Hydraulics

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Module No. # 03
Energy and Momentum Principle
Lecture No. # 02
Computation of Critical Depth

Friends, today we shall be discussing about computation of critical depth. We have already discussed regarding how to compute normal depth and also we have discussed about the concept of specific energy. Today, we shall be going to see, how we can calculate or how we can compute the critical depth, and how these things are coming into use in our different hydraulics calculations. Well, before going to this, let me just recapitulate what we did in the last class.

Well, we started with concept of specific energy. That means, just to recall, that specific energy is nothing but energy per unit weight of flowing fluid, rather per unit weight of flowing fluid with respect to channel bed. So, this is what we call as specific energy. Then, we did discuss about the specific energy diagram. Well, this specific energy is a function of the depth. So, if we plot the depth and the specific energy that diagram basically we call as specific energy diagram.

Well, then we did discuss about another diagram that we call as a depth discharge diagram. Well, for a given specific energy, we can have a relationship between the discharge and the depth. So, that diagram, that is, for a constant specific energy, we get a single line for depth and discharge a curve, we get rather and then again for a different energy level, we get different lines, a set of curve we get that we call as a depth discharge diagram.

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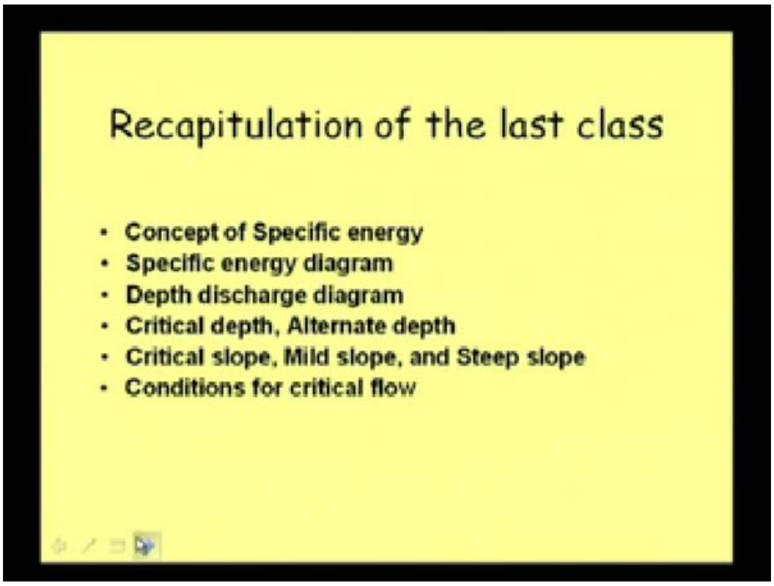

Well, then again we did discuss about critical depth, which is very important. Today, we are going to discuss about the computation of critical depth. So, critical depth is the depth which corresponds to the minimum specific energy. So, we got specific energy and then, we could see that for a particular depth of flow, for a given discharge, the specific energy in a channel become minimum. That particular depth is called a critical depth. Well, then we got another term that is called alternate depth. That also, we did discuss.

What you mean by alternate depth that for a particular specific energy. That means, in a channel, we can get a specific energy for two different depths. We can have same specific energy and these two different depths are called alternate depth. I mean, one of these two depth is in the super-critical region and another, is in the sub-critical region. So, when the flow is in sub-critical condition, then for a particular depth, suppose we are getting a specific energy. Then, say for the same specific energy, we can get another depth when the flow is in a super-critical region. So, that way these depths are called alternative. So, that also we did discuss in our last class.

Then, we talked about another term that is called critical slope. Well, when a channel is carrying a particular discharge Q, then for a particular slope, we can have uniform flow for a particular slope. We can have uniform flow at critical depth; we know that critical depth is already defined. So, we know that this is that critical depth and in a channel for a particular slope, we can adjust the slope. We can have a slope where we are having uniform flow for a particular discharge and that uniform flow depth, is equal to the critical depth. So, that particular slope we refer as critical slope. That means, critical slope is the slope where uniform flow occur with critical depth. Well, now when the slope of channel is flatter, then the critical slope, we call this as mild slope. When the slope of the channel is steeper, then the critical slope, we call that as a steep slope. So, that way we got critical slope, we got mild slope, then we got steep slope.

Well, after that, we did discuss the condition of critical flow. That means how we will understand that flow is critical. Well, we can define the minimum specific energy, means it is critical flow, but how we will understand. So, we need some definite index to show that it is the critical flow condition. Basically, one condition is definitely that when specific energy is minimum for a given discharge. That is called critical flow condition. Then, another condition is that the when Froude number, suppose in a channel, we are observing the velocity, we are observing the depth and then we can very easily calculate the Froude number. That we all know what is Froude number.

So, when that Froude number is unity, then we call, that is the critical flow condition, that we did derive in the last class. Similarly, we could see another point also for a given specific energy discharge is maximum at critical flow condition. So, that is also another condition. Then, we got another point related to that kinetic energy is half of the hydraulic depth. So, that sort of relation also, we did obtain and then we got another point, that is, of course, we did not discuss in detail that point, but still we just marked that point. That is specific force. Of course, specific force is minimum at critical flow condition. So, all those different conditions, we got for critical flow condition and which indicate that the flow is critical.

Now, after that we came to computation of critical depth rather computation of critical flow, means computation of critical depth. So, we started with that. We rather end up our last class with that computation of critical flow. There we could see that for a trapezoidal channel, if our channel is of trapezoidal shape, then we could see that we get relations which do not permit us to calculate the critical depth directly or explicitly. We can calculate and in that case, we need to go for trial and error procedure. So, let us start from that particular point, that is, computation of critical depth. So, you can concentrate into the slide.

