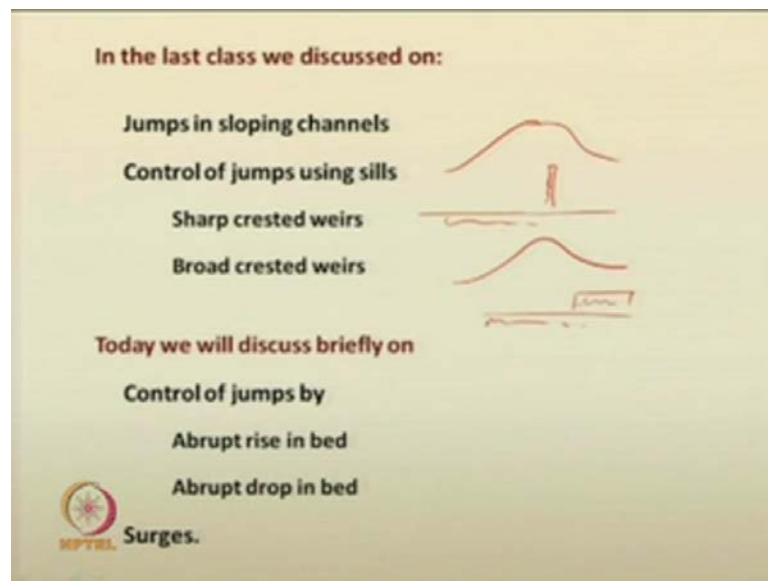


Advanced Hydraulics
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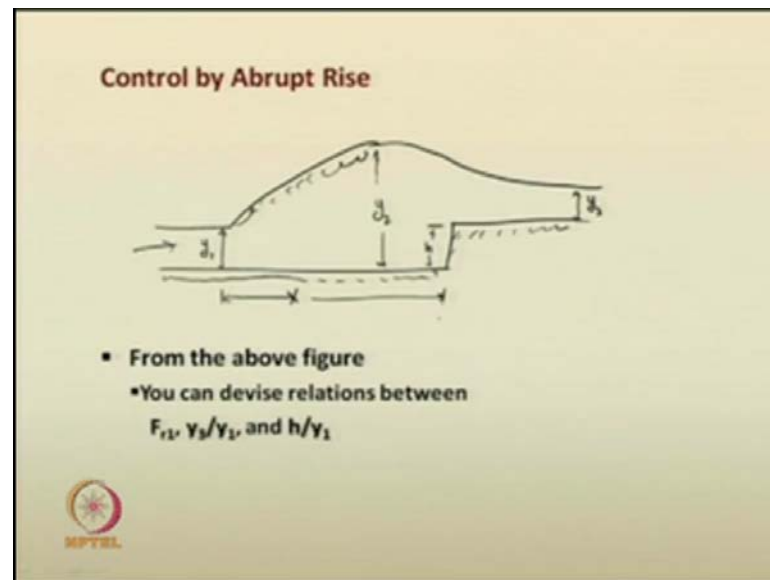
Module - 4
Hydraulic Jumps
Lecture - 7
Surges (1)

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Welcome back to our lecture series on advanced hydraulics as we are in the fourth module on hydraulics jump. We have been dealing for last four 5 lectures on this topic. Especially in the last class we had discussed on jumps in sloping channels. We have also discussed on controls of jump using sills. We have also discussed how you can control the jumps using sharp crested weir? For example, like this if you have sharp crested weir. How you can control the jump? This was discussed. Broad crested weir also we had discussed it in the last class. So, jump can be controlled using broad crested weir. So, today, we will discuss briefly on control of jumps by abrupt rise in bed also how to control the jumps by abrupt drop in bed? So, these things we will just briefly go through them, we will not have detail discussion on them. Then, we will also discuss briefly on surges.

