Hydraulics

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Module No. # 08
Pipe Flow
Lecture No. # 01
Pipe Flow: Friction Loss

Friends, welcome you all to this new topic, it is a pipe flow. In fact, till now, we were discussing most of our topic that were related to open channel flow, but today, we will be starting our new topic, that is pipe flow. As we know that, water in the nature, we find that these are flowing through open channel that is rivers, streams and all, but when we come to man made things, suppose we are trying to carry water from one point to other or may be any other fluid when we try to carry from one point to another point, like say it may be oil, it may be water, it may be other, any kind of fluid. Then many a time, we take help of the pipe, that is, we carry those things through pipe.

Of course, when for carrying water into the agricultural field, we use canal, that part we have already discussed, but when we try to carry water through some close conduit, that is suppose we do not want interaction of that water with anythings outside, then mainly, we prefer this sort of pipe flow. And there are some other advantages of course, by carrying water through pipe flow, and that is why this. When we do this design, that we are trying to carry water from one point to another point that is from the source, one point means where our source is available, and from that source we are trying to carry the water to some other point. When we talk about water distribution system, say in a city, then say our water may be available from a source, where may be we are doing treatment, we are treating water and then we are collecting the water and from that source we need to carry the water to far distance. Then, there are some important points, say we are using pipe to carry the water, that we call as a delivery pipe well.

Now, water from one point to another point will be carried, when there is a energy difference. As we know that when there is a energy difference or when there is energy at the upstream is higher than the energy at downstream, then only the water can flow. So, in pipe when the water is flowing, during that process of flow, like open channel there will be also some resistance offered by the pipe and that way some energy will get lost. And in that process the available energy will gradually reduce and if we do not design the pipe in a proper way, then our intended purpose, suppose we thought that water we will be carrying up to a certain point, the water may not be receiving that point, because water will not reach that point, because required energy may not be available. Similarly, some other problem may arise like say, when as we know that water in the pipe flows in full flow. It always flow full means, there in open channel flow the fluid flow as under atmospheric pressure.

So, it is flowing with a free surface in open channel, but in pipe, it is flowing full. So, it is under pressure. Now, in our design purpose, there is another necessary point that what is the pressure that is been created inside the pipe. If that pressure is very high and if that pipe material is not sufficient, not of sufficient strength, then it may happen that the pipe may burst. So, for all those design activities, we need to know the hydraulics of pipe flow and that is why we will be starting this topic, that pipe flow, what are the losses there and how we need to design this sort of pipe flow to carry our water from sources to our required destination.

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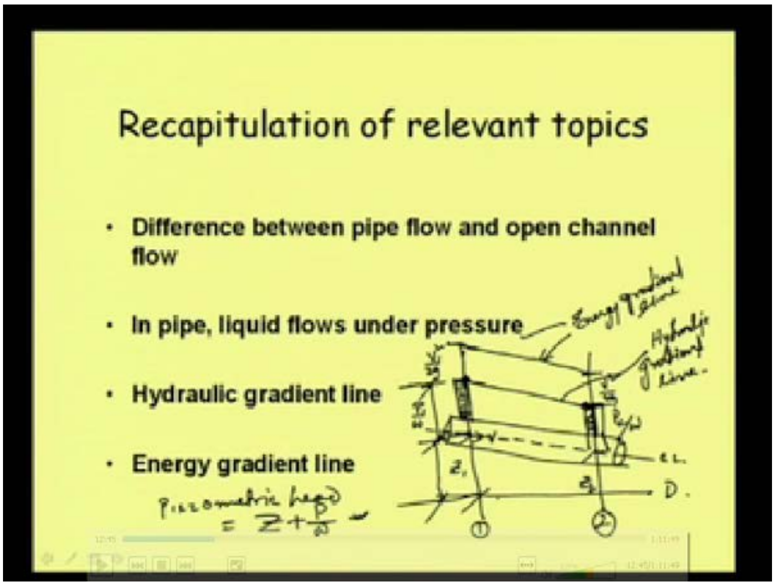

Then just to recall, what we did or what are the relevant topic that we have discussed earlier, if we just recall our very first class of open channel flow, there itself we did discuss, what are the differences of pipe flow and open channel flow. In fact, there we discuss lot of things about pipe flow also although our basic objective was to discuss open channel flow. And at least, just now as I have mentioned that point, that difference, major difference between open channel flow and pipe flow is that, in open channel flow the water flow under atmospheric pressure and in the pipe flow, the water flow under pressure. That is one of the major difference that we could see.

So, if you refer to this slide, I am just pointing out that in pipe liquid flows under pressure, this is one of the important point. And then, so far, as hydraulic gradient line is concerned, in open channel flow, we did draw our hydraulic gradient line. In fact, hydraulic gradient line in open channel flow is nothing, but the surface of the water, but here we do not have a surface of water, because the pipe is flowing full. So, we are not having a free surface. So, what will be our hydraulic gradient line? That we need to know, just we are repeating what we did discuss in our very first class of open channel flow.

If the pipe is there like that, then water is flowing, then if we consider a datum, if I take 2 section, this one and two, and from the datum, this is our datum and from the datum, generally in open channel flow, we did measure it up to the bed, but here we are not talking about any bed, because it is not like that, we do not have a free surface neither we have a bed. So, we talk about the center line, this is center line of the pipe. And then depth from the datum up to the center line is the elevation head, z 1, and then the pressure, there will be some pressure, now if we insert a peizometer here, then because there is pressure, water will start rising in the peizometer. And it will go up to that height, where pressure at this point pressure at this point will be just balanced by this height, it will be just balanced by this height. So, this height, we call as p, that is pressure by w, p by w. So, this is the difference between open channel flow and the pipe flow. And above that, you will be having, the velocity head, if the flow here is having velocity v, then the energy due to velocity is same v square by twice g, that we can have.

If the section is 1 we are marking here 1 and 2. So, for the 1, we can put p 1 by w, then in the section 2, this will be z 2, then we have up to this much the water is rising in the peizometer and then that height we refer as say p 2 by w and then above that, we have v square by twice g. This is the energy head. Ws we did discuss about this point earlier also, I am not discussing in more detail, but of course, just to recall what we did, we are just placing this point. And then, the line joining this point to that point, that is which is equivalent to the surface of water; that means, if we do not keep any barrier, water will be rising up to this point because there is a pressure. So, that point we are considering and here we are considering this point and that way the line joining this point and that point, that is we call as hydraulic gradient line. So, this will be hydraulic gradient line. And the head, if we join this total head means energy, total energy then velocity head is also coming, then if we join this 2 point, then this is the energy gradient line.

