Hydraulics
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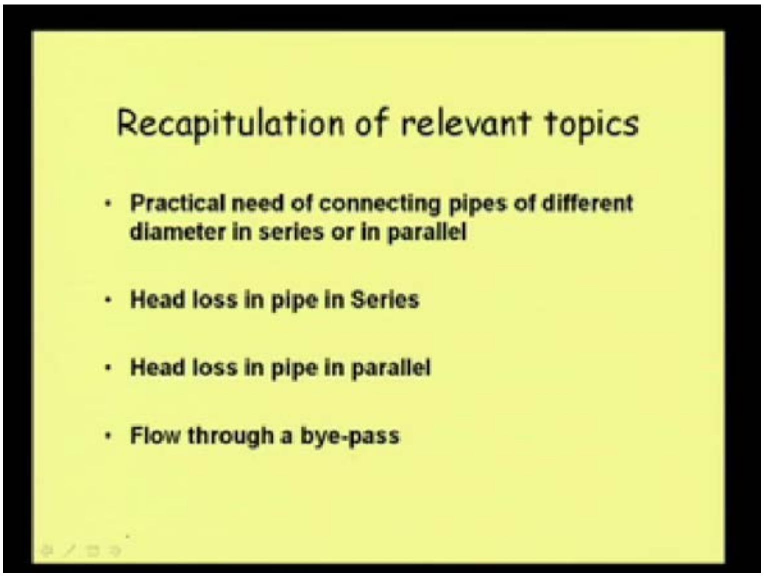
Module No # 08
Pipe Flow
Lecture No # 04
Pipe Network Analysis

Friends, today we will be starting a new topic on pipe flow, that is what we call as, pipe network analysis. In the last class itself, we did refer to this particular topic, we were discussing pipe in series or pipe in parallel, then we were saying that, a section of pipe or a stretch of pipe, suppose it is in series or in parallel,, but this may be a part of a bigger network. So, of course, not only that issue, but suppose a single pipe, several single pipes can form by joining in the form of a network. So, today we will be discussing how this pipe network is existing in different field of water distribution system and then how the hydraulics of pipe network is important and how we analyze those things. (Refer Slide Time: 01:58)



Before going to the pipe network, let us see the in the last class what we did. We were studying practical need of connecting pipes in different, pipes of different diameters in series and in parallel then of course, we did discussed, how we calculate the head loss in pipe in series. And then of course, we were studying how we calculate the pipe loss, the head loss in pipes when they are in parallel and of course, we did refer that parallel does not mean that, there should be physically parallel, but the concept of connection is parallel, that we did discuss. And of course, again we were discussing that a bypass, in a main pipe, there can be always bypass and then how by measuring the flow in a small bypass we can have idea or we can know about the discharge in the main pipe.

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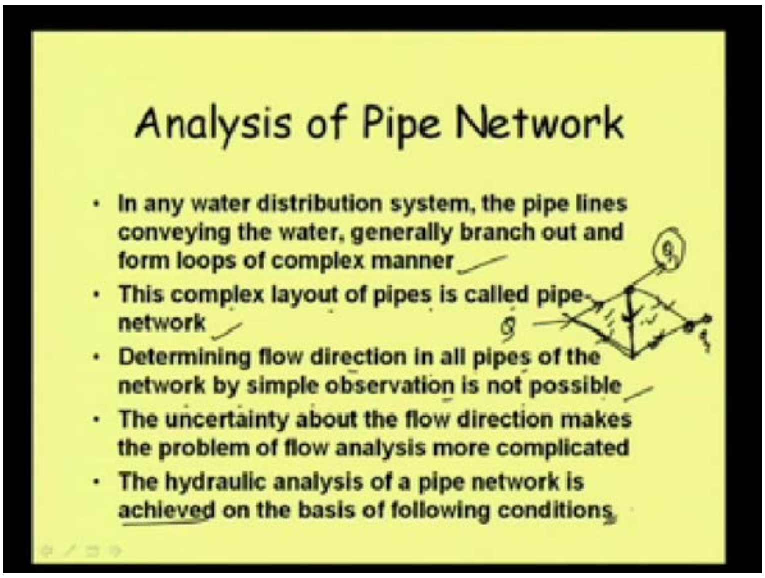

After these, now we can move on to the analysis of pipe network. In many water distribution systems, the pipe lines conveying the water generally, branch out and form loops of complex manner. So, it is our common experience that, whenever we see a pipe distribution system, suppose in city water distribution system, then we see that pipes are forming a network sort of things and a network is carrying the pipe for carrying discharge to different part. Then, this complex layout of pipe is called pipe network. Now, what is the main problem in analyzing this pipe network, the main problem is that, when there is a single pipe say when we were discussing a flow through a single pipe fiction etcetera, we were discussing then one point was very much clear to us that which is the upstream direction, which is the downstream direction then; that means, from which point to which point, the water is flowing, but even when the pipe were in series, then also it was very much clear that, from which side the flow is flowing and when the pipes were parallel, then also the direction of flow, there were no confusion about that. We were very much sure about that, but in pipe network analysis, different network can be there. Let me just draw one network of this kind, this is one simple network, water is coming from this type this part and then it is moving this way and then this way a part is going from here, suppose some water is going out also may be then this water is finally, going out from this point again and then water in this pipe is in this direction. But suppose if I ask that what will be the direction of flow in this pipe, this can be that, water may be moving this way, then it is coming this way and then it is moving in this direction or it can be either other other way also say water is flowing in this direction then it is moving this way then it is going this way and then it is going out from this particular point.

So, the direction of flow in this particular pipe is not known. This is just a network, we are drawing where, we have only 2 loop, this is one loop this is another loop, but there can be network or there will be network in real situation which will be continuing several loop having this sort of common pipe, this pipe is common to this loop as well as that loop. So, in this pipe, it is very difficult to say what will be the direction of flow in this particular pipe. And that is the major problem. So, determining flow direction in all pipes of the network by simple observation is not possible and the uncertainty about the flow direction makes the problem of hydraulic analysis of pipe network more complex. Now, what is hydraulic analysis, that will be coming later, but this makes more complex.

The hydraulic analysis of pipe network is achieved on the basis of following condition. Hydraulic analysis means, when a discharge Q is coming and we want that Q 1 discharge should be flowing in this direction and Q 2 should be the final flow, then our interest is to know that how much will be the flow in this particular pipe or in this particular pipe or in this particular pipe or in any of the pipe. This is one reason and then we may be interested to know what will be the head available at this point, suppose we want to discharge Q at this point, but this discharge will have to be raised to a certain head, then our interest is to know that how much head will be available at that point, so, that is also one of our interest.

Then another interest is that, this diameter we are not knowing suppose what should be the diameter, if we fix a very pipe of different diameter then although we need water of say Q 1 here, it may happen that this Q 1 water is not available there, and as such will we will not be getting the required amount of discharge at that point and that we can achieve by adjusting the diameter of the pipe. So, those issues, I mean those issues are important and analyzing those things and designing the pipe network, all those are coming as a part of hydraulic analysis of pipe network. And this is a shift, this hydraulic analysis is achieved on the basis of following conditions.