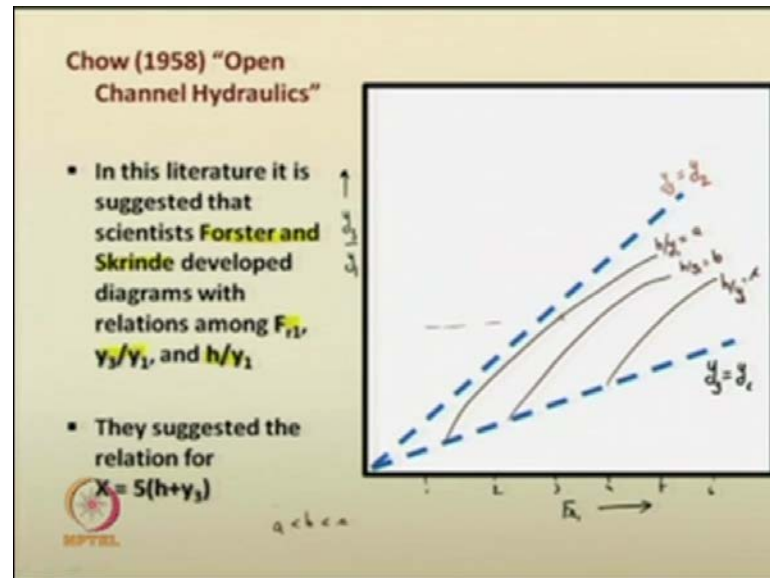
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So, control of jumps by abrupt rise, what is meant by control of jump by abrupt rise? Say if the channel flow which is being subjected to hydraulic jump. If there is a hydraulic jump then, if you are providing an abrupt rise of the bed then, how the properties behave or what are the properties in such control phenomenon? So, in this case, you have the upstream or pre jump depth as y_1 then, the jump with the proper rollers, and all it is there. You have the sequent depth of this pre jump depth as y_2 . Then, you have the height over the sill that is the depth of water, over the sill or the channel bed which the channel bed is raised here so that is y_3 . So, you have the height of the sill as h .

So, with these following parameters you can easily see that the non dimensional parameters you can visualise for such a condition is there can be an incoming for the incoming flow, what is the Froude number? You can device y_3 by y_1 ratio; you can device h by y_1 as one ratio. And based on those things also you can suggest the length up to the toe of the sill as X .

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So, from Vent Chow's book on open channel hydraulics he has suggested the some of the literature works in which certain relationships of Froude number with respect to the other non dimensional numbers were given that can be used for the design purpose, especially for control of jumps by abrupt rise in the section.

So, these phenomenon they for example, Forster and Skrinde they have suggested relationships with Froude number as well as y_3 by y_1 and h by y_1 they can be given in the following. They suggested it especially at a length X is equal to 5 times h plus y_3 . So, this thing here, if I can plot them the equations or the graph, that can be used for design. Here on the X axis on the horizontal axis you can have Froude number Froude number it can be 1 2 3 4 5 6 like that it can extend. Then, you have the non dimensional number y_3 by y_1 in the white coordinate 2 hash lines are drawn here, here the first one suggest I will just change the colour for your benefit.

