

**Advanced Hydraulics**  
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**Module - 1**  
**Open Channel Flow**  
**Lecture - 8**  
**Computation of Critical Flow**

Good morning to everybody, let us continue with our post graduate course on advanced hydraulics. So, we have studied in the first module, and it is the eight lecture of the series. In the last class, we have described about momentum equation, energy equation, specific force, the quiz related to those questions are pending. And today first we will conduct the quiz of that, then we will be going to the next part of the lecture.

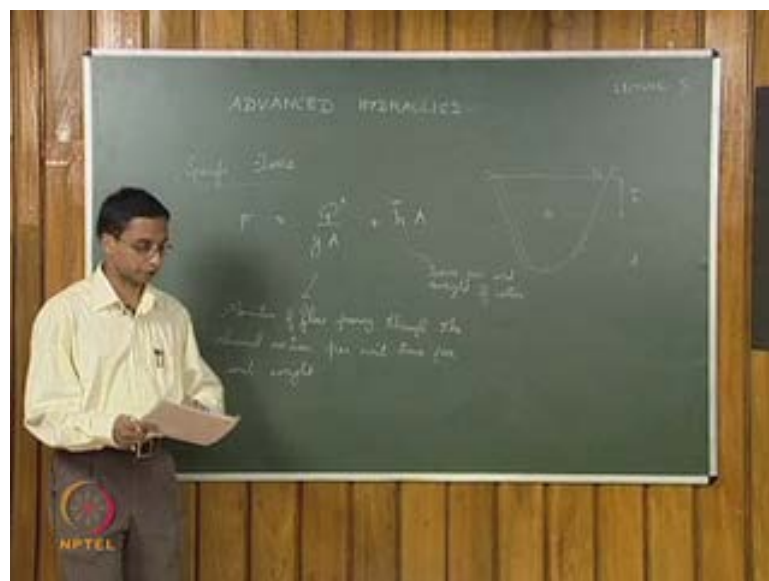
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A first question for the quiz of last class topic; give examples of the local phenomena, where state of flow changes from subcritical to super critical or vice versa. Give examples of some of the local phenomena in open channel flows, where the state of flow changes from subcritical to super critical and vice versa. So, you just give some of the examples you are aware of them. The second question, second question suggests that, if there is a reach of open channel, and 2 sections - 1 1 and 2 2 are considered, if  $y_1$  is the depth here,  $v_1$  is the average velocity across the cross section, at section 1 1.  $v_2$  is the average velocity at the cross section in section 2 2, and depth of flow is  $y_2$  here, if these

are the given conditions, in an open channel flow for a steady state prismatic channel, in a steady state prismatic flow channel, can you express discharge, express discharge now here, in terms of average parameters. The first question is actually, whether can you express discharge, in case of average parameters, if it is possible, then express state them. Third question; what is specific force, state the assumptions involved in developing specific force concept. So, these three are the questions for today's quiz, you answer them, it will take hardly 3 to 4 minutes, to solve the questions here. Let us continue with our lecture now.

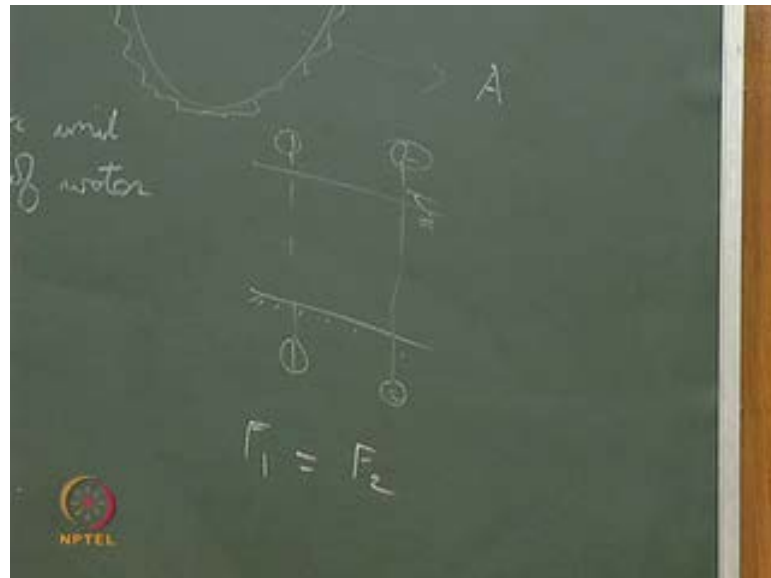
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So, last class as we mentioned we have discussed on specific force. We have developed specific force from the momentum equation, using certain assumptions, which you have right now in your quiz question. So, specific force was defined as, say  $f$  is equal to  $Q^2$  square by  $gA$  plus  $\bar{h}$  into  $A$ , if you recall them say any arbitrary cross section. The cross section area of flow is  $A$ , and if the centroid of this area, if the centroid of this area if it is at a depth  $\bar{h}$ , from the top surface of flow. Then we are using them here to compute this weak force,  $Q$  is the discharge,  $g$  acceleration due to gravity,  $A$  is the area of the cross section. So, we have also suggested what are these two terms; the first term is nothing but the momentum of flow, momentum of flow passing through the channel section, passing through the channel section per unit time per unit weight. The second term is nothing but it is the force per unit specific weight of water. So, as all of these are related to some force per unit rate of water, even the left hand side  $f$ , it is also certain

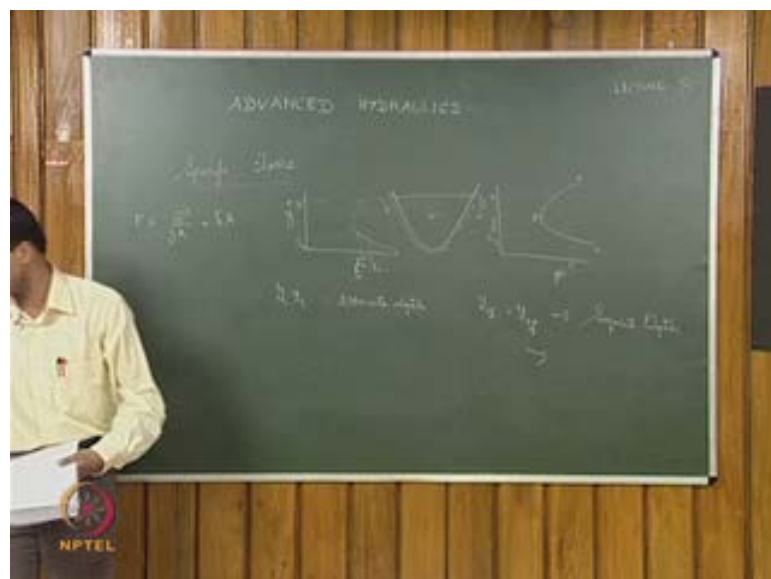
quantity of flows related to unit of water. The summation of these two quantities are therefore calls specific force, and we are applying them, in many of the open channel computations.

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We also suggested that, in a small reach in a small reach of the channel, whether slope of the channel, is negligible. Friction force can be neglected in a small reach, in such a situation, we the specific force into sections F 1 will be equal to F2, this was also derived in the last class.

