Hydraulics

Prof. Dr. Arup Kumar Sarma
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module No. # 04

Gradually Varied Flow

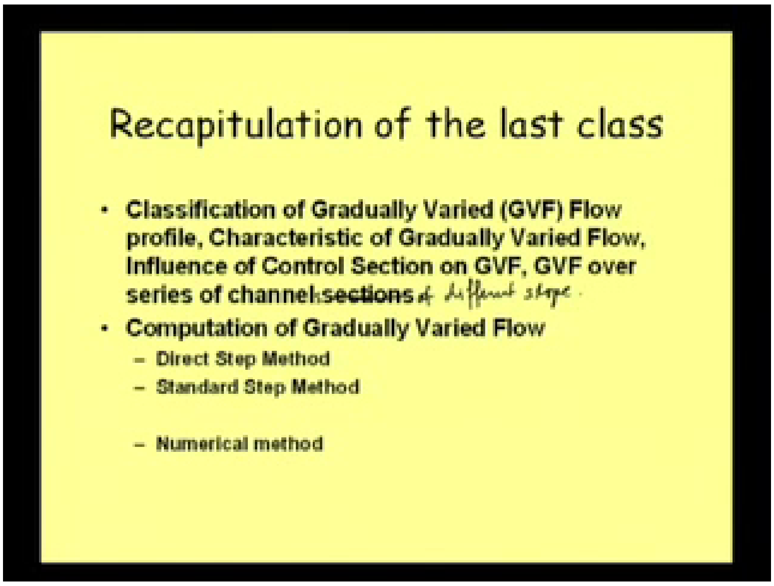
Lecture No. # 06

Gradually Varied Flow: Numerical Methods and Problem Solving

Friends, today we shall be again continuing with our earlier topic, that is the Gradually Varied Flow. Well, we were discussing about computation of gradually varied flow and in that, under that topic, we were discussing the different methods of computation and we were talking about numerical methods also.

And today, we will be going to some extent into deeper of this numerical method. Of course, not in detail, but still we will be covering some of the aspect of that, and then we will be taking up some of the hands on say, some of the numerical problem that can help us in understanding the method, we have discussed in a better way.

(Refer Slide Time: 01:56)



Well, with that thinking, let us see what we did in our last class, that is, we have discussed, just to summarize till now, what we have done? We have discussed the classification of gradually varied flow, that way we know that there are different classes. Then, we did discuss about the characteristic of gradually varied flow, different type of profile how they form and what are there their characteristic, at upstream what happens, at downstream what happens and the where profile is horizontal, whether it is rising or whether it is falling, that sort of things we have discussed and that very basic knowledge is important for computation. And then, we did discuss about the influence of control section, that is if the profile is forming in a subcritical zone, then we could see that control is at downstream, if the profile is forming at super critical zone, then the control is at upstream, so those aspect we did discuss.

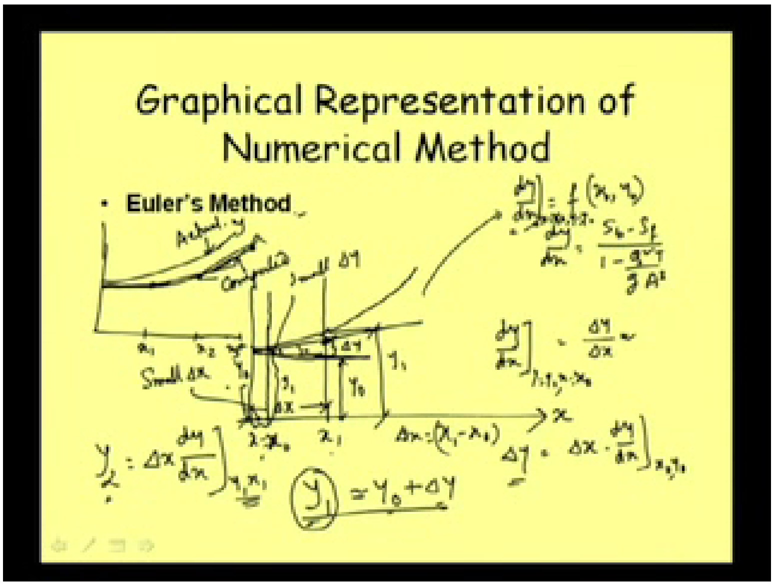
And of course, in the last class, we again did discuss about the formation of profile over series of channels, rather say channels which we can consider as series of channel of different slopes; say slope is changing from mild to milder, milder to steeper. So, that sort of things we did discuss and of course, while we discussing those things, we did not consider a slope change, which can lead to another phenomenon called hydraulic jam because we are yet to study that one. Then after that, we went for computation of gradually varied flow and in computation of gradually varied flow just to recall, we studied particularly two methods, one is that direct step method and the second one is the standard step method. Well and of course, there are some methods like graphical method, then direct integration method.

So, this sort of methods is there which need recourse to graph and which need recourse to say tables and charts. So, as now in the days of computer, we do not like to go for those things or rather we have other alternative way or say we can develop a computer program to solve this problem very easily and that is why we are not using those methods and that is why we are not discussing those methods here.

Here what we are discussing is standard step method, then direct step method and of course, we will be discussing numerical methods. And in numerical method as said earlier, there are several numerical methods which can solve this sort of governing equation; this sort of governing equation means, the governing equation of gradually varied flow which is of the form that d y d x is equal to it is a function of x and y. Of course, when it is a non prismatic channel, then only it is becoming a function of x, otherwise it is a function of y only, but it is not a function of x only and that is why we cannot solve it by analytical procedure and because this is a non-linear differential equation.

So, this non-linear differential equation, for solution of that we need to take re course to different methods and these are say one is direct step method, one is our standard step method then we need to discuss about numerical method. So, let us now discuss on these numerical methods; of course, we have discussed on direct step method already and standard step method and we will be taking up some numerical problem for solving this standard step method here, just to have feel of the actual problem. Well, so for numerical methods are concerned, in fact there are several numerical methods that we did discuss in the last class.

