Advanced Hydraulics
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Module - 6
Turbines
Lecture - 1
Application of Momentum Principles

Welcome back to our lecture series on advanced hydraulics. Till now we have covered five modules on various topics in this subject, particular subject. Today, we are going into the sixth module on turbines, or in fact it can be called turbo machinery.

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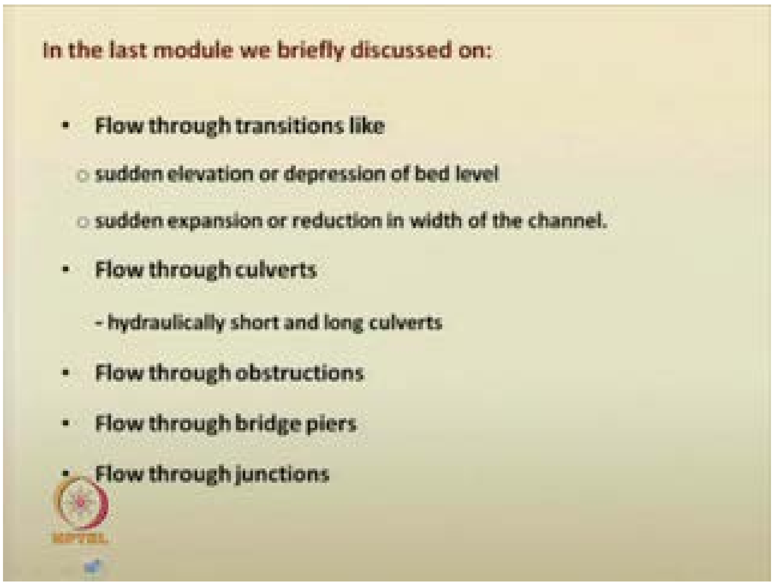

So, if you recall, in the last class, in the last module, we had briefly discussed on the various topics, like flow through transitions, especially like the sudden elevation or depression of bed level, then sudden expansion or reduction in width of the channel, flow through culverts, we have studied hydraulically short and hydraulically long culverts, then we discussed on flow through obstructions, there are various obstructions that we have discussed, flow through bridge piers was also briefly discussed, then we also discussed on flow through junctions, how the open channel flow behaves in junctions and all. So, based on these topics and all, now we can go into our next module that is turbines or turbo machineries.

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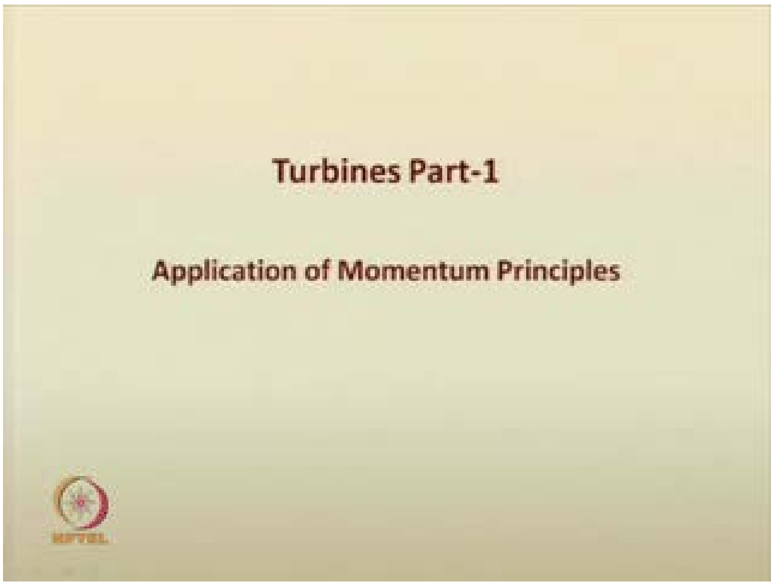

So, today is the part one of this lecture. In the first part, we will be concentrating mainly on the momentum. How we are going to apply the momentum principles? It is the simple momentum principle that we are going to apply for studying this thing. So, let me introduce this particular topic.

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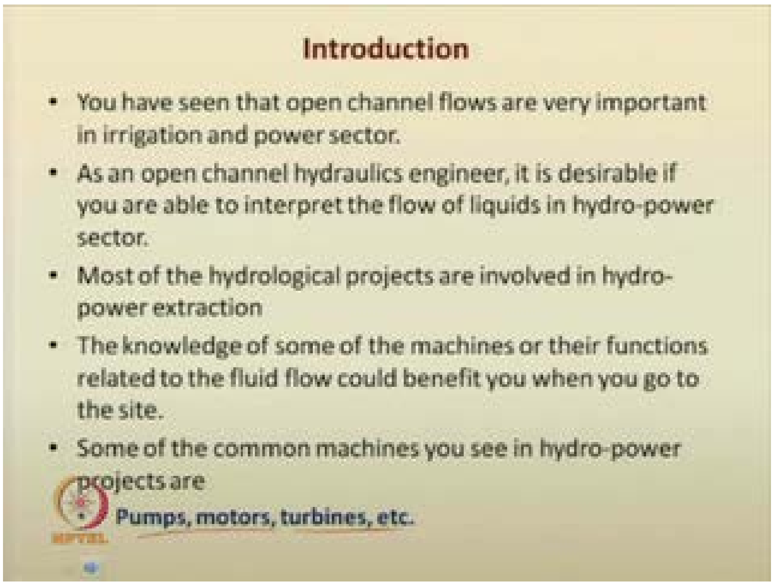

So, you have seen that the open channel flows are very important in irrigation and power sectors. So, now, when you graduate after doing this particulars course and all, you are all going to become an hydraulic engineer, or open channel means hydrology or hydraulic engineer. You will be, you, you will be given post somewhere in hydrological project sites and all, you will be asked to provide certain design aspects or even verify some of the design aspects and all, you may also have to go to the site, verify how the water is going into the project sites. For example, if the project is provided for a hydropower sector, then the hydropower sector involves lot of machinery, lot of fluid machinery works are there, like turbines, pumps and all. So, it will be desirable thing, that if you have some basic knowledge, how these machines work based on the water applied to those instruments or machines.

So, most of this hydrological projects; means, in our country, especially in our country they involve hydropower sectors and all. So, it is desirable; if you have some knowledge, some basic functions of these machines. Some of the machines that are, are pumps, motors, turbines, etcetera; we will see, based on the fluid flow, it is not the mechanical aspect, which we are going to study; it is not the manufacturing aspect of these machines; we are just going to see, how based on the fluid flow these instruments work .

