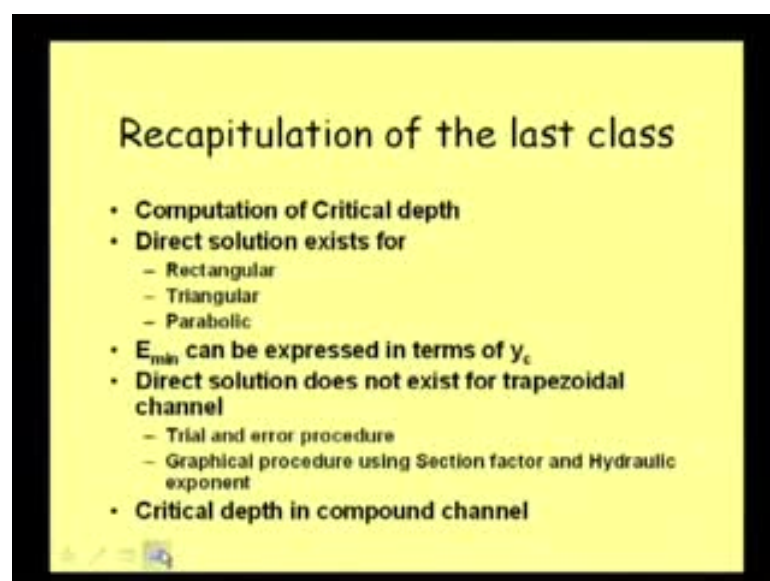


Hydraulics
Prof.Dr. Arup Kumar Sarma
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module No.# 03
Energy and Momentum Principle
Lecture No.# 03
Specific Force, Critical Depth and Sequent Depth

Friends, today we shall be discussing on specific force, what we have already discussed about specific force earlier, and we are talking more about specific energy, in the last class. And today, we shall be talking on specific force, and how critical depth is related to the specific force. And of course, we will be talking about another term or another concept of sequent depth, that we were talking about alternate depth earlier, when we were talking about specific energy, and when we will be talking about specific force, then we will be talking about two depth, that we call as sequent depth well, those thing we will be discussing in this particular class, and before going to this, let us recapitulate what we did in our last class.

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Recapitulation of the last class

- Computation of Critical depth
- Direct solution exists for
 - Rectangular
 - Triangular
 - Parabolic
- E_{min} can be expressed in terms of y_c
- Direct solution does not exist for trapezoidal channel
 - Trial and error procedure
 - Graphical procedure using Section factor and Hydraulic exponent
- Critical depth in compound channel

Well we started our last class with the computation of critical depth, well, and while discussing the computation of critical depth, we could see that direct solution to critical depth exists in case of rectangular section, that is when the channel is rectangular in case of triangular section, that is triangular channel, and in case of even parabolic section also.

So, computation of critical depth, we can say that to some extent, simpler than the computation of normal depth, of course or trapezoidal channel, we do not have direct solution that is the expression becomes us for trapezoidal channel.

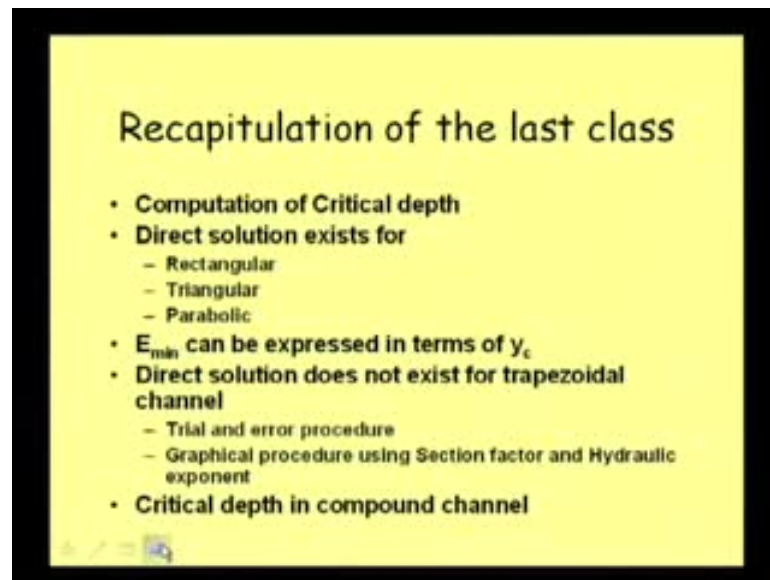
Basically, we start with the expression, that $Q^2 \text{ t by } g A^3$ equal to 1 or we can say that Froude number equal to 1, we start with that, and then we get the necessary expression for computing critical depth. And when we start with that, and we get the expression for trapezoidal section, then we could see that, for that we do not have direct solution, you cannot separate out y_c , that is the critical depth, in terms of other known parameters or other known variables. And that is why computation of critical depth in trapezoidal channel is not direct, rather **we need**, we need different other method, like say we need to go for trial and error procedure or we can go for some graphical solution, and when we go for graphical solution, then many a time, we use section factor concept, like uniform flow computation.

Here also we have a term, that we call as a section factor or we use sometimes hydraulic exponent, and hydraulic exponent, and that we also another procedure is there; of course, details of those procedures, we did not discuss in the last class, and another we will be discussing in this class also, because now a days, this graphical procedure are not that much adopted, because, say now we have computer, and even in calculator itself, we can use some sort of computation, and we can go for iterative procedure very comfortably, and that is why these graphical procedures are not, that much adopted now a days, and trial and error procedure or other method, like some direct solution has also been developed.

Of course, that is not becoming very much popular or that did not become that much of popular, and that is why we are not talking about that direct solution also, and we will be discussing more on the trial and error procedure, and one point here, I would like to say that trial and error procedure in trapezoidal channel is only required.

But when we see the national channel, say national channel, suppose if big river, we talk about, then although it may be trapezoidal, but that side slope is not that high, and for many practical purpose, we can consider that as rectangular, and as such, for most of our computation, we will be considering that as rectangular.

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And if we want to go for considering these as a trapezoidal channel, then we need to go for trial and error procedure, and in a trial and error procedure, we need to first consider one initial value, and what will be that initial value.

See we can consider the value corresponding to rectangular section, for that particular dimension, as the initial value, then we can start with that initial value, and then, after few iteration, we can get the value of trapezoidal channel. So, with this well, then another point we did discuss in our last class, that is about computation of critical depth in compound channel well, that is, of course, to some, we just could discuss a very little of that, because that topic detail discussion on that topic is beyond the scope of this particular course, but still we try to see that how computation of critical depth is a different from the different in compound channel from that of the simple channel.

Particularly, in a compound channel, when the flow depth is or discharge is suppose small and flow depth is remaining in the lower portion of the compound channel, then this is as simple as a simple channel.

But when it crosses the flood plain, when it crosses the flood plain, then only we need to take care of some other issues, and we could even see that, three possible critical depth exist in compound channel, and when the water moves above the flood plain, well, that things we did discuss in our last class.

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Specific Force

$$\text{Specific force } F = \left[\frac{A \bar{z}}{g} + \frac{Q^2}{g A^3} \right]$$

1st term
Pr. $\rho g \bar{z} A$
Pr. force of fluid = $\frac{\rho g \bar{z} A}{\rho g} = \bar{z} A$

2nd term
mass/unit time \times Velocity
 $\rho Q V \rightarrow \rho Q \frac{Q}{A} \rightarrow \frac{\rho Q^2}{A}$
Pr. force of fluid = $\frac{\rho Q^2}{A \rho g} = \frac{Q^2}{g A}$

And today, we shall be moving on to specific force, about specific force, I think we have already discussed in some of our earlier classes, what we mean by specific force, but still just to give, to bring its relation with critical depth, let us start from the equation or expression, that we generated one expression starting from the momentum equation.

