

Hydraulics
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Module No. # 08

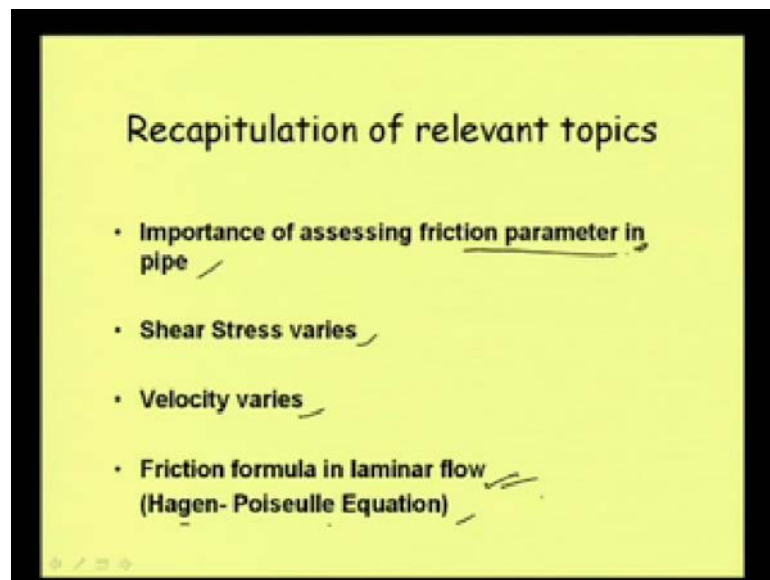
Pipe flow

Lecture No # 02

Pipe Flow: Losses in Pipes

Friends, welcome you again to this class of pipe flow. We have already discussed one class on this pipe flow and we will be continuing on our discussion on this particular issue. We started with the topic that how loss of energy in pipe is important and we will be continuing from that, how we can find some mathematical expression for that, that also we will be discussed today.

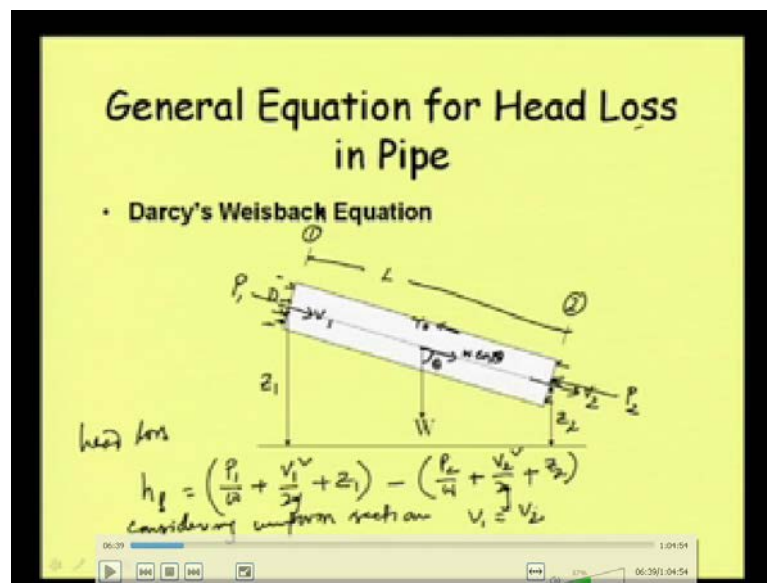
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We were stating in the last class with that importance of assessing friction parameter in pipe. Then, we were talking about for, actually in fact to get that particular topic in more detail, that assess the friction parameter, we were discussing on how shear stress varies within the pipe diameter, within the pipe section. And then, we were discussing how velocity varies within the section in a pipe and then with the help of these relationship,

that is shear stress and velocity then finally, we try to get a relationship how the friction loss can be assessed and so, we obtained the friction formula in laminar flow, that we call as Hagen-Poiseuille equation and today we will be starting with another general equation, that we call as Darcy's Weisbach equation. So, this Darcy's Weisbach equation today we shall be discussing. This can be called as a general equation of head loss in pipe.

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Let us consider this, suppose a pipe and the length of this pipe be L and the diameter of the pipe be D. On the upstream side, there will be definitely pressure force from this side and from the downstream side, there will be also pressure force from this side and so, we need to equate this pressures and then not equate, rather we need to see how much pressure force from this side, how much pressure force from the downstream side and also we need to see what is the shear force exerted, because of the shear stress τ_0 at the boundary.

And the liquid is flowing, velocity here is v_1 and velocity that is flowing at this point is v_2 . Of course, as we did discuss earlier also, in a pipe if the diameter of the pipe is not changing, it is a uniform cross section then this v_1 and v_2 will remain same and the elevation head of this particular point 1, we can refer this is 1, this is 2, the elevation for this particular point is z_1 and elevation to the second section is z_2 . And, let us consider the (()) sorry the weight of this fluid, suppose this pipe is carrying all this fluid and

weight of the fluid is acting in the vertical direction and if this angle is theta, then we know that another force that is the weight of the fluid is also acting and component of this weight in this direction, we can always write as $W \cos \theta$, because our pipe is inclined here and we are taking the angle with the vertical force. So, this force is $W \cos \theta$.

Now, let us see what is the head loss? So, total head here will be something and with that head, I forget to write about the pressure here, pressure acting here is P and pressure acting from this side is P_2 , this is P_1 and this is P_2 . Then, let us write down the head at this point and head at this point and the difference between the head in fact is the head drop or drop in head. So, what we can write and that is what the hydraulic loss in hydraulic head. So, we can write that head loss is h_l , let me write head loss h_l is equal to head at this point, we will be writing P_1 by W , P_1 by W plus v_1 square by twice g plus z_1 . This is the total head at the section 1 minus the head at the section 2.

So, this will be P_2 by W plus v_2 square by twice g plus z_2 and this is what our head loss is. Now, as the section is same so, considering uniform section what we can have, if it is if the section is uniform, uniform section then we have that v_1 is equal to v_2 because discharge is same. So, if discharge is same v_1 is equal to v_2 , then we get that head loss is equal to, let me write the head loss here, that h_l head loss is equal to P_1 by W plus z_1 minus P_2 by W plus z_2 .

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Handwritten derivation of the head loss equation:

$$h_l = \left(\frac{P_1}{\gamma} + z_1 \right) - \left(\frac{P_2}{\gamma} + z_2 \right)$$

Applying force balance for Equilibrium.

$$P_1 A - P_2 A - \gamma \pi D L + W \cos \theta = 0$$

$$(P_1 - P_2) \frac{\pi D^2}{4} - \gamma \pi D L + W \frac{\pi D}{4} L \left(\frac{z_1 - z_2}{L} \right) = 0$$

Simplifying

$$\left(\frac{P_1 - P_2}{\gamma} \right) + (z_1 - z_2) = \frac{4 \gamma L}{\omega D}$$

Thus, $h_l = \frac{4 \gamma L}{\omega D} \rightarrow \frac{4 \gamma}{\omega} = \frac{h_l \omega D}{L}$

dividing by $\frac{\rho V \omega D}{L} \rightarrow \left(\frac{4 \gamma}{\rho V \omega} \right) = \frac{1}{f} = \frac{h_l \omega D}{\rho V \omega L}$

So, this is the head loss. Now, just if we get in this form, it is not sufficient, we need to have this in a different form. So, that we can directly apply to equation. So, applying the relationship, considering balance of forces, equilibrium of forces, what we can write? So, applying force balance what we can write, for equilibrium condition. So, from the upstream side, if the sectional area, if the cross sectional area of this section is A_1 and then cross sectional area of this section is A_2 . Of course, we know that A_1 is equal to A_2 is equal to πD^2 by 4. Now, what will be the pressure force from this side P_1 into πD^2 by 4 and what is the pressure force from that side, that is downstream side will be P_2 into πD^2 by 4.

