Hydraulics
Prof. Dr. Arup Kumar Sarma
Department of Civil Engineering
Indian Institute of Technology, Guwahati

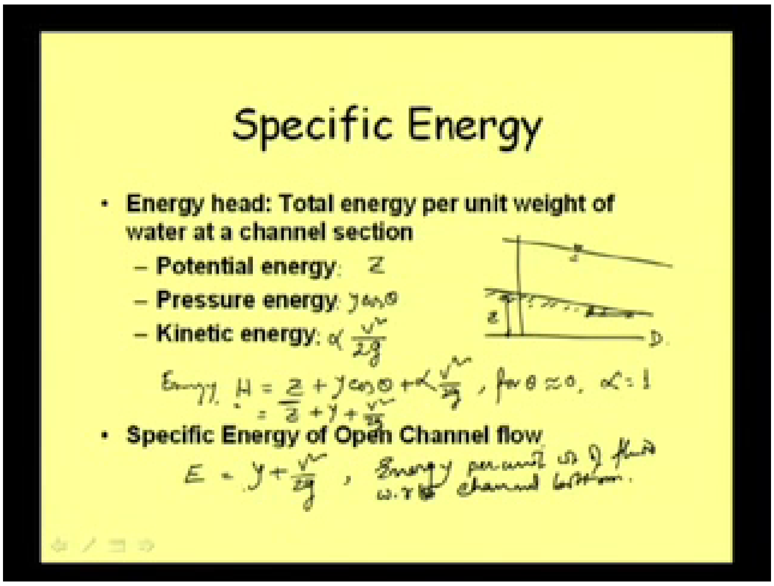
Module No. # 03
Energy and Momentum Principle
Lecture No. # 01
Concept of Specific Energy

Welcome to this third module of our course on open channel hydraulics. Here, we will be discussing in this module that how the concept of energy, and how this momentum principle, that is energy principle and momentum principle that we use for computing various works of hydraulic engineering, how these are applied to understand some of the problems of hydraulic engineering to solve those problem.

Well, to start this module, let us start with the topic that concept of specific energy; well, of course, we have already discussed some of the issues related to specific energy, but still we will start with the start from that particular point, where we left earlier that is what we did earlier that we could see what we mean by energy depth, we have already understood; well, then we could see that energy of flowing fluid that we can divide into three components like that potential energy that the energy derived due to its position; then we talk about pressure energy, means, the energy due to pressure, and then we talk about kinetic energy, that is that energy derived with the fluid due to its motion.

So, this three components of energy are there, I mean just when we talk about total energy, then we say that summation of all these energy is the total energy of the flowing fluid right; and then there is another term that we call as energy head; now, what is the difference between energy and energy head, that part also have discussed basically by energy head we mean that energy per unit weight of fluid, of course, flowing fluid we are talking about, so energy per unit weight of fluid is called as I mean head energy head.

(Refer Slide Time: 03:17)



Now, when we write the expression for this energy head, now we can concentrate on the slide, that is say we can call that energy head total energy per unit weight of water, and of course, fluid means in hydraulic engineering we talk about water at a channel section; and say, when we say energy then we always talk about that energy at a particular section, and then potential energy as we could see it is due to the position, and when we say that in a channel when we say in a channel say this is the channel then by potential energy always we talk it up to the channel bed, say, at section when we talk about potential energy then it is up to the channel bed, so it is coming as Z.

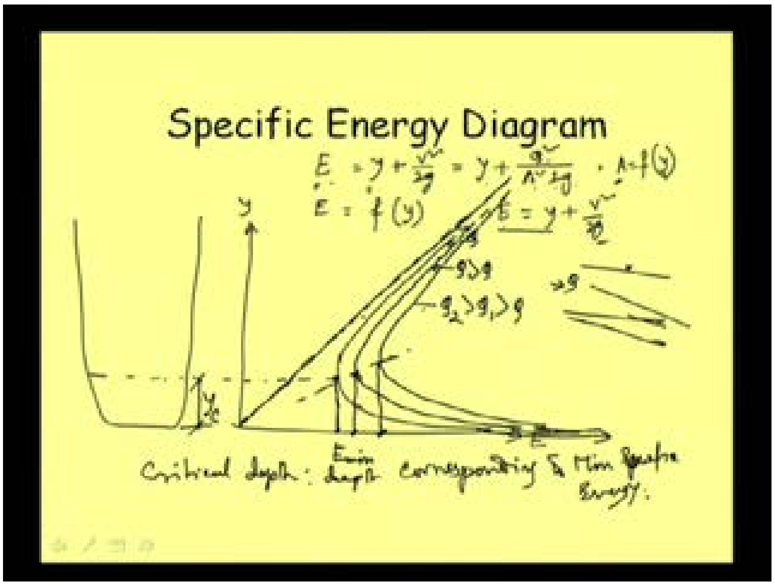
And say water is flowing up to this much level, and then as we know that point was also discussed earlier, that is depth that when we say the depth of flow in a vertical direction, this is different and when we say the depth of a flow section then it is the flow section means it is a section normal to the flow direction, so that will be different; so, from that point of view this pressure energy is basically in open channel flow it is coming as the y, but of course when there is a slope theta then we can write it as y cos theta, that was discussed, and this theta is basically nothing but the slope of the bed, that is s b that bed slope.

Well, then kinetic energy, well, that kinetic energy is again half m b square, and then when we just derived that expression for unit weight of fluid, then we get that as V square by twice g; and now, if we recall our discussion that we carried out earlier regarding that energy coefficient alpha, then we know that when we talk about energy then we know that energy is, in fact is different when we talk about energy at a particular flow section at different point it will be different, because the flow velocity is in reality it is not equal, so a coefficient comes. So, actual kinetic energy will be written as this kinetic head multiplied by that alpha energy coefficient alpha or we called it as that as co realistic coefficient. So, these total energy if we sum up, then we say that energy is equal to energy, and that is normally written as H indicating head, energy H, sometimes we write as E also, but in open channel flow we generally use this symbol H to write this total energy, that is Z plus y cos theta plus alpha t square by twice g.

Well, after recalling these things what we did discuss earlier also; then we can now try to see what we mean by specific energy of open channel flow, well, specific energy of open channel flow; again from this particular line, that is H is equal to z plus y cos theta plus alpha b square for say theta is very small, suppose for theta is approximately equal to 0, and for alpha is equal to 1, that means, by alpha is equal to 1 what we mean that the velocity that is variation velocity variation within the cross section is say very minimal very small and as such we can consider that alpha to be equal to 1, that is say velocity coefficient is not that much significant; in that case we can write this is equal to z plus y plus V square by twice g.

