

**Hydraulics**  
**Prof.Dr.Arup Kumar Sarma**  
**Department of Civil Engineering**  
**Indian Institute of Technology,Guwahati**

**Module No.# 02**

**Uniform Flow**

**Lecture No.# 06**

**Incipient Motion Condition and Regime of Flow**

Well, today we shall continue with our discussion that we carried out in our last class - that is the uniform flow in mobile boundary channel, and we will be concentrating today on the topic that incipient motion condition, that we left in last class, and then from there, we will start.

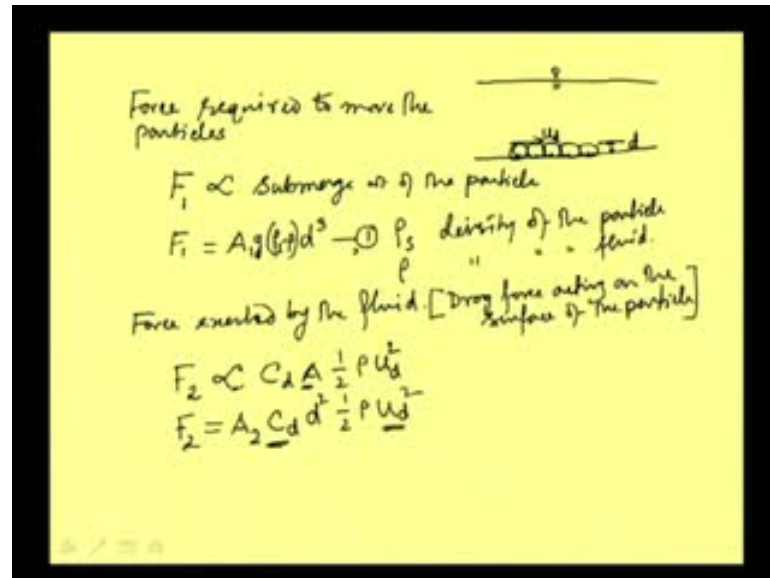
And then, how once this incipient motion condition is reached, and the sediment starts moving from the bed, then how it lead it to different regimes of flow; so, that we will be discussing. Just to recapitulate what we did in the last class, that we have seen, that to derive the equation of incipient motion condition, there are different approaches: one approach is talk about a competent velocity; then other approach **is talk** about a force, a critical force. Again, this critical force concept that we can again divide it into two parts or we can just see as **A 2**; part one is where we considered that lift force, another where we talk about the drag force.

And in hydraulic engineering particular in open channel flow, hydraulic engineers has found that the drag force approach is more logical, because the flow is moving on a surface, and the bed material that is over, which this forces is exerted is on a bed, and then, this surface is basically subjected to the force or drag force is more significant, and that way drag force approach is followed, and that way we get a term, that is referred as tractive force.

So, this tractive force, when this is just sufficient to drag the particle that is called critical tractive force, and Shield the scientist, who conducted lot of experiment, and observed data, and then he gave some theoretical understanding of the entire process, and based on this theoretical understanding, he used some observed data, and then he derive some

relationship; so that, we can call as a semi theoretical analysis, and let us start from that particular point.

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Well, incipient motion condition, suppose this is the bed, and let me just draw the particle little bigger, means, just to have, say that way the particles are there, and diameter of the particle is  $d$ , diameter of the particle is  $d$ , so we can write this as  $D$ .

And then, when we talk about the drag force, the flow is like that, flow is moving like that, and when we talk about drag force, then basically we talk about a force at this level, on the particle, and suppose, the flow velocity at this level, we can write as  $u$ , and as it is a depth  $d$ , so we can write it as  $u d$ , that is the velocity, just at the surface of the particle. Well, now, these particle are, when we are talking about erosion by the fluid, that means, these particular are always under water; so, we are interested to know the force required to move this particle, and then we will be, we first need to see that what force will be required and to which this force will be proportional.

So, if we write that force required, force required to move the particle, force required to move the particles, this force is rather than going for a definite value. First, let me write this force is always proportional to the weight, and then, when we say weight, we have to write the submerged weight, because this will be always submerged; so, we can write is proportional to the submerged weight of the particle. Now, starting with that what will be the submerged weight, that we can write, that  $F_1$ ; now, we can introduce a

proportionality constant say  $A_1$ , let it be there, this be the proportionality constant. And then weight will be proportional to the volume of the particle, and the volume of the particles will be proportional to the diameter cube; so, we can write, that it is proportional to the diameter cube of the particle, and then, as we are already using a proportionality constant  $A$ , so we can just keep it as proportional to the volume, then we write that it is the  $\rho$ , it is the mass, so we can write.

This  $\rho$  means, it is, what we can write, that this is the submerged weight, we are talking about, so we will have to write  $\rho_s - \rho$ , and what this  $\rho$ , as basically we are writing here submerged density, and then, we are multiplying this by the  $g$ , that is  $g$  into  $\rho_s - \rho$ , this is giving us the submerged unit weight, where  $\rho_s$  is the density of the particle,  **$\rho_s$  is the density of the particle**, sediment particle, and then  $\rho$  is the density of the fluid, density of the fluid, here it is water.

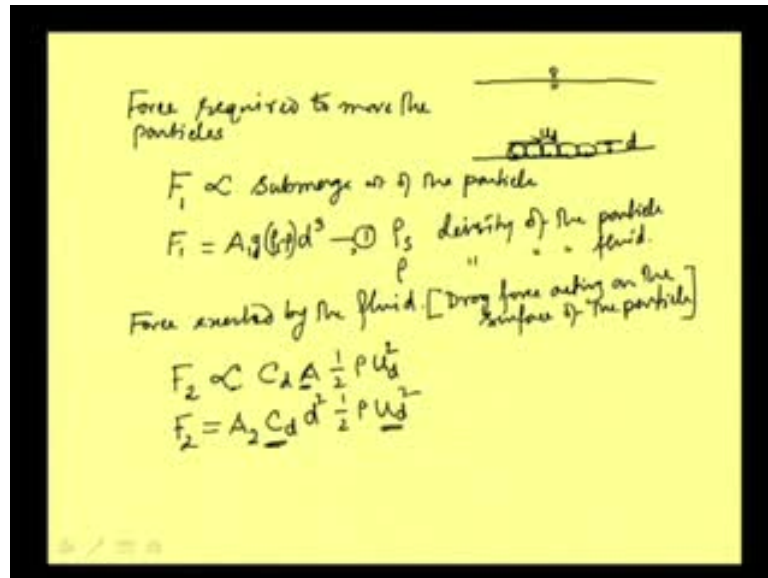
So, when we multiply it by  $g$ , we are getting unit weight, and that  $\rho_s - \rho$ , that is why we are getting submerged unit weight of the particle, and this  $F_1$ , this force will be proportional to the submerged unit weight of the particle, so we can write it like that. So, let us keep this as equation say 1, then once we have got the force required to move, then we need to know that, what will be the force exerted by the flowing fluid, just on the surface of that particle; so, we need to know the force exerted by the fluid, and that force we are coming as we explain in the last class also, we are talking about the drag force, because we are going by the tractive force concept.