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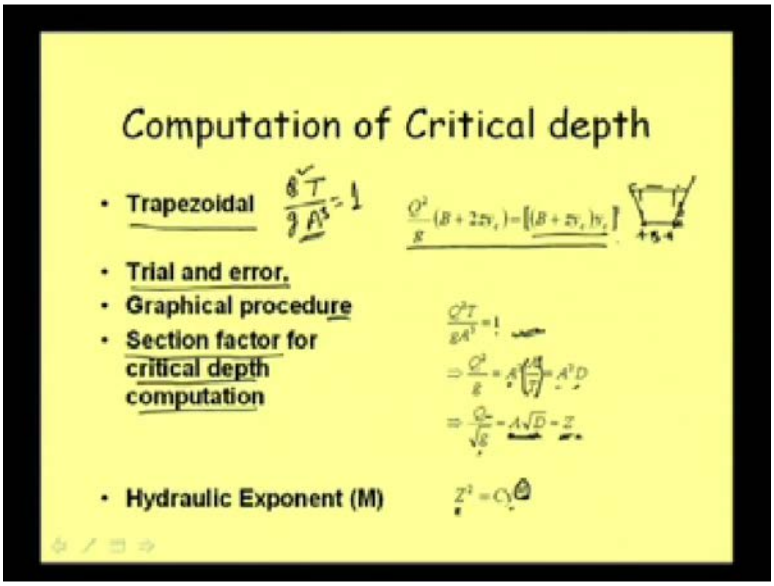

When it is trapezoidal channel, then we know that very basic relation for critical depth is Q square T by gA cube. This is equal to 1 that we did derive in the last class. If it is a trapezoidal channel like this, then we get that Q square T, that is, the top width T. If the side slope is z, then the top width T is B. That means the bottom with B plus twice of ZY c twice of ZY c. So, this part and that way, we are getting this Q square T by g and A cube. If we write on this side, then area is B plus ZY c into Y c cube, so A cube. So, that is what we are getting and from this equation, it is clear that we cannot have this in the form that Y c is equal to some other known term. So, we cannot calculate it directly.

Well, what we can do then for this sort of trapezoidal channel? We need to go for trial and error procedure. We put some critical depth value and we try to see whether the left hand side and right side is matching or not. Then, if we keep on changing this one and we try to get the value of Y c by trial and error procedure, well that procedure is definitely tiresome and time consuming. Of course, now with the advent of computer, we can get it very easily, but otherwise, if we go for hand calculation, this is really problematic. That is why some graphical procedures were also developed.

So, we have some graphical procedure and then in the graphical procedure, we use a term that is called section factor. In the computation of normal depth also, we got a term called section factor, which is basically coming from the sectional dimension of the flow section. Here, this section factor also we use, but this section factor for critical depth computation is not the same expression, that we got for normal depth. It is different expression. So, if we start from this relation Q square T by ZA cube equal to 1, then we can write it that Q square by Z equal to A square into A by T. So, that is given us. We know that A by T is nothing, but hydraulic depth D. So, as A by T is equal to hydraulic depth T.

So, we can have, that is equal to A square, that is area square into hydraulic depth. That we can write as Q by root over g taking, I mean square root of both side, we can write as Q by root over g is equal to A root over D. This particular expression is called section factor Z for critical depth computation using this section factor. In fact, we can find this section factor, if we know the Q and g is, of course, known. So, once Q is given, we can find this value. Then, we have some graphical platform which we could find the critical depth value. However, nowadays these are not used. Utmost, the basic reason is that we have computer. We can go for trial and error method very easily and that is why, these methods are not having that popularity as it had earlier.

So, then similarly from this expression, we can see, we can rewrite this expression. The Z is equal to a root over D. A is also a function of Y and D hydraulic depth is also a function of Y. So, that way, Z square we can write that A square into D. What we had? So, that we can write as C, I mean 1 co-efficiency, then Y to the power M. This is the depth. So, that way, from this relation, we are writing it in terms of one exponent M. That is called hydraulic exponent M for critical depth computation. Just as an information, I am just sharing these things that this sort of relation are using this hydraulic exponent M, then plotting it. This expression again in graphical form, we could solve for the critical depth, but as I am telling that nowadays, we have other easier procedure. So, we normally, do not go for this sort of graphical procedure.

Well, then from this particular slide, we could see that computation of critical depth, of course, we are talking about the expression what we are getting for trapezoidal channel. For this sort of channel computation of critical depth, we cannot get directly and that is why we are talking about all these graphical procedure or trial and error procedure. Is it the same situation for other shape of channel? That is, if our channel is say rectangular, if our channel is say triangular, then what will happen? So, let us just see, what sort of expression we get for solving critical depth in these different types of channels.

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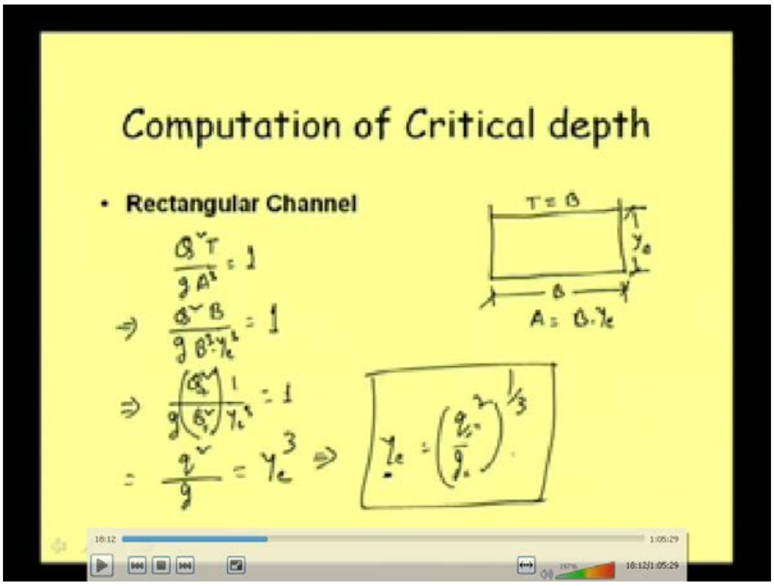

First, let us talk about rectangular channel. So, this is a rectangular channel. Then, say this is B bed width. In D, if this depth is Y c, that is the critical depth. Now, starting from the very general relation, Q square T by gA cube is equal to say 1. This is a general expression for critical depth. So, we can start from that and here, our top width T is nothing, but equal to the B. So, what we can write that this implies Q square. Then, we can write it as B and g and area is nothing, but B into Y c. So, we can write this as B cube into Y c cube. This is equal to 1.