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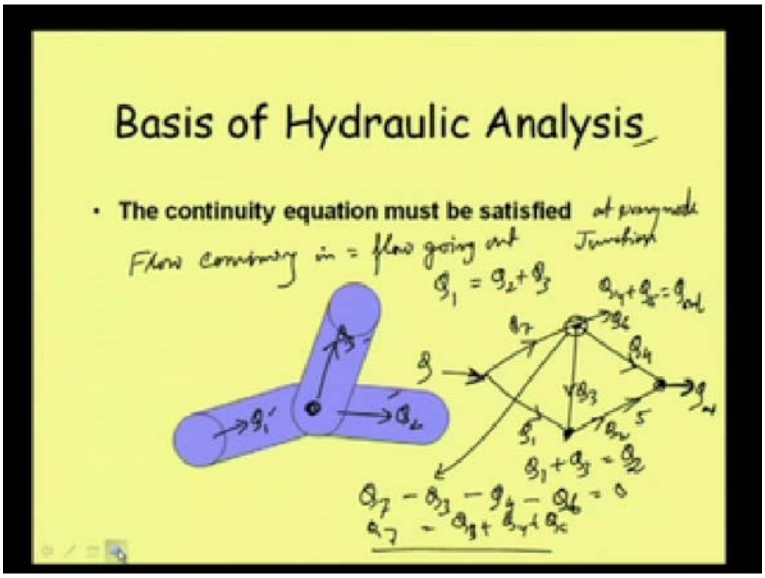

What are those conditions? Basis of hydraulic analysis, the first condition is that the continuity equation must be satisfied, now where continuity equation must be satisfied at every node, that we can write, means this is a junction point, node means let me call this as a junction at a junction point, now this is a junction and suppose a amount of Q 1 is coming in this direction, Q 2 is going in this direction and then Q 3 is going in this direction. Now, what this continuity condition is, that Q 1 that is which is coming into this particular point, this is a node suppose Q 1 is coming into this, so, Q 1 and what is going out, this must be equal Q 2 plus Q 3; that means, the flow coming into a joint junction, flow coming in is equal to flow going out. And that is true not only for a point, which is just a inner point or which is suppose its pipe are a part of the network say if I draw a network like this again, that same network, simple network, then suppose at this point, if water is coming here going there and then it is also suppose from this side, it is coming then we can say that Q 1 Q 2 Q 3, in that case we can say that at this point, Q 1 plus Q 3, both are coming in, is equal to Q 2. Then again suppose this is the discharge which is coming in Q and this is the discharge which is going out say Q out.

Now, at this point, discharge through this pipe, which is coming in and this pipe, it is coming in and going out is this point. So, this is suppose Q 4 and Q 5. Let me give this name as Q out. So, what will be the continuity condition we will at this point that Q 4 plus Q 5 must be equal to Q out. Similarly, here also, if there is a discharge like this Q 6 is going or suppose Q I mean, let me give a name Q 6, this be Q 6, then this is Q 7, then what we can write, the continuity condition if I write for this particular point, then what we can write Q 7 which is coming in minus Q 3 which is going out minus Q 4 which is going out and Q 6 is also going out minus Q 6 is equal to 0. We can say Q 7 is equal to Q 3 plus Q 7 is equal to Q 3 plus Q 4 plus Q 6. So, this is, what is continuity condition. And this must be satisfied that is the most important point and based on these continuity condition first we assigned a logical value of discharge in each of the pipe during our pipe network analysis and after that of course, we need to do iteration based on other condition. Now, let us see what are the other conditions that need to be satisfied.

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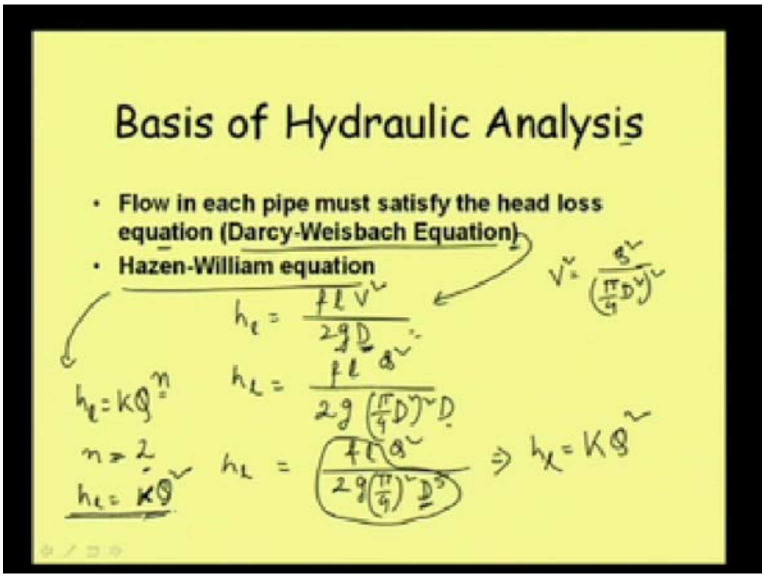

The other condition, that is the basis of hydraulic analysis based on already one condition we have discussed, then second condition that flow in each pipe satisfy the head loss equation. Flow in each pipe satisfy the head loss equation, that is the Darcy Weisbach equation and of course, we normally use the Hazen William equation here, not the Hazen partially equation, Hazen William equation and this is as just we can derive from here directly as, head loss is proportional to Q to the power n. That way we can derive. We wrote Darcy's Weisbach equation that condition, what we discussed earlier regarding Darcy's Weisbach equation for what condition it is applied. And those condition we assume that this is satisfied. Now, considering that what we have that is head loss h L is equal to say f l v square by twice g D, that is the Darcy's Weisbach equation. Now, starting from this, what we can write, that h L is equal to f l, f is the friction factor as we know, L is the length of the pipe and then V is the velocity of flow, of course, we talk about average velocity when we talk, velocity of flow in a pipe at the central line the pipe flow velocity will be more, but we are talking about the average velocity in the pipe and then D is the diameter of the pipe, g is the acceleration due to gravity, but this v square can be written as say Q square by area, area is nothing, but pi by 4 D square. So, it is whole square that we can write.

So, what we can write these value v square we can write, Q square by a square area is nothing, but pi by 4 D square whole square, so, area square. So, this is what we are writing for V and then twice g and D. Let us use the same symbol for diameter, as we are writing here capital, let me write here also capital value capital, because diameter we are writing here as capital D fine at least it is analysis.

