Hydraulics

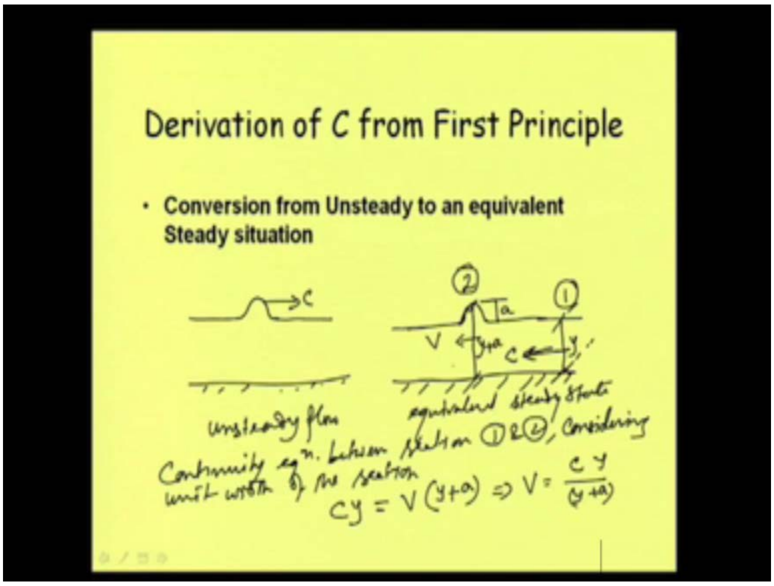
Prof. Dr. Arup Kumar Sarma
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module No. # 07
Unsteady Flow
Lecture No. # 01
Unsteady Flow Part-2

To handle this, we need to see that how we can simulate these wave flow and then how we can take protective measure, preventive measure. So, for all those understanding wave is very important and as such we started with what are the different types of wave, how we can classify waves. And then, as I am just saying that, the how the wave propagate that is very important for our various requirements, practical requirement, that is why the speed of wave propagation is also necessary. And there comes the term that we call as celerity, well.

What we mean by celerity is that, it is the speed of wave with respect to the fluid media, well. If we just simply say that celerity is the speed of the wave, then it is not complete, because we need to say that it is in reference to the fluid media. If the fluid media is it at rest, then of course the speed of the wave, absolute speed of the wave and celerity will be same. But, if the fluid is also moving and then over that fluid the wave is moving, then celerity is the speed of the wave with respect to the fluid media. And for analyzing these wave, say what is the celerity, to have an expression for the celerity, when we try to analyze it at unsteady flow, we face some complexity and this unsteady flow therefore we try to convert to an equivalent steady flow. So, that it becomes convenient for our analysis.

(Refer Slide Time: 02:50)



And in the last class, we did so how we can we could see how we can convert this to an equivalent unsteady flow and we are starting from equivalent steady flow and we are starting from that in this class, well. Say this is a wave forming here, a solitary wave, single wave and the bed is there and by say and let us first consider that this is a steel water may be in a tank and by moving a paddle, we have created a wave, a single wave is moving like that and it is moving with a speed c, well and this is a unsteady flow, this is a unsteady flow.

Now, to convert this to a steady flow situation, as I did explain in the last class, that like a walker as the belt is moving upstream or say when we walk the belt moving the opposite direction in a walker and that is why although our that we are working on a walker, but we are not moving forward. And then someone can observe us that how we are behaving when we are walking, but we are in a steady position, we are in a static position. So, those advantages we can have here, like the belt move in the opposite direction in a walker, we can put a opposite direction the velocity C, which is the actual velocity of the wave.

Then, what will happen that this is also moving in a velocity C, wave is also moving in a velocity C in this downstream direction. And as now this fluid media is moving in the opposite direction with velocity C and wave which was moving in the downstream direction with a velocity C. So, relative velocity of this will become 0 and then it will be remaining in this position and now, we can observe it carefully well.

Now, let us see how we can analyze it. Say this depth b at this section, suppose at this section depth is y and so amplitude of the wave is say a, this is the amplitude of the wave. And so what will be the depth here? Depth in the wave portion where we have the wave this will be say y plus a. Well and when we talk about velocity, then C is the velocity at this section, say section one. And if I talk about the section two, then what will be the velocity, say velocity is V, because this velocity will not be equal to the velocity C, because depth here is changing.

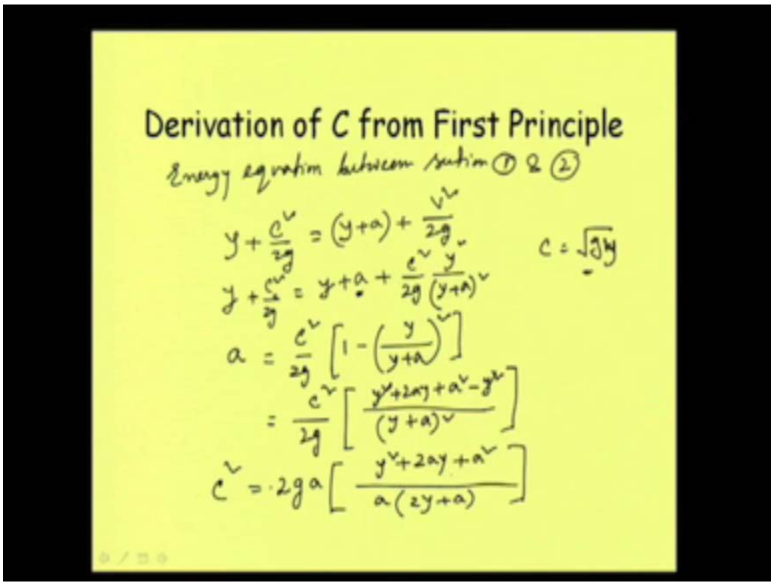
So, let us write this velocity as V, well. Now, we can write a continuity equation and from that continuity equation we can find that how we can express this V and C, how we can relate this V and C. Well, how we can write the continuity equation, say if we considered this channel to be say rectangular and suppose we are considering unit width of the channel, so this is let me just write here, this is a equivalent steady state. And now let us see the continuity equation how we can write, say continuity equation. Between sections say 1 and 2, between section 1 and 2 how we can write and that we are writing considering unit width of the section, unit width of the section.

