Hydraulics

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Module No. # 09

Hydraulic Model Study

Lecture No. # 40

Pipe Flow: Losses in Pipe

Friends, welcome you again – today's class of hydraulic model. In fact, we have already done one class on hydraulic model. They are, basically, we did discuss about dimensional analysis. And, we could see that how dimensional analysis helps us in getting more idea about some of the equations that we are using. In our entire classes of hydraulics, we have discussed different equations in different chapter in different topics. And, through dimensional analysis, we can have a better understanding of some of those equations; say for example, we were discussing in last class that coefficient of discharge. Suppose C d – when we study flow through orifice, we can otherwise also get that discharge Q is equal to say area of the orifice multiplied by root over 2 g s, which in fact, is representing velocity. And, that we need to multiply by some coefficient of discharge to get the actual discharge. But, there if you go by that, you do not understand much about what are the parameters on which this coefficient of discharge is depending. But, in the last class, we explained, we discussed and could see that how this coefficient of discharge is depending on different parameters. That we could get from dimensional analysis. So, that way, we were discussing about dimensional analysis in the last class; and then, we could see we were discussing about some of the very important dimensionless number like Froude number, Reynolds number, then Weber number, Mach number, and then, Newton number is another. So, those numbers we were discussing. And so, in hydraulic model study, these numbers – we will have to have a better understanding when you go for hydraulic model study.

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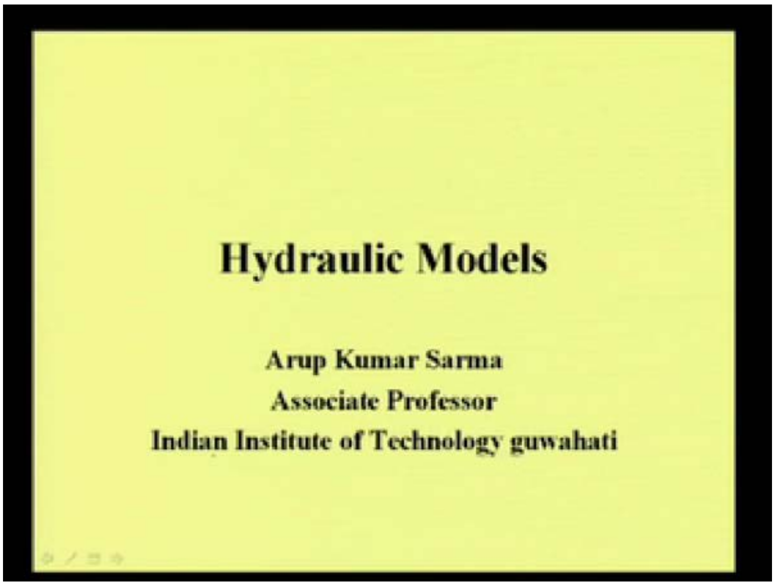

And now in this discussion, today's discussion, we will be starting with hydraulic models.

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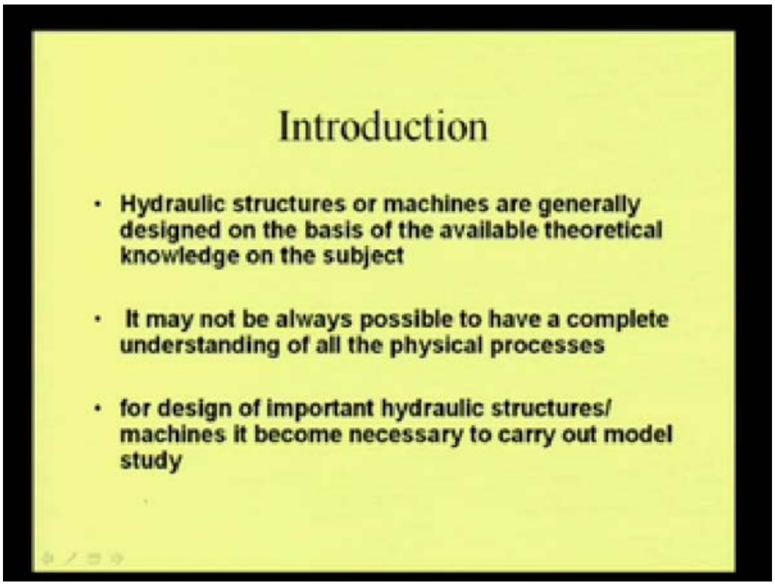

First, let us see that what we mean by hydraulic model and where we actually require hydraulic model. Hydraulic structures or machines in hydraulic engineering, we can have hydraulic structure. And, also we can have hydraulic machine structures like say dam, barrage; different type of structures we can have, say, spillway, culvert – all these are hydraulic structures. We are not discussing about those structures in detail here. Various cross drainage work can be there; say one canal is coming, then another river is going; now, canal need to cross the river; then, we can have some cross drainage work. So, that is also one hydraulic structure. And, those topics are interesting, but beyond the scope of this particular subject; but, that way, what I want to emphasize as this point that we do lot of work by applying our hydraulics in design of those hydraulic structures. Similarly, we have hydraulic machines like pumps, turbines – all those we have heard; but, we do work with all those different important machines.

Hydraulic structures or machines are generally designed on the basis of available theoretical knowledge on the subject. Now, we have some theoretical knowledge of hydraulics. Then, how the water will be flowing through these different structures or machines. So, from our basic knowledge of hydraulics, theoretical knowledge of hydraulics, we can always design this structure, say, opening of a weir; or, say a barrage we are constructing; or, say bridge we are constructing; what should be the waterway? Those things we can say design from our basis of our theoretical knowledge. But, it may not always possible to have a complete understanding of all the physical processes. In a particular phenomenon, physical processes are there and different component are interacting; different quantities are interacting. And then, to have a proper understanding or exact understanding of all these interactions, may not be possible.

Then, if only our theoretical knowledge we apply and we design, this design may not be successful. So, for design of important hydraulic structure, if the design fails, then if it has some consequence, very important consequence, then for those important structures for design of important hydraulic structures or machines, it becomes necessary to carryout model study. So, there the questions of conducting model study are coming ; that means, for those phenomenon, we do not have very clear analytical understanding that there are suppose scouring problem – we did discussed about scouring earlier in the canal design. But, it is very difficult to have a very clear cut understanding or to develop a very theoretical relationship for these things. And then, in those issues, we need to go for model study to have a better understanding.

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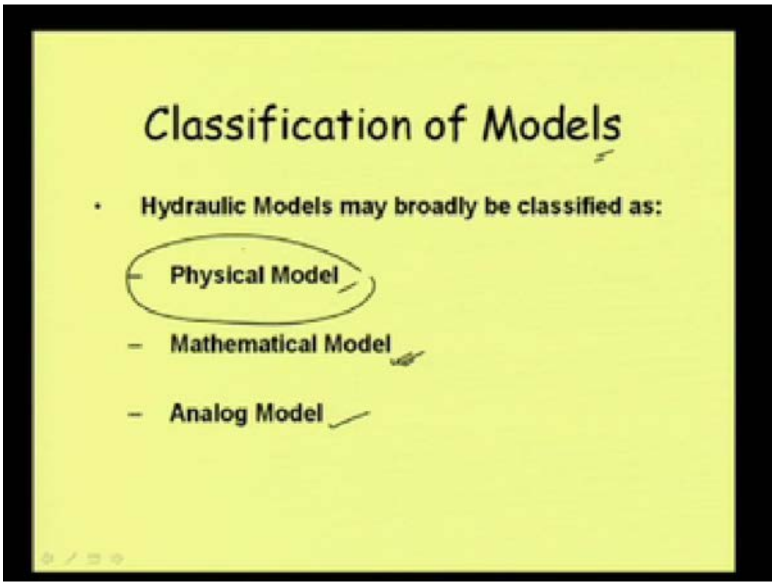

Then, let us classify the models. We are talking about model study; then, what sort of model we can have. So, hydraulic models can broadly be classified into three categories. One we can talk about say physical model. Physical model means in the laboratory, we are just producing a replica of the actual things in the field. And, actual one we call as prototype. And, what we are making the laboratory, we call as model. We will be coming to that. So, that sort of model we call as physical model. Then, there can be mathematical model, which is becoming now very popular. Why it is becoming popular? Advent of digital computer, high speed computer, is making it popular. So, we can express very complex phenomenon.