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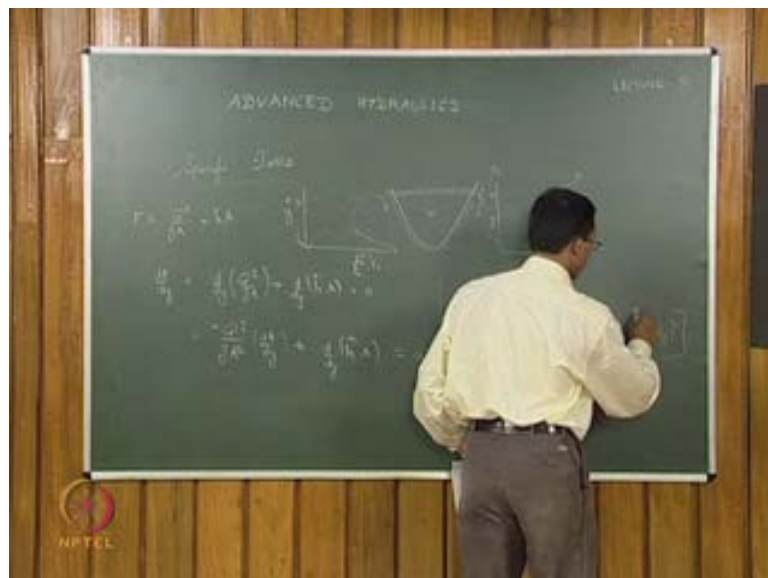
So, what do you understand, what is the peculiarity of specific force, why it will be useful. Again let me write the same equation  $f$  is equal to  $\frac{Q^2}{gA} + \bar{h}$ , using this equation. Now definitely you can plot, specific force verses depth of flow. You can easily plot specific force; say this is the cross section of the channel, I can plot now specific force verses depth of flow easily, and it is quite possible, you use this equation. Once you use this equation, it was observed that, the curve was having some shape of this form. We will be coming to that, we will discuss more on that. If you recall your energy equation if you recall your energy equation for the same channel cross section, if you plot  $E$  verses  $y$ , you have observed them that say. You got something of these nature, the energy verses depth flow equation, you had got something of these nature earlier.

So, if this is the depth of flow given to you, say  $y$ , this is the depth of flow,  $y$  given to you. Then this is the initial depth, and the corresponding, at the same energy the corresponding alternate depth will be, somewhere here. See if you call this  $y_1$  the alternate depth will be somewhere here  $y_2$ . These things are already mentioned, you are quite familiar with this things now. In this case; however, if you look into the problem now, say in this cross section, if the depth of is the depth of or from the top surface, the height at which the centroid of the area is located  $\bar{h}$  if it is at this place, and if you try to interpret the things now. Say in the specific force curve, in the specific force curve, this also I can now, make it into two portions; see  $A B$  and  $B C$ , I can just readily make it, make this curve means, we can divide this curve into 2 portions say  $A B$  and  $B C$ , where from  $B C$  means that the direction the curve changes at  $B C$ , that is what we are going to mean it here.

So, from this two limbs, initially when the depth of flow is  $y_1$ , that  $y_1$  we can easily represent in the specific force verses depth curve  $y_1$ , same as well. It has similarly in the energy curve, for this same specific force curve. For this same specific force curve, there are 2 depths of flow, there are two depths of flow for this same specific force curve, say  $f_1$ , it has 2 depths of flow. So, let me give this as  $y_1 f$  and  $y_2 f$ , when you study hydraulic germ and all, where the local phenomena occurs, at those situations, you will come into use of these two depths, where the specific force are equal. Those things we are taking in to account now, we will discuss those things later. So, these two depths  $y_1 f$  and  $y_2 f$ , if  $y_1$  and  $y_2$  in the energy equation curve, energy verses depth equation curve, if there are called as alternate depths, this we can call as sequent depths.

If you see this  $y^2 f$  is not at same height as  $y^2$  in the energy curve, there may be slight difference, that shows or if you just extra plot it here. The critical depth will be same, that we will discuss again. Here these thing you extrapolate it here, the energy location is somewhere here, and there is a loss of energy, here this was  $E_1$ , and this has now become  $E_2$ , there is a loss of energy, when the flow occurs. So, there the depth will change from  $y^2 f$  also  $y$ , if it towards it subcritical depth of  $y_1$ , then it may go to supercritical depth of  $y^2 f$ , rather than  $y^2$ , in some of the local phenomena's. We will see them in the next module, which ever deal with the local phenomena. So, let us discuss on this curve now again.

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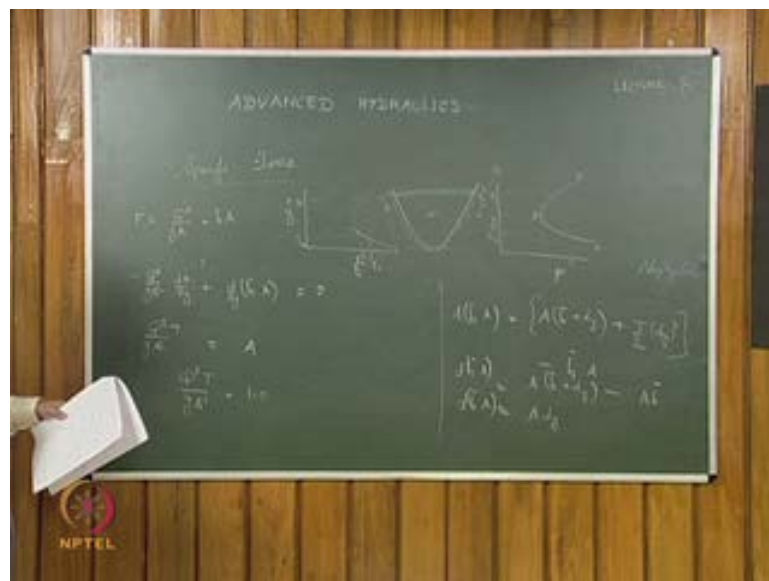


So you have this equation, so you have plotted this equation  $y$  verses  $f$ , using the same equation this thing you have plotted that, you have observed the corresponding curvature. The location of the minimum specific force, see the minimum specific force, where it is. You just try to observe that. So, minimum specific force, if you try to obtain from this curve, so this will be  $d$  by  $d y$  of  $Q$  square by  $g A$  right. So, this quantity should be zero, so that the force is minimum, specific force is minimum. How will you identify that now here, what is  $d$  by  $d y$  of  $Q$  square by  $g A$ , you know  $a$  is function of  $y$ . Similarly  $h$  bar and  $A$  both are functions of  $y$ . So, we will be dividing them appropriately, this is equal to  $Q$  square by  $g A$  square minus  $Q$  square by  $g A$  square  $d a$  by  $d y$  plus  $d$  by  $d y$  of  $h$  bar  $A$  is equal to 0, let us interpret it in this way. From this equation, how will you compute  $d$  by  $d y$   $h$  bar  $A$ . Look at into this curve, you know the

depth of the centroid from the top surface, this is  $\bar{h}$ . So, if there is a slight increase, say if there is slight change in area, say this depth of flow is now changed this quantity is  $dy$ .