Now, the head from the datum up to this point, that is z 1 plus p 1 by w or in general, we can z plus p by w. This z plus p by w total is called peizometric head. So, what is peizometric head? Peizometric head is equal to z plus p by w. In some book you will be finding the symbol gamma to represent unit weight of water and of course, it is w n gamma is used both, is used to represent unit weight of water and here we are writing w and that way z plus p by w is the peizometric head. And that is also important in the analysis of pipe flow, because in many a time, we see that what the energy here is because, the velocity, because ay why we are emphasizing on this particular point, because generally pipe will be a manmade structure. And in a manmade structure, we generally made the pipe of, diameter of the pipe at this point, section 1 and section 2 will not be different generally. If it is not a converging pipe or diverging pipe constructed or made with a particular intention, otherwise for normal pipe as you see the diameter between 2 point will be same. That is, it flows with a uniform diameter. Now, when it is flowing with a uniform diameter the point is that discharge flowing through the pipe is constant.

Now, if discharge flowing through the pipe is constant, then what about the velocity? When the diameter is uniform at every section, then the velocity will also be equal. I mean area will be equal. So, discharge by area that will also be equal. Now, therefore, the velocity between the upstream point and downstream point may not have that much difference. Mow if it is not having that much difference, main difference, main energy difference is coming, because of z plus p by w and that is why, this peizometric head what is available at upstream and downstream is more important here. With this introduction about some of the fundamentals which of course, we did discuss in our class earlier, we can now go to the actual topic of pipe flow.

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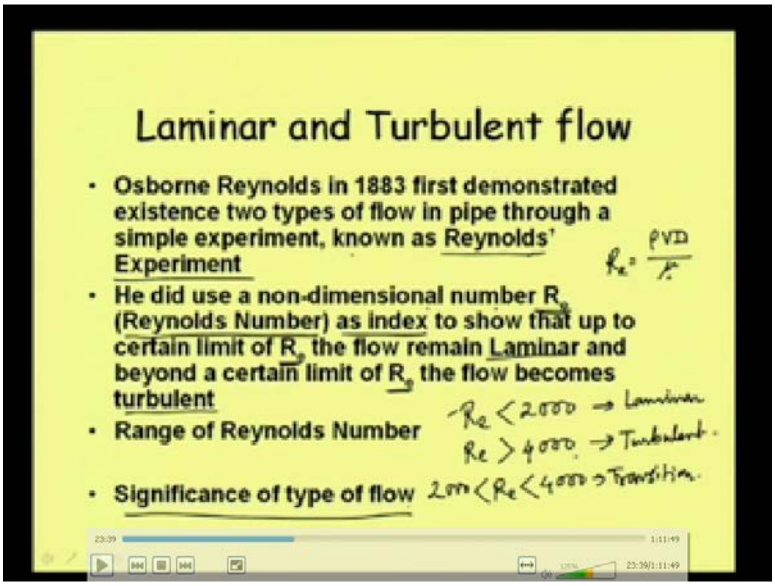

And let us see, that in pipe flow, as we are discussing that energy loss is important, I mean knowing about the energy loss is important. And pipe flow, the governing force are there in open channel flow, we were using a dimensionless number. In most of the analysis, for hydraulic jump and for other sort of analysis, we are talking about froude number, because in that flow problem, in open channel flow problem, the gravity force is more dominating, but in pipe flow, the viscous force is more dominating, and that is why we talk about reynolds number here. In most of the analysis we will be using, the Reynolds number, that will be playing a very important role, Reynolds number. So, that is why it is important and how it came basically, first Reynolds, the scientist Reynolds, Osborne Reynolds, in 1883, he first demonstrated that there exist 2 type of flow in pipe. He first observed the existence of 2 type of flow in pipe and that he did through a simple experiment, known as Reynolds' experiment.

When he was conducting this experiment this Reynolds' experiment, it is like that, he was using some dye, some colour pigment and then that dye was put. So, water was flowing through a pipe and then he allowed the water to flow through a pipe and he had the device that he can increase the velocity, he can decrease the velocity in the pipe by controlling some valve and then he applied some dye. And that dye shows the path of the fluid particle. So, when the flow is occurring through a pipe, if a dye is just added in a small quantity at a particular point, if the flow are flowing in a laminar way; that means, in a lines, the stream lines that are flowing to the pipes are parallel, then the dye will also be seen as a straight line. And then, we can understand that this is a type of flow, where the particle of the fluid are moving in a very stream line pattern.

Then if we gradually increase the fluid velocity, a time will come, when the fluid particle will be starting mixing up and then it will gradually increase and then at one point, all the fluid particle will be mixing up and although we are putting the dye at a particular point at the entry, but just when it starts flowing, it will get mix up and we will not find a trace of the dye. So, what indicate that well up to certain velocity, the flow, after certain velocity, the flow is behaving in a different way of course, although I am saying, that it is a certain velocity, but well in the experiment, we can find that velocity we are increasing and then it is happening, but velocity is not the only point, not the only point, which influence this sort of change in flow pattern, because with that velocity, suppose in a particular velocity, in a particular diameter, we are finding that the fluid is getting mixing up, but a same velocity if we allow to move with a very small diameter pipe, then we may find that that sort of mixing up is not happening; that means, it depends on the diameter of the pipe.

Also again, suppose the same velocity is flowing, water is flowing with the same velocity, and at that velocity we are finding that it is getting mixed up. Water is getting mixed up, but in place of water, if we replace this water by oil of high viscosity, then we may find that the flow is not getting mixed up. So, it indirectly mean that, if we just, the index as velocity, if we just define that if the velocity exceed this particular value, then it is a different type of flow where it is getting mixed up, which of course, we can name now, because we have already discussed, in open channel flow also, that there are two type of flow, laminar, where the flow particle moves in laminar or say fluid moves in laminar. Here we can say the stream lines are moving parallel to each other that type of flow, we call as a laminar flow. And when these stream lines gets mix up and then we call that flow as turbulent flow.

Now, if we just defined that beyond this velocity flow become turbulent this will not be sufficient. So, Reynolds did this experiment and he did use a non dimensional number R e, which we know that what R e is, R e means, that is the Reynolds number. He did use this non dimensional number as index, to show that up to certain limit of R e, if you just forgot the expression for R e, we can write it is rho v, let me write l by mu rho v l by mu is the Reynolds number, but this L is not the length of the pipe. This L basically in

Reynolds number, we use this L as characteristic length. Now, what is mean by characteristic length? In case of open channel flow, we can use to represent this characteristic length, hydraulic depth or hydraulic radius sometime and in pipe to represent this characteristic length diameter of the pipe is considered.