Suppose for expressing a physical phenomenon, the mathematical expression may become very complex. And, solution of that equation may sometime become very problematic, but now, we can solve those numerically. And, numerical solution without computer is not possible; lot of small steps of calculations are required and that will make it problem. But, with computer, we can solve those equations very easily. And, that is why, nowadays, mathematical model is becoming very much popular. They have some limitations – physical model as well as mathematical model. Then, another type of model – that we can have is analog model.

In fact, in a class, when we were discussing about pipe flow, pipe network analysis, then we did discuss about that point; that is, one type to solve the pipe network problem, rather he solved the pipe network problem using electrical analogy, because the flow around a closed pipe loop, the head loss is equal to 0. Then, similar condition we can have in electrical circuit. If it is a closed circuit, the same condition you can have. So, if we have a better understanding of the electrical system, then we can use that concept for solving a problem of pipe network. So, in hydraulic engineering, sometimes, you can use the concept of a phenomenon for which we have better understanding already. And, using the analogy between these two systems, we can have solution of which is less familiar; we can get the solution for a less familiar system using the knowledge of a more familiar system. And, that way, say for constructing flownet in hydraulic for constructing; that means, when flow through porous material we discussed. Then, flownet is a very important object. So, for constructing flownet or say for in pipe network analysis, for different phenomenon, we can have analog model. So, these are very broad classification.

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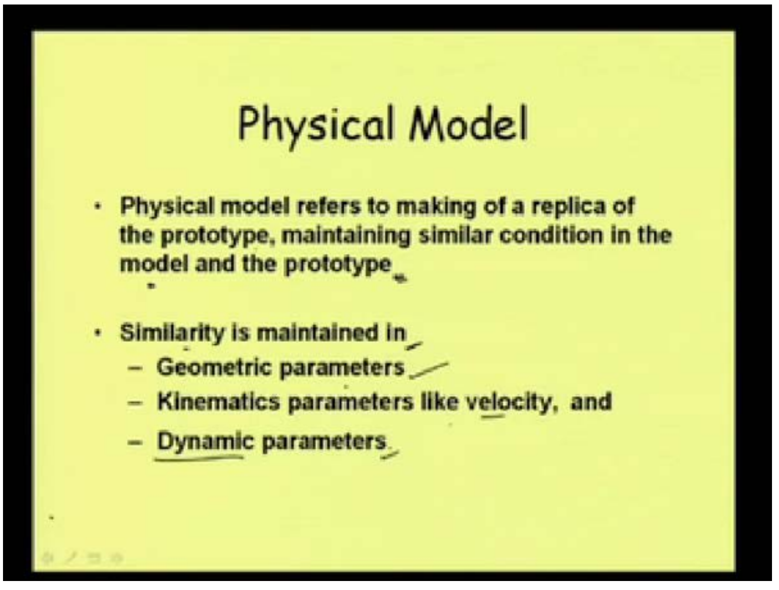

Then, what is physical model? Our interest will be more in physical model in this particular subject. We will give a brief introduction to mathematical model, which is... But, we will be concentrating more on physical model in this particular subject. So, what is physical model? Physical model refers to making of a replica of the prototype, but maintaining similar condition in the model and the prototype. Suppose for many a time, in our house itself, we can see someone is constructing a model of a house for decoration. Say a barred model of a bar someone is making or something; that sort of model people keep on making. Now, whether these we can call as model in true sense for our engineering purpose? Definitely not. So, this model will have to satisfy some of the conditions or it will have to have similar condition as that of the actual field. The field which is the actual one – that we call as prototype. And, what we are making? Replica of that; that we call as model. So, we need to have similarity between this model and the prototype. Now, when we say similarity, what sort of similarity? There can be different types of similarity between the model and the prototype; rather we need to maintain different types of similarity based on our need. So, those similarities are...

Say similarity is maintained in, first is geometric parameters. Geometric parameters means the dimension; say in the model, suppose we are making a model of a spillway; then, the dimension of the spillway, the length of the spillway in the prototype and the length of the spillway in the model, must be proportional; that means, the length of every part, each and every part of the model and prototype must be in the same proportion. Suppose we are making the width of the spillway. When we are making width of the spillway, is suppose hundred; hundred may be too large. Suppose width of the spillway in the actual field is 20 meters. And, in the model, suppose we are making is 20 centimeters and then depth or height of the spillway is suppose say 40 meters; then, here in model, the depth or height of the spillway will have to be 40 millimeters, 40 centimeters. So, that means, if we are maintaining a ratio of say 1 is to 1000 in prototype, then in every part, that ratio must be maintained; or, say 1 is to 100 if you are maintaining, if it is from meter to centimeter we are taking; suppose 1 is to 100 we are maintaining, then at each and every part, we will have to maintain that length ratio.

Similarly, area we can talk about; say area, particular area whatever we are getting in model and prototype, that ratio must be maintained. In a sense, what we can say, that if you maintain those things, then ultimately, geometrically shape wise, the model and prototype will be looking alike; but, the model may be smaller in dimension than the prototype. Why I am using the term? Maybe because generally, we have a concept that model will always be smaller than the prototype, but it may not always be the case. In some case, the model may be bigger than the prototype. Suppose we are trying to understand some of the phenomenon for which we require a bigger dimension than the model; then, we may go for a bigger size model also. In hydraulic engineering, normally, our model will be of smaller size than that of the prototype. So, this is basically called geometric similarity.

And, that kinematic similarity – that is, the similarity we call as kinematic similarity. We are writing here geometric parameters; that means, similarity of geometric parameters; that is, we call as a geometric similarity. Then, similarity of kinematic parameters – like say kinematic parameter means velocity, acceleration. These are say kinematic parameter discharge. And then, say in the model, we have some discharge or say in the actual field we have some discharge or some velocity; then, the model we will have to apply or we will have to put the proportional discharge. And further, finer things are there, which we will be discussing later, but the similarity of discharge or similarity of velocity, similarity of acceleration in model and prototype – we call that kinematic similarity. In a model, that means, we need to maintain geometric similarity; and, in some model, we need to maintain kinematic similarity. To have kinematic similarity, we will have to have geometric similarity first.

Then, we are coming about another similarity; that is called dynamic similarity; or, we call similarity in dynamic parameters. And, that is, for hydraulic model study, very important. So, dynamic parameter means we are talking about the force. Say in a hydraulic problem, where say gravitational force is very significant; then, the gravitational force that is exerted or that is exerting in the prototype, actual field, must be same as that of the... in the prototype. But, how we will judge that point? That is why, in that point, we use those dimensionless parameters like Froude number. So, Froude number of the model and Froude number of the prototype, we try to make similar. Then, we can say that yes, this dynamic similarity is maintained. But, it is not that in all the models, this gravity force will be only dominating. Sometimes, it may have viscous force. Then, we know that there is a dimensionless parameter, which is giving the idea of the viscous force; that is, Reynolds number. Say ratio of inertia force to viscous force – that is Reynolds number. Then, we can go for... In a problem, where say viscosity is more important; in those problems, we can go for similarity of Reynolds number. So, in that process, we try to maintain dynamic similarity.

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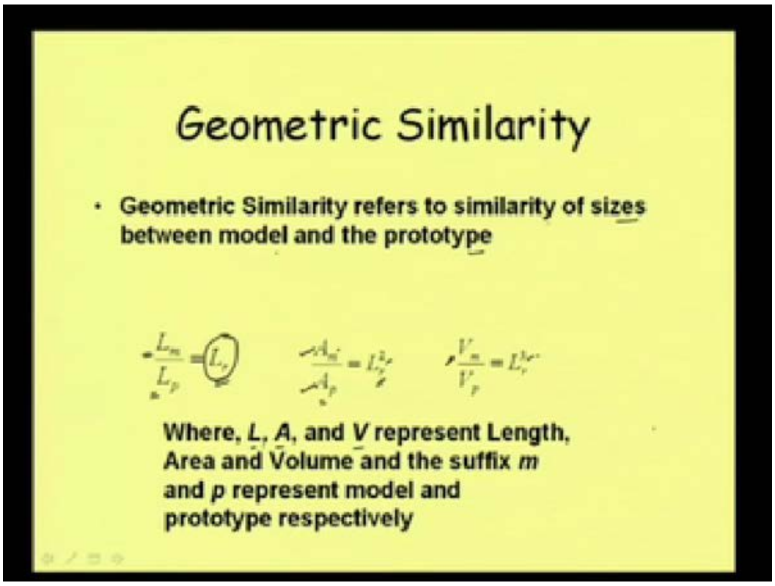

Now, to define it, say what is geometric similarity? We can say that geometric similarity refers to similarity of sizes between model and the prototype. And, similarity of size – how we can maintain? We will have to maintain some ratio. So, say if we try to maintain a ratio between length of the model and length of the prototype; that means, we will have to maintain a scale between model and the prototype. That scale we call as geometric scale. So, that geometric scale we will have to maintain. So, what will be that geometric scale or what will be that ratio? That say length of the model and length of the prototype – we call as length ratio. So, if we want to design or if we want to make a model, first, we need to decide that what will be the length ratio of the model. Then, from that length ratio, if we want to talk in terms of area; suppose in a particular model, area is very important; then, we can say that area of this and area of that model and prototype – we will have to maintain a ratio. So, if we are maintaining length ratio, then what will be the ratio of the area? Definitely, we can say that area of model and area of prototype – if you take the ratio, it will be say length ratio square, because area is nothing but L m square and this is say L p square. So, finally, the ratio will be... If we know the length ratio, we can find what will be the area ratio; that is, say length ratio square. Similarly, V m by V p is equal to... Say this is volume. So, we can say it is length ratio to the power q. And, as such, as explained say L, A, V – this represents length, area and volume. And, the suffix m and p – that we are writing for model and the prototype.