(Refer slide Time: 06:18)

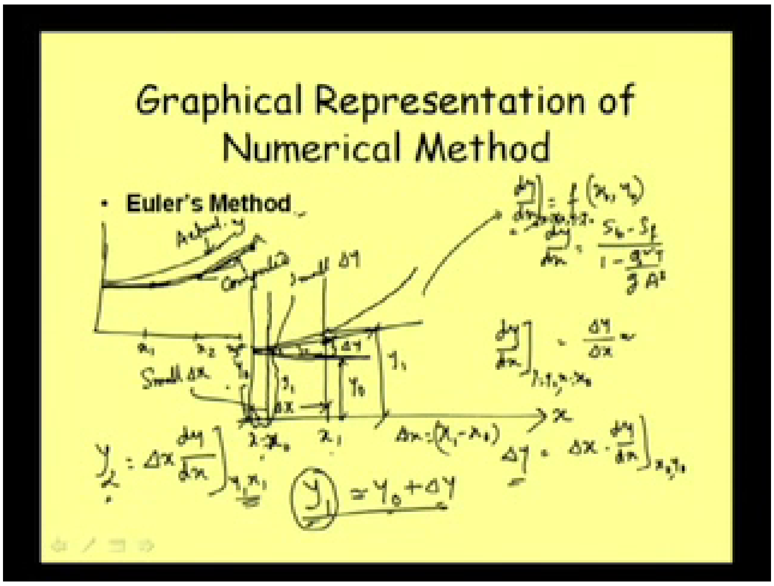


For solving this sort of equation that is d y d x is equal to rather in not writing the directly governing equation, gradually varied flow if it is f x y and for solving this sort of equation, we have different type of numerical methods. And here, we are discussing only a few of those method and we are not discussing more details of those methods, but just to understand how basically the concept of numerical applications are applied, we will be explaining that with the help of some graphical representation, well.

So, we are taking up the first one this is Euler's method say, what basically the numerical methods do? To see that suppose, we have a curve it may be gradually varied flow it may be anything well, now this curve has equation like this d y d x is equal to f of x y, it is a function of x y. In our case, this equation is d y d x is equal to say S b minus S f divided by 1 minus q square T by g A cube, well in case of gradually varied flow, otherwise it can be any equation.

Well now, if we just know the initial value, suppose in this direction we are having y, in this direction we are having x, if we know the initial value at x equal to 0, then say this is equal to y 0, suppose at x equal to 0 we know the initial value of the y. Then for this sort of situation, which you can call that initial value problem, we know this initial value and then we need to solve for the other value, then this method is quite advantageous, how? Say, when we know the initial value y 0 and we have an expression and of course, this is at x 0 that also we know and we have an expression for the slope d y d x. So, if you put these value y 0 and x equal to say x 0, I will not write it as 0, at x equal to x 0, this can be any value also it can have any value not necessarily 0.

(Refer slide Time: 06:18)



So, at x equal to x 0, so we know this value. So, if we put this y 0 and x 0, then we get the d y d x at x equal to 0 and at x equal to x 0 and y equal to y 0. So, that way this value is known, what is d y d x? It is nothing but, the slope at this point slope at this point, if I draw the tangent to this point, then of course my line is becoming almost horizontal because there should be some slope in this, let me take the curve like this. So, say at this point, if I draw a slope it is going like that and this slope is nothing but, a tan of this slope rather is called d y d x tan theta we can write. Now in numerical method, what is that, we know this value and then if we take a small distance in x direction, if we take a small distance in x direction I am taking it here, I am taking it here say this is a very small distance delta x or in many numerical method it is written as h, a small distance h.

Now, what about this distance? As we know, this slope d y d x and if we write this distance as delta y and this is say delta x, then we can write that tan theta is nothing but, delta y by delta x or we can write that d y d x at say point y is equal to y 0, x equal to x 0, this is equal to say delta y by delta x, well. So, from this we can find out what is delta y. So, delta y is nothing but, delta x into say d y d x at say point x 0 y 0, well. So, by this simple expression, we can find out delta y and this is a horizontal line we are drawing.

So, up to this much distance, this distance is nothing but y 0, now the distance y 0 plus delta y will give us up to this particular point (Refer Slide Time:12:00). So, now, if I just draw it in a enlarge form, then I can show that y 0 which is the point, our interest we are interested to know that at x equal to x 1 means, x equal to x 1 means say delta x is what, delta x is nothing but, x 1 minus x 0 at x equal to x 1 what is the y. So, our actual value for y 1 is this much, our actual value for y 1 is this much as per this curve, but if this delta x is very small, then we can write that delta x is equal to y 0 plus delta y approximately equal to you can call.

Now, of course, from this curve it is not very it is quite visible that y 1 and y 0 plus delta y these are not equal, but if I consider my delta x to be very small, now you can see suppose my delta x is here, my delta x is here this is my very small delta x small delta x. If my delta x is only this much, then with this being our delta x then, say this will be small distance will be delta y, this small distance will be small delta y say and then you can see these two points, that is the point of the curve and point on the tangent are almost at the same point, well.

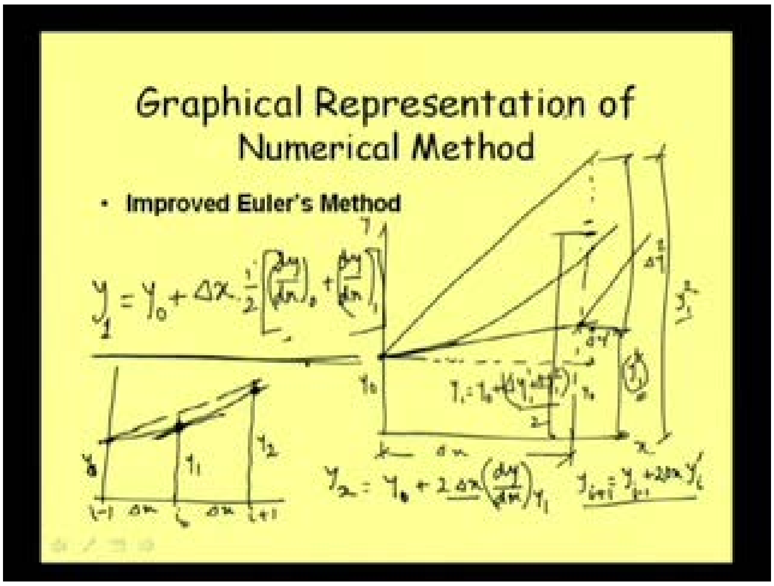
So, this delta x being very small, we can very well write that the delta x sorry this is not delta x we should have written it y 1, y 1 is equal to y 0 plus delta y, that we can write very well now (Refer Slide Time:14:00). So, this is our y 1 and we can write that y 1 is equal to y 0 plus this small delta, this is valid for a small value of delta x, but if we increase our delta x to larger value, if we increase our delta x to larger value then this is introducing some error. Now, once we get the y 1, once we get the y 1, then this y become y 1 become our known value, then we can take a slope at this point, suppose y 1 is known well let me just check it if y 1 is known, then we can find y 2 is equal to when y 1 is known, we can find out the slope say again delta x. So, next that delta x you are taking, delta x into say d y d x at y 1 x 1.

So, this value d y d x you can calculate at y 1 x 1 as now you know x 1 value and y 1 value, knowing the y 1 and x 1 you can calculate d y d x and multiply this by delta x and you can get your next y 2. That way you can continue solving for values of depth y for different x, but from this graphical diagram or from this graphical representation, one point is very clear that, if our delta x is not very small, then if our delta x is not very small then we can end up in getting suppose our actual curve is like this and with our numerical value, we can have suppose this point if you consider this point then, this point we can get suppose here.

