Hydraulics
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Module No. # 08
Pipe Flow
Lecture No. # 05
Water Hammer and Surge Tank

Energy cannot be consumed rather it can be transformed from, transformed from, one form to another, well.

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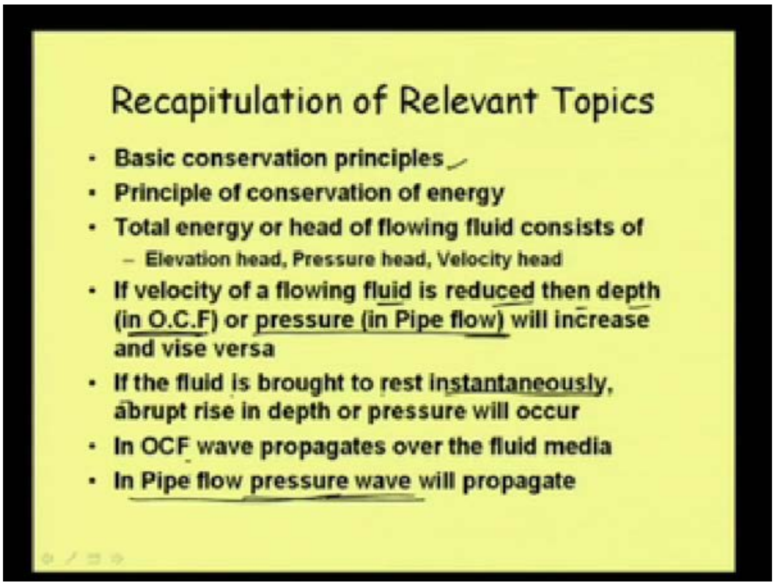

So, from that point of view, say a velocity of flowing fluid is reduced, then, the depth in open channel flow, of course if the velocity is reduced, then depth in open channel flow increases. Then, in pipe flow what will happen? As we know, that in gradually varied flow, when we reduce the velocity, then depth increases. Even in hydraulic jump also, when we reduce the speed or depth is increased, then, I mean, speed automatically get reduced at the downstream side. So, when velocity is reduced, the depth increases, well.

Because there, our pressure head was nothing but the depth, but in pipe flow, in pipe flow, as we just discussed in last few classes, that pipe flow, this pressure head is nothing but p by w, well. Our pressure is say p, so if we stop the water or if we reduce the flow velocity, that velocity head will decrease, the pressure will rise. This is a very common understanding, if velocity increases, then pressure will decrease. This is one of the understanding, that we had and the fluid in pipe flow, that means, in pipe the pressure increases in pipe flow when we reduce the velocity and vice versa, if we increase it pressure will drop, well.

If the fluid is brought to rest instantaneously, if abruptly we put a valve or do something, we put a, close a gate, that water is being stopped, that is, the flow velocity in a pipe is stopped, then the, when velocity is stopped abruptly, then rise in pressure in the pipe will also be very abrupt, well.

In open channel flow, of course, wave will form. If we stop it suddenly and that wave or say, in dam break if we release the water, if we open that thing or suppose, in a, in another canal system if we close the gate, waves are forming and that waves is moving and in pipe flow, that pressure will be rising at a point and that pressure will be in fact, moving, because in pipe flow, the flow is totally in a closed conduit, it is full flow. So, there is no question of rising other things; depth cannot be rising here that can, depth cannot rise here.

So, what will be rising is the pressure and the pressure will be propagating through the fluid media and that propagation is called pressure wave; that propagation of pressure is called pressure wave. So, in pipe flow, pressure wave will be propagating if there is a change of velocity in the flow.

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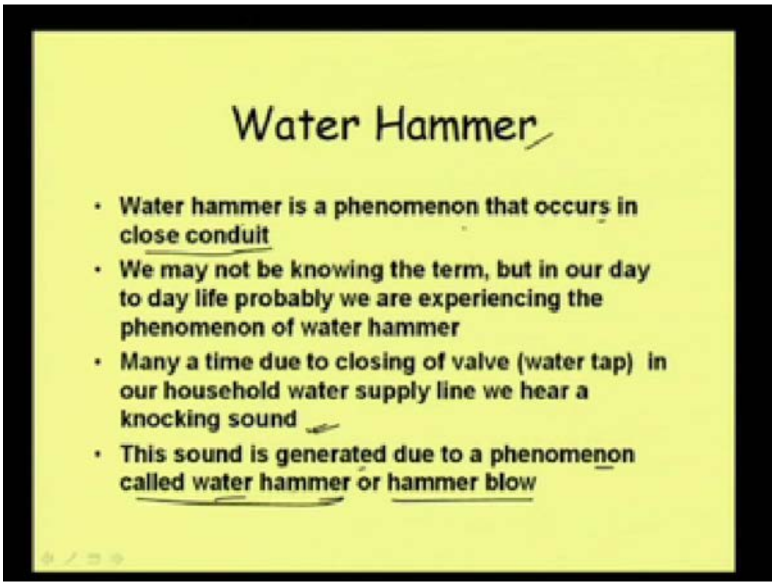

Well, now, with this very introduction, now let us talk about water hammer. So, water hammer is a phenomenon that occurs in close conduit; close conduit means, we are talking about pipe. We may not be knowing the term, term we may not be hearing till now, but in our day-to-day life, probably we are hearing this particular or we are experiencing this water hammer, say in our home itself, in our house there are lot of pipe system. In most of our house we have, we are having, say water distribution pipe or suppose in other building, institution, wherever we are studying, there also we have pipe system. Now, in the pipe system, pipe network system or may be a straight T distribution sort of system, whatever may be your pipe system, but in those pipes, suppose water is flowing through a tap and if we close the valve, means, if we close the tap, then many a time we hear a noise, a knocking sound is heard and that knocking sound is that, that, that sound or noise, that created is called knocking, at the phenomenon for which it is happening is nothing but the water hammer, well.

So, that perhaps we are experiencing and this sound is generated due to, this sound is generated due to phenomenon called water hammer or there is another name, hammer blow, but water hammer is the popular term, which is used.

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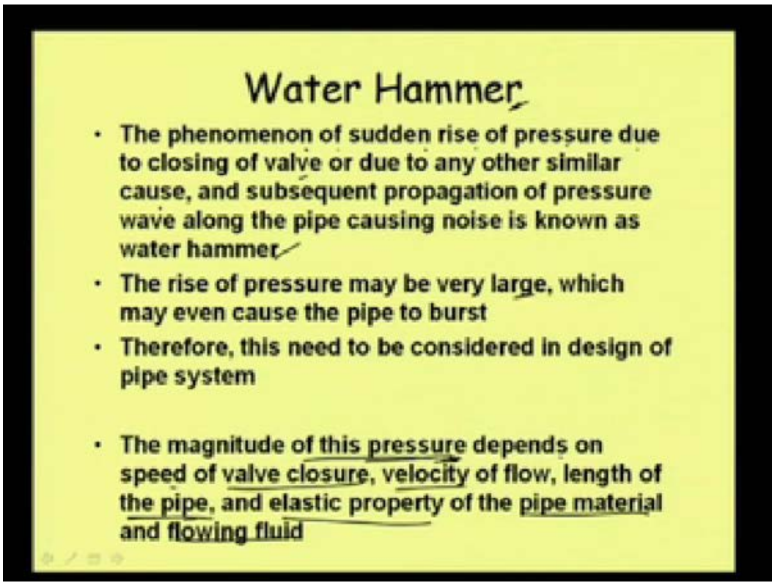

Then, let us see, what water hammer is again, the phenomenon of, in a more precise definition. If we try to give of water hammer, the phenomenon of sudden rise of pressure due to closing of valve or due to any other similar, close, cause it would, any other similar cause and subsequent propagation of pressure wave along the pipe causing noise is known as water hammer. So, that, we can consider as definition of water hammer.

