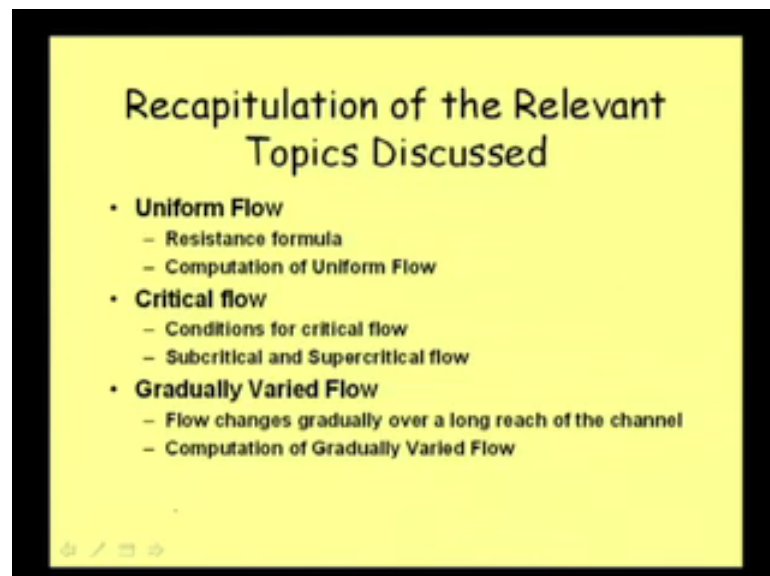


Hydraulics
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Module No. # 04
Gradually Varied Flow
Lecture No. # 07
Rapidly Varied Flow: Hydraulic Jump

Friends, today we shall be going to a new topic, one of the most interesting topics in Hydraulic Engineering that is, Rapidly Varied Flow, well. And under this rapidly varied flow, as I was telling this is most interesting topic, the Hydraulic Jump is one phenomenon which we have referred in our earlier classes. Also several times that we are getting a phenomenon, that we call as hydraulic jump, we will be getting a phenomenon, but we could not discuss that earlier. Now, today we will be discussing under the topic of rapidly varied flow the hydraulic jump, well.

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So, before going to hydraulic jump, let us see what we have discussed earlier till now. That is we started with uniform flow and there, we did discuss about resistance flow formula and then we did discussed, how using that resistance flow formula, we can

compute the uniform flow depth that is the normal depth. And then, we studied critical flow also and then critical flow is quite important or we need to recall the understanding of our critical flow for studying this hydraulic jump phenomenon; say, we did discuss about various conditions that we get at critical flow condition.

Critical flow itself is a condition and that critical flow condition corresponds to some of the very important aspect that is the specific energy is minimum at critical flow or you get specific force is minimum at critical flow like that we got different condition. **Froude number is one**, Froude number is one at critical flow all those we got. And then based on the concept of critical flow, we classified the flow into two class, one is called supercritical flow another is called subcritical flow.

Well, the supercritical flow means, when the flow depth is less than critical depth or rather we can say, when the flow velocity is greater than critical depth velocity, **that is velocity at critical depth** velocity at critical depth is something and in supercritical flow velocity will be higher than that depth. So, that is called supercritical flow or visually we can say when depth is less than critical depth then say it is supercritical, when depth is greater than critical, then it is called subcritical, well so that way we classified that things.

And then of course, we have studied gradually varied flow, then, how the flow changes gradually over the long reach of channel, long channel reach. So, when the flow depth changes because this rapidly varied flow is also a phenomenon, where the flow depth changes and gradually varied flow is also a phenomenon, where the flow depth changes. So, in gradually varied flow, the flow depth changes slowly or rather gradually within a long reach of the channel, if we recall our last class where we could solve one problem of gradually varied flow using standard step method.

There we could see that, when the flow depth were raised to 2 meter from a uniform flow depth of 0.335 meter, no from a uniform flow depth of 1.268 meter or 1.27 meter around, that is from 1.27 to 2 meter when the depth was change and of course, in a very mild slope, the slope if I remember correctly that was 1 into 10 to the power minus 4, so that was the slope, bed slope. So, when in that bed slope, we change the depth of flow in that small amount, I mean even less than 1 meter, change of depth was from 1.27 to it was 2 meter. So, it is slightly higher than 0.7 meter when it was raised, then the gradually

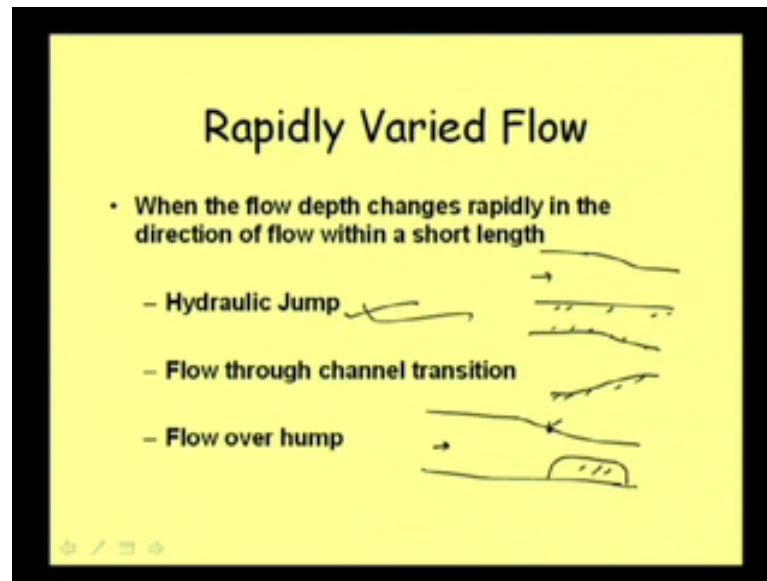
varied flow profile that extended up to about 20 kilometer. So, now we can have the feel that this flow depth change is so gradual, that it is changing in a very long reach of channel of course, it is changing very slowly; that means, at 20 meter perhaps you will not be able to appreciate **this appreciate** the change between the normal depth flow and the changing gradually depth flow, because it is very just a difference of may be millimeter so like that, but it is changing gradually and it is forming ultimately the total change we are having, but in rapidly varied flow, the flow will be changing within a very small reach of the channel.

In fact, it changes in such a small reach that, frictional effect can be neglected in such a small reach. So, it changes rapidly within a very small reach of channel and that is why it is called rapidly varied flow. And we did discuss about computation of gradually varied flow earlier and here we will have to discuss about the computation of gradually varied flow. Well, while discussing computation of gradually varied flow, while type of gradually varied flow we could see that sometimes, there is something on the upstream which is not gradually varied flow, a gradually varied flow is there on upstream of that we can have hydraulic jump.