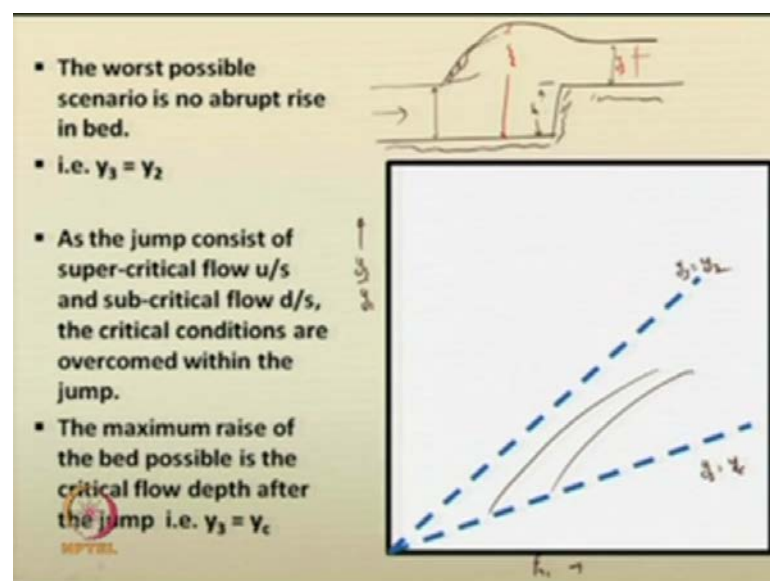
This line suggest for the depth y_3 which is equal to critical depth if you recall the figure earlier figure this is y_3 . So, if the flow over the sill if it is critical that is after the channel bed has been drawn elevated whatever flow depth if it is critical. Then that could be the minimum possible means you cannot go beyond that in such a situation. But for whatever elevation it is there for a such control for measure. Definitely it cannot go into the super critical condition means, you are providing jump for from super critical to sub

critical conditions. Later on, it cannot come back into the super critical again so that is not advisable. So, critical depth is a minimum possible condition here.

So, here y_3 is equal to y_c that condition can be incorporated then I can suggest one more line here, here the depth y_3 is same as the depth y_2 . So, just go back into the figure, this depth y_3 is same as y_2 , what does that mean? That means there is no elevation in the bed here at the section X, the elevation provided is 0 then, only you can have y_2 same as y_3 in such a situation. So, these are the maximum condition or the maximum possible phenomenon for such a thing. So, with in this you can plot, you can plot various curves that can be used for designed processes.

For example, this one particular curve I would like to draw is, say on curve it starts from here it goes on like this, another curve goes on like this. So, these are all individual curves for various ratios of h by y_1 . So, say this is h by y_1 is equal to some numerical value a h by y_1 equal to some numerical value, b h by y_1 is equal to some numerical value c . So, definitely a is less than b it is less than c in this design criteria. So, you if you have such things then, for the given condition you can easily plot, or you can easily find what could be the post jump means? What could be the tail water depth? Easily say for h by y_1 curve if one curve it is given to you. And if the incoming Froude number is somewhere here just check, what is the depth? So, you will get the corresponding y_3 by y_1 ratio or y_3 like that you can infer for the design purpose.

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So, the worst possible scenario in this case is abrupt rise in abrupt scenario in the abrupt rise in bed is y_3 is equal to y_2 that is there is no elevation in the bed now as a jump. So, this is y_3 is equal to y_2 curve, and as said earlier y_3 is equal to y_c curve these are Froude numbers and y_3 by y_1 on this X axis. So, the jump consist of super critical flow upstream. Now, if you just see for any of this curve as we draw I have drawn several curves what happens y_3 is equal to y_c at y_3 is equal to y_c . The jump consist of super critical flow at the upstream I am just again redrawing the thing here.

So, super critical flow jump occur and the subsequent flow is denoted like this. So, this is the jump so you have supercritical flow before the jump, sub critical flow after the jump. So, the critical conditions are overcome within this jump it is transforming from sub super critical to sub critical in the jump. So, the maximum raise of bed is possible is the flow depth here should be critical you cannot design any raise of the bed beyond this thing. If it goes beyond this thing, it may affect the hydraulic jump considerably. So, that is one criteria, you need to remember always.

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Control by Abrupt Drop

- The control of hydraulic jumps using
 - sharp crested weir,
 - broad crested weir,
 - abrupt rise in channel bed, etc.

are useful if the d/s depth is smaller than the sequent depth of the pre-jump depth.