So, this area cube and from that we can write it in a different form. So, this B cube and B is there. So, it can be written Q square by B square and g, let it remain like that 1 by Y c cube equal to 1. Now, this Q square by B square, that term can be written as in the term of unit discharge. So, discharge per unit width, that is Q is the discharge. So, per unit width means Q by B. That can be written as unit discharge and symbol used for, that is small q.

So, this can be written as Q square by g and that can be written as equal to Y c cube. This implies that Y c is equal to Q square by g whole to the power 1 third. So, that way for a rectangular channel, we can see that we can calculate the critical depth Yc directly. If we know the Q value, small q or say capital Q and if we know the B value bed width B, then we can directly calculate this critical depth Y c is equal to Q square by g whole to the power 1 third.

Well, one important point. Though, it is obvious that we need to mention or rather, if we mention this is better that for a channel, if someone ask that a channel is getting this much of discharge and it is having this much of slope, what is the critical depth. Normally, when someone asks to compute critical depth, many student I have seen that things about the slope here, basically critical depth is not depending on the slope. So, critical depth is a not a function of slope. Rather, we can calculate the critical depth as in terms of Q and this g value. So, that way critical depth can be calculated directly. Well, now let us see another relationship.

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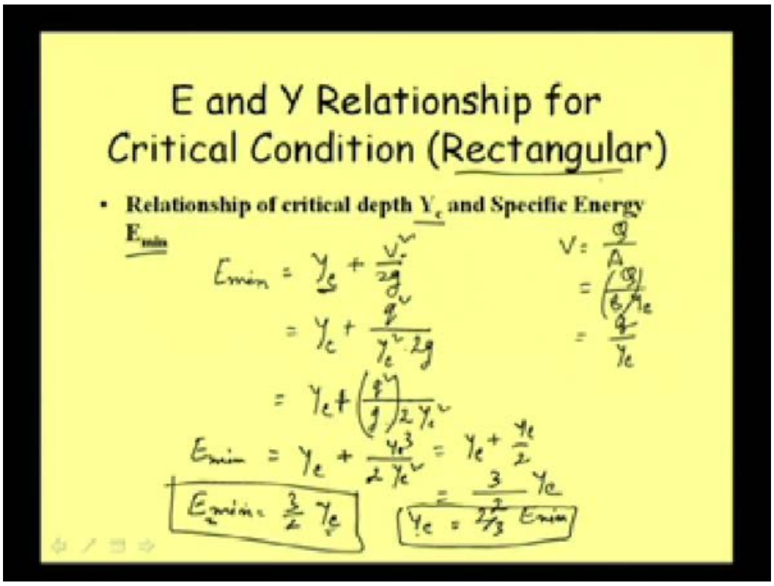

Another relationship at critical condition, suppose the channel is flowing in a critical, so water is flowing in a channel with critical condition or at a particular section. We have a critical depth. Now, what is the energy at that point? We know that energy or the specific energy is minimum, that is known, but I mean, can we express that energy in terms of that critical depth, because we know that energy is a function of Y, depth Y.

So, at critical condition, can we have a definite relationship between this specific energy and the depth? Well, let us just explore that. So, in this slide, we will be discussing about, that is E versus Y relationship for critical condition and this we are discussing for, say rectangular channel as we are starting with rectangular channel. Well, we know that, that is why, say relationship of critical depth Y c and specific energy, we are meaning, but specific energy we can always had as E minimum because this particular condition, we are talking about minimum specific energy because it is the critical flow condition.

Well, so, we can start like say E minimum is equal to Y c plus v square by twice gY c plus v square by twice g as we are writing E minimum. So, we are writing here Y c. We are writing here, critical depth. Well, then V square that we can write as, V is nothing, but equal to Q by area and for rectangular channel this is nothing, but Q by B into Y c. This is always critical depth. So, this is equal to, we can write as Q by B. That part, we can write as Q by Y c. So, this part, we can write this is equal to Y c plus V square. We can write as Q square by Y c square and then, it is twice g. From this expression, what we can write that Y c plus, we had one expression Q square by g. So, I am keeping it like Q square by g.

If we go to our previous slide, we could see that q square by g. This expression Q square by g is nothing, but Y c cube. So, this Q square by g, we can write as Y c cube. This is plus. So, this is equal to Y c plus Y c cube by 2Y c square and that will lead us to, say Y c plus Y c by 2T Y c plus Y c by 2. All these we are writing about E minimum. So, this is nothing, but we can write that 3 by 2 into Y c.

So, Yc minimum, we can write as E minimum is equal to 3 by 2 into Y c or this can be written in the form that Y c is equal to 2 by 3 E minimum. So, once we know the energy E minimum, then we can calculate the Y c directly from this or if we know the Y c, we can calculate E minimum directly from this. So, this relation shows that, there is a definite relation for critical depth computation or for computation of minimum specific energy in critical condition.

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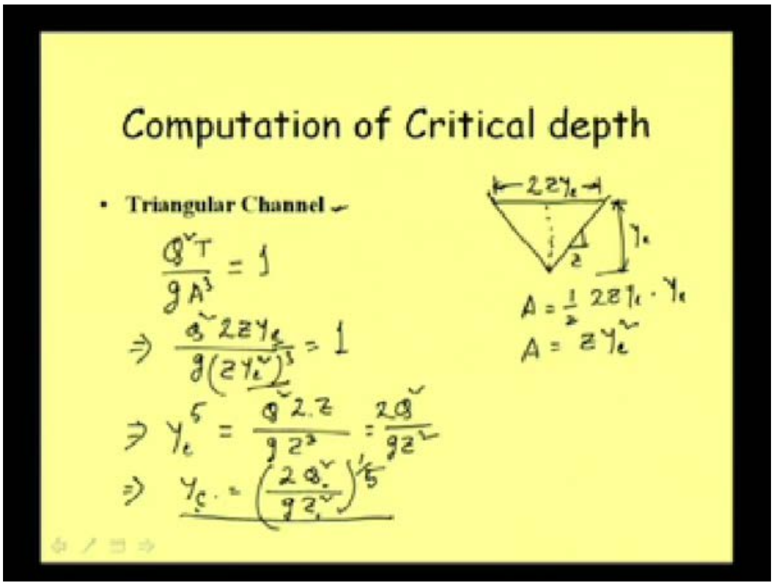

Now, let us see if our channel is not rectangular. Rather, say it is a triangular channel. So, let me draw a triangular channel. Suppose, we are having flow Y c and then, side slope is Z. So, this top width here, it will say this part will be ZY c. So, total will be twice Z into Y c and this Z is basically, we are writing about the side slope Z. So, in triangular channel, again if we start from the various basic relation, that is Q square T by gA cube and area we can write as half of base in altitude. So, base of this triangular is twice ZY c and altitude is equal to Y c.