In the last class also in the mathematical computation of one of the gradually varied flow when it was s_1 profile, then we could see that there is a hydraulic jump before that, but what is that hydraulic jump and how it happens and how we will have to introduce that hydraulic jump, where that hydraulic jump will occur, location of that hydraulic jump, all those issues need to be cleared, then only we will be able to draw the complete flow profile.

Till now we are drawing flow profile having gradually varied flow, now when hydraulic jump is also there in between the total flow profile, we will have to draw knowing the location of hydraulic jump, the change of depth between upstream and downstream of hydraulic jump and then how it will continue with the gradually varied flow lying at upstream or lying at downstream. So, all those aspect will be discussing in this particular topic of rapidly varied flow and of course, we are always talking about hydraulic jump **hydraulic jump**, but what other gradually rapidly varied flow are there.

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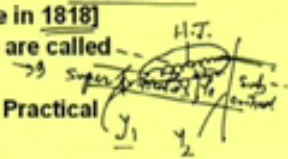
Well, there can be other type of rapidly varied flow also which is not hydraulic jump, what is hydraulic jump that I will explain again, say in a channel transition, when the channel changes from a say bigger dimension to a smaller dimension, then there can be flow variation or there can be variation of depth rapidly. So, that sort of flow phenomenon are also falling under rapidly varied flow, then some times in a channel say we are providing a hump if you refer to the slide, say a flow is like that and it was flowing like this, then we are providing a raise hump in this part; then the flow depth here will be observing that will be moving like that, this part of flow is also not a gradually varied flow, so it is a rapidly varied flow (Refer Slide Time: 08:45).

Similarly, in channel transition also we can have that means, when the channel is changing from here to here like that, say this is changing like that these are the side and if I draw it in this way, then flow is coming like this and in this portion it may come down in this way, why I am using the term may because, whether it will come down or whether it will rise, in this phenomenon also whether it will come down or rise, this will depend on what type of approaching flow is whether it is subcritical or supercritical. So, that we will be discussing of course later, but what will be discussing in this class is hydraulic jump.

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Hydraulic Jump

- When the flow changes from supercritical to subcritical condition, flow depth increases abruptly with formation of turbulent eddies and subsequent breaking up of the eddies downstream leading to air entrainment and energy loss.
- This phenomenon is known as hydraulic jump
- [First investigated by Bidone in 1818]
- Depth before and after jump are called **SEQUENT DEPTH**
- Hydraulic Jump has several Practical applications



Well, now with this introduction to hydraulic jump, let us try to define hydraulic jump, **let us try to define hydraulic jump** that is well, when hydraulic jump will occur that is the first point, when the flow changes, you can refer to the slide. When the flow changes from supercritical to subcritical condition, this is very important point to be noted that flow must change from supercritical condition to subcritical condition; at supercritical the flow is coming with a very high energy and then due to some reason, due to some obstruction at downstream or may be due to normal channel slope condition itself, at downstream depth is subcritical then, there is the flow changes from supercritical to subcritical abruptly and with lot of turbulence. So that phenomenon, when the flow changes from supercritical to subcritical condition, flow depth increases abruptly with formation of turbulent eddies.

So, lot of eddy formation will be there, lot of rolling will be there in the water and that eddy first it will be a strong eddy then, these eddies will break down and will form some smaller eddies and then in between air entrainment will be there. So, that way when it is forming some turbulent eddies and subsequent breaking up of the eddies downstream, as it is moving downstream these eddies will break up and it will form some smaller eddies and leading to air entrainments; some air will get enter into the flow bubble will be coming up and this bubbles are also moving up and then with this entire the flow, phenomenon is associated with a huge energy loss because, this eddies are forming turbulence is being created and so all these phenomenon when it is occurring within the

flow, then energy is being consumed in that part and so in hydraulic jump, there will be always loss of energy.

And this loss of energy although we think that it is not good **in any** for whatever may be the cause, but for water flow phenomenon for many a time we need to reduce the energy of flow and for those purpose we use hydraulic engineering. So, we use hydraulic jump, **we use hydraulic jump**, so hydraulic jump is a phenomenon that is used also sometime, that means we try to produce hydraulic jump so that flow energy get dissipated.

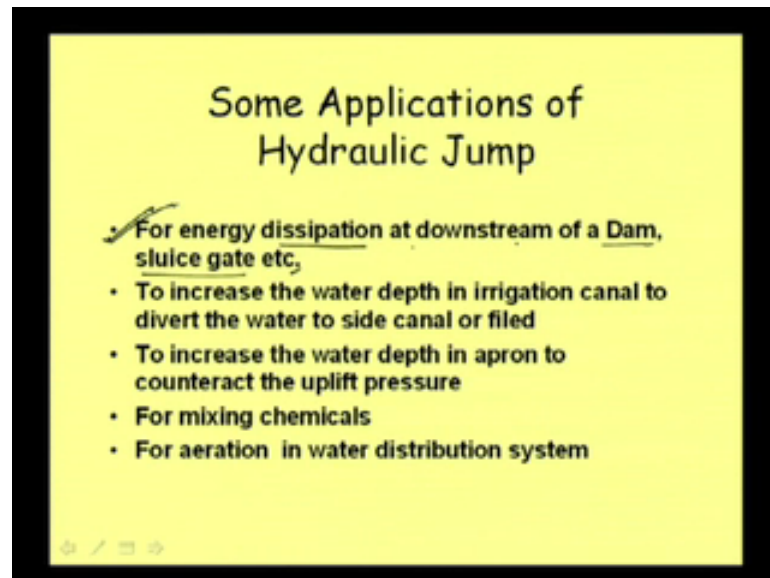
Well, so for definition of hydraulic jump is concerned, when the flow changes from supercritical to subcritical, then the flow depth changes abruptly with lot of turbulence with air entrainment **in it** into it and which is associated with loss of energy and this flow phenomenon **this phenomenon** is known as hydraulic jump. And of course, first investigation of this particular phenomenon was create out by Bidone and it was in 1818, long back this phenomenon was observed and it was investigated. Well, but still today also although it was investigated long back, but as on today also, this phenomenon is interesting and people are working on that to know much of this particular phenomenon.

