Advanced Hydraulics

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Module - 1
Open Channel Flow
Lecture - 4
Pressure Distribution

Good afternoon everybody, we are into the continuing lectures of the course Advanced Hydraulics. Today, we are going into the 4th lecture of this series till the last class we had described various properties of the channels, we have described velocity distribution.

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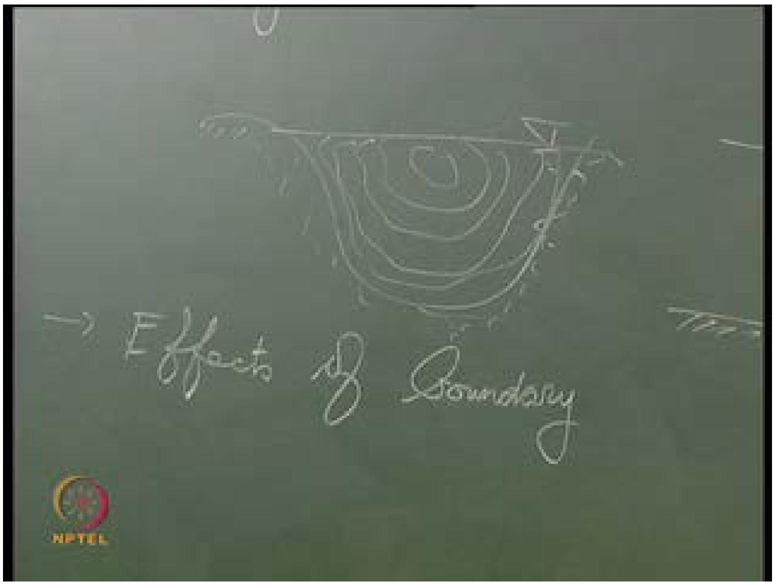

If you recall the velocity distribution in channels, if you take any cross section of the channel, any cross section of a particular channel, where the flow is in this direction. If you take the cross section you have seen that the velocity profile in any particular section, say if I am taking this section 1 1 and if I am drawing that here, the velocity you may get in some form of contours in a contour shape you will get the velocity distribution; you have seen that these are some isovels that is the line plotting equal velocity those are isovels.

And these contour lines, if you have observed that why this velocity change is there in the entire cross section; even if the flow is steady in a channel, even if the flow is steady in a channel, there will be velocity distribution in any cross section of the channel. The reason behind was that the effects of boundary, due to the boundary side walls the bottom as and all, there are many boundary effects that causes friction and there will be a distribution in the velocity pattern. One can easily identify say average velocity for such distributed velocity patterns and all, that is quite easier, why you in any cross section and all.

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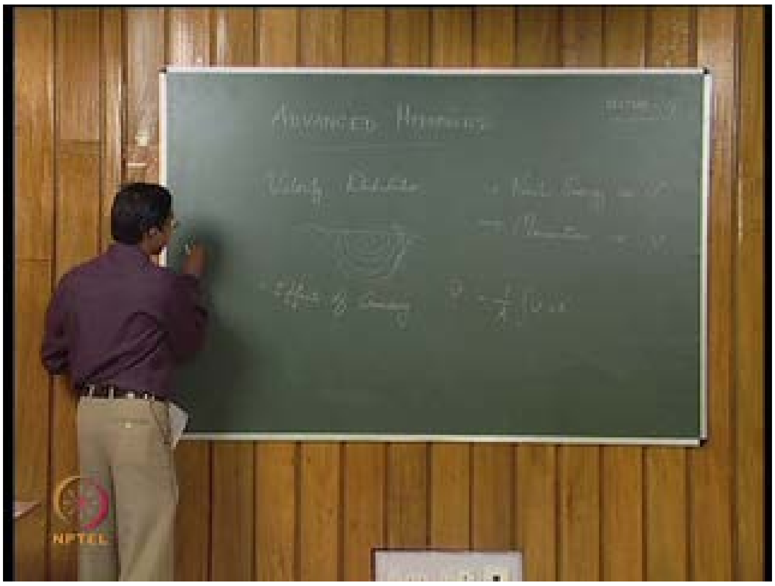

Due to these velocity distributions, you have from your physics knowledge and all you already know that, kinetic energy, momentum, etcetera these are all functions of velocity. Momentum is proportional to velocity, and kinetic energy is proportional to square of velocity, if you recall these things and all, and when if a person is taking average velocity for such a channel, which was defined in the last class such that, it is the ratio of area, that is 1 by A into v d A.

If you take any cross section, if any cross sectional elemental area in this channel, you take like this any cross sectional element d A of area d A the velocity at this position v into d A, that is integrated now for the entire cross section. And once it is once you get that integrated value, if you divide it by the entire cross sectional area you will get the average velocity. So, in most of the computational purposes, you can use average velocity term, instead of the individual velocity at various locations in the cross section, you can use average velocities for lot of computations.

Especially in open channel hydraulics, in where the most of the cases are in particular one direction and all, most of the flows f or example, channel flow predominantly it is in one particular horizontal direction and all. You can approximate the velocity of the flow using the average velocity; however, you have seen kinetic energy and momentum they are functions of velocity, this is proportional to v square, this is proportional to v.