So, now this expression we can write as, f l Q square f l Q square, another D was h there, this is coming from the area and this D is already there. So, this is there f l Q square then it is twice g, then we can write pi by 4 square, we can simplify this, but let be like that, we are not worried for that expression, then D square whole square, it is D to the power 4 then another D, So, it is d to the power 5. Now, for a given diameter of pipe, if the diameter of the pipe is given, then if the friction factor of the pipe is known and if length of the pipe is known or given, then for this particular pipe; that means, for a given pipe, what we can write that h L is equal to this part, we can consider as a constant and we can write k, k Q square of course, the Hazen William gave this expression not as h L is equal to k Q square, hazen willam gave this as h L is equal to k Q to the power n. And then of course, he said that for turbulent flow, fully turbulent flow or for when the flow is moving towards turbulent, then this n value can be taken as equal to 2 and of course, just in lower side, the n varies from 1.8 around to 2. So, that way this n can be very well for practical purpose, we can assume this as 2, because if we consider the head loss to be on little higher side. So, far distribution is concerned will be in safer side. So, we considered that h L is equal to k Q square n is equal to 2, if we consider then it becomes h L equal to k Q square. And this expression is generally use. So, this is the second condition, that we need to satisfy or that we need to know, we have discussed two condition till now, one is the continuity condition must be satisfied, second is the head loss must be, I mean, satisfy head loss equation of this Darcy's Weisbach or this Hazen William equation is valid.

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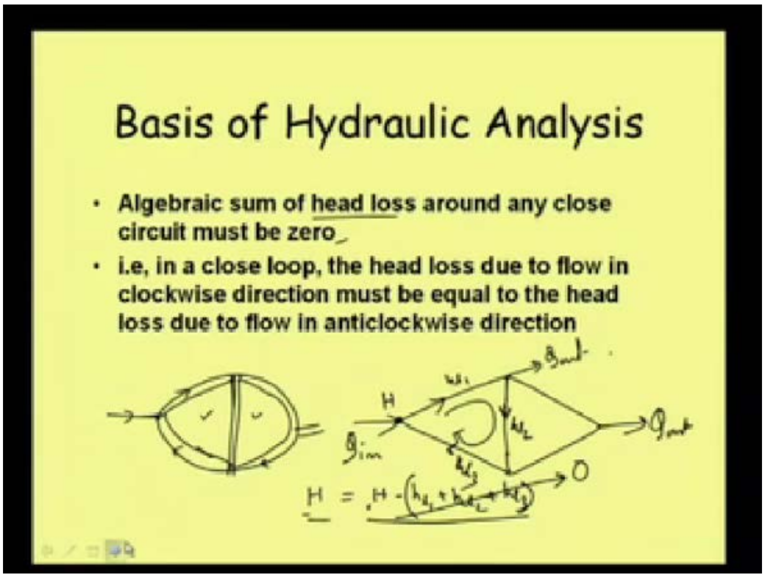

Then, the third condition and third condition is the most important condition for analyzing pipe network. What it state? That algebraic sum of head loss around any close circuit must be 0, a flow is occurring in a circuit. So, this is the circuit. Now, flow is coming from this side and it is moving this way then it is coming up this way and then it is coming this way and then it is meeting at this point. Then what will happen? Let me draw another point, this is just one loop I am drawing, again if I take this example there also, we can have loop like this and there can be suppose I can draw here also one more pipe, these are pipe like that and I can put another pipe, so, there will be two loop like that and from this side the water may be moving out also. Like that similarly here one loop and then this is one loop and this is another loop, this way the water is going out Q in Q out, and there may be another Q out with this point. Now, this loop suppose water is flowing from this side it, is going like this and it is coming like this, now in this close circuit, this is a close circuit, in this close circuit, if we calculate the head loss from here to here, we are getting a head loss from here to here, we can see that water is coming, there will be some head loss in this line then again from here to here there will be head loss.

Now, if I try to calculate the head loss around this closed loop then if we sum up the head loss here h L 1 suppose then h L 2 then h L 3, if I sum up all the head loss, then we will be getting that from here to here we have some head loss. Now, when we are seeing from the beginning, at this point we have some head available. Now, if I just go and turn around the loop and reach here, then head available here will become H minus h l 1 plus h L 2 plus h L 3, but we know one point, that in any water distribution system, at a particular section or at a particular point that cannot be 2 different head, at a particular point energy level should be same.

So, now, what will be the condition? From this point we are starting with a available head of H and then when we are returning to this point we are finding that head available is this much; that means, that head loss what we are considering to be as loss, must be equal to 0. Head loss around a closed loop must be equal to 0. Otherwise, the condition that head at this point is H initial head this should be equal to that cannot be satisfied, if it is to be satisfied, this will have to be equal to 0. So, that is a third condition and this is the very basic condition, which we apply and we try to solve the pipe network problem. In fact, as I said initially, we keep some discharge based on the continuity condition and then we see that whether the head loss around each of the closed loop, not only in one closed loop, there will be several closed loops in the entire pipe network. So, whether head loss in each of the closed loops are 0 or not, if not, if it is yes, then it is fine, but if not, then we need to apply correction.

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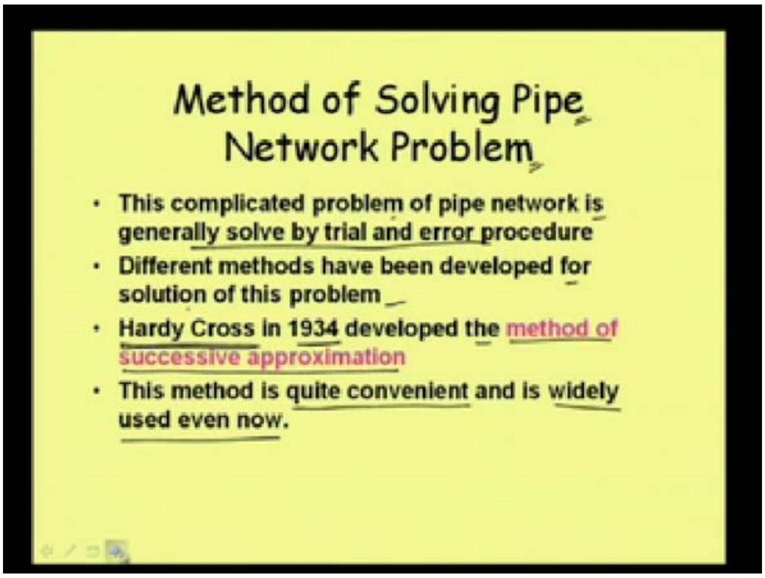

So, let us see step by step, what this step may be, that method of solving pipe network problem, different scientist have attempted solving this problem in different ways. So, this complicated problem of pipe network is generally solved by trial and error procedure, whatever may be the procedure generally it include the trial and error procedure. And different methods have been developed for solution of this particular problem and Hardy Cross, we need to remember this name Hardy Cross, in 1934 developed the method of successive approximation, they name this method as method of successive approximation; that means, it is a basically, it is a iteration, initially we assume some value that is the approximate value we are assuming, then we are solving it and we are trying it to see whether this condition is satisfied head loss equal to 0 around a closed loop and if it is not then we are getting another approximate value, but how to get this approximate value, that procedure we need to discuss, but we will be getting another approximate value, then with that approximate value with this corrected revised value, we will again check whether head loss around the closed loop is 0 or not. So, that is what the procedure, is this method is quite convenient and is widely used even now.

