. We want to discuss it briefly of course more details of this continuity equation and momentum equation that will be discussing when we will be discussing unsteady flow, and right at this moment, we will be just introducing some of the simple equation that we use.

Well, in the slide, you can see that say let me consider. First, we are talking about continuity equation. Let this be a channel one-dimensional we are talking about, and then, flow is moving like this, and let me consider two section - this is one and two, section 1 and section 2, and say let me draw the, draw the cross section not here, let me draw the cross section here. Say this is for the section 1, and on the other side, I am drawing section for the 2 and flow level is this much here. Corresponding to that and flow level here is this much corresponding to that.

Now, say just like the case, when we did earlier in the last class, let me consider say this is a small elementary area and the flow velocity through this be small v, small v, and then this area is sectional area is dA 1 and this small velocity is v 1. Here also we can consider a small elementary area dA 2 and then flow velocity through this is say v 2. Then about the from using the conservation of mass principle, how much is the volume? How much is the mass flowing through this section per unit time that we can find out that is equal to say v 1 into dA. That is the volume flowing through it per unit time and then multiplying by rho 1. Let me write rho 1 density at this particular section.

And if we integrate it over the area, we integrate it over the area, then we can get that of course, this is we can integrate it over the depth also ultimately over the area. So, this is the mass flowing through this section per unit time through this section and the mass flowing through the section 2 that we can write again the same again expression that rho 2 v 2 and dA 2 and then integrating over the area A 2.

Well, then as per our conservation principle, this two should be equal provided that there is no lateral in flow or out flow from the no lateral flow to this system through system. Our system is now this one, our system is this one. Now, this is basically the continuity equation in simple term, and supposes if it is water, for water, flow is incompressible, flow is incompressible. So, what we can do flow is incompressible means rho 1 is equal to rho 2, and so, this is equal to say rho. So, rho, rho, is not we can just cancel this each other and we can write integration of v 1 dA 1 is equal to integration of t 2 dA 2.

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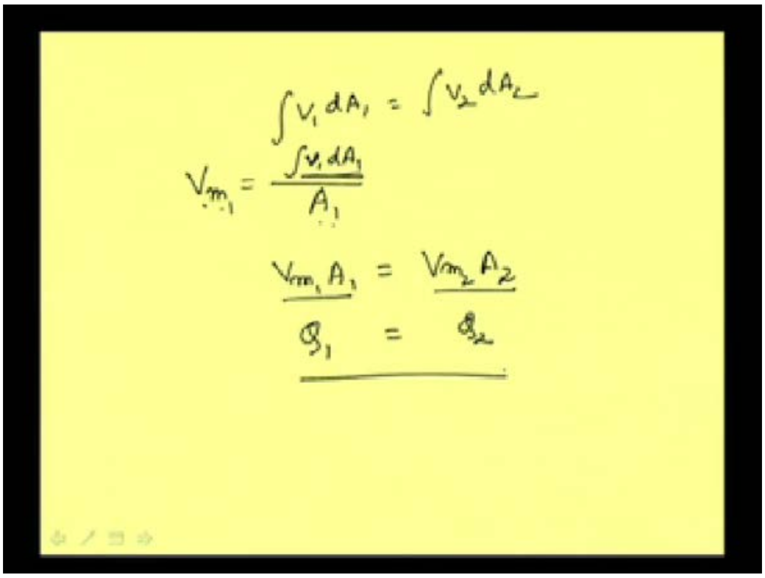

Now if we consider that, if we remember our earlier discussion, say well I am stating here again integration of v 1 dA 1 equal to integration of v 2 dA 2. Now, if we just consider that at section 1, suppose it is our mean velocity V m, mean velocity V m. Then what is that mean velocity expression that we know that, if we integrate the individual velocity at a particular section and then if we have the total flow discharge through this section divided by the say total sectional area A 1, then this our mean velocity.