Yesterday, when we were discussing momentum equation and continuity equation of flows, then we were talking about specific force, from the momentum equation, it was like that, we take a channel section channel reach, rather than taking a section, we took channel reach, and in this channel portion, we could see, that the force, that is guiding the flow or the force in the direction of flow is one, which is driving the flow, and then the force, the resistance force from the surface or from the bed or the force of air friction, even on the top of the water surface, so all those forces are acting, opposing the force, then opposing the flow.

And then the pressure force on the upstream side and the pressure force on the downstream side, difference of these two forces is also acting in the direction of flow.

And then, we could see from the Newton's equation basically, that force is equal to rate of change of momentum, because of its movement, it has some force, that is the mass into acceleration, so that force is there.

So, equating all those forces, we could derive the momentum equation, and in that momentum equation, if we neglect some of the forces, if we neglect some of the forces, like say if θ is very small, if θ is very small, then the $w \sin \theta$ that is the component of the weight force in the direction of flow that become 0.

Then the friction, if the channel reaches very small, then your frictional resistance in that portion can be neglected; similarly, if air force or resistance due to air is very small, then we can neglect, that force also, neglecting all those forces, we could see, when we equate the forces, then we find that we could get this expression.

Now, you can concentrate into the slide, we could get this expression that $A \bar{Z}$, where \bar{Z} , this \bar{Z} is nothing but the depth after the centroid of that section, and then Q^2 square by $g A$, where A is the flow section, and this A is the cross section of the flow, and Q is the discharge flow flowing through the channel, and this term at the upstream section and at the downstream section were equal.

And this particular term is having the dimension of force, this particular term is having dimension of the force, and this force is also the force of flowing water per unit weight of the water or per unit weight of the fluid. And so, we term this force as specific force, this expression as specific force, and **the** as a definition we can say, that specific force is the force per unit weight of the water of the flowing fluid.

Well, now these two terms, we can view in this way also, just writing directly from this expression, say if the water is flowing like this, then at any section, if we talk about, well say what are the forces, just initially, if we think that suppose water is not flowing nothing is there, still there are water depth.

And suppose this is the section, well let me draw it like this and this is the section, like this section, well so if you just think that water is in static condition, even without considering the flow, then also in this particular section, say section one, and this is the section one, in this section this water, because of its depth, and say it is depth up to the centroid, if we consider as \bar{Z} .

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Specific Force

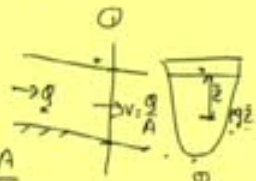
Specific force $F = \left[\frac{AZ}{g} + \frac{Q^2}{gA} \right]$

1st term

Pr. $\rho g \bar{Z} A$
 Pr. force/unit wt of fluid $= \frac{\rho g \bar{Z} A}{\rho g} = \bar{Z} A$

2nd term

mass/unit time \times Velocity
 $\rho g V \rightarrow \rho g \frac{Q}{A} \rightarrow \frac{\rho g Q}{A}$
 Pr. force/unit wt of fluid $= \frac{\rho g Q}{A \rho g} = \frac{Q^2}{gA}$



Then the average pressure, average water pressure in this section will be say $\rho g \bar{Z}$, that is, unit weight of water into \bar{Z} , that is the pressure, average pressure then what will be the force.

When we just consider this even as a still water, then also this force will be there, because there is a pressure, so the force acting in the direction of flow will be this pressure multiplied by the area; so, this pressure will be the $\rho g \bar{Z}$ into the area into the area, so first term, if we just think in this way, that the pressure force, then we are getting this one, and then if we just, say this is the pressure force, and then it is the per unit weight of area pressure force per unit weight of area per unit weight of fluid, **sorry**, per unit weight of fluid. Then, it will be say $\rho g \bar{Z} A$ divided by unit weight means γ or ρg , γ we can write as ρg , so this is equal to $\bar{Z} A$; so, this is what is our first term, it is basically representing pressure force per unit weight of water.

Now, let us think about the second term, the second term, well, we can view this term in this way, say water is flowing, Q discharge is flowing in this direction, and some amount of mass is flowing in this direction, and as the water is flowing, it has some momentum. And we know that, force is equal to rate of change of momentum, if we just try to stop this water, here the velocity will become 0, and change of momentum will be, the momentum, what it was carrying, **because when it becomes stop**, when the water is stop

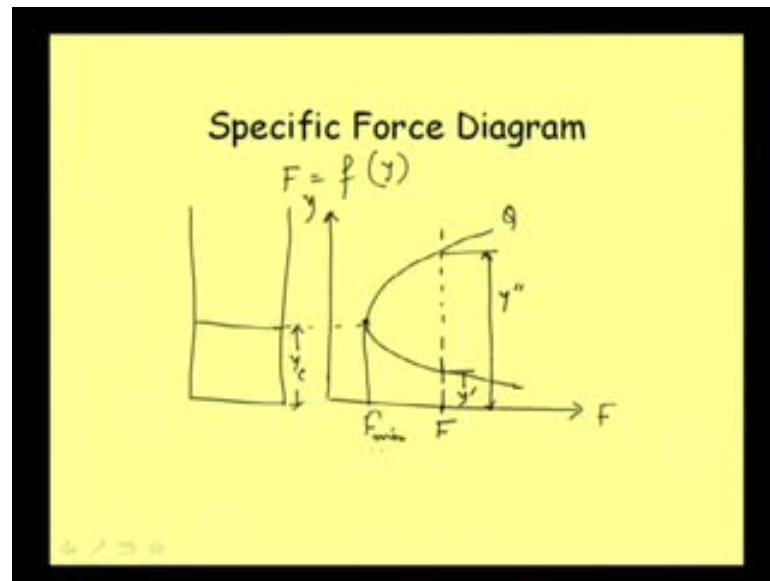
then it becomes 0, so change in momentum is equal to the momentum, that it was carrying, and momentum is always related to the mass and the velocity.

So, now, when we talk about rate of change of momentum, we need to know, what is the rate of mass flowing, that is mass flowing per unit time, so mass flowing per unit time; so, that force we can call as say mass flowing per unit time mass per unit time, and say velocity and velocity. This indirectly give us, say we can call it as mass into acceleration or rate of change of momentum, as I have explained; well, now, this term, what is the mass per unit time, that is Q is the discharge, so in unit time, Q amount of water is flowing, Q amount of water is flowing in this direction.

So, what is the mass, Q is the volume, so multiplied by ρ , so ρ into Q , this is basically the mass flowing per unit time and velocity is V ; now, this velocity V can be written as Q by sectional area; so, this can be written as ρQ , then V , we can write as Q by A , so this can be written as ρQ square by A . Then, these ρQ square by A , we are getting for the entire section, now if we write this expression per unit weight of fluid, per unit weight of fluid, so this second term again per unit weight of fluid, it will be, will have to per unit weight of fluid, if we write, then we will have to divide it by γ , and so, it will be ρQ square by $A \gamma$, γ I am always writing here as ρg so this will be Q square by $g A$.

So, this is what the second term we are getting, and both the term is basically coming as a force term, and these two terms have the dimension of force, and we refer these terms as specific force well. With this, very understanding of specific force, now we can draw a diagram, that is because from the expression it is clear, that Z bar, that is the depth, this is a function of y , if our depth increases or decreases, then this Z bar will change area, whatever maybe that channel section, whether it is rectangular, trapezoidal or whatever may be the same, but this area is a function of y , and this area is also function of y , Q is of course, we are talking about a particular discharge, so it is constant; so, what we can say that, specific force is a function of y , specific force is a function of y dept.

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And that way specific force F is a function of y , and if we plot this diagram, **this**, these two value in a graph, then it will be like that depth y and specific force F , and this diagram will be of this shape, this diagram will be of this shape. And here also, and this is of course, for a particular Q , it is for a particular Q with the increase of Q , again if we change Q , we will be getting **a**, another curve for different Q , we will be getting different curve well.

Then here also we have seen that there is a specific force minimum, that is, minimum specific force well; so, let me draw a cross section, suppose we are drawing this, for this cross section, then this depth which correspond to the minimum specific force is again called critical depth. Of course, we are just stating it, we are just stating it, at this moment, but we need to also establish, that, yes, at the minimum specific force point, it satisfy the other condition of critical depth, that is the Froude number 1, and all other condition what we had earlier, this must be satisfied will see that.