So, difference in pressure force we can definitely write as well let me write here, first as $P_1 A$ minus $P_2 A$, that is A is same, then what about the shear stress, that shear force this stress is τ_0 . So, what is the total area πD is the perimeter here. So, πD multiplied by the length that will give us the surface area as such when we talk about the shear force or the shear resistance offered this will be, we can write as say τ_0 into $\pi D L$ and then of course, another force as we did here another force is the $W \cos \theta$ which is acting in the downstream direction. So, this $W \cos \theta$ will have to be added.

So, plus $W \cos \theta$, this is equal to 0. Just rewriting this part, what we can write that P_1 minus P_2 into πD^2 by 4 minus $\tau_0 \pi D L$ plus. Now, what will be this weight? Weight is nothing, but the volume of the liquid here, which is again equal to πD^2 by 4 means sectional area into length multiplied by its unit weight. So, and that will give us the W and then its component in this direction we will have to take. So, what we can write that W , that is unit weight of the fluid and then πD^2 by 4 and length, area into length and that we can have. And of course, what we have this $\cos \theta$, that $\cos \theta$ we can write as in terms of z .

What is that θ if $\cos \theta$ we can write, there is a difference between these two level, that difference is nothing, but say z_1 minus z_2 . So, this difference is there and what is this length. So, this angle means this angle. So, this angle \cos , \cos of this angle is nothing, but say we can have this total length L , this is the hypotenuse and then we can have this z_1 minus z_2 divided. So, we will be getting the $\cos \theta$. So, that way what the $\cos \theta$ we can write into z_1 minus z_2 divided by the length is equal to 0. Then, if we simplify this relation, simplifying what we can get, P_1 minus P_2 by W , that we are writing plus z_1 minus z_2 , that will be equal to $4 \tau_0 L$ divided by W into D .

So, that will be the relation and now in fact this particular expression, $P_1 - P_2$ by W and $z_1 - z_2$, this is giving in fact the total difference of this one, then that is why this is nothing, but the head loss. What we can have, that we already wrote these things head loss is equal to this part and that if we here just we are writing P_1 by W minus P_2 by W and then plus $z_1 - z_2$. So, this is nothing, but the head loss. So, we can write thus that head loss h_l equal to $4 \tau_0 L$ by $W D$. From this expression, we can again rewrite this into a different form. That can be written as $4 \tau_0$, why we are writing that we have a definite intension is equal to h_l head loss, we are writing on this side into W into D by L $4 \tau_0$ is equal to $W D h_l$ by L . Now, this $4 \tau_0$ means, we are writing about the shear stress, this $4 \tau_0$ means, this τ_0 is coming from the property of the surface of the pipe and then other property of the fluid combined effect that is which is basically influencing the entire resistance.

So, this is giving that resistance property and as we did discuss earlier also that, this resistance depends on the square of the velocity in turbulent flow. So, that way considering all these issues, if we divide it by, let us divide the expression dividing by ρv^2 by 2, if we divide that will give us $4 \tau_0$ divided by ρv^2 by 2 and that expression we write as, in fact, f . That expression we write as f and then this is equal to here also we will have to divide it by $h_l W D$, then divided by ρv^2 by 2. Now, this expression can be now written as, h_l now our interest is from this expression, this we should know that, this is what we are writing as f and this f is called friction factor, and just from this relation, we can now write what is head loss. What is head loss that can be written.

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The image shows a handwritten derivation of Darcy's Weisbach equation. At the top, the friction factor f is defined as $f = \frac{h_L W D}{L \rho V^2}$. Below this, the head loss h_L is expressed as $h_L = \frac{f L V^2}{2 g D}$, which is boxed and labeled "Darcy's Weisbach Eqn.". To the right of this box, the text "(Laminar flow)" is written. Below the boxed equation, the derivation continues: $h_L = \frac{32 \mu V L}{W D^2}$ is written, followed by an arrow pointing to $\Rightarrow h_L = \frac{64}{\rho V D} \frac{L V^2}{2 g D}$. Another arrow points to the final simplified form: $\Rightarrow h_L = \frac{64}{Re} \frac{L V^2}{2 g D}$. To the right of this final equation, the friction factor is given as $f = \frac{64}{Re}$.

So, h_L , let me keep that expression also, here we are writing that f is equal to h_L , then $W D$ by ρV^2 by, $h_L W D$ by ρV^2 by 2 and L should have been here, L is here. So, $L \rho V^2$ by 2. So, that we can write as $L \rho V^2$ by 2 and from here we can write h_L is equal to $f L V^2$ writing capital L here. So, $f L V^2$, then W by ρ is nothing, but g , so, we can write twice $g D f L V^2$ by twice $g D$ and that is the very popular expression which is commonly used in pipe flow and this particular equation is known as Darcy's Weisbach equation. And this equation is used for determining friction loss in pipe, where friction factor is very important parameter, then how to know this friction factor that is again an issue and different scientists has in fact, work on that for finding what can be the value of friction vector.

At this point, I want to just say another point, that in laminar flow, when we were discussing, then we got one expression that h_L hasn't partial equation, that we are referring again, that we got that h_L is equal to $32 \mu V L W D^2$. We got this expression. Now, if we just write this expression, this is for laminar flow. Now, if we write this equation in the form of this equation, that is Darcy's Weisbach equation, what we will be getting. Say, h_L if we write and then first let me write here, $f L V^2$ that is f we are not writing, just we are writing $L V^2$ by twice $g D$. Now, when we are writing this that $L V^2$ by twice $g D$ then of course, in place of V , we are writing V^2 , so, we are putting one V here and then in place of diameter, this D also let me

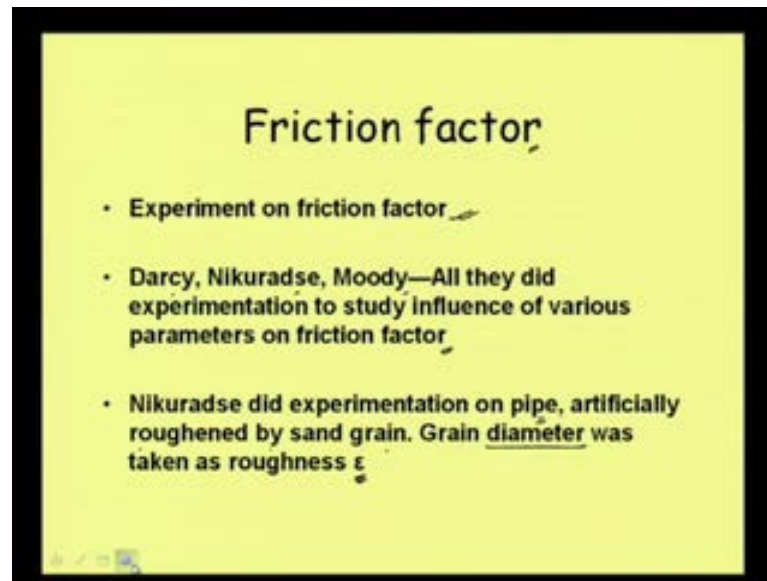
write capital, because we are writing here capital D to represent diameter, we are writing D here.