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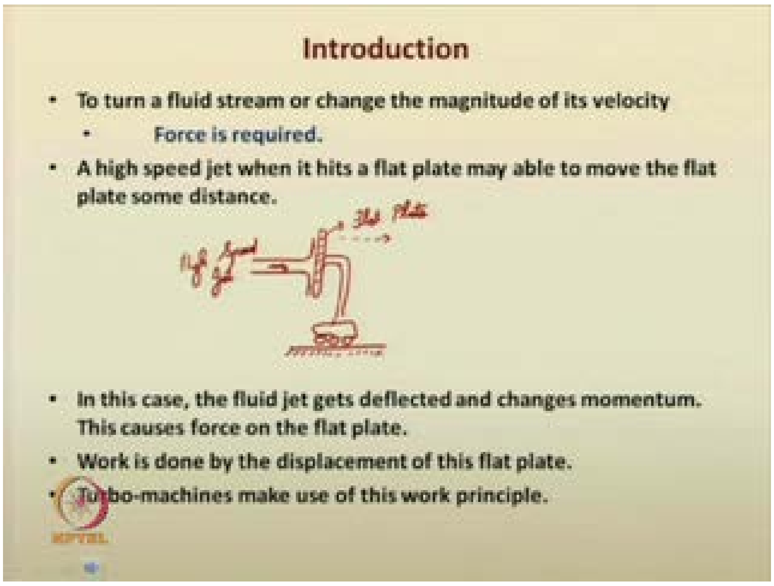

One of the basic thing is that, now to turn a fluid streams; if a gush of water is coming there or change its magnitude, if you want to change its magnitude. What is the predominant quantity? You require certain amount of force to change its aspect; that is, if a flow is coming in from one particular direction, if you want to change its flow to some other direction, some force you need to apply; or, you, means, some force is required for that change.

For example, a very simple case, in which we would like to introduce this particular topic is that, suppose a high speed jet, if it hits a flat plate; let me draw a flat plate; may be a bit better colour; say, a flat plate; I am just giving the side view, a two dimensional view of the flat plate; so plate is, the area of the plate is perpendicular to this particular screen now. So, it is moulted on a roller; say, if this thing, particular screen this is mounted on a particular flat plate, if it is moulted on a roller; now, if you apply a high speed jet on this plate, what happens?

Water get splashed along the plane, as well as there will be movement of this plate, right. Because, roller support, you have provided roller support. So, there will be no resistance to the horizontal, this thing, impact and; so it can move in the horizontal direction. So, the same thing; now, using the same principles, what we can do is that, we can now study about turbo machineries.

For example, now in this particular case, the fluid jet gets deflected and changes momentum; the fluid as it comes and impacts the plate, it changes its direction. Therefore, the momentum is also change. This causes force, right, on the flat plate. Moreover, the plate also gets displaced in the horizontal direction, as the roller is being provided. Due to this displacement, a work is done on this flat plate. It is this particular principles, that is, the work done due to the displacement; this particular principle is being employed by most of the turbo machines. So, turbo machines are machines, like pumps, turbines and all, that use the fluid, this thing.

So, it is this particular aspect; the simple concept of momentum that is impact conservation of momentum or the momentum principle which we apply, that is the running of all the turbo machineries.

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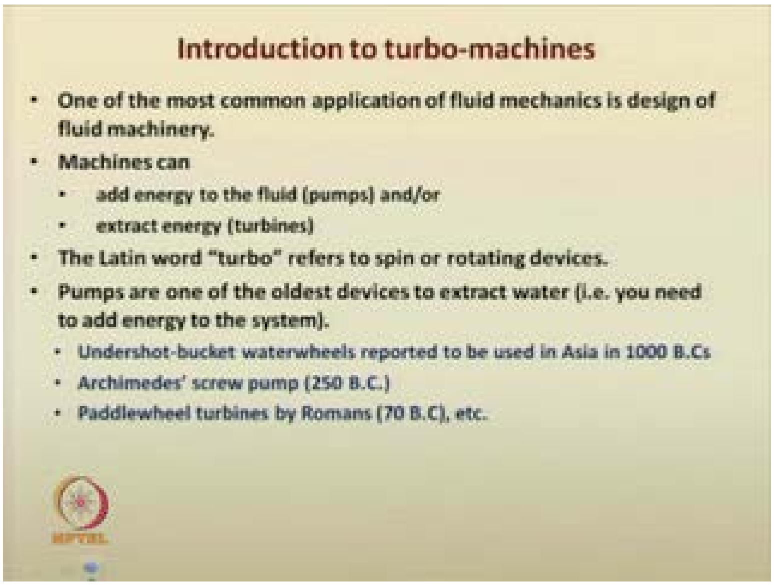

So, one of the most common application of fluid mechanics, especially in fluid mechanics and all, it is a design of fluid machinery. So, you can have various types of machines. So, machines that can add energy to the fluid flow; means, if a fluid is flowing, if you add some more energy to that, how the fluid flow occurs then? So, that types of machines are there; that is able to add energy to the system.

So, they are called pumps. You know, how from the; means, you can pump water from a particular location to another location; it is adding energy to the system, that is why it is able to extract water and pump into some other location. Similarly, you can extract energy from the water that is available. You are not going to add there, you are extracting energy that is available there, in that water body or fluid flow; and, you will be utilising it for some other purpose; especially turbines, where the energy is being extracted to create power and all.

So the exact meaning of the Latin word turbo, it refers something to spin or the rotating device, ok. So, pumps are one of the oldest devices that extract water. You can, you need, if you need to add energy to the system, or even if you want to talk in terms of turbines and all, some of the turbines or some of the pumps systems are age old; means, they have been used many civilizations before itself.

So, it is nothing new there; means, it is, now you have advanced machineries advanced techniques and all, but the concept has been evolved; means, according to the need of the society, people have started evolving. They have, say, for example, the undershort bucket waterwheels were reported to be used in Asia in 1000 B.Cs; that is, even 1000 years before christ itself, such type of undershort bucket waterwheels were used to extract water and all. Archimedes' in 250 B.C., he was using screw pump for some transfer of liquids, liquids and all. You have heard; means, Romans were using paddlewheel turbines in their civilization, around 70 B.C. and all, they were using them.

So, it is that our civilized, or now at this stage we have some advanced techniques, but people view; what we have to remember is that, people started using such concepts much much earlier, eventhough they may not be knowing the mathematical aspects and all. Now, we will try to infer it, both mathematically as well as according to the requirement of the engineering, this thing.

