Hydraulics

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Module No. # 03
Energy and Momentum Principle
Lecture No. # 04
Non Uniform flow:
Gradually Varied Flow

Welcome you all to this new topic that is non uniform flow. Till now that we have discussed all about uniform flow condition and hydraulics of uniform flow. And today we shall be starting our discussion on non uniform flow, well. And under a non uniform flow of course, in our very basic introduction to the hydraulic engineering, we were discussing or to our you know, introduction to the open channel flow, we are talking about non uniform flow, there are different type of non uniform flow; that is say gradually varied flow is one of that, where the flow depth varies gradually along the channel section, along the channel reach.

And then, there is another class that we call as rapidly varied flow, where the flow depth varies quickly, flow depth varies very quickly from within a small channel reach, and there can be another non uniform flow that we term very specially varied flow; here say discharge is changing from section to section, because of the fact that some side channel may contribute some discharge to the main channel; again some of the discharge from some channel can go part of the discharge go out from the main channel to a side channel that way also there can be some changes in the discharge or that we once the discharge changes, flow parameters are changing, velocity, depth all can change along the channel reach. So that way, we can have due to different condition, we can have different type of non uniform flow. And today, we will be discussing on one of the non uniform flow, which is called gradually varied flow. So, our discussion will be on gradually varied flow which is a type of non uniform flow.

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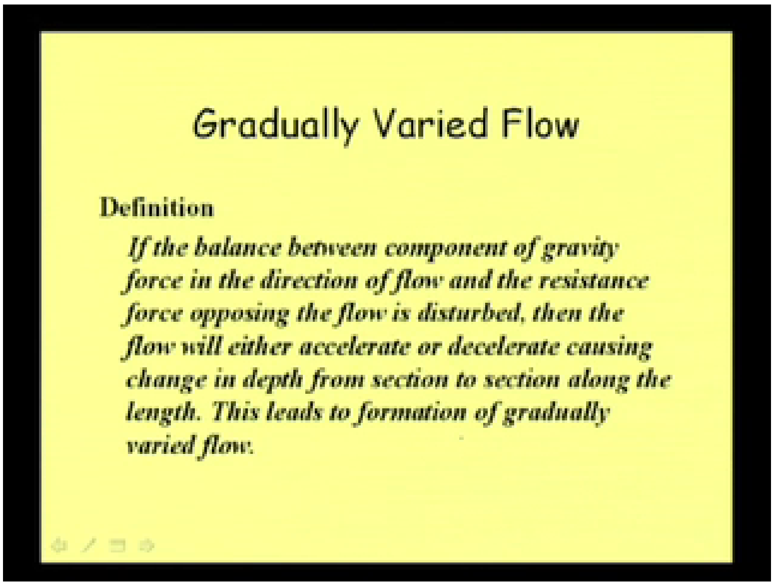

Well, now to start with let us just briefly give definition of gradually varied flow. In our initial discussion also we did discussed that, just to rephrase it, let us say; when say uniform flow is occurring, suppose uniform flow is occurring, then force the component of gravity force that is when open channel flow we are talking about, it is gravity force we are dominating flow. So, the component of gravity force that is acting in the direction of flow, and the force opposing that force flow, that is the resistance force r, suppose in balanced condition, and we are getting a uniform flow in that channel.

Now, if somehow this balance is disturbed, and that this disturbance can be due to different reason; if this balance is disturbed, then the flow earlier it was moving with a uniform flow condition means, with a uniform velocity it was moving. Now, when this is disturbed, then the velocity will change either it will accelerate or it will decelerate. And then, when the velocity is changing along the channel, then its depth will also change. So, that way, we get gradually varied flow.

So, you can concentrate on to the slide, and we can define it like that; if the balance between component of gravity force in the directional flow, always we are talking about the component of gravity force in the direction of flow, and the resistance flow opposing the flow is disturbed, if this balance is disturbed, then the flow will either accelerate or decelerate causing change in depth from section to section along the length, along the length of the channel. And this lead to formation of gradually varied flow well. So, this is how we can define gradually varied flow and this is how we define the gradually varied flow.

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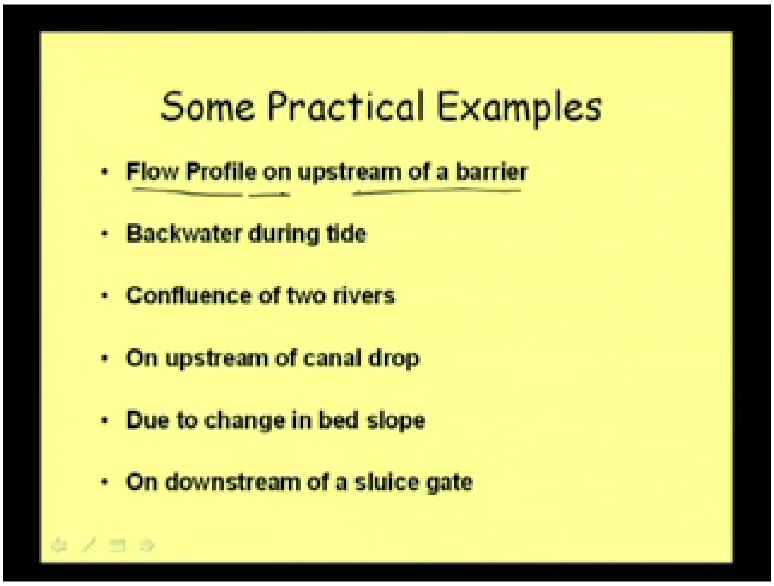

Let us now see, some of the practical cases or some of the practical situation, where we can have this sort of gradually varied flow. And the one example, the first example we can take, this is flow profile on upstream of a barrier, flow profile on upstream of a barrier; in a natural channel or you mean in a manmade canal also in a irrigation canal also, many a time we give some barrier for utilizing the water for our bettering for our need, that means, for better utilization of water, we can say.