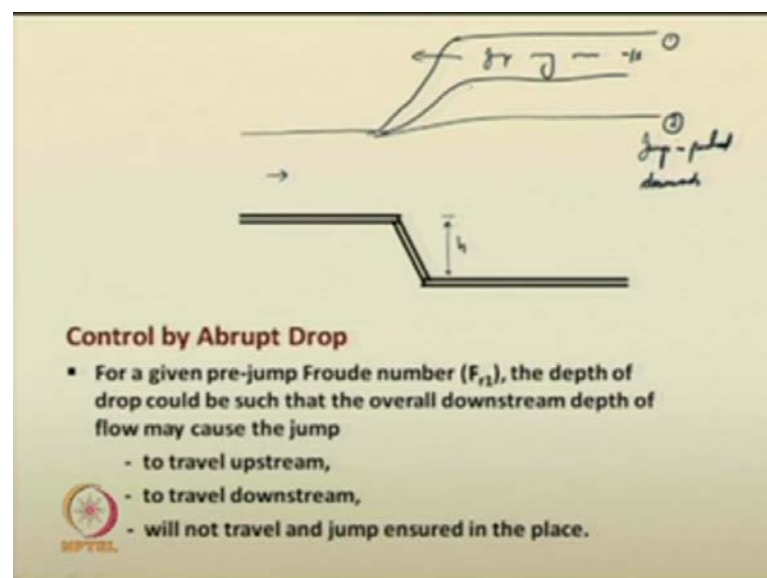
- If the d/s tailwater depth (y_3) is greater than the sequent depth of the pre-jump depth of water (y_1), then
You need to provide drop in channel floor so that jump occurs properly at that place.

Next, which we would like to deal is the next topic which we are interested is control of jump by abrupt drops as we have studied in the abrupt rise case. Similarly, abrupt drop cases are also used for controlling various hydraulic jump for example, just recall the things in the sharp crested weir, broad crested weir as well as the abrupt rise in channel bed. What were the situations that for which all situations you used these mechanisms to

control the hydraulic jump? Usually if the downstream depth for previously you just go through it. If the downstream depth these cases if they are less than the sequent depth of the hydraulic jump. You can definitely see here, here this is y_3 is less than y_2 here. So, if such situations are there, mostly in most of the cases if the downstream depth is smaller than the sequent depth then, the above mentioned sharp crested weir broad crested weir or abrupt rise in channel beds are used.

What happens if it is the other way? You have already studied how the hydraulic jumps are affected by the tail water condition so if the tail water depth or if the post jump depth, if it is higher than the sequent depth of the, if the tail water over the downstream tail water depth is greater than the sequent depth. Then, the pre jump depth of the water how it can be related with the hydraulic jump. So, you have the pre jump depth as y_1 here now this jump this depth is y_3 and the bed dropped is h . So, with these following cases you need to provide a drop in channel flow. So, that this jump occurs, if you are not providing such a drop in the channel flow like this then, this jump cannot occur in this place it may get shifted to some other place that you have already seen. So, that way to ensure proper occurrence of hydraulic jump in this location, you have to provide the drop in elevation.

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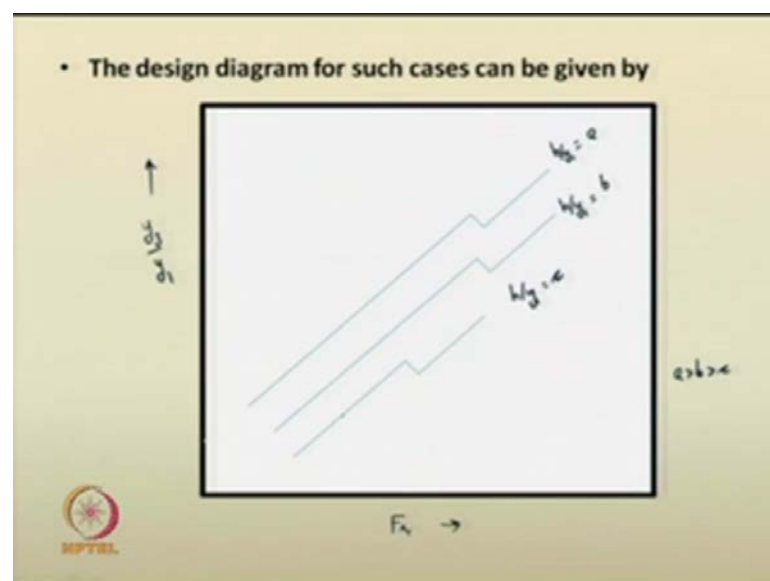
When you do the control of hydraulic jump by abrupt drop in the bed? You need to remember the following things; you need to remember for example, say if I just draw the

