Advanced Hydraulics
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Module - 1
Open Channel Flow
Lecture -6
Specific Energy and Critical Flow

Very good morning to everyone, welcome back to the course on advanced hydraulics, we are on the lecture six of this course, we are on the first module of this particular course itself. Let us continue with the discussion as was held in the last class.

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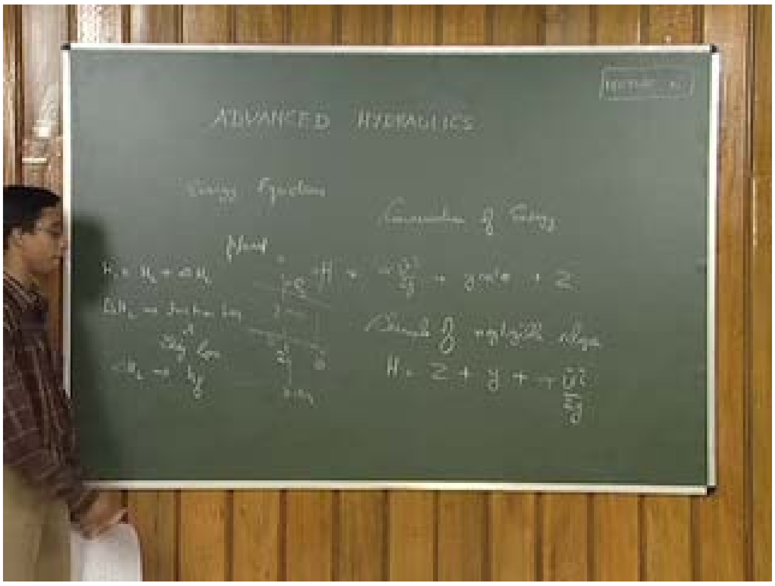

Last class we have discussed on energy equation of open channel flow, what is meant by energy, what is, how the energy is computed and all, they were discussed in the last class. Energy was represented in terms of head or you can say energy per unit weight of liquid, like that this was representing. We have even mentioned, the conservation of energy, we had represented energy, total energy at any channel section, say if any channel section is there, arbitrary channel section is there, then the total energy is represented as H. This constitutes this consist of, your velocity head, and it consist of your pressure head, it constitutes your datum head, and all this three components, that gives you, what you mean by the total energy at a channel section. So, this were given, we had normally, as we suggested at in civil engineering, we are dealing with channels of, negligible slope. Then we can give the following thing as, y is equal to z plus depth of flow y plus alpha v square by 2 g, like this we had specified that in our last class.

We had even suggested that between two sections, say if this is section 1 1 section 2 2, and flow. If the flow direction is in this right direction, right side, then the total energy at section one, and total energy at section 2, they differ by a quantity, what is the defined as head loss, or the energy lost. It is always that the energy will be loss, it cannot be gain in the, total energy can never be gain, from fluid flowing from one direction to other. So, this was also suggested between two sections we had even, suggested that H 1 is equal to H 2 plus del H L, where your del H l or the head loss constitutes is, can be or the head loss, we computed from friction loss, and eddy loss. We also suggested that, in the prismatic channels eddy losses are negligible, and most of the case, your head loss is approximately same as the, head loss due to friction.

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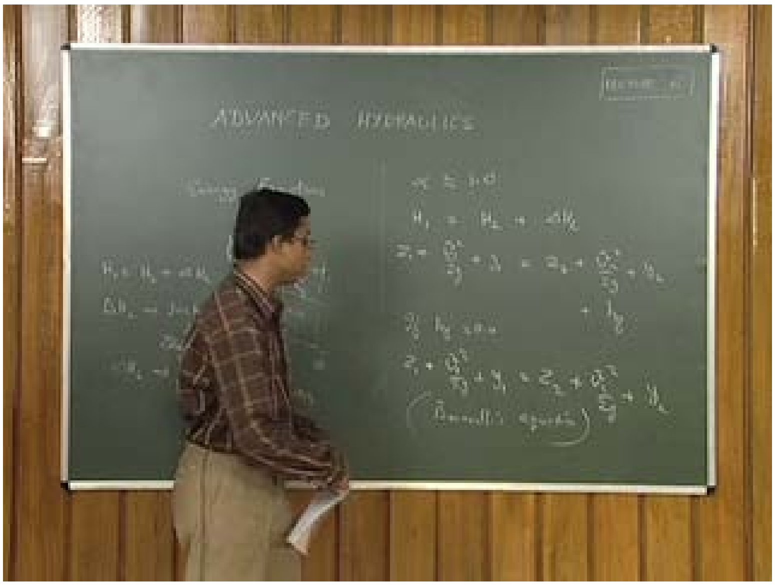

Subsequently, we suggested if you are energy correction coefficient alpha approximately if it is 1. Then H 1 is equal to H 2 plus del H l can be given as z 1 plus v 1 square by 2 g plus y 1 is equal to z 2 plus v 2 square by 2 g plus y 2 plus the friction, the loss of head due to friction, like this we had suggested. And if the loss of head due to friction, is approximately 0, then we subsequently suggested that at any two section, the total energy will be same, and this is same as your Bernoulli’s equation. So, please note that, if the head loss due to friction is the negligible, then you are getting relation between the energy equation relation in the following form. This is same as your Bernoulli's equation.