Well, then when we say about specific energy of open channel flow, then we talk about the energy with respect to the channel bottom with respect to the bed of the channel; that means, in this case we are here, say, earlier when we were doing then this Z is from the datum, say, this is our datum from that datum we got the Z; now, our datum itself is we are measuring the energy with respect to the channel bottom, so this is our datum, so this Z term is not coming at all here; and so, our specific energy we can write that E is equal to y plus V square by twice g y plus V square by twice g; that means, by definition what we can write by definition what we can write, before going to the specific energy diagram let me write the definition here, that is energy per unit weight of fluid with respect to with respect to channel bottom with respect to channel bottom; so, that way we can write what we mean by specific energy, well.

(Refer Slide Time: 09:09)



Now, with specific energy, this equation E is equal to…., again let me keep the equation here E is equal to y plus V square by twice g; now, we can draw a graph we can draw a graph relating this term, relating this E and the y, why because here this V square by twice g, V square is nothing but we can write, that is y plus it is Q square by say a square by twice g Q square by a square into twice g Q by a is equal to velocity.

Well, now, when we write this as Q square by a square; that means, again a is a function of y, now whatever may be the section this area is always a function of y with the increase of depth our area increases with the decrease of depth area decrease, so it is in some form it is also always a function of y; so, as such, that means, what we can say that E itself is a function of y, specific energy can be considered as a function of y, it is a function of y; and then so, when we plot this value, when we plot E versus y, that is change the y and calculate what E is; and if we plot these things, then we get a graph graphic plot of E versus y, and that particular plot is referred as specific energy diagram specific energy diagram.

Well, now, without going to any definite value let us take say in this direction we are plotting y, and in this direction we are plotting E, and let me draw the channel on this side, this will indicate that different depth we are here; now, from this expression that E is equal to y plus V square by twice g, from this expression we can see that what will be the line for E is equal to y, E is equal to y, this line will be definitely a line drawn at 45 degree to this axis; so, if I draw a line, 45 degree, this line is nothing but E is equal to y.

Well, then let us see what else is there; the second term is second term is V square by twice g well. So, this V square by twice g V, even V is negative, but it is V square, then it will always be positive, so V square by twice g is always a positive term it is always a positive term. So, our specific energy diagram, that is the curve of energy versus E, energy E versus depth y will be…, because when E is equal to y, then we are getting this line, but E is actually something more than y, E is something more than y, so that way it will be always on the right side of this line always on the right side of this line, because E is suppose E is y is this much plus something, so it is always coming to the right hand side; and then if we plot this curve if we plot this curve then we will see that we will be getting a line like this.

We will be getting a line like, well. So, this line we are getting a single line, this line is for a given discharge; if we just see this equation E is equal to y plus Q square by a square by twice g then we understand that if we change the Q, then entire things are changing. So, here we are talking E as a function of y, we are writing E as a function of y, when because y and a are y and a are function of y, y is obviously y and a is a function of y, but Q we are considering as constant, and that is why we are writing that E is a function of y.

So, this is for a given Q this is for a given Q, this line is for a given Q; well, now, you can just see peculiarity of this particular curve that, say, if our discharge is fix, say discharge is fix Q is like that, and it is having this much of depth of flow; now, when our depth…, if we suppose by any means, suppose let me change the slope if we are changing the slope, then our depth will come down it will reduce; and when depth come down your E from this part it will be less, but when depth come down when Q remain same our velocity increases, so this component will increase.

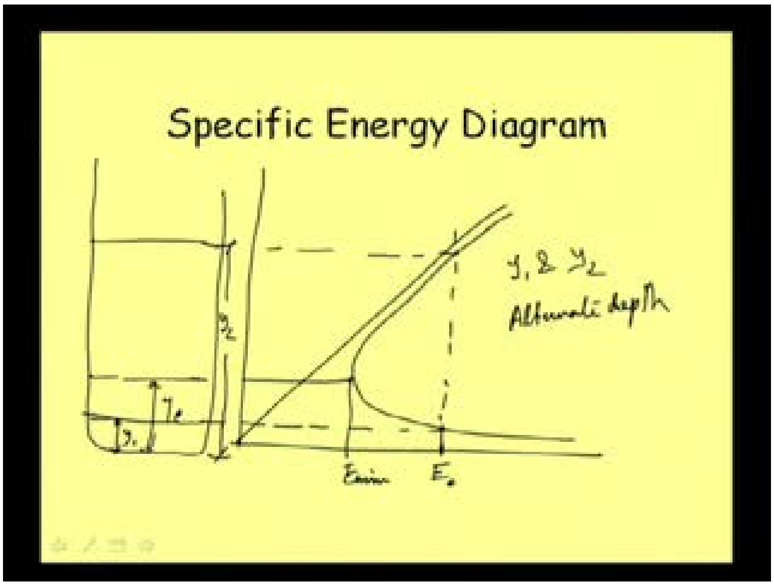
So, that way with the change of depth with the change of depth E is coming down some times and then it is again increasing, and then we get a minimum value we get a minimum value, for this Q we have specific energy minimum, we have E minimum; and for that E minimum we are getting a corresponding depth, for this depth we are getting minimum specific energy for a given discharge. Well, now, if I take another discharge, that is, if I increase the discharge, then I will be getting another curve I will be getting another curve, this curve will be coming like this.

This is going like this, and this Q 1 if I write this is greater than Q, then if I draw another.., if I increase the discharge further then I will be getting another Q I will be getting another I will be getting another curve, and this curve will be like this, this curve will be like this; and suppose, it is Q 2 then this is greater, than Q 1 this greater than Q well; so, that way with the increase of discharge with the increase of discharge we are getting our lines get shifted to little higher side, and then we are getting some point joining that all here all in this point, we have our minimum specific energy, we have minimum energy, but this minimum energy this minimum energy correspond to the discharge Q 2, and this minimum energy correspond to the discharge Q 1, and this minimum energy correspond to the discharge Q well.