So, this is written basically in pipe flow always as rho v D by mu. So, D is the diameter of the pipe, v is the velocity of the pipe, rho is density of the fluid that is flowing through it and mu is the viscosity of the fluid. So, all these, if we just combine, this forms a non dimensional number, and that was used as index, to indicate whether the flow is laminar or the flow is turbulent. So, and what he could observe that, up to certain limit of R e, the flow remain laminar and beyond a certain limit of R e, the flow becomes turbulent.

And what is the range? That range of Reynolds number, he could observe that when this Reynolds number is less than 2000 up to 2000, that means, reynolds number up to 2000, the flow is laminar. This is different from what we did discuss in open channel flow. In open channel, we could find that when Reynolds number is less than 500, then the flow remain laminar, but in pipe flow, when Reynolds number even up to 2000, the flow will remain laminar. And then, when Reynolds number is, this is the, I mean limit or limiting value in the lower side, below this point it is laminar and then it exceed this value, then flow will not be laminar, flow will no more remain laminar. We are not saying that when Reynolds number, when the Reynolds number of the flow exceed 2000, then the flow will become turbulent, we are not saying that. What we are saying that when the reynolds number exceed 2000, flow will no more become or no more remain laminar. Then there is a upper limit again, when reynolds number is greater than 4000, then the flow become turbulent.

Though these limits are given 4000, 2000, but experimentally when we observe we can have some difference again some author gives that these ranges are little bit different, but any way, we can just follow that this is the order of Reynolds number at which the flow become reynold up to which the flow remain laminar and beyond who is the flow remain, flow becomes turbulent; that means, between 2000 and 4000, what we will call that well this is lower limit and this is upper limit. Upper limit in a sense, that beyond this it is turbulent lower limit in a sense, then below this it is laminar and in between when the Reynolds number between 2000 and 4000, this range we call as transition flow.

So, Reynolds number, if it is less than 4000 and it is greater than 2000. That range we call transition.

Now, these Reynolds number why we are discussing here the ranges, the laminar and turbulent, what is the significance of that? Because, based on these reynolds number we will able to say whether the flow is laminar or it is turbulent. Now, what benefit we will be getting by saying so, the very basic intention is that, if the flow is laminar, then the resistance offered by the flow or offered in the entire system to the flow will be different from that when it is turbulent; that means, what we can say significance of type of flow, that is the resistance offered to the flow. In fact, the energy loss when we are saying that resistance offered is changing, then energy loss is also changing.

So, energy loss is our main important issue. Now, energy loss will be different, if the flow is laminar, energy loss will be different if the flow is turbulent and. So, for calculating those thing, for engineering those issues, we will be having different equation for laminar flow, for calculating loss and we will be having different equation for turbulent flow.

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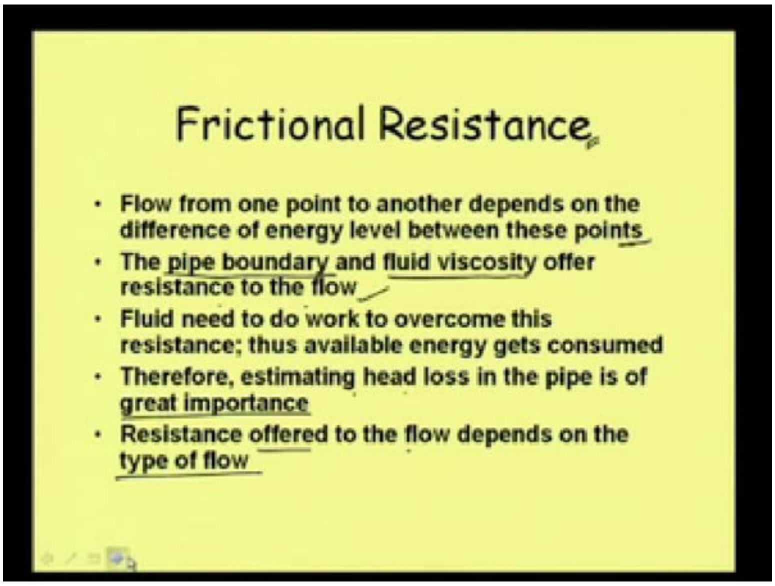

Let us see how these things varies. Frictional resistance, first let us talk that flow from one point to another depends on the difference of energy level between these points, that we have already explained, And did the pipe boundary, now resistance energy level between these point and what is energy level that we have already explained, the pipe boundary and the fluid viscosity, as I am saying that viscosity is important here. So, the resistance offered is not only coming from the pipe boundary. In fact, in some cases, the boundary friction is not that much important, rather the friction or resistance offered due to viscosity, that is the resistance offered due to shear stress between different layer within the flow is more important or more dominating sometimes. So, that way what we can say, that pipe boundary and the fluid viscosity offer resistance to the flow.

Then, when there is resistance to overcome this resistance, fluid need to do some work. So, when the fluid is doing some work; that means, some available energy, the available energy, some of the available energy get consumed and therefore, if suppose total energy, if total energy get consumed, whatever energy was available if all the energy get consumed, then the water will not move, because everything need energy to move. So, there will not be suppose energy at the upstream and downstream may become equal, then there will be no flow, like current, if there is no voltage difference, there will not be any current flow. Like that in case of water, if there is no difference in prismatic head or total energy head, then there will not be any flow. And that is why this is important, therefore, estimating head loss in the pipe is of great importance and that is why we will be discussing first in this topic of pipe flow, how we can estimate the loss due to this friction or due to this viscosity, I mean in different type, how we can estimate the losses. That is very very important, and resistance offered to the flow depends on the type of flow as we have already explained and so, now we will be discussing a very distinctly that what I mean, what are the laws that shows that in laminar flow, what are the laws of friction loss. Then in turbulent flow what are the loss, laws that we have for friction loss.

