Advanced Hydraulics
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Module - 1
Open Channel Flow
Lecture - 5
Equation of Continuity & Energy

Good afternoon everyone. So, today again we are back, in to our course is lecture series on advanced hydraulics.

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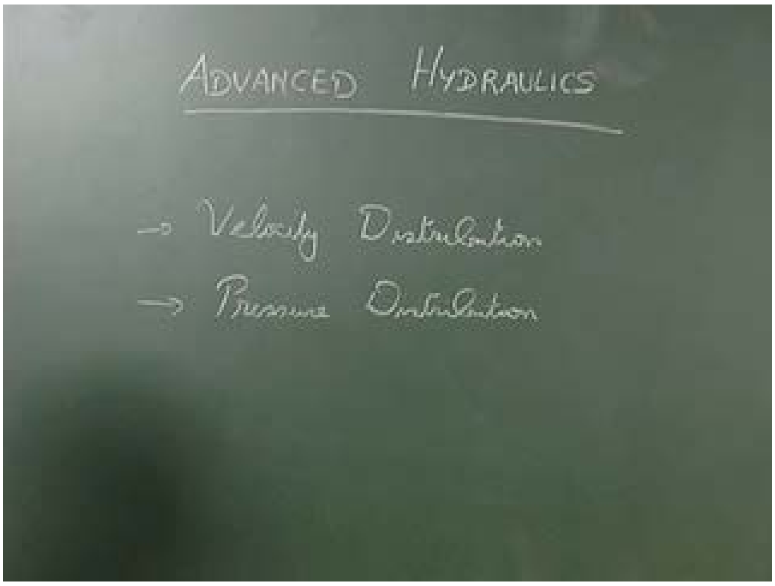

So, today we are into the fifth lecture of the series, we are in the first module of the course. Last class we discussed on velocity distribution, and pressure distribution in the open channels. In the end of last class, we had given you two quiz questions, which we did not solve them, subsequently which I did not give you the solutions are that instant. Today, now I will give you the solutions of those two quiz questions.

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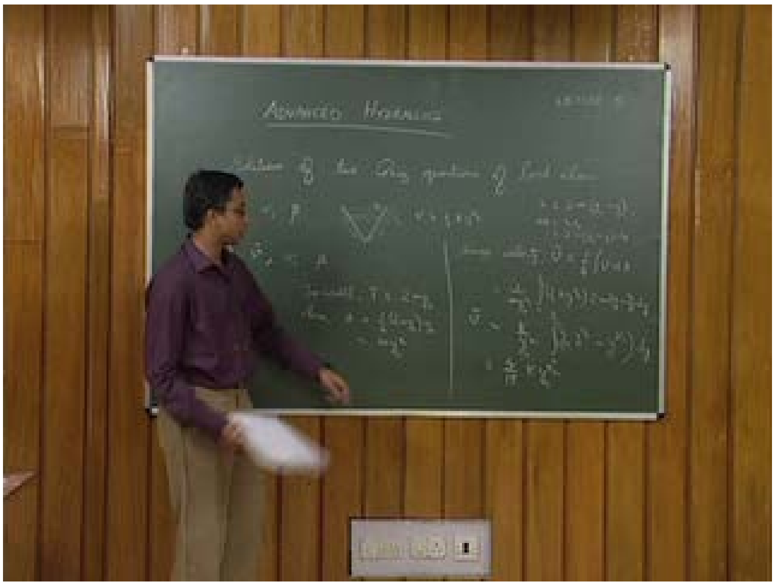

So, the first question, we asked you, we asked to determine the kinetic energy factor, and momentum correction factor for a triangular cross sectional channel, whose velocity at any point was given as half k into y to the power of half, where y is the depth from top of the surface. So, how do you solve this particular problem. I hope most of you have done that, but still for your benefit we will just briefly solve them, how to find this factors. You know that here, you need to assume, that the sides of the rectangular channel, the sides of the rectangular channel they are sloping at a ratio 1 is to m, the slope of rectangular sides of the triangular channel, the side slopes are 1 is to m. How will you find the area, and subsequently the average velocity, area, average velocity subsequently alpha beta, this are the things we need to find. One can easily find top width t, in this case, top width t in this case, will be equal to twice m y naught, area a, this will be equal to half into 2 m y naught into y naught, this is equal to m y naught square. Average velocity, this was given as 1 by a integral v d a.

This you can write it as 1 by m y naught square integral half k y to the power of half into d a, what is d a. Here you take any small section, small elementary section, of d y height, and let that top width be b in this case the width of the elemental strip be b. So, you know in this case b will be equal to, if this elemental strip b act at depth y from the top, this will be equal to this will be equal to 2 m y naught minus y. Therefore, your d a can be given as b d y or 2 m y naught minus y into d y, substitute that term here; 2 m y naught minus y d y, this integral is between the limits 0 to y naught, so what will you get here. Your average velocity v bar, will be equal to k by y naught squared, integral 0 to y naught y naught into y to the power of half minus d y, or you just substitute the things, integrate and substitute the things. Then finally, form only I will be writing here, it will be 4 by 15 into k times y 0 to the power of half. We had found for a rectangular cross sectional channel, also a similar term something related to y naught raise to half, here also we are getting a similar way.