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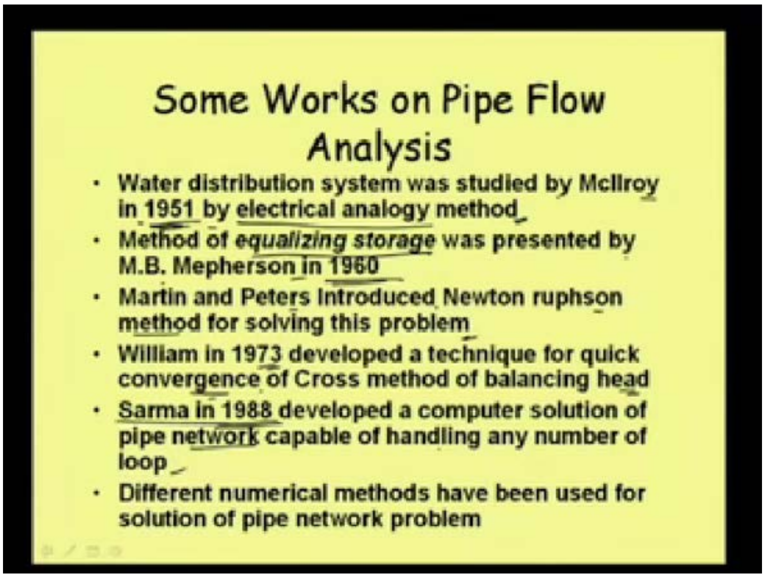

Of course, some other works are there, water distribution system was studied by Mcllory in 1951. So, he studied this Mcllory, he studied in say in 1951 by electrical analogy method. We know that various problem in our hydraulic engineering, particularly when we talk about flow through porous media, head loss in that case, that we study by electrical analogy method, that is pipe, I mean this is not pipe, what we say that flow net, flow net of course, you will have to get that the concept of flow net in fluid mechanics or even in soil mechanics, we get this concept and then there also we apply electrical analogy method and here he applied electrical analogy method and he solved this, a pipe network problem. Then method of equalizing storage was presented by M B Mepherson in 1960. Then Martin and Peters introduced Newton Ruphson method, Newton Ruphson method is a known method in mathematics and that was applied for solution of this pipe network problem. Then William in 1973 developed a technique for quick convergence of this cross method and balancing head.

So, this method was there and this method he tried to have quick convergence in this method and we in 1988 we tried to develop a computer, we did develop a computer solution of pipe network, capable of handling any number of loop. So, whatever may be the number of loop, we just need to give the loop sketch and then from that by using a computer program we develop, then it can solve the solution for the pipe network by Hardy Cross method. And then different numerical methods have been used for solution of the pipe network problem and then now a days commercially lot of software are also available, commercial software are available, I do not want to name these software, but various software are available for solution of the pipe network problem.

Because, here perhaps in this class, we will not be going in to the quite detail of this pipe network analysis, because when we are talking about pipe network, may be that we are talking about pipe, simple pipe connecting and then forming a network, but there will be a valve, there will be different other component in between the pipe network. So, in cooperating all these things, if we try to solve then of course, it become more complicated and now software's are available or simply we can apply our concept and we can develop a computer program for solving this sort of pipe network problem also.

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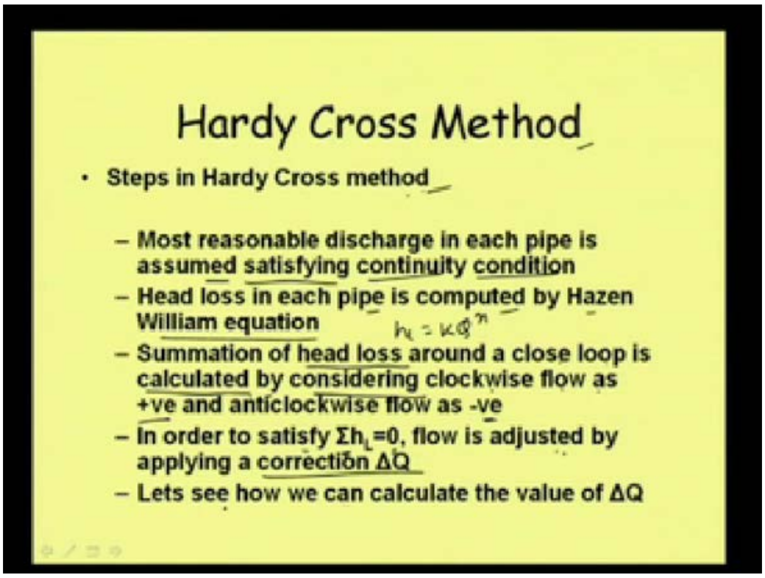

Then, let us see what is Hardy Cross method, which is widely used, and which is very popular till today. Let us start with steps in Hardy Cross method, understanding each and every steps in Hardy Cross method is, of course, important. First step is that most reasonable discharge in each pipe is assumed satisfying continuity condition. I am repeating this point from the beginning. That first assume discharge is made or considered based on the satisfaction of the continuity condition. Then, next step, that is the head loss in each pipe is computed by Hazen William equation, that is k is equal to, head loss h L is equal to k Q to the power n and this n can be considered as 2 for practical purpose. Next step is the summation of head loss, around a close loop is calculated. Now, at this point, we should be careful, if we calculate the summation of head loss around a closed loop and if we just keep on adding the head loss, it will never be equal to 0, say for example, as I did draw here first the loop here, h L 1 definitely from here to here there is a length, definitely from here to here there is a width, I mean there are diameter is there, diameter is there, length is there f value is there. So, we will be getting some head loss and if we go by that k Q square relation, then k value will be there for this, it will never be negative, k value for a pipe will never be negative, as we can see in if we go to our previous slide that k value is f l twice g pi by 4 D square, out of this nothing is negative.

So, this k value, which is this one basically and so, this will never be negative. So, k value will be there. So, k Q square, if we calculate that k Q square, then it will never be negative. So, if we try to add these value, then here also some positive value, here also some positive value, here also some positive value, if we try to add these things, then it will never be equal to 0. So, summation of head loss that way cannot become 0. So, we will have to apply a concept here, that head loss, when it is in suppose clock wise direction, when flow is in the clock wise direction, then we will consider the head loss to be positive. And when the flow in a pipe is in the anti clock wise direction, then we will be considering that head loss is negative, because we are talking about head loss around a close conduit. Now, if we are calculating the head loss from here to here, suppose a pipe is, water is flowing from here to here, from here to here, we can calculate head loss, but if the water is flowing from here to here and if we are calculating head loss in this direction; that means, from here to here, head loss will be what, this head loss is more this head this head is more at this point and this head is less, so, from here to here if we calculate head loss definitely there is no loss rather we are gaining the head and that is why the head loss in case of the flow, where the flow is in anti clock wise direction, then we can considered it as negative. Of course, this is our point that what whether we consider clock wise or anti clock wise, but generally clock wise direction flow head loss is considered positive, then anti clock wise flow in a particular pipe, we considered a head loss that we calculate as negative and that we will have to consider, otherwise our problem will not be solved. So, this point is important.