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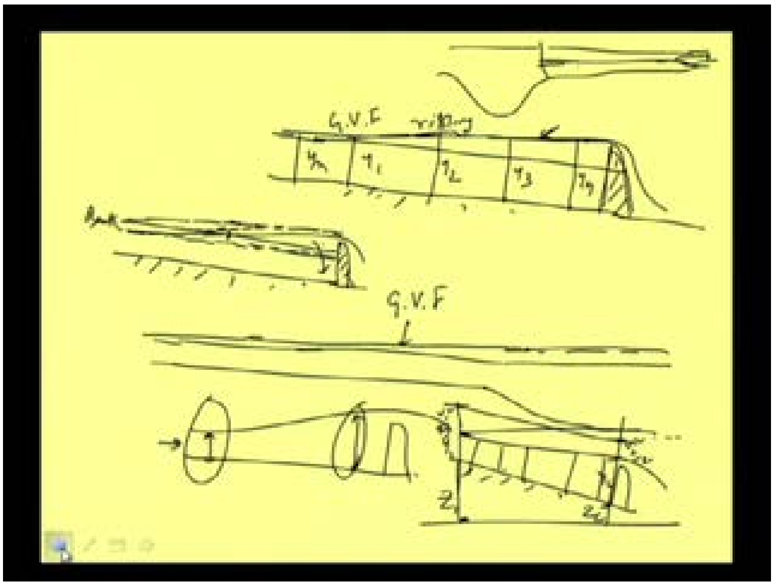

How say in a river, the water is flowing in this suppose, the uniform flow is flowing; uniform flow is of occurring, like that well. Now, we want carry the water from this river to some agricultural field for our irrigation; then now when the water is at this level, it is within the bank, and we may not be having any other device to carry the water from here to the irrigation field to the agriculture field; sometimes we may be doing pumping, sometimes we may use wheel to draw the water. But another way of doing is that if we put some barrier here, if we put some barrier or may be a large area also some times in case of dam, and varies like that; then what will happen? The water cannot flow through this level, and it will be flowing like this, it will be, it will start flowing like this; and then it is moving like this; on the downstream, it is again flowing this way.

So, that way what we have achieved that we have raised the water level to this level; now our height is increasing that is water is going to a higher level; and from this higher level as our existing level is high, we can carry this water to a much far distance maintaining the height; that means, say when water will be flowing from this level, we can carry the water, because we can draw a side, let me draw a section, cross section, suppose this is the river, generally the slope will be in this direction, and say our water is here. Now if our water level is here, we cannot carry the water after certain distance here; but if our water level we increase up to this much height, then we can carry by gravity suppose making a channel, it can be carried up to this much of level, where our level meet this one. So, we can carry to a half distance.

So that way, this is one way when we use some barrier and we raise the water level, so that it can facilitate our irrigation in agricultural field. Now this water surface this water surface that we are having here is not a uniform flow, because this depth is gradually increasing from section to section; here depth is y, and suppose on the upstream, it is normal depth say, here it is normal depth on the upstream, we can have normal depth, and then it is then water is gradually increasing, then it is gradually increasing like this.

So, at this point it is normal depth, but after that it is gradually increasing, and we are having different depth y 1, y 2, y 3, y 4 like that; and this flow is nothing but one example of gradually varied flow well. In canal also for which is made for agricultural purpose, we sometimes increase the depth to carry it to the side channel or side canal or field channel, so that way also this water level is increased and we get a gradually varied flow like that. Again this gradually varied flow can be of different type, that will be discussing later, but this one kind of gradually varied flow.

And this sort of curve is raising profile, it is rising, it is gradually rising; so this is rising and this has another name that we call it as a back water profile. Then another example is name itself is backwater during tide that we have experienced and seen that all the water that is coming is we refer as backwater of course, but then during tide what happen during tide say it is a sea here, let me draw the sea level; suppose sea level is here, and we are having uniform flow condition like that, uniform flow condition like that; and when during tide the water is rising up to this much level, then water from here will be rising like that, it will have to be continuous; so, it is rising like that. And then, this is a gradually varied profile, we write it simply as G V F. So, this is a Gradually Varied Profile.

And one point we should remember; though a many a time, we draw gradually varied profile suppose, a giving a barrier, we are giving this barrier, and we are drawing a line like this; apparently, we get this filling that, depth here is, less depth here is more; then a question may come into our mind that how the water is flowing from lower depth to higher depth, because here the depth is higher, and the discharge is moving in this direction; but of course, total energy at the upstream level is always more than the total energy at downstream level, and that is why the water is flowing. And in reality, there will be some slope, there will be some slope like this, this barrier is there, and then suppose depth of flow is there, and it is it is rising like this, it is rising like this.

But if I draw a horizontal line here, then energy line if I draw, then you will be finding that energy at this level is higher than the energy at this level. So, though the depth is increasing, depth is increasing, but if I draw from a particular datum here, then this is z, then this is y, then again due to velocity you will be getting v square by twice c, here it will z 2 say this is the slope, this is z 1, y 1, then v 1 square by twice c, it is z 2, then y 2 plus v square by twice c, again v square by twice c, suppose this much, then we will be getting energy line will be lighter, energy is low.

So, there will always have to be energy more here than energy here, and then only this flow is occurring. And in reality, it is like that, and this change in depth is so gradual, the change in depth is so gradual that when we go to a field, this is not that much of visible, that means, on the upstream of a valleys or upstream of a (( )) if we just observe the flow, we will never feel that water depth is rising, water level is rising, it is. So, this can be computationally checked, we can do the computation, and we can check; and this is very important; suppose, for some reason we are making a barrier here, for whatever may be the reason we are making a barrier here, and water depth will be rising.

And say bank level is here, let me draw a phrase diagram; say our (( )) is this one, and bank level is this one, bank, this is ban level. And then we are suppose when uniform flow is occurring with maximum discharge, then it is set this level. So, there is no question of our flooding; of course, during flood time, there may be flood, but generally the flood is not there, flooding is not occurring. Now to raise this to increase this depth, we are putting a barrier here. So, the water level from here will be rising like that, and it is flowing like that.

Now, if we do not do the calculations before hand that is to what extent the water is raising, then what will happen that if you give a barrier here, water will be coming here, and it will be over flooding, it will be crossing the bank. So, it will create problem, our target will not be achievable, water will not be rising to that level, it will be flowing out from the bank. So, we will have to construct some marginal embankment or on the sight, we will have to raise this bank. So, this is the bank level, so depth we will have to also rise like this, this is the bank level we have to rise; so to what extent? Now, we cannot rise this bank, suppose we know at this point you have bank height will have to be this much; but that we cannot carry or we not carry the same height same raising up to this point, because water is rising at this point.

So, we know that up to this point from this point water level is crossing the bank. So, we can increase our bank height like this, we can increase our bank height like this. And that way we can save our expenditure in raising the bank, and we achieve economy projects. So, for those purpose we also need to know how the gradually varied profile will form; to what extent it will go; and this sort of calculation will be becoming that will be coming in computation of gradually varied flow, well. This is just we are putting some example, where gradually varied flow can occur.