Then you are calculating based on this y 1 value, say for this x 1, x 2, x 3 like that we are calculating and for this value if you calculate the slope again, say you are getting this value first, then at this point you are taking slope and then you are getting another value here and at this point you are taking slope and then you are getting another value here (Refer Slide Time: 15:30). So, your curve will be like this, it is basically some discrete point you are getting and your actual curve is this one, and but your computed curve will be this one, this will be your computed and this will be your x value.

So, that way you can always end up introducing some amount of error, if the curvature is in downward slide, the things will be in the opposite direction, it will be little away towards upward direction. So, that is though it is very simple Euler's method, but it can have some amount of error provided your delta x is not very small, again not very small is quite blurred that, what we mean by very small, that is also we need to see into that particular aspect, well.

(Refer Slide Time: 17:55)



Then because of this particular drawback, this was improved based on some of the consideration of that that is called as improved Euler method. Well, say this is our graph and now we can say, if with d y d x at this point we are getting slope at this point and then we are suppose our delta x is this one, I am drawing a large delta x. And then for this delta x say, my delta y initially calculated will be this one delta y and this being y 0, this is y 0 and from here to here, it will be y 1 that is y 1 is equal to y 0 plus delta y.

Well, now we can see that, if we take again with this known value y 1, if we take a slope if we take a slope this slope at this point or rather for this y 1 value, the slope will be like this, slope will be like this. Now, based on this slope based on this slope if we calculate, suppose this slope we are considering that this slope is not there, rather here then if we calculate, let me draw the slope here, if we calculate here then, let me take the slope like this, if I draw the slope here then it will be going this part (Refer Slide Time: 19:00).

So, based on this slope, slope at the point y 1 and y 1, if we calculate our delta y then delta y will be large like this. This is say delta y we are calculating based on the slope at this point, but we are using the slope at this point, but putting the slope at initial point y 0. So, this time our y 1 will be quite large like this and now if we take say, this is our y 1 computed for the first time and this is say y 1 computed for the second time and this is the delta one computed for the second time, this is delta y 1 computed for the first time; the second time means with the on the basis of the slope taken at the value of y 1 which we have computed. Now, if we do like that, then we are getting the value in different way.

Now, if we take average of these two values, that is average of y 1 computed with this level and y 1 computed with the first one and if we take average, the value will be coming somewhere here. And you can see very well that is average of this means your depth is neither this one and this one you are getting average, this is your y 1 and that average y 1 is you can write this average y 1 is equal to say y 0 plus delta y 1 plus delta y 1, this is y 11; let me write little larger here, say delta y 11 plus delta y 12 divided by 2 well. So, that way we are getting this expression and what is this delta y 1 that is nothing but, you have already explained what that delta y 1 that is nothing but, delta x into d y d x.

So, what we can write here? Say y 1 is equal to y 0 plus delta x into say or you can write delta x by 2 or you can write half of d y d x at say point 0 and plus d y d x at point 1 well. So, this way we can compute the y 1 value and this y 1 value will be closer than the earlier one, more closer than the earlier one well. So, this method is referred as improved Euler method; that means, we are using the slope at the initial point and then we compute the second point and based on the slope of the second point, based on the known value of y 1 in the second point, we again compute the slope and then we take average of these two slope and then we calculate this y 1.

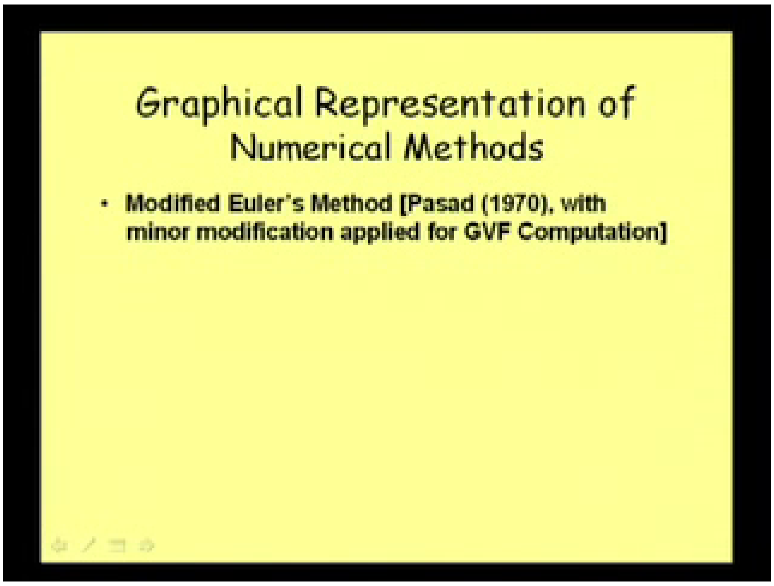
And if the variation of this slope, if the variation of this slope see slope is changing, here it is this much, here it is this much. So, slope is changing now if the change of slope between this point and that point is linear, linearly if the slope is changing, then this method will give accurate result. But, of course, the curves are not like that, that the slope will be changing linearly, it may not be this for all situation and that is why it will not give you that accurate result and of course, this method after this method, this has another advantage.

Now, what we have seen, we are taking average of this slope, average of this two slope and so once we calculate this y 1 and say once we calculate this y 0 y 0 and then we have calculate the y 1 and then we want to calculate y 2 y 2. Rather than again doing like that, taking a slope here and then taking a slope at this point, taking a slope at this point and taking half rather than doing all those things, what we can do? For computing this value for computing this value, we can see that if we use the slope at this point, if we use the slope at this point, that is at the middle point and then if we calculate from the beginning of this one using the slope at the middle point, we are reaching much nearer to the value of the second point (Refer Slide Time: 24: 30).

If we consider this as 0, than this is becoming the 2 second means rather, this actually the third point, nomenclature is becoming as y 2. So, what we can do? y 2 is equal to you can write, y 0 plus we are using this time our delta x is 2 delta x, twice of delta x into slope we are using d y d x at say y 1. So, to calculate the value of y 2, we are using y 0 and twice delta x, then d y d x at y 1 and in general, if we want to write this in general we can write that y say, if this is i this is i plus 1 we are interested to find i plus 1, i minus 1 is also known to us. So, what we can do, that for calculating i plus 1 we can write y i plus 1 is equal to y i minus 1 plus 2 into delta x into we can write this as y dash, y dash means y dash represents a d y d x f i equal to 1 sorry I mean d y d x at the point i middle point.