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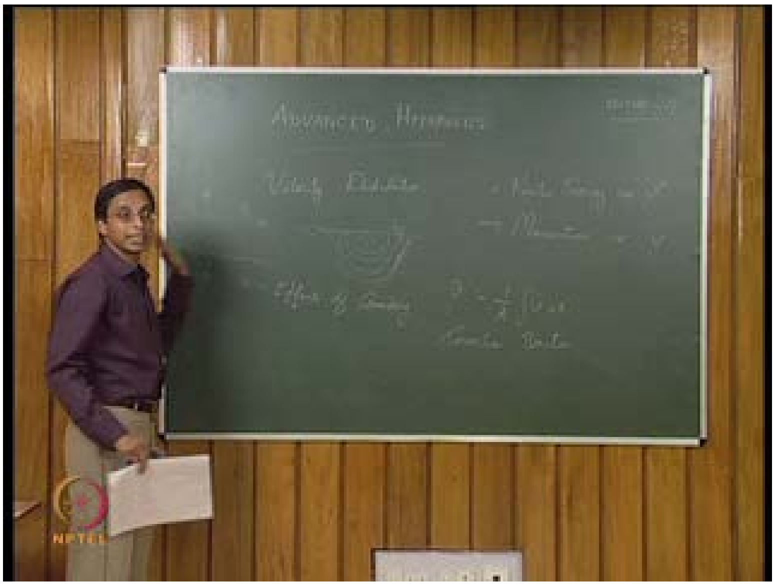

Now, if you try to compute the transfer of kinetic energy from one section to another section, if you are taking the channel stretch from one section to another section, just suggest that like that. If you want to see, say if you want to suggest that the transfer of kinetic energy from here to here or from here to here whatever be or from momentum here to here or here to here; once you try to compute those properties transfer from one section, and if you are trying to incorporate the average velocity. Then you need to incorporate or you need to put some correction factors for computing the momentum transfer, or kinetic energy transfer in such channels.

We have seen that, because due to these properties higher order properties and all, you cannot directly incorporate the velocity, average velocity for the entire cross section. You need to incorporate momentum correction factor, you need to incorporate kinetic energy correction factors, while you compute the energy transfer or the momentum transfer.

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So, what did we do, we had even given some formulas that is, this is the kinetic energy correction factor alpha, in fact we had derived it this in last class, this was given as integral of v cube d A by the average velocity cube into the area of cross section. So, this was given, you can also compute momentum correction factor beta, this was given as integral of v square d A by again you see the average velocity, the square of the average velocity into the area of the cross section like this you can compute, alpha and beta.

Just for an example case, I initially thought of giving this particular problem which you are going to do now, as a quiz problem, let us do first before starting the quiz and all, let us demonstrate one problem, let us first go through them.

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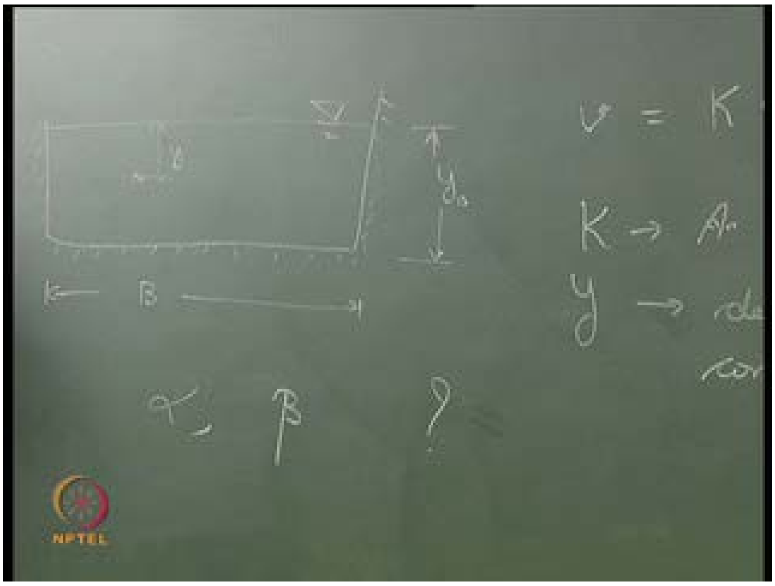

We have a rectangular channel, a rectangular channel whose depth of flow is given as y naught width of the rectangular channel is v, at any point inside the channel at any point inside the channel, the velocity distribution or velocity in the channel it is given as v is equal to k into y to the power of half. Where k is an arbitrary constant it depends on the problem, any type of flow and all; y is the depth from bottom to any concerned point, how will you compute how will you compute the kinetic energy correction factor, and the momentum energy correction factor? I hope it is quite easy for you to do that, let us demonstrate them; I will give you another problem of similar type as quiz for today's lecture.

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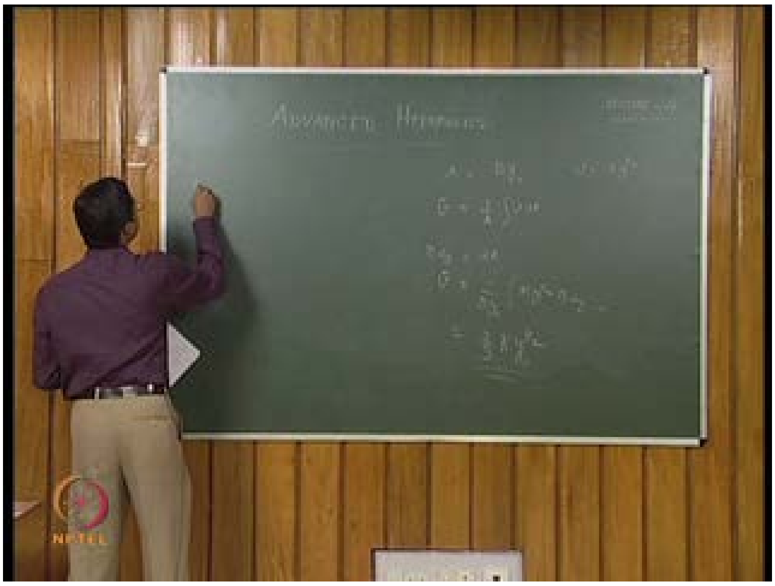

