

Hydraulics
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Module No. # 02

Uniform Flow

Lecture No. # 04

Computation of Uniform Flow (Part 02)

Welcome to this class of, again computation of uniform flow, that we will be discussing some more about computation of uniform flow in this class. We have already discussed in our last class, how that resistance flow formula is used for computation of uniform flow, and as such, we could see that for rectangular channel and for trapezoidal channel, we need to go for iterative solution. Direct solution does not exist for this sort of channel, but when our channel is, say wide rectangular channel, I mean we can be issuing that width of the channel is much larger than the depth of the channel. For that channel, we can have a direct solution for open channel uniform flow depth.

Similarly, if our channel is a triangular one, for that sort of channel also, we can have direct solution. Well, that way, we did discuss some of the aspect and then we found that when there is no direct solution, then we can either go for trial and error procedure using computer programming or excel sheet. We can use or we can go for graphical procedure. That way when we were talking about say non-dimensional equation of uniform flow, that is the uniform flow equation is basically having some dimension.

Then, the resistance parameter that we did use and is also in reality, it is having some dimension, but we can derive that expression or we can rather rewrite that expression in a form that this equation will not have dimension and that will be having some advantage. We will be having some advantage in using that sort of equation. By using those non-dimensional terms, we can derive some approximate solution for computation of uniform flow. Well, let us proceed with that.

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Non-Dimensional Equation of Computing Normal Depth

$$Q = \frac{1}{n} \frac{[(B + 2Y_N)Y_N]^{5/3}}{[B + 2Y_N\sqrt{1+Z^2}]^{3/2}} S_b^{1/2} \quad \eta \rightarrow L^{-1/2} T$$

$$\Rightarrow Q = \frac{1}{n} \frac{[(1 + 2\frac{Y_N}{B})\frac{Y_N}{B}]^{5/3} (B)^{5/3}}{[1 + 2\frac{Y_N}{B}\sqrt{1+Z^2}]^{3/2} \cdot B^{3/2}} S_b^{1/2} \quad \text{depth } Y_N \quad \text{Non-dimensional depth } Y_N = \frac{Y_N}{B}$$

$$\Rightarrow \frac{Q \cdot n}{S_b^{1/2}} = \frac{[(1 + 2Y_N)Y_N]^{5/3}}{[1 + 2Y_N\sqrt{1+Z^2}]^{3/2}} B^{2/3}$$

We were talking about non-dimensional equation of open channel flow. Then, let me take again the case of trapezoidal channel only. Just to recall what we did discuss in our last class regarding what is the dimension of N, let me just keep it on the side. The resistance coefficient, suppose manning's N, it is dimension. We can say dimension of manning's N as 1 to the power minus 1 by 3 and t. That we did discuss dimension of n is equal to this one. Now, with this, let me start with the equation of trapezoidal channel, say we can write Q is equal to 1 by N. Then, area means say B plus ZY N into Y N. Y N again, I am writing for normal depth and then to the power, we can write 5 by 3, then, this perimeter B plus twice Y N root over 1 plus Z square and that to the power 2 by 3, then S b to the power half.

So, from that expression here, as we know that all these terms are having some dimension, now if I write that Y. Y is the depth and in place of depth Y, if in place of depth Y N, if I write a depth parameter or we can make it non-dimensional, if I divide it by say b. So, I can call non-dimensional depth. I can write non-dimensional depth. How we are writing that Y n by B? Suppose this we can write as Y N, capital Y N which we can call as non-dimensional depth. Now, in this equation to have it in this form, let us divide this by B or Y N by B. So, we can write, you can concentrate into the slide that we can write it as Q is equal to 1 by N. Then, this part if we divide by B, of course, there it is a product here. E Y is also a product. So, it will also have to be Y N by B. I mean this Y N and then this Y N also will also have to be Y N by B. So, let us divide this by B and

this by B. So, it will be 1 plus Z into Y N by B. Then, we will be having Y N by B. So, in fact, we have divided it by B square.

So, we need to suppose if I keep this S to the power 5 by 3. So, we have divided it by B to B here, B square. So, we will have to multiply it by B square. So, multiplying by B square again that to the power 5 by 3 will be there. Similarly, here if I divide it by B, this will be say 1 plus twice Y N by B and root over 1 plus Z square and this one we have divided by B. So, again this is to the power 2 by 3. So, here we will have to multiply it by B. So, B to the power, it is just bringing out. So, it will be 2 by 3 and then S b to the power half is there.

Now, if I write this equation in this form that Q, then N, I am writing here Q dot N. Then, S b to the power half and this is equal to it will be 1 plus Z. This Y N by B, I am writing as non-dimensional depth, say Y N. Then, this Y N by B, I am writing as non-dimensional depth Y N, capital Y N. This is to the power 5 by 3. Well, I will write the B later. Then, here also, I can write it like say 1 plus 2 into capital Y N non-dimensional depth, then 1 plus Z square. This Z is again the side slope of the trapezoidal channel to the power 2 by 3 and then here, we had B to the power B square to the power 5 by 3, basically B to the power 10 by 3. Here, we have B to the power 2 by 3.

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$$\left(\frac{Q \cdot n}{B^{8/3} S_0^{1/2}} \right) = \frac{[(1 + Z Y_N) Y_N]^{5/3}}{[1 + 2 Y_N \sqrt{1 + Z^2}]^{2/3}}$$

$$Q_N = \frac{[(1 + Z Y_N) Y_N]^{5/3}}{[1 + 2 Y_N \sqrt{1 + Z^2}]^{2/3}}$$

$$Q_N = \frac{Q \cdot n}{B^{8/3} S_0^{1/2}} \rightarrow \frac{\overset{\text{Dimensionless}}{L^3 T^{-1} L^{-1/2}}}{\frac{L^{8/3}}{L^{8/3}}} \rightarrow \frac{L}{L^{8/3}} \text{ Dimensionless}$$

So, if we just write it combining then it will be B to the power 8 by 3. Well, from this, if we write that equation again B by B to the power 8 by 3. I can just bring in this side.

