**Advanced Hydraulics**
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**Module - 4**
**Hydraulic Jumps**
**Lecture - 8**
**Surges (2)**

Welcome back to our lecture series on advanced hydraulics. We are in the fourth module on hydraulic jump.

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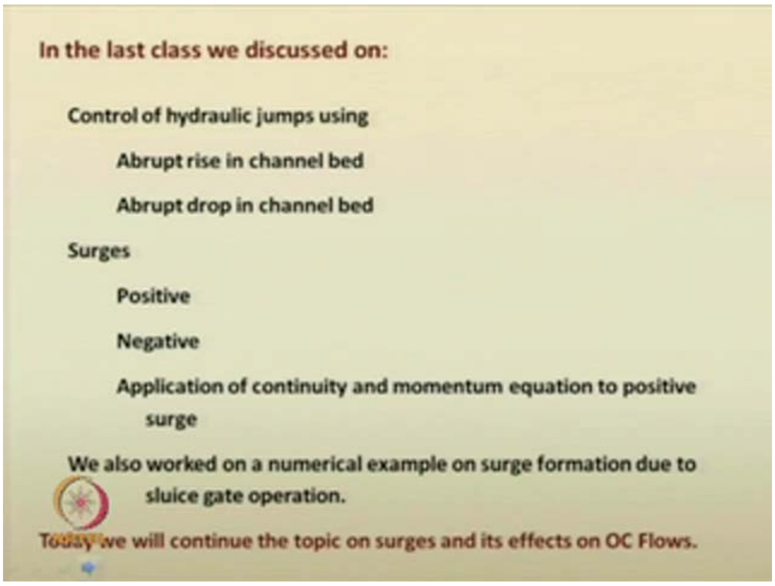


In the last class, we had discussed how the control machines are used in hydraulic jumps using abrupt rise in channel bed or abrupt drop in channel bed. We also discussed surges what is meant by surge, the positive surge, negative surge. Also we have applied the continuity and momentum equation to analyze the positive surge. We also worked on a numerical example on surge formation due to the sluice gate operation. So, today we will continue the topic on surges and probably this by today's lecture, we would like to wind up this module as well. We will discuss on surges and its effects on open channel flows. So, surges the second part of this lecture.

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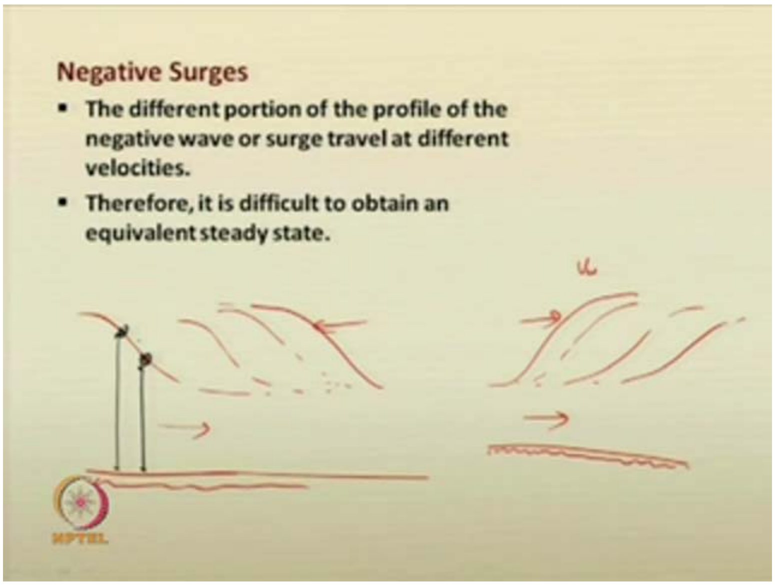


We discussed the positive surge in the last class. The negative surge, what is meant by negative surge? The different portion of the profile of the negative way or surge travel at a different velocity; negative surge means, the surge that propagates in the in any direction. Or it is negative surge is called the surge whenever that surge is propagated the depth of flow decreases. For example, if you have say, a negative surge may propagate in the upstream direction like this. So, this is the surge propagating like this. So, you have the channel bed and the wave is propagating from downstream to upstream although the flow direction is in this from left to right.

Or you have a negative surge in the same way like this also. That is from upstream it will be propagating downstream, but the depth of flow will be decreasing. See, after sometimes it goes like this. So, this is also a negative surge. So, the surge velocity is V w and the flow direction is also like this. So, what is the different means why you have to analyze differently the negative surge compared to the positive surge. In positive surge you have seen that most of the time or almost the front, the front of the wave is vertical. So, you were able to readily apply the equivalent steady state condition to the entire range of that wave.

That was quite possible. But here in the negative wave what is happening is that the increase or the profile is it is not a vertical front. So, it the velocity of the wave for example, here in this location the velocity wave as well as in this location the velocity of the wave they are quite different. So, based on the flow depth, the velocity of wave will also change. So, you will not get a constant velocity for the wave. It is quite difficult to use the equivalent steady state condition for such type of problems. So, what will we do?

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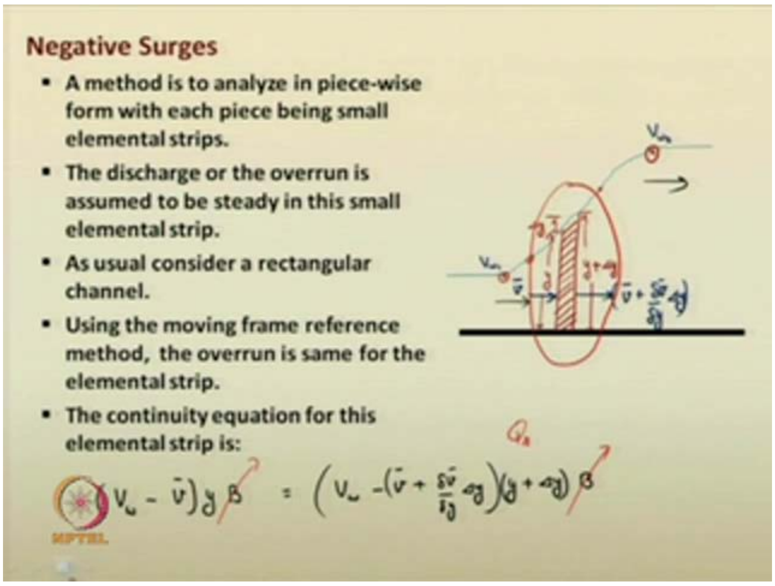


A method to analyze the negative surge or negative wave is you have to analyze it in a piece wise form. You can analyze it in a piece wise form with each piece being small elemental strips. So, if I am taking say this is a surge moving from upstream to downstream. If you have this conditions, so I will just change the colour to for your better visualization. I am taking a small elemental strip from the wave. So, you see the entire reach of the wave is from this location to this location. As you have seen previously seen for hydraulic jumps and all; so in this case the velocity of wave at this location as well as at this location at this location or at this location what will be, it will be quite different.