Well, we are considering unit width of the section, then what will be the discharge in continuity equation state, that here the discharge and here discharge will be equal discharge at section 1 and section 2, will be equal, fine. Now, discharge is equal to the velocity at this point one section 1 and the area area is equal to y into its width.

In fact, if we considered width, then width will be coming, but we are considering width to be equal to 1, so it will be C into y, so with what we can write that C into y is equal to here, we are talking the velocity as V. So, V and depth here is equal to y plus a y plus a. And from this what we can write that V is equal to C y divided by y plus a. Let us keep this expression here and now we can write another expression, this is from continuity equation we are getting this relation, that is b we can express in this form, well, but we can have another expression from the consideration of say energy equation.

Now, what will be the energy at this point, it will be y 1 plus, that means at section 1 it will be y 1 plus, sorry y, we are writing y, y plus C square by twice g. And here because velocity is C here and here it will be y plus a plus V square by twice g and let us write that expression, well.

(Refer Slide Time: 08:23)



And we can write that energy equation, energy equation between section 1 and 2, of course when we are writing energy equation between section 1 and 2, and we are saying that energy at this section at that section are equal, what we mean that there is no energy loss in between that assumption is inherent there. Well, now let us write that expression say y plus C square by twice g, that means V square by twice g. V mean velocity square by twice g, this is equal to depth in the section, two is y plus a plus say V square by twice g, let me write one step V square by twice g. Now, what this V is we know that V is equal to C, then y by y plus a.

So, we can just replace it in this form, that is y plus C square by twice g is equal to y plus a plus we can write that C square this in place of C square by y square by y plus a whole square. So, so V square we can write that V square by twice g will be V and we can write C square into y square by y plus a whole square, ok.

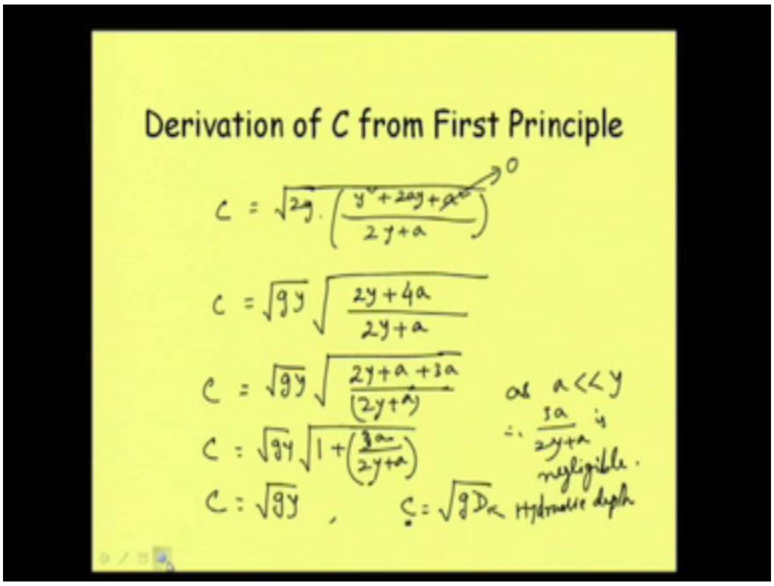
Well, this is the expression and from this let us try to write because our interest is to get an expression for C. And here what is not known and what we want to try is the C and what is our unknown thing is y, a here, the expression n. And at the same time this expression we are writing for a very for a wave where the amplitude is very small, I mean small amplitude wave. As we just if we recall our last class, then we were talking about one expression that C is equal to root over g y, for rectangular channel we obtained that and that was for small amplitude wave.

So, here also we are considering that this depth is considerably large than the amplitude a, well, let us just separate this term and see how we can write it. This y and y will of course get cancel and let me write first a in terms… a is equal to then C square by twice g, C square by twice g is here. So, we can bring C square by twice g and then we can write 1 minus say y by y plus a whole square, well. And this can be written as, say let us break up y square C square by twice g, our intension is to see that, if because in the expression of C, in fact we do not have the a term, root over say g y, that was that, that that is what we got in last class. So, let us see, by we are starting from the first principle and we are trying that yes if if we can get this expression and here we do not have a term, so let us see how we can do that.

This we can write say y plus a whole square and then this will be say we can break up y square plus twice a plus twice a y y square plus twice a y plus a square then minus y square. Well, here we are in a position to simplify now, well, so what we can do that from here let us write one expression for C. So, C is equal to, you can write that C is equal to twice g a will be coming C square, let me write C square is equal to twice g a and then this will be just, we can write in the reverse direction, reciprocal of this one. This will be say y square plus twice a y plus a square, this is there and here what will be having this and this is getting cancel already and here we have twice a y plus a square.

So, that twice a y plus a square we can write here as a, keeping common and we are writing twice y plus a, well. Now, this a and this a we can cancel at this point and let us see how we can write this expression well.

(Refer Slide Time: 13:27)



So, we can write C square is equal to now root over twice g, twice g and entire things we can put under root sign, root over twice g and then here, it will become say y square plus twice a, then twice y plus a. Let me start from this expression, again y square plus twice a y plus a square divided by a, twice a plus a this a.

And a is getting cancelled, so what we are writing that twice g and in this part, we are writing that expression, well plus a square, well. So, now, here as we know that a square is say, a itself is very small, so this a square term we can put equivalent to 0 or say it is negligible. So, if it is negligible now what we can write that, C is equal C square no, it is C, because we have written already in terms of root over.

So, c is equal to say our target is to get this term g y, so we are writing g y means from here, we are bringing a y common, we are bringing a y common here. In fact, we had the term a y, a y plus a square, right, so y we are brining common and 2 we are taking to that side. So, what we can write root over g y, then here we can write, so as one y has come, so it will become twice y 2 is going inside and then plus 4 a 2 is going and this is coming, so it will be twice y plus a.

Well, now this we can further simplify in a way that this is equal to root over g y and let me write it in this form, that twice y plus a, fine, here also let me write twice y plus a. Then, in fact, we need to write plus thrice a into write plus thrice a, well.