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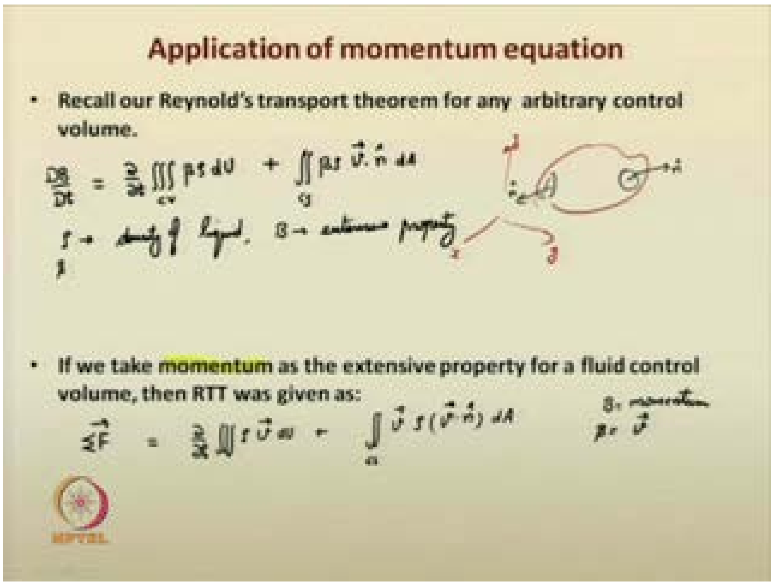

We will be using the momentum equation. So, now, recall our Reynold's transport theorem for any arbitrary control volume. So, if you have any arbitrary control volume, any volume; it can be in space, say, x, y; x, y, z direction, if any arbitrary control volume is there; you recall that control volume equation, how was it written?

The Reynold's transport theorem suggested D B by D t is equal to dou by dou t of the control volume, beta rho d u; so, that is, this is showing the volumetric integral, d u this is elementary volume in the control volume whichever you are taking into account; and, the surface integral or the, in the integration on the surfaces, beta rho velocity in vectorial form v dot n d A. So, this represents the change of this. So, I need not mention, ok. Just for your benefit I will be writing it; rho density of liquid, B is extensive property, based on mass, the extensive property was devised based on mass, right; beta is the intensive property, you recall them.

So, so, based on all these quantities now, suppose if we take fluid, if we take momentum as a extensive property for a fluid control volume, what happens? You can see that the first portion of the equation D by, D B by D t, it suggest the rate of change of that extensive property for the control volume, right. The, the first term on the right hand shows the net generation of how much amount of momentum is, extensive property is generated or destroyed inside this control volume; and, the second term on the right hand side suggests, suggest the net out flux of this extensive property through the surfaces of this control volume. That was the description; we had given it much much earlier; in many of the modules we have described them.

So, same principle, simple principle we are going to again use it for the turbine, turbo machinery as well. Now, let us take momentum as the extensive property, in this particular case. So, what we will do now? When we take momentum, what is the rate of change of momentum? Rate of change of momentum, you recall the Newton's law, it suggest that it is net force.

So, the first term, that is on the left hand side, the rate of change of momentum is equal to force this is nothing but, equal to dou by dou t of, I am writing rho first, then velocity d u plus, control v rho, v dot n d A. So, I hope you understand this n cap is nothing but, the unit; outward normal vector in any arbitrary area if you select, say, I am, if I selected this particular arbitrary area, the unit outward normal vector; say, if the area was like this here, then the unit outward normal vector would have been like this.

So, based on the control surface location it, it gives the sign or it gives the direction of that outward normal vector of that particular area. So, because, we have taken B is equal to momentum. Therefore, the intensive property beta is nothing but, equal to velocity; like that we were able to write it now, this particular form.

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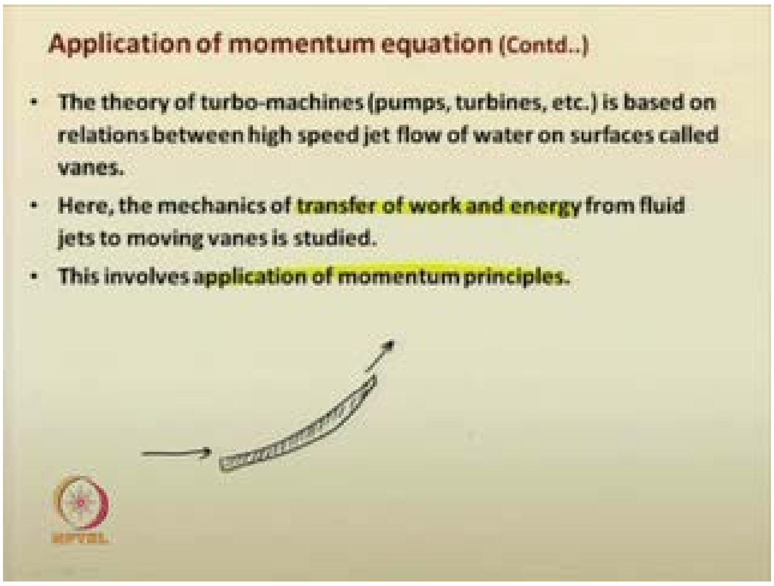

So, the same momentum equation; let us try to elaborate it. The theory of the turbo machines that is pumps, is based on the relationships between high speed jet flow of water on surface called vanes. What we want to suggest? In the previous case, we wrote this momentum equation. How we are now going to apply it for turbo machines? For that, you need to understand the theory.

The theory of turbo machines that is high speed jet flow of water on surfaces called vanes can appear; means, that is on the vanes, means, the plane. Earlier we showed the example of water high speed jet heating a flat plate. Similarly, any type of flat surfaces or curve surfaces it can be, if water is heating, such things in the turbo machineries, they are called vanes. So, they have particular design and all; that we are not interested right now.

So, if a high speed jet is heating a vane, then the work aspect or the many momentum aspect if he try to apply; means, high speed water if it is, it in, if you assume a control volume around that impact location and all, then that momentum equation whichever we have devised earlier, if we apply at that particular control volume, and whatever net force you are going to get, that can be a huge factor; means, that way you will be able to analyze, what is the force that is being impacted on the particular plate, flat plate or vane; and, that will help you to understand how much amount of electricity can be generated in the future, or how much can be pumped, etcetera, based on those phenomenon only. So, let us suggest now.

The mechanics of transfer of work and energy from the fluid jets to the moving vanes, that is being studied in turbo machinery. That is, the transfer of work and energy. You have see pumps; what they do is that, they give, they add energy to the fluid system. Whereas, turbines, they extract energy from the fluid system. So, both the, both the places you have to study about vanes and all. So, we will be applying the momentum principles to analyze the same thing.

Let me just give, how you can show a curved vane and all. Say, if I am just arbitrarily drawing a curved vane; say, you just assume this as a curved surface. So, a high speed water is impacting on this thing, then it is coming out of this vane. You can apply the momentum principles on such vanes, and you will be able to analyze the force and other quantities.