Then, it will be $Q \cdot N$ divided by B to the power $8/3$ N b to the power half. This is equal to, now you can write on the right hand side whatever was there, that is $1 + ZY$ N Y N and all these area, say $1 + Z$ and this is capital Y N multiplied by capital Y N to the power $5/3$. Then, here we can write this expression $1 + \text{twice } Y$ N root over $1 + Z$ square. So, $1 + \text{twice } Y$ N root over $1 + Z$ square to the power $2/3$. Now, this equation, in fact, is a non-dimensional form of the resistance formula, Manning's formula.

We are basically coming with the Manning's formula here, because N is there. We are using the Manning's coefficient and other parameters also. We are writing in this form. So, it is basically a Manning's equation, but in non-dimensional form. Well, now why when we are saying non-dimensional form, then well what about this particular part. This particular part that we will have to be sure because that part we are sure that Z is a slope. So, this AZ is a side slope that it does not have any dimension. Then, capital Y N that we are already mentioning, it is a non-dimensional depth. So, Y N , that is the Manning's, sorry the normal depth Y N divided by B means, this is also a non-dimensional parameter.

So, it does not have any dimension and as such. This right hand side, we are sure and this is 2. So, this right hand side is having no dimension, that is confirm, but what about this part, that is QN B to the power $8/3$ as B to the power half. Let us, if we check this thing, this we write normally as Q N . This we write as QN Q capital N . You can concentrate on the slide and that is equal to $1 + Z$ into capital Y N into Y N to the power $5/3$, $1 + 2$ into capital Y N root over $1 + Z$ square. This is to the power $2/3$.

Let me check just. What is the dimension of this Q N ? Whether, it is really dimensionless or not? Just, if you put the dimension that Q N , this is say QN B to the power $8/3$ S b to the power half. Now, dimension wise, if we put what are the dimension of this? What is the dimension of this, say Q is discharge. So, its dimension is meter cube per second. That means, length dimension cube divided by T meter cube per second, that is, the time dimension. We can write T to the power minus 1 as we did discuss already.

Well, first let me write here that B, B is equal to say, B means the bed width. So, it is the length dimension. So, 1 to the power 8 by 3, this is the dimension. As we did discuss already that what can be the dimension of N. So, dimension of N is 1 to the power minus 1 by 3 into T. So, we can write that say 1 to the power minus 1 by 3 and T 1 to the power minus 1 by 3 and T. So, this just straight way, it is coming like that. It is 1 to the power by T to the power minus 1. Then, T to the power T. This is getting cancelled and then this will become if we see it is 1 to the power 9 minus 1. So, it is 8 by 3 and 1 to the power 8 by 3. This and that it is getting cancelled. So, ultimately it is 1. So, that is non-dimensional. It implies that this is non-dimensional. It does not have any dimension **ok.**

So, that way this Q N, we write as non-dimensional discharge. Well, now using this, but just if we see that whether we can avoid that earlier condition, that is the trial requirement of trial and error method for solving. Well, if we see this equation, though it is non-dimensional, then this capital Y N, we again cannot separate out. It is remaining in the same form. So, for the other constraint requirement, that is, that it cannot be solved directly and we need to go for trial and error procedure. These are remaining as it is.

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Approximate Direct Solution

- Barr D.I.H. and DAS M.M. (1986) gave an approximate direct solution using Mannings equation for Rectangular and Trapezoidal channel

Rectangular $Y_n = \frac{Q_n}{\delta} \left[1 + 0.855 \left(\frac{Q_n}{Y_n} \right)^{1/3} \right]$ $Q_n = \frac{Q}{S^{1/2} \delta^{2/3}}$ $Y_n = Y_n \times 8$

Trapezoidal $Y_n = \frac{Q}{\delta} \left[1 + \frac{0.69}{\left(1 + \frac{3}{Q^{1/3} \delta} \right)} \right]$

Then, we can go for some approximate direct solution that was given by Professor Barr D.I.H and Das M.M that was given in 1986. Now, as I told that whether we today, in today's context whether these sort of formulas are having importance or not. Well, we have computer, so whether it is trial and error procedure or say a computer

programming, we can go very quickly and we can have the solution, but still these formula has some importance. Why? Say, even if you go by trial and error procedure and if we use a computer program, then we need to put an initial value, that is first assumed value of normal depth we need to put and from that first assumed value, it start iterating.

So, if our first assumed value is, suppose closer to the actual value, then we can reach our solution quickly. Of course, with the first computer, this time difference may be very small, but still there is a difference. We need to have that advantage. That is why I am discussing these equations, so that whenever we go for a computer programming, for solving say, normal depth, then at least we will be in a position to have the advantage of having this sort of approximate solution.

So, this solution goes like this. They get the solution for two cases. One is rectangular and one is trapezoidal channel. For rectangular solution and this, Y_N is nothing, but that we have discussed, non-dimensional depth Y_N by B . This is the actual uniform flow depth and divided by the bed width of the channel. Then, this Q_N , capital Q_N as we have already discussed, this capital Q_N is nothing, but capital Q into the Manning's roughness coefficient N , then divided by the bed slope to the power half $N B$ to the power $8/3$. So, this Q_N stands for that.

Well, now what you can do or what other we can do. Say, if in a problem, this Q is suppose given, this N is given S_b bed slope is given, then bed width B is given, we can first calculate what that Q_N . That is what the non-dimensional discharge Q_N having the value of non-dimensional discharge Q_N . This equation, you can see this equation state that non-dimensional discharge Y_N is equal to non-dimensional discharge Q_N to the power $3/5$ into $1 + 0.85$. Then, again Q_N to the power $3/5$ equation is otherwise very straight and not that difficult also to keep in memory.

So, if we calculate this Q_N first and then put the value of Q_N here and here, then we are directly getting the non-dimensional depth Y_N . Now, once we get the non-dimensional depth Y_N , we can just get the actual normal depth by multiplying that non-dimensional depth Y_N by B . So, once we get Y_N , then we can get small y_n , means actual uniform flow depth, that is equal to non-dimensional depth y_n multiplied by the width of the channel.