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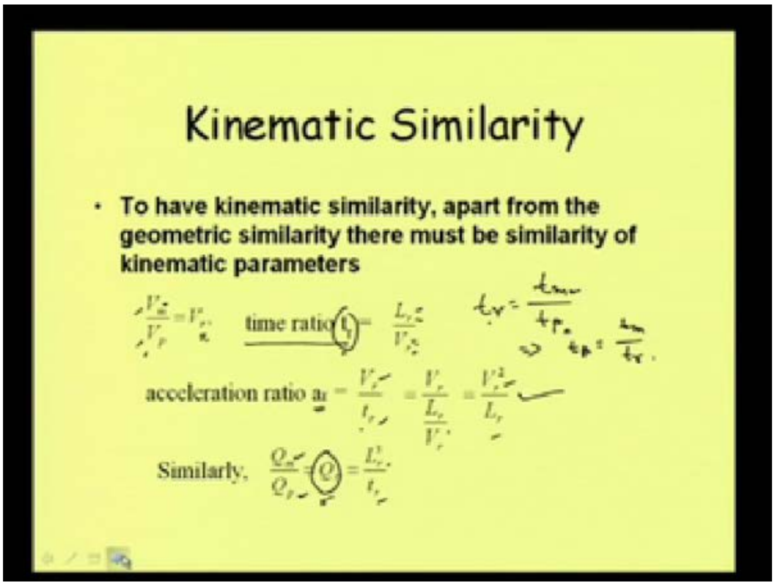

Then, we talk about kinematic similarity. Now, what is kinematic similarity? We can write that to have kinematic similarity, apart from the geometric similarity, there must be similarity of kinematic parameters; that means, to have kinematic similarity, geometric similarity must be satisfied. So, that is a prerequisite. Just satisfying the similarity of acceleration, velocity, which are kinematic parameters, we cannot receive kinematic similarity, if the geometric similarity is not satisfied. So, this is just prerequisite for having kinematic similarity. Geometric similarity – once it is achieved, after that, we will have to have similarity of kinematic parameters like say velocity, acceleration, all those.

Now, here this V m and V p (Refer Slide Time: 19:55) – this V is representing velocity. In the same lecture, we are using V here to represents volume. I can make it V cut V cut; like that, say this is volume representing; and then, here it is actually velocity. So, velocity of the model divided by the velocity of the prototype; that is, say velocity ratio. So, here when a model and we are trying to maintain kinematic similarity, then we will have to maintain a ratio of the velocity; that we find by velocity of the model divided by velocity of the prototype. And, if this is the velocity ratio, then at each corresponding point... because in the entire model, the velocity will be different at different point. And, at each corresponding point – corresponding point means that it is... Suppose we are considering a point in the prototype and each corresponding point in the model, then the velocity, if we take the ratio, it will have to have this V R value. So, this is called velocity ratio.

Now, once we have the velocity ratio then and we have length ratio already. So, we can have the ratio of some other parameters like say time ratio. So, we know that time is nothing but length by velocity. So, what will be the time ratio? Time ratio is say length ratio by velocity ratio; that means, say if we are making a model, which is kinematically similar. Suppose we are trying to find how much distance the water will be flowing or something like that; then, if we are conducting the experiment in a model; and, in the model, suppose we are finding, this is the time requirement; and, it is kinematically similar model. Then, that time what we are requiring in the model will not be same as the prototype. So, we need to know that if this is the time required in the model, how much will time will be required in the prototype. So, that is how we will be getting.

Say from the model and the prototype, we know already what is geometric similarity and what is kinematic similarity, what is the value. We know the length ratio and the velocity ratio. So, if we know the length ratio and velocity ratio, taking L r, this length ratio by velocity ratio; we are getting a different ratio; that is called time ratio. So, what is time ratio? Time ratio is equal to the time of the model and time of the prototype. So, if we want to get the time in the prototype from the known time, that we are getting from experiment in the model; we will have to divide the time obtained in the model by that time ratio; then, we will be getting the prototype time; or what we can write, that time of the prototype is equal to time of the model divided by time ratio. So, that way, we can have the time that actually will be happening in the model. With the same idea, if we try to get acceleration; say we are writing as acceleration ratio, that we can get the velocity ratio divided by time ratio. And, that if we simplify, it is becoming velocity ratio square divided by the length ratio, because time ratio – we are replacing by V r – L r by V r. So, we are getting this expression similarly.

Suppose we are interested to know what will be the discharge in the prototype if in the experiment we are getting a discharge for the model. So, for obtaining that, we need the ratio of discharge. That say we call as Q m by Q p. So, Q ratio or discharge ratio, Q r. So, how we can get this? We know that volume by time is the discharge. Now, what is volume? Volume is nothing but length cube. Already we know the length ratio. So, we will have to have that length ratio cube divided by time ratio. So, if we have the time ratio value, length ratio value, then we can have this term what is Q ratio. And then, in the model, if we are getting a Q, then from the known value of the model, we can get the value what it will be in prototype; or, in the reverse way, suppose we know that in the prototype this actual object will be having this much of discharge; then, to conduct the experiment, how much discharge we need to put there. If the model we are developing is a kinematic similar model, the discharge that we need to provide in the model to represent the discharge of the prototype; then, we can find out again utilizing this say discharge ratio. So, that way, we can apply the concept of kinematic similarity for deciding the discharge, time, velocity, acceleration – this relationship between the model and the prototype.

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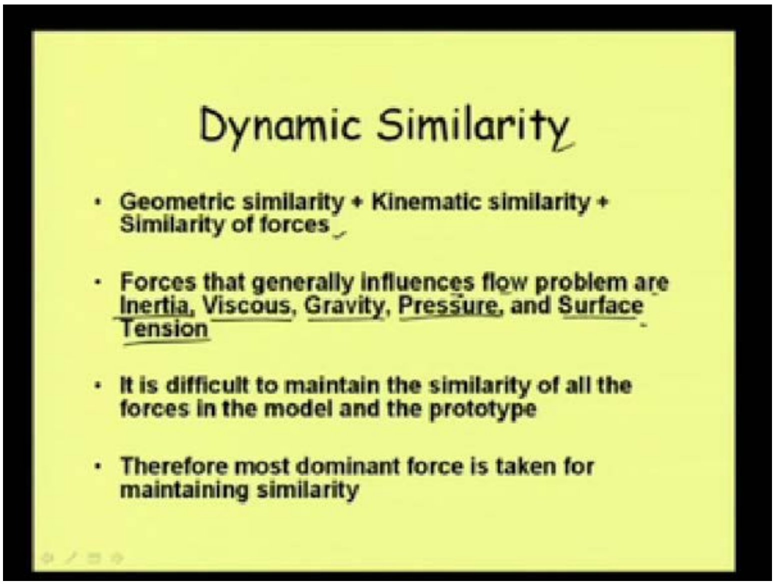

Then, we come to dynamic similarity. Now, what is dynamic similarity? Here again, we are writing in the form that dynamic similarity is that geometric similarity plus the kinematic similarity plus the similarity of forces. That means, that to have dynamic similarity, similarity of force we need to maintain that force in the model and force in the prototype must be similar. But, to have this dynamic similarity, we have two prerequisites: one is that; in fact, we can summarize that to be have one prerequisite that kinematic similarity need to be maintained, because for kinematic similarity, already the geometric similarity need to be maintained; or, explicitly, if you want to say that we can say that for having dynamic similarity, geometric similarity need to be maintained as well as that kinematic similarity need to be maintained; and then, we will have to have similarity of forces; and then, combining all these, we get dynamic similarity.