So, we cannot use V w as a constant for the entire wave reach. So, what we are doing is that we are taking a small elemental strip like this from the wave from the wave portion. Not only that including the water column that encloses the wave and all, we are taking the small elemental stream. Now, we are suggesting that on the left side here the wave is propagating from upstream to downstream. So, this negative surge is moving downstream. So, let this be the y and here the depth be y plus delta y. That means this additional depth is delta y. If that is the case let us consider in this particular situation the velocity.

So, I will change the colour. So, the velocity here and here they are quite different. So, the velocity of water here, let us assume at at depth y for the depth of flow y let the velocity be v and the average velocity of that cross section be v bar. And here we are considering v plus some change in velocity due to the change in depth of flow. So, we are incorporating it in this way. This is mathematically possible, that is del v by del y into del delta y. so, like that if you do that now here we are considering the beginning of the wave. Wave velocity is V w 1 and here the wave velocity is V w 2. So, the discharge or now when if you take this small elemental strip, which ever we are taking the discharge or the overrun is now assumed to be steady in the small elemental strip.

So, we are considering the same rectangular channel for our analysis. We are using the moving frame reference method. So, as you know in the moving frame reference method you have already studied the overrun or discharge Q r. so, the overrun from this elemental strip that will be constant. So, you can now use the continuity equation in the elemental strip in steady state condition. You can use the continuity equation. So, that can be given as follows. So, we are using the continuity equation if you remember for the positive surge. We have used the we have considered the overrun as constant in both the reaches, means within that reach the overrun is constant. For that you are taking the relative velocity based on the moving frame refer moving frame reference and all.

You are using the relative velocity. So, on the left side we can have V w minus v bar into y into the breadth of the rectangular channel, that is discharge should be same. That is the meaning related discharge based on the moving frame reference should be same, that is the meaning here. We are considering for that elemental strip it is steady. So, in that situation I can write the conditions like this. Now, on the right side we have v bar plus del v bar by del y into delta y into y plus delta y into the breadth of the rectangular channel. So, this will be your continuity equation. So, you just continue, I can write it further now same elemental strip y, y plus delta y.

So, in the same elemental strip, just rearrange the terms here. We can see that V w minus v bar into y, this b b can be cancelled off. So, we have not incorporated density here because we are considering incompressible flow. That is, for that reason the density is not incorporated here. That gets cancelled off automatically.

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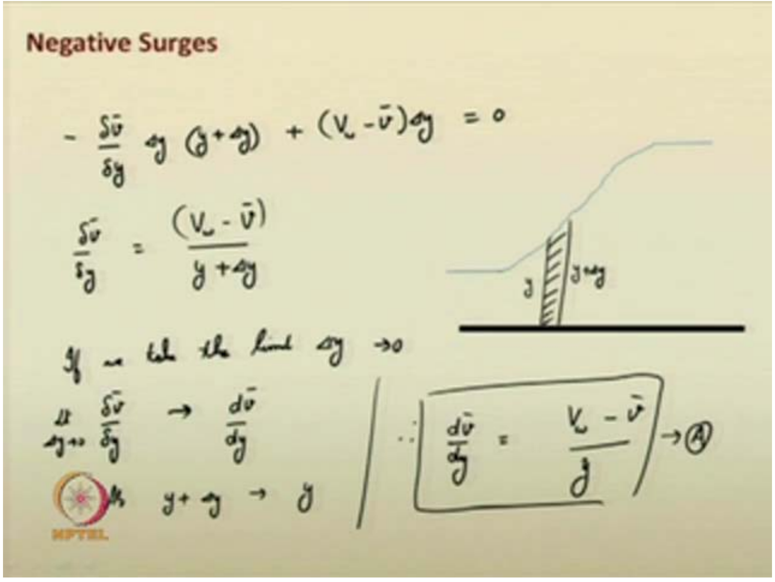


Now, you will see the subsequent simplification of the equation, means reduce those thing and I am getting the following form now; minus del v bar by delta y into delta y y plus delta y plus V w minus v bar into delta y is equal to 0. I am getting the equation like this. You can just see that you can again go through the the quantities get cancelled off and I can write it in the following form. Or you will see del delta y is nothing but equal to V w minus v bar into y plus delta y. So, if we take the limit delta y tends to 0, if delta y tends to 0 so then this becomes a more standard form. So, if you take the elemental strip by suggesting that the delta y tends to 0 then limit delta y tends to 0, the change in velocity due to change in depth delta v bar by delta y this tends to the differential d v by d y. Also y plus delta y tends to y itself.

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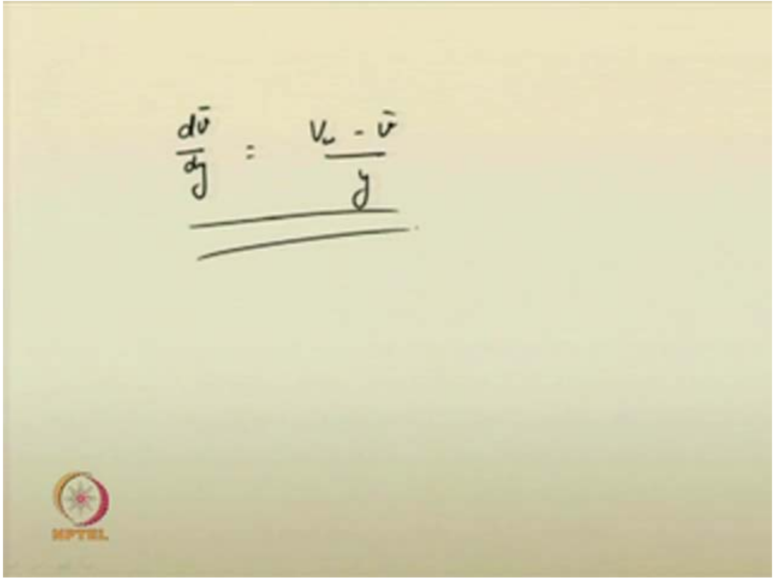

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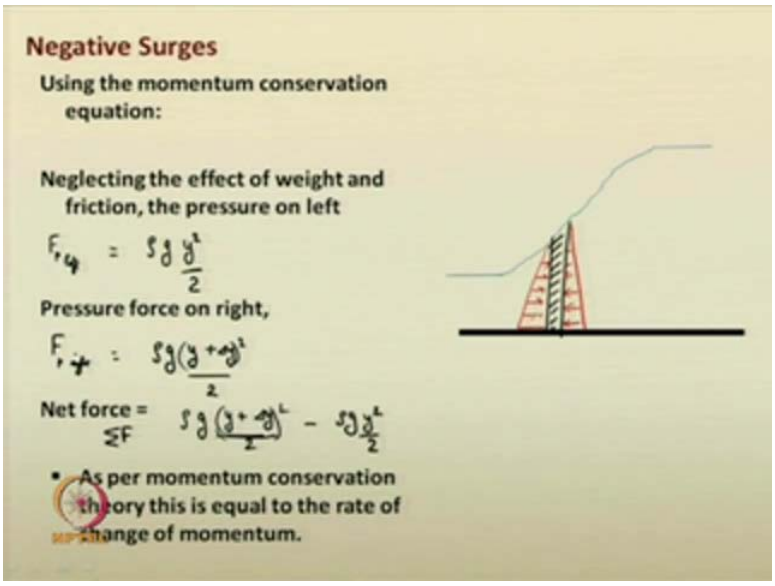


So therefore, I will get the corresponding relationship now. d v bar d y is equal to V w minus v bar by y. so, this one equation I am getting. You can if we if you want you can name it as equation A. So, you as we suggested as we worked it out d v by d y is equal to V w minus v bar by y we are getting it from the continuity equation. Similarly, use the momentum equation for the same strip for the same elemental strip here this is y this is y plus delta y you. I hope you remember the momentum conservation equation. According to the momentum conservation equation, it suggest that the net rate of change of momentum in a control volume is nothing but equal to the net force acting on that control volume.