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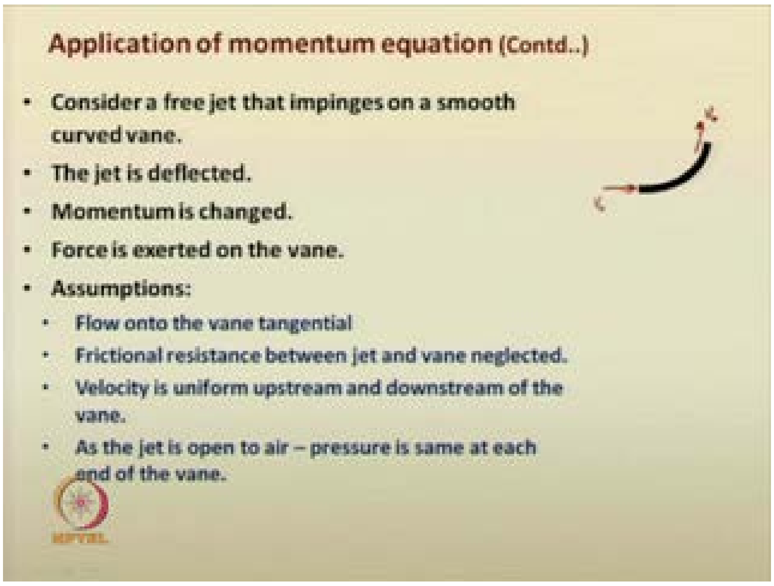

So, consider a free jet; again, let me just give it in different colour now, the impact. Let us assume water, a free jet impinges on this curve vane. So, the jet is deflected now; it goes on like this. So, it was coming like this; the high speed jet was coming like this, and it got deflected to another direction. So, the, if that direction is changed means, definitely the momentum is changed. Now, due to the change in momentum, force is now exerted on the vane, ok. So, the same thing, it can be studied in various ways.

Some assumptions which we need to incorporate while studying this thing. If you go in to the exact scientific these things and all, you may not be able to come up with proper picture. So, as an engineer we need to give, we need to apply certain assumption, so that we can engineer the entire phenomenon, or we can control the phenomenon, right. Now, the turbines hydropower, we have to control it; how much it has to be generated, or how much it has to generate every time; or, for example, pumps, how much amount of water is to be extracted; these things are controlled phenomenon, it is not a natural phenomenon.

So, for such instruments then, you have to start with certain assumptions; we can involve certain assumption. So, that, the entire physics is not going to change considerably; but, still, it is able, will be able to control in a much much better way. So, the flow on to the vane is tangential.

For example, we are assuming here, the flow onto the vane is tangential, as it is being depicted here. Then, we suggest that the frictional resistance between the jet and vane is neglected. Along, when the jet of water is flowing along this vane, the frictional resistance there it is neglected, due to this high speed; means, we can neglect it for various reasons. The velocity is uniform upstream and downstream of the vane. Another aspect we are suggesting is, suppose if v 0 is the magnitude of velocity, we are assuming the same magnitude of velocity is there in the downstream also. As the jet is open to air, pressure is same at each end of the vane. You can see here, jet is open to air; so, pressure at this location and this location, they will be same.

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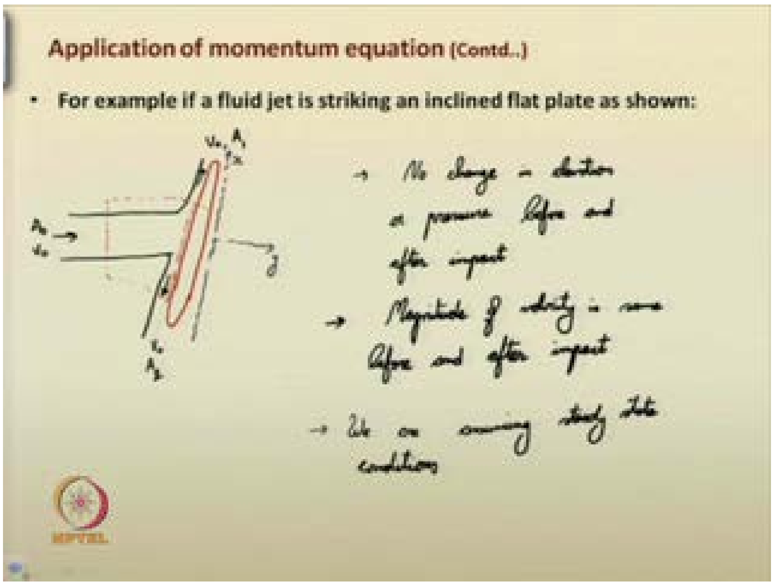

We have to apply the same momentum equation. Let us explain it with an example. Suppose, if a fluid jet is striking an inclined flat plate; I am just drawing it. Let us assume an inclined flat plate. Initially, let us suggest that this flat plate is not moving. If it is, let us consider it as a fixed quantity. So, a jet, high speed jet is impacting this particular plate. So, on its impact, it get splitted in both the directions of the plate, so along this direction as well as along this direction; this is the high speed jet flow direction.

Let us assume v 0 as the velocity. Now, as per the assumption, here also it is v 0, here also for this direction it is v 0, ok. Then, let us consider the area of cross-section for this jet as A 0, and here let us assume this as A 1, let me assume this as or let me assume this as A 2, and this one as A 1. So, you have now defined the following quantities. How are you going to apply the momentum principle?

Certain quantities you need to impact; that is, there is no change in elevation that we have to suggest; there is no change in elevation, there is no change in elevation or pressure. We have stated this thing; that pressure will be same at both ends of the vane. Similar, now this flat plate is also considered as a vane. So, there is no change in elevation or pressure, before and after impact; that is, before and after impact, you are not having any pressure change. We are also suggesting that the magnitude of velocity, the magnitude of velocity is same before and after impact, before and after impact.

So, how can we analyze, now? Ok, I am just taking that same figure again. You can now think of creating the control volume. I am just considering this following portion as the control volume now. You can also think of for the same plate, the same plate, it can be consisting; means, you have tangential direction for the plate, you can have a normal direction to the plate, right. Let me give the tangential direction as x, normal direction as y, it is upto you, means, whichever.

So, like that if you are able to define the coordinate system now, x and y; how we can further proceed it? You have the velocity, A 1, in this particular portion you have velocity A 2, in this particular portion velocity A 0. You can use the momentum principle. Again, another assumption which we need to incorporate is, we are assuming steady state conditions. So, how the momentum equation will look now? If you recall the momentum equation which was given to you earlier, or we have described it.