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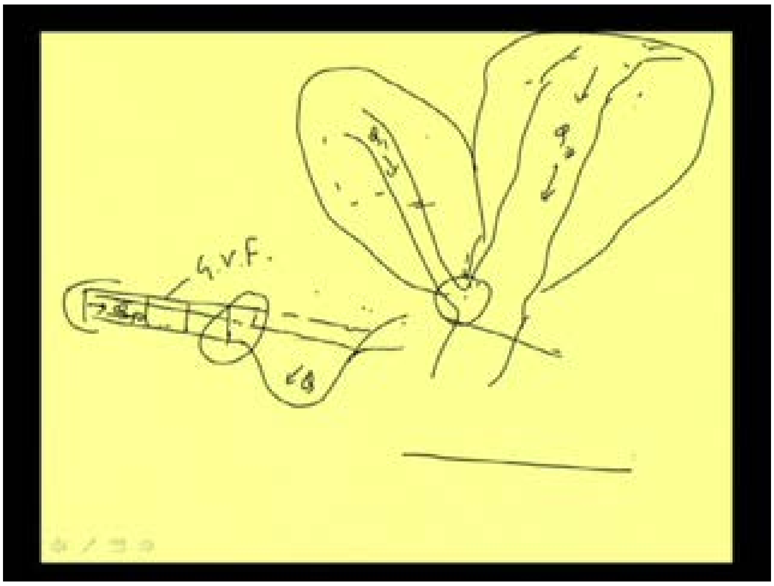

Then let me take another example that is confluence of two rivers that is also another case where gradually varied flow can occur. Confluence of two river say one river is coming like this mainstream, and another river is another tributary is joining this way, and it is flowing like that. Well, this is control coming from some discharge here; now at this counter, sometimes say this river has this catchment on the upstream of this particular section, and this river is having this catchment; catchment means whatever is occurring that is coming to this particular point; whatever rainfall is coming, it is occurring from this particular point to this particular river.

So, if suppose heavy rainfall is occurring on upstream of this river, then the depth or discharge in this stream in the main stream can increase significantly; and then depth of this stream can increase. Now, earlier suppose if I draw it like this, this is the main stream, and this the river - side river, this river is this one, it is carrying Q 1, and this river is carrying a discharge of Q here, it is flowing like that; now earlier suppose it was in this form; earlier it was like this. Now, when this level will be increasing, suppose due to increased rainfall in the upper catchment, this level is increasing; then on this channel, the water level will be rising like this, and this is not a uniform flow from section to section, this depth is increasing.

So, at confluence of two river, this is what, this is the confluence of two river, and to upstream of the tributary, there can be gradually varied flow profile there can be gradually varied profile. And knowing this is important for various activities, because when it when the water level rises, that means well, we understand that energy level, energy difference is there, but energy difference will definitely reduced from that of the original condition.

So, the flow velocity in this part will be less, in the tributary will be less; and this can happen suppose, it is a river, and it is a drain, then also this situation can happen; and then flow velocity here will be less means, less amount of water will be released or this channel will be carrying very less amount of water. So, then it was carrying earlier, because of the reduction in the energy level; and that sometimes can lead to flood congestion in the upstream area here also in this channel; and so, these are important in various analysis we use, and the concept of very gradually flow in this sort of channel conference. Then there can be another situation on upstream of canal drop; many a time, we have some canal, we have some canal like this, and there can be… suppose in a very steep slope, suppose in a very steep slope…

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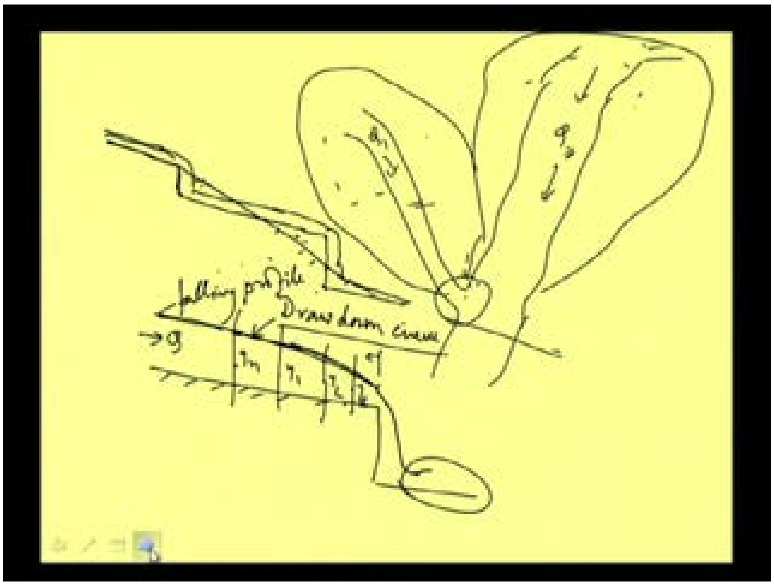

Let me use in a very steep slope, we can have this sort of situation say, natural channel is very steep. And we want to make a canal in this direction; then it may become difficult due to many reasons for constructing a channel, a canal of this slope. So, what we do? Because, a very if you construct a very steeper channel, then it may cause and then flow velocity will be very high, and then with high flow velocity, there may be erosion or there may be damage to the channel. So, what we do that suppose we construct it like that, then we cut some portion here, we make a canal drop, and then we remove this portion, and then we make another channel portion like that, this part can be deposited here, and then again there may be canal drop like that, and that way, we are negotiating the slope.

So ultimately, the canal will be carrying water like this, and then there is fall, then this is going like that, then there is a fall, then it is going like that. So water is flowing like this, though our original slope was that one. So, in this sort of canal drop, if I just draw one canal drop like this, then what will happen? Water will be flowing, suppose this is uniform flow depth for a particular discharge Q, for this bed slope; for this particular slope, we have this much of uniform flow, then when water is falling, it will be coming like this, and then it will be falling like this. And then, there may be some other phenomenon here we are not discussing that part right now, but we are talking about what is happening on upstream of this one.

So, this water surface is gradually coming down, and the flow depth is changing section to, from section to section; here it is normal depth y n, then it is y 1, y 2 y 3. Here the depth is gradually decreasing here the depth is gradually decreasing, because of the change in the flow. Now this sort of flow is also gradually varied flow, and that flow we call as draw down curve we call as draw down curve. So, basically it is draw down profile; earlier, we have seen some of the cases where the channel depth was - flow depth was rising, but here the flow depth is falling or we can call falling profile, this is a falling profile falling profile.

So, there may be two class; one is rising profile, another is falling profile. And in the falling profile, we name this as draw down curve; raising profile, we name it as backwater curve. Then let us see some other example; due to some other change in bed slope, due to change in bed slope, this is somewhat similar to that one, where we are talking about canal drop; but here say actually canal drop is not there, but there can be some change in the bed slope, say bed slope is changing this way, bed slope is changing this way.