So, the rate of change of momentum in that control volume is equal to the net force acting. So, we are neglecting some of the forces. That is effect of weight and effect of friction and all. So, you can have now the following quantities. Let me rub this. Let us, you can have on this elemental strip now the pressure force on the left side acting like this and on the right side acting like this. So, you can have pressure force. So, the net force acting on this elemental strip is nothing but the difference in the pressure force. So, the pressure force on the left how can I write it F p left.

So, the pressure force on the left is nothing but it is a rectangular channel. So, rho g y squared by 2, because you know that y by 2 is the depth from the surface of the water to the centroid of that area. So, pressure force on the right, what it will be, rho g y plus delta y whole square by 2. So, you will get the following quantities. Net force ,we are suggesting the net force as the frame is relative where we suggest the frame also moves along with the velocity of that elemental stream in that. And the velocity of wave in that elemental strip is V w. So, the frame is moving in that same direction. So, we are suggesting that the net force is more in the, more acting towards the left side towards the left side that is opposite to the flow direction.

So, net force acting is nothing but rho g y plus del y by 2 whole square minus rho g y squared by 2. So, then this is the net force sigma F, I can write it like this sigma f acting on this elemental strip. Now, this net force should be equal to the rate of change of momentum. For the rate of change of momentum you have conditions that the rate of change of momentum stored inside that elemental strip plus the net out flux of momentum through the control surfaces as per your Reynold’s transport theorem you remember that. So, we are using the same theory here. Now, for the elemental strip we have already considered the conditions are steady. So, you do not have the change in momentum stored inside that elemental strip.

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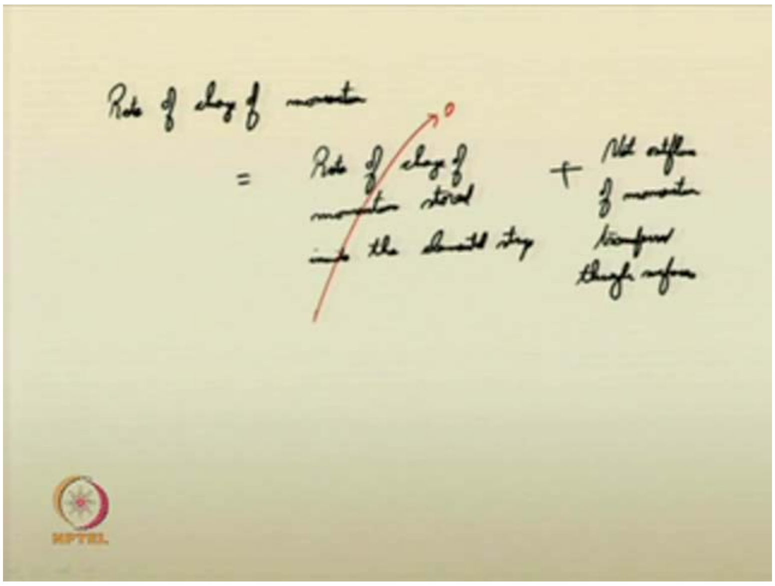

So, there will be change in momentum only due to the out flux of momentum through the control surfaces of that elemental strip. So we can write this as per the momentum conservation theory, this is equal to the rate of change of momentum. So, I can write as rate of change of momentum is equal to rate of change of momentum stored inside the elemental strip plus the net out flux of momentum transfer through surfaces through, control surfaces of that elemental strip. So, due to steady state condition this quantity is 0 or we are not taking into consideration there.

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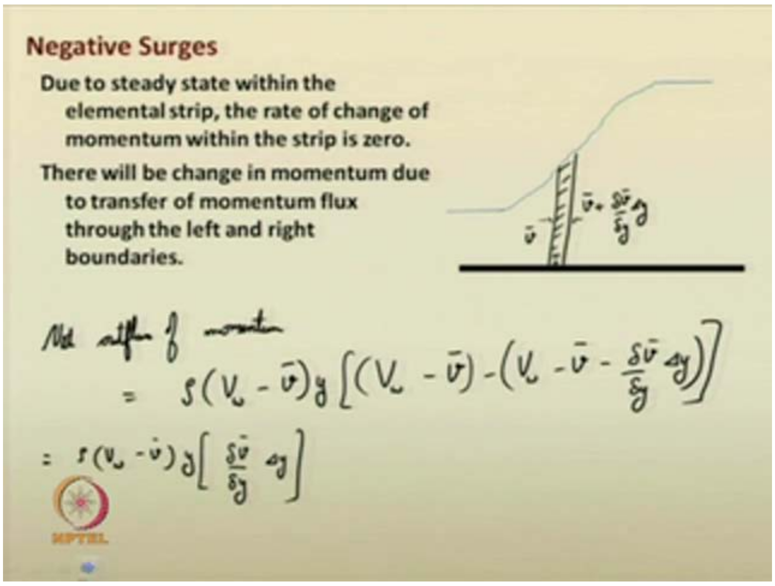

So, we can write due to the steady state within the elemental strip the rate of change of momentum is or the strip is. Therefore, in that elemental strip whatever momentums are transferred through the left and right boundaries because these are predominantly one-dimensional flow. So, only the left and right boundaries allow transfer of momentum. So, the momentum flux through the left and right boundaries that need to be taken into account. So, the net out flux of momentum, I am not going to further elaborate the Reynold's transport theorem. I am just writing the final form for this particular situation. This is nothing but rho V w minus v bar into depth y into V w minus v bar minus V w minus v bar minus delta v by, sorry del v by del y into delta y.

This is how I can write. You know the velocity at this location is v bar plus delta v bar by del into del y here this is v bar, you know those things. So, again you are using the relative frame reference or the moving frame reference, you are using them. So, you are getting the momentum equation in the following form. Cancel the terms, rearrange the term, what can you get? What will you get? So, I can write the or may be here itself I will just rewrite it. So, you will get rho V w minus v bar into y, this is plus delta v delta y into delta y. Like this you will get, V w minus v bar it is getting cancelled of isn't it. So, that I am just simplified it here. This net out flux of momentum is equal to the net force in the system.