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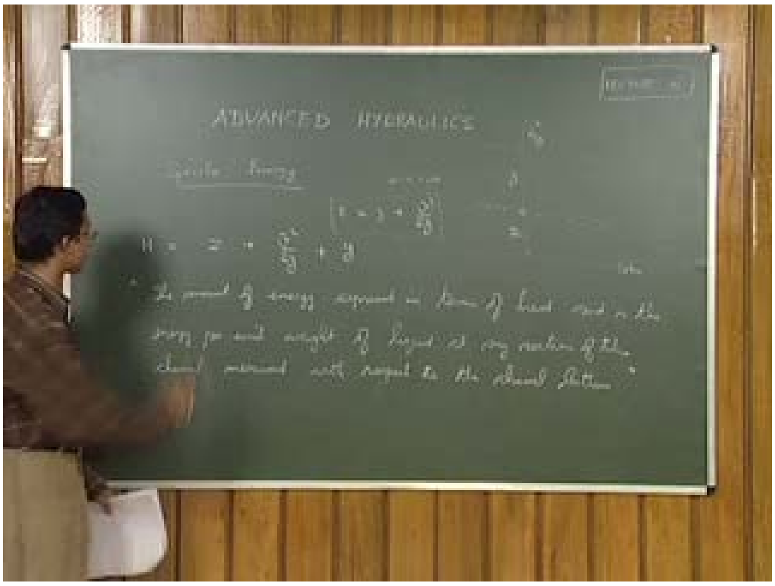

Today, we will be just going to deal with, a new concept called, specific energy. You know means, the total energy at any section H is equal to datum head plus velocity head plus depth of flow, or pressure head you can tell like that. Subsequently what we have seen that, this is for channels for small slope, and alpha approximately equal to 1. The specific energy, it is defined as, I will just write it down here. The amount of energy, expressed in terms of head, and if the energy per unit weight of liquid, at any section of the channel, measured with respect to the channel bottom. We can defined in the following form, so what do you mean by this definition. You have seen, or any channel at a height, depth from the datum line, and depth of flow y, and velocity head v square by 2 g, the total energy as given is H is equal to the following form. Now, the specific energy is the amount of energy in terms of head, and is the energy per unit weight of liquid, at any section of the channel, measured with respect to the channel bottom.

Now instead of taking your datum line, now if I consider the channel bottom, and if you measured the energy per unit weight now; that is the specific energy of the channel section, and that is called it an important parameter now in the open channel flow. You will see from here onwards; that is one of the most important terms specific energy, which you governed the fluid flow in open channel, so we will see. As per the definition now you can easily defined the term specific energy, E is equal to y plus v square by 2 g. So, that is nothing but the summation of your depth of flow, and velocity head. So, specific energy at any channel section. So, you should note that, if this is not a prismatic, if this is not a prismatic channel, or if the section of the channel varies with respect to the length. Then you can measure specific energy at specific locations only, or at specific sections. So, at each section your specific energy values may be different. Let us define specific energy in the following form.

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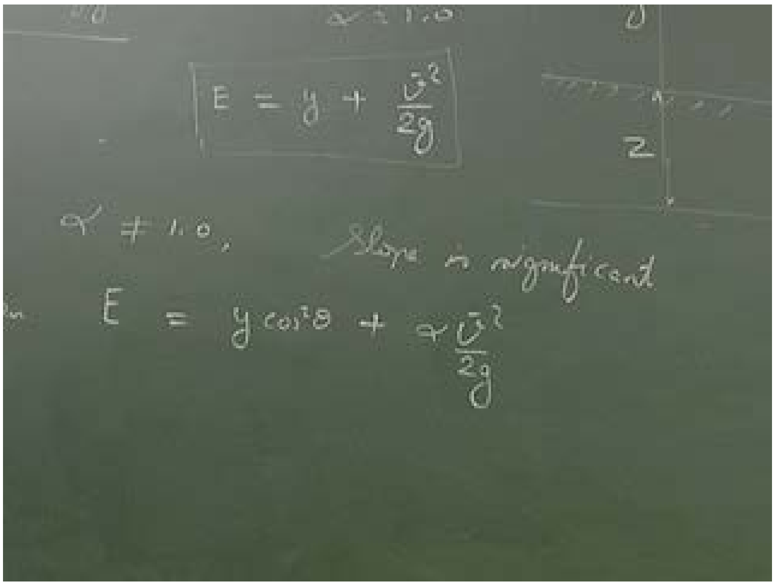

So, for any arbitrary cross section, f or any arbitrary cross section, you can use the following form E is equal to y plus v square by 2 g. Of course, we are dealing with, the small sloping channels, or where the slope of the channels are not significant, and alpha that is energy correction coefficient is approximately 1 in those situation, we can use the following form. If alpha not equal to 1, and if your slope is significant, then your E has to be given as y cos square theta plus alpha v square by 2 g.