Well, so that way, straight way we can get the value and of course, as these formulas or rather these formulas are approximate solution. So, always we can do one thing or we should do one thing, say first we are using these formula and we are getting a value of normal depth which is approximate value. That approximate value always we put in the main formula. Main formula means the original formula that we are talking about. Say, we can talk about this relation. Say, in this relation we have got one Y_N and then, this Y_N , we should put here. We should check whether the discharge we are getting by using the approximate value of Y_N using this approximate value of Y_N in this place is the discharge we are obtaining is really satisfying. Our required discharge or not, if the difference is very small, we can take that approximate solution whatever we are getting as the correct.

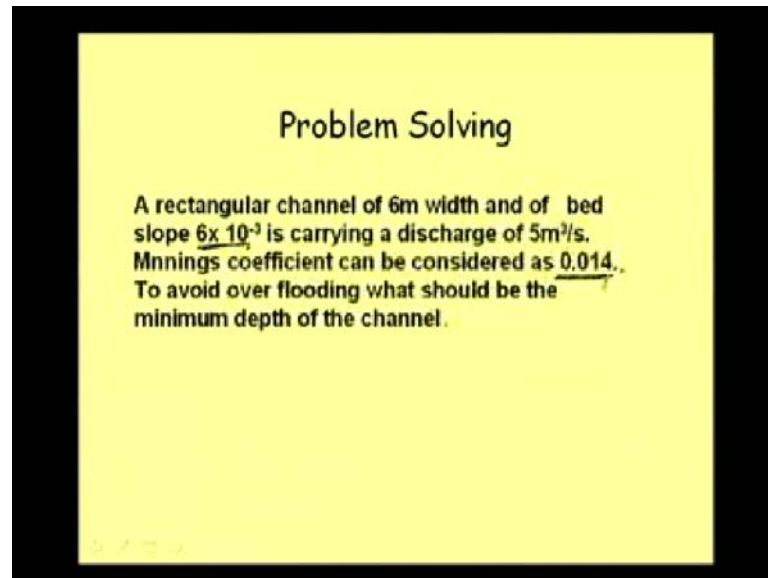
If suppose, there are difference say Q . It should have been 100 and by approximate formula, whatever we have calculated with, suppose putting the approximate value of Y_N here, whatever we are getting is not 100, but suppose 98, there is an error. Then, we need to increase this by small amount. Then, we are again doing iteration, but advantage is that now our actual discharge should be 100. We are getting a value of Y_N which is giving us 98, but without having that approximate value, if we start, we can be very far off from this actual required discharge. Then, we will be requiring lot of trial and error, but here we can go very fast. In fact, for rectangular channel, I mean by using this formula, we can get almost correct result without much error. That can be even used directly also.

For trapezoidal channel, the formula goes like this. For trapezoidal channel, you can see concentrating this slide that Y_N non-dimensional discharge is equal to again Q_N to the power 3 by 5. That is remaining same and this is becoming 1 minus 0.69 divided by 1 plus 3 by Q_N to the power 0.35 and in multiplied by the Z . So, this equation is used for trapezoidal channel and the other procedure is same. I mean for trapezoidal channel also, we can calculate the Q_N first, this non-dimensional depth and then non-dimensional discharge.

Sorry and then we will be getting the non-dimensional depth. From that we will be getting the actual depth and then in this formula, of course, there is some amount of error always. It is expected that some amount of error will be there. So, what we need to do again? We need to put it in the actual formula and if we check, whether it is satisfying

our condition or not. If it is there, will be some amount of difference and then we need to reach the correct value by again moving as trial and error procedure. So, that way this approximate direct solution given by Barr and Das is useful in taking the initial value.

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Then, with this very fundamental that we have discussed about the computation of uniform flow, now let us see how we can solve problem by using this formula. Well, let us take one problem and let us see how we can solve this. Well, to start with, we will not be taking very complex problem. First, we will be taking very simple sort of problem and then we will gradually move on to complex problem.

Well, say first problem. Let us take it like that a rectangular channel of 6 meter width and of bed width. You can concentrate to this slide and of bed slope 6 into 10 to the power minus 3 is carrying a discharge of 5 meter cube per second. Well, Manning's coefficient can be considered as 0.014 and as we know that Manning's co-efficient always give some value, but depending on a natural channel, suppose whatever it is manmade or natural made in a field situation when we go.

Suppose, based on some of the characteristic, we can say that does this Manning's roughness value should be this much, but in the entire channel reach, you may find that there are differences. Suppose, when we say that if it is a channel of concrete lining, well there may be a value for concrete lining that Manning's roughness value is suppose 0.014 or something like that. Again, when we talk about concrete lining, this concrete

surface may have difference sort of roughness. A concrete surface made in a very polished way will be quite smooth and a concrete surface made in a very raw way will be a little rough.

So, that way also, there will be some way differences in this value. Suppose, we have made one concrete channel and then with time, it may happen that some erosion has occurred, some demises occur into the concrete channel and then some part are just going away from the lining. That way, there may be some roughness in this. So, considering all those facts, this roughness, exact value of roughness is, I mean quite a critical question, but still for our computational purpose, we assume some value. This is say, Manning's coefficient can be considered as say 0.014. That is been given for this particular problem and the question is that to avoid over flooding. What should be the minimum depth of the channel? Well, that means, basically this question is of course stated like this or asked like this to avoid over flooding.

Say, in a channel, we know that this much discharge will be coming. It can be a canal or it can be drains in the city, whatever it is. So, say in that drain or suppose irrigation canal. Then, suppose this much discharge will be coming that is known to us and bed slope is given as 6 into 10 to the power minus 3. Now, always in the channel when it will be in the real field, say we have very less chance opportunity in our hand to change the bed slope because if we need to change the bed slope, we may have to go for lot of cutting in the ground or sometimes we may have to go for lot of filling up of the ground. Then only we can make, we can afford to change the bed slope to some extent.