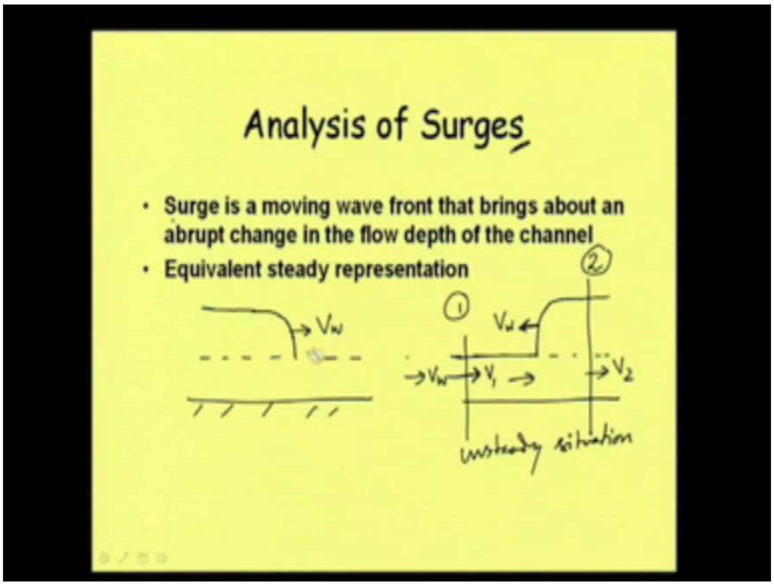
So, this twice y plus a, twice y plus a that we can have one and this will become thrice a by this expression. And so let us write C is equal to root over g y then you can write root over 1 plus thrice a by twice y plus a. Now, this expression, thrice a, a is very small and here, in the denominator, we have twice y.

So, it is quite a large expression and 2 as y is very large than as compared to a, so it this expression will become a very small term divided by a very large term, so this we can now neglect. So, if we neglect this term say as a is very very small, as a is very very small as compared to y, therefore 3 a by twice y plus a is say negligible, is negligible. And now, just neglecting this part it will become C is equal to root over g y.

Well, so that is the expression that we can derive, what we got in our last class, again starting from the first principle we could see that for a small amplitude wave we can have the expression for C as root over g y. And if it is a suppose nonrectangular channel, then what we do, that y will not represent these things properly.

So, what we write that C is equal to root over g D, what D is, D is the hydraulic depth, D is the hydraulic depth. So, using this hydraulic depth, in fact for rectangular channel this will become y, but for trapezoidal channel it will be a by t. So, using this expression also we can get the value of C, well.

(Refer Slide Time: 18:03)



So, after that let us see that, well for a wave of this type, for a wave of this type solitary wave of this kind we can have an expression for C, but if we have a surge, if we have a surge and how to analyze this surge, whether the C value will be same as or the same expression whether we can use or not. Well, that first we should know what is surge, this is also in unsteady flow, this is important, I mean important topic that is surge, we need to do lot of analysis in fact on surges. And but here we will be just doing some limited exercise on surge, that you can see that surge is a moving wave front that brings about an abrupt change in the flow depth of the channel.

So, basically what we mean by surge is that, say this is the bed and normal flow is suppose moving here and somehow a wave is coming like that, it is the moving wave front, that is why we called this is a wave front, this wave front and say then it is more. So, this water is moving like that otherwise also and then we have a moving wave front, this wave front is moving with a velocity say V w, this wave front is moving with a velocity V w and this can be this we referred as a surge. In fact, it can be in the other way also say your water level is here, water level is here and water itself is moving in this direction suppose and somehow due to closure of gate, the you have suddenly closed the gate.

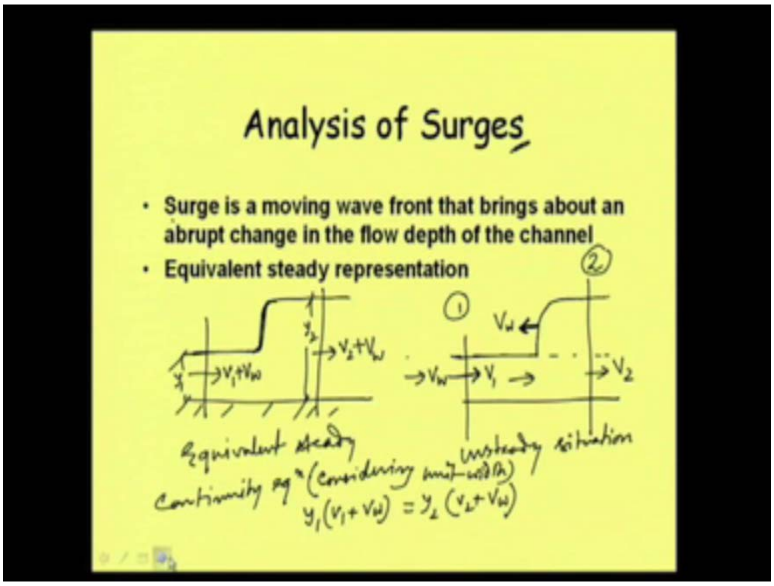
So, water level here is rising and then the wave can be moving in the opposite direction also, this is say wave moving upstream and this is a wave moving downstream. So, whatever may be the situation, we called such moving wave front, which is moving and creating a abrupt change in the depth, say depth earlier was like this and there is a abrupt change, if the depth changes gradually like this, then we will not call this as a charge, but here depth is changing abruptly and that is why we called this as a charge and surge. So, this surge also we can express in the form of say equivalent steady state for its analysis. Well, let us see how we can represent this in the form of an equivalent flow.

Well, let us consider this one, anyone we can consider, but let us consider this one, suppose this is say 1 and this is section 2, so say 1 and 2. Why we are considering because this is both in the same direction, here it is in the opposite direction, that will give us an opportunity to handle a case where the flow and waves are moving in the opposite direction, till now we have not discussed that sort of situation.

So, suppose the velocity here is V 1 and velocity here will be changing. Velocity here means in the entire section say V 2, well, now V w is the velocity which is moving in the opposite direction, this is a unsteady situation, at next movement of time this wave will be moving further, this wave will be moving further and say we will not find it here . So, let us see how we can do this to keep in a steady position.

Well, what we can do in this, we can apply an additional velocity in the opposite direction to the speed of the wave, well. So, please just see we are meaning that opposite direction to the speed of the wave, but this speed of a wave is not the celerity, celerity will be the relative speed, so we are we need to apply a velocity y in the opposite direction to the speed of the wave, otherwise it is moving with a velocity V 1. Now, if you apply another velocity say V w, if you apply another velocity say V w in this part, then total velocity at this point will become V w plus V 1 and here also it will become V 2 plus V w. And then, velocity of this in this direction will become 0, because it is moving in this direction and then at that is why the these things is not in a position to move further, it will be remaining in the same position.