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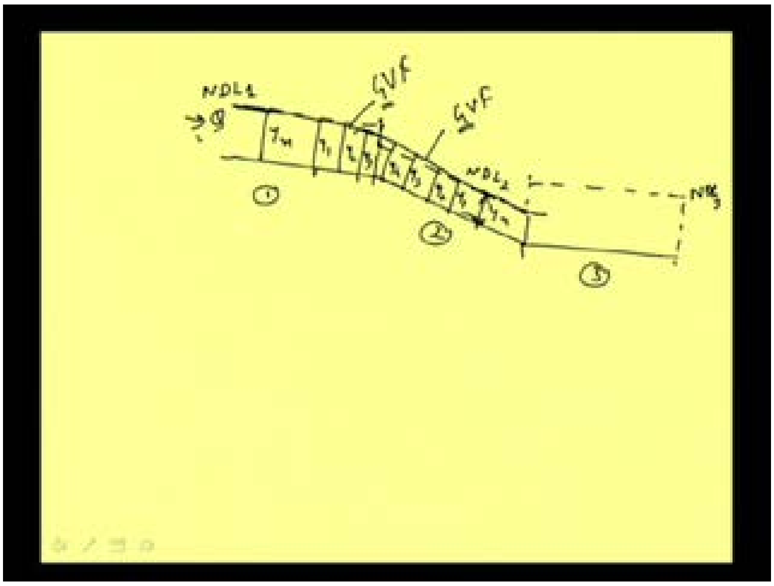

Well, now from our understanding of uniform flow, we had seen till now, but suppose a discharge Q is coming here. Then for this slope, suppose our uniform flow level is this much; NDL 1 suppose in this section 1, section 2, section 3. Now here, as the slope is increasing in the channel 2, as the slope is increasing, so obviously, the depth of flow or the uniform depth of flow will be lesser than that of the uniform depth of flow for section 1, for the channel reach 1, because discharge remaining same slope is increasing here in the 2, then 1. So, uniform flow will be becoming lower. So, this is suppose my NDL, this is NDL in this (( )) normal depth line 2.

In the third section again as the depth of flow, as the slope is flatter, then even the first one - depth of flow, normal depth of flow will be again increasing NDL. But these are all about the normal depth line we are writing; at the flow profile that the surface of water will not be moving like this, and then suppose straight moving this way, then dropping here, again moving like that, then rising here, not moving like that. But in a channel, when we allow the water to flow freely, it will always try to attain the normal depth condition; of course, if we make some disturbance there, then it will be disturbed; and if we allow it to flow continuously for a long time without any obstruction, without any drop, then it will try to attain the normal flow condition.

So, what will happen? Here if we assume the flow is coming like this, at this normal depth, but here it will try to meet this normal depth; and so, it will gradually come down, and then it will be it will be coming like this, and then of course, further detail is necessary that is whether this flow is becoming super critical or whether these, all these flow are remaining sub critical, those issues are there, and right now we are just simply considering that this is changing like this. And then from here, it will be again moving onto this particular side. Let me just talk about this part, because further detail will be… to know further detail, we will be requiring some knowledge of the super critical, sub critical combination, and that will be discussing later.

But when the change in channel slope is there, this flow depth are changing; here also the depth is, here uniform flow depth, and then it is changing gradually in this part y 1, y 2, y 3, and here it is uniform flow say y n. If this length is sufficient to form the uniform flow depth, then ultimately it will come to uniform flow depth, and then in this part, it is rising in this part it is rising; say we can write y 1, y 2, y 3, y 4 like that. For a second section, here it is normal depth then it is coming down like that.

But all these part are gradually varied flow; and when we talk about different type of gradually varied flow, then this part will be one G V F, this part will be another Gradually Varied Flow; this is a draw down curve, this is also a draw down curve, but it is if the name of this gradually varied flow profile, and name of this gradually varied flow profile, two gradually varied flow profile may be different. But right now, just we need to know that gradually varied flow profile can form on the earth surface in a channel due to change in slope, this is one example.

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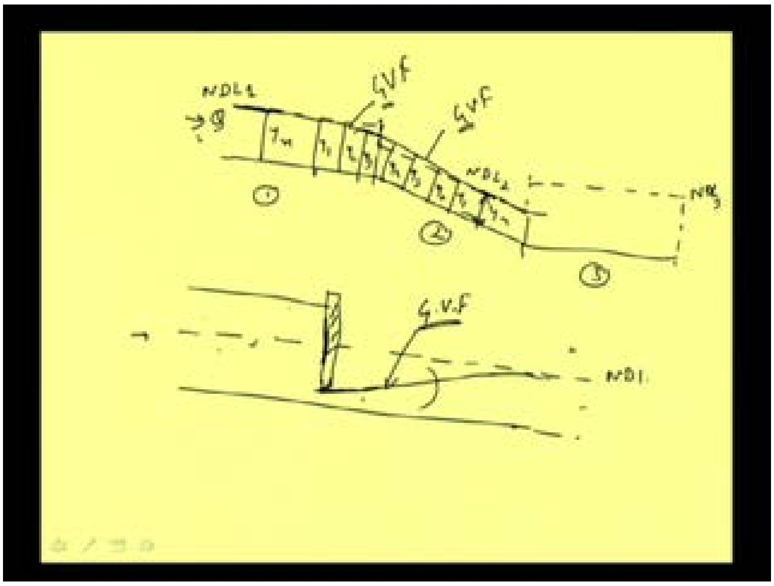

Then another example is say, on downstream of the sluice gate. So, what is sluice gate? Many a time, we provide sluice gate to control or to regulate our flow; say water depth normally is flowing in this level, let me use a dotted line; normally suppose, it flows uniform flow occur at this level. Now somehow for reason, we want to stop this water, so that we are using a sluice gate here; and of course, when we are using a sluice gate, and then we will be releasing the water also when required. So, when we block this one, water level here will be rising like that.

And then when if we close it completely, then of course, water from this side, suppose in some situation, where say there is a downstream channel and on this side; and what this is a tributary, and this is a main river; and from the main river, suppose the at high flood period, water enters into the tributary, we want to avoid that. So, we are putting a sluice gate here; so that when main channel water rises, then we are closing this, and the water cannot come into the sub channel. So, this is one example, where we are using this sluice gate. And there can be different example, and just one example I am stating.

And then when this water level will come down, after flood time, when this main channel river water has come down, then we again release the water, which again, because in this channel, in the small channel also water is was flowing, and there will be a some deposition of water due to rain water and all in this side. So, that water we need to release, once the water level comes down here. So, suppose we are leaving the water, so then we are just opening this gate, and then we are allowing the water to flow.