And then, another point, that we can see that, here for a same specific force, that is if our specific force is in this level; then, we get two different depths, two different depths, this is say y' dash, and this we can call as y'' double dash. So, this is what, some of the characteristic of specific force that we have seen; now, as I have already told you, that we have just mentioned, that this is critical depth fine, but already, we did mention that

critical depth is that, where is that, where our specific energy is minimum; well, let us see, whether the same condition, we can arrived from starting from this point or not.

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Specific Force and Critical Depth

$$\frac{d}{dy} \left(A\bar{Z} + \frac{Q^2}{gA} \right) = 0$$

$$\frac{d}{dy} (A\bar{Z}) + \frac{d}{dy} \left(\frac{Q^2}{gA} \right) = 0$$

$$\Rightarrow \frac{d}{dy} (A\bar{Z}) + \frac{Q^2}{g} \cdot \frac{-1}{A^2} \frac{dA}{dy} = 0$$

$$\frac{d}{dy} (A\bar{Z}) - \frac{Q^2}{gA^2} \frac{dA}{dy} = 0$$

$$\frac{d}{dy} (A\bar{Z}) = \frac{Q^2}{gA^2} \frac{dA}{dy}$$

$$\frac{d}{dy} (A\bar{Z}) = \frac{A(\bar{Z} + d/2) + T \cdot \frac{d}{2} - A\bar{Z}}{A \frac{dy}{dT}} = A$$

Diagram: A channel cross-section showing the water surface profile, channel width T , depth d , and the center of gravity \bar{Z} .

Well, now, **this is the expression for**, this is the expression for critical depth, and if our, this I am drawing it again at this point, where our specific force is minimum, at this point, this is y , and this is F , so at this point d/dy of specific force, d/dy of F should be equal to 0, that is d/dy of F , that is differentiation of F with respect to y should be 0 at this point. So, let us see what expression we get from this particular equation, d/dy of $A\bar{Z}$ plus Q^2 by gA , this you can write as d/dy of $A\bar{Z}$ plus d/dy of, say Q^2 by gA is equal to 0.

And we will be doing with this term later, first let me write the second term, it is Q^2 square, it is not function of y , and g it is coming, and A to the power minus 1, that we can write as minus 1 and A to the power minus 2 equal to 0, and this term remaining same say d/dy of $A\bar{Z}$ well. So, this can be written as say d/dy of $A\bar{Z}$ minus Q^2 square, when we were differentiating A , here another term will be coming dy/dA , because we are differentiating it with respect to y , but it is the term is A so dA/dy . And we know that dA/dy is nothing but the T , top with T that we have discussed several time, so we are not going back to that again, and we are writing directly that minus Q^2 square by gA^2 , and this will be TQ^2 by gA^2 , and this is equal to 0.

Now, let us see about the first term, about the first term d of $A \bar{Z}$, and that of course, we can see in a very simply simplified way; suppose, a channel section is like this, and this is the area, and depth up to this point is say \bar{Z} , this is the centroid, and depth up to this part is \bar{Z} , and say depth up to this one is y .

Now, with the little, so what this, this term is basically $A \bar{Z}$, means, this sectional area is A , this entire sectional area is A ; so, $A \bar{Z}$ means moment of this area, moment of this area cross sectional area above the surface, moment of this cross sectional area the surface well. Now, if our, this expression basically physically what it means, if y increases by a small amount dy , that is rate of change of moment of the area above the surface, which the change in y , so if the y is changed by a small amount dy ; let us see how these values will change. Then, if it is changing by dy , then this part is increasing by small amount well; now, say how this moment will now change.

Original moment, that means, when this was not changing, corresponding to y , it is $A \bar{Z}$, that we know now that change situation with that change situation, what will be the moment, it will be the moment of this portion, moment of this original area above this surface, now surface is changing, so above this surface. So that will be, say I mean difference of this two, so after change after change or after increase of depth by dy , what we are getting moment of this is A into $\bar{Z} + dy$, this is, and then, what is the moment of this small area, if this top which is T , then what we are getting moment of this small area above the surface area is equal to, so this is equal to top with T multiplied by dy and distance up to the surface this part will be dy by 2, so this distance is dy by 2.

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Specific Force and Critical Depth

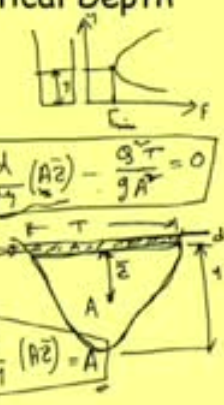
$$\frac{d}{dy} \left(A\bar{Z} + \frac{Q^2}{gA} \right) = 0$$

$$\frac{d}{dy} (A\bar{Z}) + \frac{d}{dy} \left(\frac{Q^2}{gA} \right) = 0$$

$$\Rightarrow \frac{d}{dy} (A\bar{Z}) + \frac{Q^2}{g} \cdot \frac{d}{dy} \left(\frac{1}{A} \right) = 0$$

$$\frac{d}{dy} (A\bar{Z}) - \frac{Q^2}{gA^2} = 0$$

$$\frac{d}{dy} (A\bar{Z}) = \frac{Q^2}{gA^2}$$

$$= \frac{A \cdot dy}{dy} = A \Rightarrow \frac{d}{dy} (A\bar{Z}) = A$$


So, that is what we are getting that after increase of depth by dy , the moment of area above the surface is this one, and then change we can get that, this minus area into Z bar, this is for the original, so what we are writing basically d dy of $A Z$ bar is equal to this is the change, and we want to get it in terms of dy , I mean with respect to dy , how much change, it has occurred due to change in dy .

So, this expression we can again simplify, how, that is a dy , our dy is a very small term, so product of dy that is dy square will be a much, much smaller term, so if we say that, dy tends to 0, then this term, this, where we have product of dy , and dy that term will become very small, and we can consider this to be negligible. So, if this become negligible, then our expression become, this become $A Z$ bar, and this $A Z$ bar, that will cancel each other, and then we are getting $A dy$ by dy this is equal to A .

So, finally what we can write, that d dy of $A Z$ bar is equal to A , so that way in a simple form, we can get it in this from, because dy is very small, so physically also we can just from its very basic concept, we can find this expression A . Well, now coming to our final expression; this expression in place of d dy of $A Z$ bar, we can write as A , we can write as A , well let me see.

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Specific Force and Critical Depth

$$A - \frac{Q^2 T}{2 A^3} = 0$$

$$\Rightarrow \frac{Q^2 T}{2 A^3} = 1$$

$$\Rightarrow \frac{Q^2 T}{g A^3} = 1$$

$$\Rightarrow \frac{V^2}{g D} = 1$$

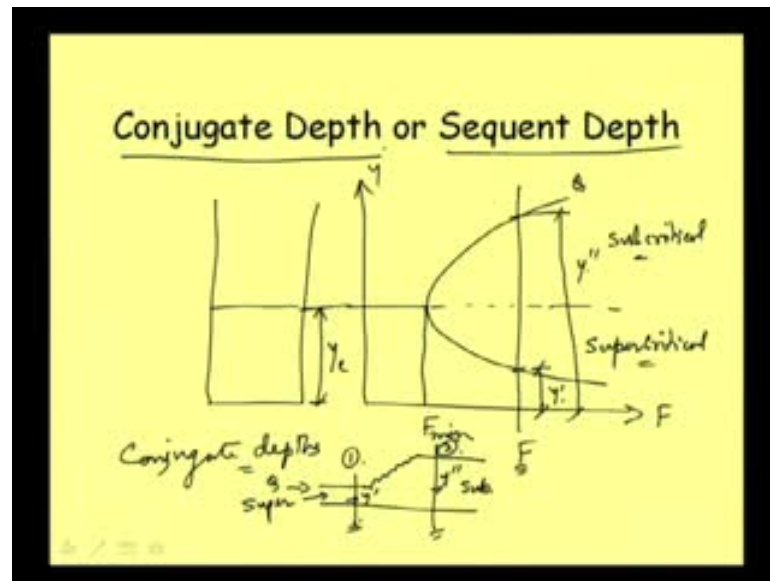
$$\Rightarrow Fr = 1 \Rightarrow Fr = 1$$

So, what we can write that A minus in place of $\frac{Q^2 T}{2 A^3}$ of $A - \frac{Q^2 T}{2 A^3}$, we are writing A minus we got $\frac{Q^2 T}{g A^3}$ is equal to 0 or that we can write as this implies that, $\frac{Q^2 T}{g A^3}$ just changing thus taking the other terms this side is equal to 1. Now, if we just recall our earlier derivation for specific force, minimum specific force, then also we could arrive at the same expression, that is $\frac{Q^2 T}{g A^3}$ is equal to 1, and from this, again we can further do it, that $\frac{Q^2}{g A^3}$ writing A again here, and T here, this can be written as 1.