The rise of pressure may be very large in many case as we are saying, that if we close the valve suddenly, instantaneously, then rise of pressure will be equally very high and that rise of pressure sometimes may be so high, that the pipe itself may get burst and because of that, analysing this water hammer is very important, where we have in a pipe system, the closure of valve, well.

In many of our systems, closure of valve is necessary, like, say in, in, in a power plant we are carrying the water from a reservoir, hydropower plant I am talking about, we are carrying the water from a reservoir to the turbine and then, turbine will not be running all throughout the day, well. Say, its capacity is 2000 kilowatt and we may be running it for 4 hours in a day at the time, we are, we are just collecting the water. I mean, this head we are trying to increase in the reservoir and then we are running this for 4 hours, that way we are producing some power and we are generating that. So, now, that means, for many reasons we need to close the turbine, shutdown, we call as turbine needs to be shutdown. Now, when we shutdown the turbine, that means, we are rejecting the water and that rejecting water will have to use some gate operation or valve operation, then water is rejected. So, suddenly, we are stopping, suddenly we are stopping the water to move into the turbine, then that sort of rise of pressure occurs. Now, due to rise of pressure, due to that rise of pressure, if the pipe carrying water to the turbine get burst, that means, we must appreciate or we must understand, that what sort of loss it can be. Entire system will have to be closed and then power generation will be stopped, there will be power crisis. So, lot of problems will be coming.

Similarly, this is in a bigger dimension, but in our day-to-day life also, suppose we are providing pipes in somewhere, now earlier, say GI pipes were provided, now we are providing PVC pipe. The new different types of pipes are coming, synthetic pipes are coming and now all those pipe, some pipe material are becoming more elastic also. Well, that that has some advantage of reducing this pressure, but anyway, the pipes of different materials are available. Now, say, if this particular system, having that sort of pipe distribution, pipe network, if we stop the valve or the system, there is a system of stopping the valve, then if we close the valve and there may be pressure rise and there may be burst of the pipe and that leads to problem, well. So, we should be careful or we need to analyse this phenomenon very carefully, so that we may avoid this sort of danger.

The magnitude of this pressure, this magnitude of this pressure, that means, due to closure of the valve there will be pressure rise, but the magnitude of this pressure depends on speed of the valve closure. With what speed we are closing the valve, that means, if we close it instantaneously, suddenly if we are closing, then the pressure rise will be more; if we are closing slowly, then the pressure rise will be less. So, this is one issue. Then, velocity of flow; if the flow velocity was high and then we have closed the valve, so high velocity has brought to rest and then, the pressure rise will be more. That means, if the flow, flow velocity is more, pressure rise will be more.

Then, length of pipe is also important. In a very long pipe, the effect may not be that much significant, but if it is a short pipe, effect may be more significant. So, length of pipe also influences this pressure rise and the elastic property of pipe material, as well as, flowing fluid sometimes, of course, this water hammer. Although here we are talking about, say water, in case of water, but this phenomenon can be in other type of fluid also.

So, if that fluid is elastic, of course, in water hammer, this fluid, the water itself we need to consider as elastic, well. So, so, fluid elasticity and the elastic property of the material, so the material by which the pipe is being made, so that elastic property of the material, these all, I mean, elastic property of the pipe flow velocity, length, then speed of valve closure, all these influence the amount of rise of water pressure.

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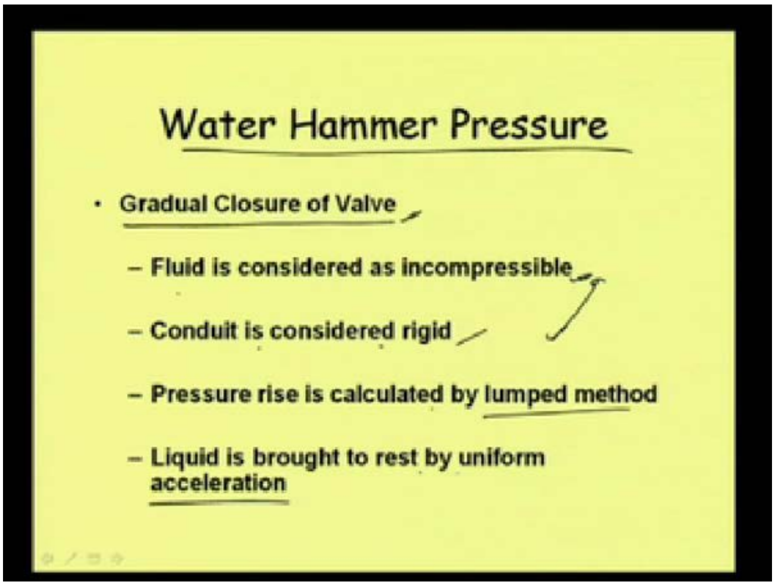

Well, let us see, how we can calculate that and let us see, what this water hammer pressure is? First, we will be discussing it for gradual closure of valve, well. If we close the valve gradually, then we can analyse this entire process in a very simplified way because when we have gradual closure of valve, we can consider, that fluid is considered as incompressible. Well, this is one advantage; here, we are not considering fluid is compressible. Then, conduit is considered rigid; the pipe that is getting the fluid is considered to be rigid.

Then, pressure rise is calculated by with these 2 points, pressure rise is calculated by lumped method, means we consider that in the entire length. Some water is there, some amount of water is there, this total water is coming to rest because it is not elastic. So, it is coming to rest and then, what will be the pressure rise. And if it is gradual closure, then with that assumption if we calculate the pressure rise, well, we will be in correct, I mean, we will be in acceptable limit of the actual pressure rise and the liquid is brought to rest by uniform acceleration. This is also one point, say we are closing the valve gradually, so water velocity will be, will become 0. Say, originally, if it is flowing with a velocity V, then it will become 0 in time T; T is the time of valve closure, in that time it will become 0. So, from V to 0, when it is coming, flow is getting, basically getting retarded and that retardation we consider as uniform; this retardation is considered as uniform.