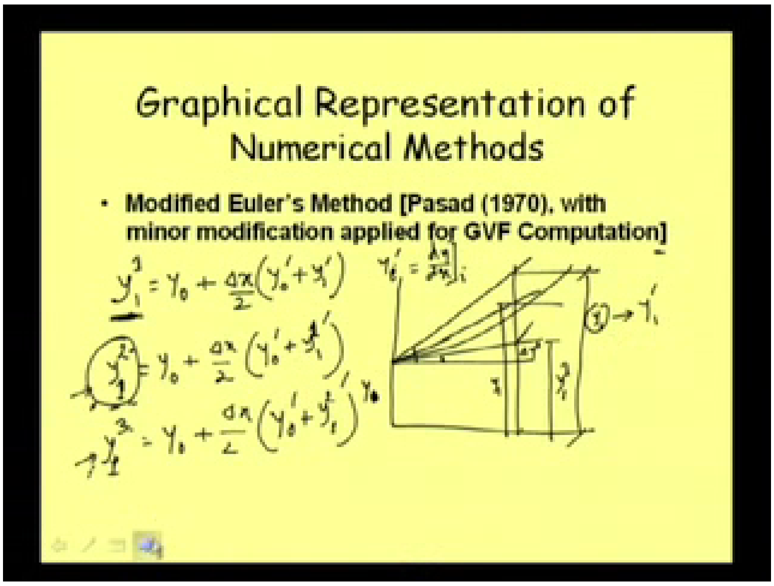
So, this procedure can be adopted this is basically the concept is similar to this one, rather than taking average we are using the slope directly of the center point, for that first we must find this initial point and once we find this initial point, then from using this slope at this point, we can and starting from this point we can find the third point. Once we find this point, then starting from this point, starting from the previous point and using the slope at the second point, we can get the third point. So, like using this way that improve Euler method can be used well, but when we have computer, we can go by this method also from one point to other point. Well, then it was found that this method as I was telling that if this is quite if this point is say linear then of course, we can get a slope which can directly lead us to this point, but because of that limitation and again this Euler method was modified.

(Refer Slide Time: 27:49)



And this modified Euler method was used by Pasad with some modification of course, and then he applied it for gradually varied flow computation.

(Refer slide Time: 28:06)



Well and what is that particular method is that say, we are trying to compute this based on this y 0, we are trying to compute the y 1, and the y 1 that we are getting, this is delta 1, and that suppose, I am now numbering as delta 1, that is the first time what we are calculating and then what value we are getting that we are numbering at y 1, then on the top we are writing one. So, that means, for the first time based on this slope when based on this known value of y 0 we are calculating slope, that is d y d x and then we are getting a value y 1 that is our say first predicted value of y 1. Then we calculate the slope at this point, we calculate the slope at this point because y 1 is known and that slope we are putting here and we are getting another value of y 1, another value of y 1 this y 1 based on this y 1, we are again calculating this is leading us to a slope y 1 dash.

So, finally, we are calculating taking the average slope of this one and that one, we are drawing a line and we are calculating a value of y 1 just like the improve Euler's method. So, this value we are getting; that means, what we are getting say y 1, our first trial y 1 is equal to y 0 plus y 0 plus delta x into y 0 dash plus y 1 dash, this dash means well the symbol dash y 0 dash or y 1 dash or any value y i dash is equal to say d y d x at i, that is what we are meaning.

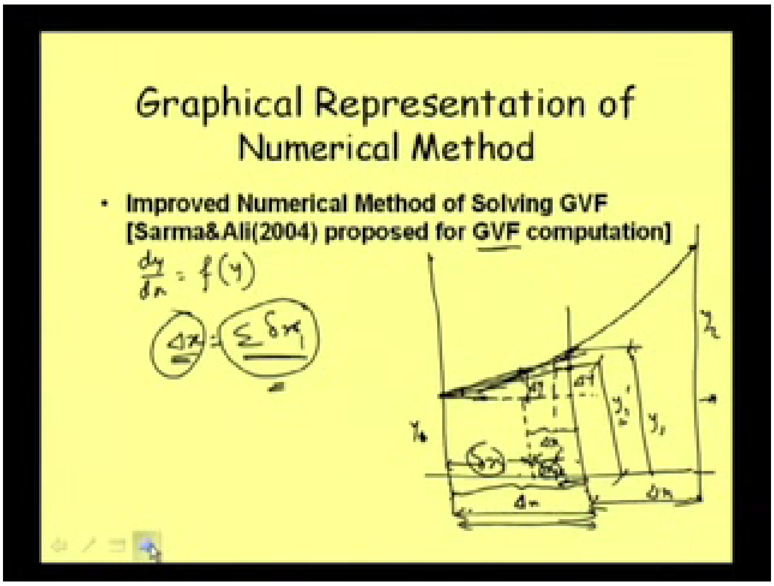
And this one represent that this is the first predicted value, divide x and this plus this divided by 2 this plus this divided by 2, then we calculate using this y 1, this we are not considering as the correct value in improve Euler method, we consider that this is the correct value but, here we are not considering this as the correct value. Using this y 1 now again we will be calculating another second trial, second trial that is we are considering iteration, but for the same value of y 1 what we are doing that y 0 plus delta x by 2 and then we are trying y 0 dash plus y 11 dash; that means, whatever we considered as y 1 in the previous step, that we are using here as the slope and here we are using that y 1 and we are calculating the slope and then we are trying a next value or next trial value of y 1.

Now, if this value and that value is almost equal, then we can consider that this is the correct value, but if it is not equal, then will go for another trial for the same y angle, but we are doing for the same y 1 value. We will go for another trial that again y 0 plus delta x by 2 into y naught dash plus y 1 2 dash, with this we are calculating a slope, with this y 1 second trial value, we are calculating the slope at that point then, we are taking half of that and then we are using this delta x and we are trying to find that. So, this procedure will be repeated till this y 12 and y 13 that is two consecutive ((drift)) calculated by this procedure become almost equal.

And once we get that, this is almost equal then we know that, our calculation is correct or and we can consider this to be correct of course, how much accurately we can approach this value that will depend again depends on whether this will lead us to almost correct value, when our variation of slope is linear, if it is not linear, again there will be some amount of error introduced there.

Well, this is what the modified Euler's method and what Pasad did rather than comparing this y 1 value that is the consecutive depth, he compared the consecutive slope that is being computed. In fact, it is indirectly the same thing if we compare the slope if it is approaching the correct value, then also you can have this correct value. So, that method was used, but in all this method we have a drawback that if we increase the delta x, then we cannot reach the correct value, because the profile equation is of the type that it will not lead you to the correct value in although you carry out all those iteration.

(Refer Slide Time: 33:53)



So, we attempted this, try to rectify this particular drawback and what was done? This is also basically a corrected initial value is calculated then iteration is carried out, but with the concept that, say this is the curve and suppose we have this. These are our delta x value, these are our delta x value, then for computing from say first delta x, this way it is going there and for computing from this known value y 0 or this y 1 in earlier method also in Pasad method also we do iteration, but here what we are doing say, if this is the slope we are arriving at this point, then this method can be applied for only gradually varied flow (Refer Slide Time: 34:11). The reason is that, the d y d x the slope is a function of y only d y d x is slope, this slope is a function of y only. So, once you know the y, then we can calculate d y d x, but if it would have been a function of y and x then this method cannot be applied because there we will have to know the x value also, here what the advantage is that, once we know the y 1 calculated, then this y 1 actually is the depth at this point.