And this set of curve this set of curve that is a specific energy diagram has the peculiarity that, this line is a synthetic to the line drawn at 45 degree, this line you can refer, this line is drawn at 45 degree to the E axis; and these lines if you just increase the Q this will keep on going, if you just increase the depth it will be just about to touch this line, but it will never touch it will be a synthetic to the to this line; and similarly, it will be synthetic in this direction, it will be synthetic to the this line this line, so it will be going like that well. And as we were explaining that when we talk about minimum specific energy, then the depth corresponding to minimum specific energy, this we call as critical depth, this is called critical depth; so, critical depth we call as critical depth, depth corresponding to minimum specific energy minimum specific energy

Well, that means, all this things we are talking about again a particular channel section, because say when we are saying area is a function of y, definitely for channel section changes that function will also change, so our same equation will not hold good, so equation means same shape will not be getting. So, that way for a given section for a given discharge we get a relation that, which says that it will give us a minimum specific energy, and that minimum specific energy corresponds to a depth particular depth, and that depth we called as a critical depth. And then there are another important point that we should know is called alternate depth; and for that let me draw this diagram again here.

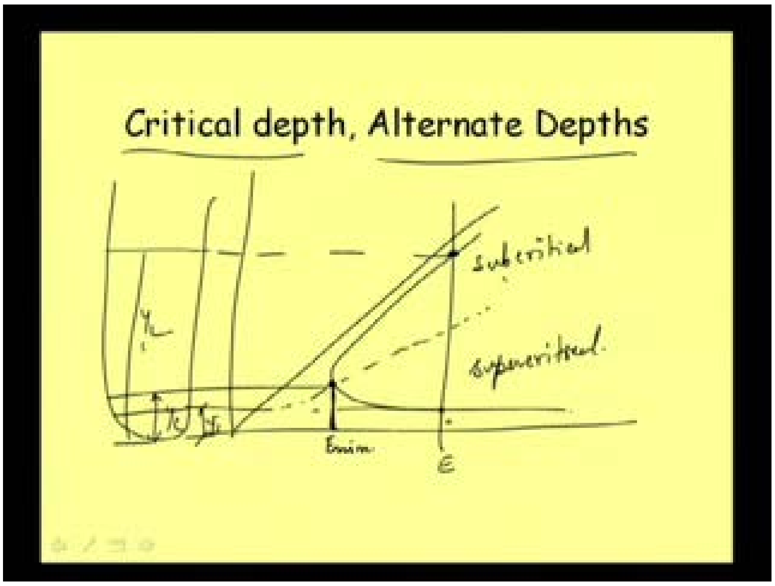
(Refer Slide Time: 19:00)



Let me draw like that, and then we got that for the minimum specific energy E minimum, we are getting a depth we are getting a depth, and that is giving us critical depth, that depth is called critical depth y c; now, for a different other than this E minimum, that is, if it is something higher than E minimum for any specific energy E for any specific energy E, we are getting a depth this one; that means, say this depth and this depth if I draw a line if I draw a vertical line through this point, then it represent that for this energy we can get corresponding to a depth this one, and the same energy we can get corresponding to a depth this one. So, now, this is say y 1, and this is y 2, this is y 2.

Well, so, this y 1 and y two we are getting 2 depth, this y 1 and y 2 we are getting two depth, and both the depth, for both the depth we are getting the same specific energy, for both the depth we are getting same specific energy, and that is why these depth has a special relationship, and these depth has a special name, these depth are called alternate depth, these depth are called alternate depth alternate depth. So, this y 1 and y 2 y 2 are called alternate depth, these are corresponding to a specific energy E; now, then what are the specialty in these.

(Refer Slide Time: 21:17)

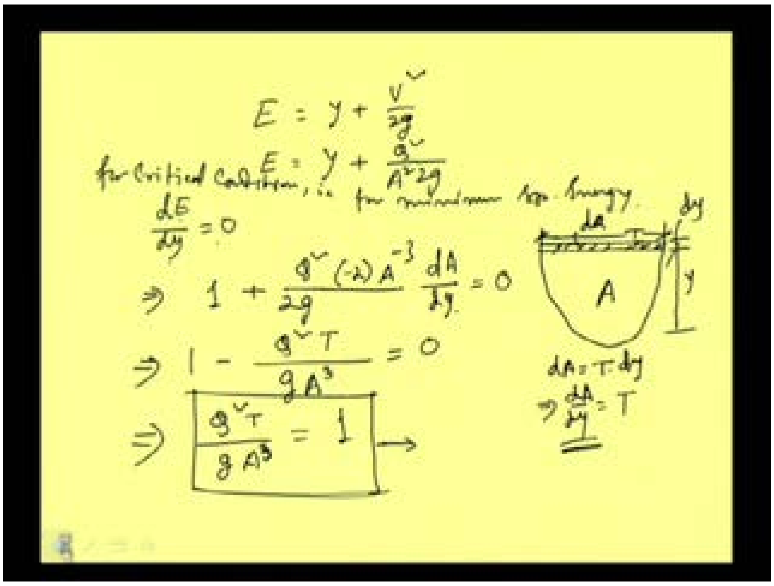


Let me just draw this diagram again, and then let me explain another point here, let us discuss say this is the specific energy diagram, and then we are having this point again with the increase of discharge this can be line can go, and then these are suppose critical depth point; and then what we can have that well this is one critical depth y c, and this is y 1 sorry, and this is y 2, y 1 and y 2.

Well, now, when say we are having this minimum depth, minimum specific energy for this depth, now for any other energy we have a depth here, and another depth here; and when this depth is less than the critical depth when this depth is less than the critical depth this zone, and where we are getting depth is called actually supercritical, this is called supercritical, and then on the top it is called subcritical; of course, that we did discuss when we were discussing the classification of flow itself, what is subcritical flow, and supercritical flow; and here also the same thing just we are recalling that this is a depth, I mean, depth y 1 is a depth in the supercritical region, and then this y 2 is a depth in the subcritical region.

Well, now, these are some of the point that we have discussed, that is the critical depth and alternate depth, now there some relationship, there are some relationship between this critical depth and the alternate depth; that means, if we know, suppose, if we know a alternate depth, if we know suppose y 1, and if we know the critical depth y c, then we are in a position to find out what will be our y 2, using that relationship or if we know about y 1 and y 2, we know that for that particular flow, what is our critical depth, so these are all possible.

(Refer Slide Time: 24:03)



Well, now, let us see that we are talking about this minimum specific energy E minimum; now, mathematically from that condition can we have some expression, well, that say we are always writing E is equal to y plus V square by twice g; well, and then from here we can definitely write that E is equal to y plus Q square by d square or say Q square by A square, that area let us make it general Q square by A square into twice g well.