Now, this is very critical, because maintaining similarity of forces in the model and prototype, now, we need to think about what force, because in a fluid problem in the flow problem there are so many forces that can influence the flow. Generally, the forces that generally influences flow problem are inertia, then viscous force, then gravity, then pressure and surface tension. Elastic force will also be dominating if it is a pipe flow case. In open channel flow, elasticity may not be that much significant. Elasticity force will be coming when we will be considering the fluid to be compressible. But, these forces are definitely quite significant depending on the type of problem. In some problem, if surface tension force is significant; in some problem, there may be say pressure force is significant. Again, in some problem, gravity force may be significant. That way, we will be discussing about that.

Now, there are different forces and it is difficult to maintain the similarity of all the forces in model and the prototype. Practically, if we try to maintain similarity of all the forces in model and prototype, that becomes almost impossible. And, that way, this maintaining dynamic similarity is in true sense, we cannot have, but still we always try to maintain the dynamics similarity by maintaining similarity of that particular force, which is dominating that particular flow phenomenon or flow problem. Therefore, most dominant force is taken for maintaining similarity – dynamic similarity basically we were talking about.

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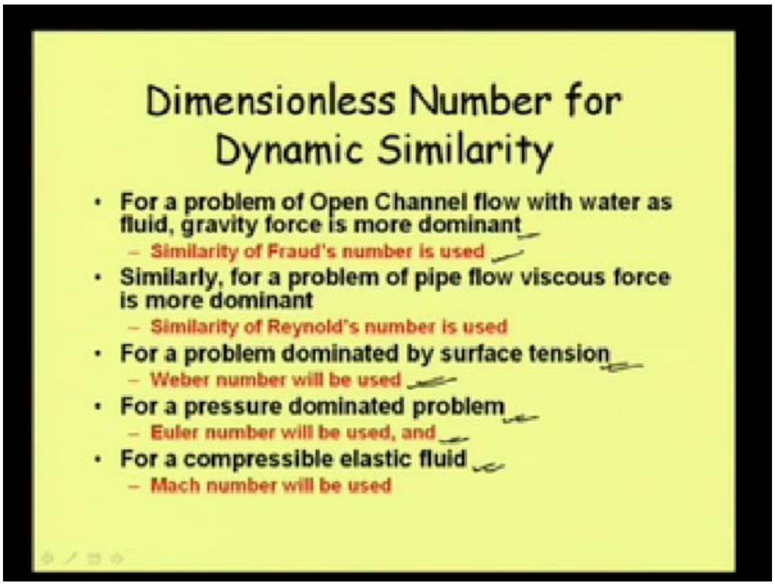

Let us see then what sort of problem, what sort of similarity we need to maintain. Say for a problem of open channel flow, which water has fluid; why we are saying that water has fluid? If some open channel flow; suppose if we talk about say viscosity or viscous fluid, then this may not be valid; but, for an open channel flow with water has fluid, that gravity force is more dominating. Gravity force is more dominant. And, that is why. We have already discussed in our last class; in our previous class, when gravity force is dominating, the dimensionless number, which represents the ratio of inertia force and gravity force. That dimensionless number is the Froude number. And, always we are using that in the form of dimensionless number, because inertia is always there. And, that is why for this sort of problem, we need to maintain similarity of Froude number.

Again, similarly, for a problem of pipe flow, in case of pipe flow, as the flow is under pressure, water is flowing under pressure. So, gravity force is not that much significant, rather viscous force is significant. And, for that sort of flow, that is why, if we try to make a model of pipe flow, then which similarity we will try to maintain? It is not the gravity; that means, it is not the Froude number, rather we will try to maintain similarity of Reynolds number, which represents the ratio of inertia force to viscous force. Similarly, for a problem, where say dominated by surface tension; as you know that, in some problem, surface tension may be dominating. There we go for similarity of Weber number. If we recall our last class or previous class, then we got what this Weber number; inertia force divided by the surface tension force. Similarly, for pressure dominated problem, we go for Euler number similarity; and, for compressible elastic fluid, we go for Mach number. Suppose our fluid problem is elastic type, the compressible fluid, then we need to maintain similarity of this Mach number. So, these are some typical cases that we are just discussing here. And, depending on the problem, we need to decide which force, which similarity we need to maintain.

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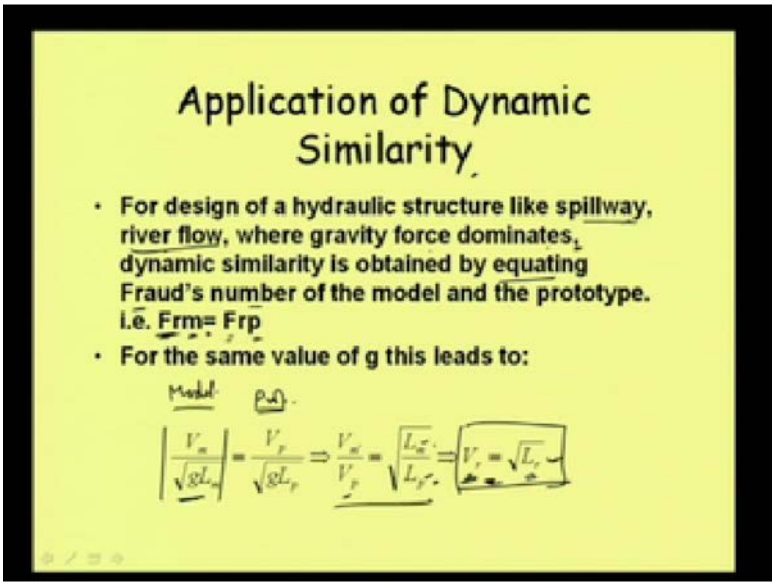

Then, application of dynamic similarity – this is very significant. Now, for design of hydraulic structures like spillway, river flow. In spillway also and in river flow also; that is, in problem of open channel flow, the gravity force dominates. Now, gravity force dominates means for maintaining dynamic similarity. What we need to do? We need to equate the Froude number of the model and the prototype; that means, see if I say F r is the Froude number, then F r of the model should be equal to F r of the prototype. Now, if we take that the experiment we are conducting – the elevation of the point, where we were conducting that experiment and where it is actually existing, is not that much difference; then, the gravity force, that is, the g – we can consider same for model and the prototype. Now, if you consider a gravity, g value to be same, when gravity is same, acceleration due to gravity is bound to be same.

When g value is considered same in model and the prototype, then we can have some interesting relation from the fact that if we try to maintain Froude number similarity, then what will be having? Say Froude number as we know, V by root over g d. So, this is what the Froude number is. Now, in case of similarity of Froude number between model and the prototype; if I am writing this for model and this is for prototype, (Refer Slide Time: 32:45) then we can see that V m by root over g L m is equal to V p by root over g L p; that is, the Froude number of the prototype is equal to Froude number of the model. Then, this will indicate if g n g are same, then from here, we will be getting that V m by V p is equal to root over L m by L p. This will indicate this.

Now, V m by V p is what? The velocity ratio. L m by L p is nothing but length ratio. So, root over L m by L p is becoming root over L r. So, this is very important relation that we are obtaining; that is, the velocity ratio is equal to square root of length ratio; that means, suppose we are constructing a model and we are trying to maintain Froude number's similarity, then in the model, first we are taking a length ratio. And, according to the length ratio, we are sizing our model; means we are shaping the model and maintaining the length ratio what we are deciding. So, that way, we have to make the model. Then, we need to flow; suppose it is a flow problem, definitely, we are discussing about flow problem only; then, say water is flowing; we are allowing the water to flow in the model. Then, we know that there is velocity in the prototype.

And then, what velocity will be maintaining in the model? If we consider or if we make the velocity ratio equal to square root of length ratio and if we are maintaining the velocity in that form; that means, we are ensuring that similarity of Froude number is now maintained. So, that we can refer as dynamically similar model; otherwise, if we are designing or if we are sizing the model based on length ratio, then if we arbitrarily decide a velocity ratio and put the velocity ratio there; say whatever velocity is the prototype; then, from that, we make that proportional, put that velocity in the model; then, it will be maintaining velocity ratio; a particular velocity ratio we are providing. It is maintaining that; length ratio is being maintained. So, that way, it becomes kinematic similarity model – model of kinematic similarity. But, if we put the velocity ratio, if we make the velocity ratio, here we can refer to the slide; if we make the velocity ratio equal to the root over length ratio and then if we are maintaining that ratio in the velocity, then this model study can give us the things, which is maintaining Froude numbers similarity; that means, it is a dynamically similar model. Now, let us see another case. So, we need to keep this; just a point that V i is equal to root over L r; when? When we are talking about Froude number similarity.