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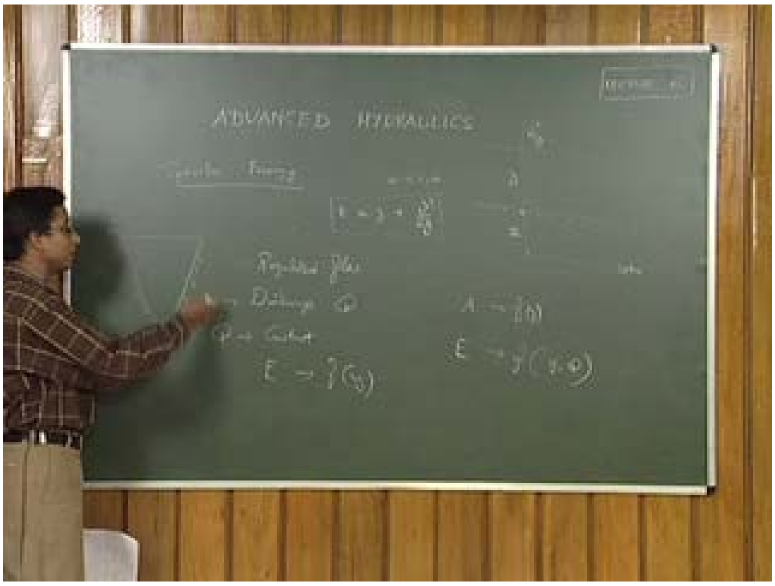

But in our analysis, let us follow the same thing, for a given discharge from the any cross section of the channel, for a given discharge. You know that the discharge can be represented as, the area of the cross section into average velocity. This is from your old physics itself you have understood that, or you can now suggest that ,the average velocity is nothing but Q by A. Substitute this quantity here, your specific energy now becomes y plus Q square by 2 g A square. However, you know that they area of the cross section A, this is the function of the depth of flow y, there is quite clear, isn’t it. So, now you are getting energy, the specific energy, now you are getting it as some function, of y and discharge Q. Once you understand that the specific energy is now, function of depth of flow and discharge.

You can easily compute; that is if the depth of flow and Q is given, you can easily compute specific energy, or specific energy is a function of following parameter. We have to see how specific energy is now affected by these parameters. As we are dealing with open channel hydraulic, mostly regulated flow is there. In canals whether it be in canals, whether it be in channels, or whether in some of the rivers and all, it is regulated flow. So, most of the time, your discharge is known, your discharge Q is a known quantity, and you can regulate Q. So, if you keep Q as a constant, if you keep Q as a constant, and then you will see that your specific energy is now only function of, depth of flow y. So, for any given cross section, you can maintain the same discharge Q, but you will see that the same discharge Q, if the depth varies, then the corresponding velocity term varies; that is how you can keep the discharge constant in a any given cross section.

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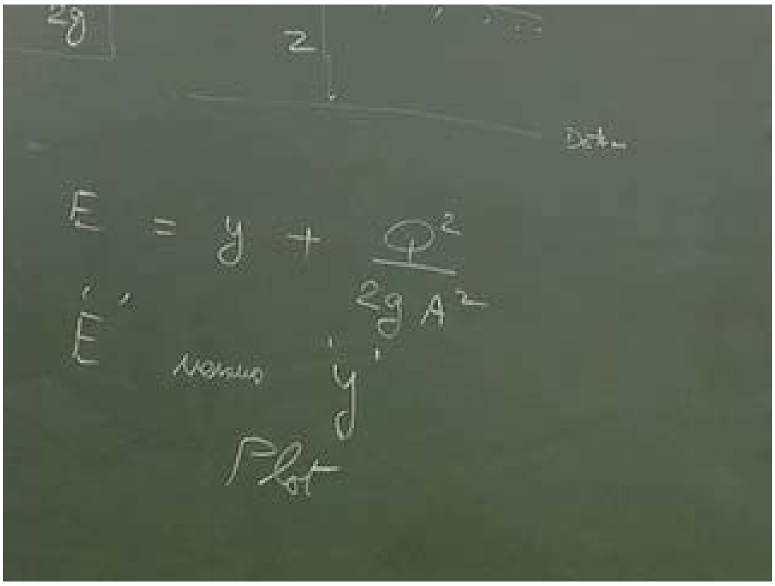

So, then in that case E is equal to function of y 2 g A square. So, Q is a constant term here, so this becomes only function of y. It can easily plot now E versus depth of flow; that is how is the depth of flow in any given cross section if its varies, how the specific energy term varies, that can now easily plot, you can plot them, we will see, say just we will just give it in a example form, that will give you better idea now.