So, in place of D, in place of D square, we are writing D here. So, one D will have to write here and then we are writing, we have not written mu here. So, we will have to write the mu here and as such if we just change this, if we just rewrite this expression, L we are writing, then v we are writing 1 . So, we are writing $1 v$ here, then mu, we are, mu here we are not writing so, we will have to write the mu divided mu. So, we will be writing it here, from this part. So, we are writing this expression here and then what are, what the value we have not written here, if we put all those in this part, then it will becoming 132 is here and 2 we are writing here. So, this will become 64 and this part will become $\rho v D$ by μ . Now, if we just check we will be finally, ultimately if we combine these things we will be getting this expression.

So, what this value is $\rho v d$ by μ , this is nothing, but the Reynolds number. So, we can write that h_l is equal to 64 by Reynolds number into $L v$ square by twice $g D$. Now, if you just compared this and that, this equation and that equation, then what we will be finding? This implies what that f is, we are replacing by 64 by Re . So, in laminar flow, we get that friction factor is equal to 64 by Re , that is friction factor is inversely proportional to the Re . So, in laminar flow, it is not influenced by frictional roughness of the pipe that, what we did discuss earlier, in the last class, that same things we can establish from this relation, that we are getting only it is related to the Reynolds number.

So, it is here the frictional characteristic is not coming. So, frictional roughness of the rather frictional roughness of the side is not coming, side pipe boundary is not coming. So, we can write f is equal to 64 by Re , it is thus inversely proportional. Then, let us see how we can get this value of friction factor, otherwise these equations when we are using so, this value of friction factor is of great important, otherwise we are not being able to apply this equation.

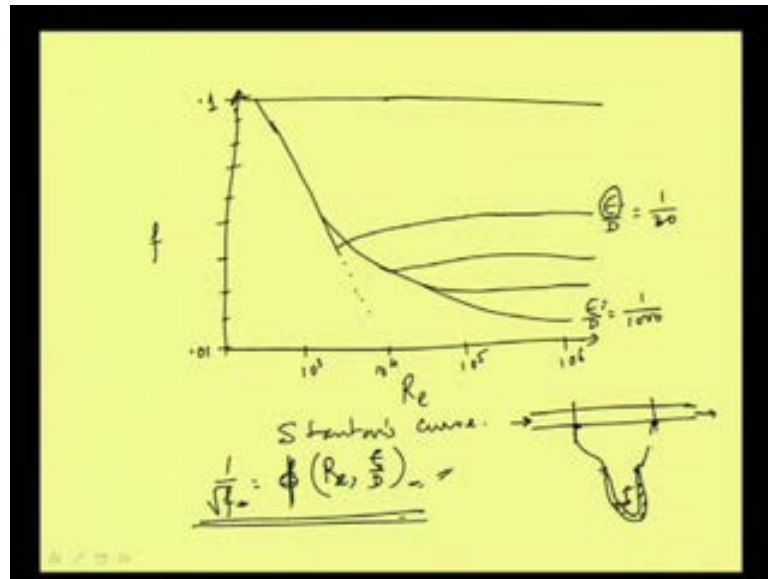
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So, experiment on friction factor that was conducted, lot of experiments were conducted rather by different scientists, out of that, we should take name of Darcy and Nikuradse, Moody, then all they did experimentation to study influence of various parameters on friction factor. And all these discussion we are not taking here, we are not covering in this particular class, because well in fluid mechanics, we are suppose to do these things in details, but just as we are discussing about friction factor, we are taking some of these issues and just we are recalling those issue which we should have studied in our fluid mechanics. And then, Nikuradse, he did experimentation on pipe, then how to take roughness artificially roughened by sand grain, he make artificial roughness into the pipe using the sand grain and then for the roughness value, generally the symbol used is epsilon that roughness value, he considered to be the diameter of the pipe.

So, the grain diameter, this grain diameter mean, the sand grain diameter, that sand use for making the pipe rough, inside of the pipe the sand layers were given and that way he could get roughness of the pipe and then what roughness he will be taking, what the actual value of the roughness he will be taking, the epsilon value what he will be taking, we took that as a diameter of the sand grain. So, that way he conducted experiment and then he found a graph relating all those different values and let me just draw the graph in a very rough way so, that we can have some idea.

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So, he draw the graph, plotted those value, on this side. In fact, Nikuradse conducted this experiment and Stanton this curves are known as Stanton curve, because he also did this work and then finally, he plotted those curve in this form, that is why it is also known as Stanton's diagram and on this side, it is say 0.01 then friction factor, that means, here is the friction factor value 0.02, then up to 0.1 it is. So, these value were there and then on this side it is Reynolds number, which is going like say 10 to the power 3, then log scale this 10 to the power 4 and 10 to the power of 5, like that 10 to the power 6 and what they could observe that when the Reynolds number is low, we did discuss up to 2000, it remains laminar. So, for a low Reynolds number the friction factor value varies linearly like that. It will be coming in this form, it is varying linearly friction factor with Reynolds number.

So, as we got in this relation that friction factor varies linearly with Reynolds number of course, inversely and that relation is indicated by our graph here and of course, we will be putting some dotted point here, because now it is in transition. And then, till this point, till Reynolds number is lower, between 2000 less than 2000, then the roughness is not influencing, but gradually this roughness will start influencing. We can have a curve like this.

So, for a value of say epsilon by D, epsilon is the roughness and D is the diameter, for suppose it is very small roughness, say 1 by 1000 around. That means, epsilon, if the

diameter of the pipe is 1000, then roughness is only 1. So, it is a very small roughness, but still roughness is becoming significant or it is influencing the curve when this Reynolds number is increasing or we can say that when the flow is becoming turbulent. So, that way this is going like that, then with more roughness, our value is increasing like that, curves are changing like that, curves will be like that. So, for ϵ by D , large ϵ by D value may be 1 by 30, if diameter is 30, then suppose roughness is 1, then we get this sort of curve.