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Alpha is equal to you know that it is v cube d a, by v cube into a, this average velocity, half k y to the power of half whole cube d a 2 m y naught minus y d y what is v bar, so 4 by 15 k y naught to the power of half whole cube, into m y naught square. So, now you can easily guess from this thing, many quantities k k cube, then numerator and denominator they get cancelled. Similarly m in the numerator and denominator they get cancelled of. You will get an expression; say 15 by 4 whole cube into 1 by 4 y naught 7, raise to 7 by 2, integral 0 to y naught, y 0 y 3 by 2 minus d y. This you can easily find it as, this alpha value, you can easily find it as around 1.5067. So, please note that, this also theatrical question for a triangular cross sectional channel, where you found the kinetic energy correction factor as 1.506. In the actual fields an all, it is quite difficult to get this much high magnitude of kinetic energy correction factor.

So, please note this may be due to, because we have assumed v is equal to half into k to the power of k into y to the power of half. May be something wrong in this assumption, or something proven error proven in this assumption, might have let to for such a high energy correction, kinetic energy correction factor. However, we have solved it according to the principles that we have mentioned. You can now compute, the momentum correction factor.

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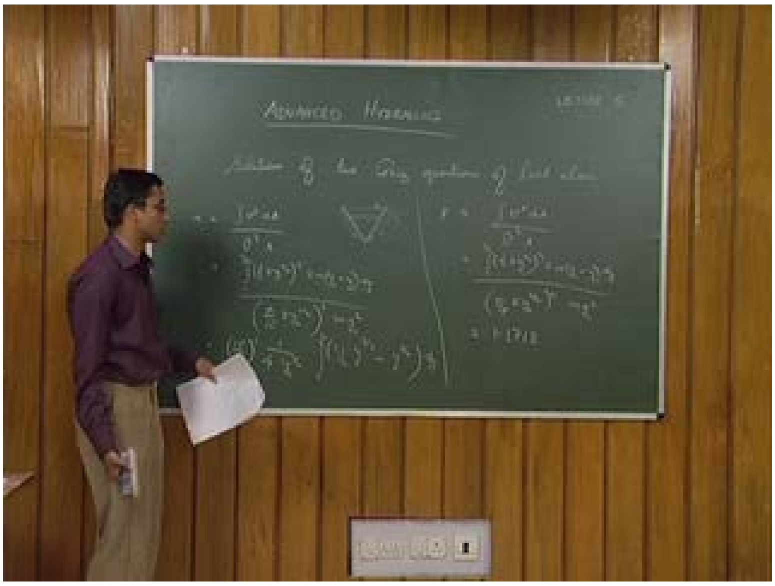

Beta, this is given as v square d a by v square A. Substitute all the terms appropriately 0 to y naught half k to the power half whole square 2 m y naught minus y d y by your average velocity 4 by 15 k y naught to the power of half, whole square into m y naught square you simplify appropriately, finally you will get the momentum correction factor as 1.1718. So, this is the solution for the first question of your last base case.

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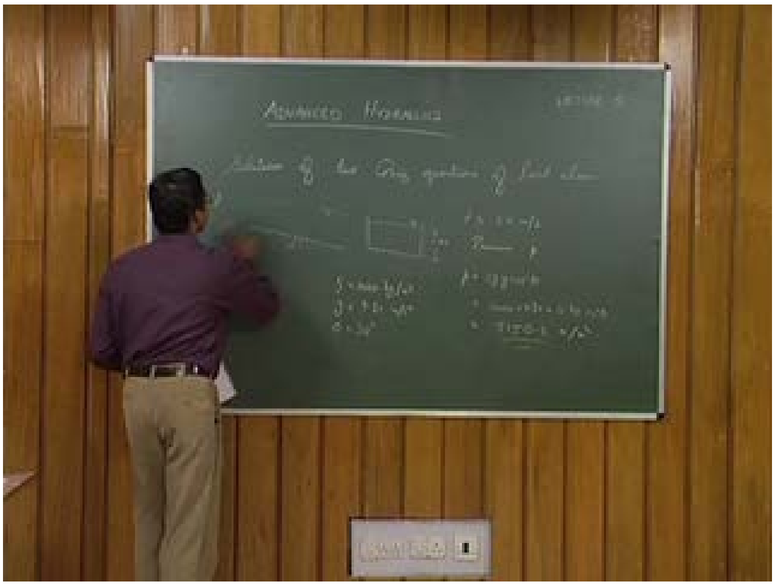

The second question, second question asked to you was. You were asked to find say for a t rectangular channel, for a steep rectangular channel, that is having angle with the horizontal as 30 degrees. You were requested to find at a particular cross section, the pressure at the bed, and at this cross section the depth of water was 0.70. You were given average velocity v bar is equal to three meter per second, you are now requested to find pressure p. If you recall the pressure formulation, this was given as rho g into y cos square theta. So, this can be suggested as; say rho is equal to, you are using the s i units, thousand kilo gram per meter cube for water, g is equal to 9.81 meter per square, theta is equal to 30 degrees given to you, substitute the terms you will get, you will get magnitude of 5150.2 Newton per meter square. So, I hope most of them have done it correctly, this particular question. So, we were discussing on the last class, on equation of continuity.