So, this is equal to ZY c square. So, area we can write like that. So, Q square T by gA cube equal to 1. That will lead us to Q square, then top width is equal to twice Z into Y c. Then, it is g and area we can write as ZY c square whole cube. Now, from this, we can express Y c in terms of the other value. I mean in terms of the other value. So, let us try doing, that is, will be Y c cube Y c to the power 6. This part will be and this is 1Y c. So, ultimately, we can write Y c to the power 5Y c to the power 5 is equal to square. Then, twice Z and here, it will be g and Z to the power cube.

So, finally, we can have it in the form that Q square, twice Q square by gZ square. So, Y c to the power 5, we are getting in the form that twice Q square by gZ square. Using this relationship again, say Y c is equal to twice Q square by gZ square whole to the power 1 fifth. Now, this relation we can use for computing critical depth directly. So, in a triangular channel, if the side slope Z is known, Q is known, then we can directly calculate the Y c. Now, let us see, can we have some relationship again for computing minimum specific energy or specific energy at the critical condition that on the basis of this critical depth.

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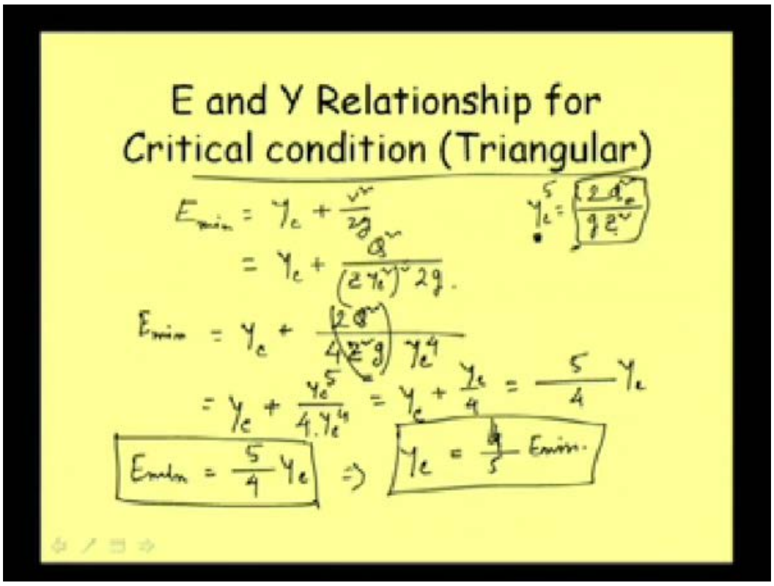

So, this slide will be discussing about E versus Y relationship for critical condition is triangular channel. Let me again start from, say E is equal to specific energy E minimum. We are talking about E minimum is equal to Y c plus v square by twice g. Starting from that we can write it as Y c plus v square is equal to Q square by a square and that a square relationship as we could get here, that we can see is ZY c square. So, we can write ZY c square whole square and then, twice g. We just need to remember what relationship we got for this Y c?

So, that is Y c is equal to twice Q square by gZ square. Y c we got as twice Q square gZ square Y c to the power 5. Rather, Y c to the power 5 is equal to twice Q square by gZ square. Now, we need to express this particular expression. To have this expression, that is this expression twice Q square by gZ square, so that we can have it terms of Y c. So, let us just rewrite this and this from Y minimum is equal to Y c plus. If we write here twice Q square, just have it in this form. In this form, we can write here another two. So, it will become 4. Then, this ZY c square is already there. So, let me write it as Z square and then g is here and Y c, I am writing separately, Y c to the power 4.

So, twice Q square by gZ square that expression we are getting here and that expression is nothing, but it is Y c to the power 5. So, we can rewrite this expression as Y c plus Y c to the power 5 divided by 4Y c to the power 4. This is nothing equal to Y c plus 1 by 4Y c or Y c by 4 and that we can write as 5 by Y c 5 by 4Y c. So, we can write as E minimum, we can write as 5 by 4Y c. So, that is another relation that we can have for triangular channel. Y c is equal to 4 by 5 E minimum.

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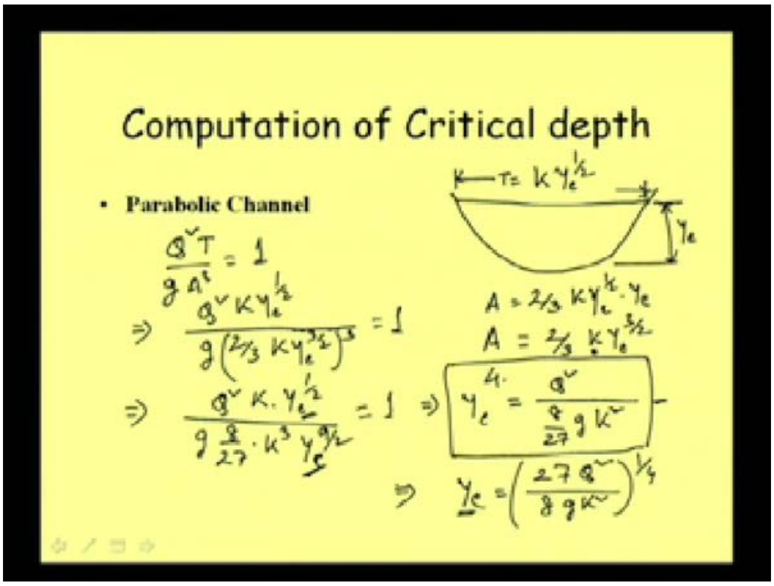

Well, from these two analyses, we could see that for triangular channel and for rectangular channel, we can express minimum specific energy, that is, the energy at critical condition in terms of Y. Now, if it is a parabolic channel, let us see what sort of expression we can have or can we solve with directly, can we solve critical depth directly for parabolic channel. So, let us take a parabolic channel like this and the in parabolic channel, if this depth is Y c, then this top width can be expressed in terms of Y c as, say K some coefficient and Y c to the power half.