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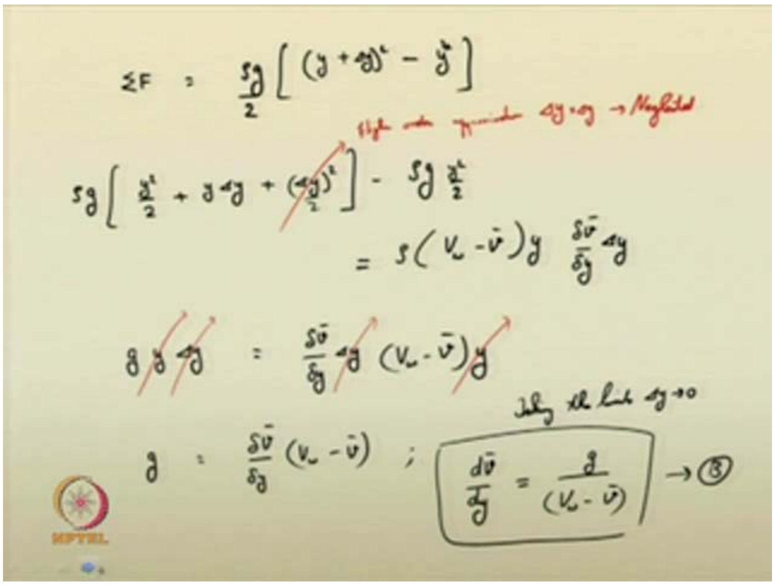

So, the net force acting in the system sigma F, I hope you recall them. This is nothing but rho g times y plus del y whole squared minus y squared. This you have already found it out. So, the same thing you have to equate it now. So, we will get the following quantity now. Equate it with the net out flux of momentum, so I will get this following thing. Rho g y squared by 2 plus y del y plus del y whole square minus rho g y squared by 2 is nothing but equal to rho times V w minus v bar into y into del v bar by del y into del y. So, you can see that y squared by 2 y square y, this 2 we have already taken it inside that is the reason.

So, those things are getting cancelled off. Even this is by 2 but any how we are neglecting this quantity. Due to second higher order this thing, we are neglecting del y's whole squared quantities. We are neglecting higher order, higher order approximation del y into del y neglected. So, I can write the equation now in the following form. What is this? This is nothing but g into y del y, g into y del y is equal to del v bar by del y del into delta y V w minus v bar into y. so, again we can cancel some of the quantities. You are able to see that this y here this y here can be cancelled of. This delta y this delta y can be cancelled off.

So, I can write this quantity now, g is equal to delta v bar by delta y V w minus v bar. So, this again rearrange the thing taking the limits del y tends to 0. So, if you take the limits del v bar by del y will become d v bar by d y. So, I am getting the following relationship now. d v bar by d y is nothing but equal to g by V w minus v bar. So, I am getting another equation in the following form which I would like to give it as B. So, using momentum equation as well as the continuity equation you got the expression for d v bar by d y. You can combine them, you can use both of them to so where velocity negative surge.

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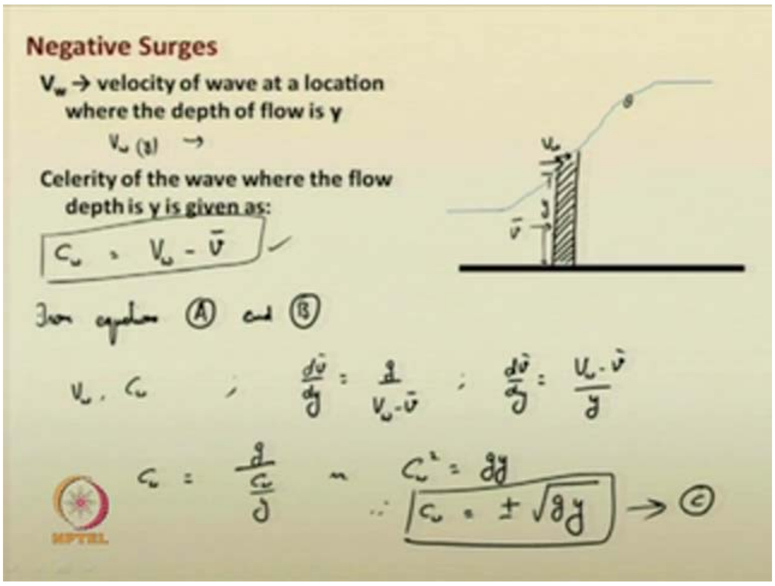


Now, V w is the velocity of wave at a location where the depth of flow is y. Celerity of the flow if you recall them, if you recall celerity of the wave, we have described it yesterday also. The celerity of the wave, where the flow depth if any flow depth is given to you what should be the celerity of the wave there. So, it depends on the depth y. Similarly, the velocity of wave V w also depends on the depth y. So, for the same elemental strip, for the same elemental strip we have seen equation A, this is equation A and equation B. So, from equations A and B what will we get? What can be infer from equations A and B?

As we suggested V w, now this v for this elemental strip the velocity of the wave is V w here. At this location the wave velocity will be different. So, this is having the depth of flow is y here. So, for that at any location where the depth of flow is y the wave velocity is V w, we are now suggesting it. So, it means that V w is function of y in this case, that is the interpretation. Celerity of the wave C w this was interpreted with respect to the moving frame reference. So, that is the velocity of the wave with respect to moving frame reference, that is the or with respect to the velocity of water.

What is the velocity of wave? That is the meaning here. So, that was given yesterday as V w minus the velocity of wave minus the velocity of water at that location. So, here the velocity of water is v. so, celerity is given as like this. So, both using V w and C w and also from equations A and B, say I will just work it out for your benefit. d v by d y is nothing but g by V w minus v bar. Similarly, d v bar by d y is nothing but equal to V w minus v bar by y. Use both the relationship, you can just take it to this side. V w minus v bar is nothing but your celerity. So, you will get celerity now. C w is nothing but equal to say g by v, I will write it in terms of celerity C w by y or C w squared is equal to g y.

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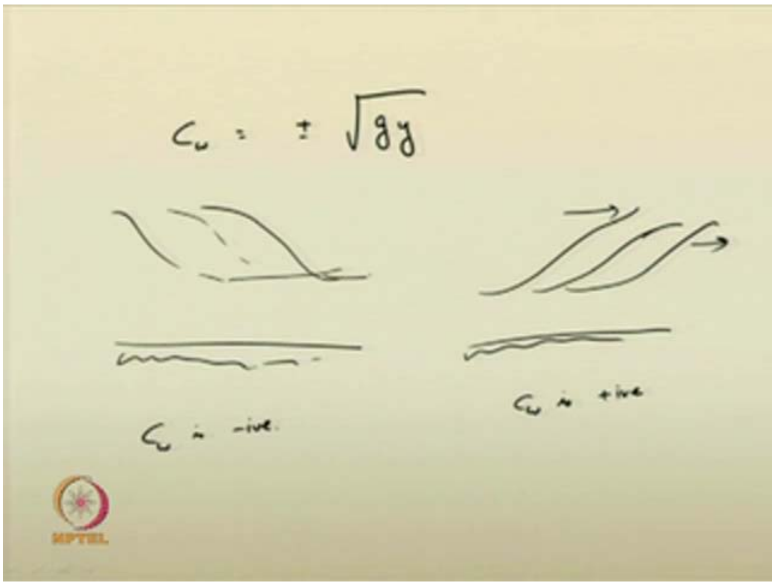

Therefore, C w celerity is nothing but equal to plus or minus root of g y. so, for a negative surge this is the standard representation of celerity or this is how you derive celerity, so for a negative surge. So, please remember it is only for negative surge. So, I can write it again here plus or minus root of g y. So, if you have a negative wave propagating upstream then C w is negative. If you have a negative wave propagating downstream like this. So, here C w is positive, here C w is negative. So, that you need to always remember.