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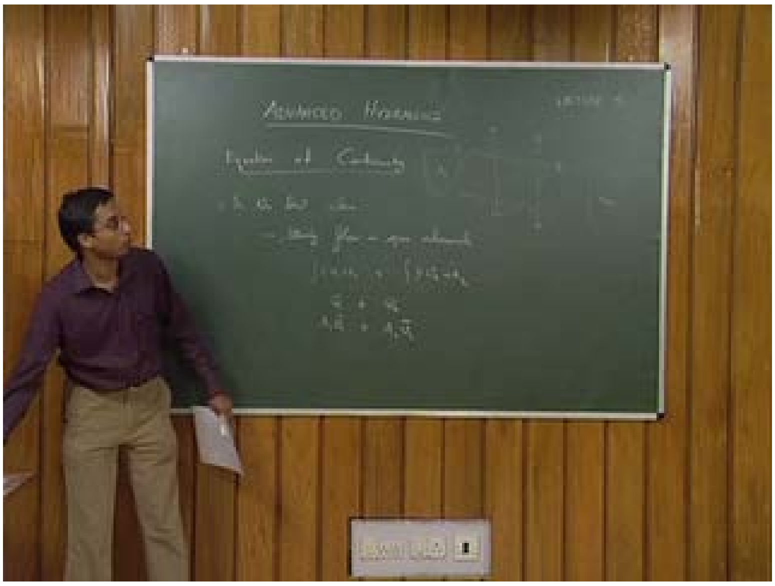

So, we will continue with the same topic today. In the last class, we had derived the continuity equation, we have defined the continuity, the confirmation of mass, definition was given to you then, subsequently we have derived the continuity equation for steady flow in open channel; say for two sections, 1 1 and 2 2. The equation of continuity, according to the equation of continuity, in steady state conditions, we had suggested that, this will be equal to in flow. The amount of or the mass of water coming into the section, will be equal to the amount of water; that is going out of this section. Subsequently we had suggested the terms discharge. So, this is q 1 is equal to q 2, or if you are using average velocity term, and if this is area A 1, this is area A 2. It was also suggested that A 1 v 1 bar will be equal to A 2 v 2 bar in this steady state condition.

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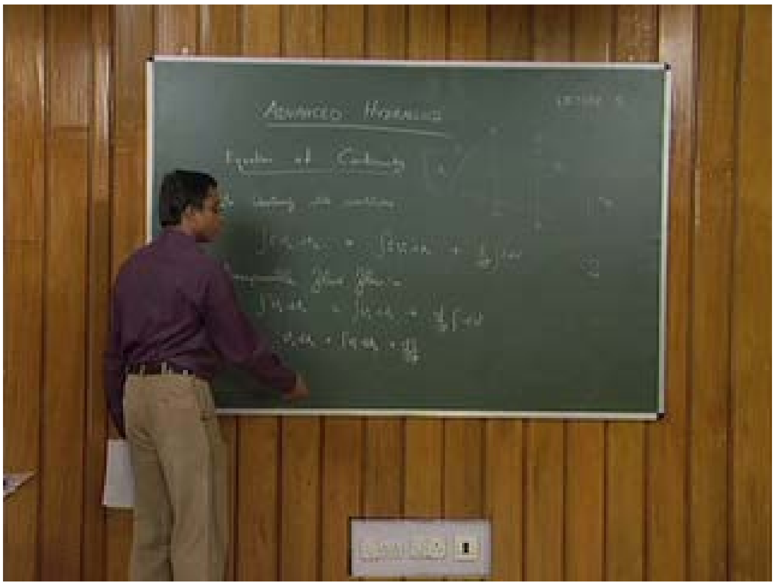

So, today we will just derive the equation of continuity for unsteady state condition in unsteady state, how will your continuity equation look like. So, we will again define means we will again give the same definition. In the last class, we had suggested that the same definition was given to you, so you can recall them, but the conservation of mass principle suggest that, the amount of water, the amount of water, the amount of water or the mass of water that is coming into the section one, and the amount of water that is going out from section two, both of them will be differed by a value; that is equal to the rate of change of storage of water, between this two section. This was given in the last class if you recall them. So, the same thing we are again applying, or I can just again restate that.

The amount of water that is living out from section 2 2, this will be equal to the amount of water that is coming in from the section 1 1, and the rate and the change of the rate of change of water stored, between this two section. So, between this two section the amount of water stored is here. So, what is the rate of change of stored water in between those two sections; that will give you the difference between discharges, in section 1 1 and section 2 2. We will again means use the same equation; that is the amount of water, or the mass of water, that is coming out from section 2 2 in a channel. You can give it as rho v 2 d A 2, this is equal to rho v 1 d A 1 plus rate of change of stored water, between the two section, so I can write it like this, density into volume will give you the mass of water between this two sections.

So, the volume, enclosed between this section 1 1 and 2 2; that is what is implied by here. So, the rate of the change in the mass of water, as this is the incompressible fluid equation. As we are dealing with incompressible fluid flow in open channel, you can easily write this as v 2 d A 2 is equal to v 1 d A 1 plus d by d t of rho d v, for our benefit we can write; say the total volume of water, this note the rho term has to be canceled out here. The total volume of water between this two sections 1 1 and 2 2, let it define as s, as the mass of water, or the volume of water stored between the sections 1 1 and 2 2. So, you can write this as integral d s by d t, proceeding the same manner as for done for the steady state condition ,you know that v 2 d A 2 this integral term v 1 d A 1 this integral term, you can approximate using the average velocity terms here.