Now, this sort of curves can be used, these curves are popularly known as Stanton's curve and these curves can be used for, suppose once we know the Reynolds number, once we know the roughness of the pipe, then we can have the value of this friction factor. For that Reynolds number, we will come to this curve and we will be seeing what the friction factor based on this particular roughness, then again another point that what this ϵ value will be. Again, Moody has conducted different experimentation on this issue and he also gave his own sets of curves, then he provided a table for in fact, this roughness value for different material, if the material is wood, if the material is steel, so for different type of, so, wood was not there, steel, if the material is concrete, then for different type of material what will be this roughness value.

So, that was also given and then, it became convenient to use this relation. So, we knowing the material we can take a value of roughness. Of course, all these basically need experimentation and so, what can be suggested, though these curves are available, fine if you do not have any other way, we will have to follow this curve to find the value of friction factor, because the friction factor is the most important parameter, it is $f L v^2$ by $2 g D$, we got the equation and here of course, there may be another confusing point that this Darcy-Weisbach equation, we called that $f L v^2$ by $2 g D$ it is fine, but in many books that as we can see that we are replacing this term, we are replacing this $4 \rho v^2$ by ρv^2 by 2, if we take this 4 on the other side, then this equation can also be written as, h_l is equal to $4 f L v^2$ by $2 g D$.

I mean, some people prefer to use this equation, that is why 4 should be, they want to show the 4 explicitly, but in a sense that friction factor also of course, we will have to give another name as f' and this equation is known as Fanning's equation. Some people prefer to show this 4 explicitly which is coming here, this 4 and however, some

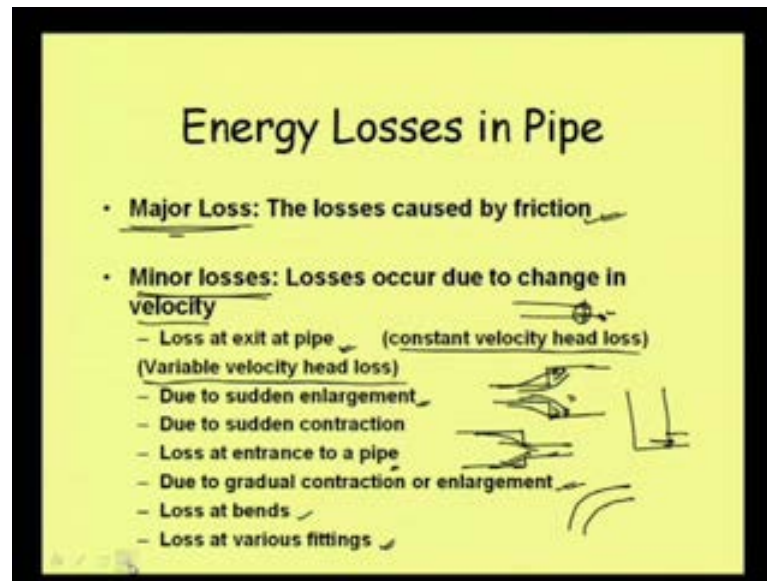
people feel that as the friction factor ultimately it is a constant, so, we want we can show it just as $f = 1$ value.

So, both the equations are used and when we are using, why I am just emphasizing this point? Because value of friction factor is very important and when we will be using the value of friction factor, then suppose we are getting the friction factor value for the expression this one and then this value if we take and apply here, then the head loss, that we will be getting will be on lower side, it will be underestimated. So, we should know that for which formula actually this friction factor value is being given and that friction factor only we should use appropriately, we should be very much careful and when we do not have any other way, we will have to take recourse to this curve. In fact, Moody has also given some curves, the Moody's curve is also very popular, that also we can use and then we have some explicit formula, that is relating Re some explicit formula of f generally, it comes in the form of $1/\sqrt{f}$ is equal to which is a function of this.

This I am writing function, I am writing it is a function of Reynolds number, then ϵ/D . So, as a function of all those some logarithmic relation can be developed and different investigators have given from their experiment different expression for that and Colebrook give one equation, like that in the Jain we get another equation in the Jain book. So, that way that is sort of expressions are there and we can use these equations also for calculating the f value using Reynolds number and this roughness ratio. So, roughness by diameter and, but now although all these things are available, what is necessary, that if we have a device with us and we know that this particular pipe will be used for a particular work, if possible we can conduct laboratory experimentation and we can find very easily what the friction loss is.

That sort of experimentation is not that difficult, we can fix the pipe and we can allow the water to flow from here to here and then from these 2 points, suppose any 2 section, we can fix a manometer and using that manometer, we can see the difference in the pressure level, mercury level here and then from that we can find out what is the head loss. So, if you do that, then we will be more convincingly we will be getting the friction factor value.

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So, just I tried to emphasize, what the value of friction factor and the reason is that energy loss in pipe, generally we divided into two parts, one is major loss, another is minor loss. And the major loss is basically coming from the friction loss. So, that is why, if we calculate this friction factor wrongly, our entire effort will go wrong. So, of course, now when some company they provide the pipe, their pipe is this one and then they specify what the friction factor value is for, that particular pipe, that way also in the market, when we purchase a pipe this may be available. Then major loss as I have explained the energy loss we can divided into two part, that is the loss cause by the friction. And then, there are some minor loss and of course, this major loss one characteristic is that this loss is due to constant velocity, when the water is flowing in a pipe, there is no change of velocity, still there we are finding that head loss is occurring. So, this sort of loss is also called as a constant velocity loss, constant velocity head loss.

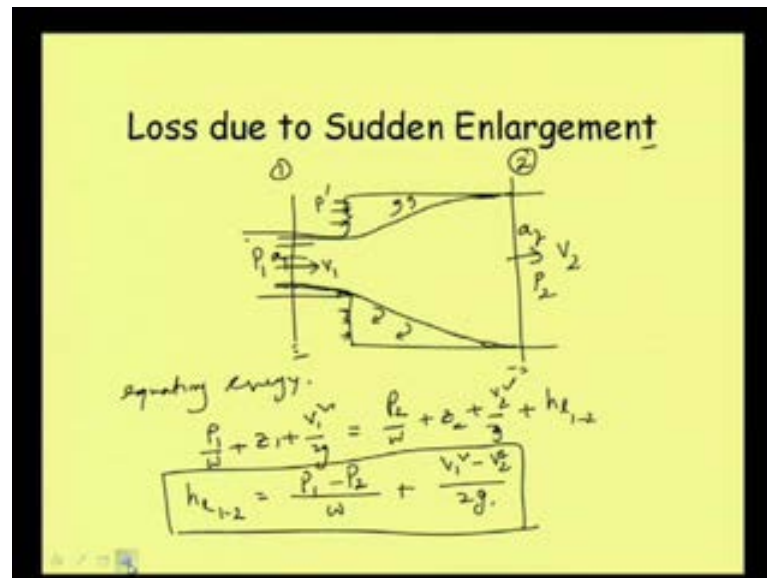
And minor loss, minor losses are losses that occur due to change in velocity. So, minor loss are that which are very small as compared to the major loss, but and these losses occurs generally due to change in velocity, in the pipe, because of many reason, the velocity can, there can be changed in velocity and because of change in velocity, this sort of loss occur of course, we can list the minor loss and there we will find that 1 loss which is called the loss at exit at the pipe. So, when a pipe is carrying water at the exit point, at the exit point here, so, ultimately the water is getting released at this point, at the exit point, there is a loss. Although in this case velocity is not changing, whatever velocity is

coming here, it is in going out at the same velocity, but still there is a loss which is small in value as compared to the friction loss and that particular loss, we call we put in minor loss and, but this loss is for constant velocity head loss and then other minor losses are variable velocity head loss. So, due to sudden enlargement of the pipe, a pipe is like this and there is a sudden enlargement in the pipe, then there will be head loss in the pipe. So, water is flowing like this and then there will be some eddy generation like that and then this will cause lot of, I mean some amount of head loss here.