So, generally the naturally available slope, I mean depending on the topography of that particular area, these sorts of slopes are decided. Of course, when we say that slope is this much, if it is a man made channel, then we talk about that in the topography, definitely there will be ups and down. In that channel, if we want to just go by the average slope, then there will be somewhere some cutting, some filling. Anyway, now in this channel, when we are saying that the bed slope is this much, means it is the bed slope that we are defining may be average bed slope which we get after some cutting, some filling up in the actual situation. Anyway, so this is the bed slope.

Well, now these problems for that particular discharge, what we need to calculate here that we need to avoid flooding. That means, suppose if we calculate the normal depth of

flow. Normal depth of flow, that is the uniform flow depth, then, we will be getting a depth of flow and our depth of the section. Suppose, our normal depth is this much, then depth of the section should be little higher than the depth of the flow. So, this water will not over flow and the channel will not be flooded. So, first for our knowledge as far is concerned that we can calculate the normal depth and after calculating the normal depth, we need to put some amount of access free board that we will be discussing in detail. When we will be doing the topic of channel design, lot of other things will be coming there and canal design. When we will be discussing, then we will be discussing that.

First, what we can see that in this problem, if you just concentrate into the slide that we can solve, say we can first calculate it by approximate solution by Barr and Das. If we go by that, then we can write that Y_n by B . We are rather than writing capital Y_N , we can write it as Y_n by B .

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The image shows a handwritten derivation on a yellow background. It starts with the equation for normal depth Y_n in terms of discharge Q , bed width B , Manning's roughness coefficient n , and bed slope S_b :

$$\frac{Y_n}{B} = \left(\frac{Q n}{B^{8/3} S_b^{1/2}} \right)^{3/5} \left[1 + 0.855 \left(\frac{Q n}{B^{8/3} S_b^{1/2}} \right)^{3/5} \right]$$

Then, it substitutes the given values $Q = 5 \times 10^4$, $B = 6$, $n = 0.014$, and $S_b = 1/100$:

$$\frac{Y_n}{6} = \left(\frac{5 \times 10^4}{6^{8/3} (0.014)^2} \right)^{3/5} \left[1 + 0.855 \left(\frac{5 \times 10^4}{6^{8/3} (0.014)^2} \right)^{3/5} \right]$$

This leads to the approximate normal depth:

$$Y_n = 0.3358 \text{ m (approximate)}$$

Next, it shows the 'Actual relation' for discharge $Q = AV$, where A is the cross-sectional area and V is the velocity. The area A is given by $A = B Y_n \left(\frac{B + 2 Y_n}{B + 2 Y_n} \right)^{2/3} S_b^{1/2}$. It then uses a trial value $Y_n = 0.3358$ and $Q = 5$ to find the final depth:

$$Y_n = 0.3358$$

Finally, it states the depth of the channel as the sum of the normal depth and the free board:

$$\text{Depth of the channel} = 0.335 \text{ m} + \text{Free board.}$$

This is equal to this is basically nothing, but the capital Y_n . What we were writing and the non-dimensional discharge that we are writing directly as Qn/B to the power $8/3$ as B to the power half to the power $3/5$. Then, we can write $1 + 0.855$, then again this non-dimensional discharge Qn/B to the power $8/3$ S_b to the power half to the power, say $3/5$.

We know that B value here, bed width of the channel is given as I think 6 meter bed width of the channel is given as 6 meter. So, if we take that 6 meter here, then we can

write that Y^n by 6 is equal to Q is given as 5 meter cube per second. Then, n value is given as 0.0146 to the power 8 by 3 as B is given as 0.006 means 6 into 10 the power minus 3. So, we can write it as 0.0006 to the power half to the power 3 by 5. Just writing the other part also 1.080, 0.855 and this part we can write the same value 5 into 0.0146 to the power 8 by 3, then, into 0.006 to the power half whole to the power 3 by 5. Now from that, the value of Y and what we will be getting, that is equal to 0.3358 meter. I am writing this to the 4th decimal place and then we are getting it as 0.3358 meter.

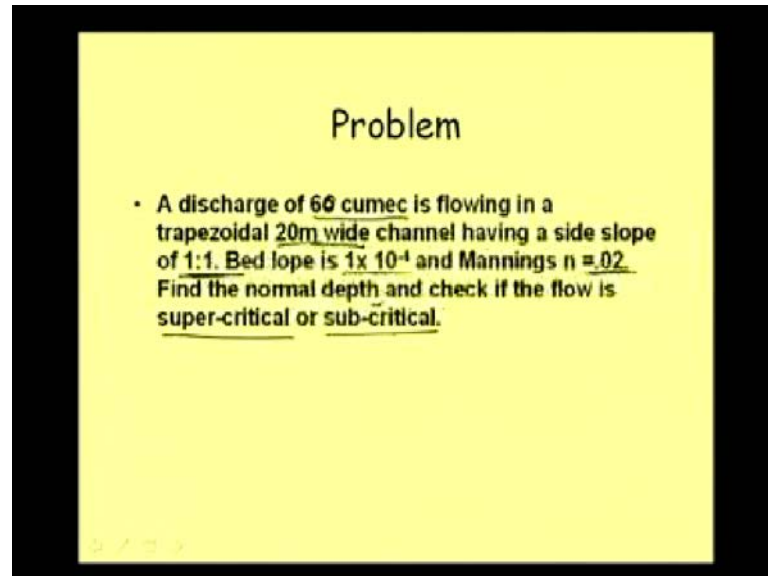
Now, of course, this is an approximate solution, but for rectangular channel, this approximate solution is almost correct, but still we need to verify whether this is giving us the required discharge or not. Then, we know again the actual formula if we use the actual relation, then we will have to write original equation, that is Q is equal to area into velocity. So, for rectangular channel, we know that area is equal to BY^n and velocity is equal to say 1 by n , writing Manning's equation area. Then, perimeter B plus twice Y^n to the power 2 by 3, then bed slope to the power half. Now, here we do not know. We know the Q , but we cannot solve for this Y^n directly.