So, this water will be flowing, and in this channel as we know that its normal depth is this line, NDL is this line; so this water will try to reach the normal depth line from this level and of course, as I have explained or as I have stated to other, as I have not explained, as I have stated that the flow condition here will depend, here means in this portion will depend on the condition whether it is super critical, and that part is sub critical or whether both the part is super critical or whether both this part is sub critical, what is combination depending on that? Sometimes we can have hydraulic jump in this portion, and sometimes we can have gradually varied profile.

And sometimes we can have a gradually varied flow profile rising, and then hydraulic jump is so kind. So, different conditions may be there, but one point is there. On the downstream of this sluice gate, we can always a gradually varied flow profile. It may be followed by a hydraulic jump or sometimes there may not be a hydraulic jump or sometimes instantaneously just after using from here, without any gradually varied flow profile also we can have a hydraulic jump like this. In that case, you will not be getting gradually varied flow profile. So, those details will be coming later, but this is one condition, where we can have gradually varied flow profile. Well, that way we have discussed some of the practical situations, where we can have gradually varied flow profile in our day to day life, in our day to day engineering activity. And then we can move on to the theory of gradually varied flow, well.

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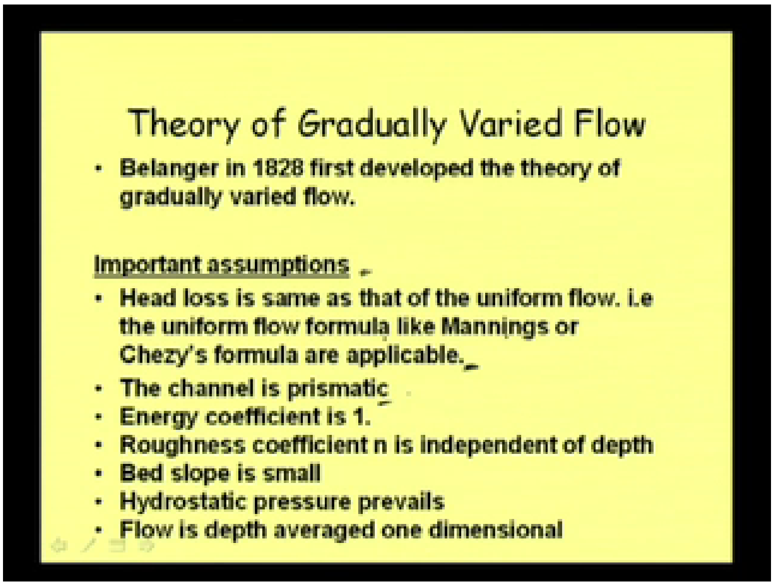

And when we talk about theory of gradually varied flow, then we must take the name of Belanger, who first develop theory in 1828. In 1828, the theory of gradually varied flow was first given by Belanger; and of course, it was for simple situation, and lot of assumptions we meet, and then still this equation is very much useful. Well, now whenever we talk some theory, whenever we talk about some theory, then to develop the theory that means, to develop a mathematical expression for a practical problem or a practical situation like gradually varied flow procedure one one one problem, and we want to derive one equation for that, we always need to make lot of assumption; because if we try to develop one equation for representing a particular flow phenomenon, considering all the practical situation, then it becomes so complex that it may not be possible or we may not have complete understanding of all the processes involved in that.

And why we make those assumptions on you to all those assumptions also; after simplifying the problem also, after simplifying the problem even whatever answers we get that have lot of, that has lot of practical importance; that is with this understanding itself, we can do lot of engineering work. And as such we are always we are always considering some assumption, and then we are developing those theories. But we should be very much aware of those assumptions, because once an equation is given once an equation is given normally we should not use it, and when in tactical field, they apply it;

and sometimes they find that what calculations they are doing is not matching with the practical situation.

Then they must know that what are the assumptions that were there in that derivation of the equation? and then only there is an understanding that is okay, this sort of differences may be there. And when we know the assumptions, then we know the limitation of that particular governing equation. And then we will be able to use it more judiciously, more wisely for our practical application; well that is why, I am just giving some of the important assumptions that are suppose very obvious; and of course, apart from these assumptions, some smaller assumptions are always there, which we are not mentioning here.

First assumption that you can concentrate on the slide that is the head loss is same as that of the uniform flow; well, head loss is same as that of the uniform flow; that is the uniform flow formula like manning's or chassis formula are applicable. Already we have discussed that manning's formula, chassis formula all these are uniform flow formula. Now these uniform flow formula basically deals with the fiction loss that is indirectly coming as head loss. So, these head losses are same as uniform flow, because in gradually varied flow, the depth is changing, and there will be difference in the head losses. But we are considering that head loss is similar to that of uniform flow, and that why for some of the calculations, we will be still using manning's formula and chassis formula in the computation of gradually varied flow. Well, this is one of the assumptions.

Then second assumption is that the channel is prismatic; in that derivation, we are considering that channel is prismatic. Now what we mean by prismatic channel that perhaps we remember, because we have discussed this already that when channel section does not change in the… it does not change in the entire channel reach or along the channel, then and then if thus channel slope is also not changing, so when channel cross sectional slope does not change, do not change, along the section, along the channel reach then we call this as a prismatic channel. So, in the derivation of gradually varied flow formula governing equation in the before for getting the final form, it was assumed that the channel is prismatic; but in real situation, channel will not be prismatic, because a channel can be always like this, channel can be always like this.

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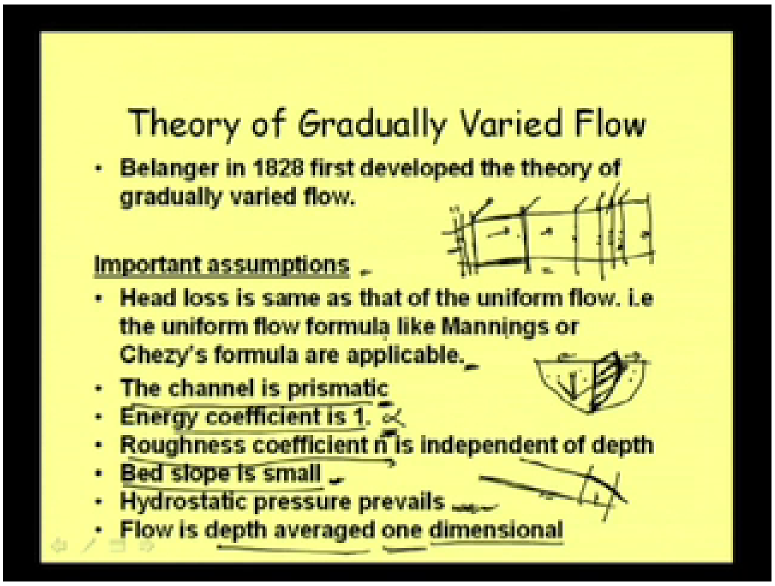

Now say a formula is given, and we want to know this is the plain view I am drawing, this is the top view of the channel, where the channel with here is this much, here is this much, here it is this much, again it is increasing to that much, well. Now if we use this formula directly, and we are suppose computing gradually varied flow in this channel; depth here, and depth here, we want to know; depth here and depth here. But if we do not remember that there is an assumption that channel is prismatic, then we will be ending up doing wrong calculation, because if we just try to use that formula, and if you compute the flow depth between these center it will not (( )), because in this portion channel is not prismatic here, it is changing.