So, what is the area of cross section, for this particular rectangular cross section say B into y naught, what is the average velocity, you know average velocity is computed as 1 by A integral v d A fine, so what is this thing here, if you just take for this particular point, just take the elemental strip, whose width is d y. And B into d y is the elemental strip area, d A therefore, your average velocity is nothing but 1 by B y naught integral v is equal to k y to the power half it is given to you, so substitute them here, k y to the power half.

So, what will you get, simple integration you cancel off B and all, you will get v is equal to 2 by 3 k y naught to the power of half. I hope there is no need of further further how to integrate those things, as this is a masters programme, by this time you should be aware of doing simple integration techniques and all.

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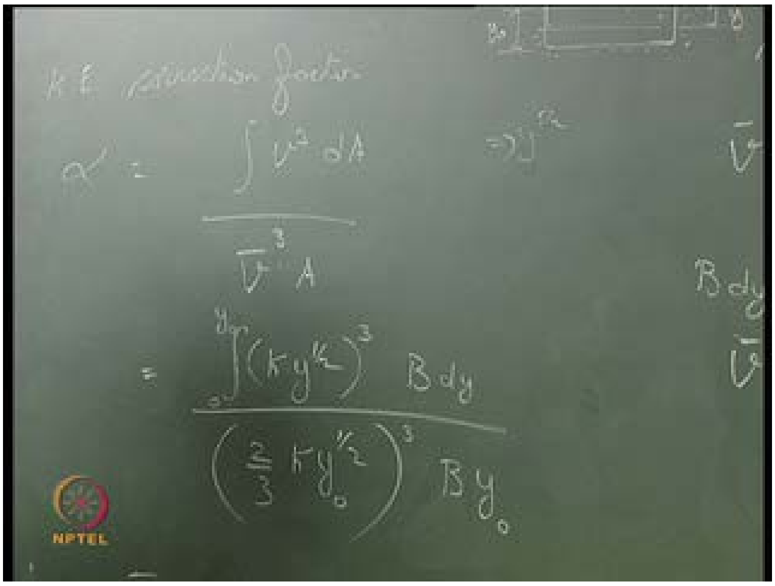

Kinetic energy correction factor, alpha nothing but v cube d A by right, what will you get this is k y to the power of half whole cube into B d y by 2 by 3 k y naught to the power of half whole cube into B y naught. Further compute the things, do the calculations, you can see k k cube is there here, k cube is there here, that will cancel off B is there, B is there here, that will also get cancelled off, ultimately you will get in terms of further on further integration, this is integrating y, y to the power of half cube that is y, y raise to 3 by 2 on integrating, you will get y in terms of y by 2.

And you know that this for this particular channel for this particular channel that the depth of flow is y naught, your arbitrary variable y that is from top surface, from top surface to any concerned point that depth is given as y, this y ranges from 0 to y naught. So, you have to give the corresponding integral limits, the same integral limits where there in this case also, when you computed the average velocity term. So, please incorporate them here, your depth of flow from the top surface that is given as y, depth of flow to a any concerned point, so that depth, total depth of flow up to the bed, that is given as y naught.

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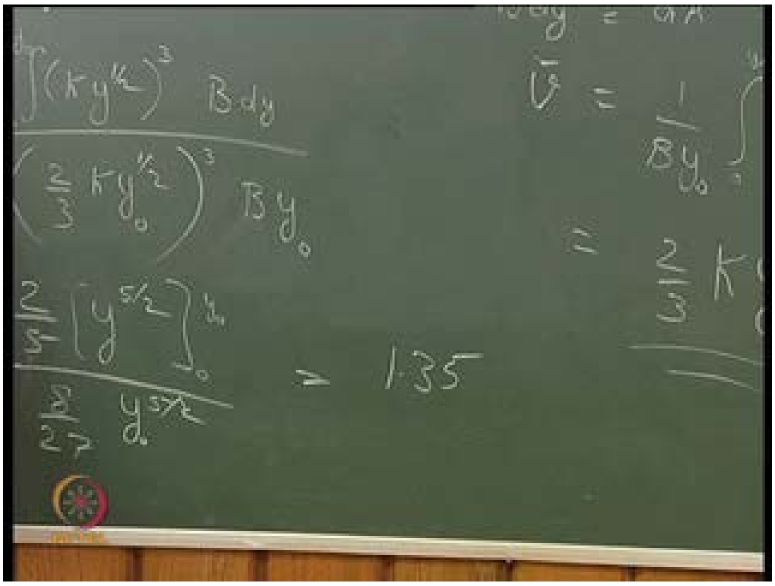

This on further simplification, you will see that, you will get, I will just give one more intermediate step, from 0 to y naught divided by 8 by 27 y naught 5 to the power of 2, this comes to be 1.35. So, what do you understand from this, your kinetic energy correction factor for this channel, it is a numerical value 1.35, it is not dependent on the arbitrary constant for velocity.