So, these value, if we put 0.3358 for this Y^n , after that writing this part, we can simplify it first. Then, after writing this, we can say that trial value of Y^n , we can go for trial. Say, trial Y^n is equal to, suppose 0.3358, then we can see what the Q value is. So, if we put 0.3358, then we find little higher. Then, this 5 meter cube, 5 point something we are getting 5 point something little higher, then I am not showing all the trial and error procedure.

For Y^n equal to 0.335, we are getting Q is approximately equal to 5 cumec or 6 cumec. What we were using? It is 5 cumec 5 meter cube per second. So, that we are getting. So, Q is approximately equal to 5 cumec for Y^n is equal to 0.335. This is exact solution. We may have to go for one more trial here and that way we are getting it. So, from this problem, we can see that approximate formula can be used very well for rectangular channel and once we get this Y^n say 0.335, now what can be our size, actual size of the channel. This we can increase by some percentage which we call as a freeboard. Then, we can say that if our channel is greater than that value, then there will not be flooding. Well, that final detail we will be going later as I have told, but this is what the procedure. That means, depth of the section if I am asked, say depth of the section, I can say as this is equal to 0.335 meter plus freeboard. Now, as we have not discussed what should be

the freeboard right. Now, I am keeping as freeboard only, so that way we can design the channel.

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Similarly, we can have another problem. The basic idea is seen just to show how the numerical value can be little different and how the erode can be little more in trapezoidal channel. Suppose, if we talk about this particular problem, say discharge of 60 cumec is flowing in a trapezoidal channel of 20 meter wide channel. Well, this is a little bigger size channel, that is 20 meter wide and then your discharge is also little higher, say 60 cumec meter cube per second. Then, this has the side slope of 1 is to 1. That means, this is not rectangular and the side slope is 1 is to 1, that is z value, small z value that we are writing, this is 1 is to 1, that is z equal to 1 and bed slope is 1 is to 10 to the power minus 4. Manning's n is equal to 0.02. Now, suppose simple question. Find the normal depth, that is the depth of uniform flow and check if the flow is super critical and sub critical.

What is super critical, sub critical? To know that, of course, we need to go back to our very, I think second or third class where we did discuss in open channel flow, where we classified the flow and that is not that important right. Now, we will be just calculating whether it is super critical or sub critical, but why I am putting this one here? To know what? Whether, it is super critical and sub critical. First, we need to know what the flow depth is. To calculate the flow depth in uniform flow condition, we need to take help of uniform flow formula. So, if you can concentrate in to the slide.

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Approximate solⁿ: $Y_n = 2.85 \text{ m}$

Actual Equation: $Q = A \cdot V = \frac{1}{n} \frac{[(B + ZY_n)Y_n]^{5/3}}{[B + 2Y_n\sqrt{1+Z^2}]^{1/3}} S^{1/2}$

Trial	Discharge
2	$Q < 60 \text{ m}^3/\text{s}$
2.85	$Q > 60 \text{ m}^3/\text{s}$
3.0	$Q \approx 60 \text{ m}^3/\text{s}$
2.93	$Q \approx 60 \text{ m}^3/\text{s}$

$F_r = \frac{V}{\sqrt{gD}} = \frac{A}{T}$

$F_r = 0.47$
 $F_r < 1$

So, if I go to the equation again, that approximate formula, I am not writing the approximate formula here. Already I have shown how the approximate formula can be written and by approximate solution, I am just writing the value for approximate solution. The same procedure you can adopt and that approximate solution gives us a value of Y_n is equal to 2.85 meter. That I am writing directly for that. In fact, we are using the approximate formula what was given here, that formula.

Using this formula, calculating this Q_n with the given Q and given n value, given bed slope and given width of the channel, we are calculating this YQ_n . Then, Z value we are using whatever is given. This Z value is basically 1 because our slope is 1 is to 1. So, this Z value, we are using as 1 and then we are calculating this Y_n . So, Y_n is again is equal to Y_n by B . So, from that, we are calculating this Y_n as Y_n is equal to capital Y_N into B . So, that Y_n , what we are getting. This, I am writing here directly that this is the value of Y_n 2.85 by approximate formula of Barr and Das.

So, if I write the direct equation or actual solution, say if I write the actual equation that will be say Q is equal to again A into V and that we can write for trapezoidal channel as say 1 by n , then area, we can write as B plus ZY_n into Y_n that whole to the power 5 by 3. The perimeter, we can write as B plus ZY_n into root over 1 plus, sorry it is not Z , it is twice Y_n into root over 1 plus Z square and whole to the power say 2 by 3. As B to the

power half, here if we put this value of 2.85 in place of Y_n , then again we need to make value trial Y_n Q.

So, if I put this value first, 2.5 discharge Q trial depth and discharge this, if I put 2.85, then we find that the Q, what we are calculating, this is becoming less. Then, the actual desired Q and what is our actual desired Q, is 20 meter cube. Sorry, it is I think 60 meter cube per second. Let me confirm. It is 60 meter cube per second. So, it will be less. It is 60 meter cube per second.

So, what we can do in the next trial, we can increase it to some higher value. Suppose, we are going to 3.00, then we will find that if you just put the value and if you try, then we will be finding that Q is becoming greater than 60 meter cube per second. That way if we carry out trial and error, then you will be finding that for a value of 2.93, we get Q is approximately equal to 60 meter cube per second. Now, we may have some feeling that well, what is the difference between this, say 2.85 and 3 or 2.93? In fact, these are in same centimeter level, these are in millimeter level. What is that difference basically? Why we should be so careful about that? Why we should be so precise?