Well and when hydraulic jump occur, then there will be a depth at upstream of the hydraulic jump and downstream of the hydraulic jump and that **two depth are called** two depths are called sequent depths. If I draw the hydraulic jump then, suppose this is supercritical flow, this is supercritical flow and say on the downstream we are having subcritical flow, it is subcritical flow. Now, how it is occurring? Subcritical flow, that aspect I am not considering right now, but somehow the depth is increasing here and somehow depth is decreasing here, discharge is same, Q discharge is coming for the same discharge, we are having supercritical flow here, subcritical flow here, then the flow from here to here critical depth may be somewhere here, this is say y_c critical depth line.

Then from this point to that point water will be moving creating lot of turbulence here, lot of eddy formation will be there at beginning and then it will be smaller eddies and then it will ultimately goes and finally, it will meet this depth and this phenomenon is called hydraulic jump. And the depth at upstream, this one say y_1 and depth at downstream, this is say y_2 , sometimes people write y_1 dash and y_2 double dash also, anyway this y_1 and y_2 are called sequent depths of hydraulic jump.

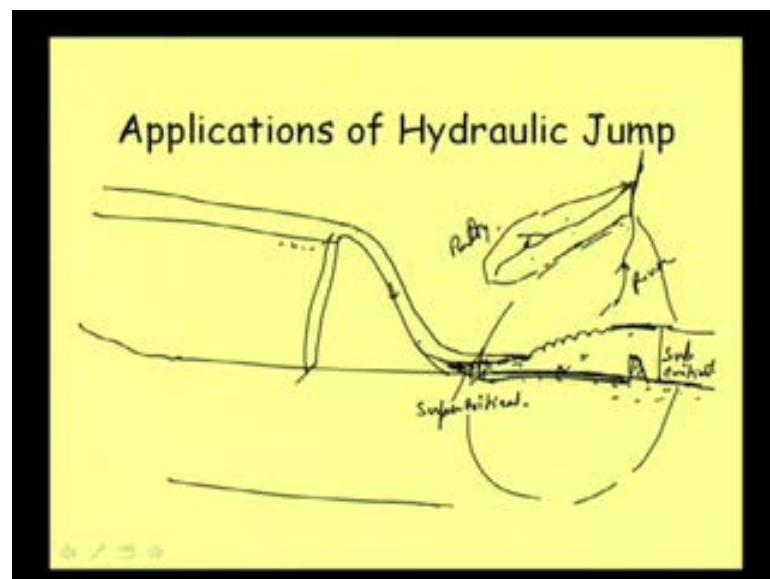
Now, this hydraulic jump before going to the theory of the hydraulic jump let us see that in hydraulic engineering, why it is becoming that much of important? Because this hydraulic jump has lot of practical application and that is why people are discussing and people are trying to understand more about this hydraulic jump, it has lot of practical application. So, what are those practical applications, first let us see.

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One practical application is that for energy dissipation at downstream of dam, sluice gate etcetera well that is one of the application.

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What that application is, say **we have a dam** we have a dam here and then of course, let me draw a spillway section, not the actual dam section, say water is being released like that, this dam is like that and we have storage of water here and then when water exceeds the spillway level, it will be moving like this and it is moving this way and then it is moving like this (Refer Slide Time: 16:40).

Now, what problem can arise and why hydraulic jump is necessary here? You can see this height may be very high, **this height may be very high** because we can have dam height varying up to say I mean 300 meter high dam, so like that also you can have and of course, smaller dam we can have 30 meter, 20 meter like that also we can have, but we can have very high dam also. Now, depending on the height of dam this water will be releasing from a very higher depth, so when it is coming from a higher depth, it is flowing all through, it is gaining lot of energy and it is moving with a very high energy in this part and this part of flow as it is moving with high speed, depth will be very low **depth here will be very low** and it will possess lot of energy.

Now, when the water with high energy is moving on the surface, this is again on the downstream we are having the river bed. So, what will happen, this high speed water can cause erosion of the river bed, rather it will slowly erode the river bed and it will cause lot of damage to the downstream. Now, if we do not do anything in the downstream part, this high speed water will be moving for a significantly longer distance and then will cause damage to the river bed and suppose if it is eroding all these things, then this eroded material will be carried downstream and somewhere it will be deposited at the downstream, that will create another problem.

So, what is expected is that we need to control this particular flow and we should not allow this water to move much downstream and if possible, we should try to form a hydraulic jump here, where its energy will be dissipated. So, in this portion we need to form a hydraulic jump, if we can do so its energy will be dissipated and then at downstream whatever water is flowing, this is flowing with less energy without causing any damage to the river bed, well. So, for that many a time some obstruction is constructed still or this is called stilling basin and this part is made concrete. So, when this part is concrete, so here suppose it is moving with high speed, this will not cause damage and water level by this sort of obstruction is raised to certain depth and this of course, we need to design properly, but here by some obstruction we raise the depth in this part and then

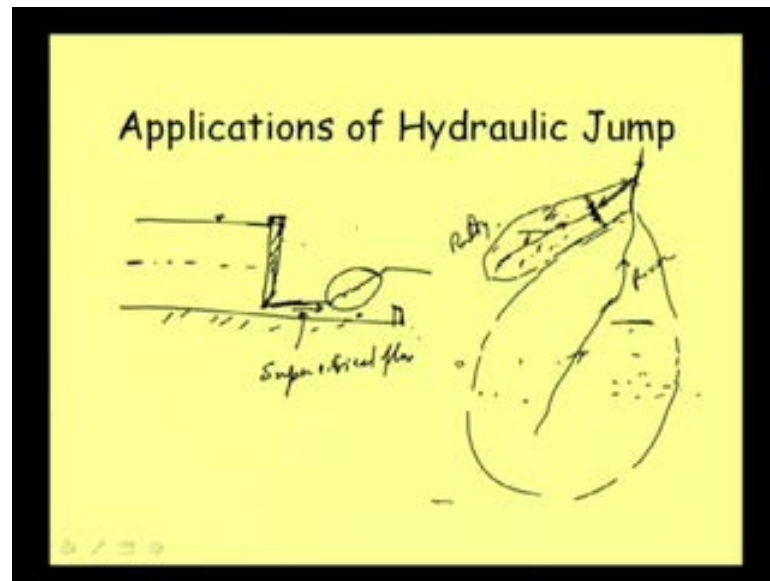
we raise the depth to such extent that it become subcritical flow here, **it become subcritical, this is supercritical**, this is supercritical.

So, ultimately between this portion and of course, some small block and other things are there, sometimes this approach is not that simple; I mean stilling basin has lot of other things to do, but of course we are not covering that in detail, lot of baffle wall that means small walls will be here in this portion itself, some wall may be should block may be there which will switch the water in the upward direction like that. So, different things can be done here, but main objective is to raise the depth and to form a hydraulic jump and that way, it is suppose switch up from this part and then it is going like this and from here a hydraulic jump is occurring.