This is the expression for mean velocity; so, that means, what we are getting that integration of v 1 dA 1 is nothing but V m into A V m into a. So, we can write that V m at 1 section 1 into A 1 is equal to, similarly here also V m 2 into A 2 and V m 1 into A 1 V m 2 into A 2 these are nothing but we can write Q 1 is equal to Q 2, and this is what our continuity equation, this is what our continuity equation. Well, now, when we write this sort of continuity equation, we should remember one point, we should remember one point that our velocity that we are using that is v 1 A 1. This v is the average velocity or mean velocity.

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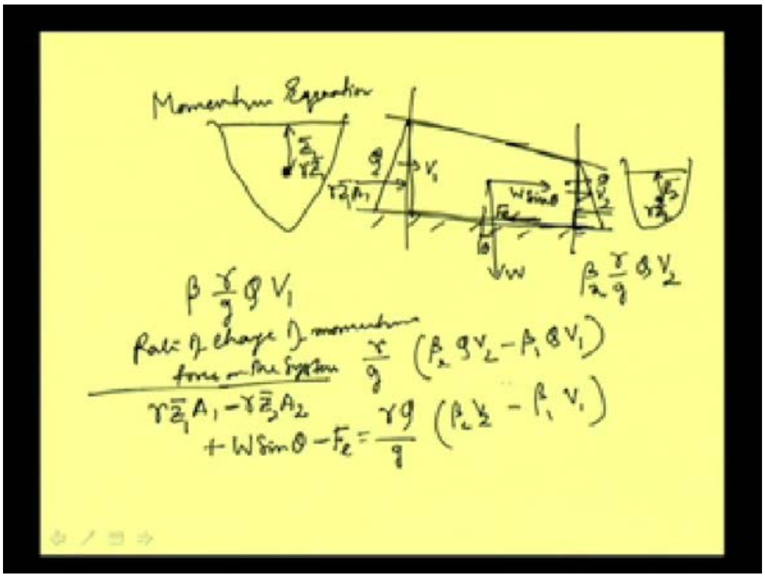

Well, now, let me discuss how in a very simple way. This is for a steady situation we are writing this equation and that is why use of all those theorem are not coming into this, this, derivation well. Now, let me see what about the momentum equation, what about the momentum equation. Momentum equation again that is for say steady state, momentum equation for a steady state we are talking about. Again let me consider the same way say this is the flow occurring and this is not uniform flow. There can be difference in flow, and this is one section; this is another section, well.

Now, in momentum equation, our interest is to first know what is the momentum here. Say the flow velocity here is v 1. Now, we know that by v 1 what we are meaning that this is the average velocity of this one, and then say flow velocity here is v 2, and suppose the pressure or we will be interested to know the force well. First, let us cover what the momentum is. So, the sectional area that this is again this one, and here, suppose it is this one, and suppose the Q discharge flowing is Q, discharge flowing is Q here and discharge flowing through this part is not changing.

So, it is also Q. Now, we know what is continuity. So, in this section, Q is flowing here, Q is flowing there also, Q is flowing there also. So, what will be the mass? Mass will be equal to rho into Q density into Q is the mass. Then if we multiply it by the velocity v 1, then we are getting the momentum. Of course, this rho we can write in terms of unit weight also as gamma by g.

These are the one way we can write, and this is of course the momentum, but with that, we need to put the momentum coefficient. That we did discussed earlier. Of course, do not confuse this as intensive property or something like that. This is representing beta is the momentum coefficient. So, this is the momentum at that point and then momentum at this point will be. So, it will be beta 2 into say momentum coefficient can be different here gamma by g and q into v 2.

Now, this momentum we are always talking about per unit time because the mass what we are taking is the volume flowing in unit time, volume flowing in unit time. So, rate of change of momentum means already we are talking about the momentum flowing per unit time here. So, rate of change of momentum we can write rate of change of momentum. That we can write as say gamma by g into beta 2 Q v 2 what the momentum we are getting here minus this gamma by g we are getting common minus beta 1 Q v 1.