And then, again this can be written as $\frac{Q^2}{A^3}$ is equal to $\frac{V^2}{g D}$, again A by T we can write as D, hydraulic depth this is equal to 1 or we can write again Fr^2 Froude number square is equal to 1 or we can say Froude number is equal to 1, taking route on both side. So, that way, this is the very basic expression, that we did derive for the condition of critical depth, that is Froude number is equal to 1, specific energy equal to 1 that we got, and from the specific force, specific energy is minimum rather, and from the specific force minimum condition also, we arrive at the same expression.

So, the statement that we made in our last class also, that the condition for critical, that is last two class I think, we were talking about these things, that one condition for critical depth is that, specific force is minimum; now, this is being established here; so, we can now consider this as also one condition, well.

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Then, we were talking about this conjugate depth; so, let me just draw the diagram again, and we have already drawn it, that this is the channel section, so the depth corresponding to F minimum, here we are plotting specific force, and here it is y is called critical depth, that is, now you can write here y_c .

And at any other specific force as not the minimum specific force, but any other specific force which is higher than the minimum specific force for a given discharge, then we are getting two depths already that have been explained y dash and y double dash, why we are writing this symbol y dash and y double dash here.

Basic reason is that, we are writing y_1 and y_2 to represent alternate depth; so, there is a difference between alternate depth, and these two depth. Alternate depth we were talking, when we were talking about specific energy, that for a energy level higher than the minimum specific energy, we get two different depth or say getting a energy level in a channel is possible within one energy level, we can get for two different depth, and these depths were called as alternate depth.

And one important point, one of those depth is in subcritical region, and other depth is in supercritical region, and similarly, here in case of specific force also, we can get other than minimum specific force; for minimum specific force, we are getting critical depth, other than that, we are getting two depth, and here this part is in supercritical portion, supercritical and this is subcritical.

So, we are getting supercritical and subcritical region, and the same specific force F we can get for a depth y dash or for a depth y double dash, and these depths are called conjugate depth, and one of these is in supercritical region, and other is in subcritical region.

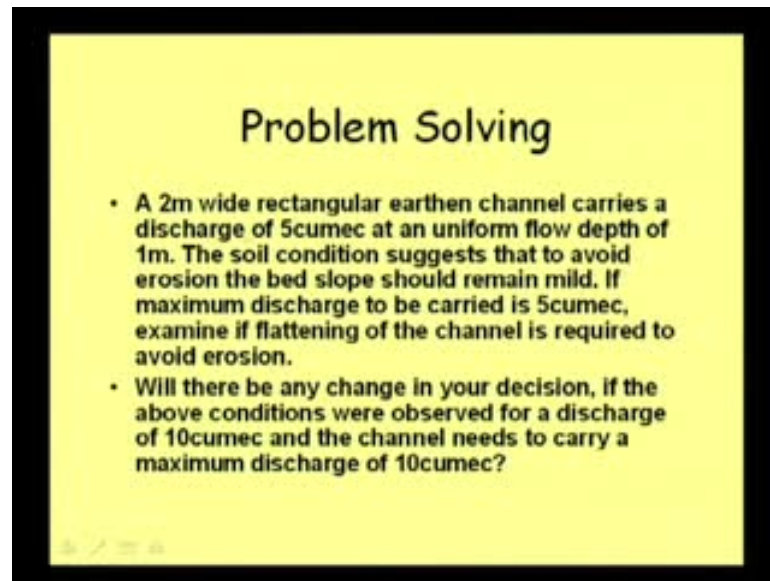
So, this way that we call as conjugate depth, and the same depth, conjugate depth has another name that we call as sequent depth; particularly, when we will be going to our next topic, that is non-uniform flow, then we will be discussing about a interesting phenomenon in hydraulic engineering, **that is called hydraulic jump**, that is called hydraulic jump. And in that hydraulic jump, we find that sometimes water for particular situation, **when**, suppose water is flowing in a supercritical condition here, this part is say supercritical, and then due to some obstruction or whatever may be the reason, the flow has changed to a higher depth, that is in subcritical condition.

Then the flow changes from supercritical to subcritical in a very small length or a small reach, and it goes like this, and lot of turbulence, **and it is**, as if the water is jumping from this point to that point, and that occurs in a very small reach, that detail we will be discussing later, but this sort of conditions occur. And then but the specific force, **if the**, as the channel reach is very small, so we can neglect the loss due to friction in this portion, and as air and length is very small, again air resistance, if it is neglected.

And similarly, if the other terms are neglected, other minor forces are neglected; then, we can say that the specific force, at this point, and specific force at this point, that is the section 1 and section 2 are equal.

Now, for a same discharge Q , if the discharge coming is Q , so for a same discharge Q , if specific force is equal, that means, this depth is y dash and this depth is y double dash, and these are called say sequent depth of hydraulic jump, that is hydraulic jump, initially we are getting this depth, and then finally we are getting this depth. So, that way this conjugate depth are also called sequent depth of hydraulic jump, that we will be discussing, of course in more detail, when we will be doing some computations on hydraulic jump.

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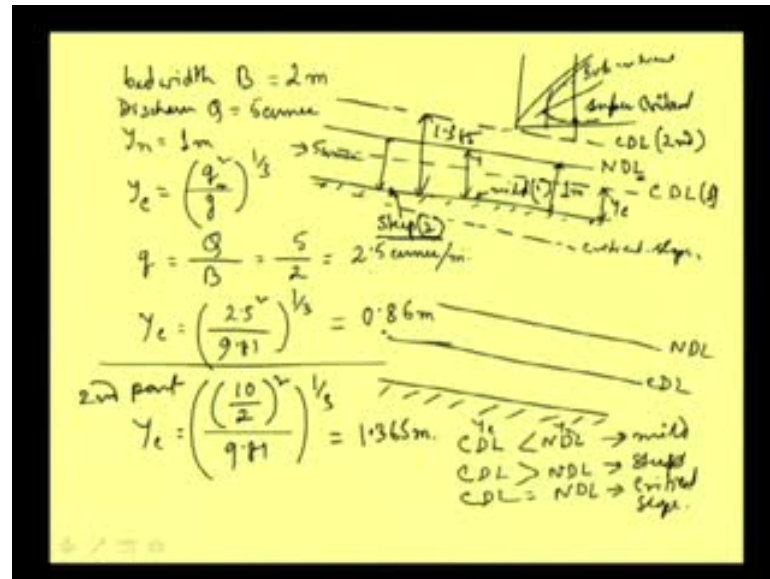
Problem Solving

- A 2m wide rectangular earthen channel carries a discharge of 5cumec at an uniform flow depth of 1m. The soil condition suggests that to avoid erosion the bed slope should remain mild. If maximum discharge to be carried is 5cumec, examine if flattening of the channel is required to avoid erosion.
- Will there be any change in your decision, if the above conditions were observed for a discharge of 10cumec and the channel needs to carry a maximum discharge of 10cumec?