Well and once this hydraulic jump occurs, this energy of flow gets dissipated at this point and then downstream portion it will remain free of danger, so it is one application of hydraulic jump, well. Then similarly, suppose we have a sluice gate, **we have a sluice gate, well** where we can have sluice gate, suppose we have a channel of this a river is flowing there, another river is coming there and this is suppose going from a paddy field just it is not a river say stream and this is a river, this is a river which is coming from hilly area and a big catchment suppose and then this river is carrying lot of water and this is having a small catchment and this is having a much bigger catchment suppose like this and this is having a small catchment (Refer Slide Time: 21:00).

This is just carrying, this river is carrying the water which is falling here or coming from precipitation into the agricultural field, now water is flowing here and it is joining this river, main river. Now, what may happen that because of heavy rainfall in this catchment, because of heavy rainfall in this catchment? Let me rub that part, because of heavy rainfall in this catchment, water this river may have flood and when this river is level of water is rising in this river, then water from this river, but suppose there is not that much rainfall and even if it is rainfall is there, from a small catchment the water in this river will not be that much.

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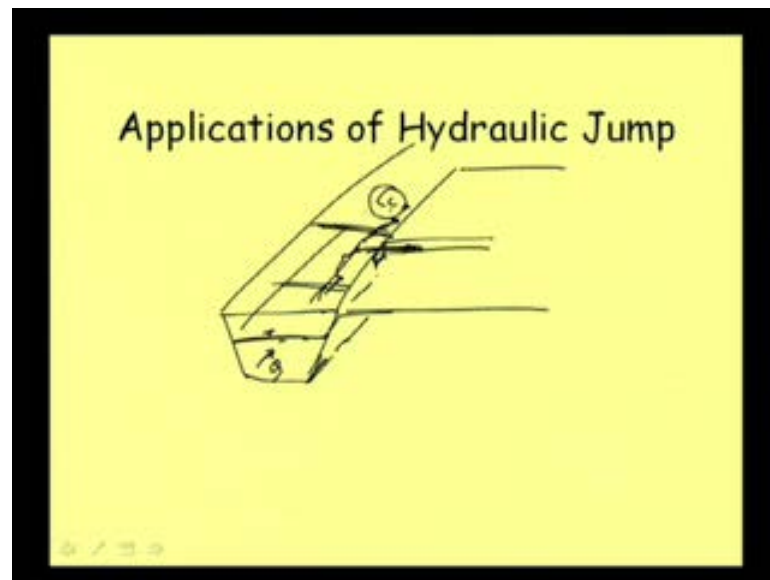
So, otherwise suppose this area this catchment is not having flood or any problem, but when this river is rising, then water from this river will flow back into this river and to this area may get flooded. So, to prevent this area from flooding, sometimes we can arrange to have a sluice gate, in this portion or gate in this portion, and then when water is rising here, we are closing the gate and water cannot come.

So, this is one way that we use for closing the gate and then we try to I mean prevent some area and we use sluice gate in this part. Then similarly there may be lot of other application where we are using sluice gate, say we want to store water, but not at dam, but we want to store water, say normally water is flowing at this depth, but we want to have a higher depth, then we can use a gate. And then this is a small gate and then water is kept up to this much, but suppose we want to maintain the water at this much level. So, some amount of water we need to release downstream, but say normally in this slope, water is moving at a normal depth, but here when we are releasing it from a smaller depth, **it will become** it will be moving with a high speed and that may move as a supercritical flow **supercritical flow**.

And that supercritical flow may cause damage in this part and as such we can put some barrier and we can do some other activity here or sometimes, the depth here we need to increase so that we get a hydraulic jump. Otherwise, this flow will move quite long and it may create or it may cause damage to the downstream side, so that way for sluice gate

for dam, we need to produce a hydraulic jump for safety of the downstream side, well so this is one of the applications. Then, let us see what is the second application, the second application is that to increase the water depth in irrigation canal to divert the water to side canal or to the field, well.

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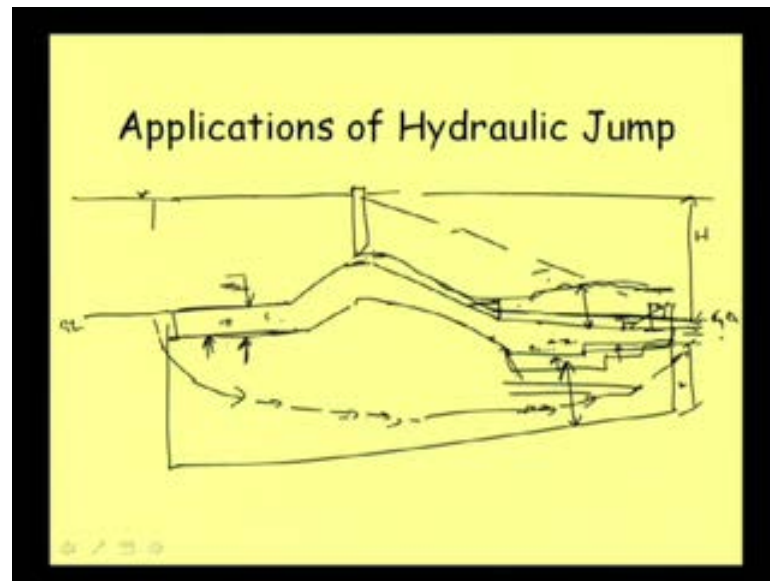


In irrigation canal, when a irrigation canal is carrying some water, say it is carrying some water and say in this side it is field and there may be some small field channels, then water depth is suppose this much and with this depth water is flowing here and then field is here field is here. Well, now suppose, so far amount of water required is concerned, we can have only this much of depth because in this normal slope or in this slope, for the required amount of discharge we are getting this much of slope, but we need to carry the water to this side channel, so there will be some opening here and our water level must cross this level (Refer Slide Time: 25:20).

So, what we can do? We can have some barrier here and then we can rise the water, we can allow or we can have some device say, we can some gate which will cause a supercritical flow here, then hydraulic jump will occur and then water depth is rising and we can carry the water in the downstream side or say we are giving a barrier here, we are giving a barrier here, then water is rising here, we are carrying this water to the downstream side. Then we need to allow the water to move downstream, then when water we are allowing again, there may be it may cause erosion here, then for that

purpose also we may have to **have** say hydraulic jump for energy dissipation. So, for this sort of activity also, we need hydraulic jump in irrigation canal, well then the third application we can mention that to increase the water depth in apron to counteract the uplift pressure, this is another important application to increase the depth of flow in the apron.