Head loss around a closed loop is calculated by considering clock wise flow as positive and anti clock wise flow as negative or we can say, head loss for a clock wise flow as positive, head loss for anti clock wise flow as negative. In order to satisfy summation of head loss equal to 0, the flow is adjusted by applying a correction delta Q. So, once we get the head loss in each and every pipe of each and every loop, then we try to check what is the summation of head loss in each of the loop and definitely with our first assumption this will never become 0, then you need to apply some correction, So, if our initial discharge, we are considering is Q 0, then we will have to apply correction delta Q, so, final revise flow will be Q minus delta Q. So, that delta Q is very critical and then we need to find out some way of calculating this delta Q. So, let us see how we can calculate the value of delta Q.

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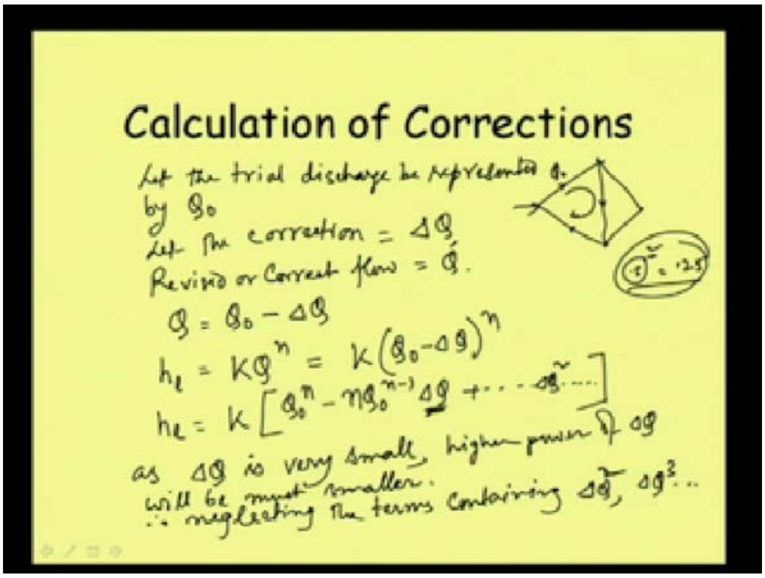

So, calculation of delta Q, in a pipe, let me always keep this diagram here. So, suppose we are talking about this loop and in a pipe, flow in this pipe, let us first write for one pipe, it is Q 0. So, let the trail discharge, so, first trail we are considering the discharge is Q 0, here it will be again Q0, I mean different value of Q 0, we can trail, so, let the trail discharge, trail discharge be represented by, we can write like this, be represented by Q 0. Let the correction is equal to delta Q and then what we can that revise or correct, revised or correct flow, initially we are considering this to be correct, I mean what after making the applying the correction. So, revised or correct flow, we can have that Q is equal to Q. So, what we can have that Q is equal to, this revised flow is equal to Q 0 minus delta Q.

Now, what is head loss? Head loss for the corrected flow what we can have, head loss is equal to k Q to the power n. Let us keep it in the form of Q to the power n first, later on we can make it two. So, k Q to the power n, n is we are keeping in general term. This can be written as, this is nothing, but this is equal to, k Q 0 minus delta Q to the power n. Now, this if we expand this in a binomial series, then what we can write this, h L, let us expand this in the form, that is equal to k, then we can write that Q 0 to the power n minus n Q 0 to the power n minus 1, then delta Q and then of course, we can keep on writing terms here. In the second term, in the next term what it will be coming, this will be n minus 2 and this will be delta Q square, then after that it will be delta Q Q. So, that way, this term will be, this power of this particular term will be increasing in our steps if we keep on writing here. Now, we know that delta Q that correction is very small as compared to the assume flow, because assume flow will have to assumed on the basis of our understanding of the diameter or the k value of a pipe, k value represent everything, I mean, loss in terms of this coefficient represent length then friction factor diameter combining everything gives a k value, which represent the head loss is proportional to Q and with this proportionality constant k.

So, that takes cares of everything. So, as we are assuming to some extent logically the value, we can consider that Q is very small, of course, that way because we are not getting explicitly. So, it will require some trial and error procedure, first let us consider that as delta Q is very small. So, when a small quantity is there, suppose 0.5 or something very small, then this small quantity to the power some power to the power square or Q this will become further smaller as 0.5 to the power square or 0.5 square become 0.25 it is smaller than this Q, it will be further smaller. So, if it is smaller value for delta Q, then we have the delta Q is very small, as delta Q is very small, so, higher power of delta Q will be much smaller. Therefore neglecting the term, neglecting the terms, considering delta Q square containing like that the term containing, delta Q square, delta Q cube or whatever it may be, we are not writing those part I mean this second term next term will be containing say delta Q square, then after that the delta Q cube like that. So, because first is delta Q. So, we are neglecting all those terms which contain delta square Q square delta Q cube. So, neglecting the terms containing this what we can write that now, we can write, h l is equal to k Q 0 to the power n minus n Q 0 to the power n minus 1 and delta Q.

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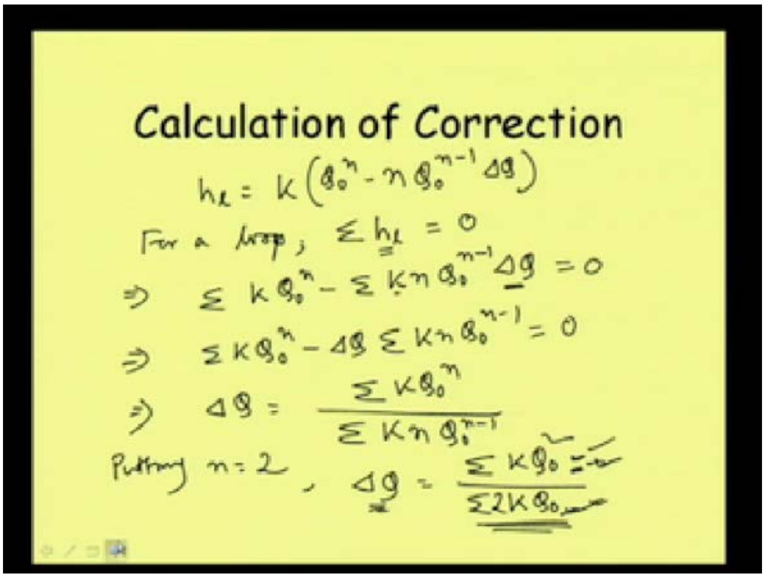

So, here we are neglecting the other part. Now, for a loop, what we have that summation of head loss, we are talking about a loop only, for a loop, summation of head loss should be equal to 0. So, summation of head loss is equal to 0, this implies that summation of k, summation of k Q 0 to the power n minus, summation of k n Q 0 to the power n minus 1 into delta Q is equal to 0. In place of h l, we are putting this term and we are getting these things. Now, in this summation expression, what is the delta Q, that is the correction what we will be applying, this correction what we will be applying, that correction is basically same for all the members of that particular loop, here we are not talking about all the members of the entire network, what we are talking about is that members of a particular loop. And in that particular loop the delta Q the correction what we will be applying is same for all the pipes in that particular loop.