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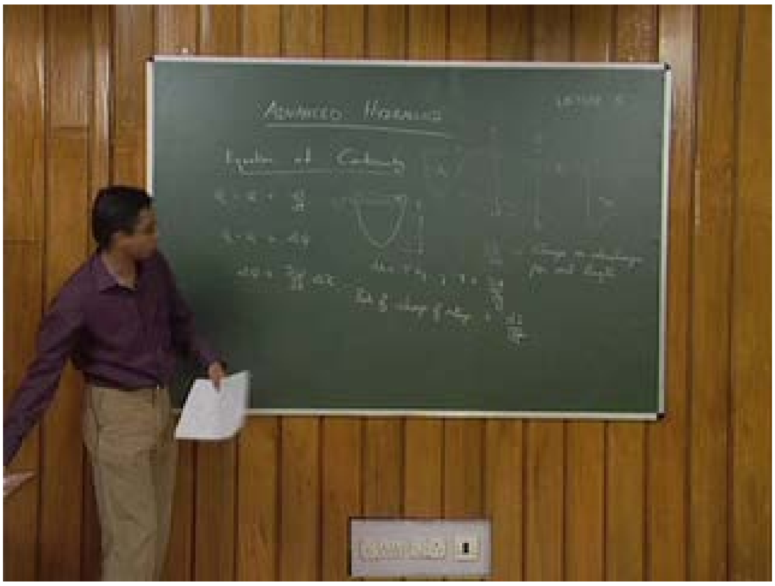

So, I can write this as A 2 into v 2 bar is equal to A 1 into v 1 bar plus d s by d t. Subsequently, you know that this is the quantity discharge q 2 is equal to q 1 plus d s by d t, or the most expression is change in storage if d s by d t is equal to q 2 minus q 1. So, that is change in storage, between two sections 1 1 and 2 2, is nothing but equal to the change in discharge between the two sections; that is how for any incompressible fluid flow in an open channel can be described. You can again redefine these things. Let us assume the distance between 2 sections 1 1 and 2 2 as delta x, or del x ,delta x. If you define in this form, you can now easily suggest that, the quantity q 2 minus q 1 is delta q; that is the change in this change in the change in the discharge between two sections 1 1 and 2 2, like this you can define that now, because we have suggested the distance between them as delta x, so you can define q 2 minus q 1 is also equal to delta q.

Subsequently you can now identify that, if discharge changes in a unsteady flow open channel, in unsteady open channel low if discharge changes, as it proceeds from one section to another section, you can easily suggest, a quantity say dou q by dou x; that is the change in discharge per unit length. This is nothing but the change in discharge per unit length in the direction of flow. If you can define this quantity, now you can easily suggest that for between sections 1 1 and section 2 2, this delta q. I can write it as delta q is nothing but dou q by dou x into delta x, because here distance between those two sections is, delta x, and this is the change in discharge per unit length. This term, if you are familiar, we can just incorporate them here.

Now consider, any cross section, any cross section of the channel. You know that the top width is t, this in the unsteady flow, in the unsteady incompressible flow, the variation in this discharge can be now easily identified by how the depth of the flow, see the depth of the flow y, and how the top width, how both the quantities changes with respect to time; that can be easily co related in same section. It will vary with respect to time in unsteady state condition. Those property you can now incorporate here. Let us take an elemental strip from the top width. Let us take this elemental state; this is of d y depth. So, you can define a quantity d a is equal to t into d y. So, whatever change in storage, and all are occurring, that can now we can now easily correlate with the change in the top width, as well as change in the depth of flow, we will come into that.

Here or you can suggest that, your top width t is dou A by dou y; that is change in area by change in the depth. Like that also you can now define t is equal to dou a by dou y, from subsequently from this relationship. This is just for our analysis .Once you define it in this form. Now you can suggest that, the rate of change of storage. We had given this as d s by d t. How can you identify through the thing here. Between this of length delta x, the volume is getting change, or the stored water is getting changed if A 1 is the area here, A 2 is the are here. Now let us suggest that, any amount of water changing, or the this area cross sectional area into del x; that will give you an approximate volume, of the stored water in this section.

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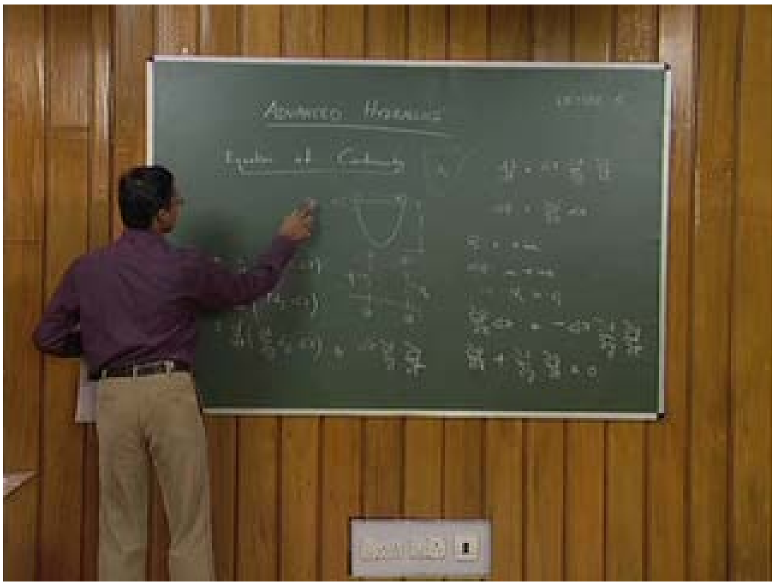