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Say many a time as we did explain, we are storing water by this sort of barriers, say some gate are there, we are raising this part to reduce the weight of the gate and also to for sediment problem, say gate is not obstructed that way we are doing it and then say, water depth is this much and then this part is called apron, because as we know that when we will be releasing water or when water will be moving above this part or we releasing it, water is coming this way and then this may be sluice gate like this and then when we are releasing water it is coming like this and then it will be moving over this bed.

Now, while designing this apron, **while designing this apron**, let me just draw it in this form now, designing this apron, what is that? This is ground level, this is ground level and I am not going into detail of this part, but what is that when water level is here, then water level we do not have any water on this part, **we do not have any water on this part** some amount of water is going and if we do not do anything here, then this water level is going in this level. Now, what is the difference between hydraulic head, this minus that,

so this is the head difference, **this is the head difference** and so as this is the difference hydraulic slope between this and that we call as this one.

Now, when we talk about this particular part of the water, in this part we do not have much problem due to uplift pressure, now when water is there, water is percolating through the ground, water is percolating through the ground and it is moving this way. Now, as the water is percolating, there will be seepage pressure and otherwise also when this is not in previous and I mean there is a communication, there will be uplift pressure because water is a continuous through the soil also.

So, water will be having some uplift pressure at this point, that uplift pressure is equal to the depth of water here and then when it is moving of course, we use different formula, like say how the energy will loss when the water is moving through the soil, energy is existing, energy will be lost and this energy will gradually reduce of course, but still it will be having substantial amount of energy and then uplift pressure will depends on that remaining energy, well.

So, the uplift pressure here will be more and then there will be like this, here is suppose less and then this is the energy loss in between and then we are getting the uplift pressure like that. For this part, suppose this is the uplift pressure, then from the upward side also same pressure is there, so for this portion of concrete we do not have any problem, we can provide nominal thickness here, because that is not I mean nominal thickness for other consideration we need to give design.

But for **so for** uplift pressure is concerned this is not that significant, but for this part of flow, **for this part of flow** we can see that uplift pressure from this side is say this much and from the top we do not have sufficient water, we have only this much of water. So, uplift pressure will be quite significant and then we need to design this flow so that, **this can resist** this flow can resist the uplift pressure. So, the thickness of this flow become quite large like that **like that** as we are going down, uplift pressure is reducing and thickness is also reducing, but it is quite large in this direction.

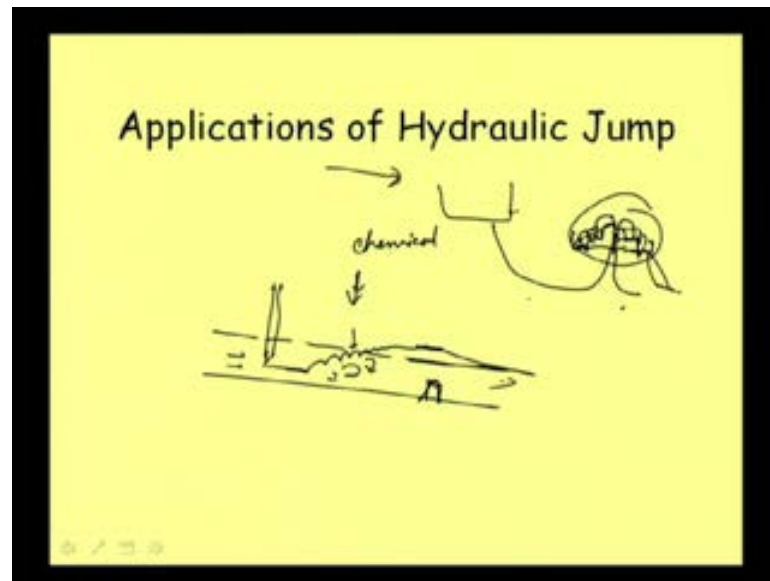
Well, this uplift pressure if we could counteract by increasing the depth of flow here, then we could reduce our depth of flow here, **we could reduce our depth of flow here**. So, for to achieve that, what we can do, that if we put some baffle wall, if we switch some switch block here, then if we put some say if we raise this part like that, then what will

happen? That water is coming like this and it is rising up to this point and it may generate a hydraulic jump **it may generate a hydraulic jump**. So, this part is supercritical that is subcritical when it is rising hydraulic jump is there, so the depth of water on the apron is also increasing. So, uplift pressure it can now counterbalance and that way the requirement of putting additional thickness to resist uplift is not coming into consideration.

Well. So, again some other problem are associated with this, if the hydraulic jump from here, then there will be suction in this part to resist that we need to do something, so the hydraulic jump is formed in a proper location, so that the suction we can avoid, well. Those are finer issues that we need to take up as a different topic as hydraulic structure; right now we just can see that, where application of hydraulic jumps is coming. So, this is one issue where hydraulic jump is applied to counterbalance the uplift pressure in case of hydraulic structure.

Well, then another we can very simple use is for mixing chemicals. Well, in many of our water distribution system, for water treatment we need to mix chemical of course, we can have some style of device and with that we can make stir up and then we can put the mixture there, but one way **suppose in a simple** suppose it is a irrigation canal and we need to mix fertilizer into water. Now, if we mix the fertilizer into the water in the canal, then it may not get mix up properly, if turbulence is not there and if you can have some portion where we have turbulence and if we put the chemical fertilizer, whatever fertilizer it may be, if you put the fertilizer in that point, then the mixing up will be proper. So, that way for mixing up of suppose fertilizer or any chemical in any water distribution system, if we can have a hydraulic jump, there will be having the turbulence and at that portion we can mix the chemical, ok.

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So, say in a irrigation canal or in any canal if we have a system of forming hydraulic jump, say normally the flow is moving in this level and it is moving as a very simple flow and if we mix chemical here, chemical or any other mixture rather if we mix here, it may be fertilizer or whatever then it is. Then it will just meet here and **it will be for** it will take time for diffusion and then it may not get mix up properly, but if we put a gate here, there may be other devices also. And if you allow the flow to move as a supercritical here, suppose normal it is subcritical flow, if we allow the water to move subcritical flow and then of course, this may keep on moving, but we will have to provide some barrier here, so that it rises little bit and then we are having a hydraulic jump and then it is moving and finally, it is again coming like this.