So, this is nothing but we are talking about the drag force acting on the surface of the particle; so, here we need to go back to our understanding of fluid mechanics, what is drag force and lift force, but of course, time will not permit us to go much back, but still, we can write down the expression for drag force here, as say this force is, if we write it as  $F_2$ , **if we write it as  $F_2$** , then we can just write in the way, that it is also proportional to, **it is also proportional to**, we are using  $C_d$ , then of course, a representative area  $A$  and then half of  $\rho$ , then  $u^2$ .

Basically, it is the velocity square, but here our velocity, we are talking about at the surface of the particle, and that we are naming as  $u_d$ ; so, this  $u_d$ , means, the velocity at this level at a height of depth  $d$ , which is basically nothing but the particle size. Again,

what should be this d, there also lot controversy as we say that d is the average particle size.

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Now, this average particle size, means, when we talk in real situation in a river bed, if we see that particles are will be of different sizes, and then, if we carry out a sieve analyses of the particle, we will be finding that gradations are different, and **then which value**, actually we will be taking some support, that well we need to take d 50, some say that d 90, like that well anyway.

Now, after lot of analysis, it is seen, that this size, though these are different, d 50 and d 90, but say it does not influence that much on the frictional characteristic ultimately, I mean, small difference, because these difference, main particles size is there, and then within that particle size, when we are talking about d 90 or d 50, it do not make much difference, and that is why people go with d 50 in most of the time, well. Now, we are considering that, suppose it is given that d is the value of the particle size, then we can go for u d and u d is the velocity at a depth d, that is by concept we are writing, that this is the force.

Now, what will be this area; again, when are talking about a particle of diameter d, then area is also not known, because this will be different, if it is a flat particle, then the area will be more, if it is a round particle area will be different. So, generally we can just consider conceptually, that this area, like that volume is proportional to the diameter

cube, area we can have as proportional to the d square, so what we can write that  $F_2$ , then let me write as equal to, and we are introducing another proportionality constant  $A_2$ , which will take care of all the proportionality. So, this is say  $C_d$ , and this  $A$ , we can write now as say  $d$  square, means, the diameter square, and then it is half rho  $u$   $d$  square.

So, this is what our expression is, then again from here, we can just express what will be the  $C_d$ , and what will be this  $u$   $d$ , here just we are writing  $u$   $d$ , but in reality, it will not be possible to know what will be the value  $u$   $d$  directly, so we need to simplify that and we need to have it in a form, which value we can have very easily.

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$$F_D = A_2 C_D d^2 \left( \frac{\rho u^2}{2} \right)$$

$$\frac{u}{u_*} = f \left( \frac{u_* d}{\nu} \right)$$

$$\Rightarrow \frac{u_* d}{\nu} = f^{-1} \left( \frac{u}{u_*} \right)$$

$$\Rightarrow u_* d = \nu f^{-1} \left( \frac{u}{u_*} \right)$$

$$C_D = f_1 \left( \frac{u_* d}{\nu} \right)$$

$$C_D = f_1 \left( \frac{\nu}{u_* d} f^{-1} \left( \frac{u}{u_*} \right) \right)$$

$$C_D = f_2 \left( \frac{u_* d}{\nu} \right)$$

Well, so, these value, we can now modify starting from this formula, well; so, we are writing that  $F_2$  is equal to  $A_2$  into  $C_d$  into  $d$  square into half of rho  $u$   $d$  square, half of rho  $u$   $d$  square, from that, if we see, that we have already one relation that we analyzed, that is what is the velocity distribution, we are interested to know what will be the  $u$   $d$  at this level, the particles are like this, what will be velocity  $u$   $d$  at this level, and we can take help of our understanding of the velocity distribution.

And that we have that  $u$  by  $u$  star, already we have discussed in our last class, this is a  $u$  by  $u$  star is a function of sheer Reynolds number, that is nothing but  $u$  star  $d$  by  $\mu$  in reality, it is  $u$  star  $y$  by  $\mu$   $u$  star, let me write it  $u$  star  $y$  by  $\mu$  first  $y$ , means, any depth  $u$  star  $y$  by  $\mu$ , this is the sheer Reynolds number.

Now, for our case here, say when  $u$  we are interested in  $u_d$ , that is what the velocity, this expression is general at any depth  $y$ , we are talking about this velocity  $u$ ; now, our interest is what is the velocity at this top, when depth is  $d$ , so we will be writing this as  $u_d$  and  $u_{star}$ , and this is a function of, that is why our depth will also be  $u_{star} d$  by  $\mu$ ,  $u_{star} d$  by  $\mu$ . And then, we can put that as we said that  $u_d$ , we do not understand what the value is or it is difficult to measure, so we are bringing it in terms of  $u_{star} f$ , then  $u_{star} d$  by  $\mu$ .

So,  $u_d$  is a function of Reynolds number, sheer Reynolds number and a product of  $u_{star}$  well. Then, again if we go back to our fluid mechanics understanding, then we have that  $C_d$ , that is the co-efficient of discharge, **sorry, this is the drag co-efficient**, generally we write this capital  $D$ , let me write it as  $C$  capital  $D$ , that is the drag co-efficient  $C_D$  is also a function of, we can write this is also a function of Reynolds number, that we can write  $u$ .

And we are talking about drag force at this level, so the corresponding velocity here is  $u_d$ , and our depth is  $d$ , so  $u_d d$  by  $\mu$ ,  $C_D$  is a function of  $u_d d$  by  $\mu$ , that means, it is a function of Reynolds number, but this Reynolds number, so we are not talking about sheer Reynolds number, we are talking about Reynolds number at a depth  $d$  for the velocity, actually  $u_d$  Reynolds number for the velocity  $u_d$ , and our depth here is  $d$ ; so, this is the drag.

Now, this, if we write, if we write  $u_d$  in place of  $u$ , if we just put  $u_{star}$  this thing, then what we can write that  $C_D$  is a function of, so  $u_d$ , I am replacing by  $u_{star}$ , and that I will be writing on the right side, and that is, say  $f$  of  $u_{star} d$  by  $\mu$ , that is there, and this part  $u_{star} u_d$ ,  $u_d$  is replaced by  $u_{star}$  into this, and then  $d$  by  $\mu$ .

So, again, what we are finding finally, that  $C_D$ , ultimately this is also a sheer Reynolds number; this is also a sheer Reynolds number, so it can be written as a function different function, say  $f_1$  of say  $u_{star} d$  by  $\mu$   $u_{star} d$  by  $\mu$ . So, we can write  $C_D$  as a function of, say  $u_{star} d$  by  $\mu$ , of course, this two functions are not equal function, so if I am writing this as this, as  $f$  this can be written as, say different function name, we can give say  $f_1$ , then we can write this as  $f_2$ , which is combining this function as well as that function, so this we can write  $C_D$  is equal to that one.