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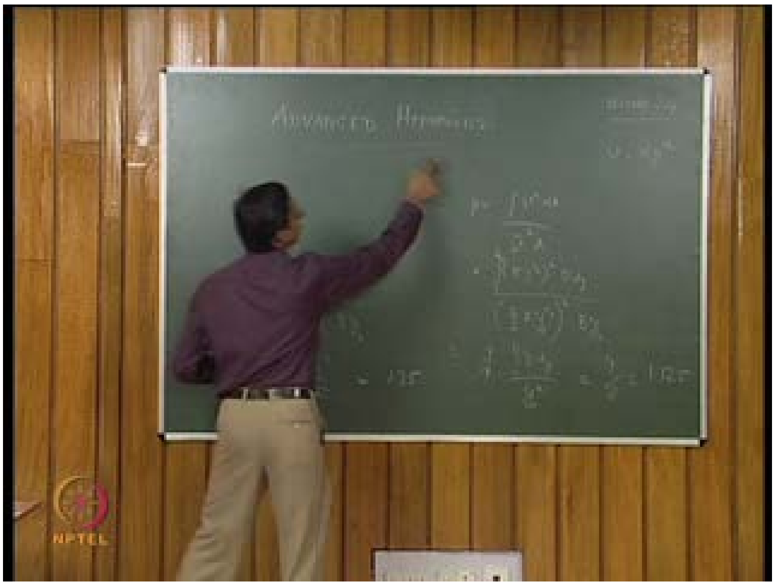

We had considered velocity is k into y to the power of half, so this k is immaterial in computing your kinetic energy correction factor; it is giving you as 1.35. Similarly, beta for this problem, you can compute this as v square d A by average velocity square into A, this is given as say again this ranges from y, 0 to y 2 y naught, k y to the power of half whole square B d y divided by 2 by 3 whole square B y naught. Do the integral procedures, cancel off the terms which that are common in the numerator and denominator, you will see that, ultimately you will get the things in the following form, 9 by 4 integral 0 to y naught y d y by y naught square; this on further simplification you will get this as 9 by 8 is equal to 1.125, so this is the way you compute alpha beta for various cross sections.

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So, like this we have studied now the velocity distributions, a similar property is now called pressure distribution. As you have seen pressure and velocity, they are the two most important terms in any fluid mechanics, or even any hydraulics course and all, they are the two most important parameters or variables that are governed, that will suggest the various distributions in the flow. So, if you are able to identify the pressure in a flow, or if you are able to identify velocity in a flow; that determines your analytical methods or that determines your way how you analyse the fluid flow and all.

So, pressure, how do you understand by pressure, what is meant by pressure, in the first class itself we suggested that pressure is a scalar quantity, this was proved by Pascal. So, if this pressure is a scalar quantity that means, it is not having any direction, how do you account for the effect of pressure, then why is this pressure that much important, see pressure is such a phenomenon, it will be there everywhere, wherever you are there will be some pressure. Wherever the pressure is there, it exerts force due to pressure, a force due to pressure will be exerted on any surface, whether it be a vertical plane, whether it be a horizontal plane, whether it be an inclined plane, whatever be forces will be exerted on those planes due to pressure.

So, whatever pressure, magnitude of pressure you have, you have to integrate or you can obtain the force created due to the that is those magnitude of pressure, that is how pressure causes various phenomenon due to especially in fluid flow and all. You will see that this pressure parameter, it creates means how the variations in pressure creates different types of flow and all, one will be seeing them; in even in courses related to fluid mechanics even in courses related to advanced hydraulics and all, you will be seeing them.

So, pressure force, how the pressure or how the force is formed due to pressure, or how the force gets appears due to pressure, you are quite aware of atmosphere here, you are all aware of the atmospheric pressure, atmospheric pressure it is a quite common term. Say let me let me give this atmosphere term as suffix here, p atmosphere, and the symbolically I am representing pressure as p, if in this class room there is atmospheric air present, so definitely a pressure, atmospheric pressure is existing in this class room.

Now, if I take, let me take this file, and if I hold this file in a horizontal plane, due to the pressure at this point, at any point in this location, normal forces due to the pressure in this room will act on this plane, please note that, forces will act normal to the surface. Now, due to the pressure here, the pressure into the area of this surface will give you the forces acting normally to this surface, so pressure will exert a force normal to the plane whichever we are taking into consideration. If you happen to keep this plane now in a vertical position the same pressure here, it is magnitude the pressure is p, same pressure is p or p atmosphere what this particular classroom, it is p atmosphere.

Now as I have kept this plane in a vertical position this also exerts, the same pressure will exert a force normal to the plane now, so here now the force will be in in the horizontal direction. In the plane was in the horizontal direction, your force due to pressure was vertically in the downwards direction, when this is in the vertical plane your force due to pressure is horizontally and into the plane. Similarly, force can exert in the backside, and into the plane like this as well, if you keep this plane inclined position the same pressure will exert a force normal to the plane, and that also is given in this direction like this.

So, you can have say, if you are placing it inclined, your force due to pressure will be normal to the surface, if it is horizontal the force due to this will be in the vertical direction, if this is vertical plane your force due to pressure will be in the appropriate directions. It does not matter in this floor of this classroom, the force due to atmosphere, atmospheric pressure it is acting vertically downward; now the same atmospheric pressure is acting on this roof as well, you see in this roof also the force will be acting normal to the roof. However, it will be in the opposite direction compared to the flow here, so in this roof it will be acting in that direction, so it does not matters. So, it depends on the plane, whichever you are taking into consideration.