(Refer Slide Time: 23:21)



Well, so let us represent this and let us see how we can derive. I am now rubbing this portion and this equivalent steady state position we can keep here, we can draw here, say it will be like this. Now, we have here the velocity will be V 1 plus V w and here the velocity will be V 2 plus V w and of course, there will not be any velocity of the wave, it is not be moving rather.

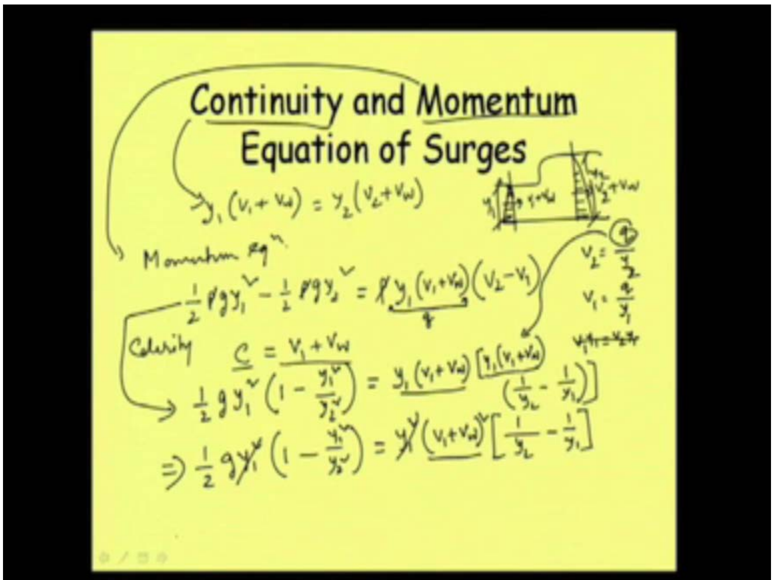
So, this is basically we can say equivalent steady situation. Well, now, again for this surge, just like we were writing earlier, we can write continuity equation and momentum equation. So, continuity equation we can write here, say continuity equation and let us put some depth value here, say depth here is y 1 and depth here is say y 2, y 2 well, now what is continuity equation?

So, again considering unit width of the channel, considering unit width, we should mention this one, considering unit width this will become say y 1 into V 1 plus V w is equal to y 2 into V 2 plus, sorry it is not V 3, it is V w V 2 plus V w, well, so that is our continuity equation. And this continuity equation can give us or from or from this continuity equation, we can express y 1 in terms of y 2 and we can do those things and then what is this V 1 plus that is also interesting.

In fact, if from this point when wave speed or absolute speed of the wave is say it is moving with a speed V w and then the fluid is moving in the opposite direction V 1, then what is its relative speed? Its relative speed will be V 1 plus V w, because both are moving in the opposite direction.

So, it will be it is in fact it will appear that it is moving faster, when V 1 is in this direction, V w in this opposite direction. So, wave speed, relative wave speed will be V 1 plus V w and this relative wave speed in fact we call as C. So, this V 1 plus V w is nothing but C, well that we can write that equation.

(Refer Slide Time: 26:09)



So, continuity equation we have seen and then let me write the continuity equation again and let me keep the diagram here, quickly, say it is V 1 plus V w, sorry V 2 plus V w and this is V 1 plus V w, that we have and depth here is y 1, this is y 2. Now, the continuity equation we have already described, so y 1 plus y 1 into V 1 plus V w is equal to y 2 into V 2 plus V w. Now, we need to write the momentum equation, we need to write the momentum equation, so for momentum equation, for writing the momentum equation, we need to see what the forces are acting. So, the pressure force from this direction and the pressure force from that direction and then the momentum, change in momentum between this point and that point.

So, pressure diagram we can draw like that and as we know, we have discussed that part a lot, so pressure here we can write as half of rho g into y 1 square, because pressure here is nothing but rho g into y 1 or w y 1, we can write in it as w or we can write rho g. So, writing in any form we can get it say rho g y 1 and then area of this triangle will be half of rho g y 1 plus multiplied by the way y, so it will become half of rho g y square. So, momentum equation what we can write, the force from this side that we can write, this is continuity equation and now, we are writing momentum equation. So, momentum equation we can write here, say half of rho g y 1 square that is the pressure force from this side and minus half of rho g y 2 square that is the pressure force from that side, that is equal to change in momentum, rather rate of change in momentum, so what we can write that mass flowing per unit time and change in velocity.

So, mass flowing per unit time how we can write, that discharge flowing per unit time, well discharge as it is not changing from here to here, because no other flow is being added, nor the flow has been taken out, so we can write it in terms of discharge of this pass. Because this is known to us, what is depth y 2 that is of course known, not known to us, because after coming of the wave this is happening, but this y 1 will be generally known to us.

So, let us write the discharge in terms of this one, that we can write discharge in fact is equal to y 1 into V 1 plus V w is the say velocity and area is actually y 1 into width, but width is 1. So, this is what the volume flowing we can say and then this volume flowing if we multiplied by rho, then it become the mass flowing, so this is what the mass flowing. And then V square or say, sorry, not V square, this is the mass flowing and the change in velocity. So, change in velocity is equal to in fact, it is V 2 plus w V w minus

V 1 plus V w, so ultimately the resulting will be V 2 minus V 1, so that is what the change in flow velocity, this is our momentum equation.

Well, from this momentum equation we can further derive some of the relation; I mean which is required for finding the expression for C in case of surge. And well before that let us write the expression for C here, this is another equation, that is C is equal to we have, we can write that C is equal to say V 1 plus V w, that we have already explained how it is.

So, this is equal to C, celerity C that is the expression for celerity C. And this expression celerity C we can further say starting from the momentum equation, we can derive it in a different form and let us see how we can do that. Well, now, we need to mention one point here that we got one expression for C is equal to root over g y and this equation is called lagrange's equation of celerity, this equation, C is equal to root over g y, this equation is called lagrange's equation of celerity.