Now, replacing this value of C D in terms of sheer Reynolds number, and then of course replacing this u star square what we can write let us see.

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The image shows a handwritten derivation on a yellow background. It starts with the equation  $F_2 = A_2 C_D d^{\frac{1}{2}} \rho u_d^2$ . This is then rewritten as  $F_2 = A_2 f_2 \left(\frac{u_d d}{\nu}\right) d^{\frac{1}{2}} \rho u_d^2$ . The next step is  $F_2 = \frac{1}{2} A_2 d^{\frac{1}{2}} f_2 \left(\frac{u_d d}{\nu}\right) \rho \frac{\tau_0}{\rho}$ . Finally, it is simplified to  $F_2 = \frac{1}{2} A_2 d^{\frac{1}{2}} \tau_0 f_2 \left(\frac{u_d d}{\nu}\right)$ , which is labeled as equation (2). To the right, there is a note  $u_* = \sqrt{\frac{\tau_0}{\rho}}$ .

Again let me just keep this equation here,  $F_2$  just for our convenient, we can keep this original equation here, we were writing that  $A_2$ , then C D, then d square, then half of rho u d square. And now, let me replace this term, that is  $F_2$ , we can write this is equal to  $A_2$  will remain as it is, and then C D, we can write as a function, say  $f_2$  into u star d by mu kinematic viscosity, well this diameter d square, we will keep as it is plus rho.

And then, this u d square, that we can write as u star because u d we know that this u star into function of u star d by mu, that we have already expressed, so when it is this part is basically u d, so when it is square, we can write u star square, and this function square, so that way we can write rho. Now, so rho is here and this part we can write like that, and then, what this value of u star, we know that u star is nothing but it is equal to root over tau 0 by rho, so here it is coming again the sheer velocity, and we are talking about the sheer stress at a particular point; so, this sheer stress at a particular point and divided by rho, sheer particular point, means, we are talking about the bed sheer, bed sheer divided by rho.

So, this we can write this is equal to  $F_2$  equal to  $A_2$ , and all these are now becoming functions of u star d, so we will keep it as a function of u star d, so d square, I am bringing here, this is not plus, this is a product or we get d square dot, so this is basically

half, this is half is there, then we are writing, that this is a function of  $u \star d$  by  $\mu$ . And then, we can write this one as  $d$ , square is already written  $u \star$  square, that we can write  $\rho$ ; now,  $u \star$  square will be  $\tau_0$  by  $\rho$ ; so, here we are introducing the  $\tau_0$ , that is just sheer stress term and this is already included there.

So, finally this expression can be written as  $F_2$  is equal to, I think half was there, so let me put this half here half, so half of  $A \cdot 2 \cdot d$  square and  $\tau_0$  this  $\rho$ , **rho** is going, and then, it is a function of  $u \star d$  by  $\mu$ , well. So, let us put this equation as equation 2.

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Force required to move the particles

$$F_1 \propto \text{Submerge} \rightarrow \rho \text{ of the particle}$$

$$F_1 = A_1 (\rho) d^3 \rightarrow \rho \text{ density of the particle}$$

Force exerted by the fluid. [Drag force acting on the surface of the particle]

$$F_2 \propto C_d A \frac{1}{2} \rho U_d^2$$

$$F_2 = A_2 C_d d^2 \frac{1}{2} \rho U_d^2$$

And already we have this equation 1, if we go to that, this is our equation 1. Now, what is that for incipient motion condition, for incipient motion condition, **we**, our basic condition is that, these two forces are equal.



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So, what we will write for incipient motion condition,  $F_1$  is equal to  $F_2$ , and of course, when  $F_1$  is equal to  $F_2$ , you are talking about what will be our bed shear stress  $\tau_0$ ,  $\tau_0$ , that we can now write as, because when these two are equal, that is what our critical condition, this is what our critical condition, so at critical condition, this bed shear stress we are writing as  $\tau_c$ , say critical bed shear stress.

So, equating these two, we can write it as say  $A_1 g \rho_s \text{ minus } \rho$ , this is the expression for our equation for  $F_1$ ,  $d^3$  is equal to half of  $A_2 d^3$ , then  $\tau_0$ , and  $F$  some function of, we are writing  $u_* d$  and  $\mu$  not  $\tau_0$ , now we will writing it as  $\tau_c$  and  $f$ , we have generalized now, it is a function of  $u_* d$  by  $\mu$ .

From here, this part are constant  $d^3$   $d^3$  square is there, so  $1/d^3$  will be there, we can just go as, say let me keep  $\tau_c$  here  $\tau_c$ , and then let me write it as  $g \rho_s \text{ minus } \rho$ , and then,  $d^3$  will be left here, and then, this is equal to we can write this is  $A_1$ , so twice  $A_1$  by  $A_2$ , we are getting that one, this one here into this, we can write as  $1$  by function of this thing or again, in a sense, we can write that, this is a function of, well to avoid confusion, let me write it as  $1$  by  $f(u_* d / \mu)$ , but it does not matter,  $1$  by these things or we can write it as a function of this one.

So, now, this expression says twice  $A_1$  by  $A_2$ , and we can say that, this is basically a function of  $u_* d$  by  $\mu$ , well. Now,  $f$  dash, suppose this a function, so what we have seen that, this critical shear stress, our interest is to know this critical shear stress, so this

critical shear stress divided by  $g$ , and then  $\rho S$  minus  $\rho$ , basically it is the submerged density, we are talking about, that is why it is coming like that, and the diameter of the, of the particle  $d$ , and then, some constant, this part is some constant, and then, **it of**, this  $u$  star  $d$  by  $\mu$ .

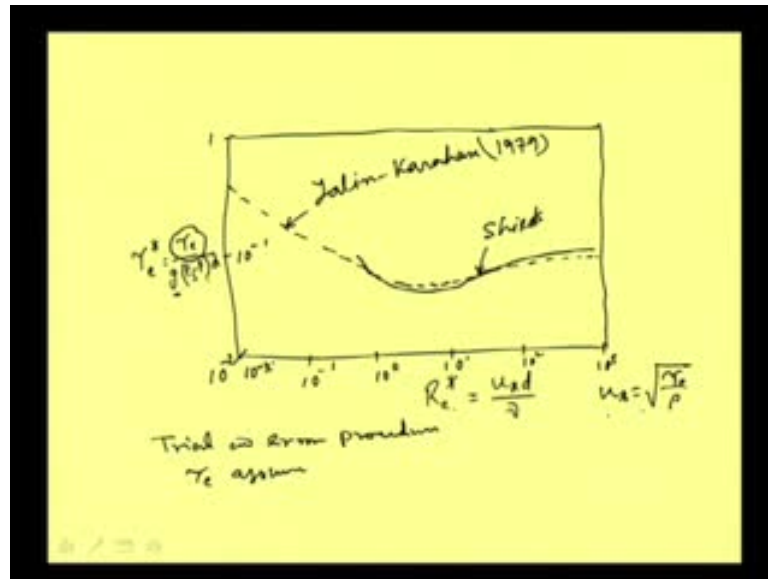
And this term, this term was expressed as that this term was expressed as  $\tau_c$  star, let me write here, this term was expressed as  $\tau_c$  star is equal to, ultimately we can call  $\tau_c$  star is equal to a function of, ultimately it is again a function, **this is**, these are constant basically; so, as say for a given dimension of the particle for a given dimension of the particle, this term will be more or less constant, so we can just consider that  $\tau_c$  star is proportional to or a function of rather not proportional is a function of  $u$  star  $d$  by  $\mu$ . And this  $u$  star  $d$  by  $\mu$ , we can write as  $R$ , so this can be written as function of  $R_c$  star; this  $R_c$  star is called shear Reynolds number, this  $R_c$  star is called shear Reynolds number.