This is our top width from the very basic expression of parabola. We can write it like, that top width is equal to KY c to the power half and the area, that we can write as area is equal to 2 by 3 KY c to the power half, that is what the top width and into Y c two third. So, this becomes equal to 2 by 3 KY c to the power 3 by 2. So, that is the expression for area and K is a constant of the parabola.

So, using this expression again, if we start from the very basic relation, that is, Q square T by gA cube is equal to 1. If we start from that again, then what we can write Q square and T top width is nothing, but KY c to the power half. So, Q square T. Then, gA cube, we can write as 2 by 3 KY c to the power 3 by 2, but this entire expression will have to be power to cube. So, this we can again simplify, rather our basic objective is to find out Y c.

Well, let me write one more step here, that is Q square K into Y c to the power half g. Then, this 2Q means 8 divided by 27 and then, we can write K cube and then, this will be Y c to the power half Y c to the power half. Sorry, this will be Y c to the power 9 by 2. Now, from this, if we write the Y c, directly this is equal to 1 and from that, let me go this side, that is Y c to the power 9 by 2. If it is coming here, then it will be 8 by 2. So, it will be Y c to the power 8 by 2 or we can write Y c to the power 4 directly. This expression, this Y c and this Y c combining, it will be Y c to the power 4. This is equal to Q square by 8 by 27g and this K, it is K square. So, that expression, we are getting, that is Y c to the power 4 is equal to Q square by 8 by 27 gK square.

Well, now using this expression, can we again just find one relationship between the minimum specific energy and Y and the critical depth Y c again. We need to recall this relationship. This can be written in this form also, Y c is equal to 27 by Q square by gK square 4 to the power one fourth. This can be written like that also ok. Now, let me go to the next slide and see how we can calculate Y c or how we can express Y c, in terms of minimum specific energy.

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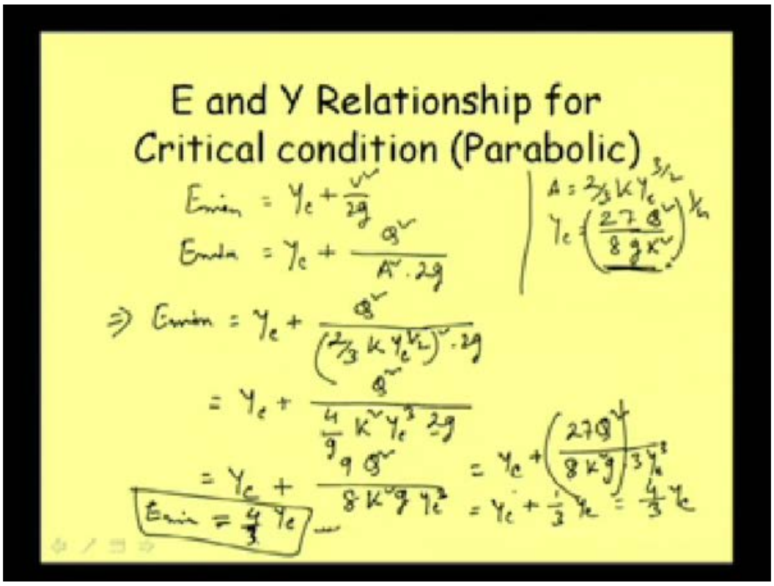

Well, again minimum specific energy E minimum is equal to Y c plus v square by twice g. So, Y c plus v square is nothing, but Q square by A square and twice g. Now, let me write this A square term and let me have this term here. So, that readily, we can write it in area terms is equal to 2 by KY c to the power 3 by 2. So, area is equal to Z square whole to the power 1 fourth. Starting from that what we can write? This implies E minimum is equal to Y c and let me again rewrite this expression in the form.

So, that we can write Y c for this particular expression, Y c plus say Q square. Let it remain like that and then, area we will be writing as 2 by 3 KY c to the power 3 by 2 square and twice g. So, this is equal to Y c plus. This we can write as 4 by 9 K square and Y c to the power 6 by 2, that is Y c cube and then, it is twice g. Then, Q square is remaining there. So, this expression, if we just try to write it in this form, then we can just write it like that Y c plus, it will be 9 by 4, but we need to get it as 27 by 8. Let me write it as 9 by 4. Then, it is Q square, then K square g is there. So, this 2 and this 4, this will become 8, 2 and 4.

This will become 8 and then, this is Y c cube, but still to express this in the form, that is 27 by 8, we can write it in the form that Y c plus. This we will write at 27 Q square, then 8k square g. This part is there and then, we will multiply it by 3Y c cube. So, finally, this expression we can combine and we can write this as Yc plus. This part is again Y c to the power 4. So, untimely it will be one third of Y c and this can be written as 4 by 3Y c. So, the expression can be written as E minimum is equal to four third of Y4 by 3Y c 4 by 3Y c.

So, this is one relationship that we can have for a parabolic channel as such. We can see now that and computation of critical depth is not always, I mean very difficult, rather for most of the cases like rectangular channel, like triangular channel and then, like parabolic channel. Even, we can get it very directly, that is if we know why, if we know the discharge, if we know the channel section, then we can get the critical depth expression directly. So, computation is rather easy as compared to the computation of normal depth. Only for trapezoidal channel, we need to go for trial and error procedure.