So, once we know this y 1, then we can find out what is the corresponding value of this x? What is the corresponding value of this say small x delta x, what is the corresponding value of this delta x for this y 1, how we can do that? Because that is very simply this is our delta y, this is our delta y and this entire depth we are first initial trial y 1 dash and then very basic concept is if our this length is small, now we are coming to this point if we solve at this point, suppose this is y 1 (Refer Slide Time: 36:00). So, slope we are getting at this point and if this portion is in this small portion, if we consider this small portion if we can consider to have linear variation of slope, then average of slope at this point and slope at this point will pass through this point obviously.

And therefore, based on this point we can find out what is our delta y value because delta y is this 1 obviously, so delta y by this small delta x is equal to actually slope this one which is average of the slope at this point and slope corresponding to this y 1. So, from that we can find out this delta x, if we mark this as delta x 1, then what we can do? When in the first trial, we were using our entire length as delta x, in the second trial we will just deduct this x 1 and our only remaining length for this first delta x first del x will be only this much.

So, considering this as our delta x, now say delta x 1, this as delta x 1 what we will do? Again we will start from this point and we will go to this particular point and then we will again come back and then we assume that within this small portion, this variation of linear slope is quite valid and then we can calculate this small delta x 2.

So, this way we are reducing the length or extend that we are using for computing the incremental length, as we are reducing our delta x in each of the repetition or each of the iteration, calculation of delta y for this small delta x is becoming more correct and the assumption of linear variation of slope between the small delta x portion is more valid rather than considering a large delta x, that is why even if we take a large delta x, say from here to here it is significantly large, but when we calculate these value, we are transferring this value to this point and our small delta x are getting reduced, these value are getting reduced and we are successively using small r delta x.

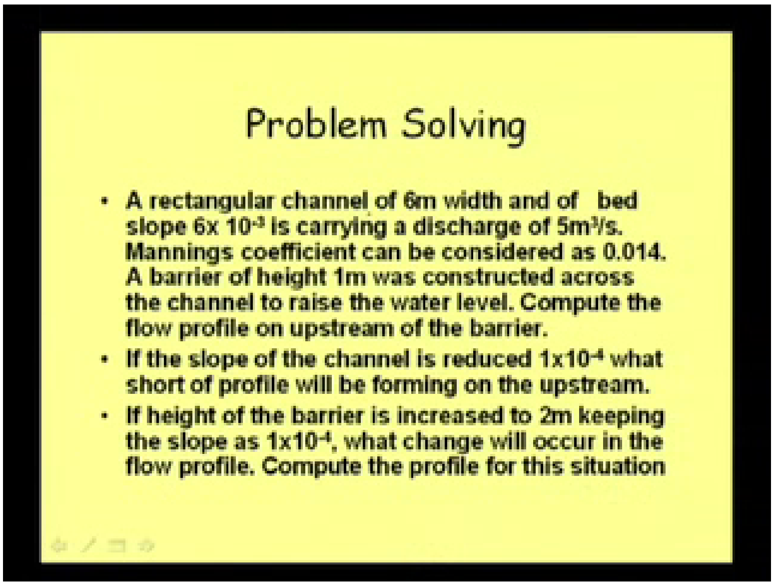
And so, this method lead us to and finally, when we see that, no more small delta x is remaining, no most that is we cannot come back that is no more small delta x is remaining then and we can see that actually our delta x should be equal to summation of all delta x 1. Then we are always checking, after each of the calculation we are checking that, if this delta x summation of delta x is becoming equal to the delta x, if not we are continuing and when the summation of delta x becoming is becoming almost equal to the delta x we can stop; that means, our entire delta x portion has been computed.

So, what value we are getting that is our final value of y 1 dash, y 1 final value of y 1. So, again from this y 1 value to compute the y 2 value, we will be proceeding in the same way dividing it into dividing means we are not dividing, rather it is automatically getting divided and finally, we are checking if this delta x is complete, then we get this value.

And then this method when we compared with Pasad method and other methods, say Euler methods and Runge Kutta method also we compared and we found that, this method is giving better result in case of gradually varied flow computation. Of course, we must know that this is not a generalized equation, which can be a generalized method, which can be used for solving any sort of I mean any sort of differential equation of this type. It can be use only when this is a function of y, when this d y d x is a function of y and as the gradually varied flow is having that advantage, when we consider one dimensional prismatic channel, then we can very well apply this method.

Well, with this introduction to numerical method and of course, as I was telling that there are methods like for Runge Kutta method is also very accurate and it gives correct result. And then let us take up some problem of course numerical problem, I mean this numerical methods we cannot apply here, because it will be quite iterative and it requires lot of time. So, let us see and let us have one problem which we can solve in this class and we can have a feel of actual gradually varied flow profile computation.

(Refer Slide Time: 41:27)



So, you can concentrate into the slide, say a rectangular channel of 6 meter width, the channel is 6 meter and of bed slope 6 into 10 to the power minus 3 is carrying a discharge of 5 meter cube per second. The manning's co-efficient of course, manning's co-efficient we need to assume, manning's co-efficient can be considered as say 0.014, well that is the end value. Then what was done if you remember this particular problem we took when we were discussing computation of uniform flow and for our purpose at that time, we were computing what is the uniform flow depth. And that is why so for the first part of the problem is concerned, I am taking the same problem. So, that computation of uniform flow part we can avoid or rather, we can simply just give the value as this was already done in the class of computation of uniform flow.

Well, so that means, up to computation of y n we know well, then what has happened? A barrier of height 1 meter was constructed across the channel. So, many a time in the channel for various purposes, say for irrigation purpose, sometimes people give a barrier at the upstream, downstream. So, that water level can be raised many a time for fishing purpose also people give barrier in the interior area, particularly in Indian condition I am talking about, in Bangladesh also even and then people give the barrier to obstruct the flow and then they can get sufficient amount of water they can go for agriculture. Some were for fishing purpose also people keep obstruction that way some purpose for many different reasons we can give obstruction. So, 1 meter of obstruction was constructed it is a small obstruction across the channel; however, small means considering the other values, this may be quite significant sometimes.