Now, Q being common, we can write it as gamma Q by g and say beta 2 v 2 minus beta 1 v 1. Now, let us see what are the forces. This is the change of momentum per unit time, and then, what are the forces that is acting on this particular section. The force is basically if it is weight is W, then one force acting in this direction that is equal to if this angle is theta, then it is equal to W sin theta that force is there; this is acting in this direction, and then, if the depth up to the centroid is z par, then pressure if we write, then what is the pressure? Average pressure we can get at this point say it is gamma into z bar; gamma into z bar is the average pressure multiplied by the area.

So, what we can write? This pressure force will be gamma into z bar, z 1 bar let me write. Here, it will be suppose depth up to the centroid may be z 2 bar. So, pressure will be gamma into z 2 bar and that is the average pressure. So, we are multiplying by the entire sectional area to get the force. So, pressure is equal to gamma z 1 bar and multiplied by the area, multiplied by the area. So, we will be getting the force.

This is the force which is acting from this direction. Say this force is gamma z bar into A 1 gamma z 1 bar A 1, and then force acting from the other side because our control volume is this one, and force acting from other side this will be just opposite to this force. So, we can write minus gamma z 2 bar A 2. This is z 1 bar A 1 and this is z 2 bar A 2. Then what are the other forces, what are the other forces?

So, forces acting in this direction is W sin theta. So, that will be coming as plus W sin theta and then the force resisting or opposing the flow that we can show by indicating that these are some external force. These external force can include the resistance due to side and surface, and if there is wind velocity, that can also be included. That of course in simple case, we are not including those things, but in any external resistance force, that is opposing the flow can be included here.

So, that is again in the opposite direction. So, we are writing minus F e. So, external force, external force, or force acting on the system is this one on the system. So, these are the force on the system. Well, we may avoid this term also. The force on the system are like this and then this should be equal to the change of momentum.

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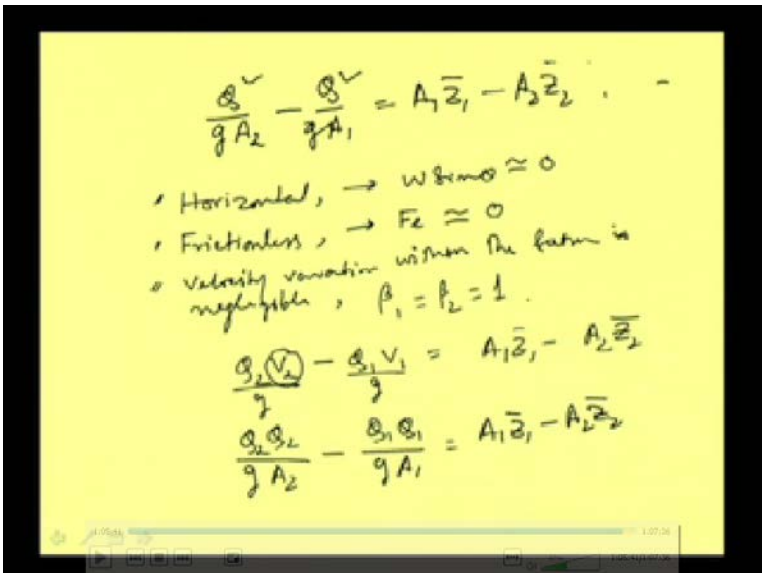

So, our equation what we can write now is say Q gamma by g then p 2 v 2 minus p 1 v 1 that is the rate of change of momentum. This is equal to gamma z 1 bar A 1 minus gamma z 2 bar A 2 plus W sin theta minus F e. This is our equation. So, this is basically a momentum equation, and that of course we can write in terms of rho also, but in this form also we can write. There is no difference between these things, and then if we make some simplification, if we just change some of the we, we, if we make some assumption, then some of the parameter can B neglected.