water surface elevations. So, you have the water surface elevation so jump, if it is goes like this very much upstream, if it is like this say if this is the maximum elevation possible. Suppose, if the jump is occurring in this way, there are chances that this jump may move upstream. So, this jump may move upstream, there are chances for that then, there are certain jumps where the jump may be like this, when you are provides a such a drop it may the jump may have just occurred like this. So, here the jump is pushed downwards so here the jump may travel upstream.

In another case, for the same Froude number that is what for the same incoming flow based on the rise, based on this elevation, the jump may be either moving upstream or it may be pushed downwards, or it may be held within this section. So, for the first condition 1 here and the second condition given here for the first and second condition definitely the control by abrupt drop cannot be used, because your jump may be either pushed downwards or it may be well it may travel upstream.

So, you have you cannot use these particular control measures. So, these jumps are only feasible for the conditions where the hydraulic jump is within certain limit. So, it is not undulating it is not too high something. So, for those case only you can use these where if the stilling basin is provide along this reach. And all you can ensure that the jump occurs in this region only so with this we would like to conclude the control of hydraulic jumps. Now the next topic which we are going to deal is.

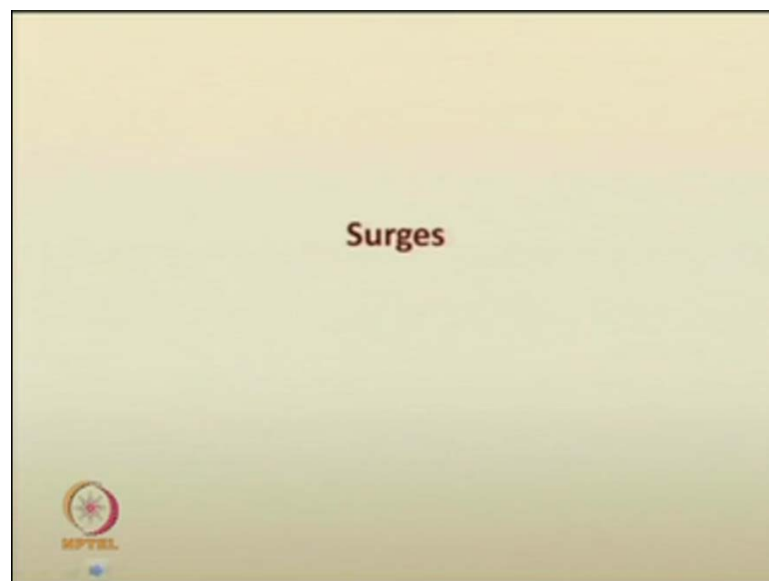
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May I beg pardon I just forgot to mention; you can also device design curves for abrupt the control of hydraulics, control of jumps by hydraulic jumps by drops. So, here also we can say for the incoming flow Froude number, or the pre jump Froude number ratio y_3 by y_1 for the various cases say h by y_1 is equal to a h by y_1 is equal to b h by y_1 equal to c for different values of this particular ratio h by y_1 a b c where a is greater than b it is greater than c like that.