So, this  $\tau_c$  star, which is related to the critical shear stress, is can be expressed as a function of shear Reynolds number  $R_c$  star. Now, with this very basic relation, let me write somewhere, it is in a very clear form, that is, say let me take this part  $\tau_c$  star is equal to a function of, say  $R_c$  star, this ultimately we are getting, well.

Upto this much the analysis, what is being carried out is, of course, this we can do by dimensional analysis, as well considering that what are the factors influencing, well I can give a brief view of that, but now, right at this moment, we can consider that  $\tau_c$  star is equal to  $f R_c$  is a function of shear Reynolds number, and then, shield he did lot of experiment, lot of data were collected by his follower also and then a graph was drawn.

So, these very basic relation with the understanding from the analytical concept was used, and lot of data were collected from experimentation, and then, these were plotted by shield, and of course, some other experiments has also been done later. Well, how the shield got this relation that you can concentrate in the slide, now.

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On this side, if we plot  $R_c^*$ , that is the shear Reynolds number, and then on this side, if we plot the  $\tau_c^*$ , and what is  $\tau_c^*$ , that we know that, this is equal to  $\tau_c$  divided by  $g \rho S - \rho$  into  $d$ , so this is what  $\tau_c^*$ . And shear Reynolds number, that is,  $R_c^*$  is nothing but this is equal to  $u_* d$  by  $\mu$ . Well, these two were plotted, can be plotted and with the scale say it is in log scale, it is 10 to the power of minus 2, starting from small value, then say here it is 10 to the power minus 1, and here it is say 1, so that way we can go.

Then, on this side, it is 10 to the power minus 2, then 10 to the power minus 1, then 10 to power 0 means 1, then 10 to the power 1, 10 to the power 2, 10 to the power 3, so that way if it is plotted. Then this observations form in a scattered diagram for different values, and then it could be drawn, it could be drawn in the form, the shield what he got, initially he was able to draw it in this form. And then, it was going like this, that means, after certain value of  $R_c^*$ , when  $R_c^*$  exceed certain value, then this part become almost straight, and we can have that, this  $R_c^*$ , for that  $R_c^*$   $\tau_c^*$ , we can have constant almost value, but this experiments were carried out, so with less number of data, so this curve we refer as shield's curve.

Then after that Yellin and Karahanin 1979, they published some work with the same experiment, but with lot of data, lot more data, and the curve could be extended up to this portion, say their curves came like that, their curves came like that, and then, it went

almost in the same direction, then it was going like this, and then, that curve we can call, that is why as per their name Yellin and Karahan in 1979 well.

So, they gave that data and from that, now we have a general relationship between this  $R_c$  star and  $\tau_c$  star for a wide range of value, **for a wide range of value**, and now question is that, how to find the  $\tau_c$  star or our interest is to know, what the value is  $\tau_c$ , how to find that.

Basically, how we do it or how it is to be done that, first it is a trial and error procedure need to be adopted; of course, some approximate network is there, but we can take trial and error procedure, means, by simplifying this relation, we can have some approximate value, generally a  $\tau_c$  star is assumed,  $\tau_c$  is assumed.

So, once we assume the  $\tau_c$ , we know what is  $u^*$   $u^*$  is nothing but  $\sqrt{\tau_c}$  by row  $u^*$  is nothing but  $\sqrt{\tau_c}$ ,  $\tau_c$  is nothing but the bed shear, so we were getting  $\tau_0$  by  $\rho$ , but it is a same thing, because it is the bed shear, but in critical condition, so we can write  $\tau_c$  by  $\rho$ ; so, that way we can get  $u^*$ , we know suppose a particular size of the bed material, the effective size we can take, then we know this new value, so for that  $\tau_c$  will be getting a  $R_c$  star value; now, from this, **our** this  $R_c$  star, **we**, from the graph want to see, what is your  $\tau_c$  star.

So, once we get the  $\tau_c$  star, again  $\rho$   $g$  and  $d$  are known, so we can find out what is  $\tau_c$ ; now, our assumed value of this critical tractive stress  $\tau_c$ , if it is matching with the value, that we are getting from the graph, then it is, that means, our assumed value is correct, if not we need to assume another value of  $\tau_c$ , calculate  $R_c$  star see, what is the value of  $\tau_c$  star and that procedure will continue.

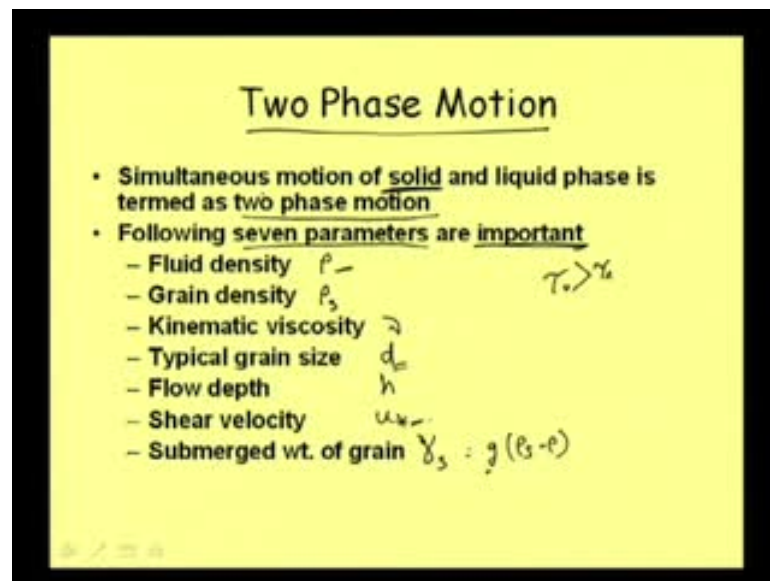
And then, once we get the two value are matching, then we can have our required value of  $\tau_c$ ; so, that way for a particular situation, we can get the value of critical tractive force, well but for irrigation channel, because generally, when we talk about irrigation channel, then these sizes are almost known to us, that it will be say around 1 to 2 meter width.