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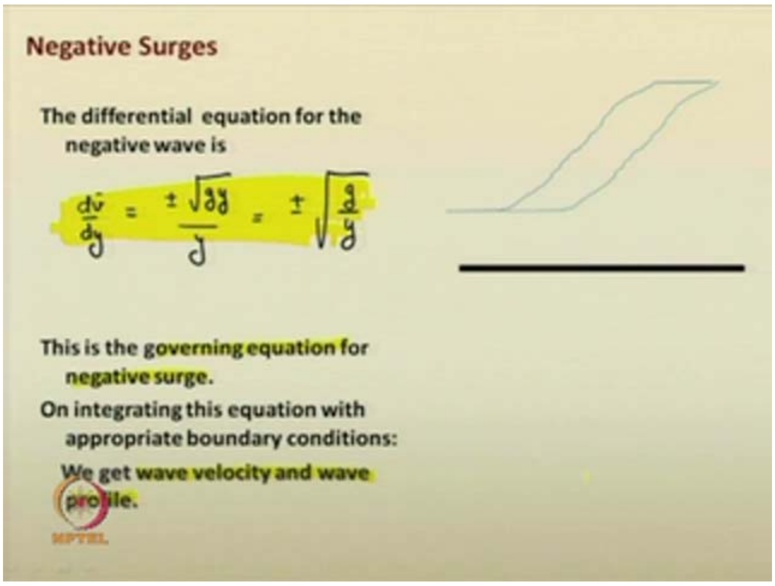


From the equation for celerity of the wave and all we can obtain a differential equation for the negative surge. So, that we will the using those differential equations and all. You will be able to interpret the physical phenomenon in a much better way. And also by solving that differential equation you will be getting the velocity of the flow as well as the profile of the wave and all at any instant. So, for that reason we are just developing the differential equation. You can see now, d v by d y, this is nothing but equal to celerity right. So, celerity which is root of g y by y or this is equal to plus or minus root of g by y.

So, this is your differential equation. So, this is the governing equation for negative surge. For a negative surge you can use this particular governing equation. So, if you use this equation solve them. So, this is fully diffrentical equation, it is a differential equation. On solving you will see that v is dependent with respect to y and solving you will get the velocity v at any depth y. Or at any flow depth where the flow depth is y at any that any location you will get that particular velocity, that is the meaning. So, we get wave velocity and wave profile by solving this particular differential equation. How do you solve this thing, how do you solve differential equation?

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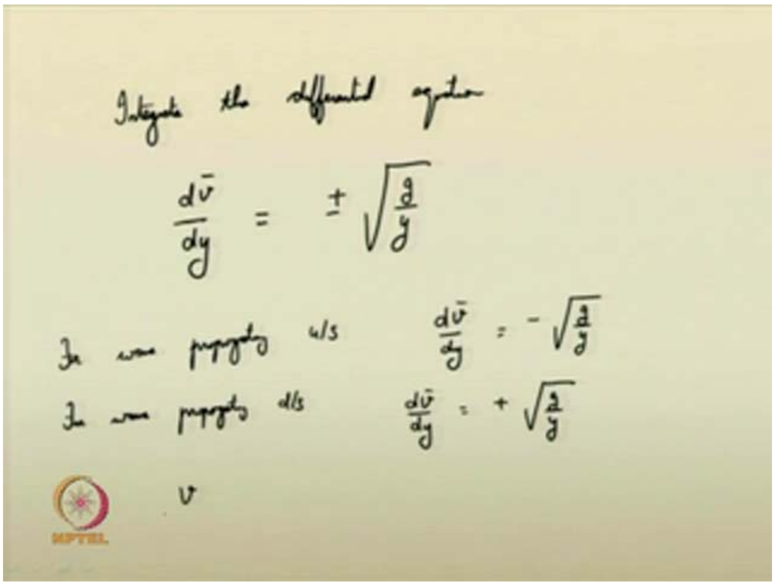

So, just integrate the differential equation. Integrate the differential equation, so your differntial equation is d v bar by d y is equal to plus or minus root of g by y. So, as we have suggested, whether if the wave celerity if the wave celerity is negative or if the wave celerity is positive based on that you have to incorporate plus or minus sign here in the equation. So, if the wave is propagating upstream you have to use negative sign here. For wave propagating upstream your differential equation will be d v by d y is equal to minus root of g by y. For wave propagating downstream your differntial equation will be d v by d y is equal to plus root of g by y. So, that for that reason you using these differential equations.

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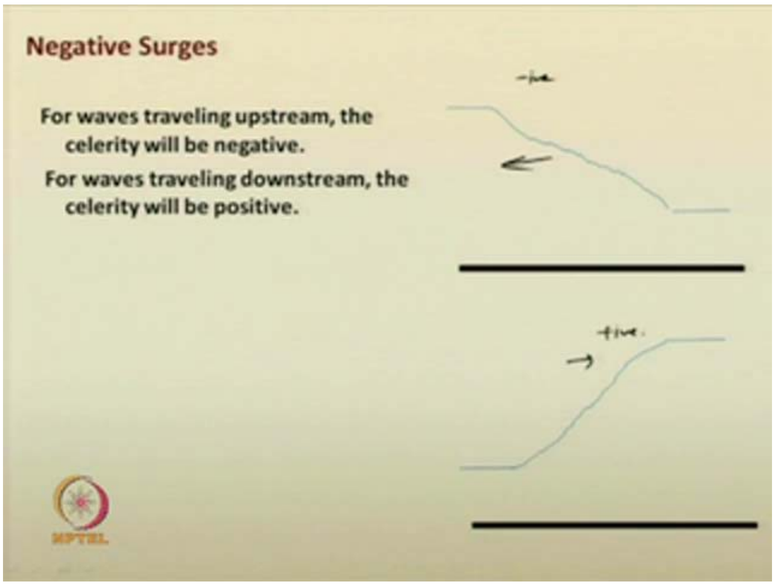


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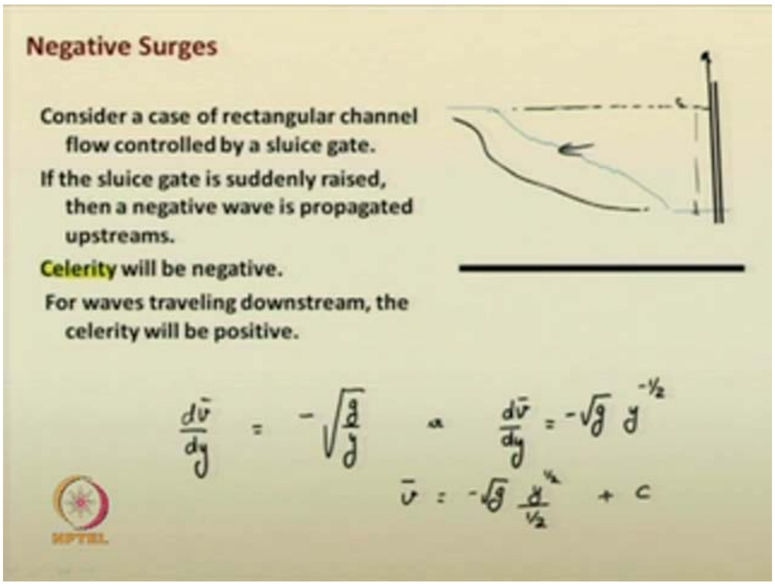


You can integrate this differntial equation to obtain v in the entire domain or entire domain of that wave. So, for waves travelling upstream we suggested that this celerity will be negative. For waves travelling downstream the celerity will be positive. Consider a rectangular, just consider a case where a sluice gate is used to control flow in a rectangular channel. So, this sluice gate is suddenly raised upwards like this. So, what happens if the gate is raised like this? The original depth, suppose if this was the original depth of flow, it gets dipped all of a sudden like this. And due to that a wave is formed in the water that gets propagated downstream like this.