Let me suggest that this small change in depth of flow is  $dy$ . Of course, you know this is the top width at the top, due to this change in  $dy$ , how this quantity  $\bar{h}$  will be affected; that is the thing you now need to compute. Going through the picture, now I can suggest that, for any small increment in  $dy$ . The increment or change in  $\bar{h}$ , it can be given as  $d\bar{h}$ , due to a small increment in  $dy$ , this can be given as, the actual area. Now  $\bar{h} + dy$ , plus I will write it again,  $A$  into  $\bar{h} + dy$  plus the change in area how its got affected  $T$  by  $2 dy$  whole square minus  $\bar{h} A$ . This is nothing, but this was the original area  $\bar{h} A$ , that the centroid, you are multiplying with this depth of the centroid in to the area, that quantity earlier, and the quantity that is occurred by this small increment in that  $dy$ ; that is this quantity. So, from here, suppose if it can be approximated that, this quantity if it is almost negligible. If this quantity, if it is almost negligible, if it can be approximated, then you can suggest that  $d\bar{h}$ , is equal to  $A dy$  minus  $A \bar{h}$ , or this is equal to  $A dy$ . So, this is actually approximation  $d\bar{h}$  can be approximated as  $A dy$ . Substitute those quantities here you will see that.

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Here if you substitute them, you will see that this is this quantity you know, it is the top width  $T$ , as it is evident, and as well as we have discussed that  $d_a$  by  $d_y$  is the top width  $T$ , in one of our earlier lectures also. So, you will get this quantity as  $Q^2$  by  $g A^3$  is equal to  $A T$ , this is  $A d_y$  by  $d_y A$ , or you can suggest that  $Q^2 T$  by  $g A^3$  is equal to 1. If you recall your energy depth relationship, the same relationship was observed; that is, this relationship  $Q^2 T$  by  $g A^3$  is equal to 1 we will occur, only in critical flow condition. So, what does this imply, if you compute minimum specific energy,  $E_s$  at the minimum specific energy, it will give you the critical flow of the section; that is what it implies. So, as the minimum specific energy also is, gives the critical flow, similarly the minimum specific force also, gives you the critical flow.

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So, next topic of ours is, computation of critical flow. So, just we call the earlier portions of our lectures, what does, how the critical flow occurs, or what are the properties in critical flow. You have observed the previous lectures and all, you might have seen that, for the critical flow to occur, for a given discharge. It is for a given discharge in a channel, specific energy should be minimum. This is one criteria for critical flow to occur. The next thing is that, specific force should be minimum, just now we have seen that, you also have seen following relationship, velocity head; that is  $v^2$  by  $2g$ . This should be equal to, half of hydraulic depth.



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Velocity head  $\left(\frac{V^2}{2g}\right) =$   
Froude Number  $= 1$   
$$\frac{Q^2 T}{g A^3} = 1$$

The image shows a chalkboard with handwritten text and equations. At the top, it says 'Velocity head  $\left(\frac{V^2}{2g}\right) =$ '. Below that, it says 'Froude Number  $= 1$ '. At the bottom, there is a large equation: 
$$\frac{Q^2 T}{g A^3} = 1$$
. In the bottom left corner, there is a small NPTEL logo.

You have also seen froude number should be, or you have also seen the following relationship,  $Q^2 T$  by  $g A^3$ , in the critical flow condition should be 1. This also you have seen.

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Or you can also suggest that, in critical flow. For a critical flow, the discharge is maximum, for a given specific energy value, this also we have proved; that is for a given discharge, specific energy should be minimum, and for a given specific energy value, the critical flow occurs when the discharge is maximum, that also we have observed that. So,



these are some of the criterion for, understanding the critical flow, or means to observe flow, these are these are the essential things required, so then how will you compute the critical flow.

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If you see that, critical flow, for critical flow, in any arbitrary channel section, whether it be the this relationship, whether it be this relationship, where  $d$  is your hydraulic depth. The critical flow depends on, the area of cross section, it depends on the area of cross section, it depends on the depth of, or area itself is function of depth, so there is no need to specify much on that. You can suggest that, all these quantities determine whether a section, allows the critical flow; means whether the flow in a section is, critical are not, these things are dependent. So, once you can identify these parameters, you can easily understand how the critical flow occurs.

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So, critical flow depends on your area of cross section, hydraulic depth of width as we discussed. You can suggest that, for any given critical flow condition, or for any prismatic channel, say prismatic channel, whose section is uniform through the reach of the channel. Then for that prismatic channel, your critical depth, will be same; isn't it. Critical depth will not be changing for prismatic channel, the critical channel will be same throughout the reach of the channel. And also, suppose, in the prismatic channel, in such a prismatic channel, it the if for the entire duration, if the for the entire duration, if the depth of flow is maintaining the critical depth, in the entire reach; that is this is prismatic channel, if the depth of flow is critical, then you can suggest that, the entire reach of the channel is, observing critical flow. The entire reach of the channel is observing critical flow, why the term critical is been called, few tried to observe that the critical flow conditions and all, especially observing your energy depth, curves and all.

You have observed the energy depth curves and all, you will see that, at the critical section, or whether the specific energy is minimum, here your curve is almost vertical, and this location, it is almost vertical. So, even if there is a small change in energy, even if there is a small change in energy that gives you considerable difference in, depth of flow, see here and here, gives you considerable depth, change in depth of flow, if this was critical depth. Then it shuffles between the alternate depths; that is super critical, and the sub critical depth. It is just alternate in between, that causes wave nature in the fluid, and the flow in the critical condition, it is mostly unstable in nature, because due to this

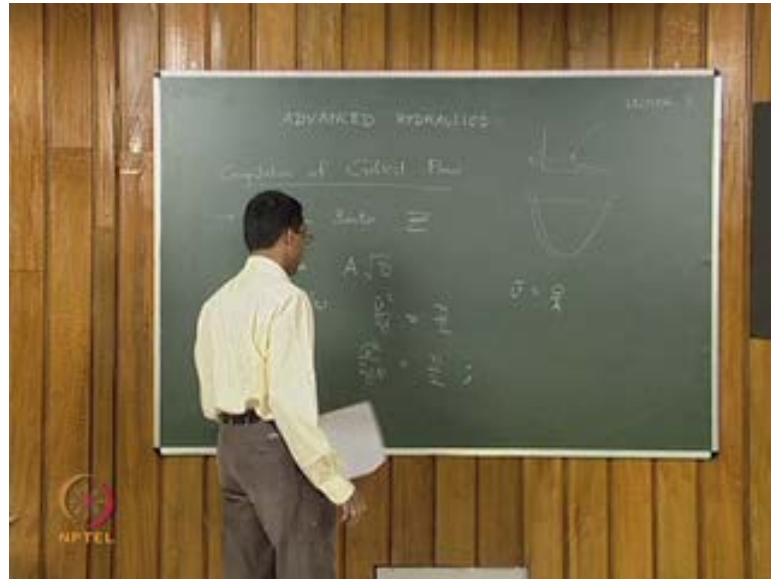
fluctuation, even a small variation in energy itself gives you censorial difference in, height and all that. So, therefore, wave nature is present, and therefore, it is generally called critical flow. So, critical flow it is not a desirable condition, if in any channel, if critical flow is existing, it is not a desirable condition, because it will be in an unstable condition, and it will not be in a friendly matter if you want to do navigation, or a if you want to do fishing, you do not want to do any particular things and all, it will be quite difficult.