And due to sudden contraction so, if there is a contraction like this, then also there is a head loss. So, in a pipe system, we should try to avoid this sort of contraction or enlargement if possible, but definitely we cannot do that, we required this sort of contraction and enlargement and now as we required this sort of contraction and enlargement. So, we will have to know that how much energy will be lost at that particular point and of course, the loss in contraction is less than the loss in this, I mean sudden enlargement, then loss at entrance to the pipe. Suppose, there is a reservoir from here, the pipe is water is entering. So, at the entry point itself some losses there. And due to gradual contraction and enlargement, we have already told about the contraction and enlargement, but if this is gradual, say it is not abrupt, rather it is expanding like this, then also there will be some, I mean loss and if contraction is gradual like this, there will be some loss, but this will be definitely different from this one, then loss at bend in a pipe, we will be having this sort of bend and if there is a bend, then there will be loss of energy at that point.

So, how many bends are there depending on that we will have to calculate the loss of energy and we should avoid bend wherever possible and loss of various fittings, as we know that to regulate the flow we need to use valve, at some point we use some t sections. So, this sort of various fittings we use there and those fittings, when the water is flowing through those fittings, there will be change in diameter at those point to some extent and those fittings also there is a loss of energy in those fittings and all these are coming under minor loss.

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Now, we can just discuss about loss due to sudden enlargement first. So, when we talk about loss due to sudden enlargement, let me draw a diagram, you can concentrate on to the slide, that this is the enlarged section, pipe section has enlarged in this way. Now, how the water will be flowing in it, water is flowing from this side and then it is moving like this and it is again going to that point. And when water get expanded like this, there will be low pressure in this portion as compared to here and there will be eddy formation and then because of that, we will be having loss in this system. And then, water is flowing like this, now if we considered just on upstream although in the diagram it is appearing that we are drawing much ahead, but this is a just on upstream of this enlargement if we take a section 1 and just at downstream of the section where it is flowing full, let me take it here, where it is flowing full again. That part is suppose section 2.

Flowing full means, here also it is flowing full, but the stream lines are touching this one, this stream lines, which is emerging from here it is touching at this point so, this is section 2. Then, water velocity here is v_2 in this part, unlike the earlier discussion velocity here is changing here, the velocity is v_1 , here the velocity is v_2 . So, head loss due to variable head is now, variable velocity is now in equation and pressure here is P_1 and pressure here is P_2 . Of course, we can write the pressure in that portion what will be, here it is P_1 , here it is P_2 . Let us write for the first case, let us take it as P dash, pressure here what is acting in the water is P dash.

Now, what we can write, that if we write the energy level between this point and that point, so, equating energy, what we can write P_1 by W plus z_1 plus, we are writing in general first v_1 square by twice g , but as we are talking about a very small section here, from here to here, z value will be in fact equal, z value will be in fact equal because this distance is not that much even if it in incline position the z value will be more or less same.

So, this is equal to P_2 by W plus z_2 plus v_2 square by twice g plus the head loss and so finally, from here what we can write that head loss and we are writing 1 to 2, head loss from 1 to 2 can be written as P_1 minus P_2 divided by W plus v_1 square minus v_2 square divided by twice g . So, that is what is our head loss is, but of course, we need a better expression. This expression is same that our head loss is the difference in pressure head and then velocity head. v_2 square by twice g . Now, from the momentum equation if we apply, that is the force balance if we do from here, some force is coming, from here some forces are there, if we try to equate those things then if you take this area as a , area suppose as a_1 and area here is a_2 , and velocity already we have written as this things.

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Handwritten derivation of the momentum equation for fluid flow in a pipe:

$$\begin{aligned} \text{Momentum/sec. through section 1} &= \rho a_1 v_1^2 \\ \text{Momentum/sec. through section 2} &= \rho a_2 v_2^2 \\ \text{Rate of change of Momentum} &= \rho (a_2 v_2^2 - a_1 v_1^2) \\ \text{Newton's eqn.} & \\ \text{Force} &= P_1 a_1 + (a_2 - a_1) P - P_2 a_2 = \rho (a_2 v_2^2 - a_1 v_1^2) \\ \text{as the fluid at the common area at } & \text{neck, } a_1 v_1 = a_2 v_2 \\ \text{considering } P_1 &= P \end{aligned}$$

$$\begin{aligned} a_2 (P_1 - P_2) &= \rho (a_2 v_2^2 - a_1 v_1^2) \\ \Rightarrow \frac{P_1 - P_2}{W} &= \frac{\rho}{W} (v_2^2 - \frac{a_1}{a_2} v_1^2) = \frac{v_2^2}{g} - \frac{v_1 v_2}{g} \\ h_{L1-2} &= \frac{v_2^2}{g} - \frac{v_1 v_2}{g} + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = \frac{v_1^2}{2g} - \frac{v_1 v_2}{g} + \frac{v_2^2}{2g} \\ &= \frac{1}{2g} (v_1^2 - 2v_1 v_2 + v_2^2) \end{aligned}$$

So, what is the momentum per second or through the section 1, what is the mass flowing if ρ is the area into velocity is the, we can write here area into velocity, a_1 into v_1 . This is the discharge coming and then if you multiplied by ρ , it is the mass coming, and then mass into velocity if you do, that means, this is $\rho a_1 v_1$ square. So, this is

what the mass momentum passing through this section 1 per second, then momentum per second through section 2, this is equal to $\rho a_2 v_2^2$, then rate of change of momentum, we are coming to Newton's equation now. So, rate of change of momentum, we can write as $\rho a_2 v_2^2 - a_1 v_1^2$. So, this is the change in momentum, which should be equal to applied force. So, from Newton's equation, what we can write, forces now what are the forces, the force we can write as, force is equal to $p_1 a_1$, this is one force, then if I consider this pressure to be p , then what is the area? This diameter is at this area is a_2 . So, a_2 is the entire bigger area. That means, this bigger area from here to here, there is no change in section.

