

**Hydraulics.**  
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**Module No. # 04**  
**Gradually Varied Flow**  
**Lecture No. # 02**  
**Characteristic of Gradually varied flow**

Friends, today, we shall be discussing on the characteristic of gradually varied flow. Well, in the last class we have discussed how we can classify gradually varied flow, and then we just started discussion on characteristic of gradually varied flow. Well, before going to characteristic of gradually varied flow let us summarize what we did in our last class, so that, we can go in a more convincing way to the characteristic of gradually varied flow.

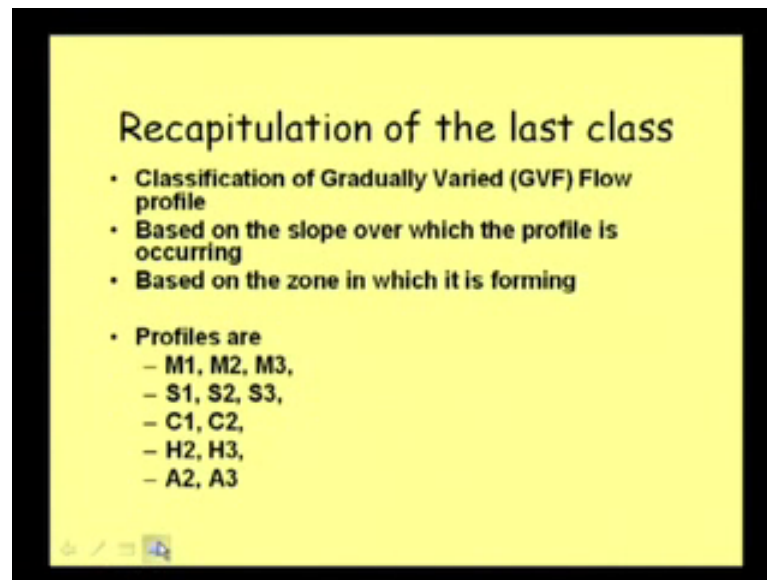
Well, in the last class, we started with the classification of gradually varied flow. Well, and we could see that how we classify the gradually varied flow, first for classifying gradually varied flow we do it on the basis of the slope on which the flow is occurring, and then we divided **the zone** the flow zone above the bed into three different zone - zone 1, zone 2, and zone 3.

And based on the zone on which this particular flow is occurring, based on that we classify; that means, we classify on the basis of slope as well as on the zone; slope means, again, there were different type of slope, say, it can be mild slope. Mild slope means, when the bed slope is less than critical slope, well, and then it can be steep slope means, the slope is steeper than critical slope, and then again critical slope of course critical slope means when the uniform flow occur for a particular discharge in that particular section, that particular channel uniform flow occur at critical depth, so that particular slope we call as a critical slope.

And then of course, we got that flow can be there gradually varied flow can be there over a horizontal bed also, so and then over adverse slope say slope is a negative direction, negative direction means slope is bed slope is not falling in the direction of flow rather it

is rising in the direction of flow, so that sort of slope we refer as adverse slope. So, in that process again zone 1, zone 2, zone 3 are always there, that we did discuss elaborately in the last class, so by that process **we could classify the channel** we could classify **sorry** gradually varied flow profile into 12 different classes and 12 different type.

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That you can see that these were M1, M2, M3, then it was S1, S2, S3, and then we had for critical slope, we have C1, and C2, and y c, well, **sorry** here it should be C3, we had C1 and C3, and why the C2 is not existing, because the zone 2 is not existing in case of critical slope, because a normal depth line and critical depth line coincide in case of critical slope, so that zone itself does not exist. So, the profiles are C1 and C3. Then in case of horizontal slope we got horizontal bed rather, we got S2, and S3 here, S1 does not exist because the depth of uniform flow in case of horizontal slope is at infinity.

So, we do not have the zone 1, which cannot be there beyond infinity, well, so that way we had only S2 and S3 profile; and similarly, in adverse slope we had again A2 and A3 profile, here also A1 **A1** is not existing the reason is that the uniform flow depth in case of adverse slope is imaginary. So, we cannot have anything when we do not know about the exact location of y, and it is imaginary depth, so beyond this we cannot have any other zone, so that way we do not have the A1 profile right, so we have A2 and A3 profile; and these are the 12 different types of gradually varied flow profile that can occur in nature; and of course, these profiles if the slope in the nature keep on changing

definitely it will keep on changing when river is flowing, then we can get combination of different type of flow profile, and that combined profile actually give us the water surface profile in a natural channel; well, with that we can now go on to the characteristic of gradually varied flow.

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**Characteristics of GVF**

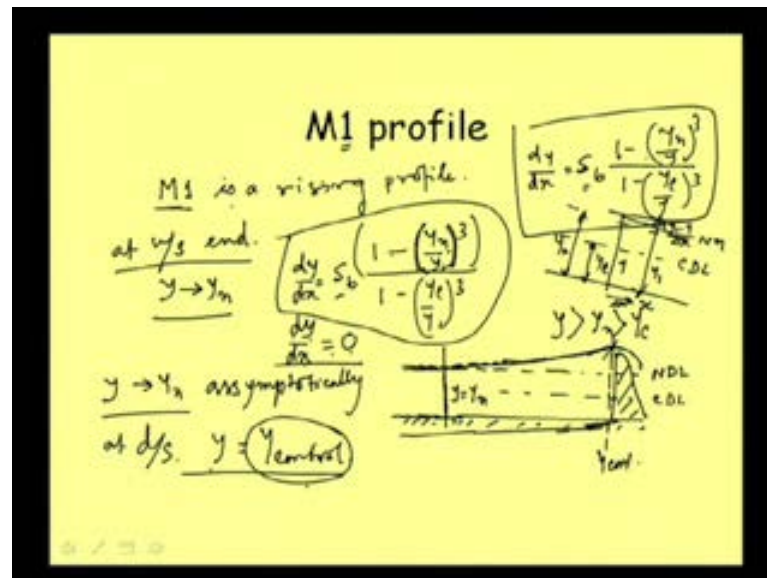
- **Governing Equation**  $\frac{dy}{dx} = S_b \frac{1 - \frac{S_f}{S_b}}{1 - \frac{Q^2 T}{g A^3}}$
- **Governing Equation for wide rectangular channel using Chezy's equation**

$$\frac{dy}{dx} = S_b \frac{1 - \left(\frac{y_c}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^5}$$