Well, but in reality, these things may not be that uniform. Say, initially, it has, we are closing the valve initially, it is coming quickly to slower value, then gradually becoming smaller and smaller, then it is becoming 0. So, it may not be that linear variation, but say, we consider this to be uniform.

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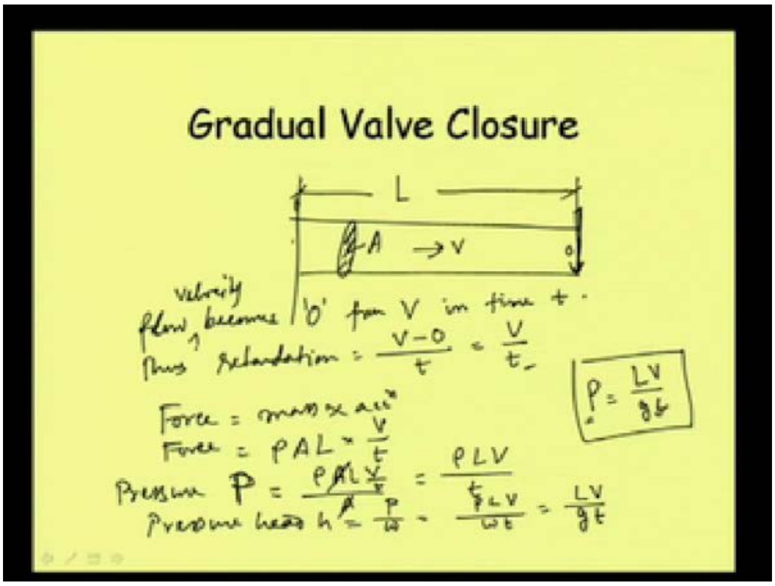

With this if we calculate the value of P, let us see how we can calculate. Suppose, this is one pipe and say length of the pipe is L, length of the pipe be L, sectional area of the flow is say, A, and the flow velocity is V, then we are closing the valve in time T; we are closing the valve in time t. So, for gradual, gradual flow, I mean, for gradual valve closure, if the time is t, then we can see the acceleration. So, closing time or we can say the flow becomes 0 from V, flow velocity rather, flow velocity becomes 0 from V. So, originally, it is flowing with V, when we stop these things, then it becomes 0 in time t. Thus, retardation is equal to say V minus 0 by t, that is equal to V by t and that we are considering, that this retardation is uniform that is why, we are getting like this.

Well, that means, if this is the acceleration, negative acceleration of course, if this is the acceleration, we are interested in the value, what will be the pressure, negative positive we are not interested at this moment. So, if this is the acceleration, then what is the mass, which mass has come or which mass has experienced that acceleration, then we will be getting, that this mass is accelerated or say, retarded by this value. So, this will make us the force, mass into acceleration, we will be getting the force. So, force is equal to say, mass into acceleration, so that we can write.

What the mass is? This area, sectional area into the length, that is the volume and multiplied by the rho, so rho into A into L, this is the mass and finally, what we can write, this mass into acceleration means, V by T, well. So, this is the force, force that required bringing it to rest. Then, what pressure is generating that force, means, that pressure is getting generated at this point. So, pressure P, we should not confuse with rho and this P, so pressure P we can write this as, say force by area. So, it will be rho ALV by T divided by area. So, this expression can be written as rho ALV, sorry, A will not be there, A and A getting cancelled, we can just remove the A from here. So, A and A getting cancelled, it will be rho LV by T, this is the expression for pressure.

Now, if we want to get it in terms of pressure head, h. So, what we can write, pressure head is, pressure head h is nothing but pressure P divided by w. So, that is equal to rho LV wt and w by rho is nothing but g. So, what we can write, that rho writing here, we can write LV by gt, so LV by gt is the expression. So, we can write finally, that pressure is equal to LV by gt that is the expression. So, if we know the L, we are knowing, that the flow is moving with a velocity V and then, if we know that at what time this valve is closing and if it is a gradual closure, then we can write this expression.

But now, when we are saying gradual closure, then we need to define that technically, otherwise if I ask someone to close it gradually, then it is very subjective, according to him closing it quite fast may also be gradual and another person may be closing it very slowly and it may take long time, which may not serve our purpose. So, we need to define it technically, but now at this moment, let us consider this as gradual valve closure.

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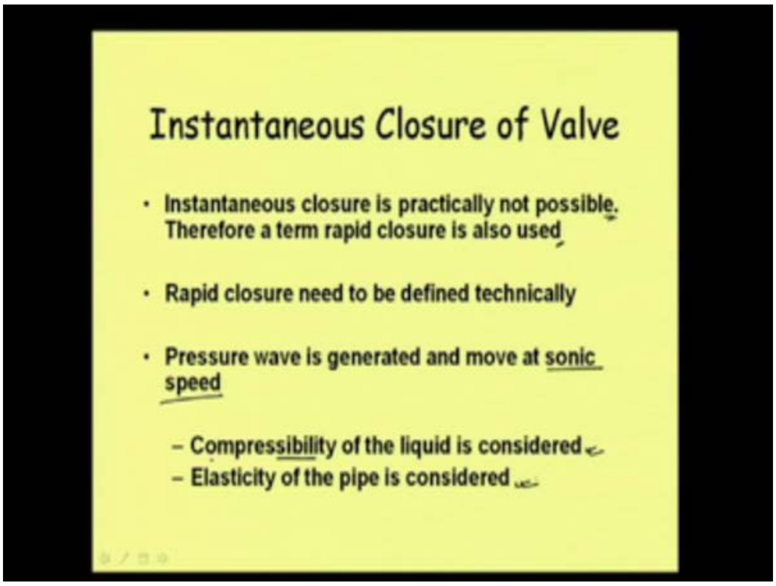

Then, if it is instantaneously closed, then what will happen? Instantaneous closure of valve; well, instantaneous closure is practically not possible because at moment, you cannot close it, whatever may be the very sophisticated device of closing, it is very difficult. Therefore, a term rapid closure is also used, again rapid, that is also relative term. So, we need to define it technically, that we will be doing.

Then, the pressure wave is generated and move at sonic speed. This pressure will be moving, pressure wave is generated and that will be moving in a sonic speed and in this case, when the pressure wave is moving, like that we need to consider, compressibility of the liquid is considered, we cannot neglect this one and elasticity of the pipe also we need to consider, because pressure rise will be more, pipe it will be somewhat elastic. So, there will be expansion of the pipe that we need to consider. Well, if we consider all those, let us see what will happen.