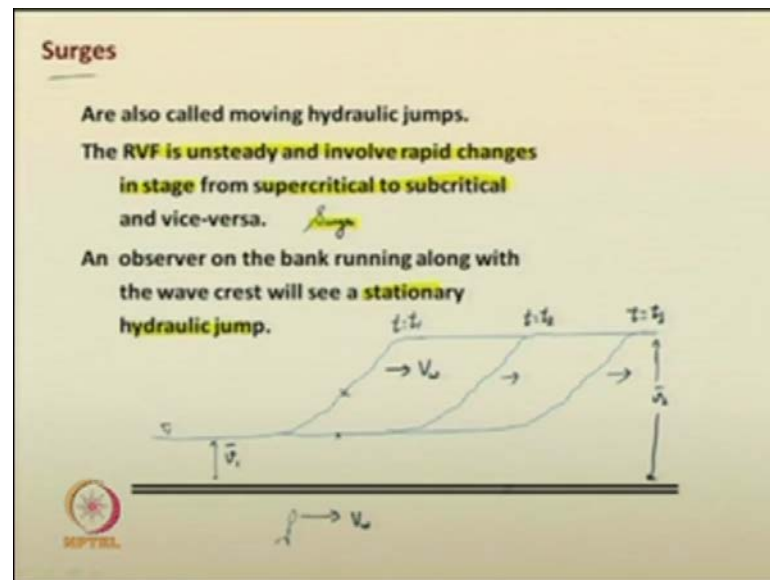
So, if you have such design, if you have such standard curves, and all these curves can be employed for the design of the design of the sills and all. So, that then jump occurs in that particular location. So, you can easily if say for the given Froude number if the h by y_1 is some values c . You can see that the corresponding y_3 by y_1 ratio is so and so the y_3 depth is like this. Like this you can easily manipulate and use for design purpose.

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So, next as I told earlier we are going to go through the concept called surges.

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What is meant by a surge? Surges are also called moving hydraulic jumps. Precisely for that particular reasons only, we have incorporated this topic in this particular module. Actually for frankly speaking, this could have been means we could have made another module of this course where unsteady flow could have been incorporated in the entire course this thing. However, as per the syllabus given or as per the syllabus device for this course and all a separate module on unsteady flow is not being suggested. But still we will be dealing with this topic called surges that are called moving hydraulic jumps.

The rapidly varied flow in the earlier cases we have studied the rapidly varied flow in steady state conditions. So, if the rapidly varied flow is in unsteady condition and rapid changes for super critical to sub critical then, such quantities are called hydraulic surges. So, such quantities, that is the rapidly varied flow is unsteady it is unsteady and involve rapid changes in stage from super critical to sub critical and vice versa. So, those are called surges or usually it is called surges. Waves in waves it is more gradual in nature. So, that we are not going to study right now here, what is the peculiarity of surge? If there is an observer, say if there is an observer standing here on the banks. And if he moves with the same velocity of a wave say here you can see that the water is rising like this at this instant. And then, it is travelling like this. So, after a few minutes this would propagate up to here right.

So, in this case, this is called a wave or the water height it is not constant, it is not steady at all the times. At this same location you will see that at time t is equal to 0, we had a water elevation here at time t is equal to something else you have a water elevation here. So, you see that some wave or some mass of water it is just flowing above a certain water surface or a certain level of water it is being pushed. So, this is called as surge or a wave surge, because the water surface profile is rapidly varying. So, after that if you say this is at time t is equal to t_1 ; this is at time t is equal to t_2 , time t is equal to t_3 the water surface profile will be like this.

So, if you have such profiles so an observer now running with the same speed, he will see a stationery hydraulic jump, he will see a stationery hydraulic jump. So, whichever topic you have studied earlier hydraulic jumps and all. So, those things will be applicable for him, because he is moving with same velocity. So, we suggest that let him move with a velocity V_w velocity at which this wave of water is moving. Then, this person is also moving with the same velocity V_w , and although the water velocity here say here it is v_1 bar, here it is v_2 bar like this you can have. Still he can observe it in a stationary form.