Well and now, let us see what will be the expression for C in case of surge, in terms of y 1 and y 2. Well, in general, we can have it as V 1 plus V w, but we then we do not know what V w is and so we need to have it in a different form, like that earlier we got in terms of depth y, so let us see what we can do. Starting from this again, starting from this momentum equation, we can write that this is equal to say rho, we can cancel in each part. Then let me take common here say half of g, then y 1 square, say half of g y 1 square if I keep here, then it become 1 minus y 1 square by y 2 square, well. And on this side we can take say y 1 is here and then V 2 minus V 1 is there, so what is the V 2 is the velocity, so velocity and this expression actually y 1 plus V 1 plus V w, this expression. This expression is nothing but, this is showing how much is the discharge I am writing this as Q, this is Q means unit discharge, because we are talking about discharge through unit width.

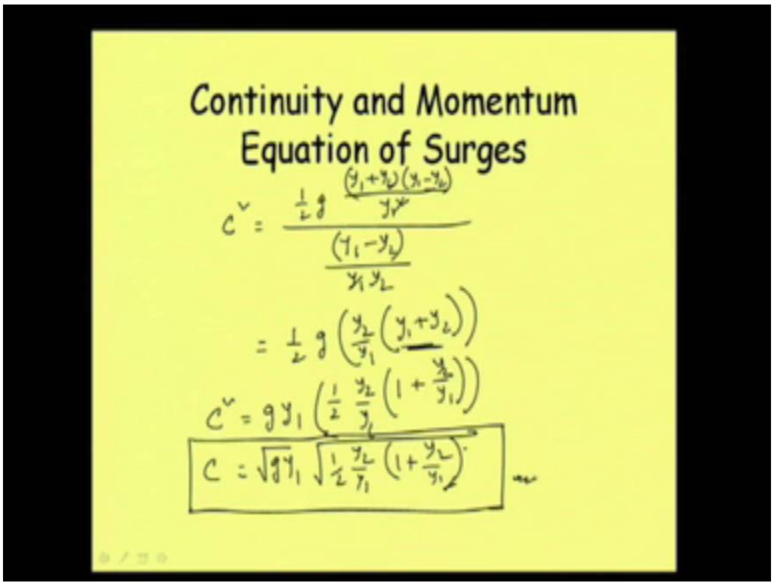
So, this is equal to Q and what is our V 2 or V 1, V 2 is nothing but Q by say y 1, because V 2 into y 1 is equal to y 2 is equal to Q and V 1 is equal to Q by y 1, because that we know, that is again from continuity equation we know that V 1 y 1 is equal to V 2 y 1, that is also known to us. So, in fact, this V 2 can be written as, V 2 can be written as Q by y 2, means Q is equal to V 1 plus V w, Q is equal to y 1 V 1 plus V w divided by y 2. So, that can be written, well, so if we bring this Q common here, let me write it in this from, here say y 1 plus V 1 plus V w and then this Q, we can let us keep rho still here, because here also rho, ok discharge will be coming.

So, this discharge is nothing but say we can again write this part as y 1 into V 1 plus V w that is what the discharge we are writing here bringing common. And then inside this will be saying 1 by that means Q, we are bringing out this Q is nothing but this expression and then, here it will be 1 by y 2 minus 1 by y 1, well. And as we can see very clearly that this term and this term is same.

So, we can just because we are just writing for discharge only, so this will be square, so this can be written as half of g y 1 square and 1 minus y 1 square by y 2 square is equal to, this now we can write y 1 square and then we can write here as say V 1 plus V w square, then within bracket we are putting say 1 by y 2 minus 1 by y 1.

Now, let us see how we can further simplify this expression here, that is the V 1 plus V w this is nothing but C. So, from that we can put the value here as C square, say y 1 square and this y 1 square we can cancel and then we can just rewrite this expression in a simple form like that.

(Refer Slide Time: 35:10)



C square we have already replaced this part, that means this we are writing as C, this one this is nothing but C, so putting that, what we can write C square is equal to say half of g, half of g, then we can write say y square, y 1 square minus y 2 square, that we can write y 1 plus y 2 into y 1 minus y 2 and this is divided by y 1 square. And then this, the denominator part we can write say y 1 minus y 2 divided by y 1 y 2, that means this part, this is become y 1, y 2, then y 1 minus y 2, so this is coming here. And this is becoming y square, y 2 square, then this y 1, y 2 square minus y 1 square, that we are writing as this, I mean this one say y 1, y 2 and y 1 plus y 2. Now, from here what we can do, this is equal to half of g, then we can write y 2 by y 1.

So, this y 1 and this y 1 is getting cancelled and we can write this as y 2 by y 1, y 2 by y 1, y 2 by y 1 into say it will be y 1 plus y 2, that will be the expression. And we can of course further simplify from this part, we can see that our target is always to get it the expression in terms of root over g y and then whether we can have it in further simplified form.

So, C square, this is equal to, we can bring one y from here, one from one y 1 from here, so it is equal to say g y 1, 1 y 1 y bringing from here and then the other part we are writing in a different way, say half we are writing here and say this y 2 by y 1 is there, y 2 by y 1 and then as 1 y 1 we have brought here, this will be 1 plus y 2 by y 1, well.

So, that is the expression we are getting and as you can see, that, oh this will be y 1. So, C square is equal to this part, then if we just try to take C, then we will be getting C is equal to root over g y 1, then root over say half of y 2 by y 1 and 1 plus y 2 by y 1.

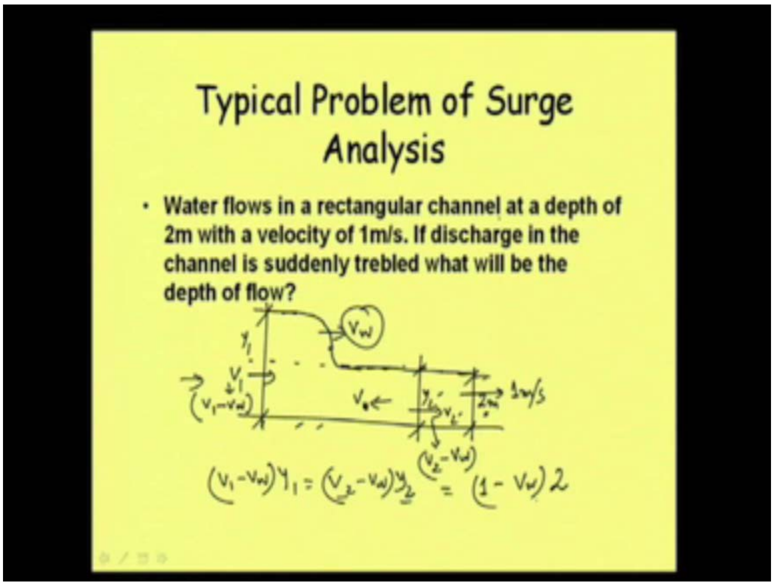
Well, now in this expression normally if we refer to our earlier diagram, this say y 2 will not be known to us, because it is coming, this wave is coming and the y 1 will be known to us. Of course, so to get the value of C, then say generally we are getting the expression in terms of y 2 by y 1 and here it is g y 1 and this y 2 we need to know in many case.