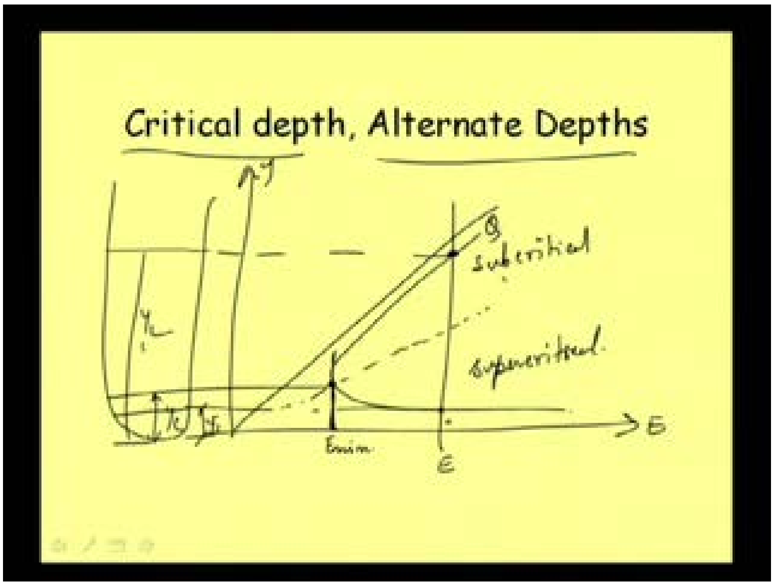
Now, at the point where specific energy is minimum, we are talking about change of specific energy with respect to y. So, at the point where specific energy is minimum at that point we get that d E d y that must be equal to 0 for critical condition; that means, for critical condition, what we can write for critical condition, that is for minimum specific energy for minimum specific energy what we are getting that d E d y is equal to 0, that means, at this point means at point, this our slope this slope is 0, so that way we are coming to that point that d E d y is equal to 0; and so, what is d E d y, this implies that if we do it, and then it is 1 y, then area is actually a function of y, so we can just differentiate it if we just differentiate it then we can have say Q square by twice g let it be then A to the power basically it is a to the power minus 2, so what we will be getting say minus 2 into A to the power minus 3 that is what the differentiation of 1 by A square, and in place of y as we have used this a as the variable, so will have to write here d A d y.

Well, now, in a channel in a channel when we talk about this area A and say depth y depth y, then by d a d y, means, if the y changes by a small amount d y, if the y changes by a small amount d y, then how much is the change in this small area d A, how much is the change in small area, basically that is what d A and d y; then if I write this as the top width top width t that from here to here. Then in fact, what is this d A this is nothing but this top width into d y rather than going for differentiation on these entire thing directly we are writing in a very straightforward way from the actual figure. So, d A is equal T into d y or we can write that d A d y is equal to the top width.

Well, so, by replacing this term what we can have, well, now we were writing d E d y is equal to 0; that means, 1 plus Q square twice g, and then this part this is equal to 0 well this implies that 1, and this minus sign is there, so it is coming 2, and 2 is getting cancel. So, we can write Q square by sorry 2 is already getting cancel, Q square by g then this a to the power minus 3 means a cube, I can write in the denominator, and this d A by d y, that is from here we can write it is equal to T, so this is equal to 0. So, this lead us to one expression that, Q square T by g A cube is equal to 1 Q square t by g A cube is equal to 1. So, this is a very very important equation or important relation, so for a specific energy is concerned; and these equation we will be using for say computing our various relation for say for computing critical depth we will be using this equation; and then for when we will be computing, say, other when we go a step ahead, suppose, we will be doing gradually varied flow, there also we will be requiring this particular relation for computing critical depth and to know then to know whether our there will be gradually varied flow or not.

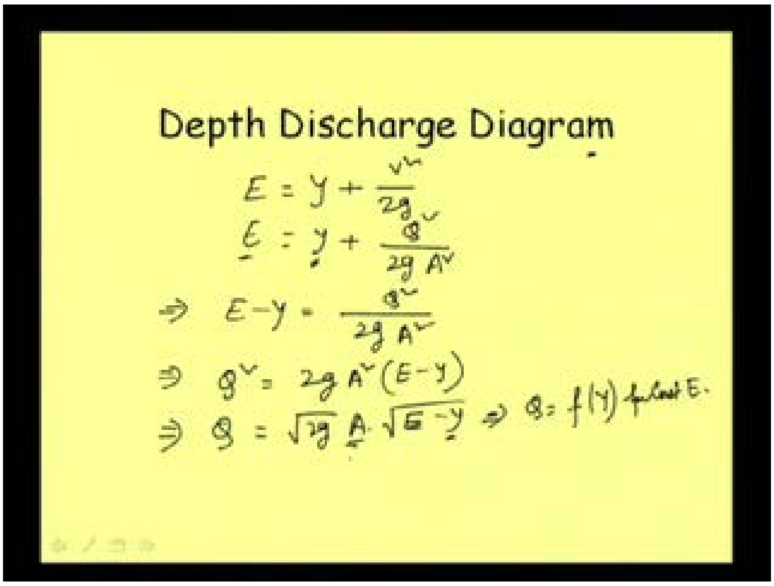
So, for different aspect we will be requiring different purpose we will be requiring this relation, and so we should always try to remember this relation; and why I am emphasizing this point because when we just simplify this equation, that means, if we write this expression this is general form; if we write this expression for a rectangular channel, then this equation will take a more simpler form; and generally, we use those expression, and we remember those expression, but in reality we should remember this original relation, which from where we will be deriving the other steps, well.

(Refer Slide Time: 29:30)



Now, again as we are finding that this specific energy is again going back to this line, this specific energy E versus y relationship and this curve is for a particular discharge for a particular discharge. Now, can we have some relation, because as we see that discharge is also say increasing; if we increase the discharge our line changes our line changes; now, there can be another relation that we can derive this is called depth discharge diagram, like specific energy diagram we can have another diagram that we call as a depth discharge diagram.