Well, with this we are concluding our discussion on computation of critical depth, and some of the finer aspect of critical depth also we have discussed, but to have more understanding on this part, we will have to do some problem, and with problem solving only we can understand, how these things are applied into our actual work; so, let us take some problem and let us see how we can solve these problems.

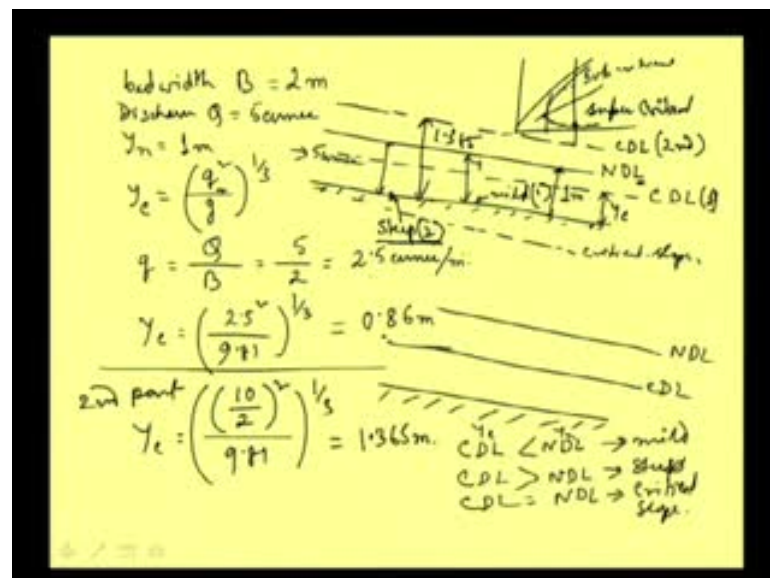
To start with, let us take one problem that a simple problem; first, you can concentrate into the slide that 2 meter wide rectangular earthen channel carries a discharge of 5 cumec, carries a discharge of 5 cumec, at an uniform flow depth of 1 meter.

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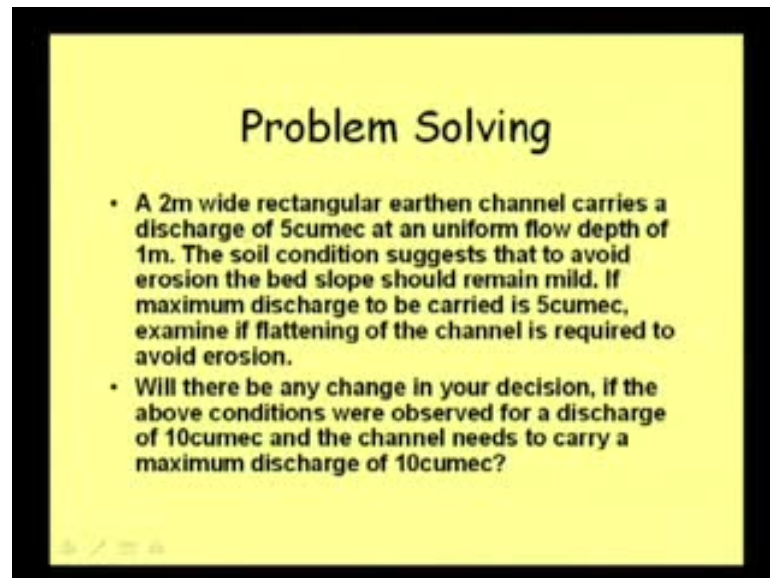
Well, now what these particular statement means, before solving the problem, we need to understand, what the problem is basically stating; so, what this problem means, that we are talking about a earthen channel, we are talking about a earthen channel, and this earthen channel is carrying a discharge of 5 cumecat an uniform flow depth of 1 meter. So, discharge flowing is 5 cumec, and this discharge is moving at the uniform flow, that means, everywhere the depth is same, uniform flow is occurring, and that uniform flow depth, is that, uniform flow depth is 1 meter.

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Now, that means, in this problem, if this is not given, suppose, we were given the bed slope of that channel, suppose if we were given the roughness of the channel, then we could have calculate what the uniform flow is, but now in this problem, uniform flow depth is directly given, that is 1 meter depth is the uniform flow depth for a discharge of 5 cumec; now, what they want to know well.

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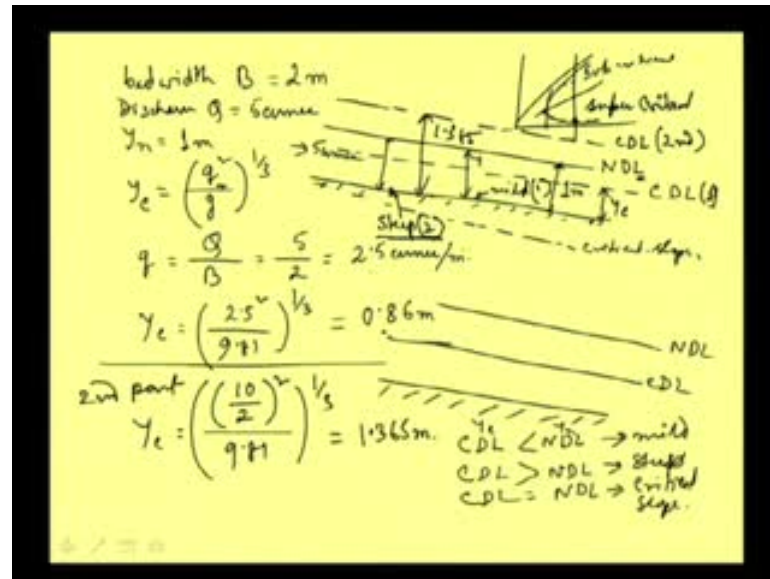


Problem Solving

- A 2m wide rectangular earthen channel carries a discharge of 5cumec at an uniform flow depth of 1m. The soil condition suggests that to avoid erosion the bed slope should remain mild. If maximum discharge to be carried is 5cumec, examine if flattening of the channel is required to avoid erosion.
- Will there be any change in your decision, if the above conditions were observed for a discharge of 10cumec and the channel needs to carry a maximum discharge of 10cumec?

This soil condition suggest that, now why this soil condition is coming, because we are talking about a earthen channel well, so if it would have been a line channel, then this question would not have come, so it is a earthen channel, and that is why, this soil condition suggest, what it suggest that, to avoid erosion, the bed slope should remain mild, why it is.

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Problem Solving

- A 2m wide rectangular earthen channel carries a discharge of 5cumec at an uniform flow depth of 1m. The soil condition suggests that to avoid erosion the bed slope should remain mild. If maximum discharge to be carried is 5cumec, examine if flattening of the channel is required to avoid erosion.
- Will there be any change in your decision, if the above conditions were observed for a discharge of 10cumec and the channel needs to carry a maximum discharge of 10cumec?

If the bed slope becomes steep, well by mild what we mean, and by steep what we mean, that we did discuss earlier also. If the bed slope become very steep, in fact, it is the steep, and mild are technically steep, and mild that means, we are meaning, that, **it is**, it is not literal meaning, but it is a technical meaning, that when our slope is greater than critical slope, that we did discuss in our last class, that is when our slope is greater than critical slope, then it is steep slope.

When our slope is lesser than critical slope, this is mild slope, and when it is greater than critical slope, that is steep slope, the flow velocity it is a same discharge, that you are placing, then the flow velocity will be very high, and when flow velocity is high, its energy will be higher, and then with higher velocity, it can erode the side and bed of the channel.

So, now, the soil condition is such that, if the flow velocity or the flow condition becomes or if the flow condition become supercritical, then there may be erosion, that is why the slope should behave as a mild slope, and again we should know, that a particular slope can be referred as a mild slope for a given discharge.