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In a channel in a channel fluid is flowing or in a river, or in a fluid is flowing in this direction you have heard about the term called free surface you have heard about the term called free surface. So, what does this free surface imply, can you guess what will be the pressure at free surface, any guess from your side the pressure at the free surface is nothing but the same as the atmospheric pressure, it is same as the atmospheric pressure. Because free surface is interacting with the atmosphere here, and at this point it is same as the atmospheric pressure, so that free surface term itself is coming at on the free surface, the pressure is same as a atmospheric pressure.

Now, in our common in our common units or even in our common analysis and all, most of the situations you take P atmosphere is equal to 0 as some sort of a datum, you have heard about mean sea level. Similarly, atmospheric pressure in almost all of the locations you consider atmospheric pressure as 0, what does that imply on the free surface your free surface is nothing but a line of 0 pressure your free surface is nothing but a line of 0 pressure. How will you analyse now, say for this particular case, how what are how the pressure is now affecting the fluid flow, how can these things be analysed, let us go into that.

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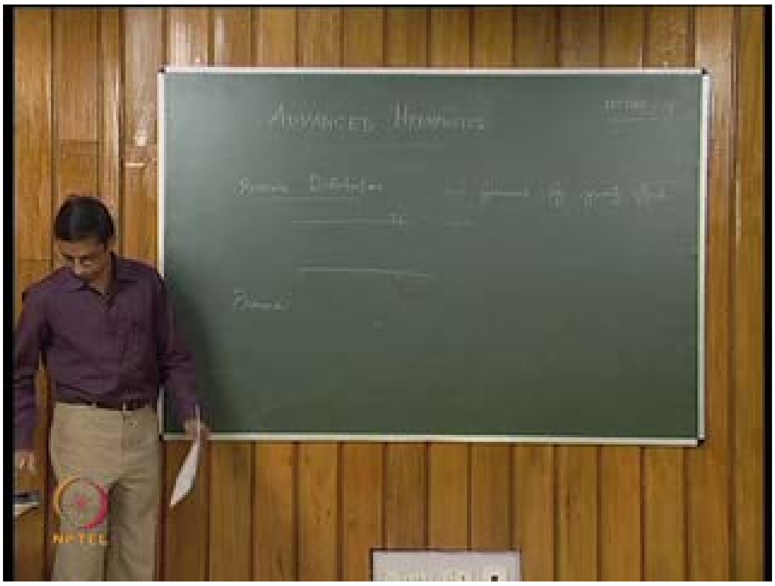

So, you know in our first lecture or I hope, so I have mentioned that most of the fluid flow in open channels, they are governed by gravity effects in free surface flow, so you know that in the open channel flow, the flow is governed by gravity effects; this we have mentioned in one of our earlier classes, I think it is in lecture 1 or 2 I cannot recall them. So, we have mentioned the predominantly in free surface flow, the effect of gravity is significant.

So, how do you account now the term pressure, we were discussing on pressure, pressure is a scalar quantity, it exerts force due to pressure or pressure force at whichever location that exists. So, how this affects the flow fluid flow in open channels or free surface flows, let us come into that picture here, let us take a horizontal channel let us take a horizontal channel.

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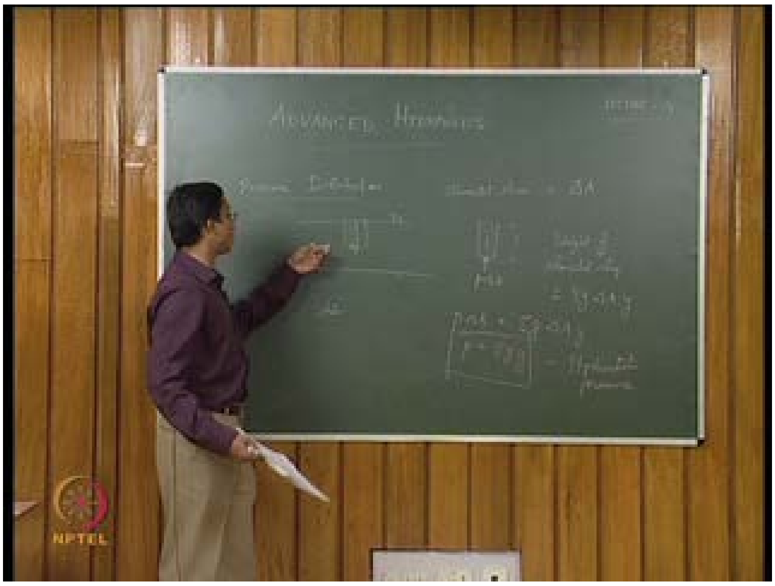

As a beginning, let us assume that a static stretch of water is there, the stretch of water it is in a static situation, I hope you understand what is meant by static that is the liquid is not moving at all. So, in this stretch of this channel, consider an elemental strip, consider an elemental strip, and at a location here where I have given the dotted point in this thing called p, the depth of water at this location it is y, the depth of water at this location up to this point, the depth of water from the free surface up to this point p, it is y.

Let us assume that the area, elemental area say if this duster is considered as the elemental strip like this, then this area at the bottom in the channel the area at the bottom of this channel that is given as elemental area, this is equal to just give this as del A. So, naturally, now when you understood the concept of pressure and all, you know that at this location, if you take this elemental strip out, at this location there will be a force due to pressure that acts in the upward direction that is nothing but pressure into del A.