And then, say it will be having a depth varying between say 1 to 21 meter or around that, now for those situation, several experimentation has been done, and then, some

investigator has given the value of  $\tau_c$  critical shear stress, as a direct expression relating this value, that is  $\rho_s d$ , using these expression we had given these value.

So, some empirical direct relations are also there, and these are also empirical, but it is more extensively done putting lot of value, and so, the  $\tau_c$  star calculated by this value will be more accurate; of course, assumption is always there in all these analysis; well, with this understanding of  $\tau_c$  star or this  $\tau_c$  shear stress value.

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Now, we can go to just see, that when our value of this  $\tau_c$  exceed, when our value of  $\tau_c$  exceed, that is **the** not  $\tau_c$ , when our value of bed shear stress  $\tau_0$  exceed, then our particle of the bed starts moving, that is when  $\tau_0$  is greater than  $\tau_c$ , then the particle starts moving.

Of course, if  $\tau_0$  exceed by small amount, what will happen,  $\tau_0$  exceed by small amount than the  $\tau_c$ , then what will happen, then when  $\tau_0$  exceed by large amount, what is happening, again this small and large very every wide, as I said earlier, so how much we should give some index.

And as I said earlier, this sort of motion, we can analyze by dimensional analysis also, and when the sediment part is moving into the fluid or water, suppose when the sediment is also in motion in the water, then the entire flow, we deal as A 2 phase motion, we deal

as A 2 phase motion, and this two phase motion, when we find, then this sort of motions are effected by several parameters, several parameters.

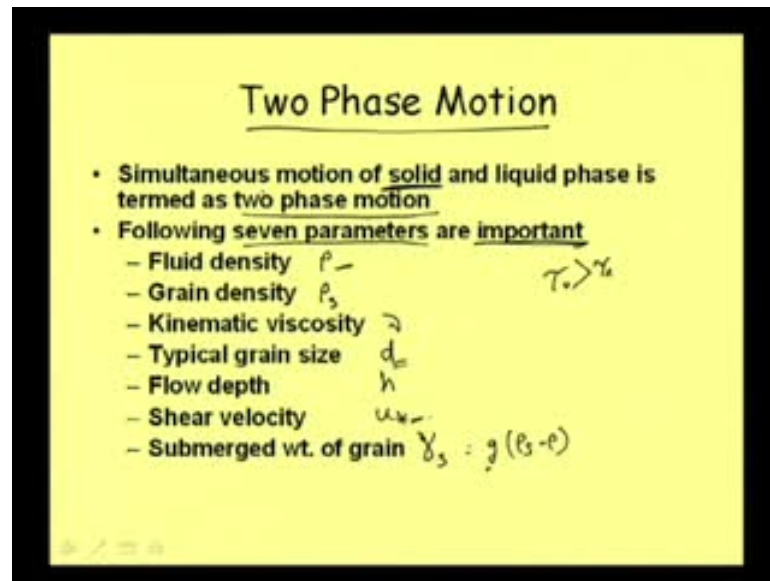
And of course, in the last discussion, we have seen how Shield could relate this relation up to incipient motion condition, and then after that, when incipient motion condition is over, I mean, we are exceeding this value, then what is happening, these are generally always in the observed form, and some relationship, some equations has been derived on that, that is again some semi theoretical relationships are there, and then generally dimensional analysis become very necessity for having understanding, what sort of parameter can be related for having that sort of equation.

So, let us have some idea on these aspects also, please concentrate into the slide, we will be discussing something on two phase motion. Well, to discuss the two phase motion, what we basically mean by two phase motion, that definition wise, we can write, that is a simultaneous motion of solid and liquid phase; by solid, actually we are meaning that the sediment part, in our hydraulics in the flow of channel, we are meaning, solid means the sediment part; of course, the sediment can be moving along the bed, it can be moving sometimes, I mean, little higher into the bed or getting completely mix up with the flow; so, that things will be coming, but in any form simultaneous motion of solid and liquid phase is termed as two phase motion, and several parameters, actually influence this sort of motion to have a proper understanding of two phase motion, we need to have proper understanding of several parameter, how they are interacting, **how they are interacting.**

And of course, when we go for deriving some relation, the physical understanding is in one hand, but sometimes, we observe something, and the physical process sometimes is so complex, that we cannot really have a proper understanding of what the internal things are going on, but still, at final level, although we sometimes may not understand, but we can at least have the understanding, that some of the parameters are obviously influencing the flow, and those parameters, if we take into consideration, then we can derive some mathematical model out of them.

Let us see that following, and that way we can summarize that following seven parameters are important; so, I emphasize that we are using the term, important it is not that other parameters are not influencing or other parameters can also be influencing, but which are significant, that way we are writing that following parameters are important.

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So, the fluid density, say rho fluid density rho, that we are talking about fluid density is definitely influencing, if a high density fluid is coming, **this will be**, this impact on erosion will be different. And then, grain size density this is obvious, if a lighter size is there, it will be eroded quickly; of course, quickly means, again there is, it is not straight way lighter, means, if it is finer, and have suppose lot of cohesive property in it, then whole phenomenon may change.

But considering these as incoherent material, that means, without having cohesive property, we are just talking about; so, without having that cohesive property, this density less means it will be eroded quickly.

Then, kinematic viscosity that we are always getting, that mu is a influencing parameter, then typical grain size, that I have already told, that typical grain size, say if we give as d, and what will be the typical, that is why typical term is used, whether it should be d 50, d 90, but we are considering this as typical, and **we will be, we are**, we can say that, right at this moment, we can consider this as d 50, say of the particle size or by the analysis, what we are getting from the field, from that we can take say d 50.

Then flow depth, flow depth h is also influencing the erosion, then shear velocity u star, this is very, very important, I mean, element this is very, very important element, and the submerged weight of the grain, **submerged weight of the grain**, and then, that is basically, so it is grain density, and I am writing as gamma s; so, this is basically nothing

but gamma s minus gamma into g or say we can write as gamma s in a single term, that is the submerged density, submerged unit weight of the grain, and that of course, we can write as g into say rho S minus rho.

So, these are the parameter, that is influencing, basically submerging density means here extra term is coming, already rho and rho S we are taking here, but this we are keeping as a separate parameter.

Well, now, we know the very fundamental concept of dimensional analysis, and so, in this part, if we carry out dimensional analysis as these are the influencing parameter, if we take repeating variable, as say rho and say depth, the typical drain size d, and if we take the u star as the repeating variable.