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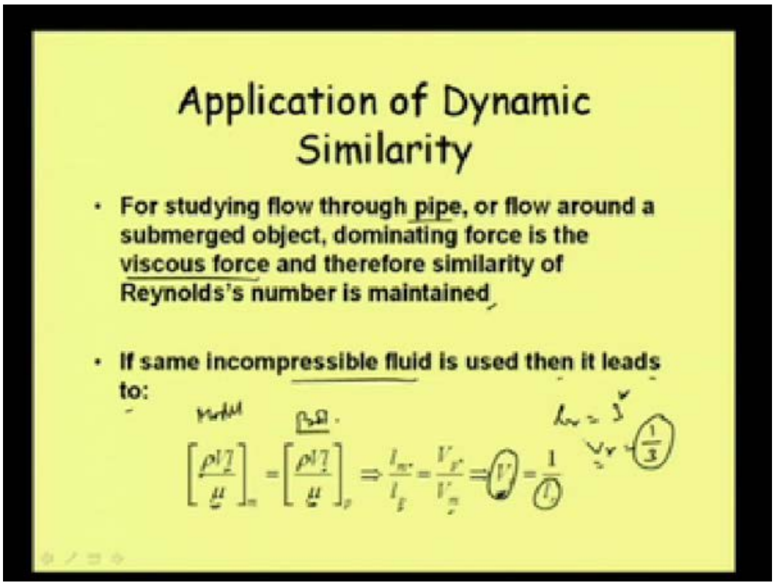

Then, we need to talk about another problem. Suppose for studying flow through pipe as explained earlier or flow around a submerged object; if our object is submerged and flow is occurring over that, flow passed the submerged object. In those cases, say dominating force is the viscous force. And therefore, similarity of Reynolds number is maintained or need to be maintained. If we want to maintain dynamic similarity in the model and prototype, we need to maintain similarity of Reynolds number. And, if same incompressible fluid is used, then it leads to... We are considering same incompressible fluid means say rho value – density of the fluid; rho value is same. If it is that, then again, starting from say model and prototype; so, model Reynolds number is rho V l by mu of the model and rho V l by mu of the prototype. So, this will give us say l m by l p; say this is l p and this is l m. So, l m by l p – if it comes here; and rho and rho is same; and, that is why and then, mu; and, mu is same. We are using the same fluid. So, mu will be same; rho will be same; or, in a sense, we can call that a dynamic viscosity and density is same; or, indirectly, we can call that kinematic viscosity, mu by rho; that is, mu is same. So, if mu is same, we are just cancelling this (Refer Slide Time: 37:35) and then we are getting say l m by l p is equal to V p by V m.

What it indicates? We know that V p by V m is nothing but velocity ratio. So, L m by L p is our...; that means, this is just reverse. One is say if it is l m by l p; say we are talking about l m by l p as length ratio as we referred to this one. Say we are talking about length ratio first, earlier; so, (Refer Slide Time: 38:10) say V m by V p we are talking as V r;

and, l m by l p as length ratio. So, here what we can say l m by l p is length ratio; V p by V m is inverse of velocity ratio; that means, velocity and length ratio are numerical; value wise these are same, but one is reciprocal of other; value wise means actually V r is equal to 1 by l r. Numerically, the value will not be same; we can say that is reciprocal of others. Say if V r is 3; then, this will be 1 by... V r should be – if length ratio is 3; then, suppose if l r is equal to 3, then V r will have to be equal to 1 by 3 . So, that way, we are getting a relationship between velocity ratio and length ratio.

Now, if we want to maintain the dynamic similarity of Reynolds number in the model and the prototype, then we need to... After deciding the length ratio or scale of the model, then we need to again have the velocity ratio with this value; that is, just reciprocal of the length ratio; we will have to take as the velocity ratio; then only, we will be finding that similarity of Reynolds number is maintained.

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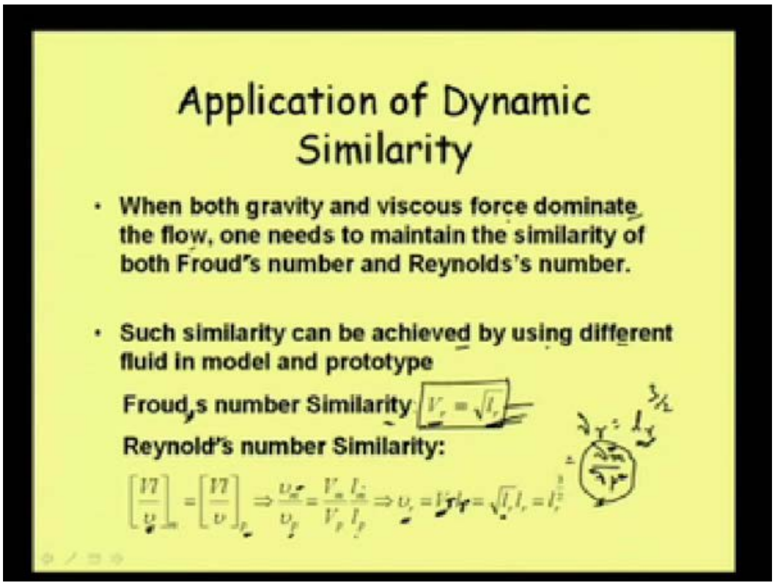

Then, let us see that we have discussed... If the similarity of Reynolds number we want to maintain, V r will be maintaining 1 by L r. This is still simple. And, if we want to make the Froude number similarity, we will be meeting V r is equal to root over L r. But, suppose in real field, there are problem, where we cannot say that this problem is not dominated that much by viscosity or viscous force; or we have say a problem where gravitational force is important, but at the same time, viscous force is also important. And, in those problems, we cannot just neglect one of these. And, in those cases, we have more problem; then, what we will do? Say when both gravity and viscous force dominate the flow – when these both dominate the flow, one needs to maintain the similarity of both Froude's numbers and Reynolds number. So, that is what. We need to maintain the similarity of Froude's number and Reynolds number. Such similarity can be achieved by using different fluid in the model and prototype.

Say if we are using the same fluid, then it is not possible; some other ways are there; but, just let us see how we maintain this sort of similarity. So, as you can see, that Froude's number similarity we can maintain by putting say V r is equal to root over l r. And then, Reynolds number similarity – we know that starting say from... Froude's number similarity – we are getting this one (Refer Slide Time: 41:25). So, this we need to maintain. Then, when we talk about Reynolds number similarity, then we have that V l by mu – this is the kinematic viscosity. Rather writing row V l by mu, we can write V l by mu of model is equal to V l by mu of the prototype. So, from that, what we are getting? That is, mu m by mu p; that is, mu of m by mu of p is equal to V of model by V of p; that is, the velocity ratio; and, l of model by l of p. So, mu r is equal to... What we are getting that V r; this is velocity ratio and this is length ratio – velocity ratio into length ratio again. So, this is the condition that we are getting for Reynolds number similarity. But, again, the same model, we will have to satisfy the Froude number similarity. If it has satisfied the Froude number similarity, we will have to have velocity ratio is equal to root over l r; that means, this is the similarity of Froude number. So, putting this V r; in place of V r, if we put root over l r; then, from again Froude number similarity, we are putting this expression; that means, we are considering both the similarity of Reynolds number as well as Froude number. This leads us to the ratio of length ratio to the power 3 by 2. So, what we are getting? mu r is equal to length ratio to the power 3 by 2; or, other way also, we can get that length ratio is equal to mu r to the power 2 by 3. That way also we can get.

Now, from this particular expression, (Refer Slide Time: 43:11) that is, mu r is equal to say l r to the power 3 by 2; what it mean? That means, mu is the property of the fluid. So, mu r is nothing but say mu of the model and mu of the prototype; that means, the fluid of the model and fluid of the prototype must not be same. It will have to have some ratio. And, that ratio is what? That length ratio – if you have already decided that this will be the scale of the model, then this length ratio to the power 3 by 2 – that value we will have to maintain in the fluid. In the fluid means in the value of kinematic viscosity of the fluid. So, kinematic viscosity of the model and kinematic viscosity of the prototype; that must be different. So, in a model, we cannot use the same fluid; we will have to get a fluid, which will be maintaining this length ratio.