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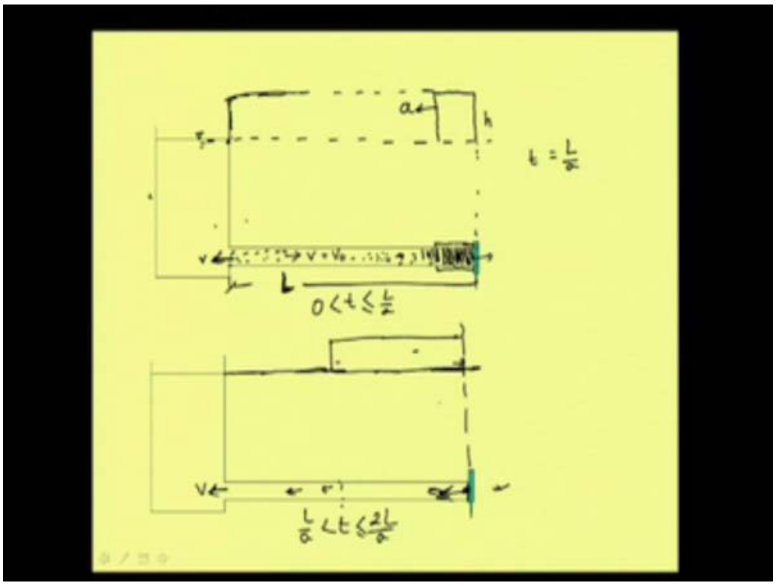

What will happen? Say, here we have a reservoir and reservoir, well, we can consider this up to infinite extent on this side, I am putting a dotted line here that is why, I say it is up to infinite extent. And then, say water is flowing in this pipe with a velocity V, water is flowing with a velocity V and say, V we can write, that this is equal to, at initial velocity V naught, V is equal to and this V is representing velocity, this is equal to V naught, it is flowing and it is flowing through this one, fine.

Now, if this is close, if this point is close, then suddenly when we are closing this point, then what is happening? That velocity is becoming, velocity is becoming 0 at this point. So, V is equal to 0. So, when V become 0 at this point, then well, when it was flowing free, when it was flowing free and if we do not consider any friction loss in this pipe, then what will be the head available at every point? Say, if I draw a line, of course there will be friction loss, there will be head loss and that way this line will not be horizontal, but of course, if we do not consider, that there is friction loss, then the head at any point in normal condition, in normal working condition, when the water is flowing with a V, say V square by g s or this pressure is only the, here the water is not moving. So, it is the only pressure rise here, velocity head, but this 2 will be equal and in normal condition, this is the head, this is the head what we have. But when it is closing and V is becoming 0, then say, as we did discuss earlier, that there will be a pressure rise. Say, at this point, at this point, suppose the pressure rise is h, h is suppose the pressure head, that is rising here, but the fluid is elastic. So, when it is just stopping, this part of fluid is suppose getting stopped, well let me just consider.

This is just we can start with a very smaller portion, this is becoming stop, but this part of fluid is still flowing. So, when this part is, initially we suppose this is very small extent of flow, small distance of flow, it is becoming 0 and then, but still this part is still flowing and so when this is becoming 0, pressure rise is up to certain extent where the flow is becoming 0. We can say like this at a particular time, say at this instant of time immediately, suppose pressure rise is this much, but gradually when this water is facing another blockage here, water is stopped here. So, this part is also becoming, now the velocity is becoming 0. So, gradually, in all the pipes the velocity is becoming 0. Everywhere the flow is now getting stop, well. And then, pressure rise is also continuously propagating in this direction; when the velocity is becoming 0 pressure is propagating in this direction. So, this pressure rise is moving, velocity is continuously stopping and pressure is extending.

So, we can say, that the pressure wave is moving. Let the velocity of pressure wave be a. So, that way, the pressure wave is moving and if the pipe is elastic, say up to this much, suppose initial pressure rise, then there is expansion of the pipe, there will be expansion of the pipe. Say, pipe is expanding because of high pressure and gradually, of course, the pipe is expanding in entire part and finally, when it is reaching this point, finally when it is reaching this point, well it has propagated and finally, it is reaching this point, then all the water is coming becoming waste, all, all water is in waste, then what will happen?

If we just see at this point, pressure here is this much, pressure here is this much and pressure just on this side is up to this much. So, pressure head or the total head we can say, total head on the right hand side is more than the left hand side. So, what will happen, the water will start moving in this direction, in this direction with a velocity V. So, this pressure is because, so because earlier, this because of stopping of this velocity V, the rise was this much. So, just reversely, because of this extra head, the flow velocity will also become proportional, flow velocity or the equivalent flow velocity will have to become V. So, water will be moving in the opposite direction. So, this part of water will start moving in the opposite direction and then, when the water is moving in the opposite direction, this velocity, then pressure will be dropping again.

So, this is the first time and this will happen, suppose our length of the pipe is l, say this length of the pipe is l, if this length is l, then well, let me write capital L and speed of the wave is a, so how much time this wave will require to reach this particular point? So, time required is, actually we can write here, the time required is equal to L by a and this phenomenon will happen within what range, from time equal to 0 to time less than equal to L by a. So, this process will happen until it reaches this point.

Once it is reaching this point, then water is moving in the opposite direction and then, flow velocity, once it is moving pressure is dropping because velocity is again, when velocity became 0 pressure rises, when velocity again from 0, it becoming v, then the pressure will drop. So, flow is moving in this direction and once the pressure, one, once the velocity is moving in this direction, so the pressure will be dropping and suppose this part is still in rest, so it will remain like that, it will remain like that of this point, well.

So, when this entire flow is moving, when this entire flow is moving from this point and it is starting, moving, when it reach this point, when it reach this point. Once it reach this point, then everywhere the pressure level will become again this one, that will not be there at that point, I am drawing up to this point, but actually when it is reaching this point, when the all the water has moved up, reached, movement of water in this direction has reached this point, then this will be everywhere, the velocity will be V everywhere, everywhere the head will be this, original head or our normal head.

Now, water is moving in this direction. So, water on this side, we do not have water. So, when this water is moving after reaching this point also means, when the movement of water in this direction has progressed up to this point, then this part of water will also start moving in this direction. Now, when this water is moving, there is no water further. So, what will happen, if there is no water, the velocity is in the negative direction and whether negative, positive, let us forget that concept; velocity water is moving in this direction, there is no further water. So, there will be negative pressure, there will be negative pressure, vacuum sort of thing will happen and negative pressure will be generated. And then, in the next step, what will happen? This process, when the water is pressure wave has gone up to this point, then finally, reached this point, the time will be equal to, will be less than twice L by a and this is greater than L by a; time is greater than L by a and less than equal to twice L by a, within that period this phenomenon is happening, well.