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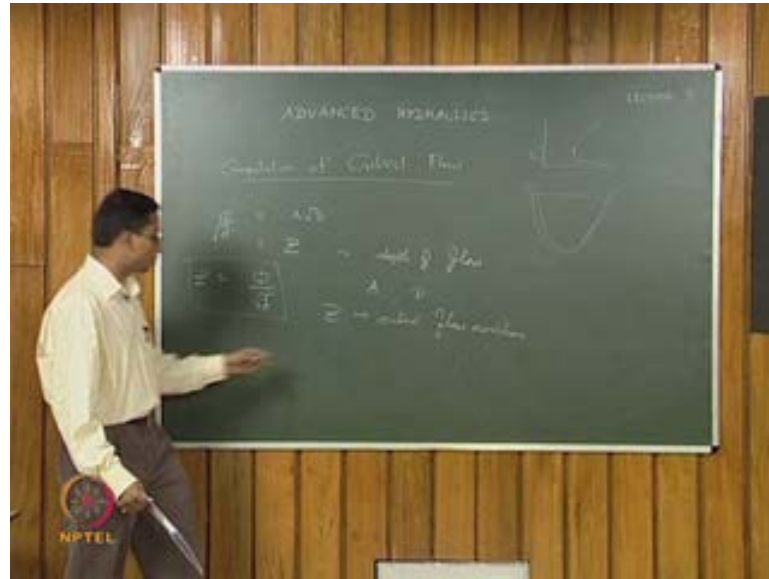
The slope of the channel same that prismatic, in the same prismatic channel the slope of the channel, that maintains critical flow, you can describe it as  $S_c$  for example, slope of the channel, that allows you the critical flow in that prismatic channel. Let us consider as  $S_c$ , then if any slope is given or a similar type of channel, and if that slope  $S$  is less than  $S_c$ , the flow will be sub critical in that channel, and that type of slope is called mild slope. If any given slope of a channel is greater than the critical slope, if it is greater than critical slope that gives you super critical flow; that is quite obvious. The depth of flow decreases velocity of flow increases in super critical, so and this is called steep slow channels. So, in open channel hydraulics and all, you will now care onwards here, you will, here fetch terms call mild slope channels, steep slopes channels and all, so these is the meaning of that.

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So, for computing critical flow, you can devise two parameters, based on your various criteria why various discussions as we have, you can devise two parameters; first one is, section factor at  $z$ , then second one is, hydraulic exponent for critical flow, usually given as symbol  $m$ , we will discuss on these thing, what is section factor. Section factor is  $z$ , this is defined as the product of, the area of your water flow, into the square root of the hydraulic depth of that section, this is how the section factor has been defined. So, why the section factor is defined, and how it is useful in computing critical flow, let us see into that. If you recall for critical flow, for your critical flow your velocity head is, equal to half of the hydraulic depth  $d$ . If this is the case, you can suggest that;  $v$  is equal to  $Q$  by  $A$ . So, that gives you  $Q$  square by  $2 g A$  square is equal to  $D$  by  $2$ , or rearrange the terms here, what will you get.

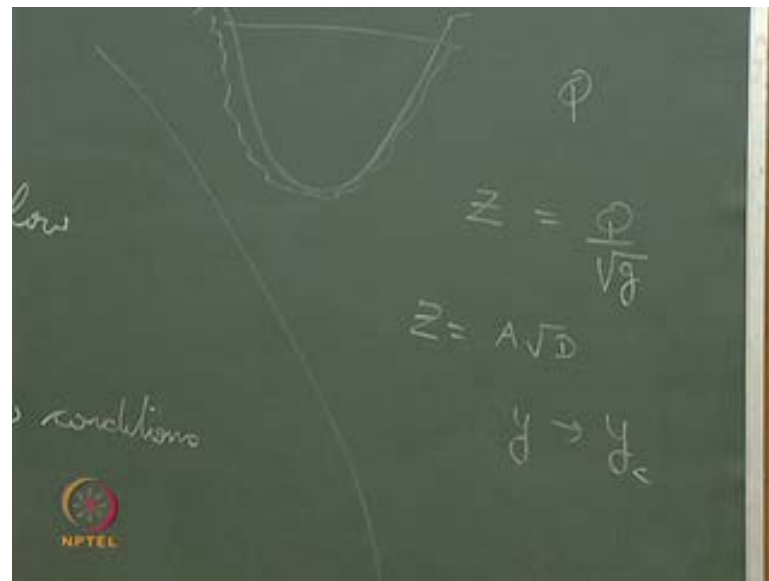
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$Q^2$  by  $g$  is equal to  $A^3 D$  or first subsequently  $Q$  by root  $g$  is equal to  $A$  root  $D$ . So, a root  $d$  we have already suggested that, this is your section factor  $z$ . So, section factor, is you can either, means it is the property of section, this gives you the property of section, and you can also compute this section factor using, the external parameter  $Q$ ,  $Q$  is the given external parameter discharge through a section. Mostly in the open channel we are considering  $Q$  as a constant value. So, therefore, if  $Q$  is given, at any reach if  $Q$  is given to you, you can now easily compute the section factor, or section factor is this quantity,  $Q$  by root  $g$ , that can be now identified. So, this section factor is or the critical flow, please note that, it is for the critical flow. So, section factor, it is function of depth of flow, because both  $A$  as well as your hydraulic depth  $D$ , they are function of depth of flow,  $z$  is available only in, means this is computed, using this relationship.

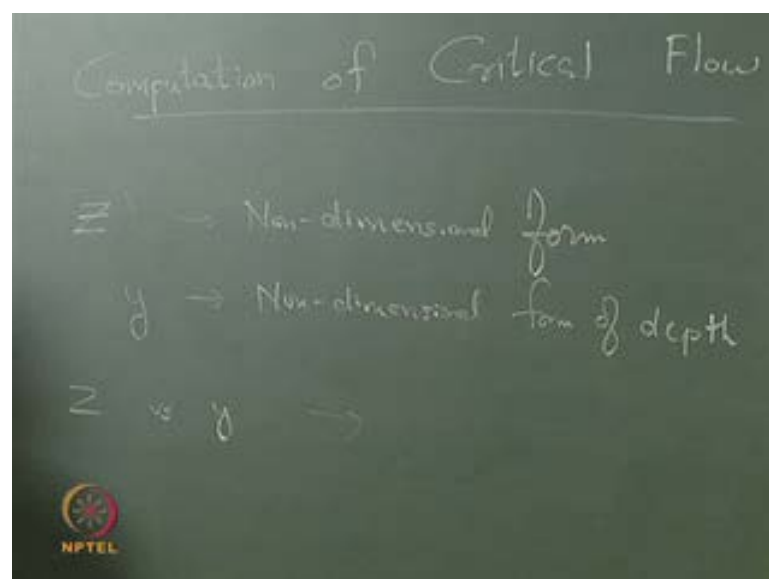
This relationship has been obtained, only for the critical flow conditions. This relationship has been obtained, only for critical flow conditions. Therefore,  $z$  whatever  $z$  you compute in a channel section, that will give you the corresponding critical depth at that section, so how will you do that. Because one value of  $z$  will yield you only one type of critical depth in that channel section. In a channel section, for a given discharge, you cannot have multiple critical depths. For a given discharge, there will be only one critical depth in a channel section, that can be easily identified from this criteria as well, we will just go through that,  $z$  the same relationship  $z$  is equal to  $Q$  by root  $g$ , once you compute that, once you compute, say in a channel section, if  $Q$  is given to you.