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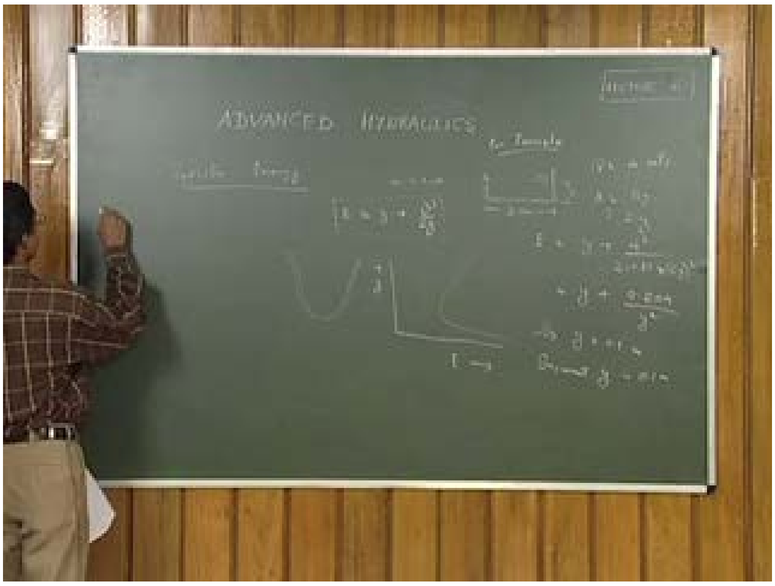

Now, you try to plot them, this is not an example, this is just for any arbitrary cross section of the channel. You can plot on the y axis, the depth of flow, you can plot on the x axis, the specific energy term, this particular curve E versus y. Now look in, it will be now in the following form, this is the form of the curve E is equal to y plus Q square by 2 g a square, where Q is a constant; that is for any particular Q, for any particular discharge value, you can easily plot E versus y, and you will get the following form of curve, how are you sure that the curve will be of the following form. You know that this is the non-linear relationship E, E is now related with y only, but it has non-linear relationship with y. Therefore this particular curve is also non-linear curve, how are you sure that it can be of the following form only. Take an example of, for example, let us take a rectangular channel, its breadth b is equal to 2 meter. So, you have your depth of flow, arbitrary depth of flow. Let us suggest that, a constant discharge Q is equal to 4 meter cube per second, is occurring to the cross section of this rectangular channel.

Now you can suggest that, area of cross section is, b into y; that is 2 y. Yours specific energy E is equal to y plus Q square by 2 g a square, what will be the depth for rectangular cross section, y plus Q is given as 4 4 square by 2 into 9.81 into a; that is 2 y whole square, or simplifying the terms you will see that this is 0.204 y square, in this relationship, for this particular rectangular channel. Now increment this relationship, now it sure that at y is equal to 0, when there is no depth of flow, this relationship is not valid. And of course, when there is no depth of flow; that means, water is not there in the channel, and we are not interested in those cases. We are dealing with when the flow exists in the channels, for those at condition, where y not equal to 0. The specific energy relationship we have in the following form. Start with the very small value of y; say y is equal to 0.1 meter onwards. Let us start and increment y in 0.1 meter steps, just try to plot that, or just try to compute them.

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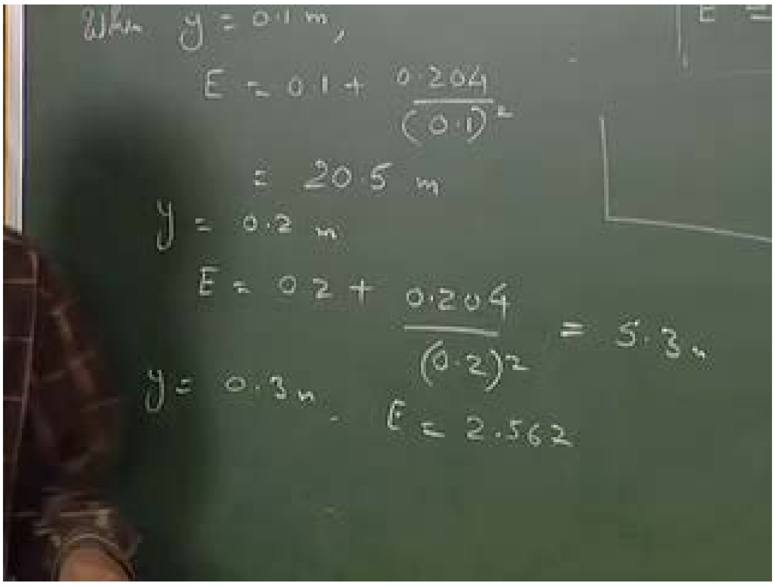

For example , when y is equal to 0.1 meter, your specific energy term, this you will get it as 0.1 plus 0.204 by 0.1 square, we just calculate them, I am getting the for fist thing as 25.5 meter, you see now, when the depth of flow in the rectangular channel, is it what 0.1 meter a very low depth. The specific energy computed was 2.5 meter, you can just plot them. For the same rectangular channel, this is the depth of flow, this is specific energy in the x axis, you can just plot that, so E is equal to 20, say let it be here when y is equal to 0.1. Then when y is equal to 0.2 meter, you are getting E is equal to 0.2 plus 0.204 by 0.2, whole square. This is around 5.3 meter. So, your specific energy is reducing now, you see, when this was 0.1, this is 0.2, and your specific energy is somewhere here, or you just extend it somewhat here. Then compute it in the similar form, when y is equal to 0.3 meter what is the corresponding E, I am getting it as 2.56.