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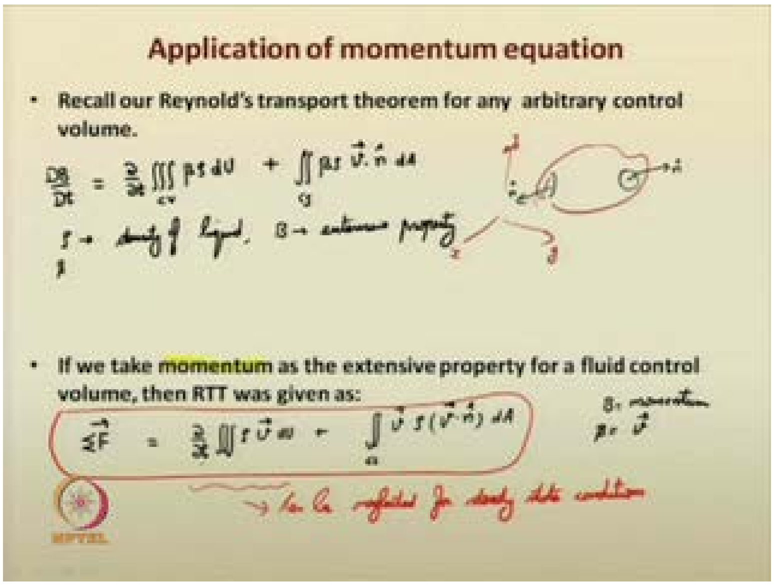

So, here in this momentum equation, if you see that; in this portion, if steady condition, state conditions prevail, this entire portion can be neglected, can be neglected for steady state conditions.

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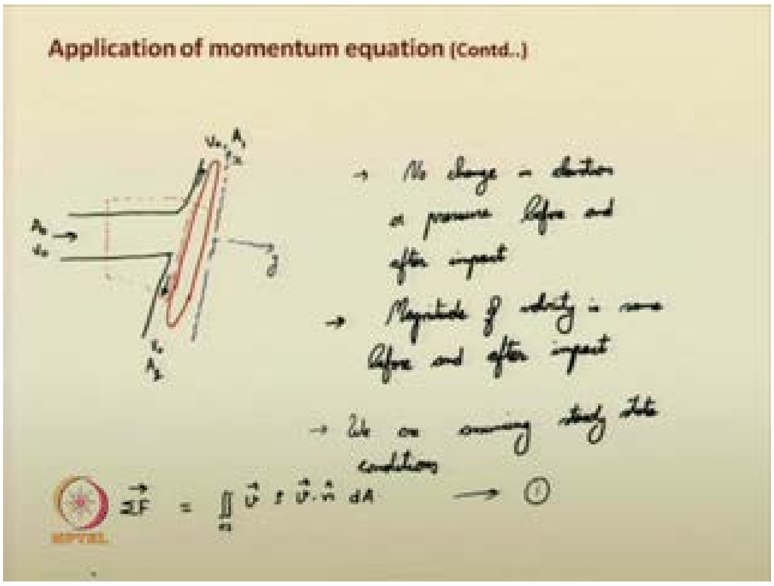

So, your equation now look like the following form. Sigma F is equal to surface integral v rho, v dot n d A. So, in that control volume given to you, you can apply this particular equation at all the control surfaces, right. This is the surface integral phenomenon. So, what are this, means, the control surfaces. You have the control surfaces; at this portion that allows water, at this portion that allows water, as well as at this particular portion where water is being allowed to transmit.

So, we can take those 3 control surfaces now, and you can apply it. If you want to see it now, each in the direction wise also, you can now devise; say, the, this is a vectorial form; please note that the force is vectorial form, because we have represented it in a vectorial way. So, here, the force can have components both in the x direction as well as y direction. So, the same thing we can utilise it, you can utilise it.

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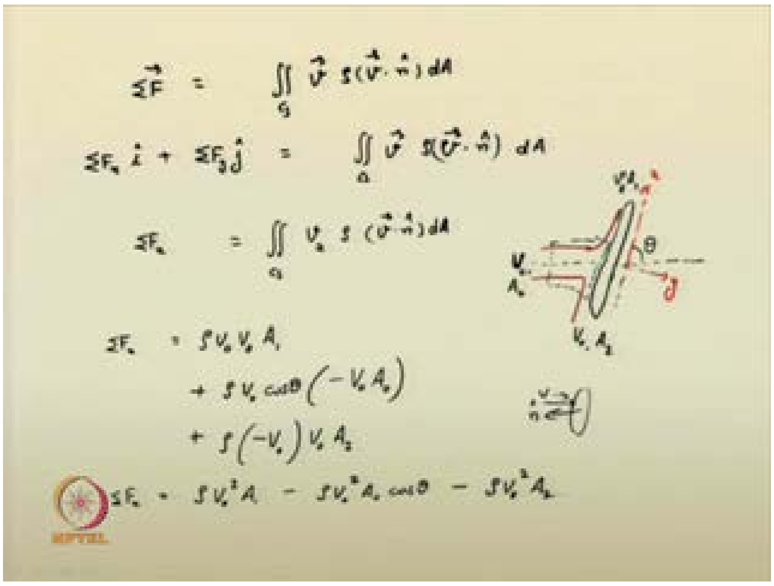

Say sigma F, again I am writing. It is equal to control v rho, v dot n d A. So, this can be sigma F x i plus sigma F y j is equal to v dot n d A. So, please note that v dot n, that is the component of velocity in, component of velocity in any particular aerial direction you are being taking into account; so, that way if we take now, the same thing now, what happens?

The sigma F x quantity, the x, particular component in the x direction, if you want to be visualize now. This will be nothing but, the same this thing; it should be the component of velocity in the x direction, so, v x rho v dot n d A. So, that way now, if I take it, here sigma F x in this particular situation, just recall the previous figure; I can just redo, redraw them again here. In the flat plate jet, high speed jet is impacting, and you are having the control volume like this. So, in this particular case, sigma F x; what could be sigma F x now? We have to take everything now in the direction of x and y. So, we have taken only in the x direction the component, right.

So, say, this is the x direction, this is the y direction. So, sigma F x, I can now write it in the following terms. This, the magnitude of velocity here is v 0, the cross sectional area is A 0; here it is v 0 and cross sectional area is A 2; in this case it is v 0, A 1, and A1. So sigma F x is nothing but, equal to rho times v 0 into v 0 into A 1, that is the control surface; at this particular location is being taken. So, the flux from that this particular portion, you we have taken now; plus rho times v 0; say, if this particular angle, if it is theta, sorry I beg pardon.

Let us draw a horizontal axis like this. Say, if this angle is theta, then we can suggest the flux at this, at this surface, at this control surface, there is a, at this surface of that control volume, where the flow is, means, out flux is allowed. So, here, the component of force, we can just write it in the following form; means, we want to write the component of force in the x direction, right. So, the net out flux or so the out flux in this particular direction it can be suggested; v 0 into cos theta is the velocity in this particular direction.

And, the influx into that portion is v 0, and you know, at this particular portion j, the outward normal is opposite direction; whereas, v is in this particular direction. So, you have a negative value. So, v 0 into A 0, then plus; at this particular control surface, we have rho times; now this is in the negative of x direction; so, there itself we are already incorporating minus v 0, then the flux is v 0; both velocity and the aerial outward normal vector, they are in same direction, so v 0 A 2, like that we can write. So, sigma F x is nothing but, equal to, I can write it, rho v 0 square A 1 minus, rho v 0 square A 0 cos theta minus, rho v 0 square A 2; like this I can easily write it.