Well, we can concentrate into the slide that we have derived the governing equation already, the governing equation is  $dy/dx$  is equal to that can be express in this form  $S_b$  then  $1 - S_f/S_b$ , then  $1 - Q^2 T / g A^3$ , so this way the governing equation we could derived, of course, it has different form. And as we explained in the last class, for analyzing characteristic of different types of profile we can have this form in a more simpler way, and in terms of the normal depth of flow, critical depth of flow, and the depth of gradually varied flow at any section, so and that we can achieve by considering the channel to be wide rectangular channel.

And we did discuss in the last class that **governing equation for wide rectangular channel** governing equation for wide rectangular channel, using Chezy's equation as resistance formula we could get the equation in this form, that is  $dy/dx$  is equal to  $S_b$  into  $1 - y_n/y$  by  $y$  whole cube divided by  $1 - y_c/y$  by  $y$  whole cube, where  $y_c$  is the critical depth,  $y_n$  is the normal depth, and  $y$  is depth at any section within the gradually varied proportion actual water surface depth, so that way  $S_b$  is the bed slope,  $S_b$  is the bed slope.

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Well, and starting from this equation, now we can see how the characteristic of M1 profile can be determined; of course, if we just think what we did in the last class, we could at least establish that M1 profile **is a rising profile** is a rising profile, that we have already established.

And we will just to recall that say  $\frac{dy}{dx}$  is equal to  $S_b$  into  $1 - \frac{y_n^3}{y^3}$  by  $y^3$  whole cube divided by  $1 - \frac{y_c^3}{y^3}$ , well, that is the equation; and in case of mild profile, that is mild slope, our CDL line is here, and NDL line is here, well, so it means that critical depth is this one  $y_c$  normal depth is  $y_n$ ; and suppose, profile is here in zone 1, because it is M1, so it is in zone 1, so depth at any section is say  $y$ , and as such we get that  $y$  is greater than  $y_n$ , that is the normal depth, depth at any section of the gradually varied proportion, and  $y_n$  is greater than  $y_c$ , so that we could get.

And then if it is like that, that means, this ratio  $\frac{y_n}{y}$  this is becoming fraction  $\frac{y_n}{y}$  is smaller than  $y$ , so this is becoming fraction, cube of the fraction is definitely less than 1,  $1 - \text{something less than 1}$  is positive here also  $\frac{y_c}{y}$  by  $y$  being greater than,  $y_c$  this is also a fraction, and that cube of the fraction will be definitely a fraction, and  $1 - \text{something less than one}$  which is a fraction, so that is also therefore positive, so numerator and denominator both are positive.

And when it is mild slope means bed slope is also positive, we talked about negative bed slope only when we talked about adverse slope falling in the direction of flow, we refer

as positive slope, that we emphasized when we were doing derivation of the gradually varied flow, perhaps we remember that; and that is why this  $S_b$  is also positive, and this means depth  $d y / d x$ , that is the change of  $y$  with respect to  $x$ , say, depth here is  $y$ , here is suppose  $y_1$ , then change of  $y$  with respect to  $x$  in this direction is positive, means, it is a rising profile, so this will always be positive.

If it is negative then only it will be a falling profile, say, this is  $y_1$  minus  $y$ , so this suppose  $y_1$  minus  $y$ , so if this distance is  $\Delta x$ , then this rate is  $y_1$  minus  $y$  divided by  $\Delta x$ , like that also we can write if we want to discredited that, but anyway this  $d y / d x$  positive means it is a rising profile; and **based on that** actually in the last class we got this statement that mild slope  $M_1$  zone 1, the profile we get is  $M_1$ , and this is a rising profile. Well, now, let us go further in to the characteristic of  $M_1$  profile, and what are the other things that we need to study. Well, now, we have seen that it is a rising profile, so if I draw this line, say this  $A$  is normal depth line, this is normal depth line and  $NDL$ ; well, and then it is a rising profile, so it will be rising like this, and  $CDL$  line will be of course here that we may not draw right now, but well and let it be here and this the bed.

Now, as it is a rising profile, means, this profile if we see on the upstream side, this will gradually fall and to what extent it will come, it will be coming to the  $NDL$  line, that is at extreme upstream, starting of the gradually varied flow profile we can call that **the** this is starting of the gradually varied flow profile this point this section and from here suppose it is rising, but at **upstream end** extreme upstream end we are getting that  $y$  is equal to  $y_n$ , that is normal depth; and the depth of profile is equal that is bound to be, because it is coming down like that. Now, if it is the situation let us see what will happen at the upstream end, so at upstream end of the gradually varied flow profile.

So, what we are getting that  $y$  is, of course, we should not say that  $y$  is exactly equal to  $y_n$ , because if we say that  $y$  is exactly equal to  $y_n$  normal depth, then it is a uniform flow, so we can call that  **$y$  tends to  $y_n$**   $y$  tends to  $y_n$  almost equal to  $y_n$ ; if this is the situation then what will happen to this equation,  $d y / d x$ , what will happen to  $d y / d x$ , it is equal to  $S_b$  then  $1 - (y_n / y)^3$  divided by  $1 - (y_n / y)^3$ ; now, in this part as we can see that when  $y_n$  and  $y$  almost equal, then this ratio when  $y$  tends to  $y_n$ , this ratio become 1 **this ratio will become 1** in the numerator, and that is why this numerator part that is  $1 - 1$ , so this will become  $0 / 1 - 1$ , that will become 0.