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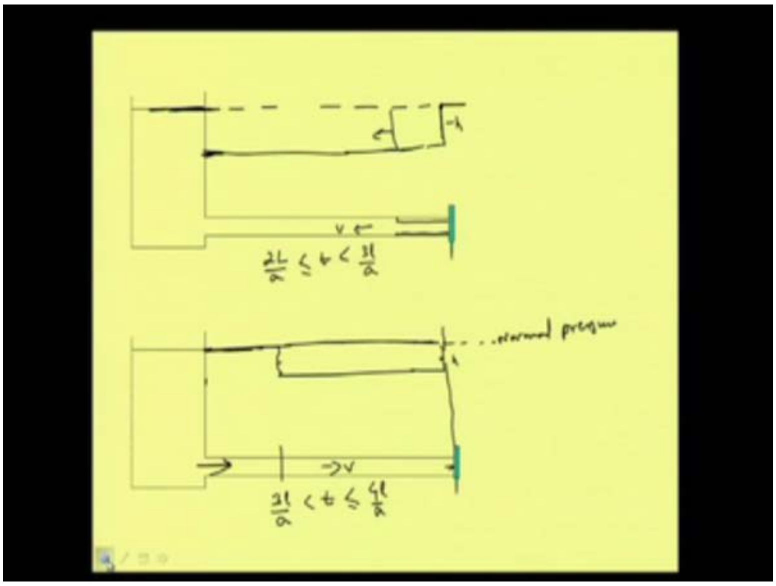

Then, once it reach this point, let me draw this line again, let me draw this line, then there will be negative pressure and suction will be there, suction will be there. So, this section, this part of water has moved. Then, gradually, this section is increasing, this head is also, pressure head is also moving in this direction, again this lower pressure, negative head, say minus h we can write and this is moving and that way, the pipe will also be actually, pipe there is suction, so pipe diameter will be decreasing, like that. Well, that way, this process is again continuing and this will be between twice L by a, this is less than and then, t less then thrice L by a. So, this process will continue.

Well, now, when all the water is, are moving up to this point, then finally, this is reaching this point. Then, again what is happening, that here is a head difference; this level is higher head pressure, total pressure, total head is at this point and here, total head is this much. So, again, water will start moving in this direction.

So, here, water will be moving in this direction, I will not show here, water will be moving in this point, water is moving in this direction, but here with a velocity V, but here water is moving in this direction. Once it moves in this direction, then pressure will again rise, I mean, not pressure, head will rise, head will rise and then, say velocity head in this point is again increasing and then pressure is again coming to normal level. So, we can show that how it is reaching this point. Say, initially, up to this much if we consider, that is going up to this point and this is still remaining and then, once it is moving in this direction, then once the entire flow is this point, then everywhere it is the normal pressure, everywhere it is the normal pressure. But once it is normal pressure, again the water is, once it reaches this point again, this flow with a velocity V is getting stop. Then, it will again be from normal it will again rise.

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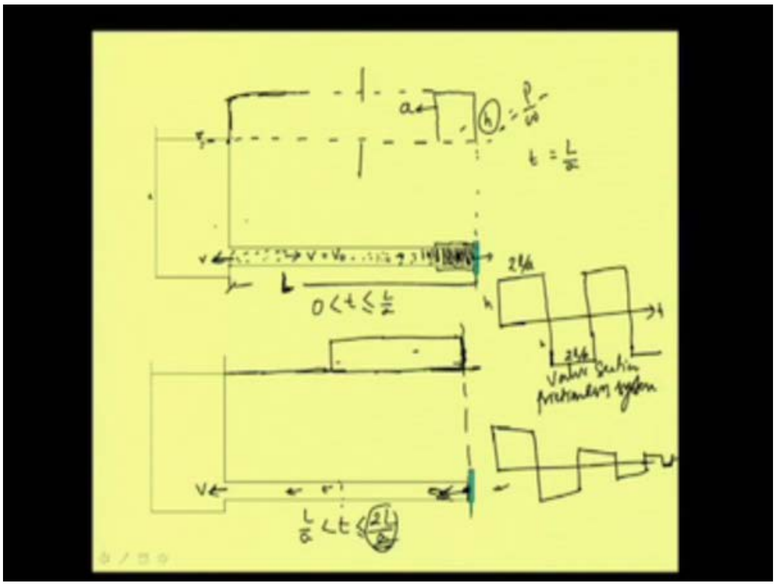

That means we are going back to our 1st condition. So, this way, this pressure wave will be moving to and fro, it will keep on moving. So, pressure rise is occurring, the water is moving, so these create a noisy sound, it creates a noise and that way, that phenomenon, that it is like hammer, it is moving like this and the water is striking the valve again, it is moving away, again it is striking. So, this striking is like hammer.

So, that process is like hammering and that is why, this phenomenon is called water hammer. Basically, technically, we can say, that rise of pressure due to closure of valve, that phenomenon we can call as water hammer. And knowing how much is h is very important, that we will have to know, otherwise we will have to design the pipes, so that it remains safe against this particular pressure head or h or say p by w, whatever we write, p by w, we need to make the pipe safe against this particular pressure.

Well, now, here we can draw how the pressure is varying for a particular section say at the valve. Let me draw a graph for the valve, how the pressure is rising. Suppose pressure is initially rising at this point by, we are drawing it for the valve section, valve section; for different section this will be different.

So, initially, at the valve, the pressure is rising by an amount h. Now, we are putting it as h, well then, h will continue to be there up to a point. When our time is, say here at this point h is there, then it is moving this way, then it is coming down, but pressure here is still remaining h and once it reach this point, then only, when the flow is, entire flow is moving in this way and once the pressure has first, rise, risen and then it is coming down. And once it is reaching this point, pressure, I mean, complete normal pressure, it is attaining, then up to that point, the pressure head here is remaining h. That means, what is the time total, time is twice L by a; for a time of twice L by a, it is remaining the same h, at that particular section. So, this time is twice L by a, say this is time and this is a, this is twice L by, let me write little lower here, this is time, fine. I am writing here this is time and this is head, pressure head.

Then, what is happening, at that moment, after twice L by a time it is here, then water has started moving in the opposite direction. At that moment, what is becoming the pressure? Pressure is just becoming negative, minus h. So, what will happen to it? The pressure will straightaway become minus h and pressure head I am talking about, then it will continue, it will continue in minus h like this, it will continue, this minus h is continuing, this everywhere it is becoming negative. Then it is, coming down, coming up again, and then, until this reach this point and the pressure become normal. Pressure in the valve section is remaining minus h, yes, everywhere it is minus h, so that time required is up to basically the, this I forgot to write here, it is thrice L by a is less than t and this is less than equal to say, 4 L by a. So, this time is again, 4 L by a, total time; total time is say, 4 L by a. And then, we can write this as twice L by a, and then, at this point again, well after reaching this point again, there will be repetition. So, it will be continuing like that.

So, if we draw a diagram, that how pressure is changing in a particular section, then we can draw like that. This is for a very ideal case where there is no friction and in this sort of system, if water hammer is occurring, that will continue to occur. I mean this is continue, this wave will continue, but this is not the practical case, in the practical situation there will be friction.

So, as such the velocity will be dropping, say initially it is coming, V is stopping to 0, but then when it is moving, V, this is also, I mean getting some frictional resistance need to overcome. So, this velocity will gradually reduce. So, pressure rise will also be less.

So, and then, this sort of diagram we can draw for any other section. It will not be identical for any other section; this diagram shape will be different. Say, initially it is 0 at this point, if I say, say, at initially this is 0 then suppose, if it is centre, then say a between half of L by a, then only it will start rising like that. So, that way, the pressure diagram, pressure distribution or pressure variation diagram will be different for different section, and this we are drawing for the valve section.