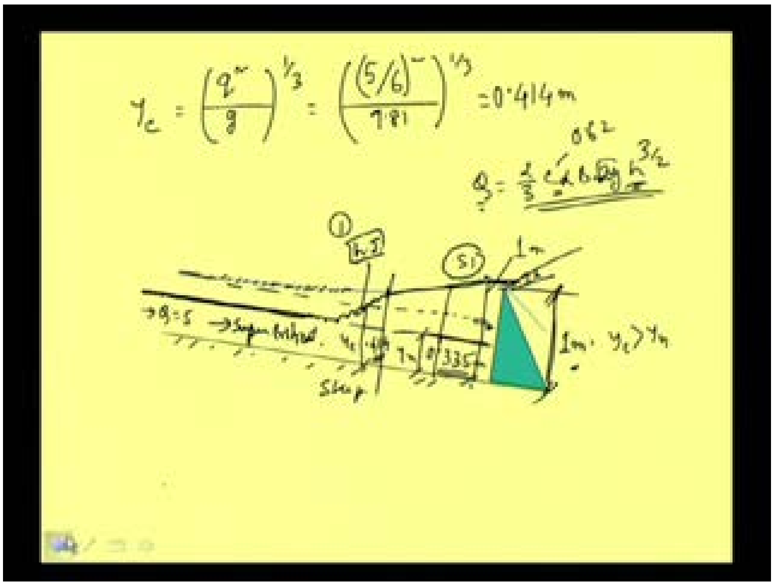
So, this 1 meter obstruction was constructed across the channel to raise the water level to raise the water level. So, objective was that will raise the water level on upstream, now when we are raising the water level, that means definitely there is a change in the water level and when there is a change in the water level, the profile will be forming. Of course, now the change can lead to another phenomenon called hydraulic channel that will be coming in the next class, but gradually varied flow profile will also be there; well, now we are of course, concerned about the gradually varied flow component.

So, compute the flow profile on upstream of the barrier. So, what sort of flow profile it will be, that is what the first part of the problem and then there is a second part. If the slope of the channel is reduced, if this slope is reduced then what sort of profile will be forming on the upstream? Many a time when we give obstruction, then if the water is carrying lot of sediment, then as the flow velocity get reduced, its sediment carrying capacity also get reduced and all the sediments may get deposited into the bed. And when the sediment start depositing into the bed, bed slope may change that is necessarily of course, but for some of our own purpose, sometimes we may reduce the slope ourselves by say putting some soil there or just we can augment the bed slope to some extent.

So, that is whatever may be the reason or whatever maybe the process, but suppose bed slope got reduced to 1 into 10 to the power minus 4, what sort of profile will be forming on upstream, then how the profile is changing? Well, then another equation is being added another part is there, if height of the barrier is increased, now suppose height of the barrier is increased, we were originally putting a barrier of 1 meter, now due to some reason to increase the depth of profile we thought that we will be increasing the depth of barrier, we will be increasing the height of the barrier so that, we can raise the water level further.

Then of course, slope suppose whatever we did change that is rare there, I mean changed slope is there and what change will occur in the flow profile? And ultimately our target is that, we need to first know what type of profile will be forming in different situation and then, we need to compute the profile for this situation. This situation means of course, taking up the entire situation may not be possible. So, we will just see for the last situation, how to compute the profile. Well, so let us go by one by one.

(Refer Slide Time: 47:12)



So, this is our say, well I need to take the error, this is the bed originally which had a slope as it was explained which had a slope of 6 into 10 to the power minus 3. Now in this slope, we had that this sort of uniform flow is occurring. So, we need to know the depth of uniform flow that we have already done in our computation of uniform flow. Now here, we are putting a bed here, then this uniform flow will not be there rather a gradually varied flow will be coming like that and then we need to compute this gradually varied flow profile.

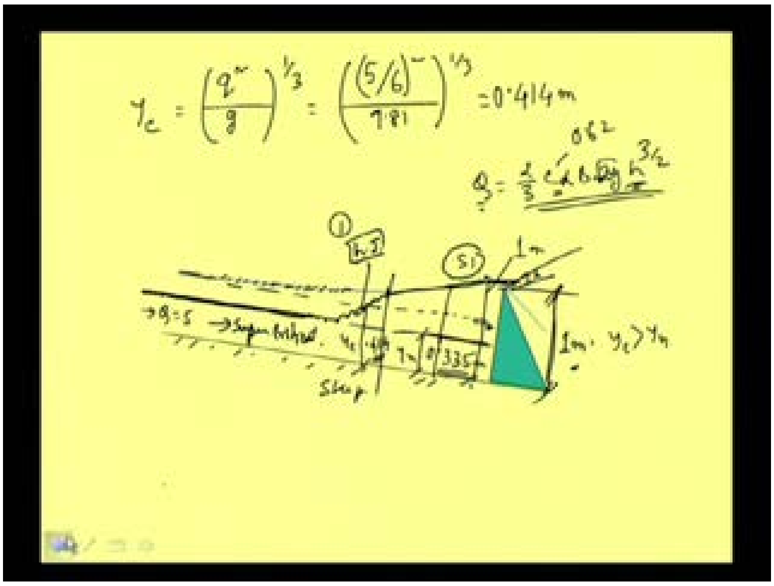
Well, now what we can see that this is the bed slope and this was the original profile, this was the original profile, original uniform flow depth and as per our computation of uniform flow depth, we had our y n that is the normal depth is equal to this normal depth was equal to 0.335, yes it was 0.335 y and I am not computing depth right (Refer Slide Time: 48:13). Now, you can use the approximate formula of dash and bar and then you can carry out some iteration to see, whether it is satisfying that required discharge or not and this discharge equal to five comic, then we can have these things. Then once we know the y n initially, in fact we do not know what about this profile, whether this profile is like that then we need to calculate, what is y c critical depth. Well, critical depth as we know it is equal to q square by g and of course, in the problem we are considering a rectangular channel, q square by g whole to the power one third.

Well, so this value we can have q square is nothing but or 5 is the comic total discharge 6 is the bed width. So, this is the unit discharge and then 9.81 and whole to the power 1 by 3. So, that way we can have the y c value as equal to 0.414 meters. Well, that means, the value is critical depth value if I draw here, this will be this will be say 0.414 y c, then the height of the barrier is given suppose 1 meter, height of the barrier is 1 meter. Now, whether our profile will be of this type or it will be of different type that we need to see; definitely this initially withdrawn from our general concept that water will be rising, but how the water will be rising whether this will be this one or it will be something else that we need to see.

So, when water need to cross this level, water need to cross this level suppose this 1 meter height of course, water will not be crossing just at 1 meter, there will have to be some depth here, which we can find suppose width of the channel is known, width of the channel is known say 6 meter. So, height we can find out using the formula of where, that is 2 by 3 that is this y also we can find out the 2 by 3 c d, 2 by 3 c d b root over twice g h to the power 3 by 2 using this relation, we can find out what depth height is q is equal to this one. So, our q is known, we know the c d we will have to assume, this is nearly equal to 0.62 for rectangular section and then b is known, g is known which is twice g, then h we need to find out.

So, from this formula we can definitely find out this h value. However, this h value will be will not be that high as compared to this 1 meter that is why for many practical purpose, we can neglect this depth also and suppose when depth of barrier is 1 meter, we can consider the flow to be also of 1 meter, flow depth to be also of 1 meter, but to be more accurate we need to calculate this. And for some of the combination, this h this h may be quite significant and as such we can check that amount also; right now in this problem we are not doing this, we are considering that depth is 1 meter means our barrier height is 1 meter means, at that point depth will be 1 meter.