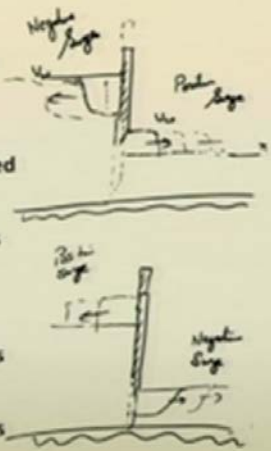
So, what are surges? Surges are formation of waves they are formation of waves. So, when a sluice gate is suddenly lowered or raised, such waves are formed. You I hope you know what is meant by sluice gate.

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Surges
Are formation of waves
 When a sluice gate is suddenly lowered or raised.

When a gate is raised
 Upstream of gate a negative surge is formed that moves upstream.
 Downstream of the gate a positive surge is formed that moves downstream.

When a gate is lowered
 Upstream of gate a positive wave (surge) is formed that moves upstream.
 Downstream of the gate a negative wave is formed that moves downstream.

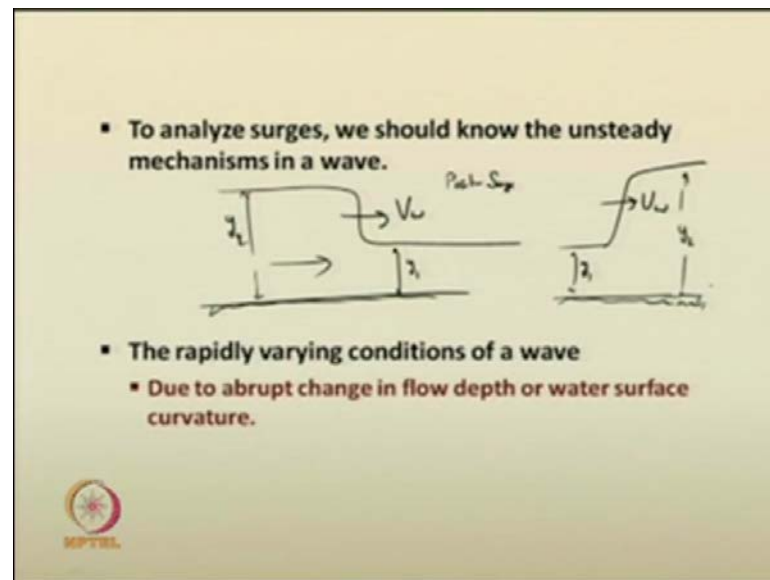


So, if a sluice gate is suddenly raised say for example, if you have a such sluice gate so either so it can be lowered or it can be raised. So, due to this phenomenon there will be a wave that will be propagated along the surface of water, a wave of wave of water will be propagated. So, this how this wave or how the open channel flow in those situations are analyzed for example, in this case surges are formed say when the gate is raised upstream like this when the gate is raised definitely the water here will get lower. So, here there will be a formation of a wave like this due to the when the gate is raised like this. And this thing will be propagated as time goes on this will propagate like this.

So, this, what is happening here? As the wave is moving from here to this position the depth of water is getting reduced. Now, the wave is propagating in this direction V w the depth of water is getting reduced so such surges are called negative surges. And what happens in the downstream? Due to the raising of this gate water which was initially at this level will now suddenly raise like this, say after a few seconds it will go like this. So a wave will be propagated in this downstream direction not only that, during the propagation of wave in the downstream direction the depth of water depth of water depth of water also increases so this is called positive wave or positive surge. So, when a gate is raised upstream of the gate a negative surge is formed that moves upstream like this. So, a negative surge is formed that will move in the upstream direction like this. Downstream of the gate a positive surge is formed that moves downstream like this so these things you need to visualize.