So, delta Q is constant in for the members of a loop. So, delta Q, when we are talking summation this Q 0 and these value will be different, k value will be different, Q 0 for each pipe will be different, but this delta Q is same. So, this delta Q we can keep out from this summation side, common rather we can take common and we can write like this, so, this implies summation of k Q 0 to the power n minus delta Q into summation of k n Q 0 to the power n minus 1 is equal to 0, and this implies that delta Q is equal to summation of k Q 0 to the power n divided by summation of k n Q 0 to the power n minus 1. Now, putting n equal to 2, we have got the expression. So, putting n equal to 2, what we can write delta Q is equal to summation of k Q 0 square and then summation of k n, n will become 1 now. It is summation of k n will become 2. So, we can write summation of twice k Q 0 n minus 1 will become 1. So, this power is 1.

So, this is how we need to calculate delta Q. Now, while calculating delta Q, there is one important point that when we will be summing up this value. So, we know that our head loss should become 0, delta Q should become actually summation of head loss should become 0 means, that correction should become smaller and it should become 0 finally. But this delta Q square, if you take that Q value of each and every pipe and were squaring it, whether we considered a direction to be negative or positive and even if it is squared up, then it will become all positive. So, that we should not do. So, we will have to specify some very specific point about that. The numerator, this value should be summed up considering numerically value is considered that k Q 0 square, that value is fine, but it will it should be considered positive or negative based on the direction of flow, but these value we should add up straight way, whether it is positive or negative I mean everything we should take the absolute value. So, taking this absolute value of this part and considering the positive and negative sign for clock wise and anti clock wise in the numerator, but denominator is absolute value we need to adopt for this twice k Q 0 then this value will become bigger finally, this ratio will become smaller and that way we will be getting the correction delta Q.

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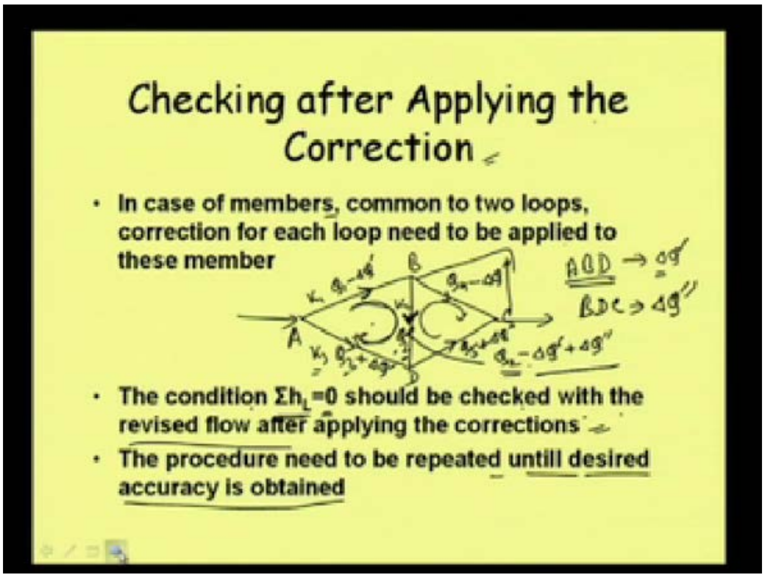

Now, after doing this correction or after calculating this delta Q, what we should do that checking after applying the correction, what sort of checking we need to do. In case of members, first point, in case of members common to two loop, common to two loops, correction for each loop need to be applied to these members. This I am not using the diagram I have drawn earlier here. Again I am drawing a fresh diagram, this is a member which is common to this loop, suppose in this loop we are getting a correction and again this is a loop, where we will be getting some; that means, let me name this say A B C and D.

So, in the loop, A B D in the loop, A B D correction is delta Q 1. So, here discharge is Q, now let me write Q 1 Q 2 and say Q 3. At the discharge, first assume discharge is Q 1 Q 2 and Q 3. Now, after all our calculations summation of k Q and k value is k 1 this is k 2 this is k 3. So, we can calculate head loss in each of the pipe k Q 1 square k Q 2 square k Q 3 square and then we are assuming flow in this direction is like that. Flow in this direction is like that and flow in this direction is this one. So, by this loop, while calculating the correction what we will be doing as the flow here is clock wise we are considering k Q 1 square is positive. Here also it is clock wise in this direction. So, our loop is this one. So, it is clock wise. So, k 2 Q 2 square is also positive, but this our loop is in this direction, it is rotating like that, but the flow is in the opposite direction. So, this we will be considering k 3 Q 3 square will be considered negative and of course, when we are calculating twice k Q, we are just taking absolute value and we are summing up, because we want to calculate the correction in head loss, correction required to making the head loss 0, that is why we are considering it like that.

Now, doing all those calculation using this expression, we will be getting a value of delta Q for this particular loop. So, after applying correction, what will be the value here. If delta Q value is positive, that we should know, if delta Q value, after calculation, if we are getting the delta Q to be positive means, our correction is positive means correction we are considering, wow, here we are considering that correction in a way that if this is the revise flow, then original flow minus the correction. So, when correction value itself we are getting positive; that means, if the flow is clock wise, this is positive. So, in clock wise flow correction if the value we are getting positive, this value will have to be deducted to get the revise value.

But, if our correction value is getting negative, then from clock wise flow, it should be added, for clock wise flow it should be added; that means, Q 0 minus then minus delta, if the value itself is negative that we will have to add. So, these are some critical points and that we need to consider. So, here let me show suppose our delta Q value is positive, we are getting some delta Q value. So, what will be the revise flow in this pipe, this is positive means and this is yet clock wise. So, correction is or corrected value will be Q 1 minus delta Q. So, this I am writing delta Q dash. Dash means this is the correction for this particular loop and then what will be this Q 2 value, this Q 2 value will be Q 2 this is also a clock wise flow is, so, Q 2 minus delta Q dash and what about this Q 3? Here flow is anti clock wise. So, here what is that Q 3 plus delta Q dash Q 3 plus delta Q dash; that means, in a sense, in a easier way in another way also we can say that if delta Q correction is positive we are applying minus delta Q; that means, we are considering the delta Q is flowing in the opposite direction.