(Refer Slide Time: 30:12)

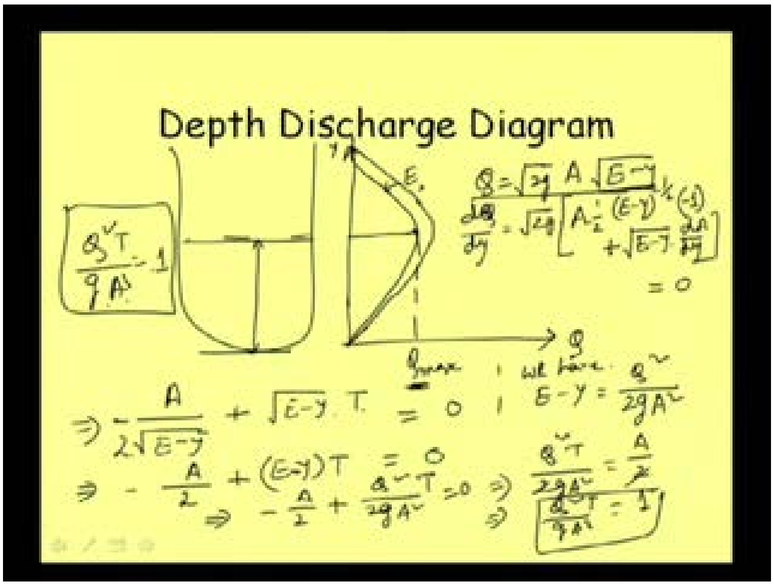


Again starting from the same relation, that is, say E is equal to y plus V square by twice g, and from there we can write E is equal to y plus say Q square by twice g by a square this part; and then from here let us just see how Q will vary if we change the depth and keeping the E constant, let us our target is that; and this diagram that is depth discharge diagram will show us that, if we keep on changing the y if we keep on changing the y keeping the E constant, then how our discharge will vary well.

So, let me write this implies that, E minus y is equal to Q square by twice g into E square; and then our target is to know Q square, so Q square is equal to twice g A square into E minus y; this will give us Q is equal to root over twice g A then root over E minus y root over E minus y. So, this is what the relation we are getting Q versus y. And then from here also it is cleared that energy is a function of y already we know, so y is there, so this area is a function of y, so rather energy although a function of y.

We are now talking about a constant energy; suppose, energy is constant then what are the other value say area and y, what are the other variable, these are area and y, and these are all function of y. So, keeping energy constant we can have a relationship that if we y changes how this Q changes, because this implies that, Q is also a function of y, Q is a function of y for constant E for constant E. Well, now, let us see that how we can derive this relation, this curve will be taking a shape, this curve will be talking a shape of this one.

(Refer Slide Time: 32:35)



It will be coming like this it will be coming like this; that means, in this direction we are writing Q, and in this direction we are plotting y, and there we are getting a Q max at this point maximum Q; and then this is for a particular energy E, this is for a particular energy E, if our energy level increases this level will rise, and that way we will be getting another curve like this; if our energy level increases we will be getting another curve like this. So, this way similar to the diagram of specific energy diagram we can have a depth discharge diagram.

Well, now, in this depth discharge diagram here also we can see mathematically how these expression for Q max we can give or that what can be the condition for this maximum Q max; well, we know till now that for minimum energy for minimum energy the depth corresponding depth is called critical depth. Now, if I draw the channel here if I draw the channel here, then for this maximum discharge for a given energy what depth we will call this one, what will be the relation, and what this depth will be will this be critical depth or this will be something else. So, let us just see that particular aspect let us see here it, let me write that is we have already got one expression, that is say Q is equal to root over twice g a E minus y.

Now, what will be for maximum discharge, for say maximum discharge, what we will get for maximum discharge, d Q d y should be equal to 0, d Q d y should be equal to 0; now, here these two a and this these two are function of y, so it is basically a…, this function we can say call as a or refer as a product of two variable. So, we can write that expression d Q d y well. So, let me write here say Q is equal to Q is equal to root over twice g into a root over twice g into a into root over E minus y, and then what will be our d Q d y, so d Q d y we can write this is equal to root over twice g will remain as it is, then we can write that this first term into the differentiation of the second term.

And let us write this as say E minus y itself and differentiate it, so it E minus y to the power half, so that will be coming half will be coming, so we can write half into root over, so E minus y to the power minus half 1 minus half, so this is minus half coming; and then let me give me put a bracket here, then again differentiation of this term, so this will be coming minus 1, because we have differentiated root over E minus 1 with respect to E minus 1, that is why we are getting this relation, and then we are just differentiating this again with respect to y, we are getting minus 1, then plus say we have already done first into differentiation of the second. So, let me do second E minus y into differentiation of the first that is d A d y, well.

Now, this relation we can write as d Q d y is equal to 0, so I am writing this equal to 0 well; so twice g already we can cancel, then this will become we can write this implies say 1 by root over E minus y 1 by root over E minus y that part, and a negative sign is there this is coming negative sign, and then this 2 is there half is there, so 2 is coming area is there, so area is there, then plus this term will be there root over E minus y into and d A and d y, we can write because d A d y we know that it is nothing but equal to T, so we can write this as T; well, and then this is equal to 0, so multiplying this by root over E minus 1, what we will be getting minus a by 2 plus E minus y E minus y into t is equal to 0.

Now, what is E minus y what is E minus y, say, we have already from our earlier expression we got that E minus y is equal to we have E minus y is nothing but from this we can see E minus y if I just square this part square this part, then it will be Q square by twice g A square Q square by twice g A square is equal to E minus 1 from this relation itself we will be getting that 1.

So, in place of E minus y we can write this expression well; so, this implies say minus A by 2 plus, then E minus y is equal to Q square by twice g A square into T, T square by g y, and this is equal to 0, or now we can write it in the form that this implies talking that to other side, Q square T by this 2 and 2 we can just…, let me write here 2 g A square is equal to a by 2, and this will lead ultimately say 2 2 cancel A is coming this way, so it is Q square T by g A cube is equal to 1.

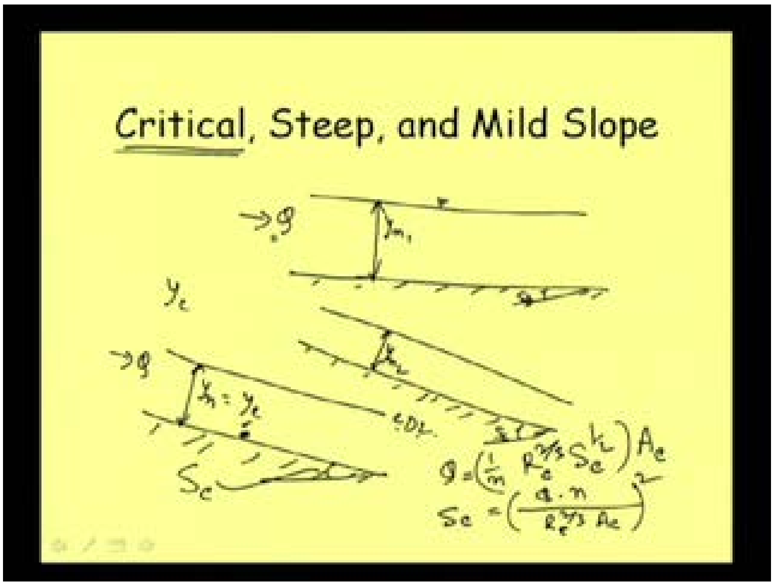
Well, so, the relation what way I am getting, let me write it here that is Q square T by g A cube is equal to 1; now, what this implies what this imply if we recall the relation that we got earlier for specific energy is Q square T by g A cube is equal to 1; and then for this discharge also what we are getting is again Q square T by g A cube is equal to 1; that means, we are getting the same condition we are getting the same condition, and same condition means this condition is nothing but the condition of critical flow. So, to get the maximum discharge what depth we are getting, this is nothing but the critical depth, again this is nothing but the critical depth; and if depth is more then we can get for any other discharge Q here, suppose, depth we are getting this one and that one, and that is also alternate depth that we are discussing earlier; and that way we can say that, here this will be subcritical flow here, this will be supercritical flow.