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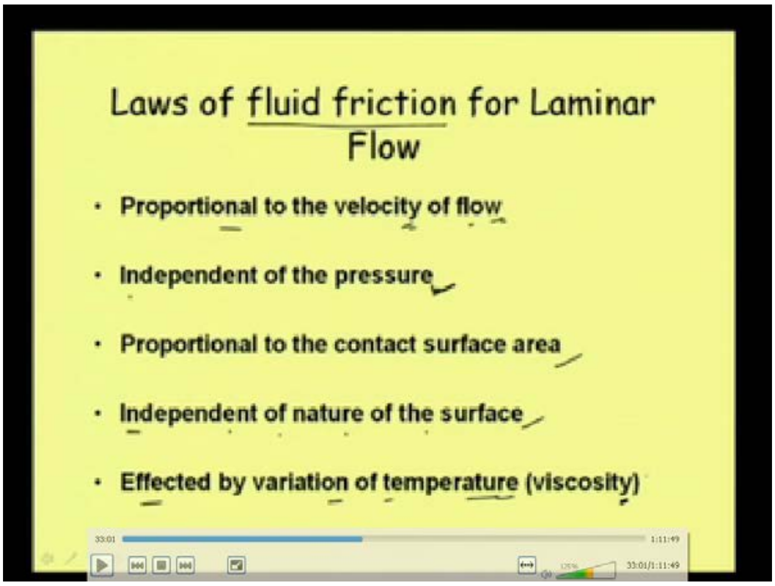

So, first we will be discussing laws of fluid friction, what are the law that we need to see that laws of fluid friction for laminar flow. Laws means how these laws are being drawn actually based on observation and experimentation. So, people are conducting experimentation and they observing how the friction loss or the laws in a pipe are behaving and from that, these can be find out, and of course, this can be analytically we will be deriving then also we will be finding that. The equation that we are getting that also says the same thing, but first we need to understand the things physically. That the laws or say if rather saying first laws we can say that fluid friction, fluid friction, it is propositional to the velocity of flow, that is in laminar flow. If the flow is flowing as a laminar flow, then the fluid friction, that will be offered, that is propositional to the velocity of flow. And it is independent of the pressure, as we know that fluid will be always under pressure, when it is moving as a pipe flow, but the amount of pressure, under what pressure the fluid is moving, that will never indicate anything about the frictional laws or fluid friction. So, that is independent of them; that means, if a fluid is having under high pressure or one is under low pressure, whatever it may be, that high pressure flow whatever laws will be there, if the flow is laminar or in a low pressure flow, whatever laws will be there, if the flow is laminar, other things remaining same that will be same. It is independent of the pressure.

Then it is propositional to the contact surface area this is of course, obvious because friction is coming from the surface as well as of course, from the viscosity, but if the surface contact surface area of the pipe is increasing; that means, if the pipe is longer, than the friction laws will also be moved. So, it is proportional to the contact surface area. Then of course, it is independent of nature of the surface. That is interesting. That, it is proportional to the contact surface area, if the law, if the if the contact surface area is more, the friction laws will be more or friction will be more, we can say, but in laminar flow, the nature of the surface, that is how rough it is, how much rough the surface is, that becomes sometimes irrelevant and the reason is that, in fact, in laminar flow, a layer forms on the boundary of the pipe, which is in rest and that always it is like that the water in laminar flow is flowing layer by layer and that which is static layer, which is not flowing at all and which is in touch with the surface. This make a situation, as if the water is flowing over that layer and that is why the frictional characteristic is not making that much difference of course, if there is a large projection, then the situation will be different.

In fact, that will create the flow turbulence at that point that is a different issue, but if laminar flow is flowing then the surface characteristic, sometimes do not influence, sometimes in general do not influence, but of course, as I am saying that if projection is very large, then it will influence the rather it will make the flow turbulence at that point. That effected by; that means, this fluid friction is effected by variation of temperature. If it is laminar flow. Then the fluid temperature is important. That is, if we change the temperature of the fluid, the fluid friction changes. That is why in experimental study, when we conduct experiment on this friction, then we say, we always do it different temperature of the fluid there in. In fact, in those experimental set up there is a device, by which we can change the temperature and that way we can see that, if we change the temperature, then how the fluid is behaving or how the resistance is generating there.

So, it is affected by the variation of temperature, why because with the change of temperature, basically the viscosity as we know that from our very fundamental knowledge of fluid mechanics, that viscosity is a property which is influenced by the temperature. In fluid, I should not say in fluid, because in gas again this will be different. I am not going into that part because, in gas, the viscosity is derived from a different phenomenon, but in fluid it is a different phenomenon. So, I mean that in case of liquid, fluid mean in case of liquid, with the increase of temperature, the viscosity decreases. So, That way in case of gas, this molecular movement is important, but here it is not like that. So, that is why in case of liquid, with the temperature increase the viscosity changes and that is why it effect the frictional resistance.

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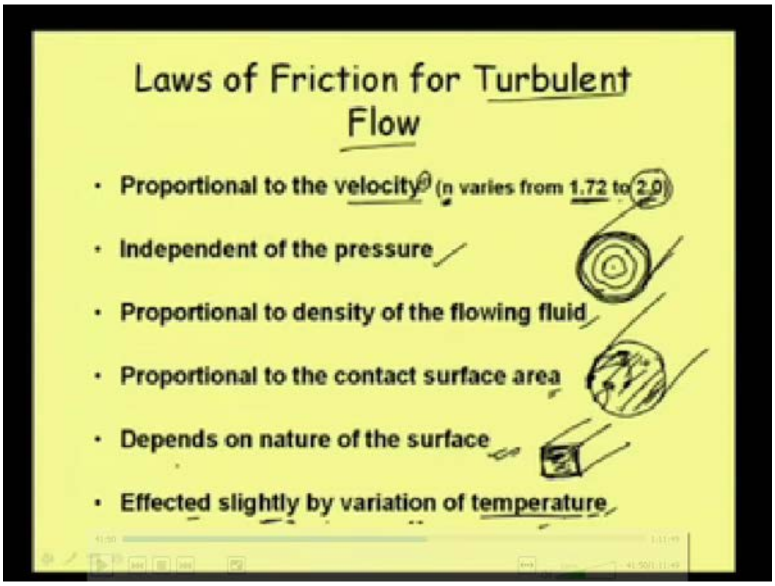

So, these are the laws that we can say that which effect the friction in a pipe in a laminar flow. Now, let us see how these laws changes, in case of turbulent flow. In case of turbulent flow, the friction is propositional to the velocity to the power n. It is not directly proportional to the velocity. It is not linearly varying with velocity. It is varying exponentially. So, the friction is proportional to the velocity to the power n and this n value varies from 1.72 to 2, from 1.72 up to 2, generally this found experimentally and for most of our actual calculation, we take it as 2, because if we consider 2, the calculated friction will be more. And then, when we are designing considering that friction will be more, then we are in a safer side, our design will be safe.