See, if you use what is the significance, is it a big channel? It is a big channel about 20 meter width and of course, in nature, we can have much bigger channel like, say in India, river Brahmaputra, river Ganges. If we talk about, then it can be width goes to 2 kilometer, 3 kilometer. Sometimes, in Brahmaputra, it goes even to 17 kilometer. So, the highest width observed is 20 kilometer around. So, that way it is end flood time. Sometimes, it flows covering everything and in this sort of situation, say this difference of depth when we calculate, something the difference of depth of even a centimeter matters a lot. With that difference of depth, our flow discharge changes grossly because the size of the channel is so large, that if our depth changes by small amount, that flow discharge changes a lot. As you know that water is now becoming very valuable, I mean it has a very high value and it is said that, I mean world war will be there for water. So, we should be very careful.

People are interested to know how much water is flowing. How much water means what is the quantity of water flowing. If somehow, our depth calculation if we do computationally, by our calculation if our depth computation is erroneous, then our whole understanding about our discharge will may go wrong. So, that way small amount

of depth variation is also significant when it is a very large channel. Of course, when it is a very narrow channel, say 2 meter, then say 1 meter channel like say irrigation canal or city drain. In those sorts of channels or canals say depth variation, if it is very small amount of depth variation in the millimeter level or centimeter level will not matter centimeter level means I am talking about millimeter level. Basically, say one say 10 millimeter or 5 millimeter, that is not making much difference, but in other cases, it will matter.

Now, what is the advantage if we go by this approximate formula? Suppose, as we are using this value 2.85 which is basically the approximate formula value and from here, we are starting, so with very limited iteration, we are arriving at a correct value. In field condition, suppose we are going into the field and we need to have some approximate idea. Then, we can calculate this. We can use the advantage of this one and then we can go for trial and error. Then, we can see that what the actual value of Y_n should be, so that way we are calculating these things. Now, the next equation is to know that.

So, Y_N is, suppose 2.93 and then next question is to whether the flow is sub-critical or super-critical. Now, if we just go back to our earlier class, then we remember that it depends on the Froude number. Of course, we will be discussing that in more detail when we will be discussing energy concept. Then, when we will be discussing, what is minimum specific energy critical flow that we will be just discussing in the next class. So, in that detail, I am not going right now.

We know that a Froude number, we calculate this is equal to V by root over gD , where D is nothing, but A by T . So, once we know the depth, then we can calculate the D and then already we are calculating, once we know the Q . The sectional area V , we can calculate or we are otherwise calculating this V here also. So, we can put the V value and we can put that root over gD value and the Froude number value. For that situation, we are getting as 0.47, and if Froude number is less than 1, then this sort of flow is called sub-critical flow. When Froude number is greater than 1, then it is super-critical. When Froude number is equal to 1, then it is called critical flow.


That we did discussed earlier also. Just here, I am putting this point of this knowing whether flow is super-critical, sub-critical that has some other issues. If it is sub-critical flow, then the flow velocity will not be very high and the chance of erosion will be less.

If it is super-critical flow velocity will be very high, chance of erosion will be high. So, sort of other things we can derive from these answers, so that we can solve this sort of problem.

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Problem

- A rectangular unlined channel of Manning's $n=0.03$ was lined to improve its carrying capacity. Manning's n for the concrete lined channel is 0.015. What will be the percentage of increase in discharge



Well then say, sometimes without going for all those detailed calculation, we can have some other problem we can do easily, like that, say a rectangular unlined channel. A rectangular unlined channel of Manning's n equal to 0.03 was lined to improve. It is carrying capacity. So, this is a very common practice that is adopted in many places. I mean like for city also, say we are having a drain which is suppose unlined. Then, we are finding that in rainy period or in monsoon, this drain cannot carry the amount of water that is coming to the drain. That is why, the depth is increasing and it is getting over flooded. That is why, flood is having there and water that is coming to the channel cannot move out quickly and it is creating some sort of problem, water stagnation problem.

So, many a time, we go for channel improvement. In fact, suppose in a channel, say this is the channel. In a channel, this we have some cue and some discharge is flowing. Then, we want to drain out the water quickly because if we cannot drain out the water quickly here, the water level will rise. Then, when this is rising, suppose another sub-channel is joining this channel, then what the water flowing here that will also be affected because once this water level rises from here, you will not be getting hydraulic gradient. Here,

you will be having higher gradient, here less, so higher depth, here less. So, flow will not get retarded in this part also.

So, once in the main channel, you have higher depth, then on the side channel also you will be having water with a slower flow. Then, you can have flooding situation in this part also as the water is not getting released quickly. To avoid those situations, we try to improve the flow condition in the channel here, here as well. Everywhere and then one option could have been very easy. Suppose, we want to release it quickly, if we increase the slope, then of course, we can release the water quickly, but as I was telling that in actual condition, the slope cannot be increased or slope cannot be I mean we cannot change the slope that easily.

So, normally we go for some other, I mean sort of devices like that we try to improve the channel characteristic, so that we can increase the flow. So, this is a problem of that kind. So, a rectangular unlined channel of Manning's n equal to 0.03 was lined to improve. It is carrying capacity. Now, Manning's n for the concrete line channel, suppose we are providing a concrete line, then concrete line channel is, say 0.015. In the unlined channel, this Manning's roughness co-efficient is 0.03 and the line canal, it has become say 0.015

Now, what will be the percentage of increase in discharge? Now, in this problem, nothing is given. So, what is bed width and other things are not given. It is a rectangular channel. Just it is asked that what will be the percentage increase in discharge. Now, we can see that, say for the first case.