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**Dimensionless parameters**

- **Grain size Reynolds number**  $X = \frac{u_* d}{\nu}$
- **Mobility number**  $Y = \frac{\mu u_*^2}{\gamma_s d}$
- **Influence of Depth**  $Z = \frac{h}{d}$
- **Influence of grain density**  $W = \frac{\rho_s}{\rho}$

$\frac{u_* d}{\nu} = \frac{u_* d}{\frac{\mu}{\rho}} = \frac{\rho u_* d}{\mu}$   
 $\frac{\mu u_*^2}{\gamma_s d} = \frac{\mu u_*^2}{\frac{\gamma_s}{g} d} = \frac{\mu g u_*^2}{\gamma_s d}$

Then we can by dimensional analyses, we can see that interaction of these parameters can be group into some non-dimensional term, and we can say that these non-dimensional terms are influencing the flow, and that is, one term is very familiar to us, already that we can name as grain size Reynolds number, that is say u star d by mu, we are using that already, we are talking these as we were talking, when we were doing the shield theory, then were just talking about that particular value as sheer Reynolds number, the same term, we can name this as a grain size Reynolds number, because in this Reynolds number, our length characteristic that is coming.



Reynolds number general expression is basically  $u$ , then  $l$ , then say  $\mu$  or say we can write  $\rho v l$  by  $u l$  by  $\mu$ , this  $l$  is basically the length characteristic, and this length characteristic in this case is coming, though in fact, when we were analyzing in our shield theory, **it is the**,  $d$  is coming because we are talking about a shear force acting at a depth of  $d$  on the top of the particle size, but in general, ultimately this length dimension is coming as the diameter, so we can name this as a grain size Reynolds number  $X$ .

And then, another number mobility number is coming, that is  $\rho u^*$  square, then  $\gamma_s d$ , again this term is nothing but the  $\tau_c^*$ , this term is nothing but the  $\tau_c^*$ , as we can see that, say  $u^*$  square is say, we know that  $u^*$  is equal to root over  $\tau_0$  by  $\rho$ ; so, this is equal to  $\tau_0$  by  $\rho u^*$  square is equal to  $\tau_0$  by  $\rho$ .

And  $\gamma_s$ , already I have written that  $\gamma_s$ , we can write as here itself, I have written  $\gamma_s$  we can write  $g \rho S \sin \theta$ . So, this term is nothing but we can write it as, say it is  $\tau_0$ ,  $\rho$  and  $\rho$  will get cancel, this is  $g \rho S \sin \theta$  into  $d$  and that is what we got as  $\tau_c^*$

So, by dimensional analysis also we are getting that, these are the term which are basically related, which are basically influencing the flow phenomenon, when it is in two phase motion, then of course, the depth factor  $z$ , that is also used, this is say  $h$  by  $d$ . And then, this grain density  $w$ , this is another dimensional list term, that is  $\rho_s$  by  $\rho$ ; these are also influencing the flow, but out of these we can see that, this can be related to that one, this can be related to that one that is for critical stage of mobile bed.

Now, we have seen that, when we are talking about this dimensionless parameter, two phase motion is influenced by all these parameter, all these parameter, and when it is in critical stage of motion, that is when just in the incipient motion condition, then this  $x$  can be related to  $y$  in critical condition.

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**Dimensionless parameters**

- For critical stage of a mobile bed,  $Y_{cr} = f(X_{cr})$ . This is in the form of shields equation

Ratio of Mobility number

$$\eta = \frac{Y}{Y_{cr}}$$
$$\tau_{c*} = f\left(\frac{R_{cr*}}{D_{cr*}}\right)$$

$\eta < 1$  - flow bed without sediment movement.

$\eta > 1$

$1 < \eta < 10$

Diagram: A cross-section of a riverbed showing 'Suspended load' and 'bedload'.

So, we can write that for critical stage of a mobile bed, **y critical**, y critical is a function of x critical, y critical is a function of x critical, and that already we have got in our earlier discussion that tau c star, which is basically, we are talking about tau c star in critical condition, and this y critical, what we are writing here is nothing but the tau c star. So, we are already getting that tau c star is equal to a function of say R c star, that we got, and this x c r is the nothing but this R c star, so same expression basically.

Well, and then, this y c r mobility number; now, this number is stated as mobility number and how much amount will be eroded or what will be the stage of erosion or say stage of covering, that can be studied in terms of a number, that we call that ratio of mobility number, because mobility number is very, very important which is influencing this sort of motion or movement of particle from the bed, and without going much detail, we can write the ration of mobility number, say eta is equal to Y by Y critical.

That means, at a particular instant of time, I mean, for a particular velocity and particular combination of all this parameter tau c star, what we are getting for a particular combination. If, you calculate this mobility number, and we know that, what is the mobility number in critical condition, which is a function of say x c, a ax critical, that is the R c star, so we can find a ratio eta. And it is seen, that when eta is less than 1, then basically we have not reached the incipient motion condition, there is no flow will be moving.

Well, here we can have a diagram, like that say this is the bed, and there can be some roughness, of course, and then, we can see that, we can draw a line, this is, we can call epsilon means up to certain depth, why we are dividing, because in some of our later period, we will be experiencing those things; on this side, we are drawing the  $y$  depth.

Now, when  $\eta$  is less than 1, **mobility**, ratio of mobility number  $\eta$  is less than 1, then our particle are not moving. So, **we are getting a plain bed**, we are getting a plain bed **without sediment motion movement**, without sediment movement. And for that situation, **our all understanding of rigid boundary channel are valid**, our all understanding of rigid boundary channel are valid, because our particles are not moving, it is just behaving like a rigid boundary, but when our  $\eta$  is greater than 1, then particle start moving, then particle start moving. Initially, it will be just leaving from the boundary, and it will be spreading like this in the bed only, it is spreading, and gradually it is moving.

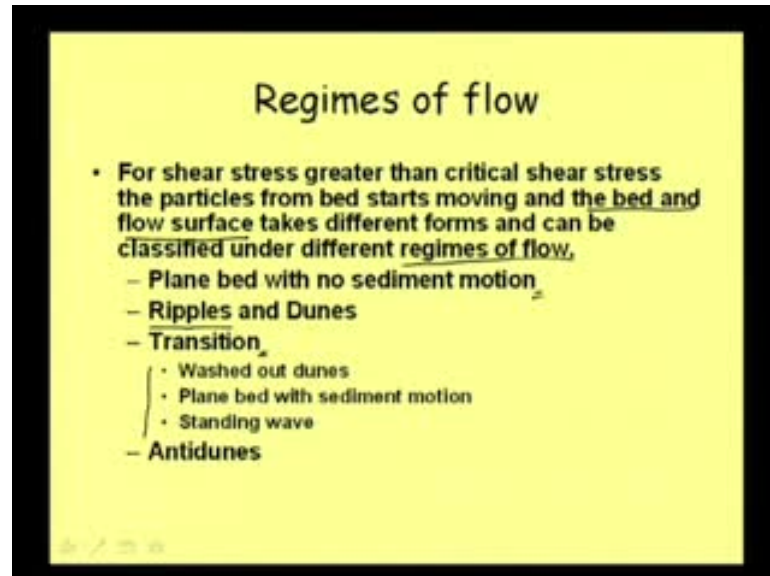
So, it is just spreading in the bed and it is moving, so that is the incipient motion condition we are getting, then when  $\eta$  gradually exceed, that value with the increase of discharge, then the stress increases, and then our  $\eta$  exceed that value, then when  $\eta$  between 1 to 10 particle, if I consider, it start jumping like this, it will follow some part like this, say we can call that particle part of the material of the bed, I am putting bed, so it is jumping, then rolling, like that particles are moving within a very small distance epsilon, that is why this distance was small distance, epsilon from the bed, and that part of sediment movement, we call as bed load, we call as bed load.