Now, these relations that is the depth discharge diagram and the specific energy diagram what we are getting, all these relation we can use for analyzing many problems that we frequently encounter in the hydraulic engineering; say, when a flow is moving, let me just give you some practical example, say flow is moving in a channel, and then there is a hump in the channel means, there is some raise in the bed in the channel, and because of the raise in the bed when we talk about say specific energy there is with respect to channel bed, so as there is a rise in the channel bed there will be changed in the energy.

Now, because of those changes how the flow depth will change that we will be knowing from this sort of relationship; and similarly, when suppose a channel is carrying water, and it is suppose somewhere does channel is narrow down, some somehow the channel is narrow down, then of course from this diagram depth discharge diagram we can derive another diagram that is depth versus unit discharge, unit discharge means discharge per unit width of the channel what it is; well, so that will be coming later, and using that relationship we can find out say when a channel is carrying some discharge.

And then when it is narrow down to a particular level, then definitely depth, suppose it was here bigger then it is coming down and is becoming narrow, then depth here and depth in the upstream side will be different; and how this depth changes, and how will compute this depth, so those part we will be doing by studying these sort of graph by studying these sort of graph, and of course, we will be deriving some mathematical relation for those things.

(Refer Slide Time: 42:36)



Well, then we need to discuss another topic that is what we called as critical slope, steep slope, and mild slope. Well, let me just explain what we mean by critical slope then these things are automatically coming up. Well, now, in a channel, let us consider a channel, then we are getting a for a particular discharge for a particular discharge we are getting a depth this much, we are getting a depth this much, and that is say uniform flow, we are getting a uniform flow depth, this is normal depth y n.

Again we are going back to our study of uniform flow, so we are getting a normal depth y n; now, having the same discharge if we change our slope, if we change our slope, then we are getting our normal depth will be different; if I write this as y n y n 1, that means, first normal depth for this slope for this slope, this is s b, this is s b for this slope my normal depth is changing, this is y n 2.

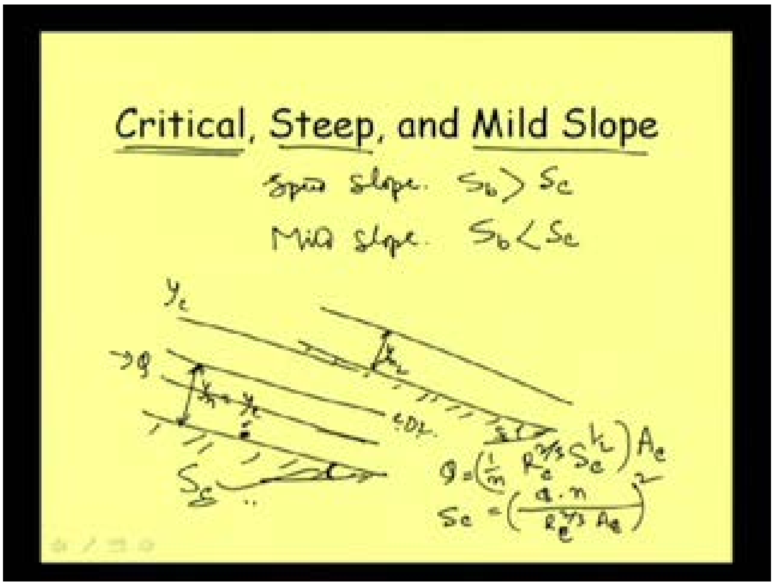
Now, we know that, that means, for difference slope we can get different depth of flow as normal depth even the other section and discharge are same. Now, suppose, for a particular slope we are in we are we we have got such a slope that for this slope we are getting a normal depth y n, and this particular depth itself is nothing but critical depth, because already we know that for a particular discharge we can have the critical depth y c; now, if we get a slope such that if we get a slope such that, for this for that particular discharge Q, we are having uniform flow depth such that this uniform flow depth y n is nothing but equal to y c, that is depth this depth is critical depth this depth is critical depth, and this particular line of course we called as critical depth line CDL; and that part we will be discussing in detail when we will be going to the our next topic, that is gradually varied flow.

But here just you know that we can have a slope, where we can have uniform flow at a depth of critical depth; that means, with minimum energy for this discharge the water is flowing with minimum energy, and that particular slope is called critical slope, if this slope we will write as critical slope S c, so that is what the concept of critical slope. And then to find it mathematically or from our other relation, if we use manning's equation then we can write say Q is equal to 1 by n, then we can write R to the power 2 by 3, then s to the power half.

Now, when we say that this equation suppose we are writing from this condition, that is when it is in critical condition, flow is critical, then our hydraulic radius we can write as R c, that is hydraulic radius corresponding to critical depth, because it is in critical condition, and then this slope we will write as S c. So, if we write it this is of course we have written this velocity expression, and a it must be multiplied by area, this must be multiplied by area to get the Q, so it is also a c areas, area means, this is also a function of y, so this a c means the depth at critical condition.

So, with this expression we can find out what is our S c. So, S c is nothing, but we can write S c to the power half is nothing, but this will be Q into n divided by R c to the power 2 by 3 and then A c. So, that way you can write or S c we can write that this is to the power 1 by this is this square if I square it then this will be square. So, this way we can write one expression for S c and here our this R is critical, and this is critical. So, using this critical term we can find out using this critical term we can find out what is our critical slope.