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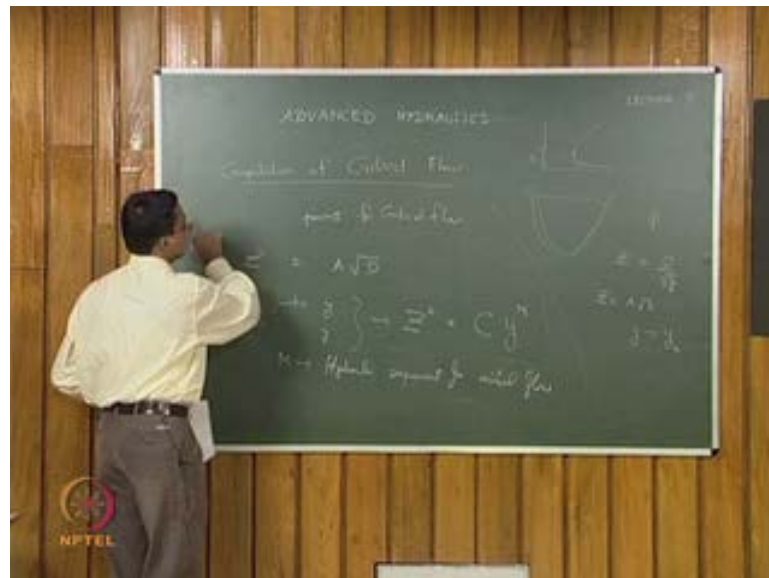
If discharge  $Q$  is given to you, now you can compute  $z$  is equal to  $Q$  by root  $g$ . So, whatever magnitude you get for  $z$ . You now write to interpret that  $z$  is equal to  $A$  root  $D$ . So, if you know the relationship of  $D$  with  $y$   $A$  with  $y$  and all, substitute it here. So, from here you will get  $y$ , what is the  $y$ , and this is nothing but the critical depth  $y$ . So, this is how you can interpret critical depth from a given discharge, this is one method, so we will tell you how  $z$  say.

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Z if it is computed, then we can make it into a non dimensional form, based on the cross section, you can obtain a non dimensionalized z. You can obtain a non dimensional form of depth of flow. If you can obtain them, using some characteristic length, or characteristic height and all, you obtain the non dimensional depth. Then z verses y if you can plot, from that graph, you can easily identify the critical flow. We will just see after this lecture, means in the concluding part of this lecture, or today's lecture, we will just do ex explain them, in a better way.

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The next parameter which we suggested is, hydraulic exponent, for critical flow. So, your z section factor A is, z is equal to A into root of D, A is function of y, D is function of y. You can interpret now the section factor z square, you can interpret section factor, in such a way that, z square is equal to some coefficient c into y to the power of m, can you understand that. We are just interpreting them, now that is we are defining that as z is function of A is function of y, D is function of y, here you know this is, this may be function of, even multiple powers of y, this may be function of y, or it may be some power of y. So, finally, we can suggest z square is equal to c into y to the power of m, is type of fitting, you type fitting your section factor, in the following form. So, if we can define it in the following form. Here m is defined as the hydraulic exponent form critical flow.



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$$z = A \sqrt{D} = A \sqrt{\frac{A}{T}}$$

$$\frac{d}{dy} (\ln z) = \frac{M}{2y}$$

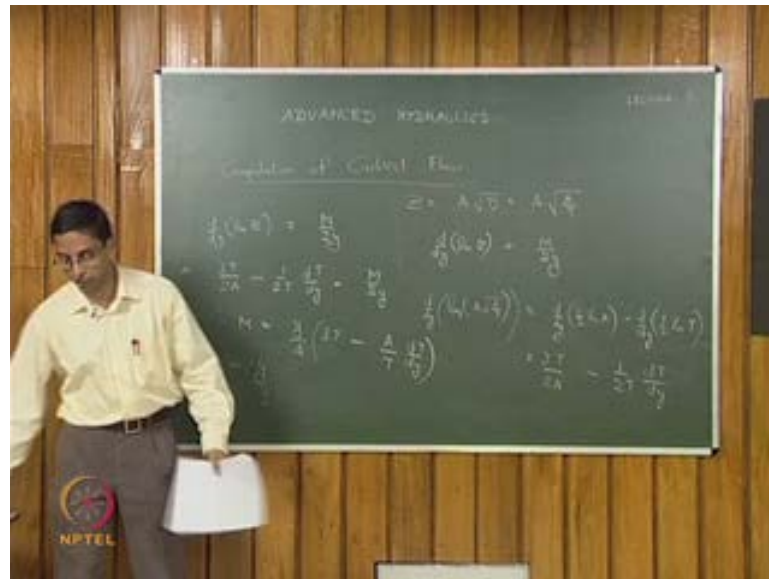
$$\frac{d}{dy} \left( \ln \left( A \sqrt{\frac{A}{T}} \right) \right) = \frac{d}{dy} \left( \frac{3}{2} \ln A \right) - \frac{d}{dy} \left( \frac{1}{2} \ln T \right)$$

$$= \frac{3}{2} \frac{1}{A} \frac{dA}{dy} - \frac{1}{2T} \frac{dT}{dy}$$

So, once you can obtain that, the motive of discussing hydraulic exponent is that, once you can obtain  $m$ , you can easily identify the critical flow, wherever it occurs. Say  $c$  is your coefficient;  $m$  as we discussed this is the thing, hydraulic exponent. So, here if I take, this is nothing but if you take the natural logarithm, you will get this thing in the following form, or you can see this thing. This is equal to half of  $\log c$  plus  $m$  by  $2 \log y$ , or you know what is  $\log y$  natural  $m$  by  $2y$ . You can write it in this form,  $z$  is equal to  $A$  into root of  $D$  this is equal to  $A$  into root of  $A$  by  $T$ .

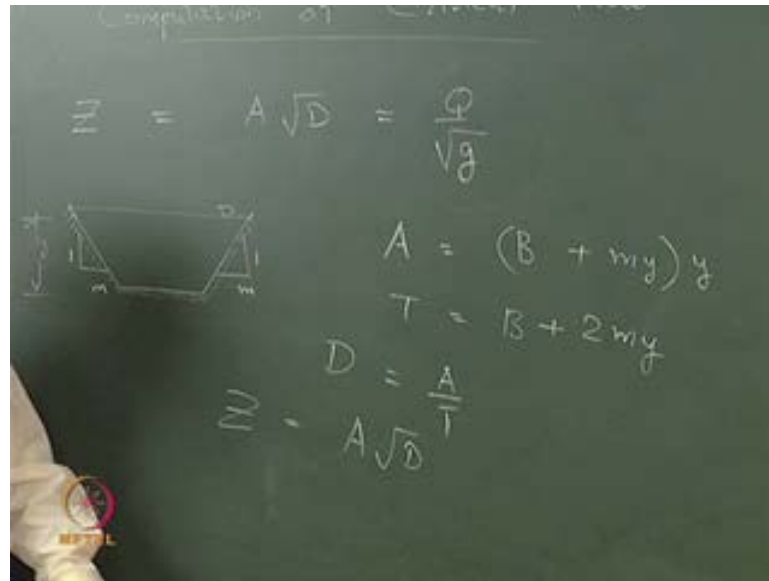
So, what will be the natural logarithm of  $z$  then, what will be the natural logarithm of  $z$ , you can just try to interpret them. If I take the derivative of this equation  $d$  by  $d y$  of  $\log z$ , this is equal to the derivative of this quantity gets cancel, so you will get. Please let me, beg me pardon. Here this is  $m$  by  $2 \log y$ , once you take that derivative, you are getting it as  $m$  by  $2y$ . So, this quantity once you get this relationship, you incorporate the following quantities here; that is  $z$  is equal to  $A$  into root of  $A$  by  $T$ , and try to derivate that. You will see that this is nothing but  $d$  by  $d y$  of logarithm of  $A$  root of  $A$  by  $T$  is equal to  $3$  by  $2 \log A$  minus  $d$  by  $d y$  of  $1$  by  $2 \log T$  or this is equal to  $3 T$  by  $2 A$  minus  $1$  by twice  $T$   $d t$  by  $d y$ .