Generally, this term will be coming in our many analysis, that how much is the bed load, how much is the suspended load like that, and then when  $\eta$  exceed further, suppose  $\eta$  is exceeding beyond 10, then the particle will start moving into this portion in a very zigzag way with lot of turbulence, and this will be the part of suspension, and we do not know exactly which is the part, **it will be**, so it is moving like that, and then, this part is actually we refer as suspended load, **suspended load**.

So, in two phase motion, we get some particle moving, just near the bed by jumping rolling, and then this sort of motion or this sort of sediment component, we call as bed load, and then, some other particle are moving into with the increase of sheer stress with the increase of force, it is moving, it is getting into the main flow, and then it is getting

completely mix up with the flow, and then, this flow, we call as a suspended load; so these two component, we will be getting in two phase motion.

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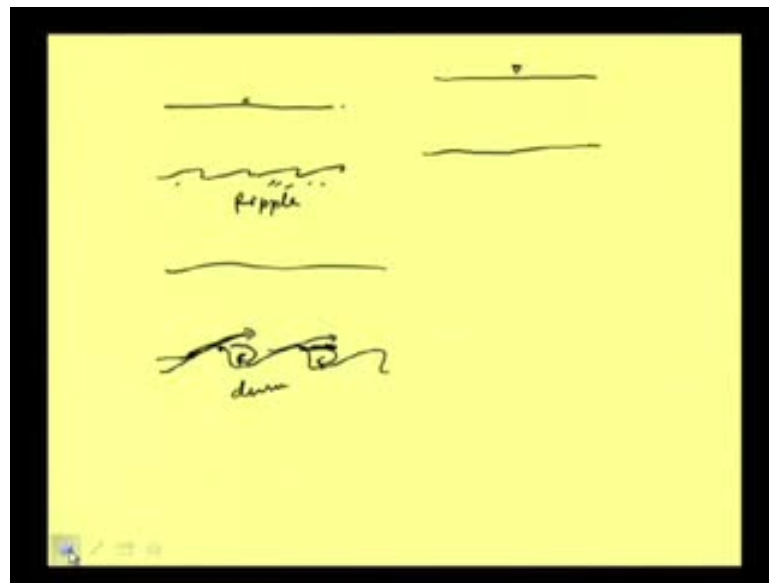
Well, with this very basic understanding of that suspended load and bed load, now we can talk about regime of flow, we can now talk about regime of flow; so, what we mean by regime of flow, that is for sheer stress greater than critical, sheer stress, sheer stress greater than critical sheer stress, the particles from bed start moving, and the bed and the flow surface takes different forms, and that is, this is important the bed, and the flow surface takes different form, and can be classified under different regime of flow, **different regimes of flow.**

So, when bed form is changing, then actually the surface of water is also changing. So, first of course, as we know, that it is this plain bed **without, with** no sediment motion, this is just as rigid, then when it start moving, it forms ripples on the bed of the river, may be many of us we are travelling in the river, and if we find that some sand deposition is there in the river, we will be finding that some undulation, some small undulations in the bed, and that sort of undulation, we call as ripple, and then, **when it is** a, this undulation gradually increase in size, and then that is called as a dune, that is a different name dune. Now, of course, these will be discussing just in a qualitative way, means, how much can be the size, **it is not that definite,** it is not that definite in laboratory level, it will be very small in the field level, sometimes it will be very small,

but for a bigger river, this sort of dune and ripples can be of very large dimensions as well.

So, just we will be talking in qualitative way, not exactly the amount how much size it can be, and then, we talk about transition, that means, between ripple, and after, and dune there will transition, and then in transition, basically there are few stages, say initially what the ripple, let me draw figure that will be better.

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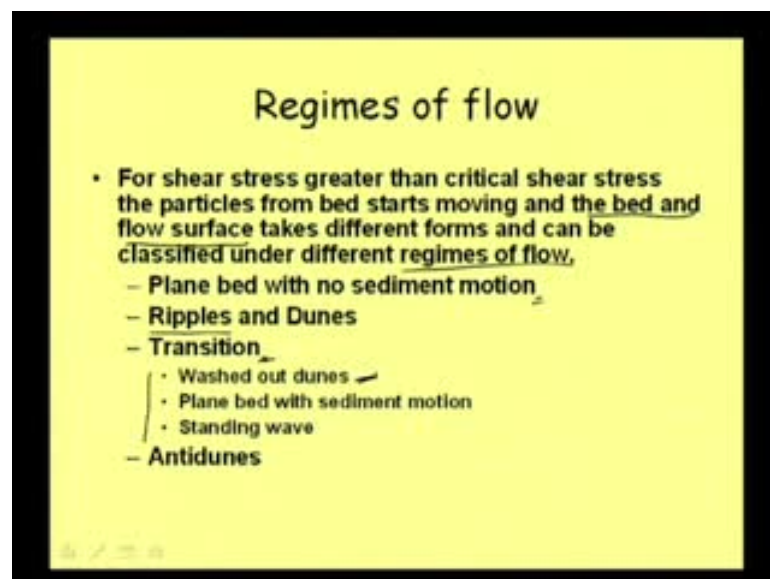
Say this is a bed, and then, initially the bed will take a form, well bed will take a form like this, and it is very most of the time, we see, and this but the surface is not that much disturbed well, that is what we call as a ripple, and then gradually this sizes are increasing, **gradually this sizes are increasing.**

And then, we are having some undulation there, and this is called dune, so this is dune and this is ripple, of course, many find that, it is very difficult to distinguish between these, well these sizes can go upto say 0.4 to 0.3 meter, say maximum, and this can be, it can go 0.4 meter 0.5 meter in smaller size, but in bigger size, it can go upto higher larger dimension also. And because the flow is moving here, the difference is that, when it is forming a dune, then the flow coming here is getting, moving like this, and there is a separation of flow, and here, there will be a separation of flow, means, there will be some deformation, and this will induce some amount of energy loss in this part.