(Refer Slide Time: 47:19)

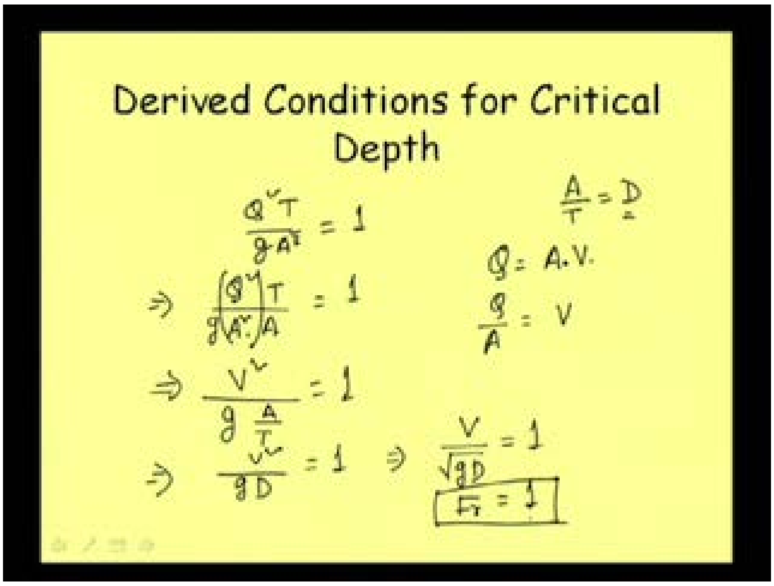


Now, to talk about this steep slope and the mild slope, this is if well let me rub this part, now what we have got that, if y is if y if S c concept we have got; now, if the bed slope is greater than critical slope, then it is called steep slope this is called steep slope; and then if bed slope is less than critical slope, then it is called mild slope, it is called mild slope; now, coming to this condition, if we talk about say you refer to this diagram, say we are talking about critical slope here S c, and for that we are getting this much of depth y n which is nothing but equal to the critical depth.

Now, if our slope exceeds these values then this steep slope, and when slope is more than that depth will definitely come down depth will definitely come down like this. And then if we reduce the slope then depth will go high; for steep slope we are getting lesser depth than the critical depth; and for mild slope we are getting depth more than the critical depth. So, these are some of the understanding regarding that critical depth, steep slope, and mild slope; well, and we will be using these mild slope and steep slope, all these in the classification of gradually varied flow when that we will be discussing in the next class.

Now, till now we have got some of the relationship that indicate the condition of critical depth; one is that specific energy is minimum for a given discharge and depth which make this critical condition, that is where we get the specific energy minimum is called the critical depth, and that is why very basic definition we are getting this one; then again we got that for a given specific energy for a given specific energy when discharge is maximum, that particular depth we refer as critical depth, but apart from this relation we can derive some other conditions that indicate that can be used as index for say critical depth.

(Refer Slide Time: 50:08)



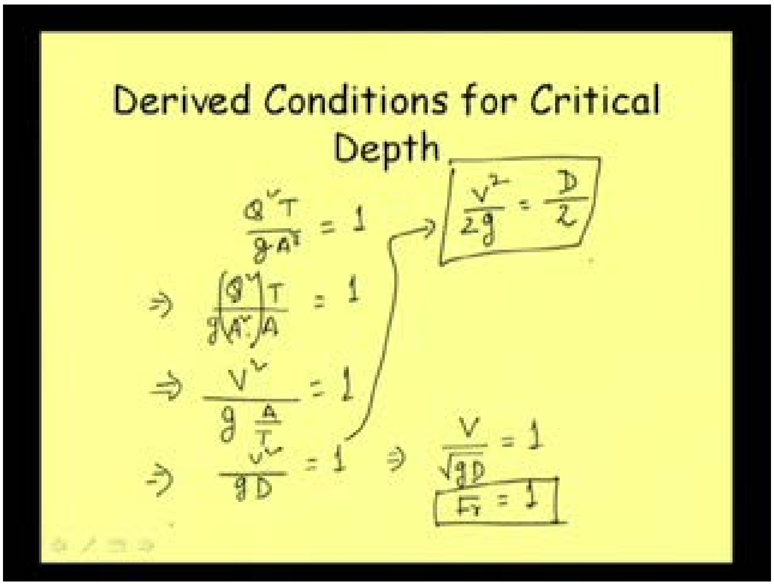
For example, let me start with this one, that is again say we got Q square T by g A cube is equal to 1, this is our first condition that we are getting; and then from this condition what we can write, this we can rewrite in the different way, because we know that A by T A by T is written as hydraulic depth D hydraulic depth D. So, using that d how we can write this, this is equal Q square, then a we can write in two part, say g is there, then A we can write as A square 1 then another A, and t is there, this is equal to 1.

Now, we know again that Q is nothing but area into velocity, so, Q by a is nothing but the velocity, so this implies from this condition what we can write that, if I put this within bracket then it will be nothing but V square and g will be there, and T by a that we can write as a by T here this is equal to 1, and this is nothing but say V square by g D equal to 1, we can write that V square by g D is equal to 1. Now, if we take square root of that expression square root of that expression, then what we are getting, that is, V by root over g D sorry V by root over g D is equal to 1, then perhaps all we remember what these expression stands for V by root over g D is nothing but the Froude number.

So, we can write that Froude number is equal to 1 Froude number is equal to 1, so that is one of the very very important relation that we get, and then we use this relation as a index sometimes when say critical depth, then we see where the fraud number is equal to

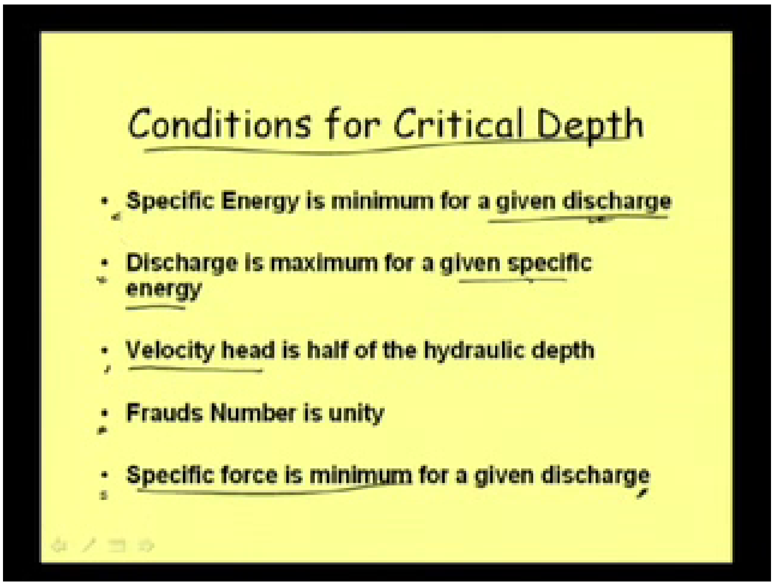
1 if it is 1 then it is critical depth; if it is more than 1 then it is supercritical; if it is less than 1 then it is subcritical. So, this is use very much as an index.