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So, substitute this quantity in the LHS here, [noise form 45:15] to [noise 45.38]  $m$  by  $2y$ , you will get the following relationship, your hydraulic exponent  $m$  is equal to  $y$ , you can take  $A$  out while by  $A^3 T$  minus  $A$  by  $T \frac{dT}{dy}$ . So, once you get this relationship, for any given cross section area, you can compute the hydraulic exponent coefficient, hydraulic first hydraulic exponent  $m$ , this is equal to  $y$  by  $A^3$  into  $3T$  minus  $A$  by  $T \frac{dT}{dy}$ . Now you can easily plot  $m$  verses  $y$ ,  $m$  is a non dimensional parameter, you can easily now plot  $y$  with something, some characteristic depth  $y$  by  $B$ , some non dimensional characteristic value. You can easily plot the curve, because you already know that, this relationship or this action factor and all, you are incorporating it only in the critical flow condition.

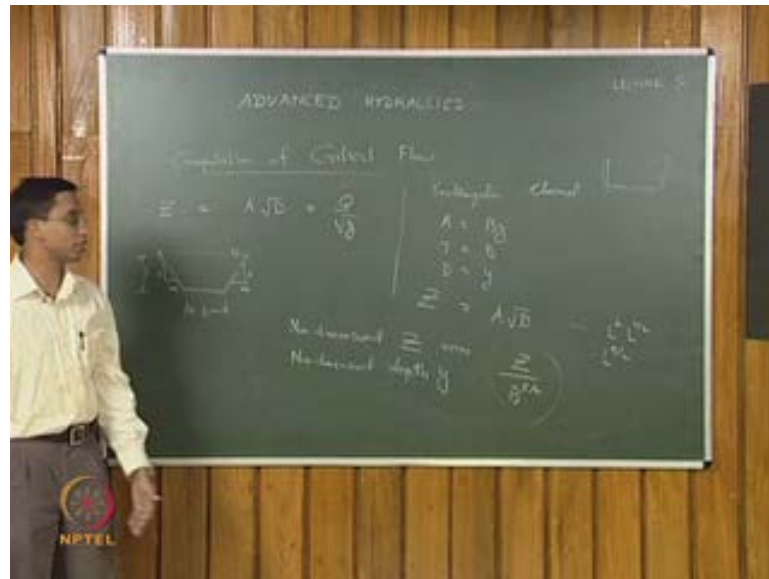
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Handwritten notes on a chalkboard showing the derivation of the critical flow condition for a trapezoidal channel. The notes include the equation  $z = A\sqrt{D} = \frac{Q}{\sqrt{g}}$ , a diagram of a trapezoidal channel cross-section, and the formulas  $A = (B + my)y$ ,  $T = B + 2my$ ,  $D = \frac{A}{T}$ , and  $z = A\sqrt{D}$ .

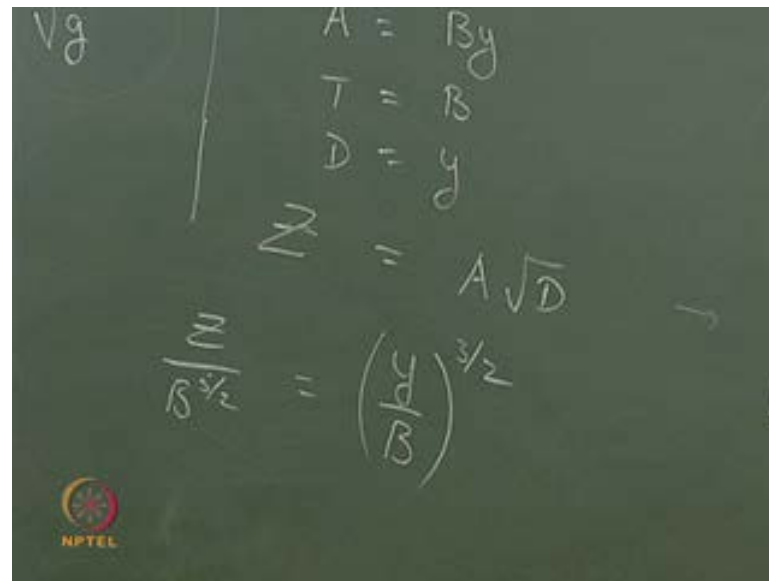
So, your section factor  $z$  is equal to  $A$  into root of  $D$ , this is also equal to given  $Q$  by root of  $g$ . Consider an arbitrary trapezoidal, consider an arbitrary trapezoidal cross section channel, its side slopes, say 1 is to  $m$ , let it be 1 is to  $m$ . Then how will you compute, whether the flow is critical or not, or how will you compute the critical flow in such a channel, how can you compute them let us see now. So, trapezoidal channel, so your  $A$  is equal to. Now this can be given as  $B$  plus  $m y$  into  $y$ , because the depth of flow is  $y$  your top width  $T$  is equal to  $B$  plus  $2 m y$ , so now, you can easily calculate  $D$ , this is equal to  $A$  by  $T$ . Once you calculate  $D$ , you can compute  $z$ ,  $z$  is equal to a root  $D$ . You have initially computed  $z$  is equal to  $Q$  by root  $g$ , you will compare this both are same, so then it is critical flow right. So, because for critical flow, this is the relationship, this relationship and all we have obtained, especially this relationship we have obtained using critical flow condition.

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We will just see here for an example, in an example, not in example. Let us consider beginning a rectangular channel; that is in the trapezoidal channel, your  $m$  is equal to 0, area is  $B y$  top width is equal to  $B$  hydraulic depth is equal to  $y$ . So, your  $z$ ,  $z$  is equal to  $A$  into root  $D$ , like this you will be, how will you compute them. You now know that  $z$  has the dimensions of  $L$  square into  $L$  to the power of half; that is  $L$  to the power 5 by 2. Now, our previous part, we have suggested that we have to use, non dimensional  $z$ , verses non dimensional depth  $y$ , this we have mentioned it earlier. So, how will you make this  $z$  as a non dimensional form. Here this  $z$  is now, if you see for the any trapezoidal channel, the fixed parameter is your width of the, bottom width of the trapezoidal channel  $B$ , it is a characteristic length then. So, if you make say  $z$  by  $B$  to the power of 5 by 2, if you obtain a parameter, if you obtain this thing then, now you can easily form a chart a series of data for different distinct. Similarly the non dimensional form of depth is given as  $y$  by  $b$ .