So, **what we can write...**, and once this part become 0, whatever may be the other value this part is of course positive as we could see, but whatever may be the value of  $S_b$ , so these are not coming in to equation, so what we are getting that  $\frac{dy}{dx}$  is equal to 0  $\frac{dy}{dx}$  is equal to 0. So, at upstream end we are getting that there is no change of  $y$  with respect to  $x$ , there is no change of  $y$  with respect to  $x$ , and we already got that in uniform flow itself  $y$  is not changing with  $x$ .

So, it is meeting the point at the end and just when it will gradually suppose we are increasing the  $y$  value the slope wise  $\frac{dy}{dx}$  is 0 here, and  $\frac{dy}{dx} = 0$  means this gradually varied flow line will meet the normal depth line a synthetically **a synthetically**. So,  $y$  tends to become  $y_n$  a synthetically that is we can see from this particular value that is  $\frac{dy}{dx}$  equal to 0 well. So, this is one characteristic of M1 profile that at upstream end the depth rises very slowly, because the depth is that is the gradually varied flow line is meeting the  $y_n$  and a synthetically. So, it will be increasing very slowly, and that is why the length of M1 profile is quite large length of because it will take lot of time to meet the  $y_n$  order uniform flow depth.

Well, then at downstream point what will happen at downstream point, of course, this is it is rising like this then to what extent it will keep on rising. In fact, when we get M1 profile, then it is above normal depth. So, normally the flow cannot rise or cannot go beyond the normal depth line are it cannot exceed, so there must be some obstruction like that, there must be some obstruction or barrier or something must be there which has cause the **water to rise** water to rise.

So, it is rising like that, and then when it will cross this barrier it will definitely fall on the downstream side, of course, what will happen on downstream right now we are not concerned about that, but we are concerned about what will be the case in the extreme right hand side, right hand side means, basically not right hand side it is the downstream side extreme downstream side. So, this section, this sort of obstruction this sort of obstruction or say artificial structures that are constructed in the river or sometimes we can have some natural barrier also, we can sometimes have some natural barrier also like that, sometimes we can have say rock outcrop which can restrict the flow zone and depth can also lead to this sort of situation or different sort of situation well.

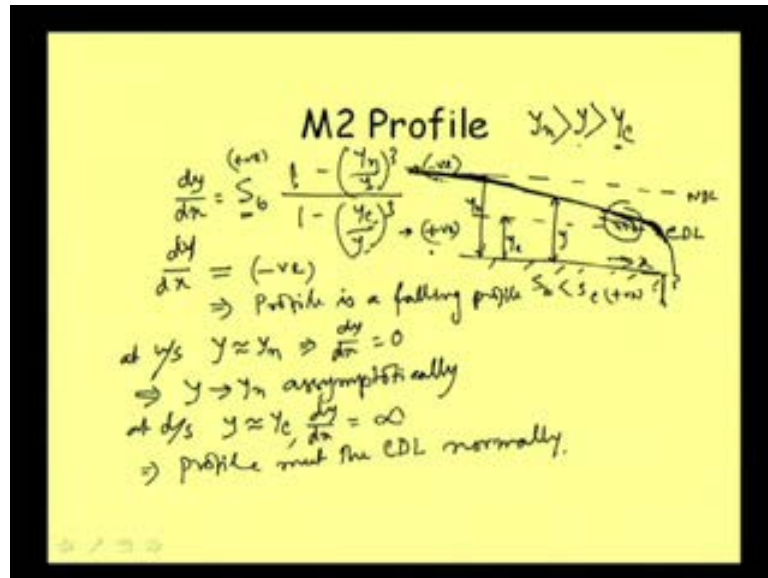
So, whether it is natural or artificial point is that that sort of structures or section we refer as control section; that means, at that section there is definite relationship between the discharge in the depth, well we will be coming to that later when we will be discussing I mean unsteady flow, then it will be more relevant discussing that part, but anyway here also we will be discussing a briefly; and at this point, that means, depth of control section we can call here is the depth of control section. So, at the downstream point we will be getting  $y$  equal to say  $y_{\text{control}}$   $y$  equal to  $y$  at the downstream point.

Of course, when it is a series of profile then that depth may be governed by what is happening at the downstream end, so like that different other situations may come, but at upstream we are very much sure that it will be equal to  $y_n$ , and it will be **it it will be** just tends to become  $y$  and a synthetically, and at downstream point the control section depth will be defining the depth of flow profile, and in between from here to here in between from here to here the flow profile is a rising profile, this will be a rising profile, and the shape of the profile what will be the shape of the profile that will be governed by the equation  $d y / d x$  this equation.

Well, if it is other type of channel section then we can go for the general governing equation of the open channel flow and gradually varied flow and well. So, then let us see what will happen to the M2 profile; well, before that when we are just getting at the upstream point  $d y / d x$  equal to 0 means  $y$  is not changing with  $x$ , well, but this  $d y / d x$  equal to 0 generally can give us a feeling that the profile is becoming horizontal, because  $d y / d x$  represents the slopes, and when we say that  $d y / d x$  is 0, that feelings may comes to our mind that the profile is becoming horizontal, but that is not correct; because here our  $y$  that is depth we are measuring with respect to channel bottom, not with respect to a horizontal that term we are not measuring with respect to a horizontal that term.

So, when we are saying when we are saying that  $d y / d x$  is equal to 0 means depth with respect to channel bed is not changing depth of flow is not changing rather with the direction of  $x$ , it will become parallel to the bed not horizontal, it will become parallel to the bed at upstream end.

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Then let us see what will happen to the M2 profile. In M2 profile this condition remain same; in practical situation when we can have M2 profile that, of course, different conditions we can have, but this CDL and the NDL - normal depth line - and NDL this will be there and suppose there is a drop something like that canal drop and say water was flowing from this point, and then it can come it will come down, weather it will come down that will just get from our mathematical expression, but right at this movement we can say M2 means it is something here, M2 means it is something here.