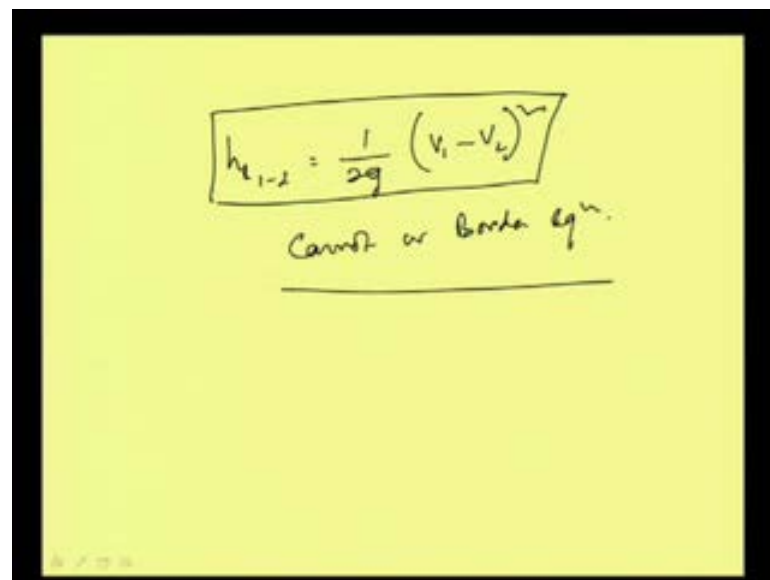
So, this is the bigger area a_2 minus so, this area minus this because smaller area will give us the area here, this portion of area and this area multiplied by p . Let us write it like that first, that is the a_2 minus a_1 into p and minus $p_2 a_2$ and this is equal to the force, so $\rho a_2 v_2^2 - a_1 v_1^2$. And as the fluid at the corner is at rest, this is at rest and then here the pressure and the pressure at this point can be considered almost equal and just that upstream although we are drawing it far part, but this p_1 is the pressure here, so as this fluid is in rest, so we can consider this pressure and that pressure to be almost same, because fluid in rest and it is in contact. So, this pressure and that pressure we can that is the pressure p_1 and p can be considered to be almost equal.

So, considering as the fluid at the corner fluid at the corner are at rest, so considering P_1 is equal to P , that we are doing and then we can just simplify this expression to say, a_2 into P_1 minus P_2 is equal to $\rho a_2 v_2^2 - a_1 v_1^2$. So, what we can write, this is equal to say, $P_1 - P_2$, our basic intention is that, we are trying to have an expression for this particular term, $P_1 - P_2$ by W . So, $P_1 - P_2$ by W , that we can write as, ρ by W , then v_2^2 minus a_1 by a_2 into v_1^2 and this we can further simplify to have a relation that this is equal to, so this ρ by W is nothing, but 1 by g . So, what we can write this as, v_2^2 by g minus, again this a_1 by a_2 , what we can write for that, we know that discharge is same.

So, we have that $a_1 v_1$ is equal to $a_2 v_2$. So, a_1 by a_2 is equal to we can write v_2 by v_1 . So, if we write it as v_2 by v_1 , so what will be having this expression will become $v_1 v_2$ by g . So, it is becoming $v_1 v_2$ by g . So, from $p_1 - p_2$ by W is now we are expressing in terms of v . Our basic intention is that, on the basis of difference in

velocity, we are trying to find an expression for head loss. So, if it is like that, this we can further simplify, say head loss from 1 to 2, now can be written as v_2^2 by g minus $v_1 v_2$ by g plus v_1^2 by $2g$ minus v_2^2 by $2g$. So now, this v_2^2 by g minus v_2^2 by $2g$, that this minus half of this that will become again, that we can write as say v_2^2 by $2g$ first, then minus $v_1 v_2$ by g plus v_1^2 by $2g$. And that can be written as now, if we take 1 by $2g$ common, so this is equal to, say 1 by $2g$ we can bring common and then what will be having v_1^2 square minus twice $v_1 v_2$ plus v_2^2 square, this is $(v_1 - v_2)^2$. So, finally, this expression is nothing, but $(v_1 - v_2)^2$ whole square. This expression is nothing but $(v_1 - v_2)^2$ whole square.

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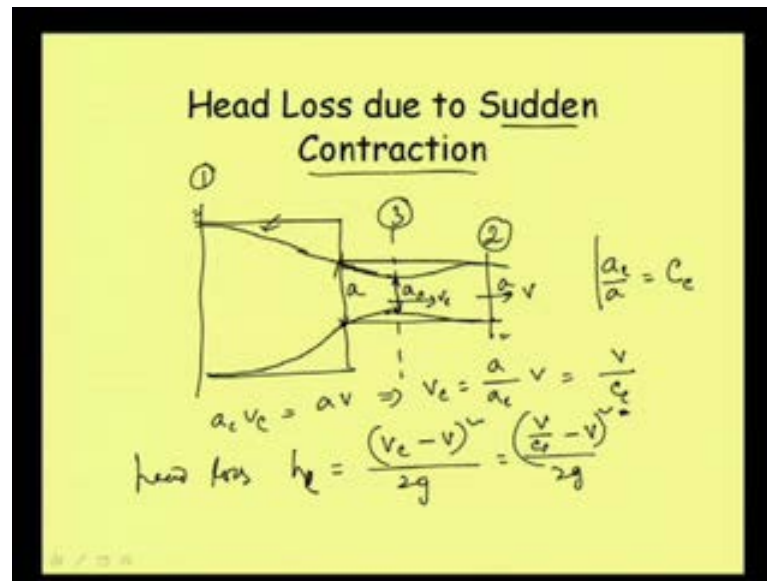


$$h_{1-2} = \frac{1}{2g} (v_1 - v_2)^2$$

Carnot or Borda eqn.

I am writing this expression here, that this can be written like that, that head loss from 1 to 2 can now finally, be written as $\frac{1}{2g} (v_1 - v_2)^2$ whole square. This equation is known as say Carnot or Borda equation, because as per as they did this. So, this equation is, popularly used for computing this head loss due to sudden expansion.

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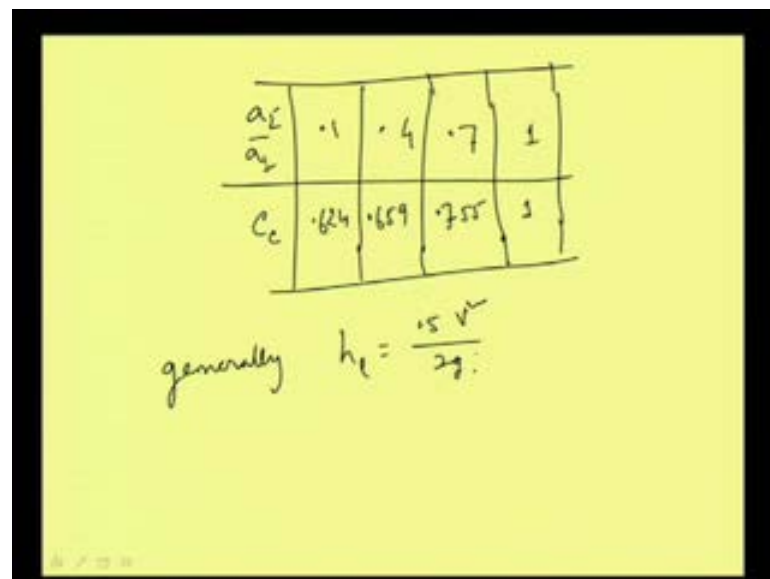
Now, when we talk about head loss due to sudden contraction, then this head loss due to sudden contraction is nothing indirectly we get it from head loss due to sudden enlargement. Let us concentrate on the slide. If we draw this particular relation for a contraction, just we can see that, this is, we can draw in this form. And here water will be coming in this way and then there will be a contraction like that, water will be, water is entering into the pipe like this and up to certain extent it will continue flow like this and then there is an expansion. It will finally meet the level here. And this contracted portion here we have the minimum area, that we write as a_c and it is like that called Vena contracta.