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Handwritten equations on a chalkboard:

$$Vg$$

$$A = By$$

$$T = B$$

$$D = y$$

$$Z = A\sqrt{D}$$

$$\frac{Z}{B^{5/2}} = \left(\frac{y}{B}\right)^{3/2}$$

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So, in any graph can now be plotted verses  $y$  by  $B$  verses  $z$  by  $B$  to the power of 5 by 2 for a trapezoidal channel. So, your  $z$   $B$  to the power of 5 by 2 for a rectangular channel, this is nothing but if you rearrange the terms here, it will come to be  $y$  by  $B$  whole to the power of 3 by 2, why I am giving this relationship here is that. We will be discussing a particular graph using this relationship now.

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A lecturer is pointing to a chalkboard. The chalkboard has the following text and equations:

ADVANCED HYDRAULICS

Optimization of Channel Flow

$$A = (B + zy)y$$

$$T = B + 2zy$$

$$D = \frac{(B + zy)y}{B + 2zy}$$

$$\frac{Z}{B^{5/2}} = \left(\frac{y}{B}\right)^{3/2} \frac{(1 + \frac{zy}{B})^3}{(1 + \frac{2zy}{B})^2}$$

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What happens, say for  $m$  is equal to one trapezoidal channel. In  $m$  in trapezoidal channel if  $m$  is equal to 1 the side slope is 1 is to 1. Then you will have your area of cross section

is equal to A is equal to B plus y into y, your top width t is equal to B plus 2 y. Your hydraulic depth D is equal to B plus y into y by B plus 2 y. Subsequently you can compute z is equal to A root D, you will see that, your non dimensional relationship z by B to the power of 5 by 2, this is nothing but y by B to the power of 3 by 2 into 1 plus y by B to the power of 3 by 2 by. So, you are now getting the following relationship. This quantity, now we will be discussing in the following graph.

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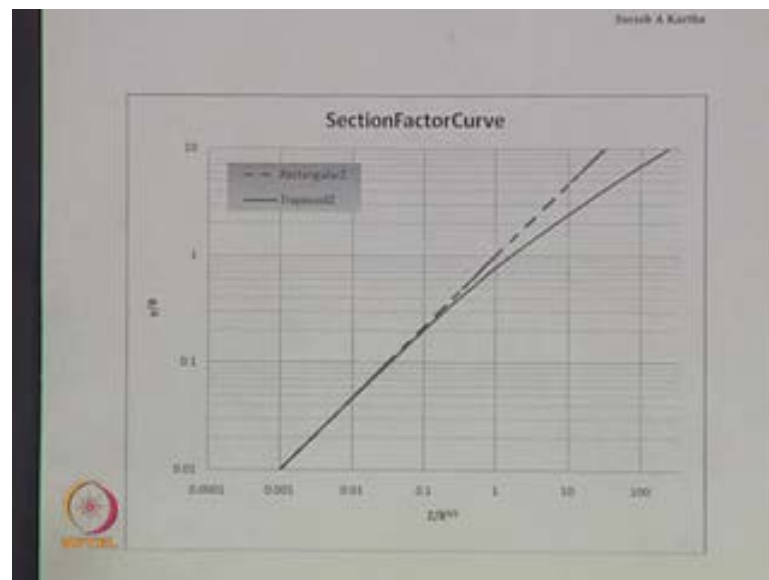
axis is the depth of flow in non-dimensional form ( $y/B$ ).

The graphs are plotted for a rectangular channel section and a trapezoidal channel section of side slope 1:1.

$y/B$	RECTANGULAR ( $z/B^{5/2}$ )	TRAPEZOIDAL ( $z/B^{5/2}$ )
0.01	0.001000000	0.001005037
0.02	0.002828427	0.002857121
0.03	0.005196152	0.005275757
0.04	0.008000000	0.008166472
0.05	0.011180340	0.011480945
0.06	0.014836938	0.015155723
0.07	0.018520259	0.019138611
0.08	0.022427417	0.023575882
0.09	0.027000000	0.028285412
0.1	0.031562777	0.033304154
0.2	0.089842729	0.099360441
0.3	0.164316767	0.192547072
0.4	0.252982221	0.312353077
0.5	0.353553191	0.459179127
0.6	0.464738002	0.634135129
0.7	0.585662028	0.837943664
0.8	0.715541753	1.071660107
0.9	0.853814968	1.336334405
1	1.000000000	1.632993162
2	2.828427125	6.57167069
3	5.196152423	15.7116881
4	8.000000000	29.8142397
5	11.180339887	49.54336343
6	14.696938457	75.49323068

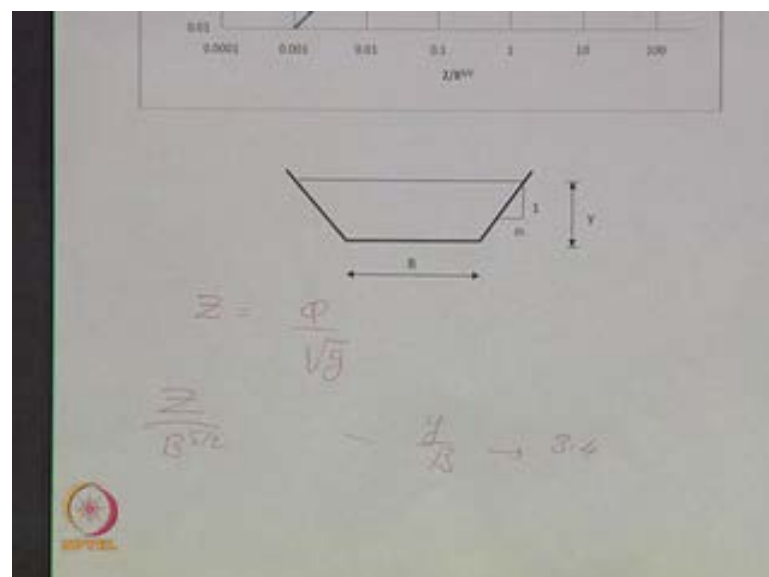
So, for a rectangular channel you can have different ranges of y by B ratios, and the corresponding section factor in the non dimensional form; that has been computed here using the formula, which we derived it here; that is computed here. Similarly for trapezoidal channel of side slope one is to one, for different values of y of y by B ratios, what are the corresponding non dimensional form of the section factor; that is also computed here. They can be easily plotted in logarithmic channel, as shown here.

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So, if we can plot the channel for different sections now, in similar way you will get different type of curves. These curves you can give this as a standard curves. Whatever section factor, or whatever discharge is available to you now, or whatever based on your discharge in a section. Now you can easily identify what will be the corresponding critical depth.