So, suppose we are trying to carry the water up to a certain extend and if we consider the friction to be less than we are in problem, we will be accessing less, but if we consider friction to be more, then we will be considering the loss will be more and then we will be putting more head and we are in the safer side. As we do not know very exactly what the n value will be for different cases. So, generally for most of our practical purpose we consider this to be 2.

So, we can say that, friction is proportional to square of the velocity. Then again it is independent of the pressure that is same as laminar, then it is proportional to density of the flowing fluid well. So, density of the flowing fluid is significant here. So, it is proportional to density of the flowing fluid. And as such like I mean laminar flow, here also, the friction is proportional to the contact surface area. It is same, whether it is turbulent or whether it is laminar, contact surface area is always important; that means, longer the pipe longer will be the loss.

So, that we should be careful and then, but here, what the difference is with laminar that here the friction in turbulent flow, the friction depends on nature of the surface, friction depends on nature of the surface. Why? In laminar flow, let me draw a diagram here. In laminar flow, we can say if it is the pipe, then velocity will be, we are not drawing velocity here, my circles become eccentric, here it should take here like this. If I consider different layer of flow, different layer of water, flowing through the pipe, because in laminar, each of these layer, we can say, that they are flowing in layer, this is they are not getting mixing up, if this is the layer flowing it will be moving like that, this is moving like that. So, these are not getting mixing up.

That is why, the layer which is just in contact with the surface is forming or it is covering the frictional roughness. Roughness of the pipe it is covering and the other things are moving it is in rest. So, that is why the nature of surface was not that much important, but in turbulent flow, if it is a turbulent flow, then the flow are not moving like that in layer, it is just moving as a single flow. Once this particle is here this next movement can go here once the particle is here his this may next movement can come here. So, that way all mix up flow is moving through this pipe. In that case, the fluid particles are always moving in a mixing way and. So, these are not getting. So, at the particle which is in touch with the surface just at a movement will be moving in to the fluid and that way they are getting mixing up. So, there is no question of having a layer form at the surface.

And as such, the frictional characteristic or the roughness characteristic of the pipe surface, inside surface of the pipe is significant. And that is why, it depends on the nature of the surface, that is why, when we will be getting the equation of friction for turbulent flow, we will be finding that this roughness parameter is coming, but when we get a equation for friction in case of laminar flow, then we will be finding that there is no such parameter. The friction parameter, the roughness parameter is not coming well.

Then this get effected slightly by variation of temperature, in case of turbulent flow, otherwise also flow is very moving with mixing up and that is why this viscosity part is already get, I mean effected by this sort of movement and in that sort of flow, the effect of temperature is not that much significant, it gets effected, but slightly, the reason is that, the viscosity become more significant when the fluid is moving in layer, when the fluid is moving in layer, when the fluid is in turbulent flow. So, this is not that much significant. And that is why we can find that it its get effected, but it gets effected slightly by the variation in temperature. So, these are some of the laws or some of the facts that is being observed by the investigator through experimentation that how the friction get influence in laminar flow and in turbulent flow.

Here we are just mentioning and we are always drawing the pipe, whenever we talk about pipe, we draw a circle or we draw a cylindrical pipe, but we should keep in mind that pipe not necessarily mean, that it should be a circular, of course, for our advantage, because of many other advantages we go for circular section, but sometimes the pipe can be square also. There is no restriction. In fact, that pipe should be circular, we can have a square pipe also. In fact, in some of the experimental setup, we find that putting square pipe is advantageous for some of the experimentation. So, that way square pipe can also be there, but for practical purpose, as we know that for handling and as some of the things are better here. So, in circular pipe, we go for circular section , but I mean, on principle, if the flow is moving full, if the flow is moving full, then this become a pipe flow. If there is a free surface, then only it is a open channel flow, when we talk about rectangular section, suppose, below a culvert, a water is flowing, below a culvert means in that portion, it is a rectangular section.

So, till the point, when we have a free surface, flowing through it, this behave like a open channel flow and then if we analyse the flow there, we will have to apply the principle what we have studied already. Friction loss or whatever it is, that we need to study or that we need to apply considering the theories and all that we have already got for open channel flow, but during flat time, if the water level increases and suppose the culvert is there and the box was there, but it is becoming full and in that case, once it become full, we should not allow or we should not rather consider that to be as a open channel flow and we should not use those equations that we have already learn in open channel flow.

So, during flat time, we should remember that if we are trying to calculate the discharge discharge through that section, which may be a circular, which may be a rectangular one, but it is flowing full then we need to apply the principle of pipe flow. That is one important point for our practical design and with this, we can now move on to some of the issues that are required, some of the facts that we need to know for calculating the head loss in laminar flow.

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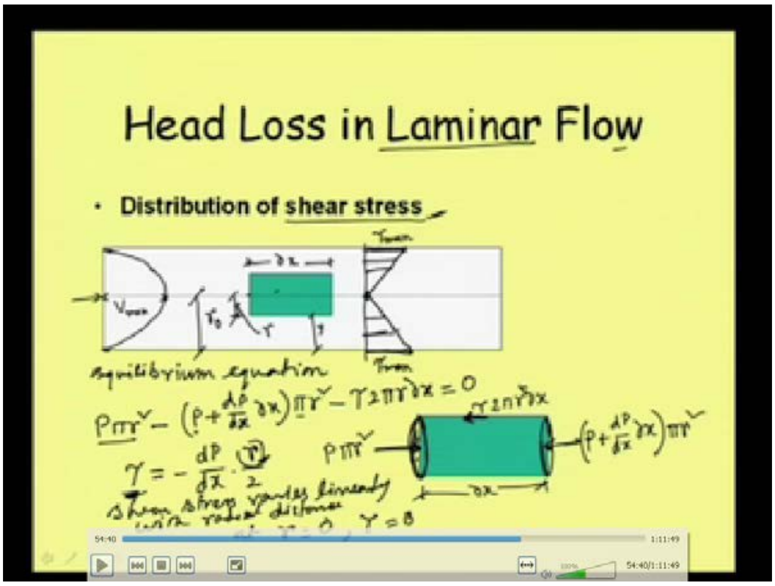


One point that we need to know is distribution of shear stress, this is a pipe, which is again I am drawing, it is a circular here, this is a pipe and it is carrying water through it and then at different level, if it is first we are discussing laminar flow as the flow is moving as layer so, at different level, this is one layer, this is another layer like that, at different layers, the shear resistance will be different and that we call as a shear stress. So, we need to know what is the shear stress or how we can have the shear stress expression. The maximum shear stress will be of course, at the side. This is here the maximum shear stress can be. So, if we say that this is the maximum shear stress at that point, then if we draw a graph, suppose let me draw a graph, suppose this is one axis I am drawing and at that axis, if we draw shear stress we brought the shear stress value, this will be maximum shear stress that will be getting here. So, I can write this is the tau max. And then shear stress as we come towards the radius, I mean towards the center line, this is the center line, this will reduce and then we do not know how much will be the shear stress at this point. Again it will be increasing here. So, at the boundary the shear stress is maximum and then at center of course, it appears that if it is to be maximum here it is to be maximum here, then we are interested to know what will be the shear stress here or we need to know the expression for shear stress.