Well, now, there is what, that if we consider friction, this is say frictionless system, this is frictionless system, if we consider friction, then what will happen, that pressure is rising like that, then it is gradually falling like that, then it is coming like that, it is gradually falling like that and then it is going like that and finally, after some time, this will come to rest; this will come to rest and this is how actually, the water is coming to rest, well.

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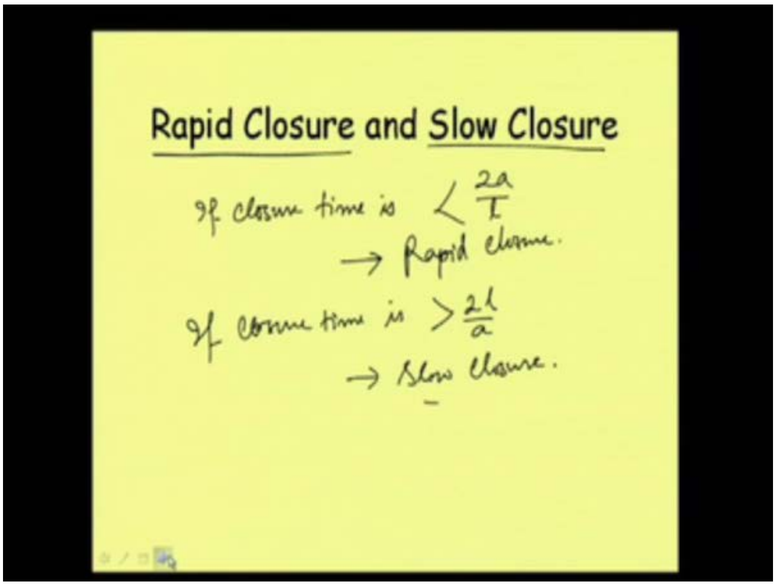

Well, then, let me talk about another point, what we mean by rapid closure and slow closure. Well, we have already discussed, that the valve closure can be rapid or it can be I mean, gradual, so slow or gradual closure, well. When the valve is closed before, suppose once we start closing, the wave is moving into the upstream direction and then, after getting the say, reservoir, it is again coming back. Now, before the wave reach and coming back, before wave, wave after coming back from the reservoir, once it reach the original point, that is where the valve is closing at that point when it is reaching, till then if the valve is completely closed, then we call this as a rapid closure. And suppose, the wave, it is reaching this point and the valve is not closing, then still some amount of water will go away through the valve and that we refer as slow closure, well.

So, by rapid closure and slow closure, what we mean, that when, if closure time is, if closure time is say less than twice a by l, then we call this as a rapid closure; then we call this as a rapid closure. And if closure time is, if closure time is greater than twice l by a, then how much greater is not a question. If it is greater than that, then itself we term as slow closure.

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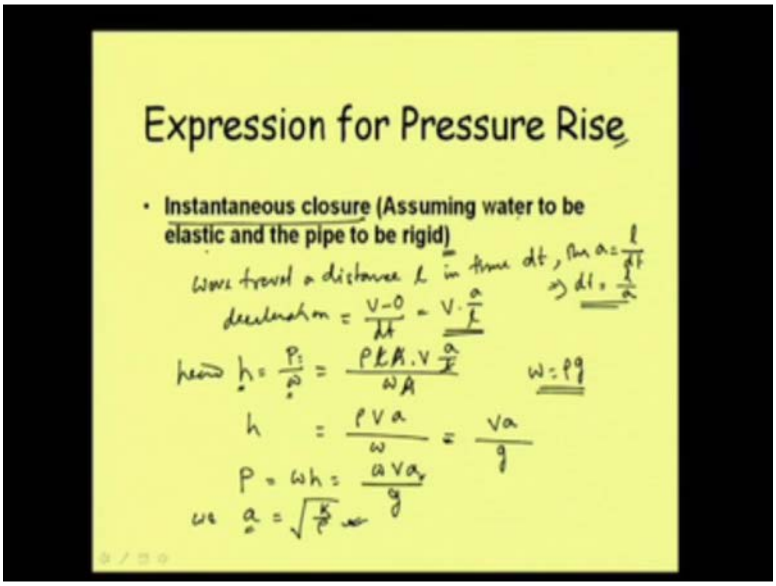

So, with this understanding, now we can move on to the calculation of an expression for pressure rise. So, how much time is there?

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Pressure rise, let us first calculate for instantaneous closure. Assuming water to be elastic and the pipe to be rigid, well, first we will be considering the water to be elastic, but the pipe is say rigid.

Stop. Start.

Start

Yeah, slide

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If the wave travels a distance, if the wave travel a distance say l in time dt. Now, we are talking in terms of travelling time of the wave. If the wave is travelling a distance l in time dt, then what we can say, that retardation or deceleration is equal to say, deceleration is equal to, we can say, from V to 0 it is becoming V minus 0 by dt. So, this is equal to V into, sorry, first let me write expressions of wave travel a distance l in time dt. So, what is the acceleration? Thus acceleration a is equal to say, we can write l by dt, l by dt or this implies, say dt we can write as l by a. So, this value now we can utilise, say deceleration is equal to V minus 0 by dt. So, this is equal to V by dt or we can write, V into say a by l, that we are getting.

Now, the head, inertia head if we write h, this is equal to nothing but P, pressure by w, w is unit weight of water. Now, pressure we have already calculated, say if the pressure, if the density of the fluid is rho and length we are saying, that it is l and area of the pipe if it is a, this is the sectional area of the pipe, then this is the, say rho l A is the mass and then mass into acceleration, acceleration is equal to this one. So, we can write v into a by l. So, this is mass into acceleration is the force and divided by the area is the pressure.

So, this pressure we are getting and then we are trying to get h. So, we are dividing it by w unit weight of water, so this w is coming. What we can have, this pressure is equal to or say, h we can write, h is equal to say, rho V a g then l, well. And from this point of course, we can simplify to have it in the form that pressure, pressure P, which is equal to h into w; pressure P, which is equal to h into w. This rho g of course, we can write as say, I mean, w as we know is equal to rho g, so we can just write, say p into h is nothing but p by w.

So, if we divide it by rho g, then what we can have, say P is equal to h into w; h into w. So, that we can write as w V a by g; w V a by g; just one minute, one minute [FL], yeah, there we need some correction here, [FL]. So, this expression, slide, slide [FL]. So, this expression can be written as, say rho l then, a a is going, well, l l is also getting cancelled. So, we can remove this one, so it is becoming rho V a by w.

Well, this expression we can write, that h we are getting, now if we try to get P. Say, P we can have it in the form, that is, P is equal to, well this we can further simplify, that w is equal to rho g we have. So, what we can write? Say, it is equal to V a by g, that way also we can write, that V a by g. Well, then what is our pressure P? If we want to write in terms of pressure, this is equal to w h that we know. So, if we put the w here, this will be w V a by g. So, that is the expression.