Say for first assumption, say we make the channel is horizontal and because it is horizontal. So, what will happen? This will lead to W sin theta become almost equal to 0; that means this term become 0 for this, and say if I consider it is fiction less, if I consider that the channel is fiction less, so when I consider this as fiction less, then your F e, suppose fiction less and air flowing is also, of, is not flowing with that speed. Then say other resistance force flow fluid is ideal almost water, so that all these other resistance force are 0. So, this can be neglected.

Well, and then, we can consider that within the section, again if we say that velocity variation within the section is negligible, velocity variation within the section in negligible. If we consider that, then what will happen? Our beta value, then our beta value will be equal to where beta 1 bet 2 this is equal to 1. So, our equation get reduced to say Q 2 v 2 minus Q 1 v 1, Q 2 v 2 minus Q 1 v 1; that means again we can divide it by gamma and then we can get it in this form and this is equal to, this is equal to, say g A 1 z 1 bar minus g A 2 z 2 bar. Well, this can be now written as say v velocity or and the velocity can be written as Q 2 is there. This velocity can be written as Q 2 by say A 2 Q 2 by A 2 g is there minus so Q 1 and this velocity can be written as say Q 1, Q 1, by A 1 and g and this is equal to A 1 z 1 bar minus A 2 z 2 bar and this can be further simplified or can be written as here say Q square by g A 2 minus Q square by g A 1 is equal to A 1 z 1 bar minus A 2 z 2 bar.

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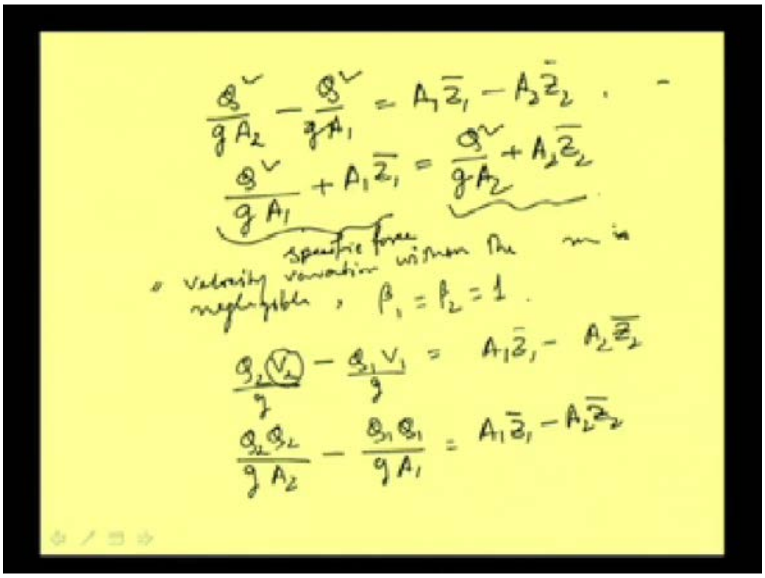

Well, now, here we can see in this expression, here we can see that this Q 1 Q 2 Q 1 is there, and if it is continuity equation is satisfied, then Q 2 and Q 1 we can write as Q simply. Already we are writing that. So, we need not put Q 1 Q 2 and this we can write like that. If we bring the two term in one side and one term in one side, then we can write that Q square by g A 1 plus A 1 z 1 bar taking this to other side and then bringing that to this side. We can write Q square by g A 2 plus A 2 z 2 bar. These two terms are equal, and this term itself is called specific force, this term itself is called specific force.

So, from the momentum equation, deriving the momentum equation, we can have that this specific force at this point and the specific force at the second point is equal, well. So, here we have discussed in a very simple situation how we can derive continuity equation; how we can derive the momentum equation, but this is just a start and we need to go into much more complex situation and we need to understand how this continuity equation, momentum equation and energy equation can be derived for such complex situation, well. Thank you very much.