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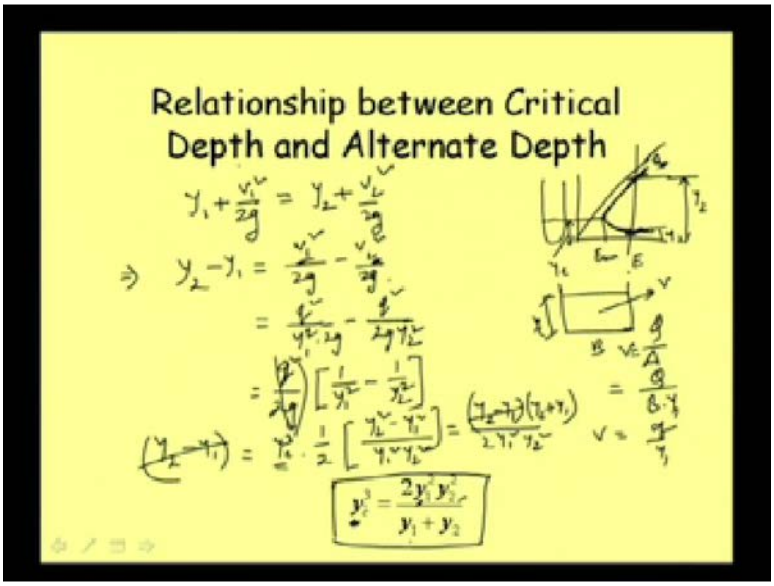

Well, let me go to another important relationship. We have seen that Y c can be calculated and thus, energy at that Y c can also be calculated and if we just recall our specific energy diagram, it was like that. Then, you can just concentrate on to the graph. Then for a specific energy E, we can have two depths, say this is Y 1 and this Y 2. These two depths, we call as alternate depth, other than the minimum specific energy.

When it is minimum specific energy, then the depth we get is called critical depth. This is what our critical depth, but other than minimum specific energy, if we go to any other energy level, then we get two different depths for a specific energy, for a given discharge Q. These two depths, one is in super-critical condition and other is in sub-critical condition. Now, let us see, can we derive some relationship between the critical depth and these alternate depths. So, in this slide, we will be discussing about relationship between critical depth and alternate depth.

Well, let me write that as in these two points, specific energy is equal. So, what we can write, that is Y 1 plus, suppose velocity corresponding to this point is this depth is V 1. So, Y 1 plus V 1 square by twice g and that is equal to Y 2 plus V 2 square by twice g. So, from this, we can say that Y 2 minus Y 1 is equal to V 2 square by twice g. Sorry, V 1 square by twice g minus V 2 square by twice g. Now, for rectangular channel, what we can write for rectangular channel? This relationship is very popular and for rectangular channel, we can have this in this form, that is if it is Y c, then this velocity V at any point can be written as Q.

Basically, first we can write it as area into, sorry Q by area and this is velocity equal to Q by area. Area is nothing but B into Y c. So, this can be written as Q by Y c. So, writing this, whatever may be the velocity, see here the velocity is V 1 and in the second section, velocity is or when the depth is higher, velocity is V 2. Definitely, it is V 2 will be less than V 1, but the discharge, there is the unit discharge or the discharge of the channel is not changing. So, if we are talking about this particular line, we are talking about this specific energy line for a given discharge.

So, when the discharge is same and if it is a prismatic channel, then our B is also same. So, that way, it is Q. This small q is not changing. So, whether it is V 1 square or V 2 square, that we can write as in terms of Q square by Y c Q square by Y c and twice g minus Q square by Y 1. Rather, at this point, it will be Y 1. At the other point, it will be Y 2 Q square by Y 1 minus Q square by twice g into Y 2.

So, taking this advantage, what we can write that we can write as 1 by twice gQ square by twice g. Rather, we can write Q square by twice g and then, it is 1 by Y 1 minus 1 by Y 2 by Y 1 minus 1 by Y 2. Oh! Here, I did a mistake because this is V square. I am writing V is equal to Q by Y 1. So, V square is equal to Q square by Y 1 square and this is equal to Y 2 square. So, it will be Y 1 square minus Y 2 square. So, this can be written as, again we know that Q square by g, that particular expression is nothing, but Y c cube, if we recall our earlier expression.

So, Q square by g can be written as Y c cube and then, it will be 1 by 2. This part is Y 1 square Y 2 square. This will be Y 2 square minus Y 1 square and that can be written as Y 2 minus Y 1 into Y 2 plus Y 1 divided by twice Y 1 square Y 2 square and this here on the left hand side, we had Y 2 minus Y 1. So, this Y 2 minus Y 1, if we cancel, then we can write that this Y c cube is equal to this part twice Y 1 square Y 2 square divided by Y 2 plus Y 1. So, this expression finally, can be written as Y c cube is equal to twice Y 1 square Y 2 square divided by Y 1 plus Y 2.

So, this is one of the very popular expressions. That is used when we want to know that what will be the, suppose if we know that a particular depth, we have Y 1, then we want to know what is the alternate depth. Then, if we know the critical depth, we can calculate that. Suppose, we know Y 1 and Y 2 at a particular energy level, we know that this is Y 1 and that is Y 2, then knowing Y 1 and Y 2 also, we can calculate critical depth. That is why while we are discussing the computation of critical depth, we are discussing this relation as well because this can be used for computing the value of critical depth.

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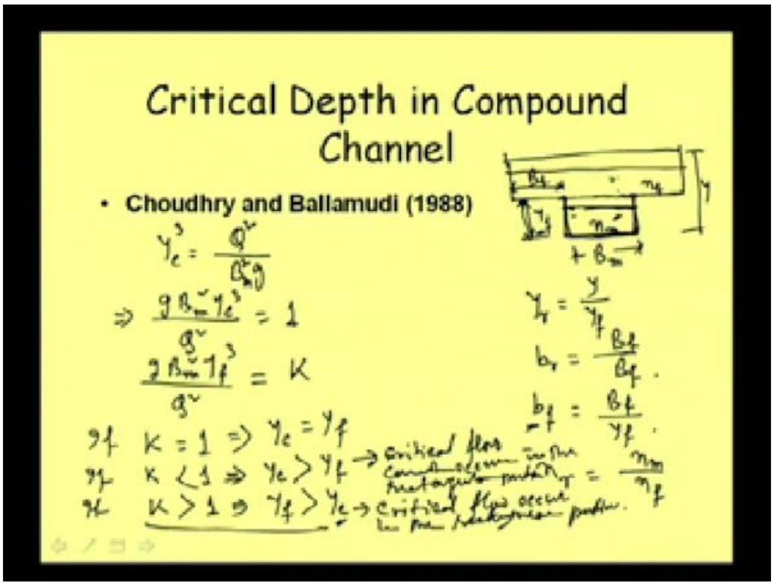

Well, then, let me discuss another topic very briefly. Of course, this topic is critical depth in compound channel and this need to be discussed very elaborately, but it is beyond the scope of this particular course. That is why, just to give you some idea or just to have some discussion on this topic, let me discuss it briefly. Well this, let us draw one compound channel and let us see how we can have some peculiar relationship for critical depth computation in compound channel.