When you raise this sluice gate the original depth which was like this much, it gets dipped here. And this wave will be propagated to the upstream. Then this, how would you analyze this propogation of this negative wave? So. you know the celerity is negative for this particular case. So, for waves travelling downstream you the celerity is positive. So, we can now write the following contiunity equation or your differential equation itself. We have already taught you the differential equation. So, d v bar d y is nothing but in this case it is equal to minus root of g by y or, for my convenience I am writing as g is a constant value.

I am writing minus root g into y to the power of minus half. So, integrating for such a problem, if you integrate the quantities on integration I will get v is equal to minus root g y to the power of half by half plus some integration constant c. So, this is how you integrate it but we have not actually reached to a conclusition. We do not know the constant of integration and all. So, for such a situation again we can eveluate it properly, how? I am just elaborating the cross section now.

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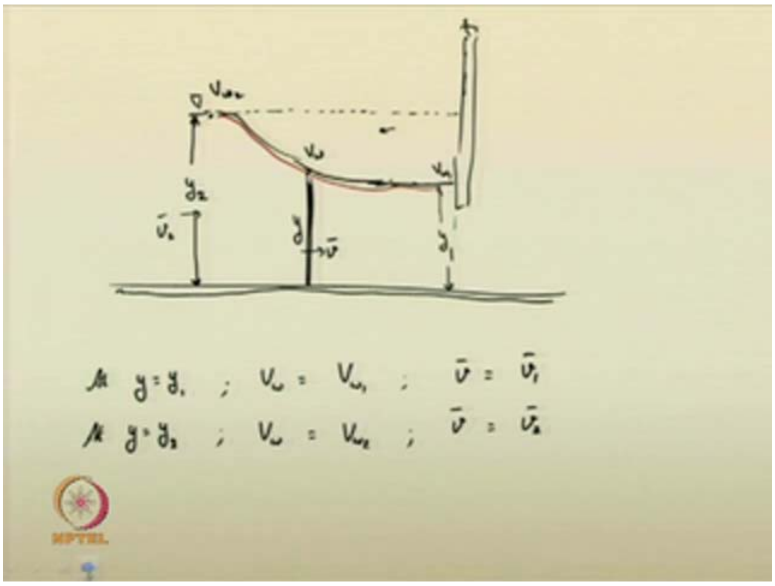

As the sluice gate is, this was theoriginal depth of flow. This is the original depth of flow in the channel. So, the sluice gate is raised like this. When it was raised the water dipped to this location and the negative surge is propagated after a few second. This negative surge reached here like that it goes on. This is how the propagation of surge occurs. So, in this case just take as we are referring any moving this thing.

So, just take a negative surge in such a way that the upstream end it is having the same depth before the raising of the bar. So, that y 2 it is already now a known value to you. y 2 this depth is named as y 2 and it is now a known value to you. So, when this thing it is here and here in the downstream, let us give beginning of the wave let us give this depth as y 1.

So, when you asign these values the profile of the wave is now from y 1 to y 2. Your profile of the wave is from y 1 to y 2 like this. This is your profile, draw the I can device a profile now using the same differential equation. Your, you have already integrated them but still you know at y is equal to y 1 you have V w is equal to V w 1 and also v is equal to v 1 bar at this location. At y is equal to y 2 we have V w is equal to V w 2 and v bar is equal to v 2 bar. That was a known quantity to you v 2 bar. It should be a known quantity because this that v 2 was the thing that was existing before the raising of the sluice gate.

I will be integrating from the known position. That is y 2 is a known position or the depth is. So, I would like to integrate it to any unknown region. Say here at this depth this is depth y. I want to find velocity of wave also there. So, V w it is an unknown quantity here here. This is V w V w 2 V w 1. So, using these boundary conditions I am just going to solve it solve the equation in the following form. d v bar d y is equal to minus root g y to the power of minus half integrating from v 2 bar to v. That is I required, I would like to find the velocity v at this particular section which is unknown to me.

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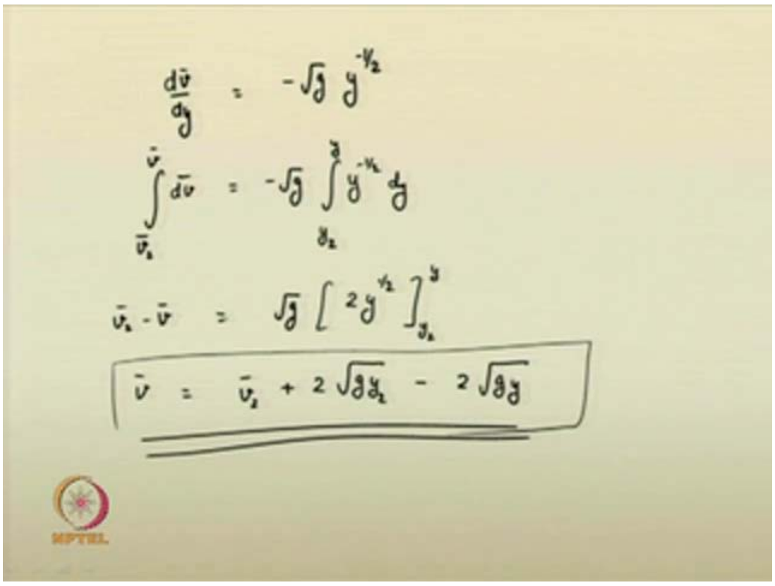

So, here v 2 is unknown quantity to you. So, based on that boundary I am integrating the thing d v bar is nothing but equal to root of g integral y 2 to y y to the power of minus half d y. So, this intergral you know this is nothing but v 2 minus v bar, it is equal to root g twice y to the power of half within the limits y 2 to y. So, this negative quantity is existing here. You can re arrange it you can put it here means you can then reallot the range from y to y 2. And I will get the velocity profile at any section or the velocity at any cross section v is equal to v 2 that is a known quantity to you, plus twice root g times y 2 minus twice root g times y.

So, this is how I am getting the velocity profile, or sorry velocity at any cross section. So, when you range the y means, you know the y ranges from y 1 to y 2. y 1 is that is same up to what height the sluice gate is raised. So, based on that y 1 is avaibale to you. So, from that portion or you can interprit once it reaches a condition y 1 the beginning of the wave arises. So, you cannot say it is the just simply as the height of the sluice gate. It is at which the height the waves starts propagating. So, that is y 1, so within those limits you will be able to. Any how y 2 is a known quantity based on that v can be evaluated. From this solution what we can infer?