To analyze that let us consider this, suppose its radius is say r 0, r 0 is the radius of the pipe. Here I want to draw another point that how the velocity will be changing, we will be discussing that later, but just here I want to draw, that if I plot a velocity diagram, then it will be like that. I am now removing this one of course, just you remember that this is a circular section we are drawing. Now, velocity if I plot, here the velocity is 0 at the side the velocity will be 0, shear stress is maximum here the velocity, but it is 0 at 0 means not only at this point, if I draw anywhere ,the velocity is 0, and velocity will be maximum at the center. That is obvious that in a pipe if we see. That velocity will be maximum at the center. And then it will be gradually changing like that and our velocity diagram maximum point is suppose up to this point and that distance is representing the maximum velocity, v max, maximum velocity.

Now, to analyze how this shear stress is changing, how this velocity is changing to have all those, what we can do. So, say let us take a fluid element, length of this fluid element is delta x. Let us consider a small fluid element and radius or the distance up to this fluid element be r, this is r 0 and that distance is r and of course, we can consider that this is at the distance of y, that may not be required here, but now let me draw this fluid element in a bigger dimension here. I am showing that this is circular here, the same fluid element, this small fluid element, I am drawing it in a little larger dimension.

Now, let us see that what is the force, that will be acting from this side. In this fluid element, everything will always be in a balance position, we are considering that this is moving in a balance position or balance condition. So, we can try to see that what will be the relations between different parameter. In case of dynamic equilibrium, we are talking about a steady state, that is acceleration is equal to 0, there is no acceleration, it is moving as a steady state and in that steady state condition, where acceleration is equal to 0, then we can say that it is in dynamic equilibrium and that is why it is moving with uniform flow velocity and in that condition what will be the equilibrium equation.

Now, if shear stress at this level, if we write as tau, then what is the total surface area? Shear stress means, it is the resistance force per unit area, resistance offered per unit area. So, if that is tau, then we will have to get the total shear stress in this direction shear force in this direction, what will be that? So, tau multiplied by the contact area. So, what is the contact area? This diameter, this radius is r. So, it will be twice pi r is the total perimeter length perimeter length is twice pi r. And then we are already writing that this length is delta x. So, twice pi r into delta x will give us the total area of this small cylindrical element. So, what will be the total force? Shear force rather which is acting in the opposite direction which is opposing the flow that will be the shear stress tau into twice pi r into delta x.

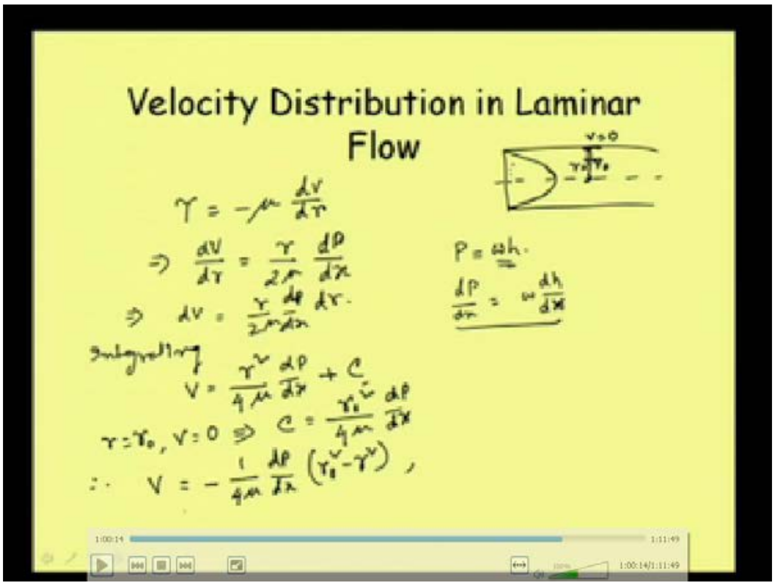
So, that we are having as resistance force, then what are the other forces acting on this fluid element? One is that if we consider the pressure at this point as p, then this pressure, pressure at this point is P. Then P multiplied by the area, sectional area. So, that will give us the pressure force from this direction. So, area is equal to pi r square and the pressure at this point is P. So, P pi r square, now of course, there will be a pressure force in the other direction also. Now, if the pressure force from the other side, we can write in terms of gradient, pressure gradient, if we say that pressure from here to here is changing; that means, from upstream to downstream is changing and the pressure gradient, if we write as d p d x, then what will be the pressure at this point? P plus pressure gradient means, we are talking about with x with downstream direction with x how the pressure is changing.

So, if it is d p d x is the pressure gradient, then at a distance of delta x, the total change in pressure will be d p d x into del x. Del x is the length of this portion. So, that will be the pressure from this side of course, we are not considering whether pressure is dropping in that direction or not right at this movement, but when we will do the equation, it will automatically come, we are saying that d p d x is the pressure gradient, it may happen that d p d x is negative means pressure is dropping in the downward direction. That we are not specifically mentioning at that point. And then the area, area is pi r square. So, now, we can draw the equilibrium equation. So, equilibrium equation, now we can write velocity distribution we will be talking later. So, equilibrium equation just we need to equate these forces from this direction and that direction. So, we can write P pi r square minus P plus d p d x into delta x into pi r square minus. This is also opposing the opposite, in the opposite direction this shear stress. So, shear force. So, tau into twice pi r into delta x, that is equal to 0; that means, all the force, if we adopt, considering its direction finally, we should get 0 when it is in dynamic equilibrium.

Then this will lead us, if we simplify these things say p pi r square and then this p pi r square that will get cancelled, d p d x will remain here d p d x will remain, d p d x and pi r square will be remaining here, that will be equal to tau twice pi r d x d x d x cancel, then we can finally, get pi pi getting cancel r one r will be remaining and then 2 will be remaining. So, it will become r by 2 and what we can have the tau is equal to minus d p d x into r by 2. We just need to simplify this part and then we will be getting this expression and of course, this relation although we are drawing for laminar flow, this also accepted to be holds good for turbulent flow also well.