Synthetically also, we can develop fluid of that ratio; this is one way. Another way – suppose exactly having this ratio l r to the power 3 by 2 in this mu m by mu p; that is definitely difficult. So, sometimes, suppose we can take two fluids; that we are deciding that we will be maintaining this two fluids in the model and prototype. So, we can find out what will be the ratio of the kinematic viscosity. So, once we get this ratio, then we can size our model; then, we can know that what is the length ratio, because length ratio we can get in that case if this ratio is fixed. For a given two different fluids, we know this ratio; and, from that, we can find out what should be our length ratio in the model and prototype. That way we can decide our length ratio later after deciding the fluid of the model once we know the fluid of the prototype. So, that way we can resize our model; and then, we can maintain say both Reynolds number similarity and Froude's number similarity. For some of the problem, we will have to do like that.

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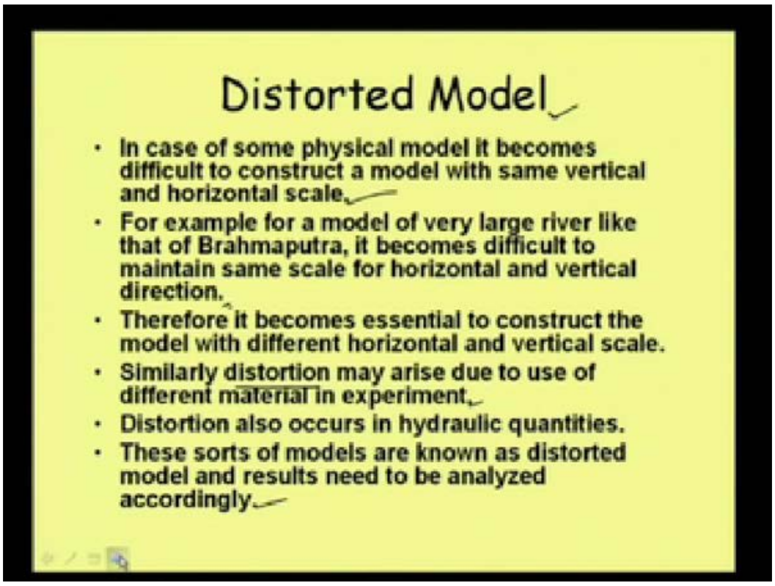

Then, we talk about another issue in the model; that is, the distorted model, because for some of the problem, when we are talking about geometric similarity, we are talking about a length ratio all the time. But, say length ratio if you try to maintain; that means, in horizontal scale, in vertical scale, we will have to maintain the length ratio. But, there may be situations. Suppose we are doing a problem, we are just modelling; develop a physical model of a very big river or maybe harbour; then, say width of the river is very large or say harbour; horizontal extent of the entire model is very large; then, to have it in a smaller model, we may have to reduce the horizontal scale too much. Say we can make the horizontal scale 1 is to 100, but say depth of flow may be. Suppose if we talk about a river; suppose horizontal scale we are reducing to 1 is to 100; and then, we are getting a model; but, the depth of flow; and, according to, we will have to reduce. If we put the same scale 1 is to 100. Suppose depth is 10 meter, then if we apply a scale of 1 is to 100 in a model, we will be left with a depth of what? Say 10 centimeters depth will be getting.

Now, that creates problem. We get this distorted model (Refer Slide Time: 47:17). Say in many a time, it becomes necessary to have distorted model. Why? Say when we talk about similarity in length or say length ratio, we are saying that the length of the model and the length of the prototype is say having a particular ratio. But, when we are saying the particular ratio, means its horizontal length. If we talk about horizontal length, the same scale we are reducing. Then, vertically also, we are reducing it by same scale. Now, there is a problem if our model is very weak. Suppose a big river like that of Brahmaputra if you have; or, say sea, harbour. We trying to model; then, to have it in a smaller dimension width, sometimes, reduce the width; maybe to 1 is to 100 scale. In that case, width wise we are reducing it.

We will be getting a smaller dimension and we can conduct experiment; but, say with 1 is to 100 scale, if we reduce the depth, you see depth in the river may be 10 meters; then, you just imagine; if you reduce it by 1 is to 100, then the depth will be very small. Similarly, in harbour, if it is very weak, we are reducing the horizontal scale by too much. And then, vertical depth also if we reduce in that proportion, then the depth in the model may be very small. And then, if we conduct experiment, then we may not get the actual result. The reasons are several. One is that when we reduce the depth to too smaller depth, then some of the forces, which are not that dominating, not that dominant in the actual field; say in a big river, the surface tension force is not that much dominating. But, in the model, if we reduce it to a very smaller depth, then the surface tension force, which is not dominating otherwise, may become a dominating force. So, that way we can face problems.

Let us see what are the problems we may have and how we can overcome those problem in distorted model. So, in case of physical model, it becomes difficult to construct a model with same vertical and horizontal scale in some of the cases; not in all. Then, for example, for a model, I have already given it for a very large river like Brahmaputra; it becomes difficult to maintain same scale for horizontal and vertical scale. Therefore, it becomes essential to construct the model with different horizontal and vertical scale. So, how we do this? We construct the horizontal scale difference and vertical scale difference. And, that makes the model distorted. This is one point. Distorted means we are not maintaining the same scale in the horizontal and vertical direction. Then, there may be another case.

Similarly, distortion may arise due to use of different material in experiment; means, say when in an experiment, suppose the bed material size; we are talking about scouring; that bed material size is again... Say suppose boulder is there and we are reducing the size. Now, to represent the boulder, if we reduce the size of the boulder by the same ratio, then the material that we will have to put – its diameter or its particle diameter may be very small. And, this may not be giving the same behaviour as the boulder will behave, because when we reduce the size to very small size, its entire behaviour changes; frictional property, then cohesive property – everything can be different. And, that way, that distortion is also one kind of distortion. So, these sorts of model are known as distorted model. And, results need to be analyzed accordingly. So, once we get a model, once we are getting the result, then if the distortion, we know that in the model, there are distortion; then, result will have to be analyzed accordingly.

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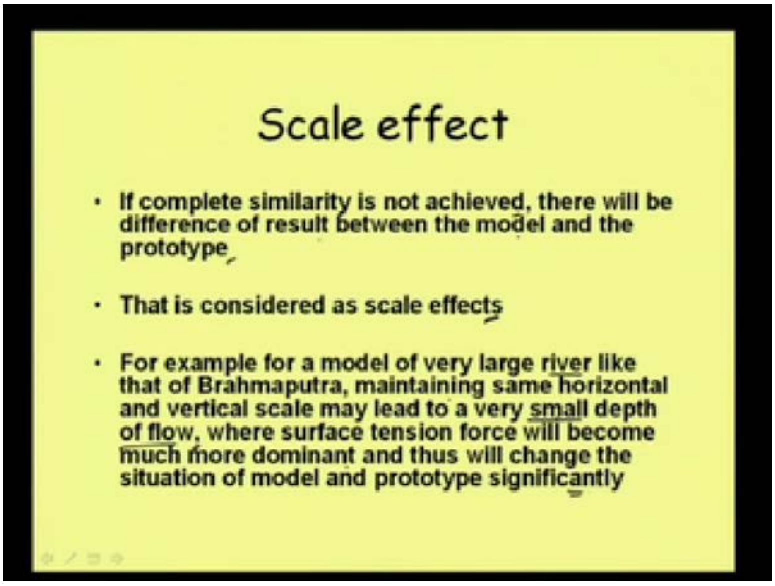

Then, we talk about scale effect. Almost these two are related. So, what we can see that if complete similarity is not achieved, there will be difference in result between the model and the prototype, because we are not able to have complete similarity all the time as we have explained that maintaining similarity of all the forces is not possible. So, that complete similarity many a time we may not achieve. Now, that is considered as scale effect. So, when complete similarity is not there, then we have this sort of problem. For example, again if we talk about the same problem; say for example, for a model of very large river like say Brahmaputra maintaining same horizontal and vertical scale mainly to very large depth of flow. So, that is another issue and where surface tension force will become much more dominant; and thus, will change the situation of model and the prototype significantly. If we maintain the same horizontal and vertical scale, then it will lead to a very small depth of flow. Yes, it will lead to a very small depth of flow as already explained. And, in that case, as I have explained already, the surface tension force may become quite significant. And, this sort of things actually we refer as scale effect.