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1st case $n = 0.03$

$$Q_1 = \frac{1}{0.03} \frac{[B Y_n^5]^{1/3}}{[B + 2Y_n^2]^{2/3}} S_b^{1/2} \quad \text{--- (1)}$$

2nd case $n = 0.015$

$$Q_2 = \frac{1}{0.015} \frac{[B Y_n^5]^{1/3}}{[B + 2Y_n^2]^{2/3}} S_b^{1/2} \quad \text{--- (2)}$$

$\frac{(1)}{(2)} \Rightarrow \frac{Q_1}{Q_2} = \frac{1/0.03}{1/0.015}$
Increase in discharge 100%

$\Rightarrow Q_2 = 2 Q_1$

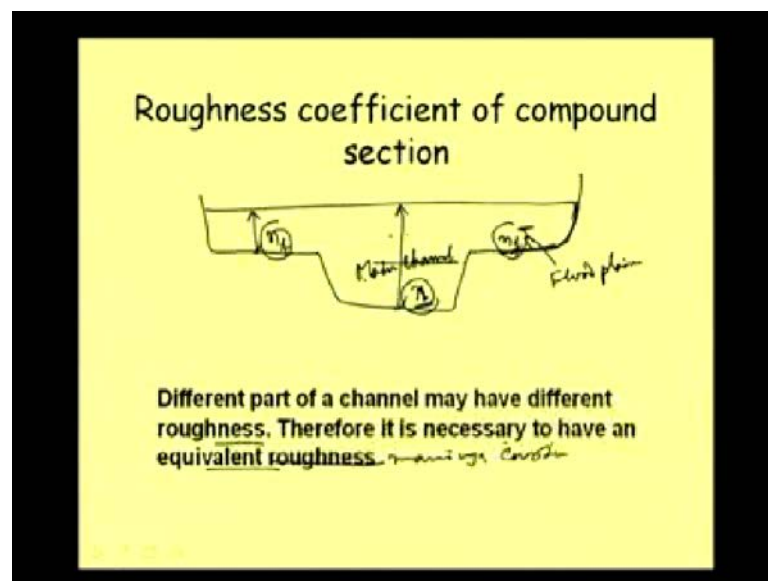
For the first case, when our n is equal to 0.015, sorry 0.03, then we can just write that Q_1 is equal to, say 1 by n 0.03. Then, it means, we can write B into Y_n to the power 5 by 3 B plus twice Y_n to the power 2 by 3 $n S_b$ to the power half. This is our, say first discharge we are calculating. Then, we are not calculating anything. We are just keeping it like that. Then, say when we have in the second case. In the second case, when n is improved, that is we are having concrete lining, then we can have Q_2 . We are having a different discharge 1.015 and of course, the section of the channel is remaining same, so $B Y_n$ to the power 5 by 3 and B plus twice Y_n to the power 2 by 3. So, that part is not changing. Then, as B to the power half, this is say equation 2. Now, our interest is to know that what will be the increase in the discharge.

So, just by making 1 divided by 2, we can have, that is Q_1 by Q_2 is equal to 1 by 0.03, other things remaining same. This will get cancelled, divided by 1 by 0.015. So, this indirectly implies that Q_2 , our second discharge is equal to, if we just calculate it this is just twice because this value is twice. So, it will be 2 time of Q_1 , 2 that means, if I am asked that how much is the percentage of increase of the actual discharge, then you can say, that it has increased by 100 percent. Earlier, it was Q , now it is twice Q . Earlier, it was Q_1 . That means, now it is twice Q_1 . That means it has increased by 100 percent. So, increase in discharge is equal to 100 percent. So, that just was directly that how by improving the channel characteristic, by providing lining, we can increase the discharge

capacity or the carrying capacity of a channel. We can have a better water management, wherever is required.

Well, now we are talking about roughness coefficient, but again when we were discussing about the computation also, then also we were discussing about compound channel. Then, we can see that this roughness coefficient and when we are using this is for a regular section we are talking about. This is rectangular trapezoidal or this sort of channel may be parabolic exponential channel, but if it is a compound channel like this, then as we know that as the depth of flow here is very less, then depth of flow here and this is very near to the bed. This is very near to the bed; this is also very near to the bed. I mean, entire flow section, if we see, this is closer to the bed and the side and this is far off from the side and from the bed if it is a compound channel.

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So, that way the roughness because roughness is coming from basically bed and the side. So, roughness characteristic of these will be definitely different. This means, suppose that we call as a flood plain and that sort of flow will be there and this is the main channel. This we call as a main channel **ok**. So, the roughness value of the main channels, say n . If I write n , this is n f say roughness of the flood plain. In the flood plain again, say this flood plain is normally do not get over flooded. Well, this will be flooded during monsoon period only and as such, when it is in a non-monsoon period, this is

already having lot of fertile soil element may be there because during flood time, it has left some of the fertile element. Then, in the non-monsoon period, flow is occurring here.

So, at that time, the vegetation in this part gets the opportunity to grow. So, in the flood plain, normally we will be having some vegetation and that is why as we know that vegetation also influences the roughness characteristic as such, the roughness value of that flood plain will be different. So, one point is clear that n, if I talk in terms of Manning's roughness coefficient, this n here and here, will be different.

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Equivalent Mannings constant

- Horton(1933), Einstein (1934)
 - Mean Flow velocity in each of the sub area is equal to the mean flow velocity of the entire section

$$V_{\text{mean}} = \frac{1}{n_e} \left(\frac{A}{P} \right)^{2/3} S^{1/2}$$

$$V_{\text{Sub}} = \frac{1}{n_i} \left(\frac{A_i}{P_i} \right)^{2/3} S^{1/2} = \frac{1}{n_e} \left(\frac{A}{P} \right)^{2/3} S^{1/2}, i=1, \dots, m$$

Now, if we try to calculate the flow depth, if we try to calculate, say what will be the discharge in this channel. Then, we need to know, we need to calculate an equivalent Manning's n. That is what we will be discussing just very briefly here. Of course, we can go for lot more discussion on this part. So, different part of channel may have different roughness. Therefore, it is necessary to have an equivalent; we will call that Manning's constant equivalent.

Well, again different investigator attempted this in a different way. First, let me talk about this development by Horton and then Einstein. That is almost parallely they did the study and they went by the assumption that mean flow velocity in each of the sub area is equal to the mean flow velocity of the entire section. That was the assumption and based on that assumption, they derived one relation for equivalent roughness means, when they

calculate the mean flow velocity of the entire section, then they did use say equivalent Manning's constant.