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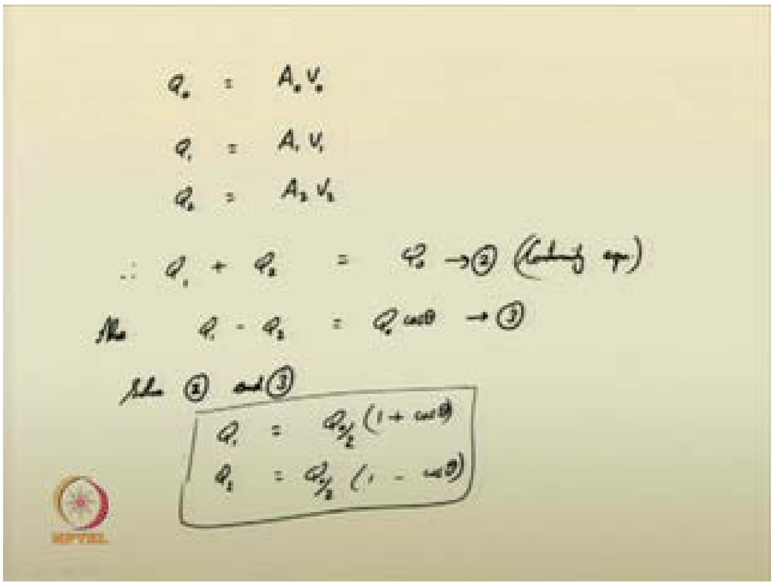

You can suggest that the Q, let Q 0 is the, at the inlet; at this particular inlet, Q 0 is the discharge. And, we are assuming steady state condition. So, discharge is nothing but, A 0 into v 0. Similarly, you can device, Q 1 is equal to nothing but, A 1 into v 1; Q 2 is equal to A 2 into v 2. So, we have assumed that there are no impact losses.

Therefore, from the continuity equation; you know that, Q 1 plus Q 2 is equal to Q 0. So, let me give this as equation number 2. Also, from the principles given earlier; from these relationships based on the velocity concepts here and all, we can suggest that Q 1 minus Q 2; what could be Q 1 minus Q 2 here? Q 1 minus Q 2 is nothing but, Q 0 cos theta; let me give this as equation number 3.

So, solve 1 and, sorry, equation number 2 and 3, we will get Q 1 is equal to Q 0 by 2 into 1 plus cos theta, and Q 2 is equal to Q 0 by 2, 1 minus cos theta. So, that is a realistic picture. We have only; how much amount of water is coming and heating the jet? That is the only known quantity to you. How much will be getting splashed in both direction? That you can now measure it using this particular theory; using the momentum principle, now you have evaluated, how to find discharges Q 1 and Q 2 from the vane, right. So, that way you found it; Q 1 and Q 2 was observed.

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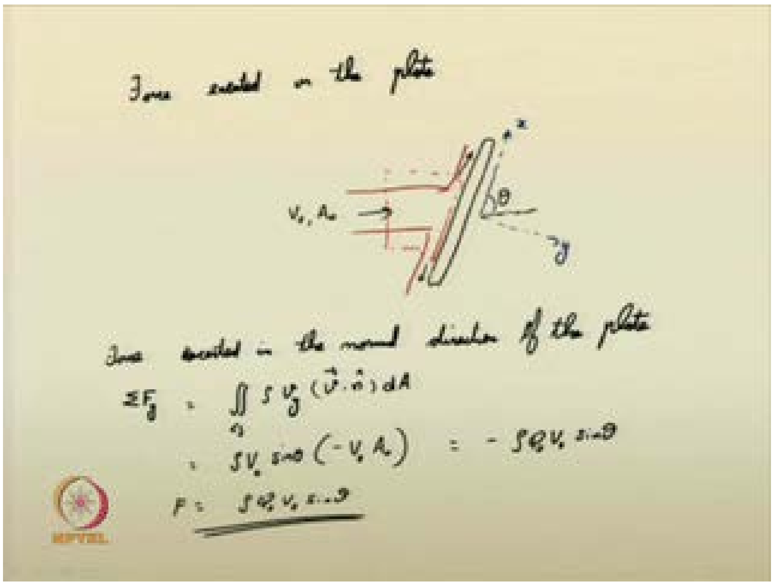

Similarly, you can now find the force exerted on the plate, force exerted on the plate, this can also be found. What could be the force exerted on the plate? Again, I am just redrawing them. Jet is coming like this, and you have the axial directions shown like this, x and y.

So, we can now suggest, force exerted in the normal direction, the force exerted in the normal direction of the plate, we suggest that force gets exerted now in the normal direction of the plate; in the tangential direction, you have already seen the discharge and all, right. We have already seen it. We are assuming that, in the tangential direction there is no force component, or sigma F x may be made as 0 and all, fine.

So, based on those thing, the force exerted in the normal direction, it can be given as sigma F y it is nothing but, equal to same thing. rho into v y, instead of v x earlier, v dot n d A. So, what will you get now? Whichever, from these control surfaces; from these control surfaces, which, which all quantities do you think will appear? We can just simplify it now. I can write this entire quantity, rho v 0 sin theta. Please note that, this particular angle is theta; so, and this is v 0 coming here, A 0 here. So, rho v 0 sin theta, in this particular direction, v 0 sin theta into minus v 0 A 0. This is the only quantity that is having component in, in the normal direction of the plate.

You can see there; the out fluxes from the, these particular directions, they are not going to create any force in the normal direction. So, the force exerted to be in the normal direction of the plate; that is in the y direction. It is being provided only by this inlet impact, in the control volume, in the in the inlet of that control volume, whatever volume of water is coming, based on that; that is the only quantity that is providing force; that is the thing. So, please bear in mind.

So, in the previous portion, I just mistakenly told it that in the horizontal sigma F x can be, it is not like that; it need not be, you need not take sigma F x as 0 and all. What we want to imply here is that the force exerted in the normal direction will be only available through the inlet quantity, not from the outlet quantities and all. So, that way you can suggest. So, this is equal to minus rho Q 0 v 0 sin theta. So, you can suggest now; force exerted on the plate is nothing but, magnitude of the forces rho Q 0 v 0 sin theta.