(Refer Slide Time: 47:12)



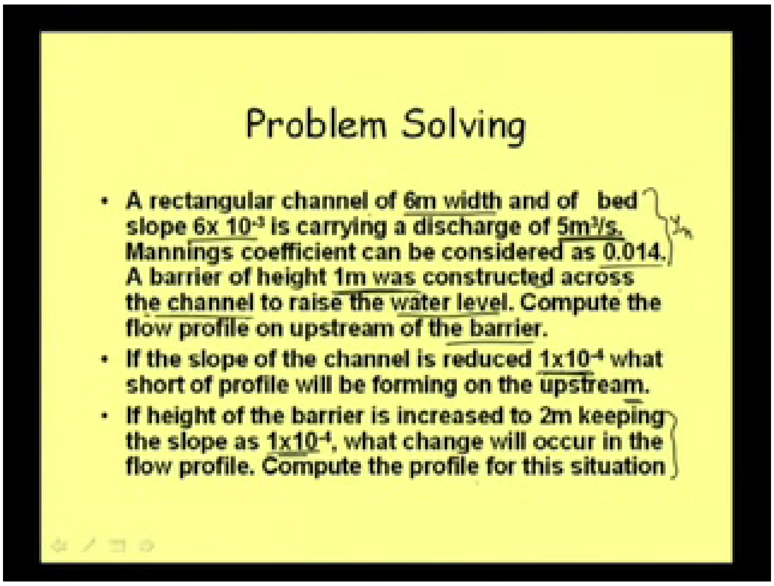
Well, now the problem is that, initially the water is coming as normal depth uniform flow at a depth of 0.335 meter. Finally, it is moving at a depth of say 1 meter finally, it is moving with a depth of 1 meter and from this we can see that critical depth is 0.1414. So, this part of flow initially it was supercritical flow, initially it was supercritical flow and after giving the barrier, it is rising to a sub critical flow. So, our profile will be here in the sub critical flow portion and normally, when flow is occurring y c is greater than y n; that means, it is a steep slope; that means, it is a steep slope the profile over steep slope and this is in zone 1.

So, it is basically S 1 profile and when it is S 1 profile, when it is S 1 profile then what we can do that this is our known depth and we can start computing from this part, it is a rising profile and we can start computing from this part and of course, although initially we thought that the profile will be like this, actually it will not be like this, because profile meet the normal depth line asymptotically not the critical depth line, it will meet critical depth line normally, so the profile is coming like this.

So, when we want to compute that profile, we will have to start from this particular depth 1 meter plus something of course then, we need to come down by direct step method or numerical method whatever it is, we can come down and we can compute it up to a certain point a critical depth point. However, in reality it will not come to the critical depth point, the reason is that, here it is super critical flow and from super critical to sub critical when the profile move, then there is lot of turbulence and it will be forming a another flow phenomenon that is called hydraulic jam that we will be discussing just after this class.

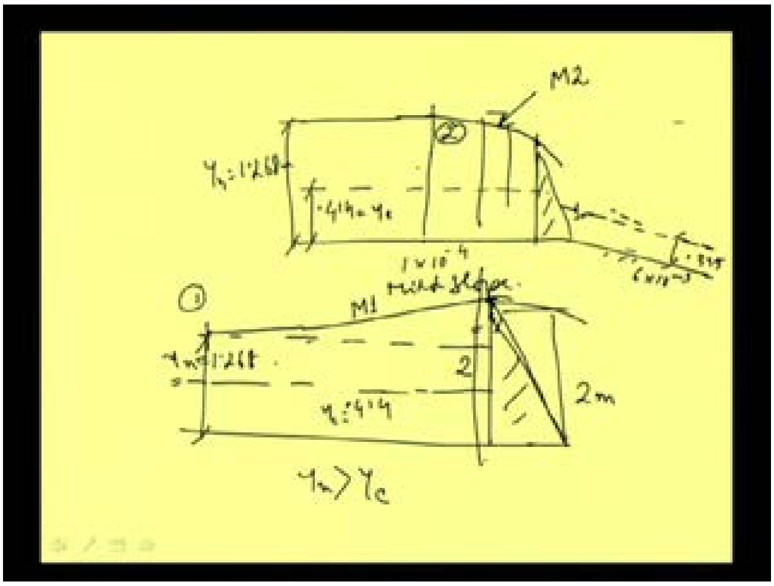
So, right now we are not talking about this part, but this will be a hydraulic jam and that part is not changing because disturbance is going on at the downstream side and this is a super critical flow, this is a super critical flow. So, this flow is not getting affected; that means, this flow surface is uniform flow, it is coming as uniform flow and from this particular point it is jumping like that. And of course, for some other situation when this point will be coming here and then there may be some other influence on this that will be coming when we will be discussing hydraulic jam and then we will be discussing more about that. Well, right now our point is that, this is S 1 profile and we can compute this profile in this way.

(Refer Slide Time: 55:52)



Now if we go to the second point, if we go to our second question, if we go to our second question that we are getting, say if the slope of the channel is reduced to 1 into 10 to the power 4.

(Refer Slide time: 56:05)



Well, if we reduce the slope like this, that slope we are changing to 1 into 10 to the power minus 4, then if we compute the normal depth y n, our critical depth is remaining as it is and critical depth is it is 0.414 and normal depth will increase. And it is found that, normal depth goes up to depth of 1 point y n computation of normal depth I am not doing here, but y n is equal to 1.268 meter. So, that way we are getting our barrier height now is 1 meter, barrier height is 1 meter.

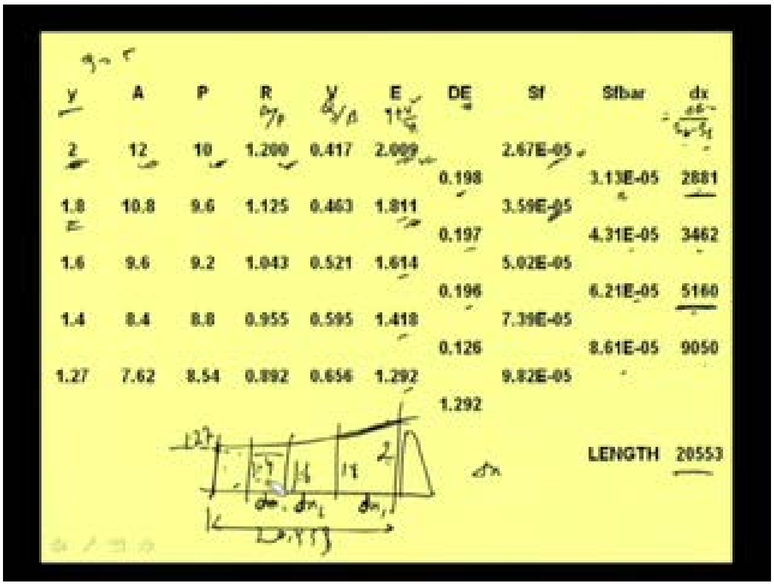
So, what is that, when barrier height is 1 meter and uniform flow on the downstream side whatever way it cross this things, say uniform flow depth will be this much and then our entire suppose it is crossing this part and then there will be some obstruction and then it is moving again ultimately it will be moving in this part. And of course, suppose if we assume that, this part slope is remaining as it is, as like the earlier, so what it was? That is it is earlier slope was earlier slope was this slope is 6 into 10 to the power minus 3. Suppose we consider that, after giving barrier we are changing this thing then it is 6 into 10 to the power minus 3; this is suppose slope in the downstream, this is remaining as the original one. Then our normal depth here will be again if we compute this will be the y n 2 what we did compute earlier this is equal to say 0.335. So, from here flow will have to come down like this and it will be falling here and then it is making this point and ultimately it is going like this.