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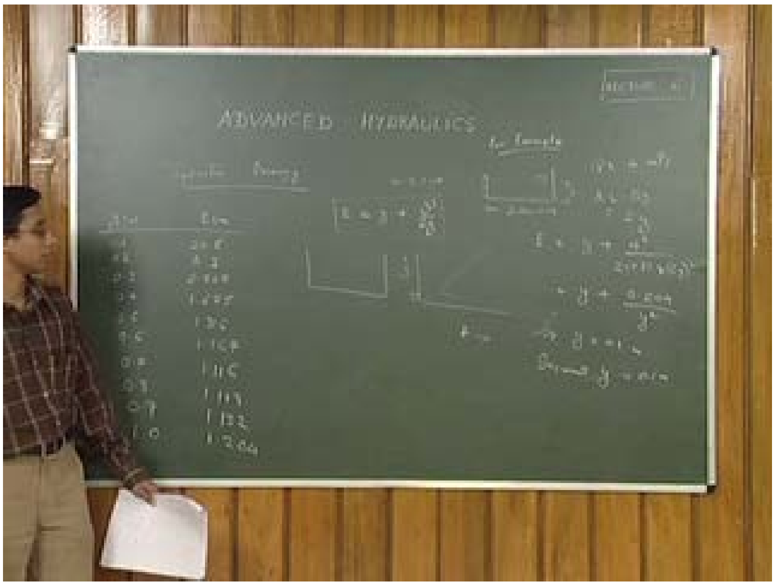

I can just give it in tabular form; can just give this as tabular form, y depth of flow and corresponding specific energy. So, when both of them are in meters. So, 0.1, this is 20.5, 0.2 this 5.3, 0.3 this is 2.567, 0.4, 1.67 5, 0.5 1.316, 0.6 1.167, 0.7 1.116, 0.8 1.119. You see that after 116 it is this started increasing now, 0.9 1.15 2, 1.204 like that you can go on computing, with respect to any depth of flow, what is the corresponding specific energy, and if you plot them you will get the curve in the following form. We continue on something of the following nature you will get. So, this is up to you to do that, you just gave that, how the specific energy curve look like, for any cross section any cross section.

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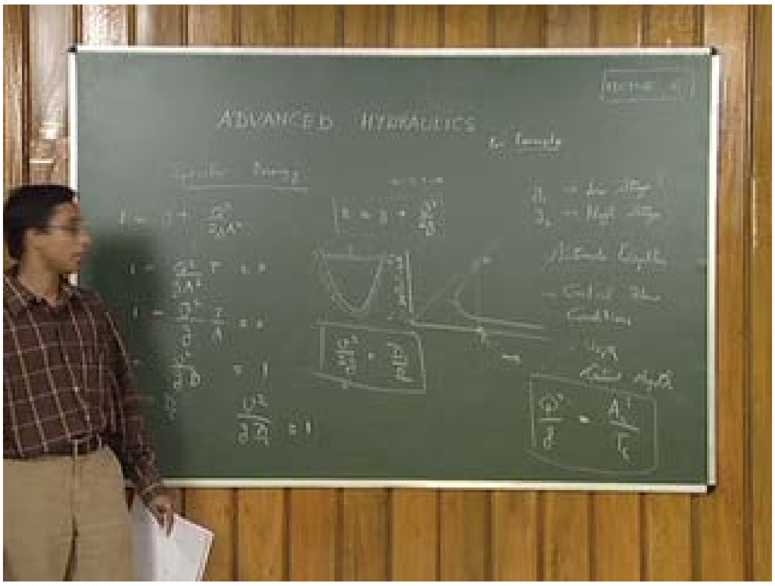

So, the same pattern is available for any cross sections, with respective of, whether it is rectangular channel, or whether it is triangular channel, or whether it is an arbitrary cross sectional channel, for any cross sectional you will get the specific energy curve in the following form. So, what you understand from that, if you look into this specific energy curve; that is E versus y. Let us now take the general case, for any arbitrary cross sectional channel, let this be the specific energy curve; E versus y. Now for any specific energy, or any point in this specific energy, say this is p, its corresponding y axis value, give you the depth of flow in the region, and the corresponding x axis will be give you the specific energy of the flow. Now just take it, say for any particular value we just extend this one. If I give this as E 1 or E at this location, for the same specific energy magnitude of E 1, or E i; any term you can give, this is just to represent that magnitude of energy here. You are now observing, two depths of flow; say one here and one here.

Let me give this as y 1 and y 2; that is you are observing two different depths of flow for same magnitude of energy, so the depth of flow y 1 and y 2. See y 1 is low stage or lower depth, y 2 is high stage or higher depth. So, you can see such two depths for any value of E, for any value of specific energy, for a particular discharge of course, for a particular discharge, for any value of E, you will see to depths of flow; that is y 1 and y 2, one is lower stage, one is the higher stage, they are called alternate depth. They are mutually called alternate depths of flow; that is for same specific energy you have two depths of fluid flow. The difference between y 1 and y 2, it may decrease as you proceed from lower to, slowly to the left side if you proceed, it may decrease initially, then it will increase gradually. So, that shows that, at a particular location, as you proceed as you decrease this specific energy for the further, there reaches the point in the curve, there reaches a point in the curve, where there is only one depth of fluid flow, there are no two alternate depth. Only one depth of fluid flow is there, for that particular specific energy, for a given discharge.