Now, this expression, from this expression what we are finding? That is, the shear stress, shear stress vary linearly in laminar flow of course, shear stress varies linearly with r and of course, this is accepted for turbulent flow also I am saying. So, that way it will be valid there also, shear stress varies linearly with r; that means, with radius, with the increase of radius, shear stress will increase, it will vary linearly; that means, from here to this point will be having a straight line and then at r is equal to 0.

So, one point what we are observing that shear stress vary linearly with r, linearly with radius or r, radial distance we can call, with radial distance. And then another point what we are getting that at r equal to 0, tau is equal to 0 shear stress equal to 0. So, shear stress here will be 0 and that is why now we can draw the shear stress diagram like this, because we know that it varies linearly. So, it will be straight line and here it is tau max and here it is 0. So, this is what the shear stress diagram, at any point if you want to find out what is the shear stress, we can calculate like this just from this equation. (Refer Slide Time: 54:42)



Now, let us see how the velocity distribution in laminar flow varies. So, let us just talk about the velocity distribution we are we will be referring to this diagram again, but for velocity distribution in laminar flow, we can start with one relation, that is we know that shear stress from our very fundamental law of viscosity that when we were talking about coefficient of viscosity in fluid mechanics, that has already been discussed, I mean not in this class, but you are suppose to know those things, which are coming in fluid mechanics. So, that way, tau we can write that shear stress tau is equal to mu into d v d r.

So, change of velocity with respect to r, this is in open channel flow we can write d v d y with depth it is changing, but here it is d v d r; that means, with distance, it is changing from that point and this here we can write as with negative sign. So, then we can move to why we are writing negative sign, because d v d r, that is the velocity if we move if I draw the pipe here, then this is the center line, if I moving if my r is increasing velocity flow velocity is decreasing, as we did draw the velocity diagram, then that will be like this velocity diagram will be like this. And then in this velocity diagram it is clear that with the increase of r, r we are taking from this point, with the increase center line. So, with the increase of r, the velocity is dropping. So, that is why we are writing tau is equal to minus sign is coming from that part that d v d r. Now, this implies that d v d r is equal to because already we have got one expression, that if we just go back the tau is equal to d p d x into r by 2. So, if we just put that here; that means, if we put that expression tau is equal to d p d x by these things, what we will be getting, that this is equal to r by twice mu into d p d x. Just we need to put the value of tau as that one then we will be getting d v d r is equal to like that. This d P, this P is also equal to w h as we know. So, for if we write in terms of h, this d p d h can be written as d p d x can also be written as w into d h d x of course, if z is coming that will be again a different issue. Here we are not talking about that part.

So, this can be replaced by that part also, but here we are help you with this one let us proceed from this point. So, what we can do now that is integrating, now we can integrate this part that d r we will be taking to that side, this implies d v is equal to r by twice mu d p d x into d r. So, integrating what we will get. So, integrating this expression, what we will be getting, that v is equal to just integration of this one. So, it is r square by 4 mu and d p d x. So, then a constant of integration will be coming cC, but we need to know what this value of C will be for that we need to take help of boundary condition well.

Now, what is our boundary condition here? When we are talking about velocity v at the boundary this will become 0, at boundary what is our r value, this will become r 0, from here if we go, this become r 0. So, at r equal to r 0, our v is equal to 0. So, with that boundary condition, we know that r equal to r 0, v is equal to 0. So, this will imply that C is equal to, we can write, r 0 square by 4 mu into d p d x. Then what will be our expression final expression? So, we can write that putting this value, putting this value of c therefore, we can write that v is equal to this minus that of course, we can write this at the beginning. So, we can write minus 1 by 4 mu into d p d x into r 0 square minus r square. Why we are writing like this, because we know that r 0 is the radius, which will be always larger than distance to any section within the pipe. So, we are putting r 0 square minus r square here and then we are putting a negative sign here.

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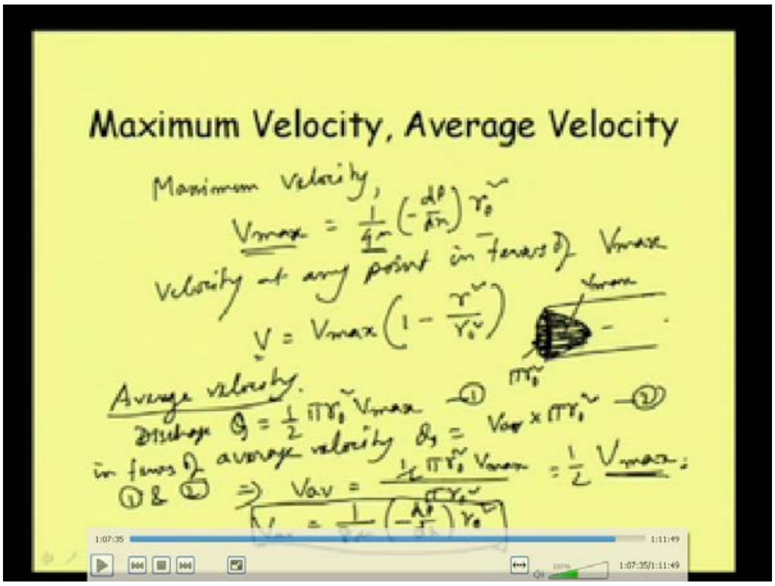

So, that way we can get the velocity expression for velocity. Now, what will be the maximum velocity, we can write velocity we have got, now let us see what will be the maximum velocity that we can write, that maximum velocity, v max and maximum velocity, we will be getting, as we can see this expression when our r will become 0, that will become maximum velocity. So, we can write the maximum velocity expression as, 1 by 4 mu and minus d p d x, the negative sign we are bringing here into r 0 square. So, that is what the expression, and as we remember that earlier we were putting d p d x straight without considering or writing that as a positive, but in reality d p d x will become negative, because the pressure will always be dropping in the downward direction, but we did not consider when we started the x derivation as we are not considering that condition that d p d x we are writing positive, but in reality the velocity will be, I mean the pressure will be dropping in the downward direction. So, it is finally, coming as negative, at that negative sign will make the entire things positive. So, that will give us the v maximum the value will be d p d x coming as negative. So, this minus sign ultimately v max, we will be getting as maximum at this point.