So, this y 2 oh sorry, this is this should have been y 2 square by y 1 square, yes, we are taking common y 1 square here and as such it will be y 2 square by y 1 square and this is y 2 square by y 1 square, right. And that is why we are getting this expression in this form like that one, this one and so this is coming in this form, well, fine.

So, that way C is equal to root over g y 1 and this expression we are getting in terms of y 2 by y 1. And this is the expression normally we use for computing surge, surge means a celerity in surge. And then this help us in various in solving various numerical problems, small numerical problems, we can solve by using this particular expression, well.

[FL] [FL] expression for C 2 variable continuity and momentum equation [FL] 39 [FL] 39 [FL] [FL]

(Refer Slide Time: 39:57)



Let us take a problem, typical problem. In fact, we will not we will not be solving the problem as such, but we will just show how the steps need to be followed and how we can get the result. So, the problem suppose it is like that the water flows in a rectangular channel at a depth of 2 meter, with a velocity of 1 meter per second, well, say this is the channel and here, water is flowing, first I am drawing by dotted line is flowing with a depth of 2 meter, this depth is 2 meter, this is a typical problem taken from the book of Rangaraju open channel flow.

So, this is say depth is 2 meter and the velocity of flow is 1 meter per second, well and if discharge in the channel is suddenly trebled what will be the depth of flow? So, here it was a depth of flow, suppose in the discharge if we suddenly trebled, then what will be the depth of flow? And suppose depth of flow is increasing like that and this will in fact continue to increase, this will continue to increase, this will continue to increase and then wave is moving in this direction and after some time the depth will be increasing.

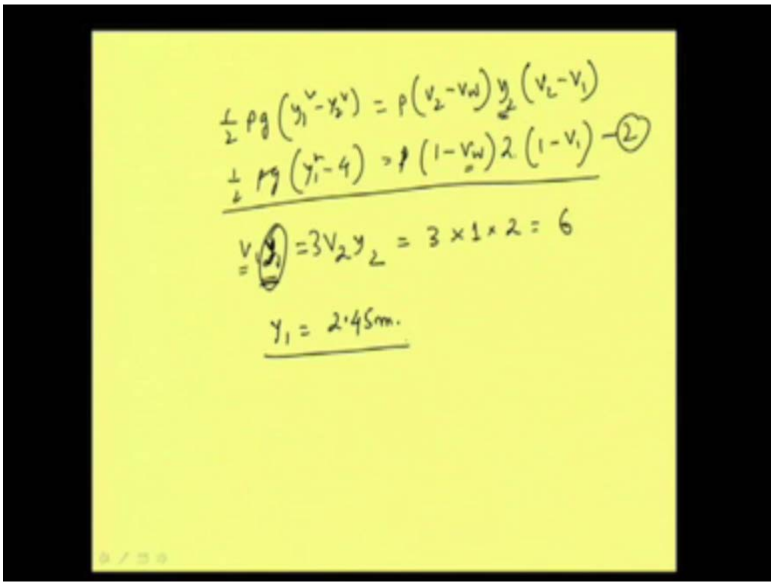
Now, here this is in fact our say we can put as y 2 and this is our y 1, of course we can write either way also and then velocity here is say V 1, velocity here is V 2 and then wave velocity let us put as V w. Then how we can write the continuity equation and momentum equation for this case and what we know here is the depth y 2 and this V 2 is known, velocity is 1 meter per second and depth is say 2 meter.

So, that part is known and other things are not known and what we know is the discharge, what was earlier flowing is just trebled. So, this discharge what will be getting here y 1 V 1, say considering unit width, what will be getting y 1 V 1 that will be three time of that of the say y 2 V 2, well.

So, how we can write the continuity equation again to first make it steady, what we can do to make it steady? We can apply a velocity opposite to this V w, so we will be applying here V w. And then if we apply V w, what will be the velocity here, ultimately this velocity will be changing to say V 1 minus V w, after applying this one and this will be changing to, this velocity will be changing to V 2 minus V w.

Well and then continuity equation we can write say V 1 minus V w into y 1 is equal to V 2 minus V w into y 2, so that is what we can write the continuity equation. Now, here, we can put the value of V 2 and y 2, so what we can write that this is nothing but say V 2 is equal to 2, so 2, sorry V 2 is equal to 1 minus V w into 2. So, this equation will give us some information and then we can write the momentum equation, we can write the momentum equation say.

(Refer Slide Time: 44:56)



So, momentum equation can be written as say half of well… let me give a number to this equation, say this equation is 1, then momentum equation is equal to half of rho g say y 1 square minus y 2 square is equal to rho V 2 minus V w. Here, this is the velocity, then we are getting y 2 and I mean this part is actually the discharge and then it is V 2 minus V 1 that is what we are getting. And then this expression also we can simplify, simplify means we can put the value like half of say rho g y 1 square minus y 2 square, means 4, this is equal to rho, you can write this as rho, then V 2 minus Vw that we can again write as say V 2 was our 1 minus V w.