This particular energy, this and give this particular energy, and this particular depth, they are classified as, critical flow conditions. So, please note that, the depth of the specific energy, when you reduces the specific energy gradually, a point will be reach, where both the depth coincide; that is both the alternate coincide. That is the minimum specific energy available, and beyond that the specific energy will increase further. So, the minimum specific energy, where there is only one flow of depth, sorry where there is only one depth for the fluid flow; that is called critical flow condition, and that energy, means that specific energy is minimum in critical flow condition, that depth is also call critical depth . So, I can just write this as y c, critical depth of fluid flow. So, what do you understand by critical depth of fluid flow.

In the critical depth or in the critical flow condition, there will be minimum specific energy; that is specific energy will be minimum, for a given discharge Q, in critical flow conditions. So, let us inform from this, you know E is equal to y by v square 2 g, or E is equal to y plus Q squared by 2 g A square. For critical flow condition, what do you observe here. For critical flow conditions, you will observe that, the two alternate depth; y 1 and y 2, one will be below the critical depth, one will be above the critical depth. So, the y c which is given as the critical depth, whatever flow occurs, or whatever fluid flow is there, say for any arbitrary specific energy E 1, it was having two alternate that y 1 and y 2, y 1 is 1less than y c. Therefore, as discharge it's constant, the velocity will higher in this situation, where as y 2 is greater than y c, velocity will less in this situation.

So, the y 1 depth condition, it is called super critical flow, and y 2 depth condition it is called subcritical flow; that is mainly regarded to the velocity, at the critical depth velocity will be in critical condition, or the flow will be in critical condition. Whatever flow occurs of depth of flow, if it is greater than the critical depth, then the velocity will be less than the, it will be less than the critical flow velocity. Therefore, it is call subcritical flow condition, when the depth of flow is less than the critical depth, velocity of the flow will be more. Therefore, we can suggest that as a super critical flow condition. What are the criterion for, critical flow conditions. Just let us look it look this into the mathematical form. For critical flow condition, your specific energy should be minimum for a given discharge; that is what we have identified from this particular curve; that means, d E by d y should be equal to 0, just try to do that.

You know d E by d y, so this is one minus Q square by g A cube into d a by d y, this consider a top element its depth is d y elemental depth is d y, top width is t and that elemental area is given as t into d y, or d a by d y can be suggested as the top width of the flow also. Let us substitute it here. So, you will get d e by d y is equal to 0 is equal to 1 minus Q square by g A Q into t. So, for the critical flow condition, the corresponding terms will be related in the following form, or you can suggest here again, 0 is equal to 1 minus Q square by g, that area whatever will be measured or computed, that will be critical flow area, and whatever topic will computed that will be the topic computed in critical condition t c. Like that we can just give as a suffix, or the top width and area. This implies Q square by g is equal to A c cube by T c. This is one relation for one condition, that suggest critical flow condition, or for critical flow condition, the following relationship should hold good. Please note that this only for the critical condition, the following terms follow good. You can again rearrange the terms here, you can rearrange the terms in the following form also.

I will just give it here, in this relationship, one minus, let me take this as v square by g into t by a equal to 0; that is you can suggest that v square by g. If you recall the term a by t that was suggested earlier; that is area of flow by t that was defined as your hydraulic depth D, this should be equal to 1, or may I beg pardon, this is v square by g d. So, this D is in critical condition. So, D c let me give it as critical hydraulic depth in critical state of flow. So, v square by g D c is equal to 1, or you can suggest that v square by 2 g is equal to D c by 2. This is just, it is to note that your velocity head is equal to half of your hydraulic depth. Like this you can study, you can understand the concept in such a way that, in critical state condition your velocity head is equal to, half of your hydraulic depth.

Similarly this can be again rearrange; v square by g d c is equal to 1, it was suggested, or this can be given as v square by root of g D c is equal to 1. If you recall of 1 of our earlier explanation also, this quantity v by root of g D c, this can be defined as Froude number. You can defined the left hand side of this particular relationship as Froude number, and Froude number for critical flow condition, it should be always 1. For critical flow condition the Froude number should be always 1. If Froude number is less than 1, if Froude number is greater than 1, what is that imply. See Froude number greater than 1 means, the velocity should be greater, that implies supercritical flow. Froude number less than one means, velocity should be less that implies the critical flow. So, these are the criterion for critical flow conditions.