So, let us come into that; the rate of discharge. Now I can just give this as d by d t of area of cross section into del x. We are just suggesting this, in this following form. If you recall the top width, and the area delay, the change in this d a, now can be correlated here. You can subsequently bring it that quantity here. I can suggest now d s by d t is nothing but d by d t of, the change in area d a into del x. So, the same phenomenon, you know this is t d y del x, or you can write it this as d by d t of dou a by dou y into d y del x. This can be approximated now, we are suggesting, because we have just taken an arbitrary length delta x, which is not vary with time; that is delta x it is for our convenience we have taken some length. You can this can be approximated now subsequently as delta x dou a by dou y into dou y by dou t; that is the change in depth of flow, your storage is now, become a function of change in depth of the flow. So, d s by d t is now approximated as delta x dou a by dou y into dou y by dou t. So, continue the formulations, you got d s by d t is equal to del x dou a by dou y into dou y by dou t.

You also have seen that, delta q, this can be given as dou q by dou x into delta x. So, in the section, before I just rubbed that, in the channel stretch, for section 1 1 section 2 2, in this channel stretch, we have suggested same, say if q 2 is the discharge from this portion, q 1 discharge entering here, d s by d t is the change in storage, between the two sections. If in our analysis, if you are taking the out flow from the section as positive, and if you want delta q as positive, if you want the delta q as positive; that means, q 2 greater than q 1; that implies if q 2 is greater than q 1 that naturally implies that, the amount of water stored, between the 2 sections 1 2 and 2 2 it is getting reduce. So, the amount the volume of water if it's getting reduced, that is definitely negative quantity.

So, we will suggest now here; delta q or dou q by dou x into delta x. This is equal to minus of delta x dou a by dou y dou y by dou t. I hope you understood that why I took the negative sign here, because volume, this quantity is for the volume, increase in volume is positive and decrease in volume is negative. If you want if you decide that the out flow, or the out flow from section two to discharge from section two to as a positive quantity, or if you want delta q as positive. Then accordingly the volume of water stored, it will get reduced therefore, I am writing it the negative side here.

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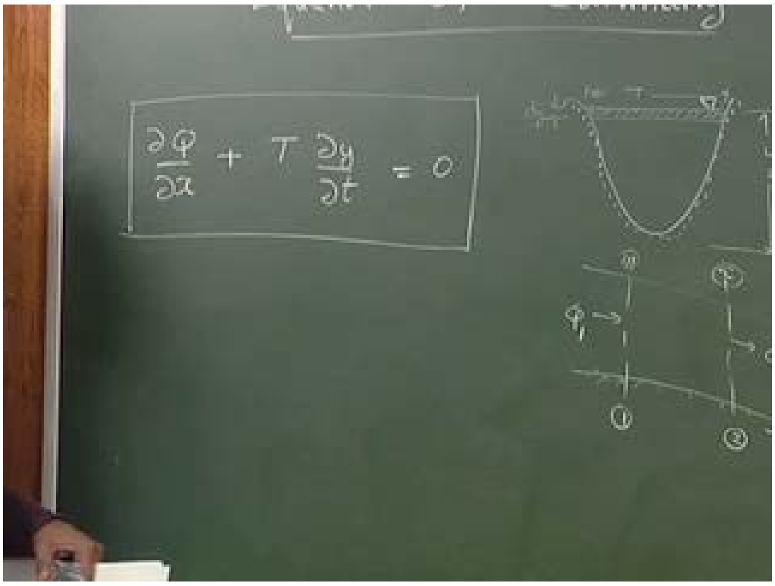

You can easily form the equation here, cancel out the repeating quantities dou q by dou x is plus dou a by dou y into dou y by dou t, this is equal to 0. Or in a most famous way, I can write the unsteady flow continuity equation as; dou q by dou x plus T into dou y by dou t is equal to 0, where t is also function of y. So, this is the famous continuity equation in unsteady fluid flow.