Similarly, there will be a force due to gravity that acts in the downward direction, this elemental strip it has the depth y, it has the elemental area del A at the bottom, so your weight of elemental strip this is nothing but rho g del A into y. So, again let us come back into the, as this is a static fluid, there is no force both in the horizontal direction as well as in the vertical direction; there is no acceleration predominantly, what I want to mean here is that, there is no acceleration in any of the directions, so the fluid is in static condition.

So, if you if you equilibrium with the vertical forces and all, you will see that this force, force due to pressure and the force due to the mass or weight of the elemental strip, they should be same. Therefore, what will you get, P del A is equal to rho g del A y cancel out the repeating terms P is equal to rho g y, so this is the formula for the distribution of pressure in a static fluid; so that is P is equal to rho g into y. If you happen to analyse or if you happen to measure the depth of flow at any locations, or depth of fluid at any location in a static fluid, you will be able to easily measure what is the pressure at that location that is what by this formula suggest, P is equal to rho g into y, this pressure is called hydrostatic pressure this pressure is called hydrostatic pressure.

So, it irrespective of the planes whichever you are taking into consideration, from the free surface whatever is the depth y, at those locations the pressure will be P, so say at if this is location one then it will be P 1, if this is location two it will be P 2, like that you can easily analyse them. If I keep any plane here, now due to this hydrostatic pressure there will be a force in the downward direction that is hydrostatic pressure force. If I keep a plane like this, it will this hydrostatic pressure at this location, at this place it will exert a hydrostatic force in the direction normal to this particular plane, so that is how you need to analyse the problem.

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In a similar analogy, you can easily suggest now, say instead of static fluid if there is a horizontal parallel fluid flow, the term parallel fluid flow we want to mean here is that, the stream lines are parallel in this location that is what the meaning here suggests, so horizontal parallel fluid flow. If you take the same if you take the same element, elemental, now in this case the fluid fluid is flowing parallely, so let us assume that or let us consider that, if there is a parallel flow, it suggests that the uniform velocity distribution at various locations, they will be uniform that is what you want to suggest it here.

If that is the case, there is no change in velocity between two cross sections that is whatever average velocity you are taking at this section, that will be same as this section, that is what parallel fluid flow suggest. If you happen to take those things into consideration now, let us see here what happens, there is no acceleration in the horizontal direction, velocity is uniform, so if there is no acceleration, the net force in the horizontal direction that is also 0.

Now, there is no movement of liquid in the vertical direction, acceleration is 0 there, so the net force in the vertical direction that is also 0, so you are if the same analogy as you have taken for the static fluid, here again you equate the forces. You will see that your pressure distribution for horizontal parallel fluid flow, this also will be same as your hydrostatic pressure, this I do not want to derive simple, simple mathematics coming in to picture, same as the things here.

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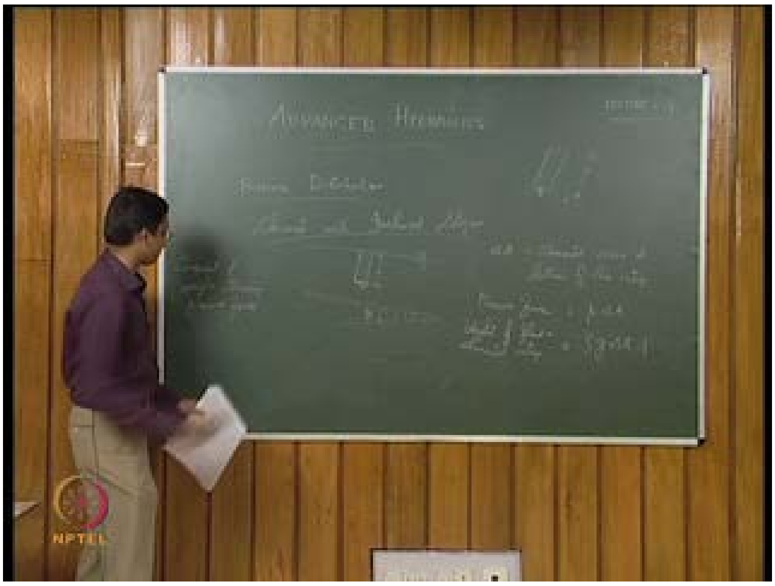

What happens, if there is inclined fluid flow, if your channel is inclined if your channel is inclined then what happens, here again the channel bed, we are suggesting that the channel bed it is inclined to the horizontal at an angle theta, I will mark it with a different colour. The channel has a free surface flow, how will you analyse the pressure distribution for such a situation, you have already seen for a static fluid, you have already seen for a horizontal parallel flow, how the pressure distribution is there.

Now, how will you determine the pressure distribution in inclined channels, again use the same analogy, here I am taking a small element small element, you see the element itself, this will be normal to the bed of the channel, so it has a area at the bottom d A at bottom of the strip. In this case, let us consider say the point P, the point P is at a perpendicular distance d from the free surface, the point P is at a vertical at a perpendicular distance d from the free surface.

If you take this in to consideration now, you again use the same concepts whichever you have done in the static fluid, and horizontal parallel fluid, we will just use the same thing, take the same strip now, so at the bottom pressure force, this will be P into del A instead of d A please correct this as del A, you are using let us be consistent with our symbols. So, this is the distance d, it is inclination of theta, in this case now let us consider this thing here, if you closely mark the thing here, this particular point P, we can balance the thing, weight of fluid in elemental strip, this is rho g del A into d.