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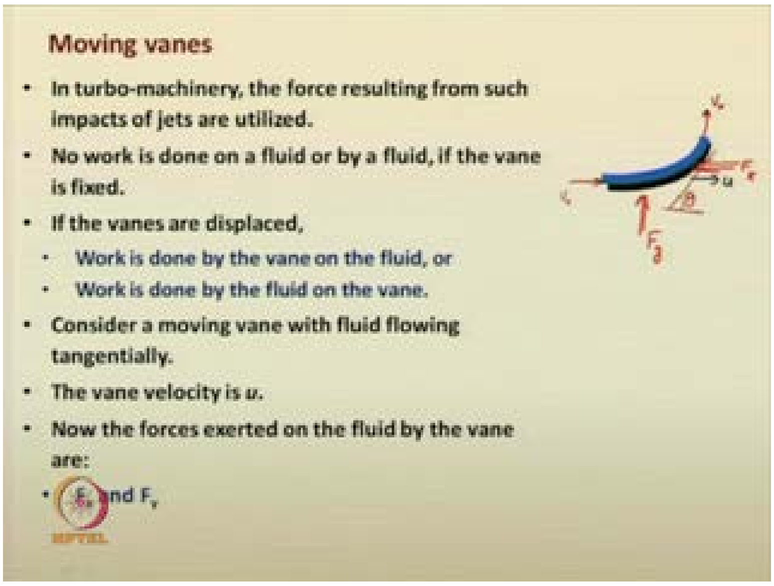

Now, we will discuss on moving vanes. How the forces are exerted on moving vanes? How we can use the same principle to analyze the moving vanes and all? So, usually, as suggested earlier also, in the turbo machinery, the force that is being resulted due to the impacts of just, jets, or the impact of vanes on the water, these are being utilized for various purposes, right.

So, you cannot do any work on a fluid, if the vane is stationary. Similarly, the, if the vane is fixed, no work can be done on the fluid or by the fluid into the system. So, that has to be clear; means, we have made it clear in the last portion also. So, if this particular vane, if it is moving in the, if it is stationary, there cannot be any work done. So, if the vanes are displaced, then you can say, work is done by the, either by the vane on the fluid, or you can say, the work is done by the fluid on the vane. So, this blue colour is the fluid, this is the vane. So, it can be either way. So, where the, it depends on what type of work it is being done.

Now consider a moving vane with fluid flowing tangentially. So, we have just described a figure here. This black colour is a vane. A fluid is moving tangentially, right. The vane velocity is u. Let us consider a velocity u for the vane. Let us assume this particular angle now. This particular angle as theta, let us say; let us assume that. So, the vane velocity is u. Water is coming at a tangential velocity v 0. It is exciting at same velocity v 0, because we have assumed steady state conditions, right.

Now, the forces exerted on the fluid by the vane are- we can suggest 2 forces; that is, what are the forces exerted by the vane on the fluid; that is the reaction components, right. So, you can say the vane exerts a force; maybe, I can give a different colour. Now, F x like this and F y like this; let us assume the vane exerts these forces.

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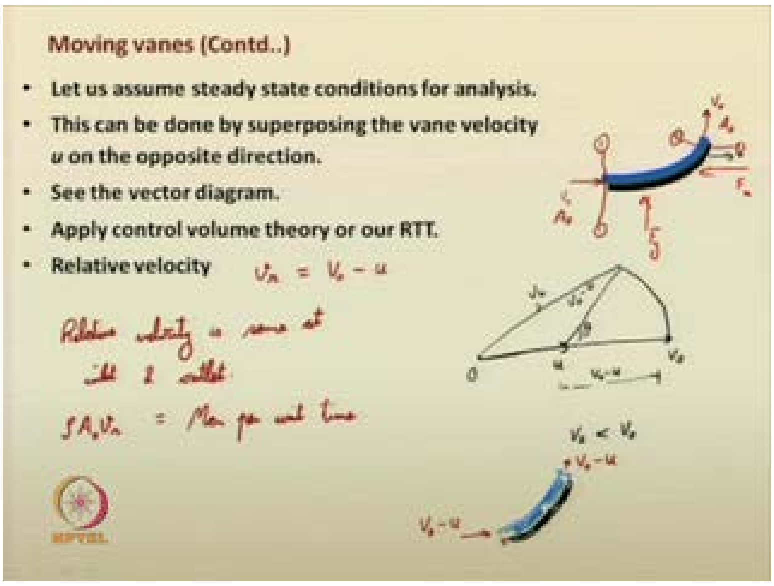

So, if that is a case, how can we continue the analysis? So, that is, you are having F y like this, F x like this, and the vane velocity is u. So, if that is a case, we can now use the same approach; that is, this can be done, means, we can assume the steady state condition. If we consider the vane, vane velocity and the actual velocity of liquid, if they are superimposed; that is, if the velocity of the vane, if it is a constant parameter, suppose if it is a constant quantity, then you can now easily assume that. Because, we already suggested v 0 is constant.

So, both the things, you can now incorporate in a relative manner; that is, you can superimpose the velocity in the opposite direction of the flow of liquid. If you can superimpose them, so we will get a relative velocity now, a relative term will be obtained. So, you can see the vector diagram. That is, say, from any particular origin o, if you want to draw the magnitude of the velocity of water, say, v 0. So, let us say that this is the magnitude, this much length could be there for that particular velocity.

Now, from the same origin o, if we draw a velocity magnitude for the vane; so, it may come something around here. Let us assume that; this is the case. So, this is velocity u. If you are aware; means, if you know these quantities; now, if you superimpose or if you find the relative velocity, what is meant by relative velocity now? This velocity, this difference in quantity, this will be the relative velocity in this direction; but, at the inlet both are having same directions, that is fine; and outlet, we have already suggested the velocity of water is v 0, ok.

So, you can now draw a relative velocity of same magnitude, v 0 minus u in this direction. So, that is; so, they are part of the same curve of a circle, right. So, like this you can draw the quantity. Now, the absolute velocity at the exit; at the exit point, at this particular exit point, the absolute velocity would have been this much; it can be given as v 2 and all. So, naturally, as per this diagram, this is the theta in curve suggesting that particular angle; then, you can now see that v 2 is less than v 0. Because, here, the water is impacting water, with a velocity v 0; it is impacting a particular vane that is moving in the same direction of the flow of water.

So, there will be some reduction in velocity as per this vectorial diagram, you can see that. Because, you can see that this is the, this curve is part of the same circle. So, naturally, we, the maximum available quantity will be v 0 here; in this, that is both v 0 and u in the same direction if it is there, that will be the maximum quantity here; this v 2 will be less than v 0.

So, in that way, when we draw that; when we can take these quantities and all, now let us apply the control volume theory. So, we will be having sections, same sections, say, this is section 1. So, this is section 1, this is section 2; so, 1 1, 2 2; like this if you can suggest 2 sections, you are now going to apply the control volume theory, you take the same thing.

What is the relative velocity? Now, the relative velocity term, I can write it in terms of equation, v r is nothing but, equal to v 0 minus u. Like this I can write, isn’t it? So, the same quantity is now written. So, this same quantity will appear both in the inlet and outlet. So, I can now; as we want to make it a steady condition, I can now write this quantity here; v zero minus, or I can just redraw the entire thing; say, this is the vane, and I am just drawing the water. So, this particular quantity, here, I can suggest, this is tangential; please note that this is a tangential quantity; so, v 0 minus; or, I will, so this velocity, relative velocity will be giving at the inlet v 0 minus u, and at the outlet also v 0 minus u; like this we are going to give the relative velocity.