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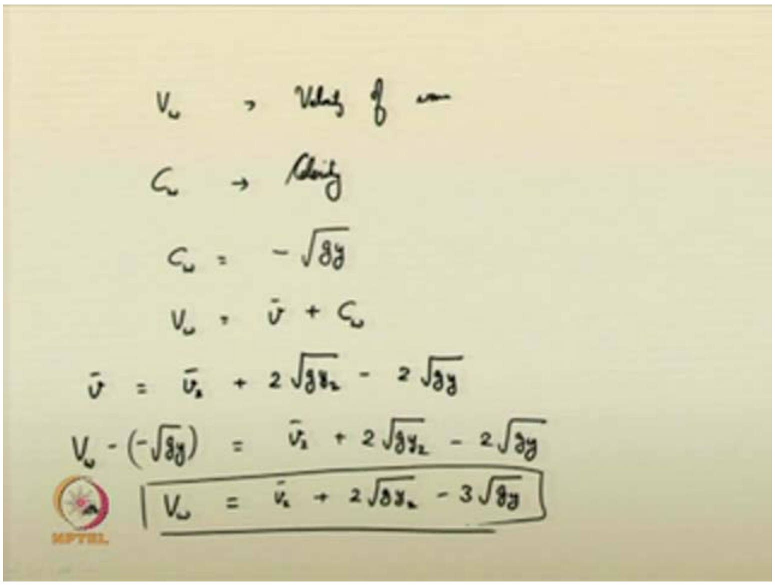


V w velocity of wave whereas, C w celerity. So, C w I can get you know that this is a negative surge. That is this negative search is propagating to the upstream. So, celerity is negative. So, I have C w is equal to minus root g y. And also V w is equal to v bar plus C w, this is also you are quite aware. The velocity of wave at any section is equal to the velocity of water there plus celerity. Just substitute this quantity in the previous equation. V, previously derived equation v bar is equal to v 2 plus twice root g y 2 minus 2 root g y. So, here v bar we are substituting it by V w and C w. So, that is v bar is nothing but V w minus C w.

So, I am just substituting this quantity here V w minus C w is nothing but root of g y is equal to v 2 plus twice or the velocity of wave at any section. That is the important thing which we require here. That is nothing but, equal to v 2 plus twice root g y 2 minus thrice root g y. So, like this I can derive the expression for velocity of wave at any section. Why these things are studied, why negative surge is surge is studied? You might have heard about lot of issues like dam break problems and all, dam break problems.

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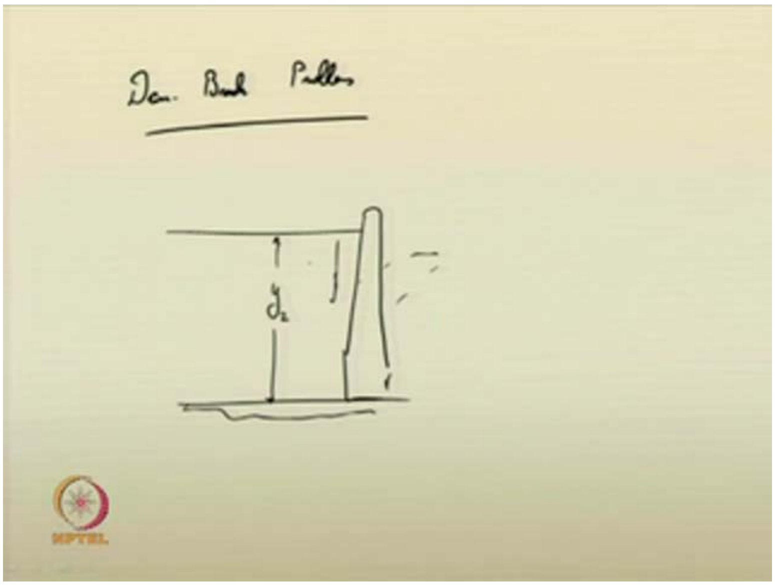

So, whatever dam break problems you have studied or what means whatever dam break problems occur. Here say if there is a dam like this and that is impounding a water. Say of height known height y 2. All of a sudden if this dam collapses like this, if it collapses this water will go down. So, negative surge will be propagated to the upstream of the reservoir, to the upstream of the reservoir the negative surge will be propagated as well as a gushing of water occurs into the downstream. So, how this negative wave will affect the upstream of the reservoir due to dam break problem and all that can be analysed using this negative surge analysis. So, you know the known depth y 2 here.

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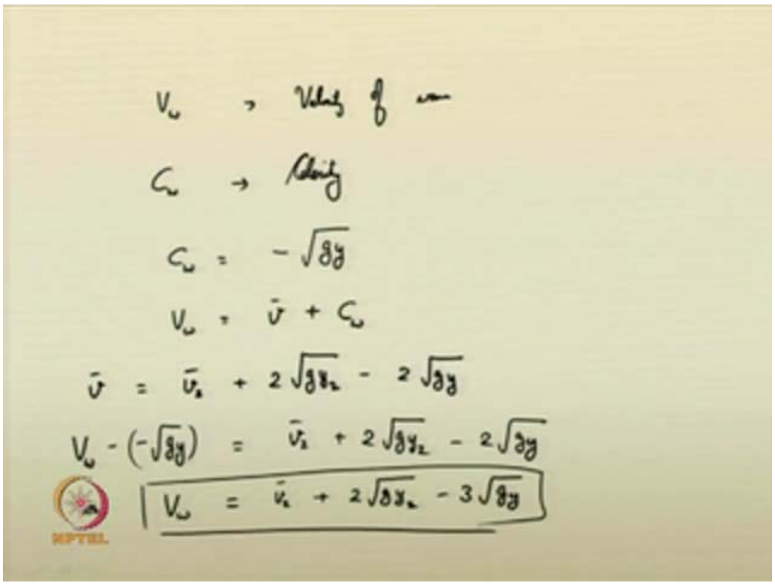

So, how severe is the wave or how severe is the wave velocity. We have already studied the wave velocity V w. So, this using this quantity you can now learn means you can infer due to the break of dam how fast the wave is being propagated in the upstream of the reservoir. So, that can hamper the ecological systems there or it can hamper many of the monuments or lot of issues are there. So, these things you can analyze. So, it is a quite interesting challenging problem,s dam break problem.

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We often see surges in power canals, surge in navigation canals. We also see surge in transitions where the channels get trance tranced from. Either the bed is increased, bed level is increased or the width of the channel is dicreased or increased. So, such transition in channels canal how the surge surges are encountered, that also is being studied, surges in channel junctions are also studied. Now, all these things we were interested to take in detail. But as almost eight lectures are conducted for this particular module itself. We would like to wind it up. I would prefer you people going, referring these books. Especially the standard books given by professor Ven te chow, Rajesh Shrivastav and all.