So, what we can do? For our practical purpose, we can consider that from here to here, a small reach we can take; and from here to here were it is gradually even we can make it smaller, and we can consider that from here to here this channel is prismatic, because it is a distance, and change in the for soft cross section is not that much significant, we can take average section. Then from up to this much suppose we are talking like this, and from here to here you can see the channel is more or less of uniform size, and if slope is also not changing, this portion we can consider as prismatic; and from here to here, we can again consider as prismatic; from here to here, we can consider as prismatic. Then again in this expanding portion, we can some smaller section, and we can consider that channel is prismatic within these sections.

So, when we will be using the formula, the governing equation of gradually varied flow, between this section and that section say, between this section and this section, then it will be giving more or less characteristics. We will be using governing equation between this section and that section again, so this will be giving us correct results. And we can use this section and that section directly; then we can apply the same equation between this section and that section, because more or less this part is prismatic.

So, that understanding is very important what assumption we have, this is what the channel is prismatic; then another assumption is that energy coefficient is one; we were talking about alpha below energy coefficient I mean, the velocity coefficient; within velocity coefficient, we have energy coefficient, and momentum coefficient, this energy coefficient alpha is 1; that is the variation of velocity within the section, within the cross section is suppose we are computing gradually varied flow, the velocity may be different here, but we are assuming that this is uniform.

So, in the actual field, if this difference is quite significant or computed result maybe varied from the external result that we need to know. And we need to know that if this variation is small, then well, we are applying with more confident our equation; and if it is quite high, then of course, we need to consider the value of alpha. In the final form, we may not be having that alpha, and if this variation is very high, we need to consider this alpha value. Then another important assumption that this assumption roughness coefficient n, that is a Manning’s roughness coefficient or it may be chezy’s roughness coefficient as well, each independent of depth.

Now, when we were discussing our uniform flow formula, then we very specifically pointed out that Manning’s roughness coefficient varies with the depth; if our depth increases, then our roughness value changes. But for computing the gradually varied flow between two sections, if you apply that equation, we will be considering a single and valued or single roughness parameter; means, that we are violating that very basic condition of Manning’s roughness, that is it changes with depth. But however, we are assuming the change in the roughness parameter is not that significant for the variation that is occurring within the gradually varied flow portion, and that is why we are considering that n value is not changing.

And then of course, if we are aware about that then what n value will have to use, that is the n value for a smaller depth if we use, then it may not be applicable for a higher depth. And similarly if we use the n value for a higher depth that may not be representing very correctly for a very smaller depth; then we can use the n value intermediate between these two. And so, knowing these two assumptions, we should walk; that assumption is there.

Then another assumptions that bed slope is very small, bed slope is very small; generally in natural channel, when we find this gradually varied flow in most of the cases of course. Some exceptional cases will be there, but in most of the cases when we find this gradually varied flow problem, as we have already discussed some of the cases, the bed slope is generally not dead high and we can consider this to be very small. And when we consider bed slope to be very small means, theta is small, and some other considerations are automatically coming like if we remember we talking about the pressure, water pressure, hydro static pressure then also a pressure of the flowing fluid. In steep slope, the pressure is different, the pressure is different, but in flat slope pressure theta if we neglect, this expression is different. So, that is the hydrostatic pressure condition we give.

Now this ultimately leads to hydro static pressure prevails, it leads to the condition that hydro static pressure prevails; and the pressure correction is not coming into picture, but in a flow situation where slope is very high or even, suppose in a gradually varied flow in this portion, suppose this is very small, but the surface slope is very steep, this portion. Here also, in fact, the flow is moving as a curved flow as we discussed in our earlier classes, and there hydrostatic pressure may not be exactly valid; so, we mainly to apply connection here. And similarly when the slope is steep, we may have to apply the correction progression; and if we do not apply, at least we should know that some amount of (( )) we are introducing layer. So, this assumption is also important.

And then we will be deriving this equation for flow depth, considering the flow as one-dimensional and depth averaged; depth averaged means the velocity that we are talking about is the velocity considering the average depth, with the average velocity for the entire depth, because we know the depth velocity here will be more, here will be less, and there may be a velocity profile like this of course, highest velocity may not be at the surface, it will be, generally highest velocity will be little lower than this, that we have already discussed, but that velocity variation we are not considering in this particular derivation.

And then we are having depth, for the entire depth we are considering a single velocity V, which we refer as depth averaged velocity; so velocity averaged over the depth. And of course, this equation is one-dimensional, we are considering two-dimensional, but we can have two-dimensional expression also. Well, with these assumptions being on the background, and understanding these assumptions, now we can move to derivation of the governing equation, well.

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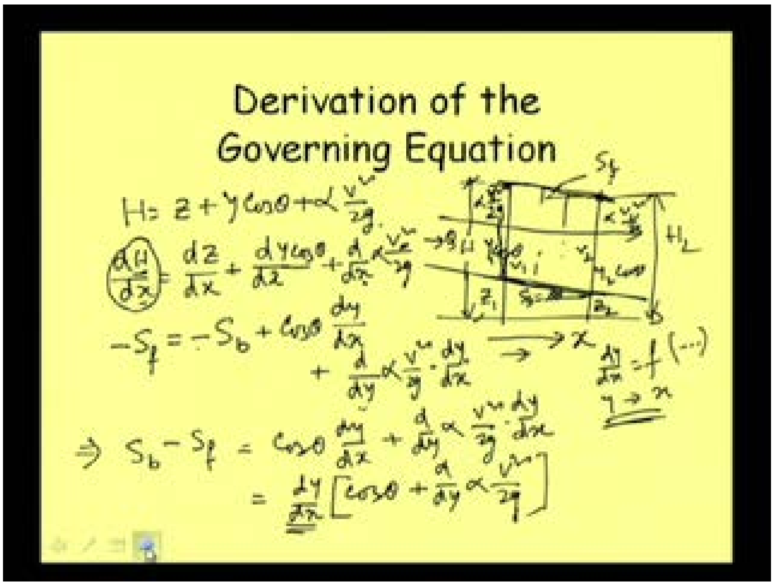

So, let me draw a channel like this, and let us see that this flow is moving like that; and let me write the expression for a section here, and of course, we can have a section here, and this is the datum of course, say uniform, they are changing like that. Then let us see what will be the energy, total energy H, total energy H in a section 1. As we have already written that part, say this is z 1, this is say y cos theta, if this slope is theta, in reality it is y cos theta means, depth if I draw like this. And then it is alpha V square by twice g that part we have already discussed in one of our classes. So, I am not going to discuss this again, but this is alpha V square by twice c.