So, when it is facing delta Q in the opposite direction flow will become negative, flow will become Q 2 minus delta Q. Here it is Q 1 minus delta Q, but here it is in the direction of the flow correction is we are giving in the opposite direction, because it is minus delta Q we are starting, in the starting itself we have considered. So, it is Q 3 plus delta Q. Then this value is this one, now for this common member, what is happening, for this loop also for the loop B D C, our correction is say delta Q double dash. Now, this correction when we are applying in this loop also suppose it is Q 4, this is Q 5. Then this will become if the delta Q is positive this will become Q 4 minus Q 5 and this will become Q 4 minus delta Q double dash, this is Q 5 minus delta Q, this flow is in this direction, then it will be plus, because this is loop is like that and this is in the anti clock wise direction, so, it will be Q 5 plus delta Q double dash. And then what about this one? In this pipe, original value is Q 2, now correction from this particular loop is Q 2 minus delta Q dash, but again for correction from this particular loop is there, that correction is in fact, this is in the direction of opposite in it is clockwise direction is like that. So, flow is in the opposite direction, so, this will be again plus delta Q double dash.

So, this flow finally, will be Q 2 minus delta Q dash plus delta Q double dash, like that we need to apply the correction from both the loop to this particular pipe to have the corrected value, but this procedure is followed to get the corrected value and then the condition a summation of h l equal to 0 should be checked with the revised flow after applying corrections. So, once we apply the correction to each and every loop, there may be several loop again connecting to this one there may be several loop like that. So, applying the correction to these loops, we need to again recheck, whether we are now having the condition that summation of h l equal to 0 or not.

Once we get this value summation of h l equal to 0. Then we should stop, that we have got the correct value., If not then we need to again repeat the procedure starting from calculation of a new delta Q 0 and then solving it for a new and then deducting or adding based on whether it is positive or negative we can get the solution. So, this procedure is followed in Hardy Cross method and this procedure need to be repeated until desired accuracy is obtained. So, here we do not say that we will be going up to the 0 value, we will not be going up to the 0 value of delta Q, but if our discharge is of the order say 50 cumec or I mean this order may be 5 cumec 6 cumec, that is order then when our if in that case suppose delta Q value is coming in decimal 0.2 say if it is 5 or 5.2, it does not make much difference that correction we may not apply. So, based on our total flow or say approximate flow, what it should be or order of flow that we are getting, based on that we should stop once our correction become smaller. So, significantly smaller to influence the revise value. Once our delta q is very small, even if we apply the correction, the revise value will not change and with that suppose revise value will not change significantly, then again if we repeat, we will be getting same order of correction and revise value will not change, it will be fluctuating between that. So, we should not come to 0, but we should come to a desired accuracy level. That is what the procedure of solving the pipe network by Hardy Cross method.

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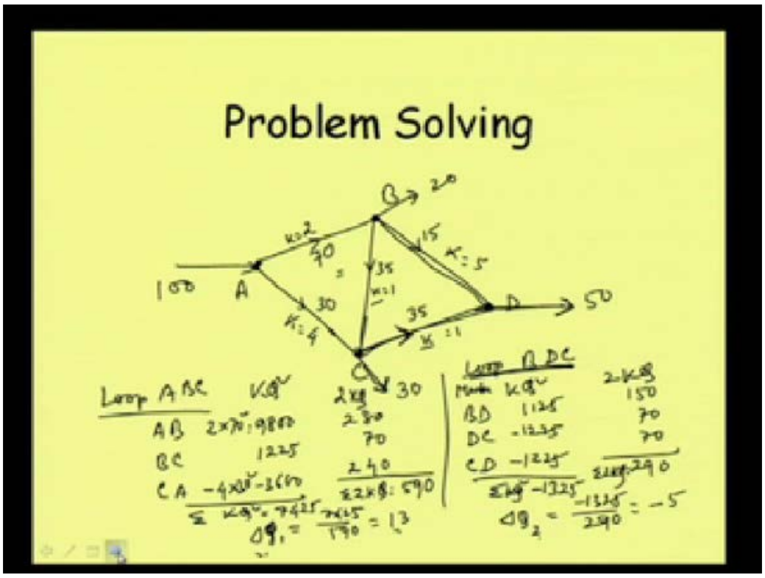

Now, let us see that in quickly, let us just see quickly, taking a problem and to see that how we can solve this problem. I am taking a solved problem from the book, but just to explain how this problem is solved and let us see say the problem is again this typical loop what we are talking about already, this loop is there. A discharge of 100 is coming, whether it is cumec or whatever it is that we are not interested, I get this movement. Discharge of 100 is coming and this loop name is A B then C and D and then from this point from the point B, there is a out flow of 20, then from D there is a out flow of 50, then from again the entire continuity also must be satisfied, if hundred cumec is coming, 20 is going from this side, 50 is going from this side, if nothing is given and just say something is going out from this side also, that point, that it must be equal to 30, because the total going out and total coming in to the loop must be same first. Then let us apply our just understanding or general experience and let us see how we can divide the discharge into different pipe.

Before that let us keep some k value, k here is equal to 5, well for this is 2 k here is equal to k here is equal to 5, then k this value is equal to 1, then k of a c is equal to 4 k of C D is equal to again 1. Now, let us assume the first approximation, as this k value is smaller; that means, head loss k Q to the power n order or rather when k value is smaller, we can assumed the rough, we can consider that roughness is less or total resistance offer is less or head loss will be less in this direction, So, for a smaller k value, we can always consider the higher discharge. So, let us first assume that 100 will be divided like this, here it will be going 70 and here it is coming 30, then from this point 20 is going out. So, remaining is now we need to check continuity at each point, here the continuity is satisfied, 100 coming in 70 and 30 going out means 100 going out now here 70 is coming in. So, from 70, we need to see the 20 is must 20 must be going out because it is already given in the problem. So, remaining is 50. Now, we need to just divide this 50. So, this k value is very large 5 as compared to this k value. So, what we can assume that this k value is less means let these be 35 out of 50, then this is 15, then 35 is coming in this direction and 30 is coming from this direction.

So, this value must be, let us calculate 35 is coming from this direction, 35 is coming from this direction, 30 is again going out from this point that we need to check 30 is going out from this point. So, here in this direction, it must be 35 then only 30 plus 35 must be equal to 30, which is going out this point and 35 which is going out from this node. So, that way now it is satisfying all the condition let us verify if our assumptions are correct 15 is coming from this point 35 is going. So, 15 plus 35 is equal to 50 and finally, 50 is going out from this point/ So, that way first approximation of the discharge we have put now we will be able to show you one step of calculation let us see and then we can of course, do practice with the other steps. We can solve these in the form of a tables.

First let us consider the loop A B C. So, in the loop A B C, what are the member A B then B C and then C A. Now, our interest is to find k Q square value and of course, we will be calculating twice k Q this value. Let me write k Q square here and twice k Q here. Then if we put the value of k and Q square, I am just directly writing this value, this value one step the here A B k is equal to 2 and 70 square this value is equal to 9800. The next value will be 1225, mention this, these are positive because clock wise, but this value which is in the opposite direction will become negative.