So, now you cannot balance both the things, because weight of the fluid it is normal or it is perpendicular in the vertical direction, whereas your pressure force it is normal to the bed which already is inclined at an angle theta with the horizontal. So, you cannot directly balance it off, you need to take component of appropriately, the component of pressure force, component of weight balancing the pressure force; of course, while writing this thing, you have to suggest that, you are still taking even in the inclined position the flow, there is no particular acceleration in any direction, whether it is normal to the bed or horizontal or parallel to the bed, there is no acceleration.

Based on those situations, we are now taking this thing into account, so weight of fluid in the elemental strip rho g del A d is there. Now, component of weight in the direction component of weight in the direction normal to the bed, so this is nothing but rho g del A d you know that into cos theta, so your p delta a should be equal to rho g del A d cos theta in this case.

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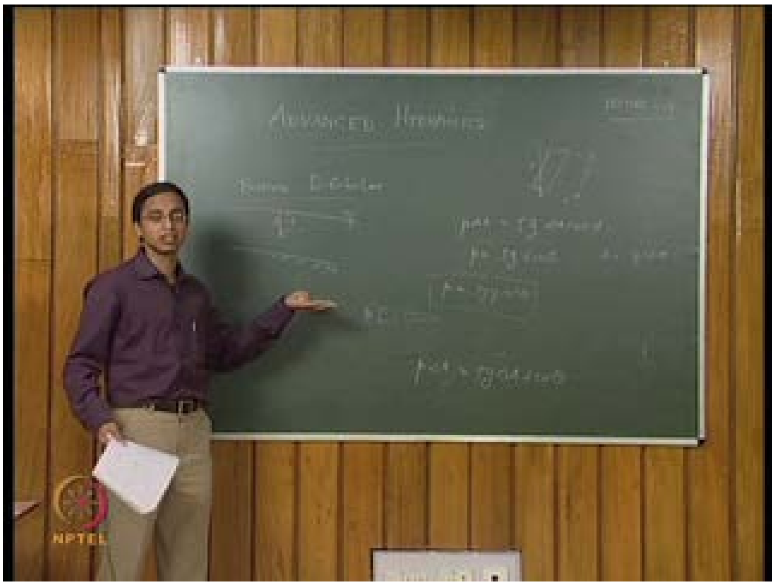

So, let me repeat here, P del A is equal to rho g del A cos theta into d cancel off the terms you get, P is equal to rho g d cos theta, so this is the pressure distribution now in a inclined plane. But if you see for any inclined planes as an scientist, or as an engineer you will be more interested say from any location on the free surface, you are more interested in the vertical distance, the vertical distance the vertical distance to those planes. Say in this case, the point P is inside the water, and this is the location of free surface right, now this point P, how much far it is from the free surface how much far it is from the free surface in the vertical direction, if you happen to mark this like this.

Now, you are getting a better picture, this point P is this much distance from the free surface of water, let me mark that quantity as y, then you can now easily suggest that your quantity d is nothing but y into cos theta, is it not, it your depth your depth of flow. I again redraw then, see in any point P, from the free surface the normal distance to the point, from the free surface that is d, but the vertical distance from the vertical distance from this point P to the surface, this is y, and y and d can be now easily related as d is equal to y cos theta you substitute that thing here, you will get P is equal to rho g y cos square theta. So, this is the general relationship you use for finding pressure distribution in an inclined channel.

y is the depth of flow to any concerned point, it is the depth of flow to any concerned, point even one can easily find now, if your channel happens to be a mild slope channel or if the slope is considerably negligible or if it is negligible, if it is too means that slope if it is not significant, you will see that your cos square theta is approximate to 1. Then again your in those situation, you will again get your hydrostatic pressure distribution in the channel fine, this is how you measure the pressure distribution in channels.

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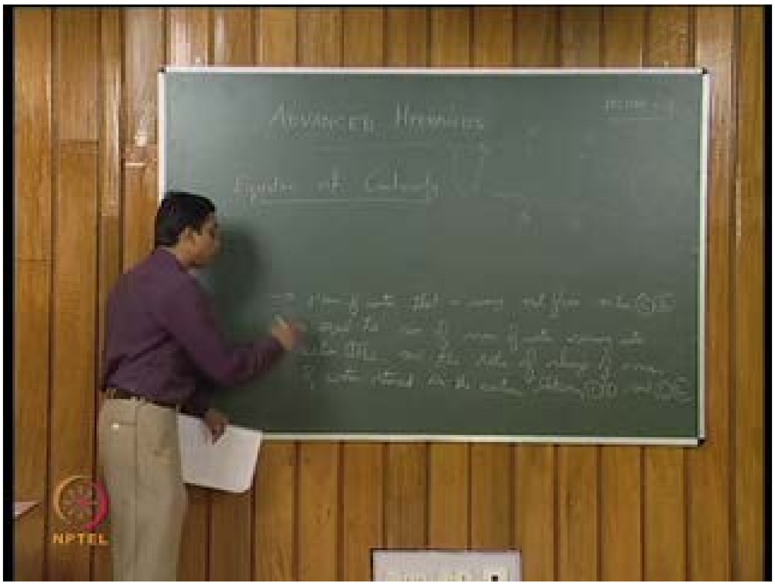

So, we will stop the pressure distribution here, next topic is equation of continuity, so this as any physics student is interested, who is very much by hearting many of the physics thing, they will be readily able to raise their hand and say the definition of these methods and all. Equation of continuity means nothing but it depends it is the satisfaction of flow of conservation of mass, and in our case the mass of water is the quantity that is getting conserved in a channel, just take a stretch of the channel, consider two cross sections 1 1, 2 2.