So, total heat, we can write as z 1 plus y cos theta plus alpha V square by twice g; this is the total heat, this is the total heat at any section, well. To derive that equation, what we can do? We want to see that how this heat is changing with respect to the flow direction.

If flow direction we put here as x, then we can write what is d H d x that is how this H is changing with respect to x. So, we can write it as say d H d x, there is derive taking derivatives with respect to x, this is equal to what we will be getting the d z d x; I am not using the term one, because it is I am writing in general for a particular section, then plus we can write say d d x of y cos theta plus say the d d x of alpha V square by twice g.

Now in this expression, this d H by d x term that is the heat; now if I draw here z 2 y 2 cos theta, this is y 1 cos theta, and alpha say V 2 square by twice g, then here I am getting a total energy level is 2; and then if I join this, then I am getting a line, which we call as a friction slope S f, and this theta in fact, we write as bed slope S b; now this line should be horizontal of course, this is bed slope is b, and this is the slope of the fiction line S f. So, rate of change of heat with respect to x that is nothing but slope of this particular line, because we are drawing this joining the total heat at two different points.

And one point again here we need to mention that if our section, we are considering 2, and heat here, and heat here, we are joining, we are getting a slope. If I take another section in between total heat will be different; and S f between this point to that point and from again this point to that point, both may not be same. So, here we are taking basically average friction slope, because friction slope at this point; and friction slope at this point may be different, but it is varying as our section will be smaller, as this section should be smaller, we can consider this as average friction slope.

And as heat is falling in this direction, heat is falling, because of heat is more, so heat is falling in this direction; so, we are writing this as minus S f minus S f. Well, if we want to give the value of S f somehow, then that numerical value of S f, we need to be positive; because in the equation, we are using the negative sign to indicate that it is falling. So, in the equation itself, we are using this sign concept and so, numerical value when we calculate for S f, this we should calculate positive, this S f, because already we are assigning the sign to that particular, considering the direction.

Similarly, this d z d x again z 1 is dropping to z 2. So, it is also falling, we are considering that it is falling from this direction to that direction; of course, there may be just adverse slope also, but here we are considering that it is falling from upstream to downstream. So, this d z d x is nothing but bed slope, and this also we are assigning a negative sign. So, numerical value of S b when we are considering positive, when it is falling, because we are already putting a sign; so numerical value we need to put positive for falling one; sign is already there; and if it adverse slope suppose the bed is rising like that, numerical value itself we have to put negative; if then only with this minus sign it will be coming as positive that is the rising one.

So, these things we should be careful well; in the derivation what we are doing that if we know then we will be putting the sign correctly. And then we can write this as cos theta, this has nothing to do with x, if we consider the slope is not changing in this portion; then cos theta we can just bring here and we can write d y by d x, plus this particular expression we can write in a form that, because this V square velocity is changing, velocity is changing from section to section here it is V 1, here it will be V 2, velocity is changing from section to section. But these velocity change, we can directly get in terms of y, because depth is changing means depth is changing means we can get it the discharge remaining same, Q we are talking about the same Q. So, we can have with the change of depth, it is velocity is changing.

So, we can get it in the form that we can write it as plus say, let us write this change in velocity in terms of y we can calculate, because x of course, in the direction x it is changing, but how it is changing with x depth, it is not known to us right at this moment. So, let us see how it is changing with y, then we can take it, again multiply it, how y is changing with x. So, that will ultimately give us the d d x of this part. So, what we will, write d d y of alpha v square by twice g into say d y by d x, because we are differentiated with respect to y. So, let us differentiate y with respect to x again, original differentiation our target is with respect to x. So, we are writing this.

Now this equation we can have in this form that S b minus S f S b I am bringing on this side, this equal to say cos theta into d y d x cos theta into d y d x plus say d d y of alpha b square by twice g into d y d x. And this we can further simplify now putting all those assumption; well, this let me write this d y d x, because our target is to write one expression for gradually varied flow profile well; gradually varied flow profile means, if we can get how y is changing, how this depth are changing with respect to x, how this depth are changing with respect to x, that is d y d x; how these are changing.

And then if we can get these in terms of quantity on the right hand side, then definitely we are getting our answer; we are getting a equation, and from that equation, if we solve it, then we will be getting we can interrogate it, we can solve it by different way; if you solve it, we are definitely getting depth at different section. Once d y by d x is known, once d y by d x is known as function or some known parameter, if we solve we will be getting y at different x; and that is what our target; if we know that, then we will be able to solve what type of profile it will be, how the profile is changing.

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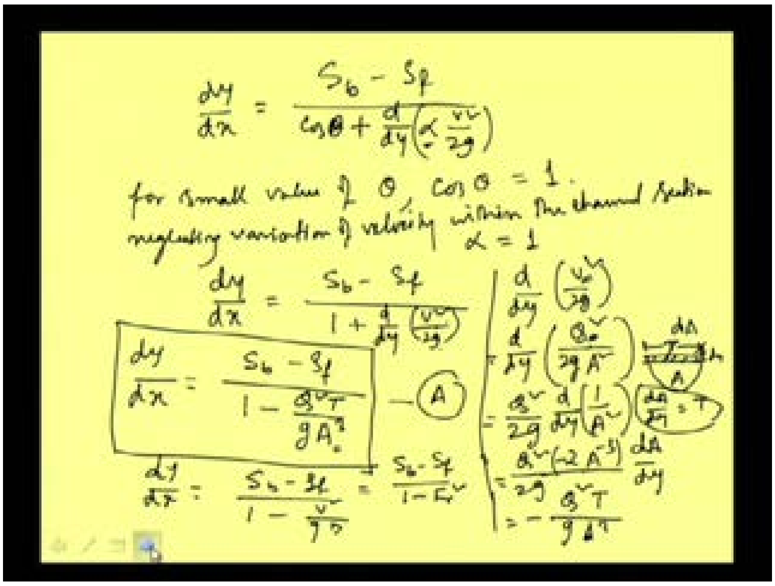

So, let me write it in this form dy dx into cos theta plus d dy of alpha V square by twice g. Well from these dy dx will be, we can write that dy dx, dy dx is equal to S b minus S f divided by 1 sorry cos theta plus cos theta plus d dy of alpha V square by twice g. Now, we can just introduce some of the assumptions that we had, just before derivation of this equation. So, those assumptions are first the theta is very small - for small value of theta for small value of theta. We can have cos theta is equal to. Then again for alpha equal to one, that is we are considering say variation neglecting, variation of velocity within the channel section.