So, the objective of putting such barrier is to just fix the location of hydraulic jump, otherwise hydraulic jump can keep on moving, that will be coming later we will be discussing how it can keep on moving, but anyway if we can form such hydraulic jump and then if we can mix the chemical here; if it is mixed here, the mixing will be proper because, here we have lot of turbulence and that mixture will be perfect and then it is moving in the downstream direction.

So, in this process this is one application of hydraulic jump where it can be used for mixing chemical. Again another application is that sometimes in our say water supply system, the water may be having less amount of oxygen or air entrainment may be

required at that time point, generally erosion tank is always there, erosion system is there where we allow the water to move some turbulence now of course, different devices are there. Say if we have some where a tank here, say water distribution water supply system, in any water supply system, say water in water treatment plant rather from here, we allow the water to come this way and then it is going like this, water is moving and then say we can have some platform here and then this water may be allowed to just move as z and then it is coming like this and then we can make some step here. So that water is moving over the step and lot of turbulence are getting created and then from here, this is a tank suppose from here, this water is carried to the next section this is for erosion we are using.

Now, similarly this erosion can also be achieved by forming hydraulic jump. So, where this sort of devices are costly and in a system, we need erosion for different purpose may be for say biota like I mean may be for (()) or whatever may be the reason. Suppose we need that, we can form some hydraulic jump where it will be taking some air into it and air entrainment occurs and this become helpful for a many activities or many purposes. So, that way that is another use of hydraulic jump.

Well. So, that way we have seen that hydraulic jump can be used for various purposes, hydraulic jump can be used for various purposes and I have just mentioned few of them; and there can be few more uses of hydraulic jump well, but these are some of the prime use of hydraulic jump which are commonly adopted.

Well, then we can now see, with this knowing the its practical importance in our engineering activity, now we can see that how we know the phenomenon and how we can express the relationship of different parameters involved in the phenomenon into mathematical form and using the theory of hydraulic jump, how we can now we are telling that we need to control hydraulic jumps should occur here and then what should be the depth at the downstream point, all those things we need to know.

So, for that we need to know the very basics of those theories and let us just go into the equation of hydraulic jump, it is basically coming from momentum equation. In our earlier class also may be in our second or third class, we did discuss about the momentum equation, where we did discuss to some extent about this part and so we will

not be going into much detail, but we are starting from without explaining the very basics, we are going like this.

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The slide is titled "Equation of Hydraulic Jump". It features a diagram of a hydraulic jump in a channel with a slope θ . The flow is from left to right. At the upstream section (1), the pressure is P_1 , velocity is v_1 , and depth is y_1 . At the downstream section (2), the pressure is P_2 , velocity is v_2 , and depth is y_2 . The weight of the fluid W acts vertically downwards. The component of weight acting along the slope is $W \sin \theta$. Friction force F_f acts opposite to the flow direction. Air resistance F_a also acts opposite to the flow direction. The discharge Q is constant across the jump.

$$P_1 - P_2 + W \sin \theta - F_f - F_a = \rho Q (v_2 - v_1)$$

for small value of θ , $\sin \theta \approx 0 \Rightarrow W \sin \theta \approx 0$
for short length of the channel $F_f, F_a = 0$

$$P_1 - P_2 = \rho Q (v_2 - v_1)$$

prismatic rectangular channel.

Suppose, a flow phenomenon is there, say flow phenomenon is there may be a hydraulic jump in this part and then, we have that here what are the forces acting on this part, say p_1 pressure force, pressure force from this side is p_2 , then frictional resistance we have F_f then, suppose air is blowing then there may be F_a air resistance is also there and then it is flowing with a velocity v_1 and it is flowing out with a velocity v_2 , the water coming is Q . So, these are suppose the system and then it has a slope θ , it has a slope θ , so weight of the fluid is w and then component of the weight acting in this direction is say, $w \sin \theta$, this angle is θ , this is $w \sin \theta$. So, what we can have using the momentum equation, the force acting in this part, in this part is equal to the rate of change of momentum. So, rate of change of momentum means momentum per unit time how it is changing.

Well, so that we can write as say force from this direction we can say p_1 , then minus p_2 well at this and what are the other forces acting in this direction, this is $w \sin \theta$ and then opposing force is minus F_f friction force, then minus F_a that way all these opposing forces are there and that is equal to momentum means, say Q is the discharge flowing in unit time, so volume flowing, so mass is equal to ρQ and mass into change in the velocity is equal to $v_2 - v_1$. Now, from this expressions we have already discussed this equation, so I am just writing it directly, when θ is small so

when we are discussing about hydraulic jump then, say **theta for small theta for small value of theta** for small value of theta, what we can do say sin theta is equal to 0 or we can say that $w \sin \theta$ can be neglected. Then as we know that hydraulic jump is occurring in a very small length of the channel, so **in that small length** in that small length also there will be flow is turbulence. So, because of that this small length we can consider that, friction is not that much significant, if it is a long channel friction has something to do, but if it is a very short portion, in that short portion the effect of friction can be neglected.

So, that way for **short length of the channel** short length of the channel, what we can say that F_f there is a friction and F_a all can be considered equal to 0. So, what we are getting that $P_1 - P_2$ is equal to $\rho Q (v_2 - v_1)$. Now, this expression is basically the relation of momentum equation and from that, we could derive the specific force expression also and of course, let us see for rectangular channel, if it is a prismatic rectangular channel, how we can have this equation. Suppose it is a prismatic rectangular channel, rectangular channel for that situation, **how we can well** how we can just re write this equation to have a relationship between the depth at this point say y_1 and depth after the jump y_2 , how we can have this relation let us see, **ok**.

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$$\begin{aligned}
 & \left(\frac{1}{2} \rho g y_1^2 - \frac{1}{2} \rho g y_2^2 \right) B = \rho Q (v_2 - v_1) \\
 & \Rightarrow \frac{B}{2} g [y_1^2 - y_2^2] = \frac{v_1 y_1 B}{g} (v_2 - v_1) \quad \left| \begin{array}{l} Q = v_1 y_1 = v_2 y_2 \\ \Rightarrow v_2 = \frac{v_1 y_1}{y_2} \end{array} \right. \\
 & \Rightarrow (y_1 + y_2)(y_1 - y_2) = \frac{2 v_1 y_1}{g} \left(\frac{y_1 - y_2}{y_2} \right) \\
 & \Rightarrow (y_1 + y_2) = \frac{2 v_1^2}{g} \frac{y_1}{y_2}
 \end{aligned}$$

So, starting from this equation now for a rectangular channel we can have this p_1 , say p_1 pressure is nothing but, say we can have $\rho g h$ is the pressure here, here it is 0. So,

total pressure will be half of base into altitude this is well, let me write we are writing perhaps y , yes we are writing y , so this is y , so how we can write this, half of these things. So, half of $\rho g y$ square is the pressure force and it is we are talking about per unit width, if we talk about width of B , this will have to be multiplied by B and this is y 1. So, P_1 minus P_2 we can write say half of $\rho g y_1$ square minus half of $\rho g y_2$ square and this B as it is suppose here also p_2 is also the same, half of $\rho g y_2$ square into B , B is same because it is prismatic. So, we can write this p_1 minus p_2 in this form and then this is equal to Q is Q here Q is same, so what we can write, mass is equal to ρQ and then v_2 minus v_1 ok.