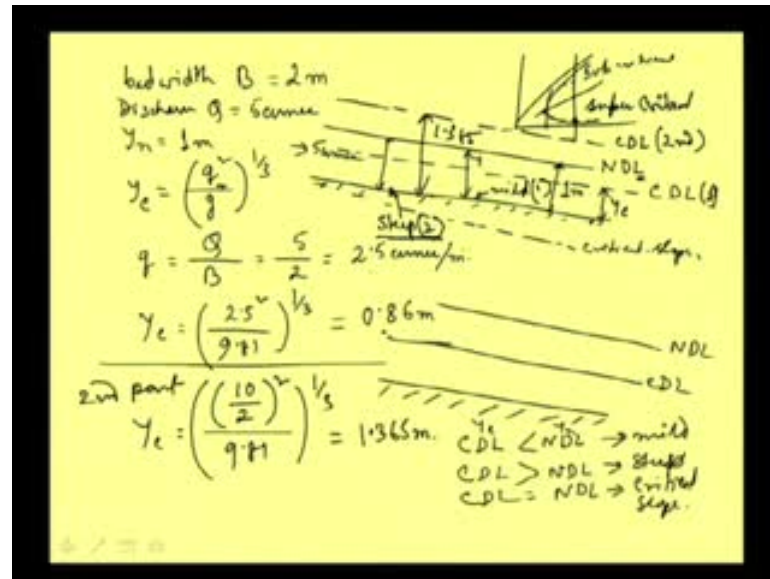
And again the same slope, suppose if discharge increases, then this can behave as a steep slope also, basically we need to see whether for that particular discharge, this slope is behaving as a mild slope or not, whether it is becoming the flow is subcritical or not, so that is the condition, we need to see.

So, the soil condition suggest that to avoid erosion, the bed slope should remain mild; now, if maximum discharge to be carried is 5 cumec, that means, this channel, suppose we are constructing or this channel is being made, to carry a maximum discharge of 5 cumec, examine if flattening of the channel is required to avoid erosion.

Well, that means, now we are observing, that this is carrying 5 cumec of discharge at a depth of 1 meter which is flowing as a uniform flow; now, this channel if it is meant to carry 5 cumec of discharge, maximum of 5 cumec of discharge, sometimes, it may be less than that, so maximum of 5 cumec of discharge.

And then, if the soil condition is suggesting that, this would be mild slope or say the flow should not be supercritical, in that case, to make it mild, should we reduce the slope, that is now existing in natural condition, so that is the problem.

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Well now how to check this, the problem though it looks somewhat complicated, but it is not like that, it is very simple one, say in this given problem, **our first**, we let us mark what are the things, say bed width is given, bed width B is given as 2 meter, and discharge Q is given as 5 cumec, and normal depth y_n what we are observing is 1 meter.

Now, right at this moment or in this condition, whether this uniform flow is moving as a subcritical flow or supercritical flow or whether this is behaving as a steep slope or mild slope, because **it is**, we are talking about total uniform flow condition, in this entire reach, so let us see that, to examine that, let us calculate what is the critical depth, what is the critical depth for this particular channel.

For that what we need, we know, that it is a rectangular channel, so y_c can be calculated as q^2 square by g whole to the power 1 by 3, and what is q value, here q means discharge per unit weight; so, this q is equal to total discharge divided by the width, that is equal to 5 by 2, so this is equal to 2.5 cumec per meter, well this is what our small q . And then y_c critical depth, if we calculate it 2.5 square divided by g is 9.81 whole to the power 1 by 3, then this value we will be getting as 0.86 meter well.

That means, if I draw here the depth critical depth, then I can indicate this by a dotted line, because in reality, we are not having any flow here, but this line is indicating that critical depth line or the depth of critical flow is y_c , and this line, we call as critical

depth line; so, by dotted line, I am representing that, and this uniform flow depth that is y_n , that we write as NDL means normal depth line.

Now, this condition after calculation we are finding, that critical depth is lower and normal depth is higher. Now, if we want, suppose when we can say that the flow is critical, **when our uniform flow would have been**, if uniform flow would have been in the line of critical depth line, then we can call that flow is critical, and the slope which can make the normal depth to flow in critical depth line is called critical slope.

So, to get the critical slope, what we can do, in fact, if we steepen this channel, then only the flow depth will come down, if we just steepen the channel, then only if we make the channel more steep steeper, then only this flow depth will come down, and say we can have uniform flow in critical depth line. As such, now, that means, so in the present condition, this is flatter than the critical slope condition, so critical slope will be little steeper than, that because we are getting critical depth line lower than normal depth line.

As such the present slope is milder, **the present slope is**, we can say that present slope is mild, it is not steep, present slope is mild well. So, mild slope means, we are having depth of flow above critical depth, and that means, it is subcritical flow, if we remember our energy diagram, then we remember that, if this is the critical depth and if the depth is above critical depth, these are all subcritical flow, **and as such, these are in**, that is the slope is mild slope.

So, we can conclude that for a present condition, more flattening of the channel is not required, because the channel is already in the mild slope condition. And of course, from this we can summarize two things, that is when we just directly can refer this as a mild or steep, say if this is the critical depth line, if this is the normal depth line, **if CDL is greater than**, if CDL is less than NDL, normal depth line, it means that, this is mild slope; if CDL is greater than NDL, then it is steep slope, we are not getting this condition, and if CDL is equal to NDL, actually we can write this as CDL means y_c , we are talking about, and this is y_n , if it is equal to these things, then it is critical slope, **critical slope**. Well, now there is another part to this particular problem.

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Problem Solving

- A 2m wide rectangular earthen channel carries a discharge of 5 cumec at an uniform flow depth of 1m. The soil condition suggests that to avoid erosion the bed slope should remain mild. If maximum discharge to be carried is 5 cumec, examine if flattening of the channel is required to avoid erosion.
- Will there be any change in your decision, if the above conditions were observed for a discharge of 10 cumec and the channel needs to carry a maximum discharge of 10 cumec?

Will there be any change in your decision, if the above conditions were observed for a discharge of 10 cumec, if our discharge would have been 10 cumec, that means, say rather than 5 cumec for a 10 cumec of discharge, if we were getting 1 meter of depth uniform flow depth, and if the channel need to carry 10 cumec of discharge, then will my decision change or will our decision change.

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bed width $B = 2\text{ m}$
Discharge $Q = 5\text{ cumec}$
 $Y_m = 1\text{ m}$
 $Y_c = \left(\frac{q^2}{g}\right)^{1/3}$
 $q = \frac{Q}{B} = \frac{5}{2} = 2.5\text{ cumec/m}$
 $Y_c = \left(\frac{2.5^2}{9.81}\right)^{1/3} = 0.86\text{ m}$
2m part $Y_c = \left(\frac{\left(\frac{10}{2}\right)^2}{9.81}\right)^{1/3} = 1.365\text{ m}$

Diagram illustrating the relationship between Critical Depth (CDL), Normal Depth (NDL), and Mild Slope (MS) for a rectangular channel.

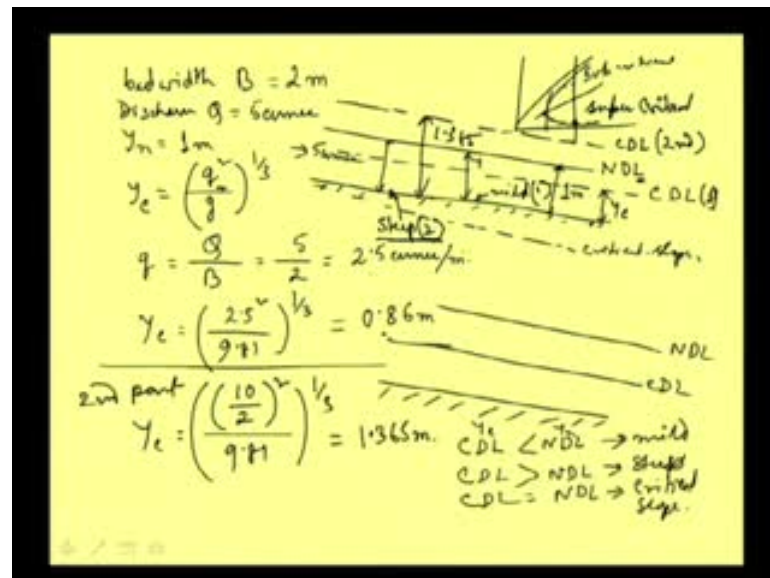
The diagram shows a cross-section of a channel with a bed width $B = 2\text{ m}$ and a discharge $Q = 5\text{ cumec}$. The water surface profile is shown with a depth of 1.365 m at the entrance. The critical depth Y_c is marked at 0.86 m . The Normal Depth (NDL) is shown as a dashed line. The Critical Depth Line (CDL) is shown as a solid line. The Mild Slope (MS) is indicated by the relationship $Y_c < NDL$.