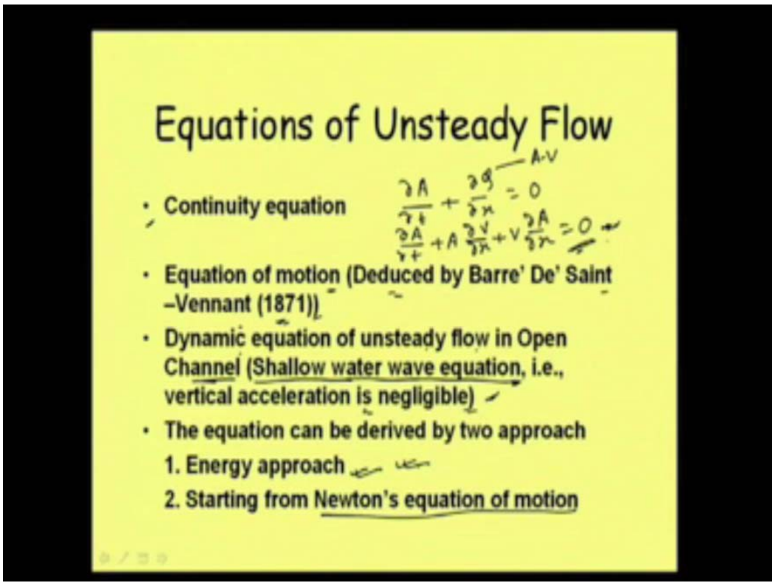
And this y 2 is already known to us, so this will this is 2, so this two depth is, we got the depth as, sorry we got the depth as 2 meter. So, that we can put here, say 2 and then this V 2 minus V 1, again this we can write as 1 minus V 1, now this is say equation 2.

Well, that what information we have, we have a relationship between, what information we have that you can refer to the slide, we have one information like that V 1 y 1, which is the discharge, which was the discharge after the charge is coming, that is V 1 y 1. This is the actual discharge what is coming here and this discharge is trebled or three times of that V 2 y 2, so we can write it like that.

V 1 y 1 is equal to say three time of V 2 y 2 and this value we can have that is equal to 3 into say one is the velocity into to the depth 2, so this is equal to 6. So, that way we can have, I am just writing the numerical value, not writing the unit here, so this is one expression and then we have this expression. And solving these expressions we can get ultimately the value of y 1, because our target is to know the value of y 1 and so if we know this value of y 1 our problem is solved. And to get the value of V 1 we have these equations, say this is one equation and then, here of course we have this unknown V w and this is one equation where we have this unknown V w.

So, V w and y 1 are the unknown that we can solve by trial and error method and we can get say y 1 is equal to 2.45 meter. This is just a solved problem already in the book of rangaraju and just I am taking here, just to show how the surge problem can be solved, well. Then, we will be taking up the next topic rather the equation of unsteady flow, well.

(Refer Slide Time: 48:16)



Also, equation of unsteady flow that we know, that already we have discussed some of the equations of unsteady flow when we were discussing our earlier discussion on say governing equation of open channel flow. For unsteady state, we were discussing about the continuity equation, the continuity equation that as we know it is say del A del t plus del Q del x is equal to 0, that we could get. And this of course we can write as say del A del t plus this Q, if it is Q is written as a into V this can be written as say a del V del x plus V del A del x is equal to 0, well.

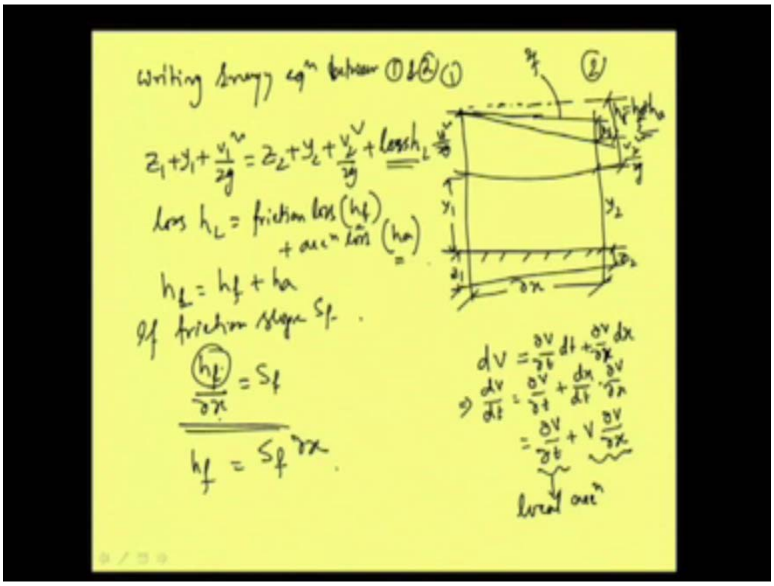
So, here, I am not taking up again this continuity equation, that you can refer or rather we will be knowing by referring to our earlier classes. And then, the two governing equations are required basically for solving the problem of unsteady flow and then, one of the most popular equation, of course continuity equation is there, then another equation that we take is the equation of motion. And this can be of course we use momentum principle or energy principle to get this equation of motion and for some case we use say continuity and momentum equation in couple for solving the problem, sometimes we use continuity and energy equation for solving the problem, depending on the situation where suppose energy is, energy loss is there, there we go for momentum couple, continuity momentum couple.

So, that way these considerations are there, of course in this class we are not going for all those details, but we will be just discussing the equation of motion that was given, that was first deduced by barre de saint vennant and popularly known this equation as saint vennant equation. And that is say 1879, long back this equation was derived, well and this is a dynamic equation of unsteady flow in open channel and for what condition it is the shallow water wave equation? Shallow water wave equation?

That means, why we are giving this as shallow water wave equation, because when this equation was derived this vertical acceleration was considered to be negligible, so vertical acceleration is negligible.

And then we should know that for what condition we can apply this and derivation of this equation not this equation, this is continuity equation, derivation of the equation of motion can be done by two approach, one is energy approach and another is that we can start from newton's equation of motion. So, by both this approach we can have the derivation and here, we will be discussing how by energy approach we can derive the equation in a simple way, ok.

(Refer Slide Time: 51:27)



And then, let us see what is the energy equation, say if this is the flow and this is going like that, well let me take some space, because we will have to write here, say this is section 1 and section 2, so section 1 and 2, well. In this section, 1 and 2 let us take a small length, let us take a small length and let us write this as say as this is small, let us write this as say del x, this length is and that term is here, that term is here. So, this depth is say z 1 and this depth is say z 2 that is the elevation head up to the bed. And then up to this bed this is y 1, we are considering slope to be small and that is why we are not writing y 1 cos theta or those term we are writing straight way it is y 1.