And again, what is a, what is a, say a we can have, well, from our knowledge of elastic, this things, elastic flow, what we can have, that this acceleration a, acceleration of the, this, this is, this is the velocity not acceleration, this is the velocity of the, I mean pressure wave. So, this pressure wave velocity we can have in terms of k by rho; we can have it in terms of k by rho. So, using that expression, that is, the pressure wave velocity a is equal to k by rho, this is from our very knowledge of, basic knowledge of physics, we can write like that.

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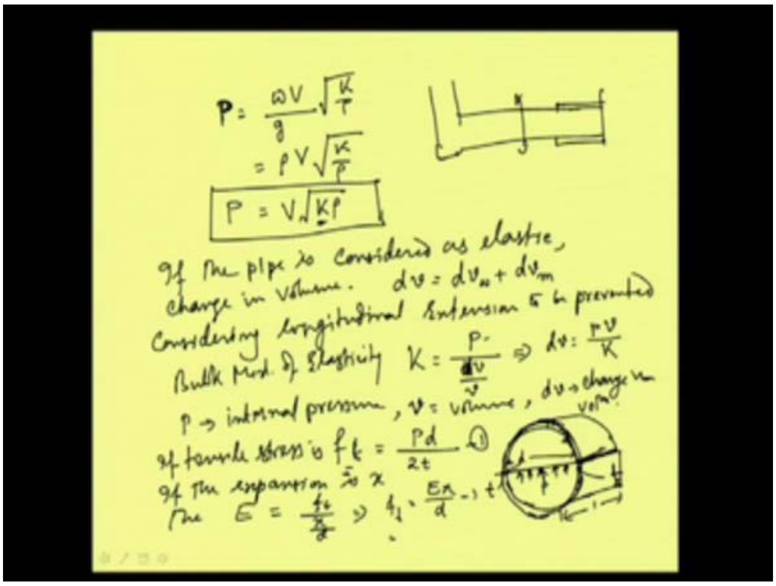

And then, using these things, we can write P is equal to, we can write P is equal to w by g V, then root over k by rho, well. And this can be written as, rho w by g is equal to rho. So, popularly it is written rho V root over k by rho. Then, we can have P, again this symbol should not be confused, this pressure P is equal to say, V into k, this rho if we take inside, it will be rho square. So, rho and rho getting cancelled, it can be written as root over k rho. So, this is one of the popular expressions. Then, k value we need to find out, of course k is say, modulus of elasticity that we need to find out. That means, for different condition, for different condition we need to find out this k.

If we consider the, I mean, fluid to be compressible, if we consider the pipe to be say expansion, expansion of pipe if we consider, then we can write this expression for k in a different way, well. This expression we are writing only considering the fluid to be compressible, not the pipe. Well, so if we consider the pipe to be consider compressible, that we can write. If the pipe is considered, say as elastic, as elastic, then if we consider, that is elastic, then change in volume, change in volume. So, our interest is to find out this k, bulk modulus of elasticity, so that we need to find out.

So, considering change in volume, suppose dv is the change in volume, that we can write in 2 parts, one is that change in volume of the water plus change in volume of the material. There will be change in volume of the water as well as there will be change in volume of the water; material also, water also, so these 2 we can keep like that.

Now, when, suppose water is, we are closing the water at this point, we are closing the water at this point and there will be change in volume as, as we did, so that the pipe can expand and then water is getting compressed. So, that way the, there is a volume change in the water as well as in the pipe material. So, these things are there.

And, but if we consider, that change in volume in the longitudinal extent, in the longitudinal, longitudinal extension, if we prevent. Suppose, there is, as a pipe cannot extend longitudinally, but it can expand in this direction, in this direction, so if we consider, that longitudinal extension is prevented. So, considering longitudinal extension to be prevented, extension to be prevented, then we can write it in a simple way, that bulk modulus of elasticity, say bulk modulus of elasticity.

Well, we know, that modulus of elasticity is nothing but stress by strain; stress by strain. Strain means, when we are talking about bulk modulus of elasticity, we will have to talk about volumetric strain, strain. So, stress means here is the pressure. So, what we can write, k that is equal to the pressure, that is exerted and then change in volume, dv, change in volume dv divided by the v. So, this is what the expression for strain, volumetric strain and this implies that what this volumetric strain, this dv is equal to P into v divided by k. Let us keep this expression; we will be using it later.

Well, now what this P is? P is nothing but the internal pressure, internal pressure, internal pressure in the pipe and v is the volume; v is the volume. We are writing v, small v, so that we should not confuse this with velocity, volume and dv is the change in, velocity, volume, that already we have written; dv is the change in volume.

Well, if it is like that, then let us see it from a different point of view, that is, say in a pipe, if I draw a circular pipe like this, then, say if I consider a section plane here, then the pressure P will be suppose acting on this line. And let us consider just one section, so that it becomes just two-dimensional analysis we can do. Say, this is the pressure P and there is a say, tensile stress in the pipe. When it is subjected to a pressure, this will try to expand and that way this material will be subjected to a tensile stress. Let the tensile stress to which it is subjected be f t, ft and because it is subjected to tensile stress, so this pipe will expand and then diameter, suppose it is d, then this diameter d will be expanding from d to say d plus x, if extension or expansion is x. So, that way, this diameter may be expanding. If due to these things if it expand, then diameter will be expanding to, say if this small distance is x, then it is, say we can have the diameter, will be d plus x of course, it can be in this direction and that way a pressure is P.

Well, so what we can write? If tensile stress, if tensile stress is say f t, f t, let me write this as f t, that tensile stress f t in the pipe, how we can write. Say, how much will be, how much will be the stress if the thickness of the pipe, let me consider thickness of the pipe to be say t, well. This, this line I was drawing just to show, that expansion of the pipe d, but here we can explain it in a different way because this thickness, otherwise you will be confused. Because that let us consider this as the thickness, this as the thickness and this is the thickness of the pipe and expansion we are not showing in the diagram because if I show this as also expansion and the, this as thickness, then there may be confusion. Suppose, t is the thickness and let us just consider, that x is the expansion.

Now, say, we can write, if f t is the tensile stress, then how much is the tensile stress generated. Well, say P is the pressure at any point here inside the pipe and d is the length. So, total force will be equal to P into d. In fact, let us consider unit area. Suppose, unit area of the unit length of the pipe, suppose then P into d into unit length in this section, suppose we are considering unit length and that way, this unit length we are considering and suppose this length is 1, then we can say, that P into this d into 1 will become the total pressure. So, this is P into t dot 1, I am not showing here P into d and then this stress or this force is being resisted by the thickness.

So, material will be subjected to what amount of stress if t is the thickness then there are 2 side t. So, total area that is resisting the pressure, suppose if I consider in this direction thickness 2, that is, twice t into, of course, if I consider unit length, this is 1. So, that way this pipe is subjected to pressure. If need to be rough side, then this stress, which is producing it should be less than the permissible rough, I mean, the, the, the value of at which this will be rough side, so that way we are writing this as P d by twice d.