So, that we are getting here and diameter at this point is or say area, area at this point is say a and the area at this point or this point is same. So, this is the area a at this point, say this is section 2 and this is an important section here, this is 3 and let this section be section 1. In fact, the loss is basically due to not because of this contraction part, but because of this expansion part that is water is expanding from here to here, that is the very basic reason of the head loss here and well as this is the reason. So, let us first find what will be the velocity at this point, we know that say velocity here is suppose is v and that what will be the velocity here, say, a_c and suppose velocity here we are writing v_c .

So, a_2 into v_2 is equal to a_1 into v_1 , and this implies that our v_2 is equal to a_1 by a_2 into v_1 and this a_1 by a_2 in fact, of course, not a_1 by a_2 , a_2 by a_1 is called, I mean this is written a_2 by a_1 that is given as a coefficient C_c , contraction coefficient, say.

So, this C_c , we can have here that a_2 by a_1 and that is why we can write this as v_2 by C_c , well, this is what we can have and then of course, we have standard value for this C_c , we have some value for this C_c for different ratio of say this area and this area. Say, if we give this as a_2 by a_1 , then, that as portion to this ratio we can have some value for this C_c and as such, this can be used for calculating the value. Now, head loss as we were saying that head loss is equal to head loss h_L is equal to, so, our velocity at this point v_2 minus v_1 , because just like it is expansion, $1 - \frac{1}{2} \frac{v_2^2}{g}$ and this velocity we should write, say, well let me write first v_2 minus v_1 whole square divided by twice g , just like expansion loss from here to here and v_2 is nothing, but what we can write v_2 by say, C_c minus v_1 whole square divided by twice g . Well, this expression can be used for computing the loss due to sudden contraction.

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$\frac{a_2}{a_1}$	0.1	0.4	0.7	1
C_c	0.624	0.859	0.955	1

generally $h_L = \frac{v_1^2}{2g} (1 - C_c^2)$

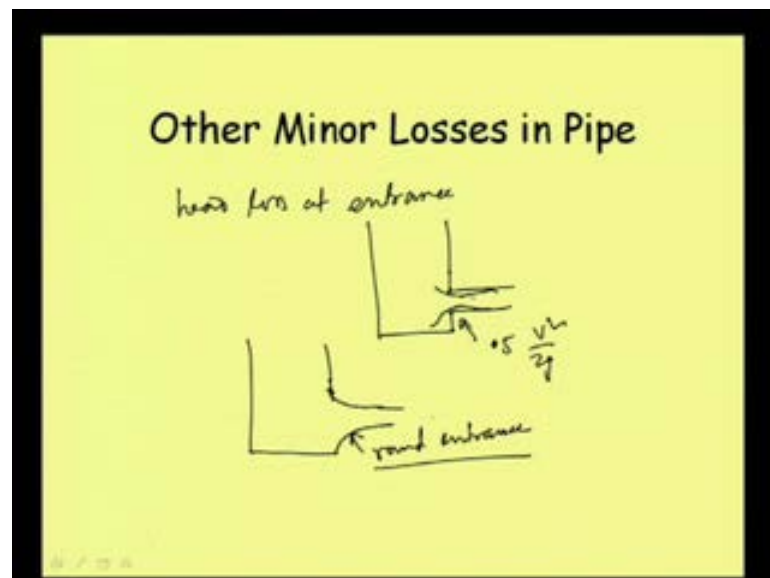
And then, we can have as I said, that we have this sort of ratio say, a_2 by a_1 , if we put say, and then we can have the value of C_c , there is a 0.1, 0.4, 0.7, 1. 1 means this pipe is not having any contraction. So, if we get this sort of a table and then this table gives that this is equal to 0.624 when it is 0.1, a_2 by a_1 , a_2 is the downstream point that one and

that mean whatever it is, this is smaller one, this is bigger one a 2 by a 1. So, that is why it is becoming suppose 10 time thus bigger one is 10 time larger. So, we are getting 0.1.

So, it is 0.624 this value is C_c and this 0.659, this is 0.755, with the decrease of this ratio this coefficient is basically increasing and then, **sorry**, with the increase of this ratio this coefficient is increasing then when it is 1 then there is no value of this C_c , because there is no contraction. So, using that relation we can finally, find the value of course, generally for most of the cases if we do not have this sort of I mean, chart for a particular situation then and for simplicity because this losses as said that these will be these are minor loss and very small.

So, generally this head loss h_l due to contraction is taken as equal to say, $0.5 v^2$ by twice g , v is the velocity. That means, in directly what we can say here this is v and C_c value if we put, we will be getting ultimately this, in terms of v^2 by twice g and of course, some other term will be there this twice v^2 , v^2 that will also becomes say, v^2 square and some coefficient will be there. So, it will be a I mean putting those value we will be getting nearly equal to this expression and then the value can be written approximated at $0.5 v^2$ by twice g .

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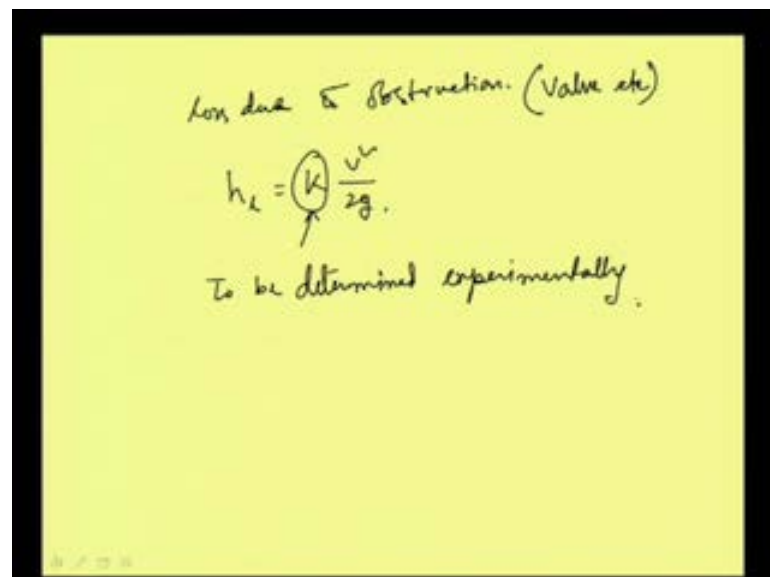


Similarly, now when we talk about say head loss in entrance. So, head loss at entrance So, head loss at entrance, if the entrance point is like that, say, entrance point is like that

sharp is. So, water will be entering like this and then it is flowing here and this can be considered as $0.5 v^2$ by twice g .