And then here this energy is V square by twice g, V 1 square by twice g energy head up to that point. And then here, in the second section section 2, this is the y 2 and the V square by twice g is suppose that much, V square by twice g, V 2 square by twice g. And then we can draw the energy gradient line as this one energy gradient line, as this one and there is definitely some energy loss, this we refer as…

So, when we draw it parallel to the datum then we will be finding that there is a head loss and this head loss we can write as h L. And in fact, this head loss is having two components, one is that, one is that its friction loss, that is known to us because we did discuss about this friction loss during our discussion of gradually varied flow itself, but here there will be another component. Because, in case of unsteady flow that local acceleration exists, local acceleration exists just to recall briefly. Say, when we say V, then it is a velocity, which is a function of x and T, of course in steady flow it become a function of a it it is not a function of T, but in unsteady this become a function of T.

Now, when we talk about, so this change in V say d v, that we can write as del v del t into d t plus del v del t into sorry del v del x into d x, del v del x into d x. Now, if we write that it is a function of x and t, so if we just write it as d v d t, d v d t, then we are getting del v del t plus say this d x d t, so d x d t divided into del v del x. And this d x d t is nothing but V, so now perhaps you can recall that we did discuss earlier also d x d t is equal to V, del v del x. And then this is called local acceleration, this is called convective acceleration.

Now, in case of say steady flow, this term is not existing, so that problem becomes little simpler, but here we have this term. So, this local acceleration exist local acceleration exist. So, because a acceleration is there in any whether this fluid or in a solid body, if acceleration is there that is a mass moving and that mass is having some acceleration means, some force is being utilized to have that acceleration, otherwise that acceleration will not be there. So, some force is being utilized to have this acceleration and when some force is being utilized and things are moving, then there must be some work done, force is applied and it is moving, so some distance has been moved.

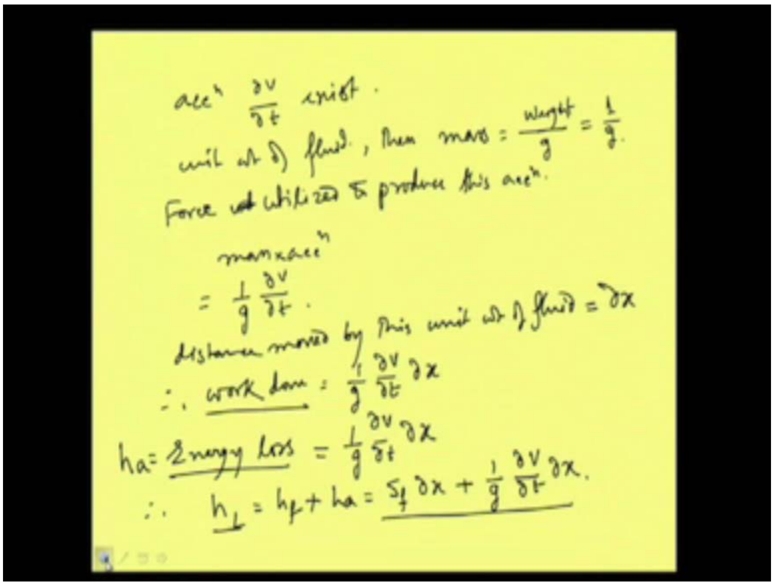
So, some work done will be there and some work done is there means then some energy loss is there. So, we can say that in this sort of flow there is two types of loss, one is h f and other is h a, so this two laws will be there, this h a is say loss due to acceleration.

Well, now if we write the energy equation, so writing energy equation between say 1 and 2, between 1 and 2, what we can have, between 1 and 2 this we can write say z 1 plus y 1 plus V 1 square by twice g is equal to z 2 plus y 2 plus V 2 square by twice g plus h L, well, plus h L or say we can say loss loss h L. Now, this loss as I have explained that let me write here say loss h L is equal to say friction loss, friction loss that we write as h f and plus the acceleration loss, acceleration loss that is say h a.

So, our expression become, now of course in this last term we need to put this two, so this is equivalent to h f plus h L, already we have written here, now we need to have an expression for these two. So, h L is equal to, we know that it is h f plus h a. Now, how we can get the h f, because friction slope we know that s f, if friction slope is s f, if friction slope is say s f, then we know that h f after that, that is friction loss divided by delta x delta x is equal to s f or we can say that s f the slope into distance, s f into this delta x will give us the h f.

So, friction slope may be this like that and we can have s f up to certain point h f and then this slope is actually s f and then other part of loss is due to say this acceleration. So, that is cleared to us, that we have discussed earlier also, so in place of h f by delta x, sorry in place of h f, we can write that h f is equal to s f into delta x, well, now that is for our next acceleration lost is there, how we can calculate that that?

(Refer Slide Time: 58:58)



So, acceleration del v del t, local acceleration del v del t exist here. So, if this is the acceleration, then if this is the acceleration then the mass suppose if we considered that unit weight of fluid unit weight of fluid, then mass is equal to say weight by actually weight by the g, so this is here as weight is equal to 1.

So, this is equal to 1 by g and force utilize force utilize to produce this acceleration, force utilize to produced to produce this acceleration that is equal to mass into acceleration, mass into acceleration, so that we can write as 1 by g into del v del t. So, this is what force required, force utilized, then this force is working for how much distance, this force is working and then acceleration is being there and then the fluid is moving, this mass is moving from here to here, so the distance of say delta x it is being moving.

So, what is the work done? And distance moved distance moved by this unit mass, by this unit weight rather by this unit weight of fluid is equal to delta x, that is what we are considering. Therefore, the work done, how much work is being done? Work done is equal to 1 by g del v del t into del x, del v del t into del x. Well, so this is the work done and what this work done is that is the energy loss energy loss. So, energy loss is equal to; that means, 1 by g del v del t into delta x.

So, finally, how we can write the head loss, so this this energy loss is nothing but h a. So, this energy loss due to acceleration that is the work done due to produce that acceleration and that is the energy loss we can write like that. Therefore, say what we can write that head loss h L is equal to h f what we can write, h f plus h a h f plus h a and that h f we can write s f into delta x and then this h a we can write that is the acceleration loss 1 by g and del v del t into delta x.

So, putting this value of h L in our equation of energy this one, then we can get a simplified expression and then manipulating with that expression finally we will be able to derive equation of motion and that we will be doing in our next class. And then from that we will be also trying to see how this equation can be solved and then by solving this equation how we can get solution to our real life problem of open channel flow. So, thank you very much.