Then, the e value, that is the, if the extension, if the expansion, it is not failing, but there may be expansion. If the expansion is, if the expansion is x, then the modulus of elasticity E is equal to stress by strain, stress is equal to f t and strain will be equal to, x is the expansion from the original dimension d. So, this is the expansion, so f t by x by d. And so, what we can write? This implies f t is equal to say, E x by d; E x by d, well. Now, so, this is f t we are having and this is also f t we are having, if we equate these two, say this is 1 and this is 2, this is 2.

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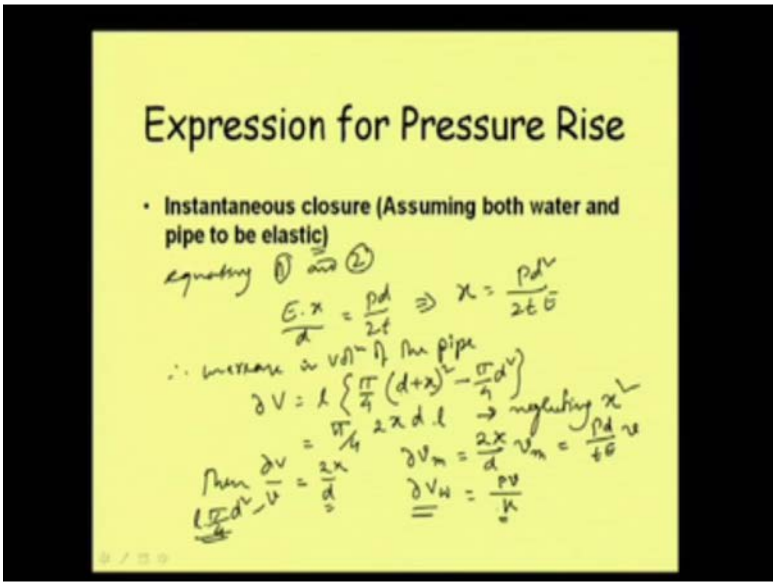

So, equating 1 and 2, equating 1 and 2, in fact, we are already considering both water and pipe to be elastic and then we are doing it. So, equating 1 and 2, what we can have, that is E into x by d is equal to P d by twice t and this implies what? The x is, x is equal to P d square by twice t; E P d square by twice t E.

So, therefore, increase in volume if we find out, therefore, increase in volume in the pipe, say increase in volume of the pipe, increase in volume of the pipe, what we can have.

Say, del v is equal to, well l this is the length and then we are talking about the area that will be say, pi by 4. Now, increase in volume means, say d plus x is the ultimate volume. So, pi by 4 d plus x whole square, this is the final and initial is, pi by 4 d square. So, if we simplify this one that will become pi by 4 twice x d into l. Here, basically we are neglecting, say this will be a smaller terms of square. Say, if we just see, this will become d square plus twice d x plus x square. So, pi by 4 d square and pi by 4 d square is getting cancelled. Then, pi by 4 twice x d that term is here, but the x square term we are neglecting. So, neglecting x square, neglecting x square we are getting this expression and then utilising that expression, what we can have, this is our del v.

So, then del v by v is equal to, we can write twice x by del v by v, v already, we have an expression for v, total volume. So, total volume, if we divide this by total volume, we will be having twice x by d; total volume will be pi by 4 d square into l. So, that way we will be getting this one. So, v is nothing but pi by 4, sorry, it is pi by 4 d square into l. So, if we put this part here, then we are getting twice x by d and thus, what we can have, that del v m, del v m that we can write as, that is the material volume, that has been, that has expanded, that we can write as twice x by d. This expression into the V m, basically it is this we are talking about, material and that we are writing like this, well.

And then, the increase in volume of water, del V w, that we have already got one expression. If we just go back to our this expression, d v is equal to P v by rho P v by k, so that we will be writing P v by k. Well, then putting these expressions finally, what we can write and this x value we need to put here. So, if we put the x value, then we will be getting here, this expression will become P d by t E into v, this expression and already for d v w, we are having this term.

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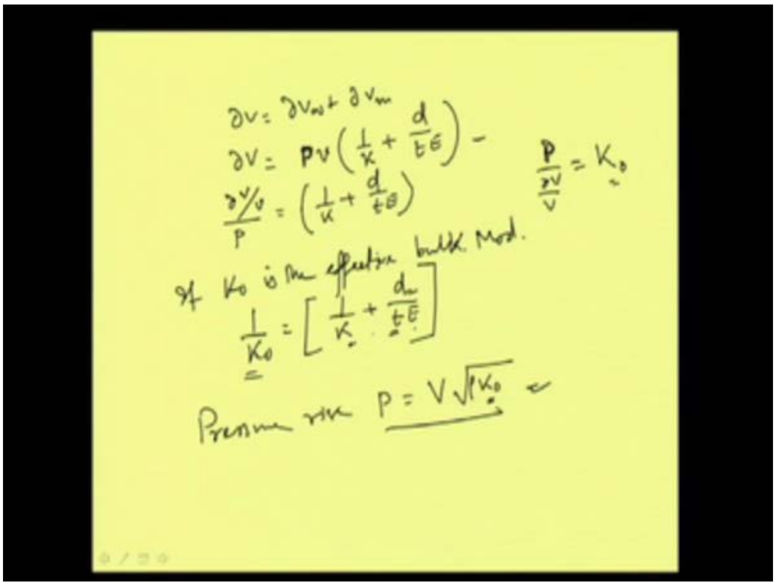

So, combining these 2 values, if we see what is del v? This is equal to del v w plus del v m and that we can write. Now, as, say P into V into 1 by k plus d by t into E, if we just sum up this 2 values, if we just sum up this 2 values, then we are getting this expression P everywhere, P we are taking common and then we are writing this expression.

Well, well, so this expression now we can rewrite it in the form, that del v by say v, we are writing here and divided by P stress, this is equal to 1 by k plus d by t E. Now, we know, that stress by strain, P by del v by v is equal to basically bulk modulus and this is suppose, total bulk modulus we are talking about. So, we are writing this k 0, which we can refer as effective bulk modulus. If k 0 is the effective bulk modulus, then what we can write, say this expression will be basically 1 by k. So, 1 by k 0 is equal to, we can write 1 by k plus d by t E. So, we will be knowing this value of E, we will be knowing and k we will be knowing for the material and the fluid we are knowing these value. So, t thickness we know, d we know, so this value we can find out.

Once we can find out this value, then pressure rise can be calculated. Say, pressure rise P can be calculated as V root over k 0 rho k 0; v root over rho k 0. This expression is nothing but what we did use earlier, at the beginning we are getting, that P pressure rise is equal to v root over k rho, the same expression we are using for calculating the pressure. But difference is that in place of k, here we are using k 0, which is coming from the material property and the fluid property.

So, that way, we can calculate the pressure rise in case of water hammer and knowing this pressure rise is very important for calculating our required size of the pipe and to make the entire pipe system safe.

So, thank you very much.