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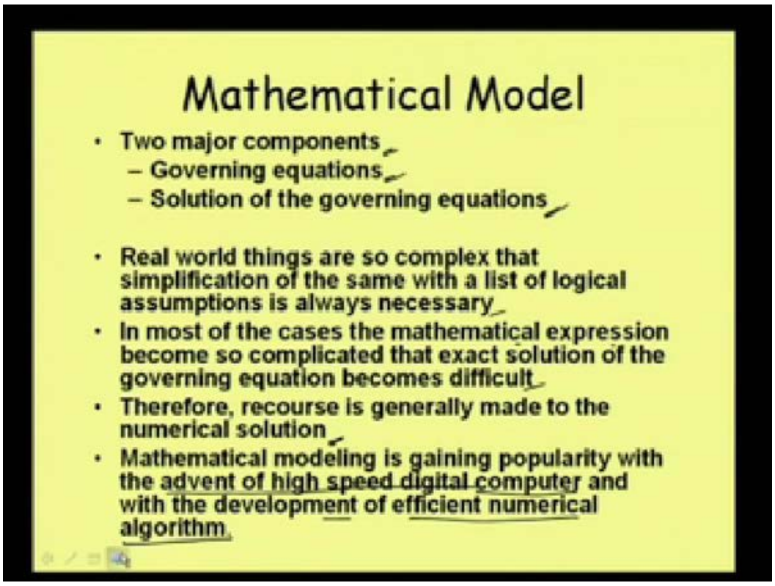

This is what we have spoken about the hydraulic model. And particularly, we are talking about say physical model; but, there can be model, which we develop as mathematical model. Already we have explained what mathematical model is. So, this mathematical model – we can just look as having two major components. What we mean by this two measure components; that means, in the process of developing and then after implementation, we basically deal with two things; one is that this two measure components; one is that governing equations. What you mean by governing equations? Say first, we are making the model to represent a physical phenomenon. And, after understanding that physical phenomenon, we need to express the relationship of the physical parameter involved in that physical phenomenon in terms of some mathematical equations. And, that equation we call as governing equation; but, for a particular problem, we may not have a single equation; like for example, say when we were talking about unsteady flow in open channel, then the governing equation we were deriving; and, to represent that unsteady flow, we need to have two governing equations when this is one dimensional problem; that is, one is continuity equation and another is say momentum equation or energy equation. We can have three equations: the continuity, momentum and energy. But, when we have two unknown variables for the problem, then we will be using say two equations. It should be continuity momentum couple; or, couple means we are using two equations. So, we are talking this as a couple; or, it can be continuity energy couple.

Now, this governing equation – understanding of these things are very important, because in a problem, where we know that – suppose energy loss is quite significant. In that case, we should not use an equation relating the energy of upstream equal to energy of downstream. If we know about the loss in between, then we can use energy equation. That energy at upstream minus loss is equal to energy at downstream; that we can do; otherwise, we should not. So, depending on that, we will have to have the governing equation. Then, in different problems, we can have one equation, two equations; like that. After that, next step – once we know governing equation, the next point is the solution of the governing equation. How we will solve the governing equations? Mathematically; we can solve it analytically if possible; if not, sometimes we go for its graphical solution; if not, sometimes we go for numerical solution.

Now, if we see that real world's things are so complex; that simplification of the same; that means, the problem – simplification of the problem with a list of logical assumptions is always necessary. When in all the previous classes, we could see that when we are talking about derivation of some equations, we are always talking about some assumption involving it. So, those assumptions we are always including in the governing equation. And, that is why, when we are using a governing equation, in physical model, we have some problem as we have discussed. But, in mathematical model also, we are using those equations with some assumptions. But, those assumptions may not be fully valid.

Now, in most of the cases, the mathematical expression becomes so complicated that exact solution of the governing equation becomes difficult. So, in that process, the governing equation becomes so complicated. Most of the cases, that we hardly can get exact solution of that equation. Therefore, recourse is generally made to the numerical solution, because as we know now that we have discussed some of the numerical issue in our earlier classes also. And, from our very basic mathematics, we might be knowing what numerical solution means. So, that way, using those numerical solutions, we can now get solution for... Say partial differential equation we can get solved by numerical method. Whatever complex the equation may be, say non-linear equation or whatever it may be, we can get solution – non-linear differential equation. Generally, many a time, we get this sort of equation in hydraulic engineering.

Then, mathematical modelling is gaining popularity nowadays, because earlier, the solutions of those equations were problematic. Now, with the advent of digital computer, this solution is becoming quite efficient. And again, people are developing with development of efficient numerical algorithm. One is that gaining popularity with two things: one is that advent of high speed digital computer – yes, we have this; and with the development of efficient numerical algorithm. So, numerical solution also when we talk about, there will be always some error introducing in this. But, nowadays, lot of developed numerical algorithms are coming up, which is making our numerical error reduced. And, they are robust. With those numerical schemes, we can solve the problem for different situations. Robust means, for say, if we make our time space larger or time scale if we make larger, then also, we can solve those equations; some different type of schemes are there. In this course, we cannot go into detail of those topics.

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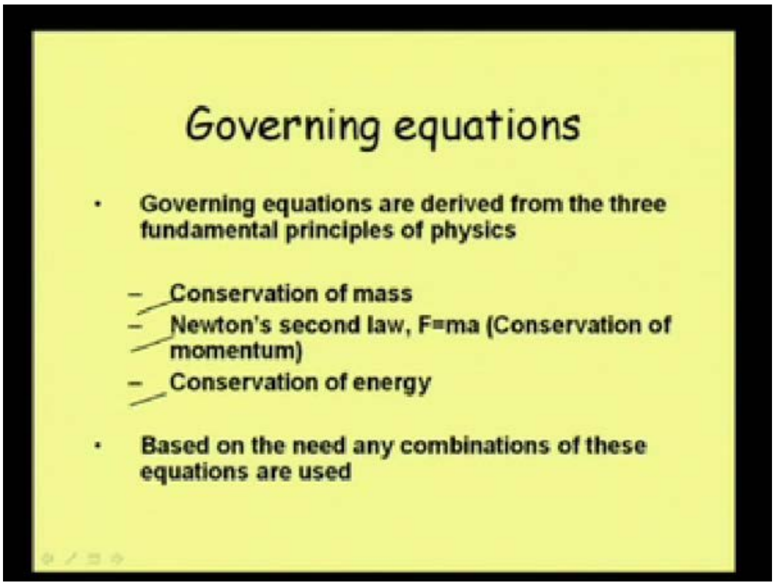

But, still we can say that governing equation means this conservation of mass, then momentum and conservation of energy. These things can be used. And, based on our need, we use different combinations.

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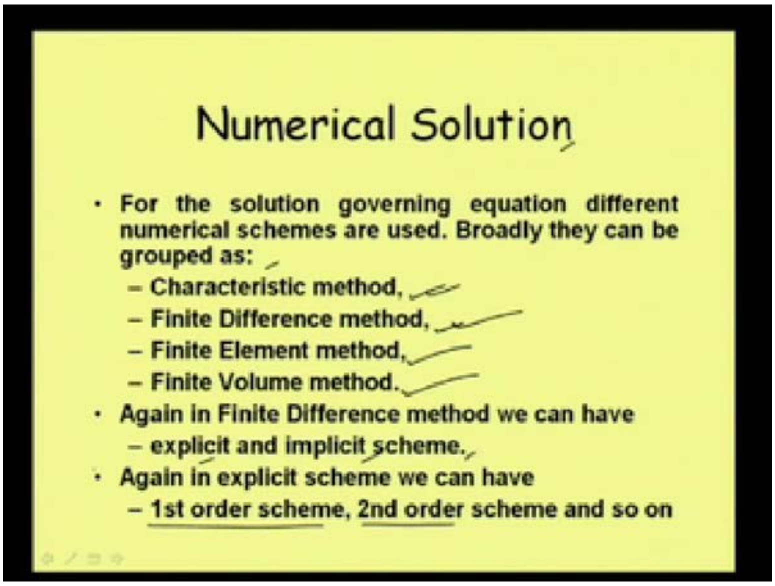

Then, we are going for numerical solution. We have discussed the numerical solution; this we did mention earlier also when we were discussing about unsteady flow; that characteristic method, finite difference method, finite element method, finite volume method – all these different methods we can use. Again, suppose in this method itself, say, in finite difference method, for example, we have say explicit and implicit scheme. Explicit scheme – we can get the value of the variable directly; implicit we need iterative procedure. Then, explicit scheme sometimes may become unstable; but, implicit schemes are stable. But, implicit scheme though stable may give erroneous result. But, explicit scheme – when it becomes unstable, we are not getting any result at all. But, if we maintain the stability of the scheme, we get accurate results. So, that way, some advantages, disadvantages are there. Again, in explicit scheme, we can go for first order scheme, second order scheme depending on how many terms of the Taylor series we are using. Those sort of different things are there, which we do not have the scope of discussing all these in detail in this particular course. But, still as just introduction, we are giving this part.