(Refer Slide Time: 52:51)



Well, then apart from this we can have some other relation; well, just let me take this part, and let me see how we can rewrite this expression from here, from here I could have gone this direction also, say V square by g D equal to 1, that I can write as say V square by let me write a 2 here and g let me write here this is equal to D I am putting on that side and as I am writing 2 here I am writing another two here. So, what this relation state that is V square by twice g is nothing but the kinetic energy that is kinetic head; and what is D by 2? It is half of the hydraulic depth. So, now, if someone asks that what is the kinetic energy at the critical flow condition then if we know the hydraulic depth, then we can say that yes kinetic energy at critical flow condition is equal to half of the hydraulic depth. So, that is also another relation.

(Refer Slide Time: 53:55)



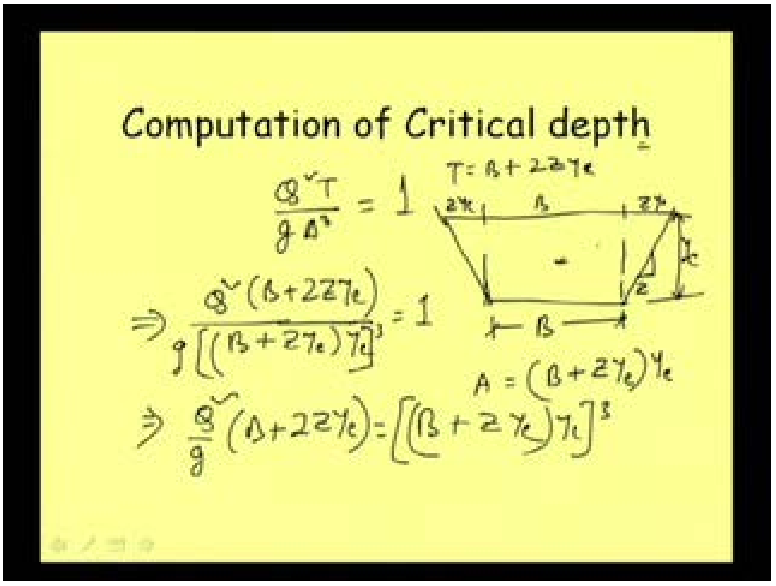
So, combining all this relation combining all this relation, now we are in a position to summarize that what are the different relation that we have got, and this can be summarized as that conditions that we have for critical depth. So, what are the conditions for critical depth first, of course, it is the specific energy is minimum for a given discharge, and always we should remember to mention this particular point that is specific energy is minimum fine specific energy is minimum, but this is for a given discharge, and that depth is called as critical depth.

Well, and second condition as we could see that discharge is maximum, then here when we say discharge is maximum, here we need to speak about discharge is maximum for a given specific energy, given specific energy, this is also important. So, all these things when the first condition we use to talk about, then we need to mention about specific for a given discharge when we talk about the second condition, it is for a given specific energy; then the third condition just we have seen that critical depth is that depth at which velocity head is half of the hydraulic depth, velocity head is half of the hydraulic depth, and then another very important condition that we derive is called Froude number is unity that is Froude number is one for critical condition.

Then another relation we have we are yet to go into this one, but till now when, but as we are summarizing this part, then we need to…, I feel that we need to mention this point just like the specific energy we got another term that is called as specific force, that is called as specific force; and that we did discuss when we were discussing the momentum concept and momentum equation, rather when we were starting from the momentum equation then we were discussing that we got one term which was which was indicating that this is this term is containing force component, and that term we gave as a specific force.

So, specific force we got that specific force in two consecutive section if we talk about say any two section, we were able to equate distance, of course, when our all the forces are in balance condition; then so that specific force if I draw then for specific force also we can have a diagram, we can have a diagram, that is depth versus specific force just as we started with energy equation here; if we start with the momentum equation or the expression for momentum, if we start then we can derive that relation that is the specific force diagram we will derive, and that will be doing in the next class, but still the relation what we will be the getting is that specific force is minimum for a given discharge specific force is minimum for a given discharge. So, both specific energy and specific force is minimum at critical condition, well. So, these are some of the important point that we have derived.

(Refer Slide Time: 57:21)



Then we need to go for computation of critical depth, because we are talking all these that critical depth is that even, this condition we get critical depth, then for many computation we will be requiring to know what is the critical depth; that means, we need to know what the critical depth is, and that is why…; suppose, the discharge is given a channel section is given then we need to compute the critical depth for that particular channel, and that is what we refer as computation of critical depth; and always we will start from this relation that Q square T by g A cube Q square T by g A cube is equal to 1 Q square T by g A cube is equal to 1.

Well, if we take a channel, say if we take a trapezoidal channel, just for example, then the side loop is suppose z, and if we say this is a critical condition of flow is occurring, that means, this depth we are referring as critical depth y c. Well, now, when it is y c, what will be this side, it is z y c, and that side is z y c, and then this bed width being b this bed width being b the top width we can write as, top width is equal to B plus twice z y c, and area as we know that, area of this section is equal to B plus z y c into y c B plus z y c into y c. So, that way we can write that, because this plus that divided by 2 means basically we are getting B plus z y c and multiplied by the depth y c, so we can write the area.

Now, putting these expression here, what we can get that, this is equal to Q square, then say T is equal to we can write B plus twice z y c twice z y c divided by say B plus z y c into y c, this is equal to 1 or we can this is a this should be cube, again this should be cube, this is cube Q square T by g A cube g will be here; so, this implies that Q square by g we can keep in one side, and then we can write that B plus twice z y c is equal to we can write the other part here that is B plus z y c into y c whole cube is equal to 1 sorry these two are equal.

Now, from here as we can see that we cannot separate out this y c if it is a trapezoidal channel, and it means again like our computation of uniform flow for computation of critical flow also we cannot separate out y c for this sort of trapezoidal channel, and we need to go for trial and error procedure here for computing critical depth; and of course, some approximate solution has been derived for this purpose also; and then for some simple cases like rectangular triangular we need to see whether we should go for trial and error or we can go for a direct solution or whether we have direct solution, so all those we will be discussing in the next class in more detail. Thank you very much.