So, a very common expression that we use is $0.5 v^2$ by twice g . These are fine, but if we try to reduce, if we think that we need to reduce this head loss then what we can do at in where we have the entry point then, there we need to make rounding of this pipe. So, if we can have our connection in this form then our head loss will be minimum. There will be some head loss, but, it will be if we can round it off. So, round entrance, the head loss will be less. So, if possible we should make our entry point in this form.

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Loss due to obstruction. (Valve etc)

$$h_L = \left(K \right) \frac{v^2}{2g}$$

To be determined experimentally.

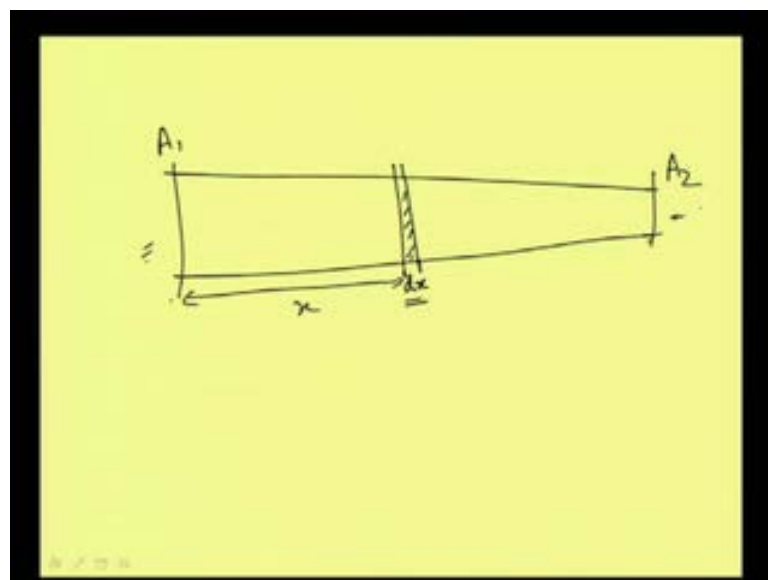
Then we can have some other losses like loss due to abstraction. Abstraction means, this abstraction by what the what we can mean, that it is the fittings suppose, any fittings are coming and in the pipe as we said that there can be valve and other things.

So, all those when you combine then, we totally in totality we can say that these are some abstraction to the entire pipe flow system and for all those abstraction what we can take the head loss as, this is normally taking a very definite value is difficult. Generally, from our experience from observation we should try to estimate what sort of loss that can be and generally, a K value we can say, we are knowing that a coefficient will be definitely coming in terms of change in velocity or normal velocity, if the abstraction is not there then in the pipe we can have the velocity suppose v then the loss will be always say proportional to v^2 by twice g , but there will be some coefficient multiplied by

that. So, we expressed this loss, any loss in this form say valve etcetera, loss due to abstraction means, what we are meaning it can be valve etcetera and that is, h_l head loss we give as say $K v^2$ by twice g and this K is the coefficient that we need to determine, that is determine to be determined experimentally.

So, that way we did discuss already about some of the losses, the pattern of losses and we have listed some minor losses, if we go back to our this point of loss due to entrance in pipe that we have discussed explicitly. Loss then, due to gradual contraction and enlargement that point what we did mention in fact, as is as if we are rounding off. So, when it is a gradual contraction then this sort of abrupt change is not there. So, eddy formation and those sort of things are coming less and when in a pipe system we can make a gradual change whether it is contraction or expansion then, we can reduce the head loss. And of course, if a pipe is tapering this is thus changes if the if a pipe is tapering here we should not confuse with this part that gradual expansion and enlargement, if the pipe is tapering, well loss in bend is also like that we need to keep a K value and then we need to determine this loss in bend and by experimentally we should determine it and fittings already we have mentioned then and say, what when we are talking about loss due to say gradual contraction.

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Then there is another issue that, if a pipe itself is contracting like this. If a pipe itself is contracting, tapering pipe. Well, the pipe is tapering and in that case the friction formula

that we are using that, $f L v^2$ by twice $g D$ will not be straight way applicable. We need to take care of the area, because area is changing. Here suppose area is A_1 and here the area is A_2 then, this sort of changes we need to consider that it is tapering and that we can find out by applying the principle that for a small elementary length say dx , we can find out what is the dx for a small elementary line say dx we can find out the what will be the area from A_1 to A_2 , if it is like that at a distance dx at a distance x for small elementary area dx , what will be the area we can find out, and that way for a length of dx how much will be the head loss that we can find out and then to find out the total head loss from here to here and for finding the head loss here we can use the formula $f L v^2$ by twice g because it is a small length and change in say area between these 2 point is not that much significant.

So, we can write $f L v^2$ by twice g here, but the L , what is that L will be dx here and then velocity of course, we will have to take v based on the area ratio and then, we can find out the with the friction characteristic f we can find out these value and then finally, to get the total head loss we need to integrate it from this area to that area and then we will be getting what is the head loss in thus in this sort of pipe.

So, in this particular class, we have tried to see that in a pipe system, apart from the friction loss what are the other kind of loss that we can have and of course, initially we have discussed the loss due to friction which is a major loss and we have a general formula of friction loss that is $f L v^2$ by twice g which can be applied, but while applying it is easy formula we can apply, but for applying this formula the value of f is very important and that is why we have also discussed from where we can have these value of f ? Sets of curves are available, Moody's curve or say, curve given that we have already discussed here the Stanton's curve and so, that way curves are available. Similarly, some formula are also available, explicit formulas are available that if we know the reynolds number value, if we know the roughness value then we can calculate the friction factor directly by using some explicit formula.

So, with all those formula we can find the value of f , but still if we are not convenient confident and suppose, our problem is very much important and very sensitive then we should know about the friction value directly by experimentation and we can develop a setup, developing a setup for conducting that experiment is also not that difficult. So, we can develop a setup for conducting experiment, for finding the friction factor value of

different pipes and then that we can use, of course, company which produce pipe they also provide some frictional characteristic of the pipe, friction factor value they also provide, but for very precise work we need to know this friction factor our self and then knowing the friction factor we will be getting the energy loss due to friction and apart from that in our entire system there will be lots of bends, there will be lots of enlargement, there will be lots of valves. And for all those enlargement, contraction valves and bends, we will (()) will be having some sort of loss that are of course, very small as compared to the friction loss in the pipe, but those losses we termed as minor loss and at the exit at the entry also we have some loss, but one important point is that minor loss are of course, not that significant when our length of the pipe is very large.

In a very large pipe system where length is very large this minor loss, if the joints and these things are less then we can neglect, but if the pipe system is not that long, if our length of the pipe is not that long and then in that case minor loss does not become insignificant. Then we will have to considered the minor loss because, major loss is less when the pipe length is less and that is why then the minor loss also become dominant.

Well, with this we have understood some of the issues related to friction factor and calculating head loss and then using these in the next class we will be going for discussing some more topic in pipe flow. Thank you very much.