So, let the cross section be here, and let the cross section at this location be, so now according to our channel hydraulics, the conservation of mass, what it is meaning is that if the fluid is flowing in this direction? Then the amount of water that is the mass of water, that is coming out from section 2 2 is nothing but the summation of the amount of water that is coming from in to the section 1 1 here, plus the rate of change of mass that is mass of water that is stored within this section 1 1 and 2 2. That is what the definition of continuity is, or that is what you mean by conservation of mass.

Let me write it here, it is nothing but the amount of instead of amount let me write it, mass of water that is coming out from section 2 2 is equal to sum of mass of water coming into section 1 1, and the rate of change of mass of water stored in stored in the sections between 1 1 and 2 2 fine. So, you can use this definitions, as the definition suggest, there is a particular quantity called the rate of change of mass of water stored in the section. So, how much amount of water that is stored here between sections 1 1 and 2 2 in the channel, how much amount of water is getting changed with respect to time that is also a dependent quantity; now in a simple case, wherever you are dealing with steady state conditions, this particular term gets vanishes off in your steady state conditions.

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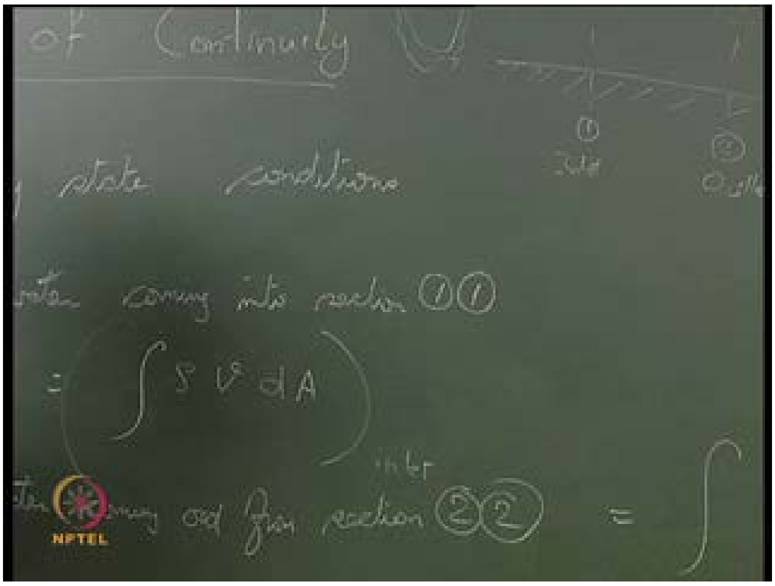

You will see that mass of water that is coming in, or mass of water that is coming out from section 2 2, is same as mass of water that is coming into the section 1 1, so we will be proceeding with the same analogy here. I can write mass of water say in this particular case, you just take any elemental strip at depth y from the free surface, and here also you can take of course, both the y is are different, this is y at the section 1 1 that you can define it as inlet.

And this section 2 2 you can define it as outlet, so this is the depth of flow or depth at a concerned point at outlet, and here this is at inlet, using this you can just integrate to find the entire area, and the mass coming out we can just suggest that the mass of water coming into section 1 1. This I can write for any arbitrary cross section, integral of rho v d A in the inlet or 1 1 like that also you can write, mass of water coming out from section 2 2. This again you can integrate the things, why this integral sign is given you is that you are taking arbitrary elemental area d A, for the elemental strip, you are integrating the d A for the entire cross section now, for that reason we are just given the integral simple.

So, let us proceed now, so rho you can equate both the quantities, rho v in d A in is equal to rho v out d A out, or you can as we are dealing with incompressible flow, you can cancel off rho, if you are taking average velocity for the cross sections you can suggest that v bar in into area of inlet, this is equal to v bar out area of outlet fine. This particular product, this is called discharge, so the discharge at inlet is equal to discharge at outlet for steady state fluid flow fine, so you can just compute it appropriately. Now, let us we will continue with the equation of continuity for this unsteady cases and all in our next class, let me stop it here for today's lecture, before that there will be a mini quiz.

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The first question is find alpha and beta for triangular cross sections depth of flow is given as y naught, and for any arbitrary thing the depth will be y, your velocity is here half K y to the power of half, so you determine alpha and beta. So, you will be given 1 minute time for solving this thing, and I hope it is quite easy now, you have already done it for rectangular cross-section, worked it out for rectangular cross-section, you determine it for the triangular cross-section.

The next question is suppose a steep rectangular channel is there, steep rectangular channel slope equal to 30 degrees with the horizontal, it has the slope is 30 degrees with horizontal. So, at a section the depth of flow is 0.7 meter, at a particular section the depth of flow is in the rectangular section, at a particular location the depth of flow is 0.7 meter, velocity average velocity was observed to be 3 meter per second. Now, your question is, find the pressure at the channel bed at this section, find the pressure at this location channel bed, this in 30 seconds you can compute right. So, any how I am not going to tell the solutions of this quiz right now here, it is up to you, you work it out if you can do it right now it is better, or submit it in the next class.