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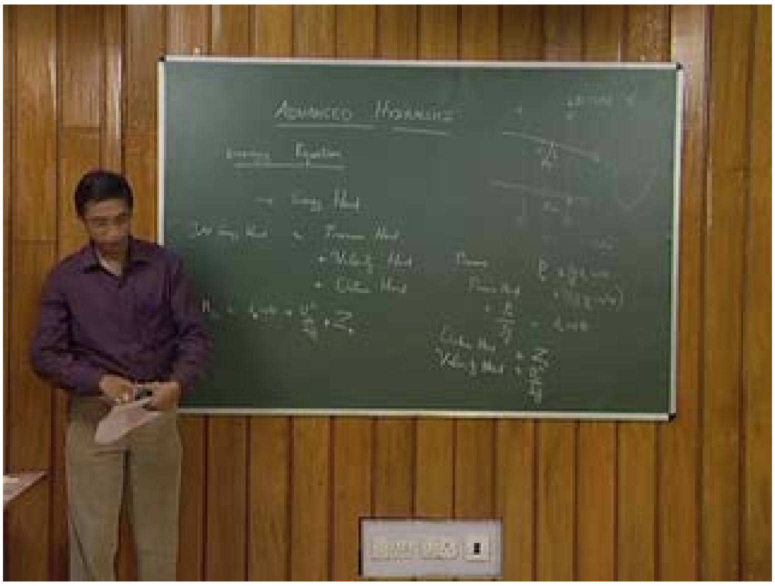

So, our next topic, next topic is one energy equation. Till now we discussed on the equation of continuity, next is energy equation. So, energy, the term energy and all you are quite familiar with those quantities and all. Even in your elementary hydraulics, or even in your school physics and all, you have studied various energy terms; kinetic energy, potential energy and all. You have even studied the terms, how energy can be represented as energy head. Most of you should be aware of that; the energy can be easily represented as energy head. So, what do you mean by representing energy as energy head. Let us take a channel stretch of inclined sloop. Let us suggest this as the water surface.

Now there are two sections here; section 1 1 section 2 2. Please note that, I have drawn the section based on, considering the depth of flow, and not the normal distant from the water surface to the bed, that is not the case given here. You can define the stream lines for fluid flow in the open channel. If I take, say at any stream line of fluid flow occurring there, and any point say o, if you consider any point o. This can be now suggested say, if it as a normal distance, d o from the free surface. From the free surface, if you draw a normal distance to the bed, and if this point t at a distance d naught, if this stream line, is at a distance d naught from the free surface. Naturally we have done in the other quantities and all, you know that this can be given as y naught and all, we will coming to those discussion.

Let us consider at datum line, and this point t is at an elevation z o, from the datum line. If this is at a z o, the slope of the channel it is theta with the horizontal plain, your energy head, or total energy head, this can be now given as submission of, pressure head, plus velocity head, plus datum head. So, one can easily give, their total energy head in terms of pressure head, velocity head, and datum head, these things, most of you have studied in your school days and all. How do you account the things here, how you want to represent now energy equation for open channels and all, what does the things here suggest. What is pressure head now, for this particular case, say for this at this point in the stream line, at this point in the stream line, what is the pressure head. Pressure is p 0, pressure at o at point o it is p 0, your pressure. This is…

This you can give the give then give it as say; rho g d 0 cos theta or even you studied like this rho g y 0 cos square theta, if this is y 0 y 0, that also you have studied. So, let us what is the pressure head quantity, pressure divided by rho g, so you will get the quantity d 0 cos theta. Your datum head this is equal to z 0; that is from your datum line, how much elevation is there for that particular point in the stream line. Your velocity head, which is given as, the velocity at the stream line, at this point, this stream line v o square by 2 g. So, your total energy head H at the point o in the stream line, this can be given as d 0 cos theta plus v, this is not 0, it is o, d o cos theta v o square by 2 g plus z o. Similarly you can define energy head for different stream lines in the fluid flow in the open channel, you can define them, if you take any cross section now, just take any cross section, may be section 1 1 or section 2 2.

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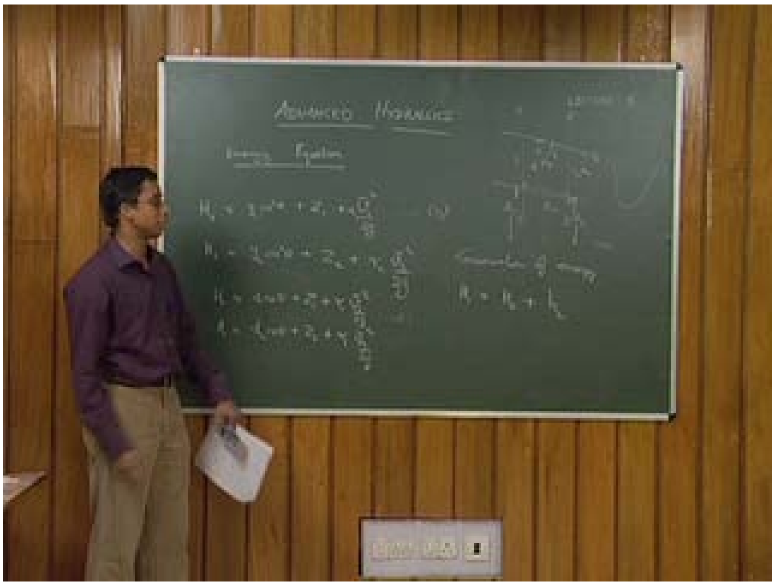

There also now you can define energy head; say if this is z 1, this is z 2, then subsequently you know that, this is y 1, this is y 2. H 1; that is the energy head for the channel section one, this can be given as y 1 cos square theta plus plus z 1, plus here you need to in cooperate, average velocity term, and once you in cooperate the average velocity at this section 1 1, you need to incorp, as means you need to provide the kinetic energy correction factor here. So, therefore, I have to multiply this energy equation, in velocity head, you have to multiply the term alpha. Similarly, at section 2 2, you can easily obtain this thing y 2 cos square theta plus z 2 plus, if this is alpha 1, let this be alpha 2 v 2 square by 2 g, or if you in cause suggesting in terms of d.