Then of course, we can have a relation between, S what is the relation between or we can velocity at any point in terms of or rather at any distance in terms of say v max, that we can write now straight way , that is v at any time is equal to if we know v max, which in real condition also we can measure, v max at the center, if we measure then, we will be getting the v max into 1 minus r square by r 0 square. So, just v max by v if we do, both the equation are remaining in different feel, you can observe this equation and then you can see this equation if we divide this by that and taking r 0 common if I if we bring r 0 common here, this will be 1 minus r by r square by r 0 square. So, that r square and these other part will be getting cancel and this will remaining. So, that way we can get what the expression for v is.

Now, we can talk about average velocity, what is the expression for average velocity? What is average velocity, when we talk about this pipe, when we talk about this pipe, if we see that this is the velocity distribution diagram, it is almost triangular and then we know that this area is equal to pi r square pi r 0 square rather, this area is equal to pi r 0 square. So, if it is pi r 0 square, then the total discharge will be equal to discharge is equal to just to explain that, it will be suppose, if I consider a small area and at that small area, the velocity is this much. So, in unit time, how much discharge is flowing this area multiplied by this velocity. So, that is the total discharge flowing. So, that means, area into this velocity will give us the discharge. So, that way if I consider another small area, this area into this velocity will give us the discharge. That way, if I consider all the smaller area, then we will find that this will give us the discharge; that means, what indirectly means, that if I consider all the area, the entire covering area, that is the triangular area, what we are getting nearly triangular area, that will give us the discharge.

So, discharge q we can write, that this is pi r square. So, what we can write discharge Q is equal to, this triangular area means half of base into altitude this base is equal to pi r square, I mean pi, it is conical basically pi r square and it is into height. So, half of pi r 0 square into the height means we are talking about v max height means we are talking about this v max, this is the height. So, let me put, this is the v max here. So, this discharge Q is equal to half of pi r square into v max.

Now, we are interested in average velocity, if we talk about average velocity, then average velocity, again we can have in terms of average velocity what we can have in terms of average velocity, we can write the discharge as Q is equal to average velocity v average into the area, area is equal to pi r 0 square and this two must be same. So, from that, this is, if it is 1, this is 2. So, 1 and 2 implies that v average is equal to this divided by half of pi r r 0 square v max divided by pi r 0 square. So, this is equal to half of v max. So, that is of course, we have done some approximation here, but still we can get very conveniently that average velocity is equal to half of the v max well. So, that is how we can get an expression for average velocity.

Now, considering these equation or now using the expression for v max here v max already we have one expression. So, we can write v average is equal to half of v max that we can write as 1 by 8 mu; that means, from this expression 1 by 8 mu and other part will remain same 1 by 8 mu minus d p d x into r 0 square. So, this is the expression for velocity in pipe, because when we are writing v as a velocity at any section, we must write about the average velocity. So, though here we are writing average velocity from this point onward when we will be talking about velocity at a particular section v means, we are meaning the average velocity, because for calculation of discharge also, we are using the same velocity and that way it will be convenient for us to have all other discussion in that line. And from this point what we can do that v average is equal to this part, then this lead us to one expression which is called Hagen Poiseuille equation which is of quite important in, I mean, very importance in our study of pipe flow.

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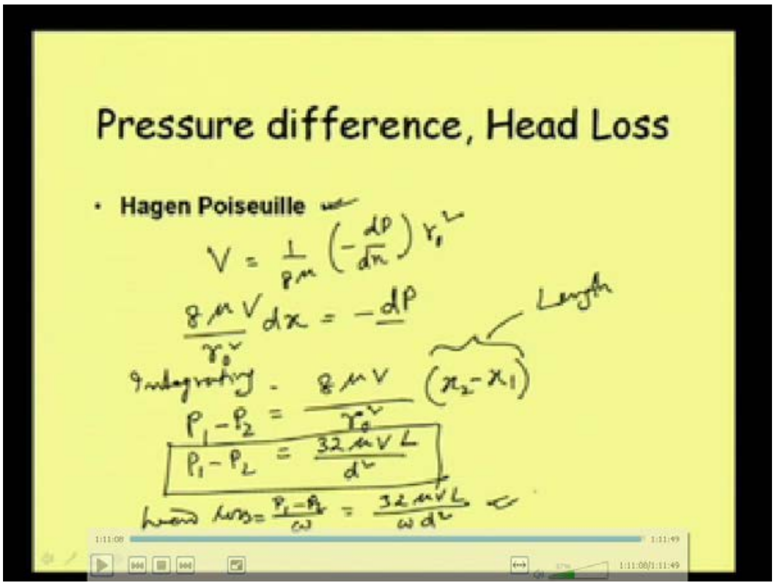

So, this Hagen poiseuille equation, how we can derive, already we have got that v average, now we are writing as v, v is equal to say 1 by 8 mu, then minus d p d x into r 0 square. So, pressure difference now we can just have it in different form and what we can write, that 8 mu v just changing it to this side, 8 mu v and then on the right hand side divided by r 0 square of course r 0 square, then we are keeping the d x here, is equal to minus d p. Now, if we integrate this one, integrating, what we will be getting integrating d p if we integrate, we will be getting p 1 minus p 2 and that is equal to say 8 mu v r 0 square into d x, we can replace by x 2 minus x 1 and what is x 2 minus x 1, this part is nothing, but the length of the pipe, length of the pipe. So, what we can write finally, and that p 2 p 1 minus p 2 p 1 minus p 2. In fact, once we integrate, it will be coming as p 2 minus p 1, but it is a negative. So, we are getting as p 1 minus p 2 and p 1 minus p 2 is equal to, now we can write in terms of L, so, n suppose if we want to replace this r 0, that is generally in case of pipe, we will be getting diameter. So, if we want to replace this as diameter then this is basically d by 2 square. So, we will be what we will be getting d by d square by 4. So, replacing this by d, so, it will become d square by 4 and that 4 we are writing here. So, 4 into 8 this will become 32 mu b into L.

So, that is one important equation and is called Hagen poiseuille equation and starting from that equation, now we can have what is head loss, our interest to know head loss, head loss is equal to basically p 1 minus p 2 by L because z we are considering, we are not considering here. So, if we divide it by w p 1 minus p 2 by w that will give us the head loss is equal to, we can write 32 mu v l divided by w d square. So, we have got one expression for head loss ultimately after doing all this calculation and all this rather derivation and that head loss is for laminar flow. We will be discussing further more on that and we will see if it is turbulent flow how we can have the equation for head loss and can we have a general expression which can be used for calculating head loss in pipe, that we will be seen in the next class.

Thank you very much.