So, say  $y_n$  is this depth,  $y_n$ ,  $y_c$  is this depth, and then depth of flow  $y$ , right now I am drawing it in the draw down safe or in the down word direction, because I know that otherwise we need to proof it mathematically; well, that  $y$  let this be the depth, and what the condition we are getting in M2 profile, the condition we are getting is that  $y_n$  is greater than  $y$ , greater than  $y$ ; that means, it is because the profile will always be between  $y_n$  and  $y_c$ , so  $y$  is found to be less than  $y_n$ , but it is again greater than this  $y$  depth  $y$  is always greater than  $y_c$ , because the lowest limit of the M2 profile is or rather we can say the lowest limit of the zone 2 itself is the critical depth line, well, so  $y_n$  is greater than  $y$  and this is greater than critical depth  $y_c$ .

Now, if we referred the same equation that is  $\frac{dy}{dx} = \frac{S_b}{1 - \left(\frac{y}{y_c}\right)^3}$  is equal to  $S_b$  into  $1 - \frac{y_n}{y}$  by  $y$  whole cube divided by  $1 - \frac{y_c}{y}$  by  $y$  whole cube, let us refer the same equation well; and here also as it is mild slope, so  $S_b$  is positive  $S_b$  is less than  $S_c$ , but it is positive it is positive well. Now, when if we see the numerator then we can see that  $y_n$  is greater than  $y$ , so this ratio  $y_n$  by  $y$  will be greater than 1  $y_n$  by  $y$  will be greater than 1, and



then cube of that that will be, of course, when one point something to the power cube it will be also greater than 1. So, this ratio is always greater than 1, then 1 minus you can concentrate on slide then 1 minus this ratio which is greater than 1 will lead to negative value of the numerator. So, what we can say that numerator become..., this become negative, and what about the denominator that we can see that  $y_c$  is less than  $y$ ,  $y_c$  is less than  $y$ .

So, this part again become fraction, and cube of that fraction will always be a smaller than one term, so that way 1 minus smaller than 1 will lead to some positive value. So, that way this ratio one is in negative on the top numerator, and this denominator is positive, so negative by positive, and this  $S_b$  is positive  $S_b$  is positive that way  $\frac{dy}{dx}$   $\frac{dy}{dx}$  we are getting that this is equal to negative, whatever may be the value, but this is equal to negative.

So,  $\frac{dy}{dx}$  negative means, this implies that the profile is **profile is a falling profile falling profile**; so, depth is depth  $y$  is falling with respect to  $x$ , it is decreasing with respect to  $x$ . So, in the direct  $x$  means direction of flow definitely and so with the direction of flow the depth is decreasing and that is why this profile now I can draw more confidently that it is like this. Now, well, up to that much it is all right that it is falling profile that is 1 characteristic we are getting of M2 profile, but then what will happen at the upstream end and what will happen at the downstream end.

Now, let us see at upstream end at upstream end as it is falling, means, if you just go back like this, it is bound to meet the normal depth line  $y_n$ . So, at upstream  $y$  is say approximately equal to  $y_n$  and as we know that when  $y$  become approximately equal to  $y_n$  this ratio become 1, and our  $\frac{dy}{dx}$  become equal to 0, that we have already discussed in the last case of M1 profile, where you could see that **when  $d$  when  $y$  approach  $y_n$** , when  $y$  approaches  $y_n$  normal depth, then  $\frac{dy}{dx}$  become 0, and  $\frac{dy}{dx}$  is 0, that means,  $y$  tends to become  $y_n$  a synthetically again.

So, the profile this gives us the condition that  $y$  tends to become  $y_n$  a synthetically, and so the profile will be on the upstream side it will be like that this profile will be gradually going down, and it will be meeting like that will be meeting the normal depth line like that it is gradually going down.

Now, whether let us see so upstream part is clear, now what will happen at downstream point, what will happen at downstream point, whether the same situation will be there in the downstream, means, whether this line is also meeting CDL a synthetically like that or something else will happen there, but at downstream point what will happen as it is coming down, so  $y$  is approximately equal to  $y_c$  well why I am writing approximately, because if I write  $y$  is equal to  $y_c$  means, that is the critical depth, and then **we are having** we are on the critical depth point, we are not on the gradually varied flow part, here the flow should vary.

Well, so, I am writing  $y$  is at approaching  $y_c$ . Now, when  $y$  is approximately equal to  $y_c$ , then you can see that  $y_c$  by  $y$ , that is the ratio  $y_c$  by  $y$  of the denominator is becoming 1, and then  $1^3$  is also equal to 1, then  $1 - 1$  that is the denominator become 0, **denominator become 0**; now, whatever may be the numerator, numerator value will be definitely I mean negative or whatever it is, but anything divided by 0, anything divided by 0 that become infinity, **so  $\frac{dy}{dx}$  is equal to...** we can write infinity.

So, at downstream point when  $y$  is almost equal to  $y_c$   $\frac{dy}{dx}$  is infinity, well, by that what do you mean that is change in  $y$  due to small change in  $x$  is infinity, means, this is change it by small amount,  $y$  change is very large, means that can be possible only when this line is coming like that, and it is meeting it perpendicularly, because with little change in  $x$   $y$  is changing to a very large value; that means, when  $\frac{dy}{dx}$  is infinity we can say that the profile will meet the CDL line normally, profile will meet the CDL line perpendicularly well normally, that is why the profile shape will be like that on this part it is meeting synthetically it is coming smoothly and here it is meeting perpendicularly.