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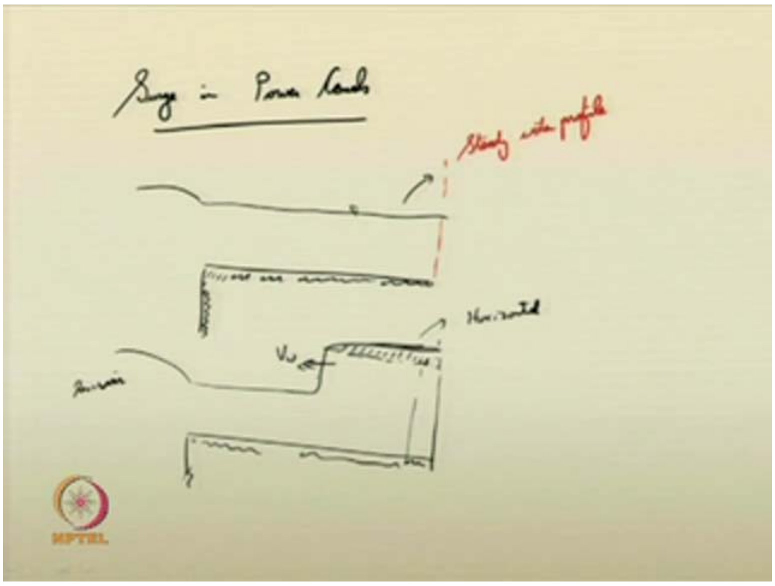


If their books if you refer them you will get more details on those aspects. Just as a brief thing I would just like to inform you how the surge in power canals occur. So, for the surge in power canals I will just quickly demostrate it through some figures. Say, from a reservoir you have a channel that supplies water to a power plant. If it is a, so this is a reservoir. So, from that a steady flow is being supplied to the reservoir. So, let us consider this as steady water profile that is being supplied to the power plant. Now, what happens? How the surges are developed in such power canals? Due to various factors like the closing of generators or tripping or lot of issues there may be chances that the water supply have to be blocked all of a sudden to those generators or to the power plants.

So, if you stop the supply of water all of a sudden at this location if it if you stop them, then what happens? Water starts building up, also a surge a positive surge will be propagated into the reservoir, so that you need to take into account. So, what happens is the same channel if I just draw it. So, this is reservoir and in this section what happens is a positive surge will be formed. We have clearly shown that there are some slopes in the channel here. So, bed slope is present there and theoretically speaking a positive surge will be moving in this direction to the reservoir. Now, the there is a slope here whereas, you may see that this is quite horizontal, this is horizontal.

So, there will be some additional quantites that will be present here. If you try to analyze these place as parllel then there will be some additional quantites here that you need to take into account. So, due to friction as well as means you have neglected friction earlier, but in this case by incorporating proper friction and friction as well as the slope of the channel. And to consider this additional volume you may see that this profile or this this surge, it may not go with a vertical front and reach the reservoir entirely like that. Theoritically speaking it should have reached like this.

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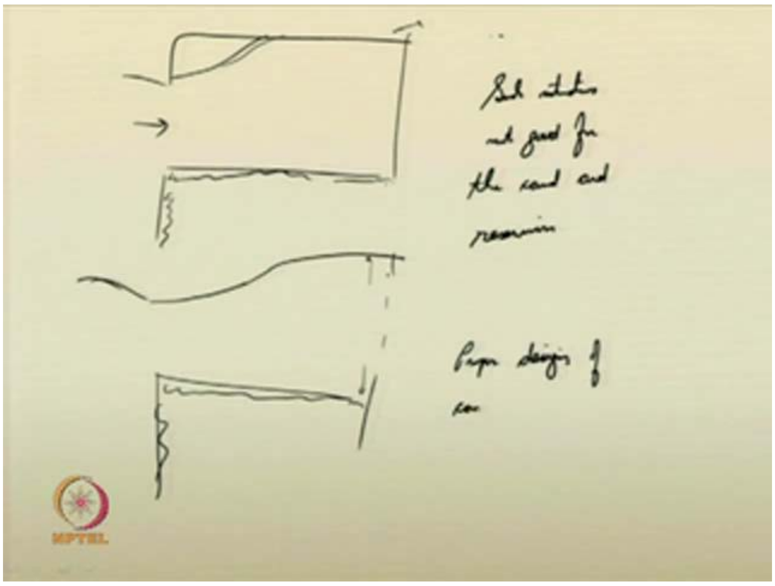

It should have reached like this, but that may not happen. And this is also not good for the canal, such situations not good for the canal and reservoir. So, we need a situation where the reservoir is still supplying the water to the canal, but due to the blockage here it starts building up here and then it has the profile has to come like this to the reservoir. So, that is being taken care and you may see the surges in canals like of the following form. You may see that, so here it will go like this and then like this. So, it will build up more, so the maximum height will be at the location where it is being blocked. Then it will decrease after a certain distance.

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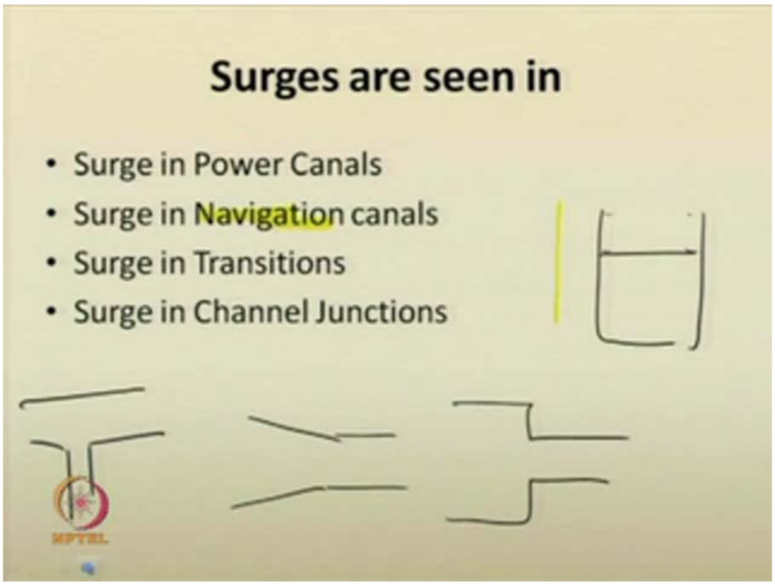


So, you can such situations are required for proper designing of canal. So, surge analysis is required in canal designing also. Proper designs of canals for power supply. Similarly, you can use you can analyze surge for navigational purposes also. Due to navigational purpose, say if there is any hindrance, any traffic block or any materal that gets jammed into the canal and all, surges may be developed that may propagate in the canal. So, you need to have proper say the depth of that canal should be such way that you have proper free board that will allow excess water. Surge in transitions are also there.

Say if there is a canal change in or width of the channel all of a sudden, how the you use the continuity equation there to analyze those things continuity and momentum equations. How in the channel junctions surges are formed. That is also we analyze. So, with this way we would like to wind up this particular module. So, we have studied a lot of aspects on the hydraulic jumps. Also the surge, we considered surge as a moving hydraulic jump. There, related to the topics in this particular module we request you, you have to go through the proper reference books whichever are given and all elaborately. Thereby, you will be able to enhance your knowledge appropriately because in the course, we have limited time to cover all the aspects.

Thank you.