If this is d 1, if this is d 2 depth, your H 1 will be b 1 cos theta plus z 1 plus alpha 1 v 1 square by 2 g H 2 is equal to d 2 cos theta plus z 2 plus alpha 2 v 2 square by 2 g. So, please note that, when you are taking the whole channel cross section, not a particle point in the stream line and all, when you are taking the whole channel cross section. There you were taking the average velocity term, and subsequently you have incorporated the correction factor. So, what does the energy equation suggest. Just now I have given the expression only for the total energy head term.

So, what does the energy equation suggest, or energy, conservation of energy suggest. Conservation of energy, or the principles of energy, what does that suggest. It suggest that, the energy total energy at any section 1 1, and the total energy at any section 2 2, they will be differed, only by at quantity where, the energy lose is occurring, or you can suggest that the total energy head at 1 1, is equal to the submission of the total energy head at the section 2 2, plus the loss of energy, that has occurred between sections 1 1 and section 2 2. So, that can be given as H 1 is equal to H 2 plus some lose in energy, lose in head or lose in energy. So, energy you are expressing in terms of head here. I hope most of you have understood that the term head has units of length, this has units of length, even though this is the energy equation, the dimensions are in terms of length. So, here some units of head loses, is occurring there between the two flows, when the flow occurs from one channel section of the channel to the other section, there will be some head lose, you need to account that also, and the conservation of the energy suggest H 1 is equal to H 2 plus, whatever lose in energy has been there.

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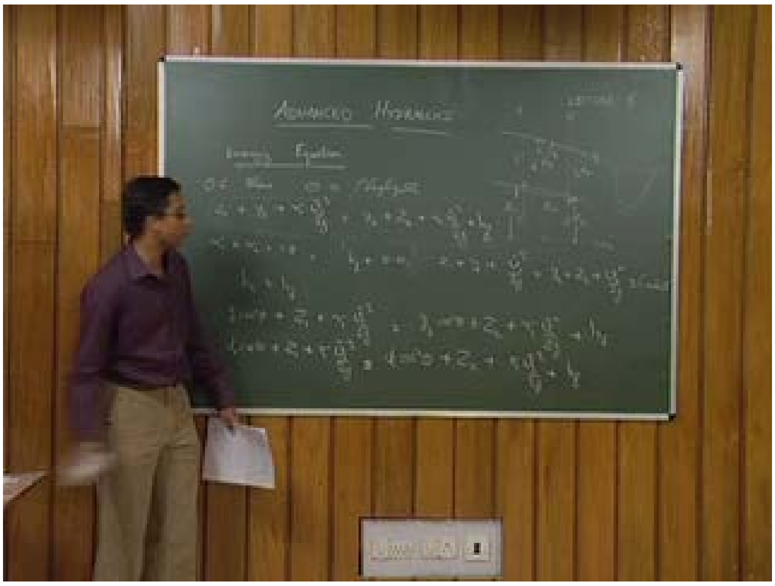

So, you can now easily write this three terms as H 1, is equal to again i m just reinstating that; H 2 plus H l, various head losses are, head loss due to friction, and head loss due to eddy formation. So, in most of the... You can give this as H f, and this as H e. In most of the preximatic channel, this quantity is provided, eddy formations are almost negligible. So, H l in most of the cases is approximately same as your, loss due to friction. So, that quantity if you suggest it, you will see that H 1 is equal to H 2 plus H l, you can express them in the expanded form, which are given that earlier. This is either you can write it in terms of say y 1 cos square theta plus z 1 plus alpha 1 v 1 square by 2 g, this is equal to y

2 cos square theta plus z 2 plus plus H f, because we are considering that the channels are mostly cosmetic here.

Once you write the equation in this form, this is called the energy equation, in a more expanded form, when you write it in head energy head form, this is the energy equation or if you write it, write this particular quantity in terms of the depth, the normal depth from the surface to the bed d ; that will be d 1 cos theta plus z 1 plus alpha 1 v 1 square by 2 g is equal to d 2 cos square theta plus z 2 plus or most of the open channel, open channel flows, here theta is negligible. In most of the cases theta is negligible, this equation, then subsequently becomes z 1 plus y 1 alpha 1 v 1 square by 2 g this is equal to y 2 plus z 2 plus alpha 2 v 2 square by 2 g plus H f. So, in most of our open channel flow, we can use this particular energy equation, in most of the cases, exception means; however, for higher slope channels, please note that you need to account the flow of appropriate cos theta values.

Otherwise you can use this particular relationship, you will see that this equations. Suppose if your energy correction factor, if it is 1, if head loss due to friction, if it is 0, then what that does that suggest. Suppose in such cases, you will find that z 1 plus y 1 plus v 1 square by 2 g is equal to y 2 plus z 2 plus v 2 square by 2 g; that is, this is equal to constant, and this is your famous Bernoulli’s equation; that is applied in your fluid flow problems and all. So, Bernoulli’s equation is also a energy equation.