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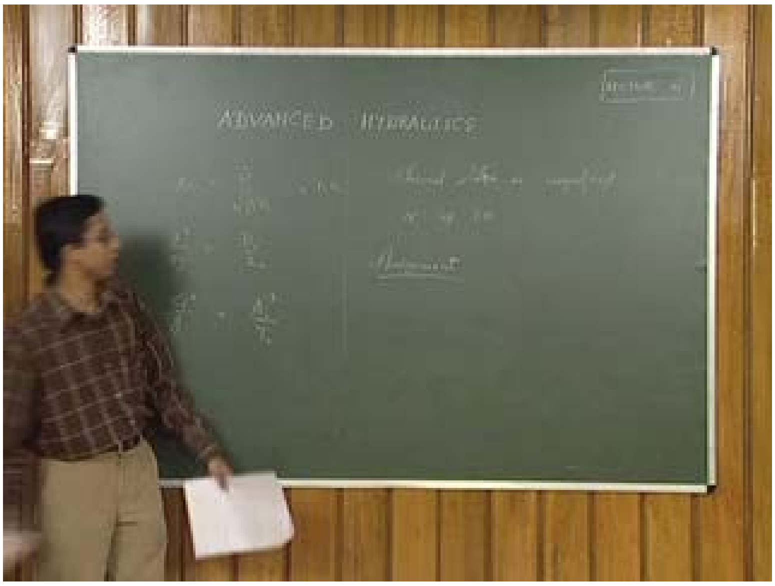

So, we have seen that, for critical flow conditions, your fluid number, this should be one, or you can give the other relationship, v squared by 2 g is equal to half of your hydraulic depth, or you can give specified Q square by g is equal to A c cube by T c, like this these are the sum of criterion you can use for critical flow condition, we suggested, then we derived it. What happens if you are channels slope, if channel slope is, or channel bed slope is significant, and if alpha; that is energy correction coefficient, if this is not equal to 1, if such situation exist, how can you defined the critical flow condition. Please do this as assignment. Now this is up to you to do as assignment, how to define the, how to define the critical flow conditions, for a channel section, whose beds slope is significant, and the energy correction coefficient is not one, for such a condition, how you can define the critical flow condition, in a similar form, it’s up to you to do that you complete that as an assignment.

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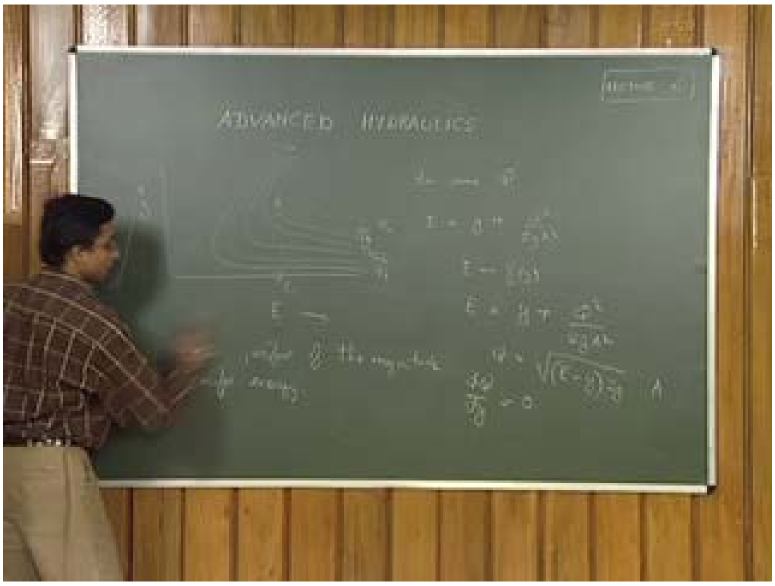

We had suggested for same discharge Q, how your specific energy curves look, this was just discussed few minutes ago. We suggested that E is equal to y plus Q square by 2 g A square here, this E is only function of y. We are stating this for constant discharge condition in a channel section, what happens if you are varying the discharge in the channel section, any arbitrary channel section. If you are varying the discharge quantity, how will your specific energy curves look like, and from this relationship you know, for a particular discharge Q, you will get a specific energy curve in the following form. You can draw now arbitrary curve, say for different values of Q; Q 1 Q 2 Q 3 different values of Q, you can get different specific energy curves; maybe say Q 1 is less Q 2 less Q 3, like this if you want to draw the curves, then the following specific the specific energy curves will look in the following form. Say this is for Q 1, this is for Q 2, for Q 3, Q 4.

So, you have not ask me how I can draw, how do I know that Q 1; the specific energy curve of Q 1 is like this, specific energy curve of Q 2 is like this, for Q 4 it is like this. You have to adopt the same principle, we have done how to compute, or how to compute the specific energy, for a same discharge condition, in rectangular cross section you have already done that. Similarly you now vary the discharge, we just give saying the same, condition same problem itself, you just vary the discharge, instead of 4 metre Q per second, now use to 2 meter Q per second, then you use 6 meter Q per second, then you use 8 meter Q per second. Subsequently you will see that, the specific energy curve, means for which discharges minimum that curve, will be shown like this Q 1. Q 1 is having least discharge, Q 2 greater the, means Q 2 is greater than Q 1, Q 3 is greater than Q 2, Q 4 is greater than Q 3 like this, you will get the specific energy discharge, it is up to you to do it, work it out and find the proof, we are not going to discuss. We have already done for a simple case earlier, you have to analyze on your own. So, if you can plot it in the following form, what you import from the Figurama E versus y is here. If I give any particular, if a person happens to go through; say any particular arbitrary specific energy, he just observe that, where is a specific energy value here E 1.