Legend:

- $Y_c < NDL \rightarrow \text{mild}$
- $Y_c > NDL \rightarrow \text{steep}$
- $Y_c = NDL \rightarrow \text{critical slope}$

Well again, here, another point I must mention, that here for this condition, of course, it is not required to flatten, because it is already in mild slope condition; now, our discharge that the channel need to carry is maximum of 5 cumec.

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Now, if it is less than that, will there be any problem, say if q is less than 5, definitely this will be smaller, this value will be further smaller, so y_c will be further smaller means, it will be coming much lower, so NDL will be relatively much higher, what we are getting right now.

As such there is no need of again flattening this channel, for if the our q is less, so we are talking about maximum q is 5 cumec, and that is why, we are checking it for that condition; now, let us check, if it is 10 cumec, so second part say y_c is equal to this time, the small q is equal to capital Q is 10 by b , that means, b is 2 and this is 9 point, this Q square divided by 9.81, we are writing at a time to power 1 by 3, and that value if we calculate, this become 1.365 meter that value become 1.365 meter.

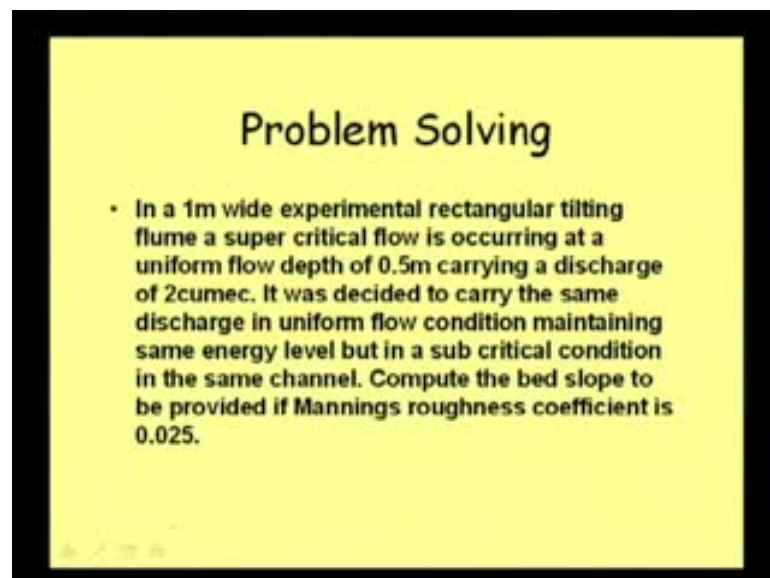
So, what it indicate that this condition will lead to a different situation, that our in second case CDL, this is say first case; in the second case it is 1.365, this is CDL for the second case, this is for the first case.

So, for the second case we have seen, that CDL is higher than the NDL, and as CDL is higher than NDL, means, flow is actually occurring **below the critical depth**, below the

critical depth, means, it is in the supercritical region, critical depth is this one; when it is below the supercritical, critical depth flow is occurring here in this portion; so, it is supercritical flow.

So, for this condition we see that this slope will behave as a steep slope for the second, so mild for 1, and for the second case, it will be behaving as a steep slope, this is for the second case, and as such we need to change the slope or we need to flatten the slope, if we want to bring it to mild slope condition. So, the answer to our question will be for the first case, no, further flattening of the channel is required; for the second case, yes, flattening of the channel will be required, because we are having steeper channel and we are having say supercritical flow in depth.

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Problem Solving

- In a 1m wide experimental rectangular tilting flume a super critical flow is occurring at a uniform flow depth of 0.5m carrying a discharge of 2cumec. It was decided to carry the same discharge in uniform flow condition maintaining same energy level but in a sub critical condition in the same channel. Compute the bed slope to be provided if Mannings roughness coefficient is 0.025.

Well, so let us take another problem; in a 1 meter wide experimental rectangular tilting flume, now, we are talking about a experimental tilting flume, a supercritical flow is occurring at a uniform flow depth of 0.5 meter carrying a discharge of 2 cumec.

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bed width $B = 1\text{m}$
 $y_1 = 0.5\text{m}$
 $Q = 2\text{m}^3/\text{s}$
 $q = \frac{Q}{B} = \frac{2}{1} = 2\text{m}^3/\text{s}$
 $y_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.74\text{m}$
 we have
 $y_c^3 = \frac{2y_1 y_2}{y_1 + y_2}$
 $0.74^3 = \frac{2 \times 0.5 \times y_2}{0.5 + y_2}$
 $\Rightarrow 0.405 y_2 + 2.025 = 0.5 y_2^2$
 $\Rightarrow 0.5 y_2^2 - 0.405 y_2 - 2.025 = 0$
 $\Rightarrow y_2 = 1.16\text{m}$ (not considering -ve value)

Well, so let me draw this condition first, say this is an experimental channel, well by experimental channel what we mean that, this channel we can tilt it is a tilting flow, means, say there will be some hinge portion supported here, and then there will be some jack system or some pulley system, different system can be there, by which we can tilt the channel, we can bring it to any required slope; so, that sort of experimental channel it is. And that, in this channel we are having water flowing at a depth of 0.5 meter, carrying a discharge of 2 cumec, the discharge flowing is 2 meter cube per second.

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Problem Solving

- In a 1m wide experimental rectangular tilting flume a super critical flow is occurring at a uniform flow depth of 0.5m carrying a discharge of 2cumec. It was decided to carry the same discharge in uniform flow condition maintaining same energy level but in a sub critical condition in the same channel. Compute the bed slope to be provided if Mannings roughness coefficient is 0.025.

Well, now the question is, and the channel which is of course 1 meter, it was decided to carry the same discharge, means, 2 cumec of discharge, same discharge in uniform flow condition; again, in uniform flow condition, maintaining same energy level; so, energy at this level, initially when it is flowing with a 0.5 meter depth, then it has some energy 2 cumec of discharge is flowing, so it has some energy, so the same energy level, energy level should not change, and in the same energy level, but in a subcritical condition.

Earlier, say it was in a supercritical condition, now we need to carry the same discharge in the same channel, **in the same channel**, that is width is 1 meter, but we need to carry it in the subcritical condition. Now, how we can achieve this, compute the bed slope to be provided, if manning's roughness coefficient is 0.025, say in the experimental channel, our manning's roughness coefficient is 0.025.

Then, we need to decide or before conducting the experiment, we need to calculate that what slope, at what slope, we must keep the channel; so that, we can have the same discharge flowing through the channel with the same energy level, what it had earlier, but the flow is in subcritical condition, of course, before that we need to see what the critical flow and all.

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bed width $B = 1\text{ m}$
 $y_n = 0.5\text{ m}$
 $Q = 2\text{ m}^3/\text{s}$
 $q = \frac{Q}{B} = \frac{2}{1} = 2\text{ m}^3/\text{s}$
 $y_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.74\text{ m}$
 we have
 $y_c^3 = \frac{2y_1 y_2}{y_1 + y_2}$
 $0.74^3 = \frac{2 \times 0.5 \times y_2}{0.5 + y_2}$
 $\Rightarrow 0.405 y_2 + 2.025 = 0.5 y_2^2$
 $\Rightarrow 0.5 y_2^2 - 0.405 y_2 - 2.025 = 0$
 $\Rightarrow y_2 = \frac{0.405 \pm 1.754}{1} = 1.16\text{ m}$ (neg. considering -ve value)

Well, let us again write down these what are been given; so, here bed width, bed width B is equal to 1 meter, and normal depth y_n is given as 0.5 meter well and Q discharge is given as 2 meter.