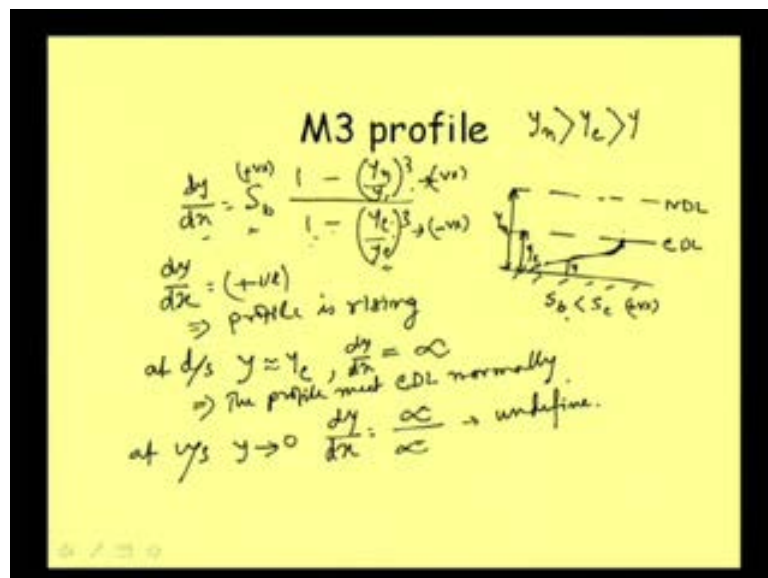
So, that is how we can define the characteristic of say M2 profile at downstream; so, this will indicate this implies that  $y$  tends to become  $y_c$  or rather we can write the profile meet profile meet the CDL line CDL that is critical depth line normally perpendicularly it means, perpendicular means, this perpendicular is to the CDL line not to the horizontal, because again CDL is parallel to the bed line, so it is with that respect we are saying it is meeting perpendicularly, so that way we are getting characteristic of M2 profile; and there are some other finer points here, because when it is meeting perpendicularly then at that point when flow is occurring like this, of course, what will happen in the downstream means after that it is entering into the zone, after that its

entering into the zone 3 or it may happen that when it is a fall then it is coming like this, and then it is meeting perpendicularly, and then it is falling down on this part.

So, this part we are not talking about, but we are talking about the profile, what is happening here in the zone 2, but one important point I want to highlight here is that in the governing equation of gradually varied flow we did make one assumption that hydrostatic pressure prevail; that means, pressure when we compute, then we are computing with respect to considering as if the flow is static, and that is true when the stream lines are parallel, when the stream lines are parallel we can have it.

But when it is meeting perpendicularly the flow stream line **will be having** will be having curvature significant curvature at this point. So, when we compute the flow profile at this point, of course, this zone is very narrow, this zone is very small, because change in slope is infinite means it is at very first rate changes. So, a small portion at the downstream end of the M2 profile, here the condition of hydrostatic pressure may not prevail will not prevail rather, because of the curvature, and we may end up while computing we may end up with making some error at the point, of course, this zone is, so small that for practical purpose we can neglect that well.

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Then let us see we are left it one more profile in mild slope that is M3 profile. Now, in M3 profile say this is bed slope is always mild  $S_b$  is less than  $S_c$ , and of course, it is positive, and as explained already this is CDL, and this is NDL, this is NDL, and profile

here is line in this portion; and right now I do not want to say that it is rising or falling, but it is a profile here, and then depth here is  $y$ , critical depth line up to critical depth line it is  $y_c$ , and depth here is say  $y_n$ .

Now, so, what condition we are getting, that is  $y_n$  is greater than  $y_c$  and is greater than  $y$ . So,  $y$  is the smallest value here, depth of profile is the smallest one, and other values are meagre this normal depth and critical depth, then going back to our equation again say  $\frac{dy}{dx}$  is equal to  $S_b$  into  $1 - \frac{y_n^3}{y^3}$  divided by  $1 - \frac{y_c^3}{y^3}$ . Well, again if you see this equation then we can see that  $y_n$  is greater in..., the numerator  $y_n$  is greater than  $y$ . So, this fraction will be something greater than 1 cube is greater than 1, so  $1 - \text{something greater than 1}$  will be negative, so this will be negative this will lead to negative value, and then in the denominator this is also  $y_c$  is this time again greater than  $y$  unlike the M2 profile, here  $y_c$  is also greater than  $y$ .

So, **this value is also a fraction rather** this ratio is greater than 1, so  $1 - \text{cube greater than 1}$  cube, so  $1 - \text{something greater than 1}$  that will also lead to negative value, and  $S_b$  value is of course positive, because we are talking always about mild slope which is positive; then this negative in the numerator, negative in the denominator that will make the ratio total ratio again positive negative by negative, so it is positive,  $S_b$  is positive, so what we are getting that  $\frac{dy}{dx}$  is positive.

Now, when  $\frac{dy}{dx}$  is positive, what it means again this implies the **profile is rising** profile is rising, so we are getting again a rising profile now. So, this will be a rising profile like this; now, I can draw this shape as a rising profile, then what will happen at upstream and downstream, again that question rise, in between of course, in between the shape of the profile we can determine from the governing equation by solving the governing equation, but what will happen to the **boundary value** boundary limit, that is at upstream and downstream.

Let us see at downstream what will happen; first let us see at downstream, because already we have down done this at downstream  $y$  is approximately equal to  $y_c$ , and when  $y$  is approximately equal to  $y_c$ , that point we know that this value become 1, and then  $1 - \text{I mean this ratio of the denominator this become one } y_c \text{ by } y$ , and then  $1 - \text{something which is also equal to 1}$ , then this become equal to 0, denominator become 0.

So, when this become 0, the ratio  $\frac{dy}{dx}$  become infinity,  $\frac{dy}{dx}$  become infinity, and of course, if we think about  $y^n$  by  $y$  this time, then  $y^n$  is of course greater than 1, and this will become negative, and we will be having some definite value, and  $S_b$  is positive, but anyway this is becoming ultimately minus infinity or so anything divided by 0 we can call, this as an infinity, our basic requirement is that what will be the safe of the profile. So, it is raising negative means  $\frac{dy}{dx}$  is raising like that, and **at this point** at this point kit is meeting the CDL line normally, so this implies that the profile meets CDL normally, so that is relation already we obtained.