Now, we can try to simplify this expression ρ and ρ will get cancel, so what we can write that say, ρ here ρ here ρ is getting cancel and as it is let me write here this part, that half of say g we are bringing common, half of $g B$, suppose B by 2 into g then this y_1 square minus y_2 square, so y_1 square minus y_2 square. This is equal to, this Q we can write as $P_1 y_1$ into B , that is y_1 into B is the area and then velocity into the depth at this point and then we can write is v_2 minus v_1 . Now again, we can cancel B and B and then we can write it in this form, that is y_1 minus y_2 is y_1 minus y_2 into y_1 plus y_2 ; this is equal to we can write it as 2 will be coming here, twice $v_1 y_1$ by g and then this v_2 minus v_1 that we have here, we have found continuity equation Q , we have $v_1 y_1$ equal to $v_2 y_2$. So, v_2 can be written as v_2 can be written as say $v_1 y_1$ by y_2 , so we can write v_2 as $v_1 y_1 y_2$ minus v_1 .

Well, now this will lead us to say twice v_1 square, we are bringing twice v_1 square here and y_1 here and then it is y_2 and it is y_1 minus y_2 . So, this y_1 minus y_2 again, this y_1 minus y_2 and this y_1 minus y_2 we can cancel, this will lead to y_1 plus y_2 is equal to say twice v_1 square and y_1 by y_2 , we had one g here right g .

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$$\frac{y_2}{y_1} \left(y_1 + y_2 \right) = \frac{2 v^2}{g y_1}$$

$$\Rightarrow \frac{y_2}{y_1} \left(1 + \frac{y_2}{y_1} \right) = 2 F_1^2$$

$$\Rightarrow \left(\frac{y_2}{y_1} \right)^2 + \left(\frac{y_2}{y_1} \right) - 2 F_1^2 = 0$$

$$\Rightarrow \frac{y_2}{y_1} = \frac{-1 \pm \sqrt{1 + 4(2 F_1^2)}}{2 \cdot 1}$$

$$\Rightarrow \frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8 F_1^2} - 1 \right] \quad \text{--- (A)}$$

Well, now this expression, we can further simplify this expression, we can further simplify to say y_2 by y_1 square if we write here, then it will become y_1 plus y_2 and this will lead to $2 v^2$ square by $g y_1$, why we are writing y_1 square because here earlier we had $1 y_1$ sorry y_1 , here we had y_1 , we will have to write it as y_1 , y_2 by y_1 square and this is y_1 plus y_2 v^2 square by $g y_1$. Why we are writing in this form that because, we are getting here one term v^2 square by v^2 square by g . Now, we know one very important term that is called Froude number and that Froude number is nothing but, v^2 square by $g y$ that is $g d$, we can call $g y$ call that is a depth term, so v^2 square by $g y$ we can have and that is why we are writing it this expression particularly to have this term.

Well, then starting from this expression what we can have, our basic intension is to have it in this form that is y_2 by y_1 , y_1 y_1 I am writing here, another y_1 if I take this inside this bracket, then I will be getting 1 plus y_2 by y_1 . Well and this is equal to twice F_1 square, this is equal to twice F_1 square F is what, the Froude number, F is Froude number at the point one, F is Froude number at the point one. So, it is Froude number square actually Froude number is equal to v by root over $g y_1$ v by root over $g y_1$.

So, we are writing just Froude number square and this ultimately we can write as say sorry this is y_2 by y_1 , we can write as y_2 by y_1 square this one, plus y_2 by y_1 minus twice F_1 square is equal to 0 . Now, this is just becoming a quadratic equation, this is just becoming a quadratic equation and this quadratic equation can be written to have

suppose it is x^2 , this part is $x^2 + x - A$ is equal to suppose that means, $Ax^2 + Bx + C$ is equal to 0. So, we can write say x is equal to means y_2 by y_1 is our variable here, y_2 by y_1 is equal to say minus B plus minus root over A square. So, the popular expression we can write minus B means minus 1 here, B is minus 1 then plus minus root over B square, B square means again 1, B square minus 4 $A C$. So, minus 4 is 4 then A is 1 and C is minus 2, so it is minus 2, so into 1 4 into 1 into actually minus 2. So, we can basically this will be plus 8, it is not minus 2, C is equal to minus 2 $F 1$ square.

Well, so that can be divided by twice A means 2 into 1, divided by twice A 2 into 1. So, that can be written as y_2 by y_1 is equal to half, half we are bringing here and then this term I am writing first, root over 1 plus this is total bracket 1 plus 2 into 4, so minus minus plus, so 8 $F 1$ square and then out of this plus minus sign, we will have to use this plus sign here, we cannot write plus minus. The reason is that, if we write minus sign, then it is minus and then again minus, so minus minus it individually be this ratio y_2 by y_1 cannot become negative, in no way it can become negative, so this we cannot use the minus sign.

So, this we are using the plus sign only, minus sign is not practically possible because there is minus 1 if we use minus sign, then minus and minus ultimately this ratio will become negative, which is practically not possible. So, **this is the very popular expression**, this is the very popular expression for relating this y_2 and y_1 , which are the sequent depth of hydraulic jump, this is y_1 and this is y_2 . And in free flow condition, that is the jump will just start from this point, where the depth is 1 and it will end at this point whether depth is y_2 . And we know that, when we draw specific force diagram and energy is not equal between this point and that point, because there is lot of loss of energy in this portion.