Well, what we can write, that means, equivalent mean flow velocity as we use that say, V mean for the entire section, then we can write that V mean of the entire section. That we can write as 1 by n and then A by P to the power 2 by 3 , then S to the power half. Well, this S again, we can consider say same for all the section. This S we can consider as same for all the means on the sub area here. What is the bed slope? Here, what is the friction slope? Here, what is the friction slope and here, what is the friction slope that we can consider same?

Then, say we are starting from that and then, for the sub area, when we write this will be say 1 by n_i , then, A_i by P_i to the power 2 by 3 , S to the power half. Well, now by this assumption, what we can write by this assumption? What can be written? Well, what can be written that 1 by n into A by P to the power 2 by 3 , S to the power half. That is equal to this is mean flow velocity and that is the velocity flow, velocity of the sub area. We can write as 1 by n_i , then A_i by P_i to the power 2 by 3 and S to the power half. In fact, I can go for equal to 1 and any value. Well, now this can be written in a form that well, let me rub this part from this. What we can do? Say, S being if I consider same, we can cancel it and taking A power to 3 by 2 .

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Equivalent Mannings constant

- Horton(1933), Einstein (1934)
 - Mean Flow velocity in each of the sub area is equal to the mean flow velocity of the entire section

$$\frac{1}{n_c^{3/2}} \frac{A_c}{P_c^{5/2}} = \frac{1}{n_i^{3/2}} \frac{A_i}{P_i^{5/2}} = \frac{\sum A_i}{\sum n_i^{3/2} P_i^{5/2}}$$

$$n_c = \left(\frac{\sum n_i^{3/2} P_i^{5/2}}{\sum A_i} \right)^{2/3}$$

We can write that $1/n$ to the power $3/2$. Then, we are writing this as A/P and this is equal to $1/n$ to the power $3/2$ and then, I am writing as A_i/P_i . Well, I have already cancelled and when we have a series like that, this is equal to that. Then, if we can take the summation of the numerator and denominator, then it becomes same.

So, we can write summation of A_i divided by summation of n_i to the power $3/2$ into P_i . So, that also can be written and then simplifying that expression, what can be written. So, combining this and that we can write as, say n is equal to summation of P_i/n_i to the power $3/2$. Then, this P , we can write also as summation of P_i . This can be written as summation of P_i and then, A and summation of A_i is same. So, this will get cancelled and we can write like that summation of P_i . Then, enter to the power $2/3$. So, that way we get an expression for equivalent n .

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Equivalent Mannings constant

- Mulhhofer(1933),
- Einstein and Banks(1951)

Total resistance force = sum of the resistance
force of the sub area

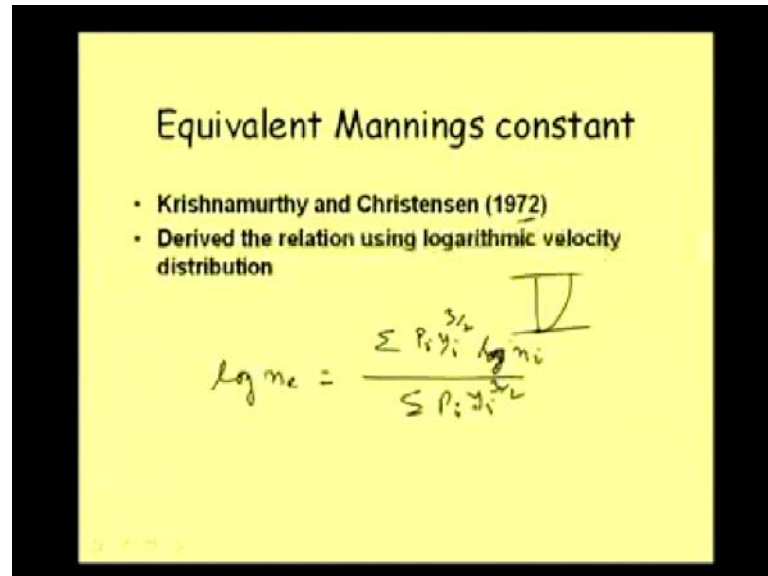
$$n_2 = \frac{\left(\sum P_i n_i^2 \right)^{1/2}}{\sum P_i^{1/2}}$$

Then, some other scientist, of course, did that, tried for getting the value of n and suppose, he did this expression. That discharge is equal to the sum of the discharge for the sub area by following that he got that n is equal to P perimeter. Then, R to the power $5/3$ and summation of P_i , that is the perimeter of each individual, then R_i to the power $5/3$ divided by n_i . So, this is from that concept.

Then, again in 1993, Einstein and they also did in 1955. There they were starting with the assumption that total resistance force is equal to sum of the resistance force of the sub area. By this assumption, they could get the value that n is equal to summation of

say $P_i n_i^2$ and whole to the power half divided by summation of P_i to the power half. So, that can be one expression.

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Equivalent Mannings constant

- Krishnamurthy and Christensen (1972)
- Derived the relation using logarithmic velocity distribution

$$\log n_e = \frac{\sum P_i Y_i^{\frac{3}{2}} \log n_i}{\sum P_i Y_i^{\frac{3}{2}}}$$

Krishnamurthy and Christensen, it is of course, latest, but I mean it is not that most accurate. So, in 1972, they derived by using that logarithmic velocity distribution. That means, velocity distribution is considered as logarithmic and that way, they could get a value that log of n_e is equal to that summation of $P_i Y_i$. Here, Y_i is basically nothing, but the flow depth and log of n_i and divided by summation of $P_i Y_i$ to the power $\frac{3}{2}$, where Y_i is nothing, but the flow depth. So, that way, different formula were developed but in 1980, in fact, a study was conducted with all these formulas and then, it was found that the first formula what I have given, that gives in fact more accurate result as compared to others.

So, that way, we can conclude that equivalent Manning's constant can be used for trapezoidal. Sorry, for compound section and then, we have seen how we can carry out our computational work. Well, then with this, we are concluding our discussion on the computation of open channel flow and we will be going for some more discussion when the channel carry sediment and other things. That is when the channel is erodible channel. That part we have not discussed. That we will be discussing in the next class. Thank you very much.