In fact, Chaudhry and Ballamudi in 1988 did this sort of analysis by symmetrical compound section. What we mean that these are identical. This left hand side, right sides are identical. This is a compound channel and then, say depth flow at any level, may be

Y here, then this B we can write as B m. That is the main channel and always we know that in a compound channel, when the depth of flow will be less or discharge will be less, then flow will be concentrated, sorry the flow will be flowing through the main channel. Of course, concentrated flow goes through. The flow concentration is always in the central part and then, on the side, the flow will be less. Rather, the flow velocity will be less. Then, this level we call as depth up to the floodplain Y f and this portion is call width of the floodplain B f. As it is symmetrical, so this side also it will be floodplain width, the same bf.

So far, Manning's roughness is concerned; here we can write that this is nm, main channel roughness. This is, say floodplain roughness and then in the compound channel, using this term, some non-dimensional term can be derived say Y r, that is, equal to Y depth of flow at any point divided by Y f. This is say non-dimensional term regarding depth. Then, we can have breadth related non-dimensional term, that is, equal to floodplain width divided by say, sorry floodplain width divided by width of the main channel B m.

Then, we can have another term B f. This is equal to B f divided by Y f. Then, we can have this n r, that is, equal to nm divided by n f. Now, this is just non-dimensional term which is used for obtaining or for analyzing this critical flow computation. Of course, as I have already mentioned that we do not have that much of scope to go in detail of these analyses, but still we can have some of the understanding.

Well, now we know that Y c, as we have already discussed say Y c is equal to, we can have Q square by gY c cube is equal to Q square by g. So, if it is Q square by g, means small q is equal to q by B and when we talked about this rectangular portion, that is the lower portion of the compound channel, this particular portion. Then, it is just simple rectangular channel and we can have this expression, that is, Q square by g. That Q square we can write Q square by B square small q square. So, q square by g and Y c cube is equal to Q square by g. So, this is say, B m. Now, this we can write as at critical condition, it is like that Y c cube is equal to, so this can be written as gB m square. Then, Y c cube by Q square is equal to 1. I mean from that we can write this one.

Now, let us see, if for a particular discharge, if our critical depth lies below this channel, below this floodplain depth, then we can use this expression directly because up to this point, we do not have anything, nothing like compound channel. Here, it is just simple rectangular channel. Now, we can calculate if in place of Y, say that means, upto Y c is equal to Y f. If your Y c remains Y f, then we can have this sort of relationship.

So, we can write one expression that gB m square, then say, Y f cube by Q square. Let us define this as K. Now, from this expression, we can have some understanding that if our K is equal to 1. So, K is equal to 1 means Y c. This implies that Y c is equal to Y f. Now, if K is less than 1, then 1 means Y c, that is it is less than y means Y f is less than Y c. So, Y c is greater than Y f. Well, now what it indicate Y c greater than Y f means critical depth.

If you just refer this diagram, critical depth will be greater than Y f. That means, greater than this particular depth. So, when critical depth is greater than this particular depth, that means we cannot have critical depth below this main channel, below the floodplain level. In that case, that will not be valid because up to this portion only, we can have rectangular. So, this we can consider, this is rectangular channel. So, this implies that critical flow cannot occur within the, let me write critical flow, cannot occur in the rectangular portion.

Now, if we have another condition that K is greater than 1. So, K greater than 1 will indicate Y f is greater than Y c. Now, if our floodplain depth B f, sorry if our floodplain depth Y f is greater than critical depth, that means the critical depth will occur or critical flow will occur within the rectangular channel. So, this will lead us to that condition that critical flow occurs in the rectangular portion.

So, when critical flow occurs in the rectangular portion, then this we can calculate directly. Now, when critical flow is occurring somewhere here, when the flow is over floodplain that means from this analysis, what we are getting that, if the flow is critical depth is occurring, what the flow is occurring above the floodplain. Then only, that is when the flow depth somewhere here, then we can have some complexities in computing critical flow, when it is below this. Yes, we can do it directly and for that another term was defined.

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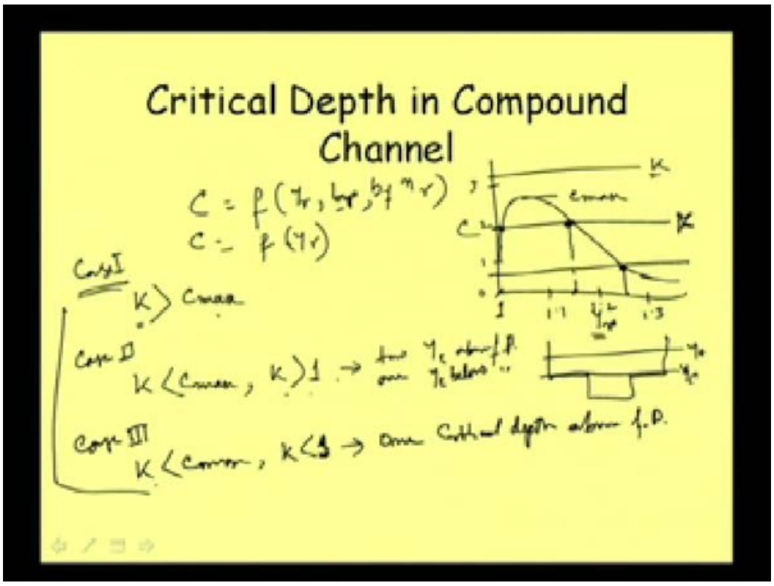

That term is called C, which basically relates, that is Y r b r. Those non-dimensional relationships, B f and N r and this C 1 mean, when this K expression is equal to C, then we can have critical flow condition. Well, that C, we can plot like this. That means, on this side, if we plot Y r, basically for a section for other things remaining the same. Say, our B r, as we could see, just to again go back, B r is equal to B f by B f by B m. It should be written B m; sorry B r is equal to B f by B m.