So, what this negative value is, it is say minus we can write 4 into 30 square. So, this value will become minus 3600. Now, if we sum up this value, then it become that summation of k Q square, this is becoming 7425, then what about twice k Q, is twice k Q is becoming 280 for first loop, first then this is 70, here we are not considering positive negative sign, everything is absolute value 240. So, this value is becoming 590. So, summation of twice k Q is becoming 590. Then if we calculate the delta Q, so, this delta q is becoming equal to say 7425 divided by 590, this is equal to 13. So, this delta Q is the correction for this particular loop. Then loop means A B C. Then if we consider loop say B D C, B D C means B D C. So, in this B D C loop, again we can calculate k Q square and twice k Q and the member we can write down here, member we can write member B D, then D C and C B. B D from here to here D C and C D. Now, k Q square here it will be 1125, then this will be negative, because it is in the opposite direction, so, it is minus 1225, I am not showing the calculation, this value will also be minus 1225, because here also 35 in the anti clock wise direction, here also 35 in the anti clock wise direction, k value is 1 k value is 1 here. So, for both the pipe it will be the same, then we can calculate what is summation of k Q square, summation of k Q square is becoming minus 1325 and whether twice k Q this value will be 150, then this is 70 and it is 70 again, so, you can check the calculation later and this is 290 twice k Q. So, these value summation of twice k Q. So, here the correction, delta Q is equal to, we are having minus 1325 divided by 290 and this is equal to, we are getting minus 5. So, this correction we are getting minus 5 and this correction we are getting plus 5. We can write this as 1 and this as 2. Now, if we apply this correction how this flow will change. Now, this delta Q 1 and delta Q we are getting the value.

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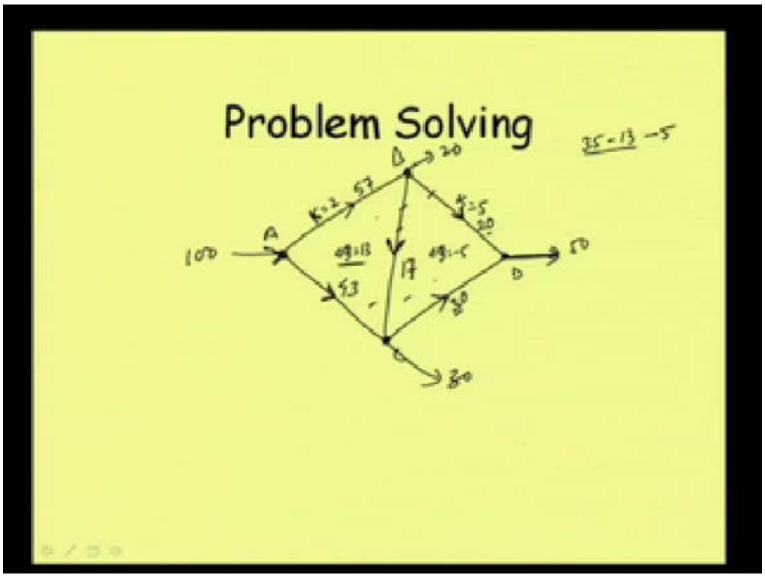

Now, let me see or let us see how we can apply this correction to this our earlier loop. So, here in this and here we are putting the delta Q is equal to 13. So, plus 13 means that we will have to deduct from the clock wise flow and minus 5 means that we will have to add to the clock wise flow and just the reverse things we will have to do here. So, delta Q is equal to 13, so, if we just recall what was the our original assume value it was 70. So, this value will be 70 minus 13. So, this value will become now 57 and then if we see this particular pipe, here our first assume value was 35. So, first assume value it was 35. So, now, it will be, this is also clock wise. So, from 35 we need to deduct 13. So, right now we are not deducting this, we know that because there is this is a member common to this one as well as this one.

So, what we will do here it should be 35 minus 13, but we are not deducting that right now, we are keeping it here. And then this pipe as we can see, here if we apply the correction then it is anti clock wise, this flow is anti clock wise, so, this 13 we will have to add here, if we add this 13, then earlier this flow was 30. So, now, if we add 13, then it will become 43, then similarly in this loop it is minus 5 this correction is minus 5. So, earlier flow it is clock wise, so, correction is minus means we will have to add to the clock wise flow, So, earlier this flow was first assume flow was 15. So, now this flow will be 15 plus 5 that will be 20. And then here this earlier assume flow was 35. So, now, this new flow will be 35, here, but the flow is anti clock wise. So, it is anti clock wise and correction is negative. So, from anti clock wise we will have to deduct, so, it was earlier, we have this earlier flow was this one 35. So, now, it will be 30 here after correction. So, it will be 30 and what about this? This flow earlier was our 35. So, here this 35 and then our this flow is say in the opposite direction.

So, we need t, this flow it is in anti clock wise direction our flow is this direction clock wise is this one this is anti clock wise direction. So, from anti clock wise flow, the negative flow should be deducted, positive flow should be added to anti clock wise flow. So, it is negative flow. So, that we will have to deduct from here. So, earlier it was 35 that should be again minus 5. So, what we will be getting, 35 then again minus 5 will be coming. So, it will be, total will be minus 5. So, this will be 30 minus 13 finally, this will become 17, this will become 17. So, like that we are getting the different flow distributed in this entire network. And now we can of course, verify whether our condition is satisfied or not because this is very important, we can check, 100 is coming 57 is going out. 43 is going out, 43 plus 57 again it is 100. So, at this point, continuity condition is satisfied. At this condition 57 is coming in, 17 is going out, 20 is going out and 20 is going out, so, 20 plus 20, 40 plus 17, 57. So, total 57 is going out from this and this 57 is coming in. Here also if we check, our continuity condition will be satisfied. Then again if we check here, 20 is coming in 30 is coming in 50 is going out, our continuity condition is satisfied. But this head loss condition again may not be satisfied. So, how to check, we again need to check because k value are known again, k value are not changing, these are renaming same.

Of course, if we consider in this pipe flow problem, if we consider f l v square by twice g divided by f l Q square, those formula if we consider and if f value, we consider to be varying with the Reynolds number, then of course, our k value can keep on changing, we can use a variable k value or this roughness value, then then of course, the every time we need to calculate Reynolds number and indirectly we need to check that value, but here of course, for simple situation we are considering k to be constant and that way we can again calculate the same we can repeat the procedure and we can see whether our summation of head loss is satisfied rather what we are calculating, we should calculate the delta Q in both the loop. And when we will be getting delta Q in both the loop to be very small as compared to that order of the discharge that is flowing in different pipe. Then we can say that our flow what we are getting is correct flow and that repetition of course, we do not have sufficient time to complete the entire repetition, because this will require 3 4 repetition to make the delta Q very small.

But, I hope that we tried to explain the procedure and I hope that this procedure we have explained and so, if you try to do the same procedure, if you try for the getting for calculating the discharge value in the next few step again and then if you check it after 2 3 trail, I hope you will be getting the correct result. So, with this we are concluding our class on this pipe network analysis, in the next class we will be going for some more interesting topic of pipe flow that is, so, which are very much important in water power sector, like water hammer and those things we will be discussing in our next class.

Thank you very much.