So, neglecting variation of velocity, variation of velocity within the channel section within the channel section, what we can have within the channel section, if we neglect this part, then our alpha is equal to 1. So, our equation reduced to say dy dx is equal to S b minus S f divided by 1 plus d dy of V square by twice g, d dy of V square by twice g.

Now, we can further simplify this expression, let us see how we can write d dy of V square by twice g; so V square, again this V is nothing but depth average velocity.

So, as we know the Q, this V can be written as d dy of say Q square twice g by A square. Now, V is varying with depth, but Q is not varying with sorry V is varying with x and y, but Q is not varying with y. So, Q is constant. So far, this derivation is concerned, so we can write this as d dy of sorry, we can write Q square by twice g is a constant. So, d dy of 1 by A square. Now, this part if we just do the derivation, we can write Q square by twice g, first A square A is definitely varying with y, but of course, we can derive it with respect to A, and we can write dA dy. It is do that. So, that it will be A to the power minus 2.

So, it is minus 2 into A to the power minus 3 that derivation, and then we can write dA dy. And as we know in a section that, if this is the area for a small increase in y, if the change in area is dA, then if top which is T, we can write dA actually dy into top with T is nothing but dA. So, dA by dy is nothing but T. So, putting this as T, what we can write this expression. All these are coming as Q square, this T will be T, 2 and 2 is getting cancel minus sign is remaining here, Q square T by g A cube.

So, putting this expression for d dy of V square by twice g. We can write say, dy dx is equal to S b minus S f divided by 1 minus Q square d by g A cube. And this is, what is called as governing equation of gradually varied flow. This equation number, I am just putting as A, this is very very important expression, and this equation is used for various practical purposes. And of course, this Q square T by g A cube as we did earlier also, we have done several times. This can be expressed in terms of fraud number also, and that is why we can write say dy dx, we can write it in the form of dy dx is equal to S b minus S f divided by 1 minus that is Q square by A square will become say V square. So, we can write it V square, and A by T that will become d. So, it is g d, and that can be written as S b minus S f divided by 1 minus F r square.

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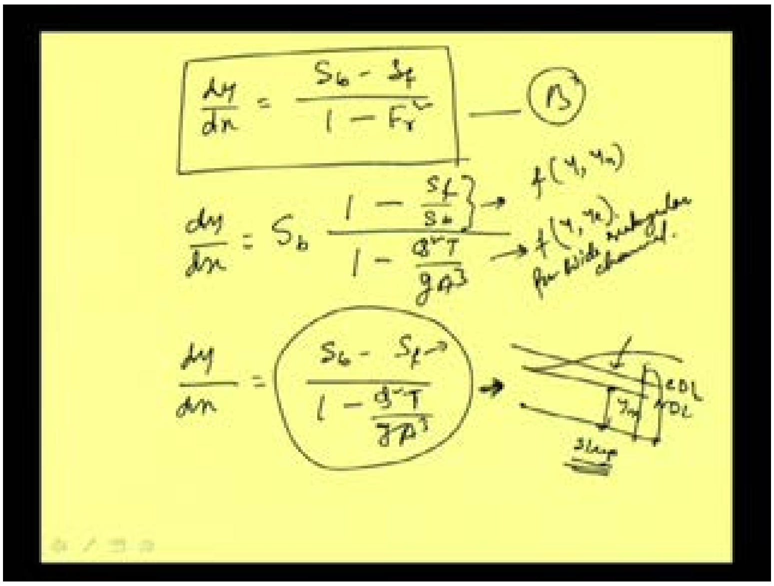

So, that is also another form of governing equation that dy dx is equal to S b minus S f divided by 1 minus F r square. So, this is also one popular expression, I am writing S b. So, that way this particular governing equation, we can have in different form. And this is, this expression for a rectangular channel or for a wide rectangular channel, we can again express it in a different form; in terms of normal depth, and critical depth. So, that will be coming, and that will be discussing many a time, we write this expression as dy dx is equal to…

Suppose S b, we are bringing here and then we write 1 minus S f by S b divided by 1 minus. Let me write it as Q square T by g A cube, and then we can have this S f by S b in terms of as a function of y c and y n. And this also, we can have and in terms of y c and y n no sorry. This this this we can have in terms of y and y n; in this we can have in terms of y and y c. Like that, we can for rectangular channel, these are for rectangular channel we can have it, for wide rectangular channel we can have it.

So, for different channel, we can simplify this particular equation into different form. And then, we can from that we can derive lot of our important understanding or we can see how the flow can be classified - how the gradually varied flow can be computed, and can be classified well. One important point just I want to mention before going to all these, it is this equation dy dx is equal to S b minus S f divided by 1 minus Q square T by g a cube, it is a non-linear differential equation. Because this S f is also a function of y, and from these expression you cannot separate out you cannot separate out, your y value.

That is we cannot solve it directly by integration, if this solution of this equation by direct integration is which occurs, (( )) it is possible, but we cannot solve it directly. And that is why for solution of this equation, we generally take recourse to numerical method, and some other approximate solutions are there. All those different methods existing for solution of gradually varied flow equation, computation of gradually varied flow that we will be discussing in the next class, and as we have already mentioned, there are different classes or classification of gradually varied flow are also there.

So, when the flow is occurring on a mild slope, and we are getting a gradually varied flow then we name it in a different way. When the gradually varied flow is occurring on a steep slope, then we name it in a different way. And sometimes it may happen when we are talking about uniform flow, then we found that when it is a uniform flow occurring, if it is on a steep slope. Then this uniform flow depth y n is always less than the CDL line; NDL is always than the CDL line, but in gradually varied flow we are putting some obstruction here. Though the slope is steep slope, water may be rising like that. And so, it is going to this level, then it is a depth of flow at any point may be more than our critical depth though the slope is steep.

So, based on all those condition, that if A, flow is occurring above critical depth in a steep slope the name will be different. So, in a very systematic way, we classify the gradually varied flow considering all these different different concepts, and then we name them. And then after that we will be going for computation of gradually varied flow. So, next class, we will be doing all those for today this much is sufficient, thank you very much.