Well, that means, on the downstream part it is cleared that it will be meeting CDL normally; now, what will happen to the upstream part, what will happen to the upstream part that is at upstream **at upstream**  $y$  what will be the  $y$  the depth of flow coming down. So, as it is coming down its it appears that it will meet the bed, and when it is meeting the bed the depth will become 0; now, of course, for practical purpose that upstream **depth cannot become 0** depth cannot become 0, because say 0 depth from, 0 depth it cannot start; if some discharges coming there must be come discharge some depth must be whatever may be the speed of flow, whatever may be the speed of flow very high velocity, but if some discharge is coming with very high velocity, depth may be very small, but there must be some amount of depth; practically, it cannot become 0.

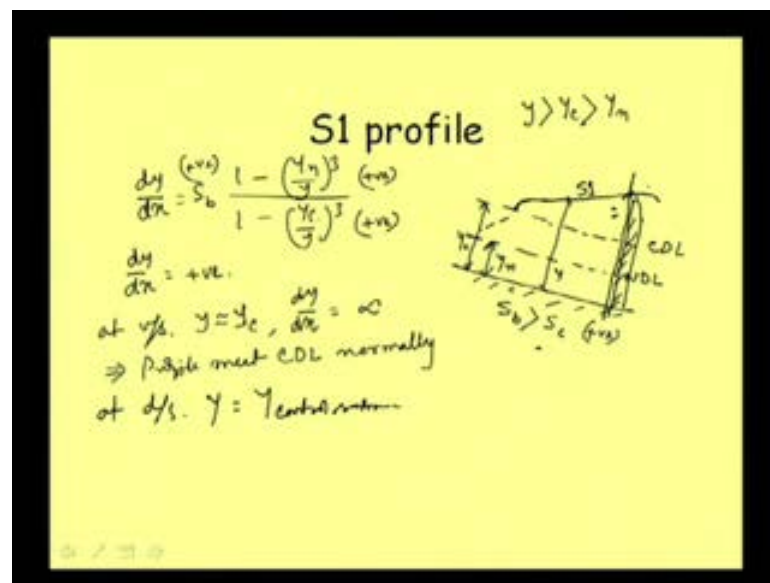
But if we see the downstream shape if we see the shape of the profile, and if we talk in terms of mathematical relation then we can say that  **$y$  approaches 0**  $y$  approaches 0. Now, if  $y$  become 0 or when  $y$  is approaching 0, we can now just mathematically see that in the equation you can see in the slide, that is  $y^n$  by  $y$  this is 0, so this term become infinity 1 minus infinity, again we can call this is infinity; and then the lower terms, that is the denominator, this also become infinity because  $y$  is approaching 0.

So, **when our  $\frac{dy}{dx}$** , we can write as  $\frac{dy}{dx}$  we can write as infinity by infinity, now infinity by infinity is mathematically we can see it is undefined, so we cannot really have a very definite idea about the point, what will happen that just at the extreme upstream end it will depend on basically the depth of flow here, and this will be coming from this particular depth, and this profile cannot go up to the 0 depth, that is what we can say and it is undefined. So, if you want to draw something here, we can just put some dot here indicating that we are not sure what will happen if it is approaching 0; so, it is undefined, that is why we can put some dotted line **dotted** dot here indicating that this is undefined,

and then this profile will be rising like that, and it is meeting the CDL line in this form that is normally.

So, that is what the characteristic of M2, M3 profile, and this is of course a rising profile. So, following this procedure we can analyze the characteristic of different type of profile. Now, let us see what will happen in case of S1 profile, that **S1 profile**, till now we were discussing about M1, that is M2, M3 all are mild slope; and now, we are going to the steep slope.

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Well, the S1 profile means it is a steep slope like this, and in case of steep slope this  $S_b$  is greater than  $S_c$  critical slope, and **of course positive**, of course positive, but it is greater than  $S_c$ ; and when it is greater than  $S_c$ , then our normal depth line will be somewhere here, say NDL, and critical depth line will be above the normal depth line, this is what CDL. Now, when we talk about zone 1 that means the profile will be here.

And so this is our  $y$ , this depth is  $y_n$ , and this is critical depth  $y_c$ , and the condition that we are getting is that  $y$  is greater than  $y_c$ , and  $y_c$  is greater than  $y_n$  here; and then if we go back to our equation again say  $\frac{dy}{dx}$  is equal to  $S_b$ , then  $1 - \frac{y_n^3}{y^3}$  by  $1 - \frac{y_c^3}{y^3}$ , well the same equation; here we can see that as the profile is in zone 1, so  $y$  is greater than  $y_n$ , means, this ratio that is the ratio of  $y_n$  by  $y$  in the numerator is a fraction, and that will be less than 1, so  $1 - \text{less than } 1$  will become positive.

And in the denominator also the  $y_c$  by  $y$  that is also a fraction, and cube will be again a less than 1, and  $1$  minus something less than  $1$  that term will become positive, and  $S_b$  is also positive, so that way these are the terms we are having and that way we are getting that  $\frac{dy}{dx}$  is positive well; that means, it is a rising profile,  $\frac{dy}{dx}$  is positive. Now, if it is raising profile, then let us see at upstream in what will happen, well, at upstream end as we know here CDL line is on the top, that is critical depth line is above the normal depth line.

So, when the profile is coming down it will be meeting the critical depth line not the normal depth line in case of S1 profile; well, so, at upstream point  $y$  is approximately equal to  $y_c$ , so when  $y$  is approximately equal to  $y_c$ . then we can have that is this ratio  $\frac{y_c}{y}$  will be equal to  $1$ , and then  $1$  minus  $1$  will become  $0$ , and that means, again we are coming back to the original point when  $y$  meet  $y_c$ , then  $\frac{dy}{dx}$  is becoming infinity  $\frac{dy}{dx}$  is becoming infinity; and when  $\frac{dy}{dx}$  is becoming infinity, what it mean that the profile this implies the profile meet CDL normally.