So, energy is not equal, but the specific force we are starting from specific force equation and we are starting with the condition that, specific force at upstream and downstream are equal. So, this y_1 and this y_2 we are talking about, these are for same specific force, this specific force, this is depth y and this depth is of course critical, this is critical, this is critical depth y_C . And one of the depth is that is this depth is in subcritical reason, this depth is greater than critical depth y_2 and this initial depth is lower than critical depth. So, that we need to know y_1 is called initial depth, y_2 is called sequent depth and of

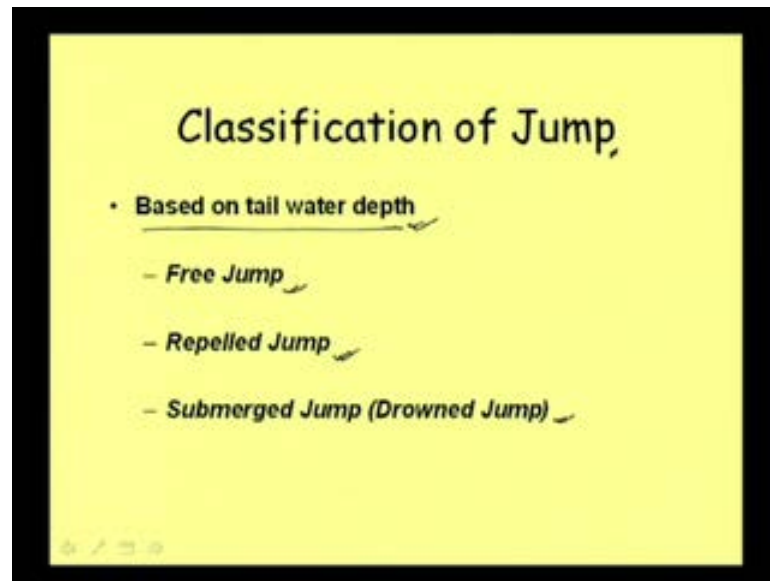
course as it together, y_1 and y_2 we called sequent depth of hydraulic jump and conjugate depth also it is same, that these are called conjugate depth **ok**.

With this very introduction to the equation of this, I mean equation of the hydraulic jump, here we should remember one point that this equation we are writing very popular, but it is for rectangular channel. If it is not a rectangular channel, rather a trapezoidal channel then, we cannot have our equation in this form, then we will have to go, we will have to derive of course, till we can derive the relationship between y_1 and y_2 for trapezoidal section.

But, in that case we will have to start from the very basic principle, that specific force is equal, specific force at upstream here and downstream that is what we are writing here, at the specific force at upstream and downstream are equal, based on that we will have to start and then from there we need to compute the depth of upstream and downstream. Once we know the upstream depth, we can compute the downstream depth and that we can do by trial and error procedure for trapezoidal channel, of course some graphical procedures were developed and some approximate solutions were also developed.

In fact, we also tried to develop one approximate solution, empirical solution generating lot of data in laboratory channel and then of course, by computation also by mathematical model, we did develop lot of data for y_1 and y_2 and we then tried to see how these can be related empirically and we could relate it based on the discharge value of course, this equation is valid for a definite range of channel, may be in the level of irrigation canal this is valid, but we cannot say in general, that is why I am not putting that here anyway, so these relations can be used.

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Now, with this understanding of hydraulic jump, now we can go for the classification of hydraulic jump, we can go for classification of hydraulic jump. Well, hydraulic jump can be classified in different way on the basis of different, rather on different basis. So, one classification of hydraulic jump that we carried out, this is based on the tail water depth and **understanding** proper understanding of this particular classification is very important. Well, this way we classify the jump as free jump, then next is repelled jump and then we talk about another jump which is called as submerged jump or drowned jump.

Well, just let us see how this tail water depth is significant and how it influences the hydraulic jump, well. Say, we have a y_1 here, now let us refer to the specific force diagram, this is the specific force diagram and this is y_1 , this is y_2 . Now, what it indicate, this is specific force in this side, that is specific force F we are talking about and on this side, we are putting the depth y well and sequent depth means this y_1 and y_2 , well. Now, this means that if this depth is y_1 we are fixing and we know that this is supercritical flow, this is supercritical flow and at tail water means at a downstream side, at a downstream side this depth must be subcritical, **this depth must be subcritical** then only hydraulic jump will form; otherwise if the under downstream also, suppose we have supercritical flow, then there will not be hydraulic jump, there will be a gradually varied flow profile in that case.

Well, now somehow here the tail water depth is say this much, **tail water depth is this much** and we are writing this tail water depth as y_2 dash, **y_2 dash** we are not writing this as y_2 , but y_2 dash because this tail water depth, this tail water depth will be governed by some other condition that is in the side, in the side whatever we are barrier we are putting how we are increasing the depth of the tail water, may be some barrier we are putting here and increasing these things or may be the channel is of the type that here by some sluice gate on the upstream, suppose we are creating this y_1 , but the normal channel flow will be like that say downstream uniform flow will be this much.

So, by various reasons due to various reasons, we can have the tail water depth this is coming from the physical process and that is what is y_2 dash. Now, we know that hydraulic jump will occur from this y_1 depth to its sequent depth y_2 to its sequent depth y_2 . Now, if this y_2 dash, **that is the tail water depth is equal to** if this tail water depth y_2 dash is equal to the sequent depth, we do not know what this sequent depth is, if this tail water depth is equal to the sequent depth y_2 , but this y_2 is sequent depth corresponding to corresponding to y_1 .

So, for different y_1 , suppose if our y_1 is here, **if our y_1 is here** our sequent corresponding depth or its corresponding sequent depth, its corresponding sequent depth will be this one. So, for each of our y_1 , which is in supercritical condition it has a corresponding sequent depth say this one and one interesting point is that, when our this is critical flow, when our initial depth increases, say initial depth is increasing like this is y_1 , then it can come here. When our initial depth is increasing, then its corresponding sequent depth is decreasing, so for one particular initial depth, there will be a sequent depth. Now, if **tail water depth is equal to** tail water depth is equal to the sequent depth then say, this y_2 dash or now we can write like this, if this y_2 dash tail water depth is equal to y_2 , then from here that it is corresponding sequent depth is this one, so it will jump like this and this part we are getting lot of turbulence and this sort of jump, we name as free hydraulic jump, free hydraulic jump.

We are not forcing the hydraulic jump to occur in a particular place, we are not doing anything, our tail water depth is just equal to the required sequent depth for this initial depth and that is why the jump is occurring freely. And we call this jump as free jump, now question is that if this depth is tail water depth is higher than this one, now for this y_1 sequent depth is y_2 that is fix, but tail water depth may be more than that, tail water

depth may be less than that, then how the hydraulic jump will occur that we need to understand and understanding that only we can classify the hydraulic jump into other type that is what is repelled jump, what is submerged jump. So that we will be discussing in the next class and then we will continue with our most interesting topic, hydraulic jump in the next class also, thank you very much.