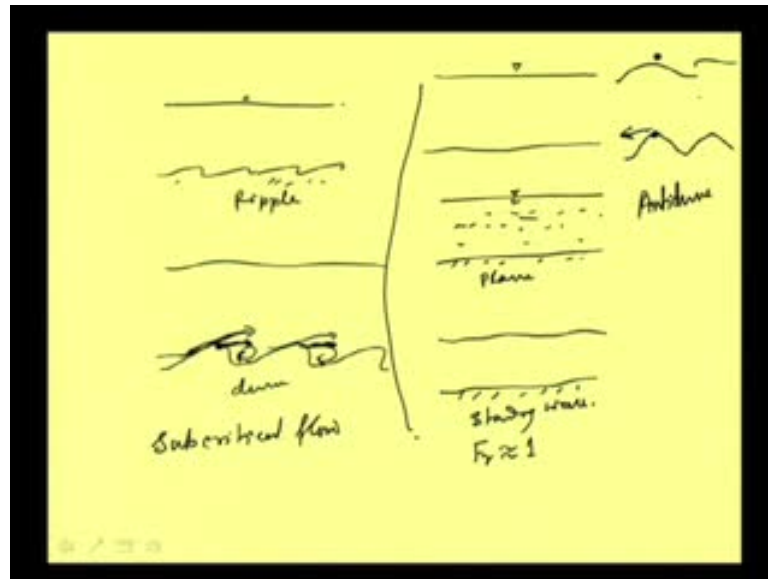
And because of this movement, say flow is moving like this, and here also the flow is moving like this, and then, it will be coming down, because that will be a low pressure zone, and then, flow will be just coming down like that, some turbulence will be created, and this is again just inducing some erosion in this part and that part.

That way in dune, this the crest of the dune is always moving in the forward direction, in the forward direction; sediment is, of course, moving in forward direction, but the crest of the dune also, in a very slow speed, again I do not want to quantify, that means, suppose today it is there, after several days it may be moving few meter or few centimeter in the downstream directions. So, with a very slow speed, it is moving in the downstream direction. Then, after that a time will come, when velocity is increasing, discharge is increasing beyond certain limit, then we talk about transition means initially the dunes will be washed away, **dunes will be washed away.**

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So, that is what we were talking about this part, that transition say washed away dune, **washed away dune**, then after that, these are completely plain bed form, this is completely plain bed form, but the difference is that, when we were talking about first stage, say plain bed without sediment motion, here we will be having a plain bed, but there will be sediment motion, so there will be sediment moving, and that is the difference.

And one important point is that, in plain bed, though it is also a plain bed and when our  $\tau_c$  is critical shear stress, I mean, our bed shear stress is less than critical shear stress, no sediment motion, then also we are getting a plain bed, but the resistance that is, we were discussing resistance parameter earlier.

This resistance to the flow resistance offered by this sort of flow will be much higher almost double than that sort of flow, when there is no sediment motion, that is, how understanding these things are very important. And then, we get some standing wave like this, this is some standing wave like this, we get some standing wave like this, that is called standing wave.

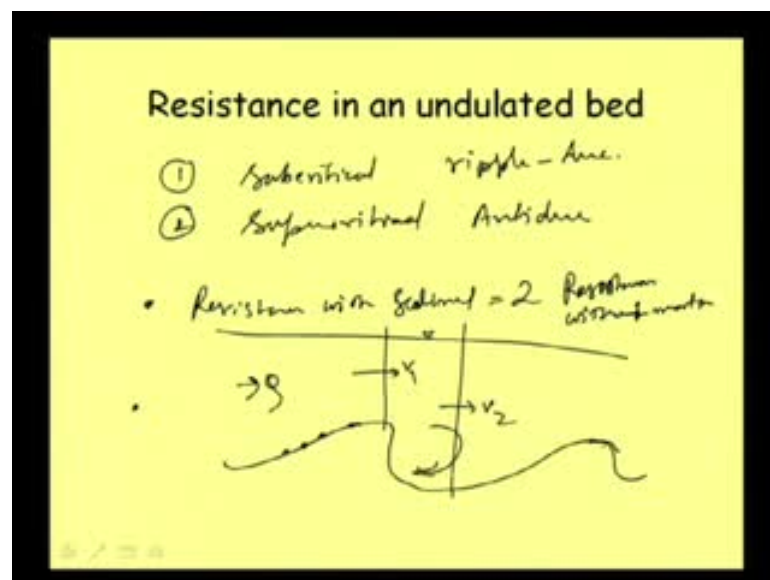
So, these are just in transition stage, and one more important point, this sort of flow that is the ripple and dune, generally occur; generally, means, it is occur in the subcritical flow condition, in the subcritical flow condition, very subcritical flow condition, that is Froude number will be very less here, and this transition take place, this transition take

place this sort of things, when say gradually, it is increasing fraud number is gradually increasing, and it is say fraudnumber is becoming nearly equal to say 1, of course, it will not be exceeding 1, but nearly equal to 1.

Then, after this situation, after this situation, the flow will be just another stage that will be that is called anti-dune, it will be very big size dune, and it is symmetrical, this was just forward direction, leaning in a forward direction, this will be symmetrical like this, it will be symmetrical like this, and of larger size, and those sediments are moving, and here the surface can also be break like that, surface can also have some discontinuity.

Well now the one difference is there, when this crest, in case of ripple, this crest is moving in the forward direction, but in case of dune, although this sediment is moving in the forward direction, but this crest is moving with a very slow speed in the reverse direction; so, that is why the name is called anti-dune, it is called anti-dune, where the surface can also break. Well, like that we can have different sort of, I mean, different sort of flow regime, as we could see all these different type of flow regime we can have.

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Now, if you concentrate in the slide, we can discuss something on the resistance in a undulating bed, some of the critical issue, we need to discuss. Well, already we have discussed that in what sort of flow, in what sort of condition, we can have, and then, so that means, when the ripple, and this things forms in the sub critical zone, that already



we have discussed, then we discussed that in super critical flow only, we can have the anti-dune and this we can have ripple and dune.

And then, resistance to the flow is higher, when it is a plain bed with sediment motion, that also we have summarized that is almost double; resistance is almost double is equal resistance with sediment motion, is almost double, that of the resistance without sediment motion, without sediment; so that is also one important aspect.

Then another aspect that we need to know that, when sediments are moving, and suppose, these bed forms are occurring like this, then here, when the flow is occurring like that, the resistance offered is not only due to the roughness of the bed, but also due to the energy, that is lost or the resistance offered by this sort of undulation.

As we can see that, here suppose it is coming with a velocity  $v$  in this portion; the velocity will gradually drop to  $v_2$ , if it is  $v_1$ , this will be  $v_2$ ; if discharge  $q$  is same, then it is dropping, then it is a case of suddenly velocity is dropping, and then we can find that, how the velocity due to expansion of that velocity, how the energy get lost.

And that is why this loss of energy, means, some energy get consumed in overcoming these sort of situations, some dune will be forming like that, and so considering all these losses ultimately, our entire resistance to the flow phenomena will be different from the resistance of the plain form without sediment or we can say that resistance to the rigid boundary channel.

And that is the very basic need of discussing; all these cases that we have discussed that, flow in a mobile boundary channel, means, uniform flow in a mobile boundary channel, because we are talking about a resistance flow formula and resistance behave in a different way in mobile boundary channel.

And of course, when we will be going for practical work, we need to have understanding of this phenomenon, and when we are using resistance parameter, then we have to consider all these different factors in it, and then only we will able to come out with a proper calculation, and proper design of various river structure, various irrigation structure; so, with that we are concluding today's discussion. Thank you very much.