So, here the profile will be meeting the critical depth line normally, and it will be going like this, it will be meeting like this, and it going like this. So, the shape of the profile at the upstream end this will not go continuously extend too far distance by this way, and rather it will be just rising from the critical depth line in this way; of course, this is just we are discussing about the characteristic of S1 profile, and in nature it is not depth flow is coming in critical depth line and it is jumping like that, basically it will be coming in the normal depth line, always the normal flow will be occurring or without any obstruction without any barrier to the flow **the flow** will always tend to move with normal depth line.

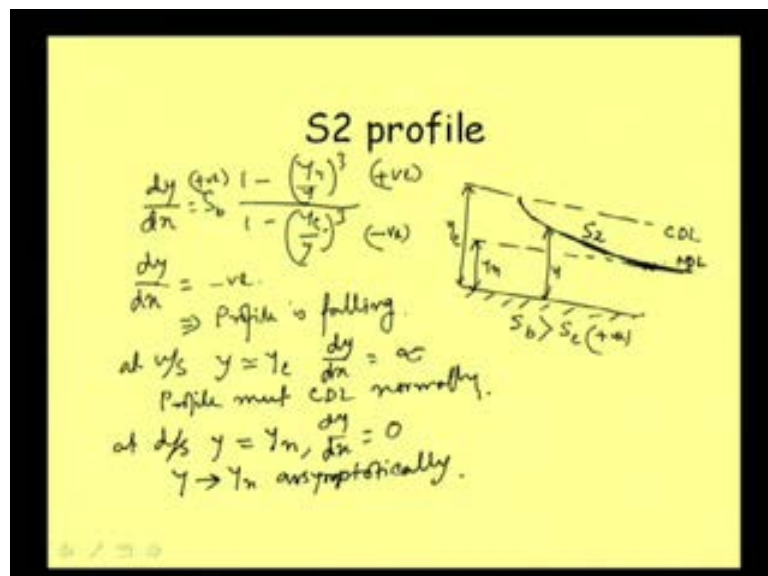
So, it will be coming as a normal depth line then there must be something some disturbances may be some obstruction like this, then sort of things or anything it can be some obstruction, and then the flow is rising, say flow is crossing this part, it may happen that the flow is crossing this part, and it is moving like this; again, there will be some control section at the downstream, there will be some control section at the downstream, and before reaching or before starting of the S1 profile, there must be some flow profile here, which we will lead the flow to move from normal depth to the critical depth line; of course, there are may be some other combination, what that will be discussing, when will be discussing about the flow profile on series of channel slopes,

say, if different channel slopes are changing, then how profile is occurring that will be discussing, there will be discussing this things in more detail.

But here so far S1 profile is concerned, it will meet the CDL line normally, and then it is always rising like that, and at downstream at downstream what will happen that the depth will be  $y$  is equal to  $y$  control, so this a sub critical flow; and although it is on steep slope although it is on steep slope do not take it like that if the flow is on steep slope, it is always super category, it is not like that if it is flowing normally uniform flow then only it will be supercritical, but here the flow has somehow gone up to above the critical depth line.

So, when here it is flowing it is the subcritical flow, and this control section is here, and this will just governed what should be the depth of flow at the downstream point, well that we will be discussing in more detail in our later part; well, but this downstream  $y$  is equal to  $y$  of the control section, so that way we can say that characteristic of S1 profile that it is a rising profile it which the CDL line normally at upstream point, and it raises till the control section is arrived, and of course, something else can be there on the downstream.

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And let us discuss about the **characteristic of S2 profile** characteristic of S2 profile; here the bed slope  $S_b$  is greater than  $S_c$ , the same thing this is normal depth line NDL, and this is critical depth line CDL, and our profile will be here; and let me draw, it as a



falling because I know this; in fact, now we all can be confirmed about one point, that is when a flow profile **when a flow profile** is occurring in the zone 2, it will always be a falling profile, because from this equation it is very cleared; and now, whenever we get a flow profile in the zone 2 we can draw it as a drawn down profile, because this equation  $\frac{dy}{dx}$  is equal to  $S_b (1 - \frac{y}{y_c})^3$  and by  $y^3 (1 - \frac{y}{y_c})^3$ .

From this equation it is cleared that, in zone 2 this normal depth either of this things I mean if it is in zone 2, if it is steep slope then your critical depth is greater than  $y$ , and normal depth is less than  $y$ . So, **1** if 1 ratio is positive then other ratio is negative, and **if it is say mild profile**, if it is mild slope profile then your normal depth will be top that is normal depth will be greater than depth  $y$ , and critical depth will be less than the depth  $y$ .

If critical depth is less than the depth  $y$  then this ratio will be fraction and this will be positive denominator will be positive, but numerator will be negative; that means, either numerator or denominator will be if one is positive other will be negative, so ultimate expression for  $\frac{dy}{dx}$  will always be negative, and that is why this profile is always a falling profile, any profile in zone 2 will be a falling profile.

So, now we can draw it very clearly, **so this indicates that, so, we can...**; well, **in this case** in this case our  $y_c$  is in the denominator  $y_c$  is greater than  $y$ , so this ratio is larger than 1, and so this  $1 - \text{something larger than 1}$  greater than 1 will lead to negative, and this part is positive on the numerator it is positive, this  $S_b$  being positive. So, we can have that  $\frac{dy}{dx}$   **$\frac{dy}{dx}$**  is negative finally, and this indicate that **profile is falling** profile is a falling profile fine.