So, as we can see that this B f B m, suppose for a given section, if these are constant B f Y f these are constant, but this Y r will be varying with that depth, that means when depth changes, Y r is varying N m and N f. This nr ratio is also remaining the same. So, ultimately from this expression, if for a given section, for a given roughness, everything remaining the same C becomes a function of Y r. So, we can plot Y r versus C and C and Y r. Y r will start from 1. If you just go back to this expression, Y r is nothing, but Y by Y f.

So, when Y is equal to Y f, this Y is equal to Y f. The depth flow is this much. We are getting it below the rectangular portion. Now, our point or we are now interested when it is above this thing. So, when Y will be exceeding this Y f, then Y r become higher than 1. So, when it is higher than 1, then we are interested what will happen then. We are talking about this term C.

So, this is say 1. This is 0123. If we get like that, depending on the section, we can have different sort of curve. This will be 1, say 1.1, 1.2. That way the ratio is gradually increasing and then, we can have a curve like this. We can have a curve like this and then, when we calculate the K value, that K value, what we got earlier, this K value. Then, if we see that in this curve, K can be greater than, there we are getting a value C max, maximum value of C for a given section. If our K is, suppose K value we are calculating and K is greater than C max, this can be one condition. Say, this is case 1.

Case 1, we are getting K is greater than C max. Then, we can have another condition that now, here we can see when case 1, K is greater than C max means, this on the floodplain, when the flow is above floodplain, we are not getting any intercept of this K value with the C max. That means, it directly means that no critical flow will occur in the upper portion of the compound channel because we are talking about upper portion of the compound channel as Y r ratio is higher. So, no critical depth will occur in the upper portion of the compound channel.

Well, critical flow will occur only in the lower portion of the compound channel, that is, the condition. When we have, say if our case 2, if we can have this K is less than C max and we can have K is greater than 1. That means, it is above 1 and is less than C max means somewhere here, this K value. Now, if it is like that, that means, we are finding there will be one critical depth where K is equal to C. Here, one point and this is above the floodplain Y r ratio 1.1 and 1.2 between. So, it is higher than that. So, at some point, it will be there and another critical depth we are getting here, where it is just likely higher than Y r. Y r is slightly higher than 1 means just near the floodplain, we are getting this thing.

If I draw this compound section, that means, Y r is just greater than 1 means just here somewhere, we are getting one critical depth, say Y c. We are getting 1Y c and another Yc we will be getting here which correspond to this value. So, this condition indicates two critical depths. One will be there above the floodplain, 2Y c above the floodplain. So, that is what we are getting. Of course, as K is greater than 1, from this condition, we got when K is greater than 1, there will be another critical depth below the floodplain. So, 2Y c and 1Y c below floodplain, so that also we are getting.

Then, we can have another case 3. When our K is less than C max and also K is less than equal to 1. That means, if we are coming below this 1, then it is going like this and we are getting one critical depth above the floodplain. So, this will lead to one critical depth above the floodplain and obviously as K is less than 1, we will not get any critical depth below the floodplain level. That is why from these conditions, we can see that there will be a, if draw the specific energy diagram, this is a floodplain level.

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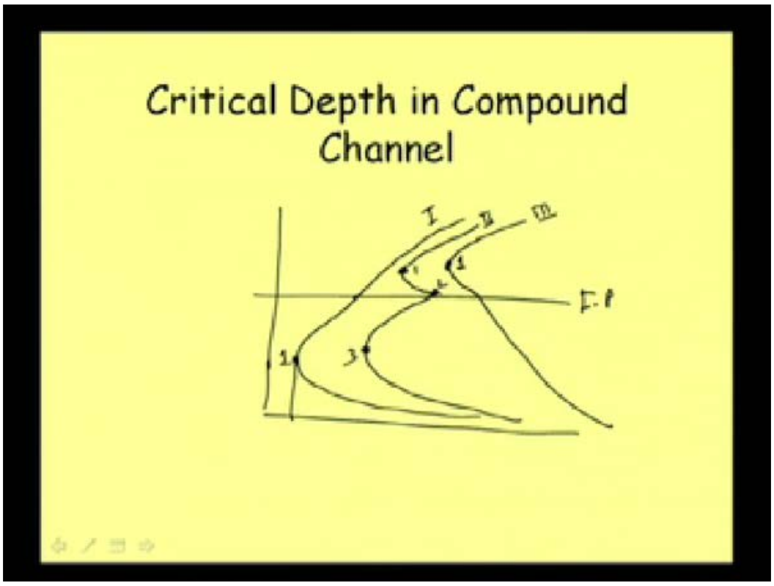

For case 1, there will be no critical depth here and there will be a change in the diagram. This is the critical depth. This is case 1. Then, for case 2 what will happen, there will be one critical depth here. Then, energy diagram is going like that and then, it is coming like that. So, there will be three critical depths. Two above the floodplain and one below the floodplain 1 2 3 and here, it is only one. For the third, this is case 2 and for the third case, what will happen, there will be a critical depth above the floodplain, but there will be no critical depth below the floodplain. This is what one critical depth. So, that is why in compound section, we can have three critical depth conditions, three critical flow conditions. For all these specific energy is of course minimum. Here, specific energy is minimum in the case 1. Specific energy is minimum for two cases. In the case two, but for case 1, specific energy is not even minimum. So, this sort of different conditions, we can have for computing critical flow in compound channel.

Well, this much is, I think sufficient who have some understanding of critical depth computation in compound channel. In the next class, we will be going in more detail into the critical flow computation. We will be solving some numerical. That way, we will try to see how critical depth computation can be done for the different types of channel and how this critical flow computation helps us in studying other aspect of hydraulic engineering. Thank you very much.