So, in this case, once you identify the relative velocity; relative velocity is same at inlet and outlet. So, you can now, say, the quantity rho into A into A 0, that is this is the cross sectional area, right; that we have already said, seen earlier also. So, this is equal to the net mass rate discharge from the nozzle and all; you can, it is not equal to from, from the nozzle, but you can suggest that this is mass per unit time, fine.

If you connect some series of vanes like this; say, in, in an circle circular disk, if you connect such vanes and all and if you rotate it, and all the jets that is coming into this portion, that will be captured by any, any one of the vanes and all; then you can subsequently suggest the discharge from the nozzle and these quantities are same.

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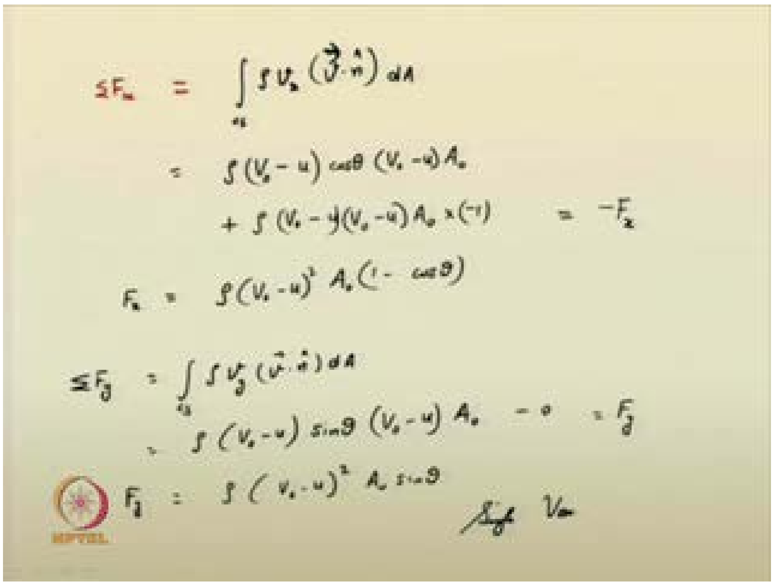

So, we will be analyzing using the same control volume approach now. What is sigma F x? Sigma F x is nothing but, steady state condition. So, rho into v x, then v dot n vectorial, so v dot n d A. For this particular vane, you know, what is this quantity? In the x direction at the outlet v 0 minus u; just go to the previous portion at the outlet; this is v 0 minus u, and this particular angle is theta. So, v 0 minus u cos theta into v 0 minus u A 0 plus at the inlet, we are talking about the net momentum out flux, right. So, v 0 minus u into v 0 minus u into A 0 into minus 1, we have to give, why, because at that inlet you know v dot n will be a negative quantity, is it not? So, therefore, we are giving it into minus 1 like this.

So, you can say this is nothing but equal to the force exerted by the fluid, right. So, it will be minus F x, because we have suggested F x in the other direction, if you recall them. So, F x can be now written as rho into v 0 minus u square A 0 into 1 minus cos theta, fine. So, this is the x component of the force. Similarly, the y component of the force can also be evaluated using the same method rho v y v dot n d A. So, this is nothing but, equal to rho v 0 minus u sin theta v 0 minus u into A 0 minus 0. Because there is, you know that, at the, at the outlet, sorry, at the inlet v 0 minus u component is not available in the; there is no component of velocity in the y direction. So, that is the reason.

So, this is 0 here. This is same as F y now, because we have assumed velocity in the positive direction of y; so force, it will be same like that. So, you can write F y is equal to rho into v 0 minus u square A 0 sin theta, fine; like this you can. So, you can get it for; this is for single way.

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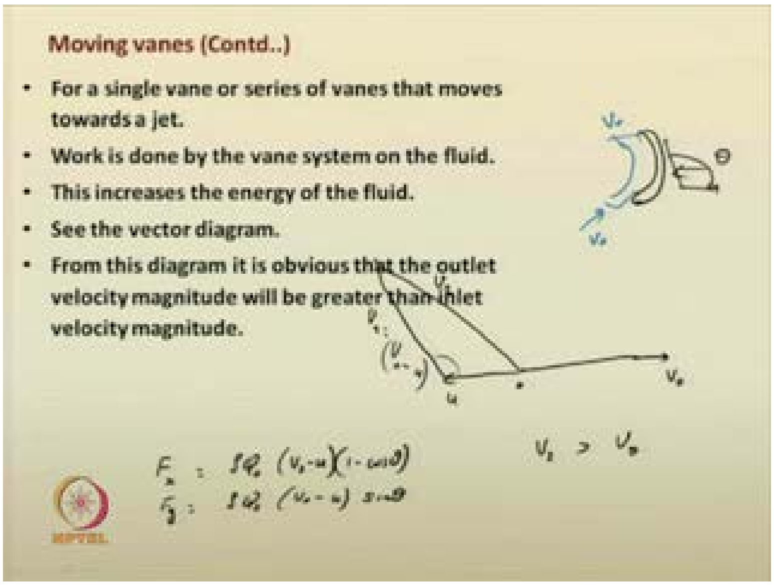

Similarly, if you consider for a series of vanes; for a single vane or series of vanes, there for, if you consider, say, for series of vanes, these things can be further integrated. I will just show it later. Let me continue with for a single vane or a series of vane that moves towards a jet, what happens now? If a single vane or series of vanes, series of vanes that moves towards a jet, the other way; so, there is a vane like this; it is moving with a velocity u in this opposite direction; whereas, there is a jet of fluid that came like this v 0, and this is v 0 like this.

So, you can use the same vectorial diagram, ok, same vectorial. Now, the work is done by the vane system on the fluid. So, it increases the energy of the fluid. We can use the same vector diagram as suggested earlier, an origin o is being selected, this is the magnitude of velocity v 0 at the inlet. Now, from this origin, it is in the opposite direction; the velocity of vane u is being occurring there. So, you can now see that based on this particular angle, based on this particular angle here, the same angle is being suggested.

So, this is your relative velocity v r is equal to, fine. So, this quantity, you will see; the magnitude of the actual velocity at the exit of the vane that is v 2; v 2 will be greater than v 0, in this particular case, it can be greater than v 0. So, we will be getting higher velocity when such an impact of vane occurs on the flow fluid, flowing fluid. So, the exit velocity will be created.

As suggested earlier, I can write for a series of vanes; if it is connected, this can be given as rho Q 0 v zero minus u into, 1 minus cos theta, like this; and, F y is equal to rho Q 0, because we are considering the discharge is same, so you can write it like that, fine. So, from both the velocity diagrams and all, we can see the exit velocity is greater.

Thank you.