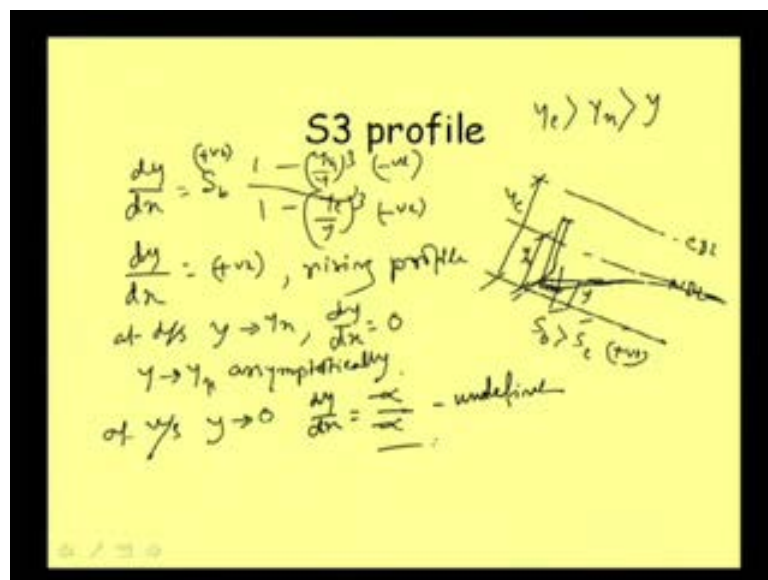
Then what will happen at upstream, and what will happen at downstream, once we do that then only our analyzing or analysis on characteristic of S2 profile will be complete. So, let us see at upstream what happen, **at upstream** at upstream  **$y$  is approximately  $y_c$**  equal to  $y_c$  that is it is meeting the  $y_c$ . So, when  $y$  meet  $y_c$  that we have got several time that when  $y$  meet  $y_c$ , then  $\frac{dy}{dx}$  is infinity, so we need not explain it anymore, because we have this we have discussed this issue.

So, at  $y$  approaching  $y_c$  means at upstream  $\frac{dy}{dx}$  is equal to infinity, so it indicate the profile meet CDL normally. And at downstream what will happen, **at downstream  $y$  is....**, so let me complete this part, this is say meeting normally means, it will be coming like this; and then it is moving in the downstream side, so what will happen at the

downstream point, at downstream  $y$  is equal to  $y_n$ , then when  $y$  is equal to  $y_n$  then this part become 0, the numerator become 0, numerator of the equation expressing  $\frac{dy}{dx}$  that become 0, so this indicate that  $\frac{dy}{dx}$  is equal to 0.

And mathematically saying  $\frac{dy}{dx}$  equal to 0 physically means that the profile will meet the NDL a synthetically. Well, so, what will happen here this line will be meeting this normal depth line a synthetically, it will keep on going like that, and at far distance it will be ultimately **margin with the NDL** margin with the n d l. So, we can just see that this  $\frac{dy}{dx} = 0$  means profile or we can write simply that is  $y$  tends to become  $y_n$  a synthetically; well, so, this completes the shape of the S2 profile **say of the S2 profile**, well, then it is going like that.

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Now, let us see what will be the characteristic of S3 profile. Well, in S3 profile this slope is steep as well, and this is our NDL, this is CDL, and profile is somewhere here, again I am drawing it is a rising one, because now we can write it confident that this is always rising, if it is somewhere in zone 3. Well, this condition is  $y_c$  is greater than  $y_n$  and is greater than  $y$ ; so, referring to our equation say  $\frac{dy}{dx}$  equal to  $S_b$  into  $1 - \frac{y_n^3}{y^3}$  by  $y_c - y$  whole cube  $1 - \frac{y_n^3}{y^3}$ ; well, then going to this equation again here  $S_b$  is greater than equal to  $S_c$ , and this is positive, so we can write as it is positive.

And then  $\frac{y_n}{y}$  this you can see that normal depth is greater than  $y$ , so this ratio will be in numerator the ratio  $\frac{y_n}{y}$  will be greater than 1, so  $1 - \text{something greater}$

than 1 will be negative; and in denominator also  $y_c$  is greater than  $y$ , so it will also be negative; and finally, what we are getting that  $\frac{dy}{dx}$  is positive is positive, so it is a rising profile, it is a rising profile. Well, so, this rising profile again if we extend it on the downstream side at boundary value what will happen that we need to know, so whether it will be of this shape or it will be of something else let me draw it at downstream point.

At downstream point  $y$  tends to become  $y_n$   $y$  tends to become  $y_n$  approximately equal to  $y_n$ , so we are discussing already, and we have discussed several time then when  $y$  approaches  $y_n$  then  $\frac{dy}{dx}$  become 0, and this physically mean that  $y$  tends to become  $y_n$  a synthetically a synthetically. So, this statement indicated the profile will be coming like this, and it will be meeting the normal depth line like that profile is meeting the normal depth line like, that it is a synthetically not normally it is coming like this, and it is coming in the upstream direction like that. And at upstream what will happen, at upstream again if we see that  $y$  if it approaches 0, if it approaches 0, if  $y$  approaches 0, then as per this expression in the numerator also  $y_n$  by  $y$  become infinity 1 minus infinity, so this is remaining say minus infinity, and this part is also denominator also  $y$  is very large than  $y_c$ , so this is also infinity.

And finally, what we can have the  $\frac{dy}{dx}$  is equal to say minus infinity by minus infinity, this it is basically infinity by infinity, so this becomes again undefined; well, so, this becomes undefined means we cannot define this part properly, so we should draw it like that just putting some dot here then we are continuously extending; and in reality this undefined means, in reality we cannot have this situation, because at upstream point depth cannot become 0, so we cannot have this situation it will always be there will be some amount of depth and up to that depth we can always start computing or we can always continue our computation up to that depth.

And our depth computation of upstream depth will continue up to the depth, where we have some definite depth at the upstream point; this case may be suppose you have the sluice gate here, and you are releasing the water, then it is going like this, and then it is rising like this sort of profile, and then our depth of computational will be up to the depth somewhere here, that part will be discussing in more detail of course, but super characteristic is concerned we can discuss the characteristic in this form. Well, so, today in this class we have discussed our characteristic of mild slope profile, and steep slope

profile, and all the three profile M1 M2 M3 and S1 S2 S3, and we need to discuss about characteristic of C1.