If the straight line is plotted, the straight line is plotted, he will get different types of alternate depth, say for the discharge Q 1, he will get the corresponding alternate depths, for discharge Q 2, for same magnitude of energy, for this different discharge, he will get different type of alternate depth. This is the (( )) you can analyze that, from the same channel section if, so and so discharge occur, this will be the alternate depth, so this will be the depth of flow like that, you can visualize now. These curves suggest those parameters, or those properties. So, it will very good to analyze the things channel properties and all, still we very good to understand those things. Say here if I look it here Q 3, for discharge, for the discharge Q 3, the depth is like this, and for the discharge Q 4, you see the depths are so and so. You will reach a particular point, this curves can be further elaborated, like this on increasing the values of Q, this curves will go out Q 5 Q 6 and all.

So, you will see that, this energy, specific energy whatever magnitude of specific energy you have taken as of interest that will become the critical energy condition, or critical flow condition, when it is. This will become a critical flow condition for a particular case. So, I just draw the curve properly here, this become a critical energy, or it will become, this energy will become condition for critical flow, only for a particular situation. So, if that curve happens to some where here let us suggest that. You can now identify the corresponding discharge, isn’t it, trying to identify the corresponding discharge. If you are given, if you are specified of the magnitude of specific energy, then this magnitude of specific energy can determine a particular flow, where critical flow condition occurs. You can identify that from the following curves. You have to interpret them, how to interpret them, use the same mathematical analysis now; E is equal to y plus Q square by 2 g A square.

So, now in this case what you are doing, you have specified E, E is not a variable now, if you have specified E, what is y what is Q, and what is A, let us see that now. Your analysis it can given in the following form, I can write the relation of Q now, Q is equal to root of E minus y into 2 g. The whole quantity multiplied by the area of cross section, what does this suggest now. For the given flow condition, if it has to be critical flow condition, what could be your magnitude of Q, how will you understand that. Let me ask you here. Here the Q 5 specific curve energy is going like this, Q 6 is going like this. Do you think that, those discharges have any influence on the specific energy term here. They are not having any influence; you see this line, mark inside this line, goes on.

The specific energy for Q 5 and Q 6 are on the right side, it is not having any influence; that is it is not having any depth of flow; means this particular energy is not determining any alternate depths for Q 5 and Q 6 for higher magnitudes of discharge. So, it is not having any influence on the discharge there. However, the same magnitude of energy, it is determining alternate depths for discharges, which are less than Q 5 or whichever are less than this particular value of Q, which we do not know. So, that means, whatever are on the left they are having less discharge, and this is the point, where maximum discharge occurs, can you understand this is the point, where for the given energy maximum discharge occurs, so just that relationship here. So, d Q by d y will be equal to 0 for the maxima condition, d Q will be d y for the maxima condition.

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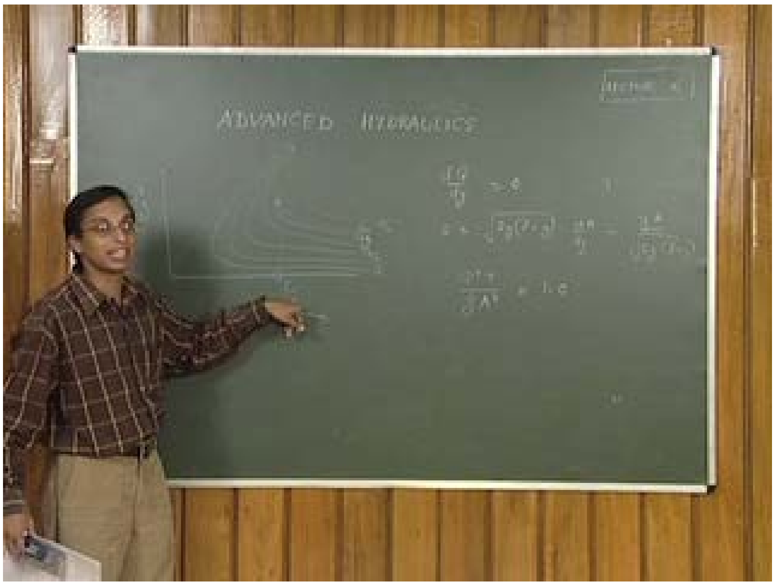

You see what will be the relationship now, so this will be equal to 2 g E minus y of root d a by d y minus g A by. So, this quantity is 0, this is T you know that, or you will get the following relationship now same that is Q square T by g a cube is equal to one. The same relationships are identified, when you determine the flow, means depth of flow condition also isn't it. So, that mean for a given specific energy maximum discharge occurs, when the flow is in critical condition right, or for given discharge Q, the minimum specific energy for the given discharge Q, that will determine the critical flow condition. So, that quite principles quite good relationship, which you can infer, and further analyze fluid flow in open channels and all. Based on to continuing with this lectures and all, in the next class we will determine, what are the other things, what are the things that suggest for the specific energy, related to specific energy, some of the other concepts that come into picture and all, we will be discussing them. We also be conducting the quiz for the today's topic, in the same series. So, let us wind up today's lecture.

Thank you.