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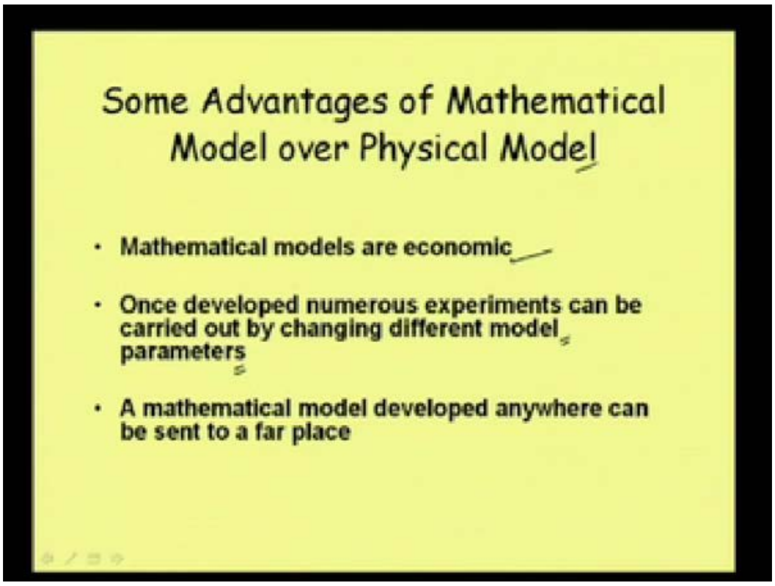

Then, knowing that mathematical model, what we do, we take a governing equation; then, we get the solution of that. And then, with computer, now, we solve that problem and the result we can develop graphically; and then, that we can create animation out of those results we are getting. So, exactly, how in the nature it will be? Suppose a flow problem will be occurring; how it will be flowing; we can just simulate the same thing there in the mathematical in the computer. So, lot of software are also available for solution of different types of problems.

Now, what are the advantages of mathematical model over physical model? One is that mathematical models are economic. If we want to make physical model, we need to spend lot of money. But, if we want to make mathematical model, a computer if you take, if we know the algorithm very well, we can write a program and then we can get the result. So, it is not that costly. If we try to purchase the software, this may have some initial investment, but we need not purchase the software; we can develop the model ourselves if we know what numerical technique is, what the governing equations are. And, we need to do for developing those sorts of models. And then, once developed, numerous experiments can be carried out by changing different model parameters. Say in the model, suppose in a river model, we want to see that if the width is reduced, what will happen? If the depth of flow is increased, if the discharge is increasing, how the model will behave? If you want to do it in physical model; suppose you want to see if this band is increased, then we need to reconstruct the model by making say band larger, width larger; or, if you want to make the width smaller, we need to again reconstruct the entire model. But, in mathematical model, we need not do all those things. What we need to do is that we need to just change the value of the parameter in the model. Just in one click we can make everything change; and then, we can see how the model will behave for different situations. This is one Advantage.

Then, a mathematical model developed anywhere can be sent to a far place. If we say we are developing a model in IIT Guwahati, that can be sent to any other place simply through the web now. So, that is one great advantage. But, if we develop a model in this institute; then physical model if we develop; if someone asks from say the far place that we need this model; transporting that physical model – if it is of bigger dimension, will be big problem. But, that is not happening in case of mathematical model; we can send the code straightaway. And, people are sending; people are sharing their knowledge and people are using with great advantage this mathematical model. For different problem, even for reservoir operation, also now, we are using model; and, for river model study, we are using model. So, that way, mathematical model – people are using in a big way.

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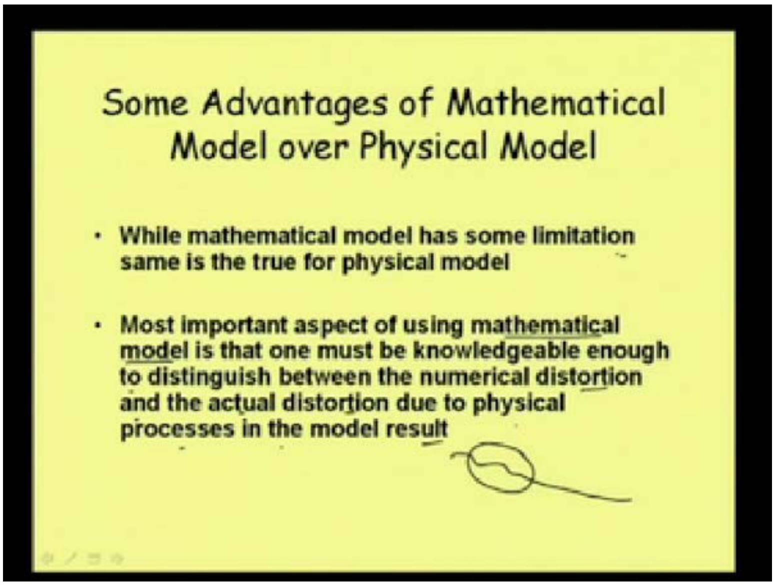

Then, this mathematical model has some limitations. Some of those we have already explained; assumption is there; stability problem is there. But, we cannot say that physical model is not having any limitations; physical model is also having limitations. So, both are having some limitations, some advantages. And, most important aspect in mathematical modelling is that suppose we are doing mathematical model, due to numerical problem, there may be some distortions particular in fluid problem. Suppose surface – we are finding that somewhere some distortions in the computational profile; this sort of... Now, this sort of distortion sometimes may be due to numerical error. But, one must know very deeply what the numerical problem can be. And, that is why, if he just do the mistake by thinking that in the actual physical problem also, this sort of disturbance will be there, then he will end up in doing error and mistake. So, this is very important.

The most important aspects of using mathematical model is that one must be knowledgeable enough to distinguish between the numerical distortion and the actual distortion due to physical processes in the model (( )). So, whatever he is getting, he will have to understand. So, without knowing blindly, people should not use mathematical model. So, we will have to be thorough about the process of development, about the governing equation. Again, in governing equation also, there are different types as we did discuss in our very beginning classes, some of our classes of the beginning; say conservative form, non-conservative form. All those forms he should be very clear; and then, what the numerical methods he is applying, he should be very clear about that. Then, after understanding all, one should go for applying mathematical model.

We have discussed today about the hydraulic model. There we have seen that the model can be physical model, mathematical model; and, it can be envelope model as well. But, we have discussed more detail on the physical model and the mathematical model. And, though we are saying that we have discussed in detail, this is just a preliminary discussion we can say on models, because to cover all the issues that are there in physical model and as well as mathematical model, it will require much more discussions. But, still with this limited time, we have tried our level best to discuss the issues of physical model and mathematical model. And, we have seen that in physical model, we have some issues like distorted model or say scale effect. And, similarly, in mathematical model, though it has lot of advantages that we have discussed here; several advantages are there; but, still we should be very much careful. If we get suppose in a surface, water surface, we are getting some distortion that it may be due to numerical distortion. But, if we just take it as the distortion of actual physical distortion, then we may be misleading by that. So, in all those issues, we should be very careful.

With this, we are concluding our topic on hydraulic model; and, not only hydraulic model. Today, we are concluding our entire course on hydraulics. In our entire course, we have discussed several topics starting from uniform flow, non-uniform flow, hydraulic jump, and then pipe flow, then canal design. So, several topics we have discussed. In all those topics, we have basically tried to give concept of the hydraulics of all those issues. In some of the topic, only we have done some numerical problem to do some exercise. But, our basic intention was to give the concept. Why? Once we have the concept, then we can do exercise. Numerical – we can take book and we can do exercise to have expertise on those issues. Expertise means say practically, if we do the numerical problem, then we will be gaining more understanding of those issues. But, the understanding the very basic concept is more important. Once you can understand the concept, then with that understanding, we can solve our actual problem in a much better way to become a good fuel engineer; or, we can do our research in this particular field to become a good researcher. We hope that if we could explain the topic, explain the concept in a better way, then we will be able to become... If we have understood that, then we will be able to become... If you want to become an engineer – fuel engineer, a good professional fuel engineer; or, if you want to become a researcher, you will be able to become a good professional researcher.

With that, best wishes. Thank you very much.