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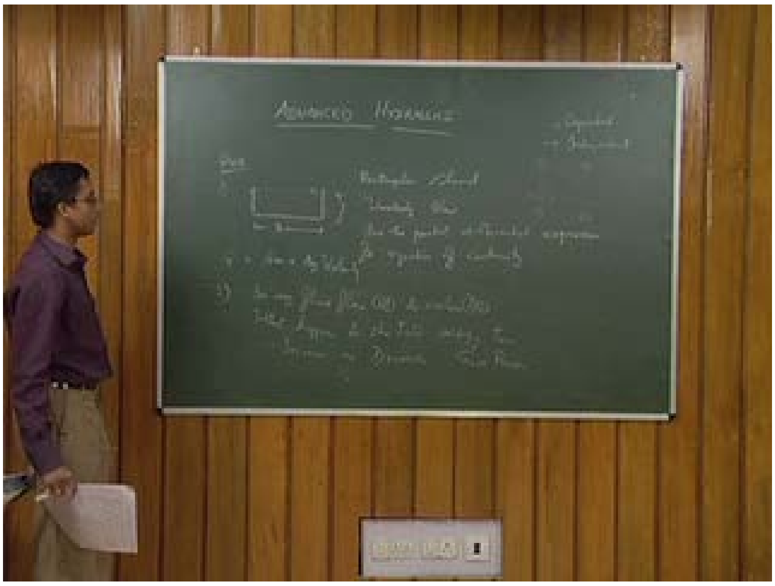

So, now we will come into the quiz for the following today's lecture. Let me ask you, say if there is rectangular channel, if there is a rectangular channel the depth of flow is y, width of the channel is b. There is an unsteady flow in this particular rectangular channel, unsteady fluid flow in this particular rectangular fluid flow channel. Find the partial differentially expression, find the partial differentially expression for equation of continuity in this rectangular channel, find the partial differentially expression for equation of continuity, in this rectangular channel. Hint is that, discharge through a cross section, this is equal to, discharge is equal to area into, area of cross section into average velocity, I am just giving this particular hint.

Now you derive the, or you have already derived the unsteady flow equation of continuity, using that just get an expression, partial difference expression, for the equation of continuity for rectangular channel. Hardly it will take thirty seconds I hope so and it is quite easy. Can you just tell me what type of equation it is, how many dependent variables are there, and how many independent variables are there. In a partial differentially equation, what is meant by dependent variables, what is meant by independent variable. These things are clear I hope so for you people, because this is masters program, or you should need to refer your under graduate books again, what is meant by dependent variable in a partially differentially equation, what is meant by independent variable in a partial differential equation. Your second question for the quiz, your second question is, for any fluid flow, from section 1 2 to section 2 2, in a channel, for any fluid flow, in the channel, from section 1 2 to section 2 2, what happens to the total energy term, whether it increases or decreases, give reason. Say you have this computed what is meant by total energy, so from section 1 1 to section 2 2 when a fluid flow occur, whether the total energy term increases or decreases, just tell me.

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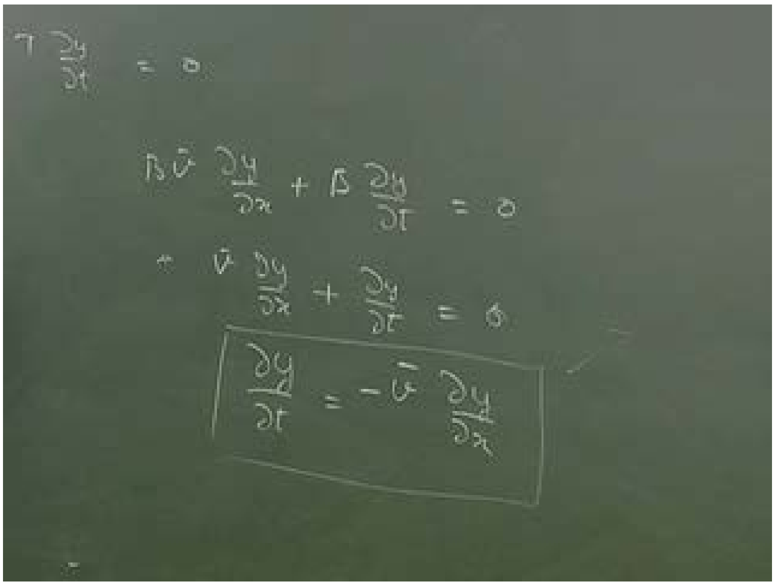

The quiz solutions for the first question, you know the unsteady flow equation dou q by dou x into yes tell me dou q by dou x plus t into dou y by dou t is equal to 0 q is equal to a v bar t is equal to same as your b a is equal to b y substitute the terms here you will get your partial differential equation in the following form b v bar dou y by dou x plus b dou y by dou t is equal to 0 or v bar dou y by dou x or you can say dou y by dou t is equal to minus v bar minus v bar dou y by dou x. So, you just suggest me as an assignment, what type of partial differential equation is this one. Now, solution for question number two is that, energy will always decreases, from one section another flow, in the direction of flow. In the direction of flow, energy will decrease. There is no point in gaining energy, if there is no external source of applying energy and all. So, that way we are concluding today's lecture. So, in the next lecture we will be starting this specific energy terms and all.

Thank you