So, this part of flow it is in mild slope, now it is in mild slope because your critical depth is this one and normal depth is this one (Refer Slide Time: 58:00). So, normal depth being greater than critical depth, it is mild slope and it is in zone 2, this zone is zone 2. So, this profile is M 2 and the depth here will be this much.

So, we can start computing from this point and again we can come back and we can compute up to the point, where it is becoming almost equal to y n. Now, let me take a third case, in the third case it is said that the barrier is increased to barrier height is increased to say 2 meter, barrier height is increased to 2 meter and the other things remaining same. Our critical depth here will be critical depth here will be say 0.414 and normal depth is now say if I draw the normal depth line, it will be 1.268 and barrier height is 2 meter. So, as the barrier height is 2 meter, the profile here will be rising like that and then it is crossing this part crossing this part.

So, this profile is as y n is greater than y c, y n is greater than y c. So, this is mild slope and this is zone 1 because it is above this one. So, it will be M 1 profile. Now for computing this profile, we need to start from this one say again it will also be greater than 2 by that small amount, but if we start from 2, then we need to compute up to just little higher than this y n.

(Refer Slide Time: 59:58)



y	A	P	R	V	E	DE	Sf	Sfbar	dx
2	12	10	1.200	0.417	2.009		2.67E-05		
						0.198		3.13E-05	2881
1.8	10.8	9.6	1.125	0.463	1.811		3.59E-05		
						0.197		4.31E-05	3462
1.6	9.6	9.2	1.043	0.521	1.614		5.02E-05		
						0.196		6.21E-05	5160
1.4	8.4	8.8	0.955	0.595	1.418		7.39E-05		
						0.126		8.61E-05	9050
1.27	7.62	8.54	0.892	0.656	1.292		9.82E-05		
						1.292			
								LENGTH	20553

Now, how we can achieve this by standard steep method, let us see briefly. Say y 1 we are first we are starting from the depth 2 well, then for 2 as the depth is known width is known we can calculate the area, this is being calculated 6 into 2 12, then perimeter we can calculate, then hydraulic radius a by p we can calculate, then velocity we can calculate q by a, q is equal to 0.5 that is known. So, we can calculate this one, then energy we can calculate y plus v square by twice g. So, this we can calculate that, so up to this much. Suppose we are taking that at two basis of the point at 1.8, next depth suppose we are considering as 1.8, this is at 2, it is at 2 and y n is 1.268 (Refer Slide Time: 1:00:00).

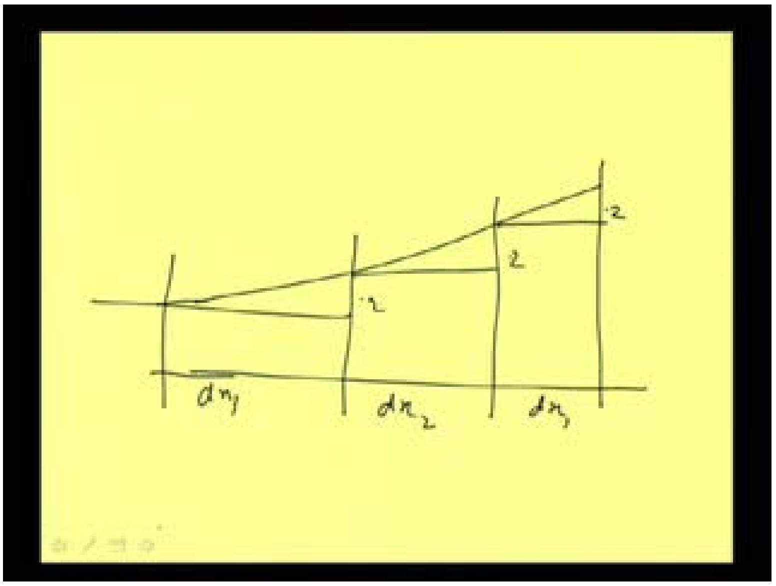
So, we are starting from 2 and then at this point we have calculated everything, then we take let us take another step 1.8, then 1.6 will be next, 1.4 will be next, then it is just higher than the last depth, it will be say 1.27 little more than 21.1268, little more than 1.268, we are taken 1.27 which is just higher than y m.

So, for 2 we are getting everything, then for 1.8 up to this much we can get now knowing this things and then energy delta E because our delta x is equal to this delta x we can calculate this delta x is equal to delta E divided by S b minus S f. So, this delta E we can calculate by this minus that will give us this one this minus that will give us this one.

So, at every section for each section we are calculating the E and delta E then, friction slope by using manning’s formula we can calculate for each of the depth, then average friction slope we can calculate as this plus this plus this divided by 2. So, that way average friction slope we are getting and then we can find out this delta x by using this relation, that is delta E is equal to delta x is equal to E divided by S b minus S f this divided by S b minus S f. So, we are getting these are the delta x. So, delta x 1 delta x 2, I am writing d x there d x 1 d x 2 like that d x 3.

And then you can see for this small rise of two meter depth in a 6 meter wide channel, what is our extend total length of the gradually varied flow profile, entire length is becoming 20553; that means, 20 kilometer 205 and then for the first part as it is rising in any at this point it is rising. So, for lowering of 0.2 meter, extent of slope is 2.8 kilometer or 2881meter, but here as we are coming here, it is making asymptotically means for the same drop, here also the drop is 0.2 meter from 1.6 to 1.4, but in this part, in this part the extent is length is delta x is quite large.

(Refer Slide Time: 01:03:25)



So, actually though I am drawing it like that, the profile will not be of this sort, it can be it will be of the type that for equal amount of drop for equal amount of drop this is gradually increasing; for equal amount of drop is may be 0.2 everywhere, this drop is 0.2, but this d x is increasing d x 1 d x 2 d x 3 like that it is increasing. Finally, for all if we sum of all these things, we are getting total length of gradually varied flow profile.

Well, we have just tried our level best to show how a problem can be solved, it is definitely difficult for taking up solution of problem in this sort of class, but still we have tried our level best. And in the next class, we will be moving to rapidly varied flow; there we will see how we can compute for rapidly varied flow and how with gradually varied flow and rapidly varied flow being in the same situation, we can have combination of this sort of flow. So, thank you very much.