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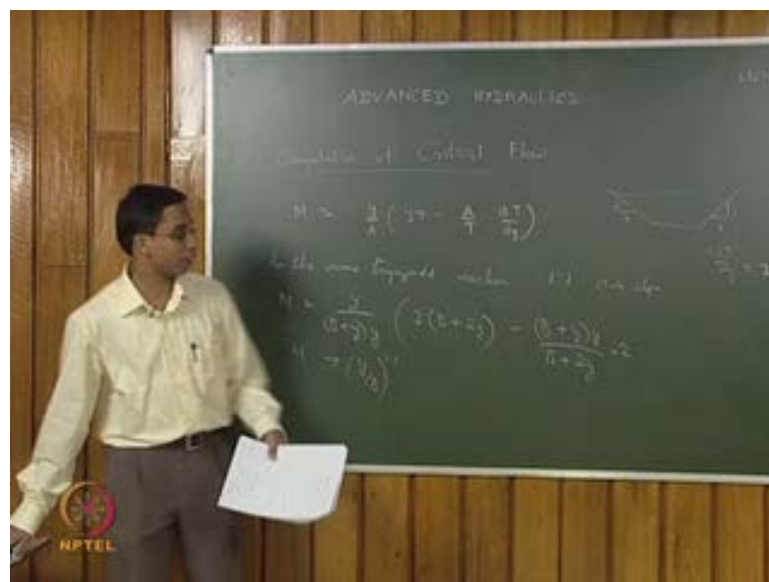


For example, if in a particular channel, or for a trapezoidal channel of sides loop one, you got a section factor value somewhere here, if you got somewhere here. Then just go



and interpolate that, find that, what is the corresponding non dimensional depth. Whatever is there here 2.3, between 3 and 4; that is 3.5 or 3.66 what 3.4, let us take 3.4. So, in this case 3.4 is the non dimensional depth, that can be now easily interpreted, according to your requirement. So, this will give you the critical flow for the given, say  $z$  is equal to  $Q$  by  $\sqrt{g}$ , your first computing it for a cross section channel, because  $Q$  is a given parameter,  $Q$  by  $B$  by root of  $g$  you are computing it. Using this you are trying to identify, means this you will subsequently obtain  $z$  by  $B$  to the power of  $5/2$ , because  $B$  is also given parameter. You have identified that, that values of somewhere here. Now you are trying to interpret the corresponding  $y$  by  $B$ . So, you got  $y$  by  $B$  value as 3.4 in this particular case, interpret what will be the critical depth. So, critical depth will be  $y$  is equal to  $B$  into 3.4, that is all, that is how you compute critical flow using the charts.

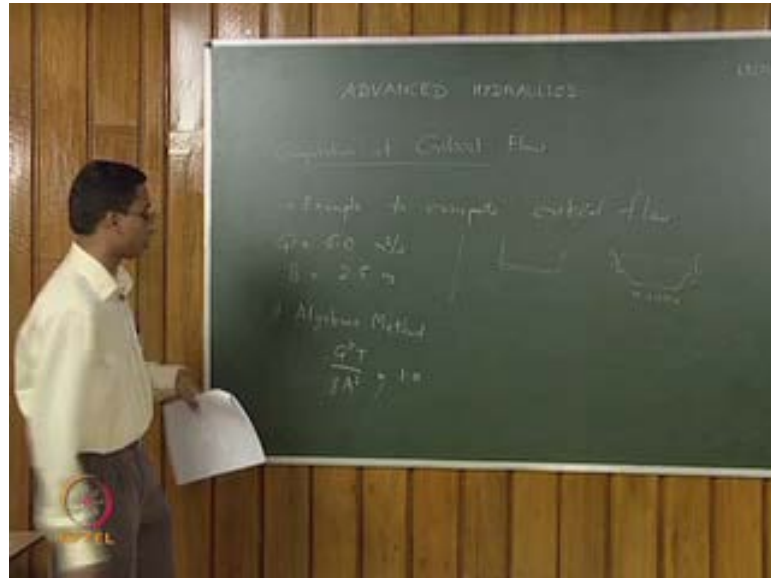
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Now, for the same the hydraulic exponent  $m$ ,  $m$  you have suggested that,  $m$  is equal to  $y$  by  $A^3 T$  minus  $A$  by  $T$   $dt$  by  $dy$ , or the same trapezoidal section of 1 is to 1 side slope. You can now easily compute  $m$ ,  $m$  is equal to  $y$  by thrice  $B$  plus  $2y$  minus  $B$  plus  $y$  into  $y$  by  $B$  plus  $2y$  and  $dt$  by  $dy$  for 1 is to 1 side slope  $dt$  by  $dy$  is equal to 2, so that 2 has to be multiplied here. You can subsequently rearrange the terms, in a non dimensional form here if you. So, you will get  $m$  related to, something with  $y$  by  $B$  to the power of some quantity, you will get such a relationship, even this can be plotted, as we have plotted non dimensional section factor verses non dimensional depth. Similarly you can

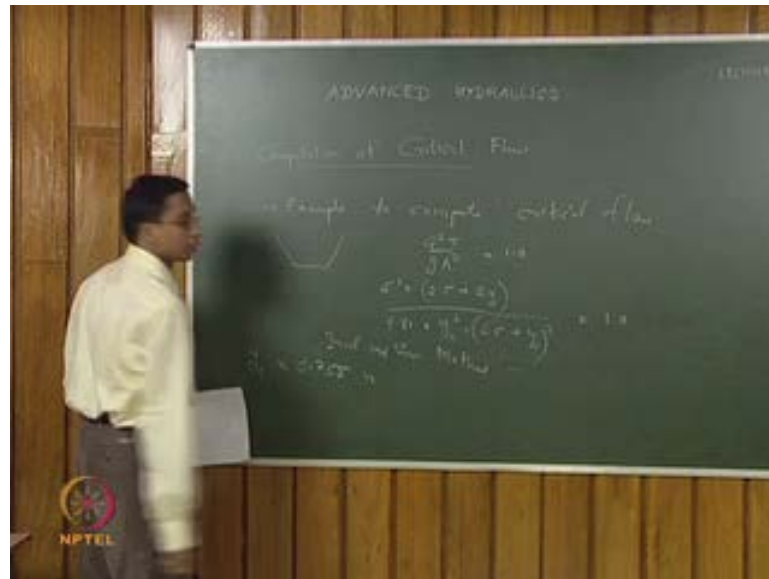
plot hydraulic exponent verses non dimensional depth, like that curve also we will yield you, how to identify the critical flow.

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We will just go through an example, to compute, to example to compute critical flow, given discharge 6 meter cube per second. You are given a discharge  $Q$  is equal to 6 meter cube per second, bottom width  $B$  is equal to 2.5 meters. You are now requested; say if it is a rectangular channel, what is the critical depth? If it is a trapezoidal channel of side slope one is to one, what is the critical flow depth, you are now requested to find that. We are now following the algebraic method first, before using the curves, which ever we have utilize, or whichever which ever we have derived, before utilizing them. Let us go through the algebraic method to compute the critical flow, because this rectangular channel, quite easier one. This is the criteria for critical flow, substitute the quantities here, you will get 6 squared into  $t$  is your, breadth itself 2.5 g 9.81 into what is your  $A$ , this is 2.5 cube into  $y$  cube this is equal to 1. Form this equation, you will get  $y$ ; that is your critical depth  $y_c$  is equal to 0.837 meters, from this relationship you will get that.

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for your trapezoidal channel, you can use the trial and error method, in the same formula  $Q^3 T$  by  $g A^3$  is equal to 1, substitute the quantities 6 squared into 2.5 plus 2  $y_c$  by 9.81 into  $y_c^3$  into 2.5 plus  $y_c$  whole cube is equal to 1. Rearrange the terms, you will get some relationship, you have to use trial and error method, you can use that to solve this equation, this equation, you can use the trial and error method to solve this thing. I have used a trial and error method, through several iteration I have obtained around 0.755 meters. Suppose if you are getting something different, you please let me know that, may be your approach may be better or your hit and trial method may be better, compare to mine. This is how you solve or you compute critical flow depth using algebraic method. In the next class we will discuss for the same thing, using the charts which ever we have shown it here, we will discuss them how to use them, and compute the critical flow.

Thank you.