Now, if we calculate the critical depth, then again we need to know unit discharge q is equal to Q by B that will remain same, because this is 2 meter, this is 1 meter; so, it again remain 2 meter cube per second per meter. Then, what is our y_c , critical depth y_c is equal to Q^2 by g whole to the power one-third, we are always writing about rectangular channel, we are talking about rectangular channel; of course, our channel can be triangular or channel can be trapezoidal, in those case, in trapezoidal, we need trial and error procedure, but in triangular channel, we do not require that, and that of course, this expression Q^2 by g to the power one-third that will be different, that will be different; so, y_c is equal to Q^2 by g to the power one-third, and that value will become 0.74 meter, that value is becoming 0.74 meter; so, here our critical depth line is above the normal depth; so, this is CDL, this is NDL, 0.74 meter.

Well as our CDL is higher than the NDL, so it means, the flow what is occurring is supercritical flow that is, of course, given in the problem, in the statement itself it is mentioned.

Now, we need to calculate how much slope we need to provide, at present what slope is there, that is not known, we are not been given that information, and that is not even required also, we need to know that, at what slope we must put it, so that, this water will be flowing with the same energy, carrying the same discharge but at subcritical condition. That means, we are talking about this depth y_2 , we are talking about this depth y_2 , we want to find what is the y_2 , we know now the y_1 , y_1 is known to us, and also we know the y_c .

And if you remember, that we did discuss earlier, that there is a relationship between y_c , y_1 and y_2 , that we have discussed in our last class. So, using that expression, taking advantage of that expression, we can calculate what will be our y_2 , because y_2 is the depth y_2 is the depth, which occur for a same discharge in the same energy level, this is the energy level, same energy level same discharge, but in subcritical condition, when this is in supercritical condition. So, we want to know this y_2 first, well, for that we have this relation, that we have say y_c^3 is equal to twice $y_1^2 y_2^2$ divided by $y_1 + y_2$.

Sir, (()).

Administration, (()) , ok.

So, this y^3 is equal to this 1, and then we know already y , so we can write 0.74^3 is equal to $2y$, y is already known to us, that is equal to 0.5 , so 0.5^2 , we want to know and this is $0.5y + 2$. At this expression, if we simplify, it will become $0.405y + 0.2025$ is equal to $0.5y + 2$ square, we can just cross multiplication, and then finally writing this in this expression, then this will take a form of quadratic; this is basic in a quadratic equation form, and we can write it in this form, that $0.5y^2$ minus $0.405y$ minus 0.2025 is equal to 0 .

And this will lead to y is equal to rather y^2 , y^2 it is a quadratic equation, so y^2 is equal to, if we write that and we will be getting 0.405 plus minus 0.754 , well that means, say it is minus B plus minus route over B square minus $4ac$, that part is giving this one.

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$bed\ width\ B = 5m$
 $y_1 = 0.5m$
 $Q = 2m^3/s$
 $q = \frac{Q}{B} = \frac{2}{5} = 0.4m^2/s$
 $y_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.74m$
 we have
 $y_c^3 = \frac{2y_1y_2}{y_1 + y_2}$
 $0.74^3 = \frac{2 \times 0.5 \times y_2}{0.5 + y_2}$
 $\Rightarrow 0.405y_2 + 0.2025 = 0.5y_2^2$
 $\Rightarrow 0.5y_2^2 - 0.405y_2 - 0.2025 = 0$
 $\Rightarrow y_2 = \frac{0.405 \pm 0.754}{1} = 1.16m$ (not considering -ve value)

Now, this depth y_2 we know from physical consideration, that this cannot be negative, if we consider a negative sign, this will become minus 0.35 something, so that way, **this cannot be**, so we will have to consider the positive sign, so this is equal to this y_2 is equal to 1.16 meter, considering or rather we can say ignoring or not considering negative value, because it is practically not possible, so we are not considering the negative value. So, this is our depth, so one point is already known, that y_2 , what we are getting is 1.16 , when y_1 is equal to 0.5 .

Now, our next task is to find out what bed slope, we will have to adjust at what slope, we will have to put our channel, so that, we can get a depth of 1.16 here, we need the computation of normal depth or uniform flow using manning's equation.

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bed width $B = 1 \text{ m}$
 $q = 2 \text{ m}^3/\text{s}$
 $q = \frac{Q}{B} = \frac{2}{1} = 2 \text{ m}^3/\text{s}$
 $y_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.74 \text{ m}$
 we have
 $y_c^3 = \frac{2 \times 0.5 \times y_2}{0.5 + y_2}$
 $0.74^3 = \frac{2 \times 0.5 \times y_2}{0.5 + y_2}$
 $\Rightarrow .405 y_2 + .2025 = 0.5 y_2$
 $\Rightarrow 0.5 y_2 - .405 y_2 - .2025 = 0$
 $\Rightarrow y_2 = .405 \pm .754 = 1.16 \text{ m}$ (not considering -ve value)

Because already we have done this, and from that, say now our required depth, so for the second phase of the problem, say our required depth y what we need to calculate, this is equal to 1.16. Then, in the channel discharge flowing is already given discharge flowing is 2 cumec, that will have to be like that, and then B bed width of the channel is also given 1. So, what we can have that relation Q is equal to, again manning's n is also given as 0.025.

So, using all this expression, Q is equal to 1 by n A R to the power 2 by 3 S to the power half, because it is uniform flow, we are writing S to the power half, and this implies that, S is equal to you can write Q square, then n square A square divided by A square R to the power 4 by 3 . Now, for this channel with a depth of 1.16, our area is nothing but B into y so B into y , means, it is 1 into 1.16 , so this is equal to 1.16 , then perimeter is equal to bed width is 1 plus twice of y , that means, 1.16 ; so, that way perimeter is 3.32 meter, then hydraulic radius R , hydraulic radius R is equal to area by perimeter that will become 0.35 .

Well, now getting all this information, if we put this value here, then we will be getting S is equal to say 2 square, 2 is our discharge, 2 square n is equal to 0.025 square area, as

we are getting 1.16 square, hydraulic radius we are getting 0.35 to the power 4 by 3, and this will give us a value S_b is equal to 7.53×10^{-3} , and this of course, we can write as S_b as 1 in 133; so, this is the bed slope that we need to put.

Now, if we put the channel, if we exhaust the channel bed to be in this ratio, that 1 in 133, then we will be getting the flow depth of 1.16 meter, in that channel of 1 meter width, when a discharge of q is flowing, and this depth will be flowing in the subcritical region.

Now, here if we want to check, if we want to check, whether the energy level E_1 is equal to E_2 , that also we can do, say as now, we know the depth of flow energy level at E_1 , we can write that $y_1 + \frac{V_1^2}{2g}$ by $y_2 + \frac{V_2^2}{2g}$. So, V_1 we can calculate q is known to us and at y_1 level, that is the y_1 level, we know the area because depth is known width is known; so, you can calculate area, so you can calculate velocity.

And then, on the right hand side, it will be $y_2 + \frac{V_2^2}{2g}$ that V_2 also we can calculate for this one, here our kinetic energy component will be less V_2^2 square by twice g , because our velocity is less, but this pressure energy term will be more because y_2 is more. So, here, this will be more y_1 is more, because velocity will be higher, and this will be less because our depth is less, that is a pressure component will be less, but both will have to be equal.

And this, if we check, we will be finding that both are equal, and I am leaving this task to you, so that, you can check it, and you can know that, yes, what we are doing is correct, and then, we will be developing more confidence; this is the two simple problem, we have discussed, and we will be going to more and more problem, when we will be taking up some more problem, when we will be discussing computation of gradually varied flow, in our next module, in our next class, when we will be going for non-uniform flow, there we will be discussing gradually varied flow, there we will be discussing hydraulic jump.

And then, we will be seeing that how this computation of critical flow will help us in understanding the condition, that is, whether there will be a hydraulic jump or not, what sort of gradually varied flow will be there; in all those calculations, computation of critical flow will be very, very important. So, thank you very much, we will be meeting in the next class again.