Similarly, when a gate is lowered, what happens? You are lowering the gate now initially its position was like this. Now, due to lowering of the gate suddenly this water height will increase and it will raise like this, and simultaneously earlier the water level was like this it will get dipped and it will go like this. So, this wave will propagate like this in the downstream direction, this wave will propagate in the upstream direction like this. So, here when a gate is lowered, positive surge will be formed that will propagate in the upstream direction it will go like this. And negative surges form in the downstream of the gate, and that will propagate in the downstream direction. So, these phenomenon's occur so surges are formed due to sudden raise or lowering of the sluice gates.

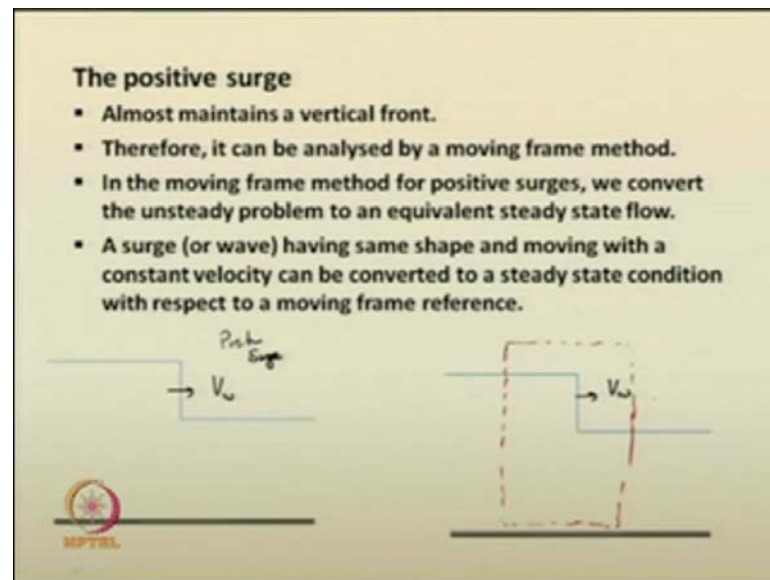
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So, how do you analyze these surges? You need to understand the unsteady mechanism in a wave. So, you have, say all of a sudden wave travelling like this in the flow direction. So, this is a positive surge or you have a negative surge that is the wave is getting pushed downstream. So, as the water flows in the downstream direction the flow or the, the depth of water get reduced. So, such criteria's you can easily visualize y_1 or y_2 I will tell you why this is called y_1 here, y_2 here and all. So, how do you analyze this thing? You can see us rapidly varying condition, say the front of the water in the wave it is almost vertical that is why it is called a surge.

So, due to abrupt change in flow depth or the water surface curvature the unsteady mechanisms are prevailing in those situations. And you need to analyze surge using unsteady flow mechanisms, but how far is the requirement of unsteady conditions? Therefore, the analysis of open channel flow for surges.

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For example, consider the positive surge I am just giving a positive surge here consider a positive surge. So, it has almost a vertical front. Now, just by simplifying the situation, as we told earlier in this particular case, as the wave is getting propagated in the downstream direction. And the person if it is he is moving with the same velocity of wave then, he will be observing a stationary hydraulic jump. So, the same crude phenomenon or the crude principle we are going to adopt it. And it will be now more scientifically called as moving reference method.

So, here the person if he was stationary he would have seen different fluctuations in the water surface at that location, but as he is also moving. Now, he is the reference object and as he is a moving object, moving reference object you were able to identify the wave or the surge in the water in the channel as a stationary hydraulic jump. So, the same phenomenon if he incorporate again. For example, in the positive surge using the moving frame method we can analyze the positive surge, say this is a positive surge. So, in the moving frame method we what we are doing is that we are converting the unsteady problem to an equivalent steady state problem. You know the stationary hydraulic jump it is a steady state problem. So, as the reference of frame now the frame reference as it is moving you were able to visualize a equivalent steady state condition for the surge, say surge having same shape and moving with a constant velocity can be converted to a steady state condition with respect to the moving frame reference. So, if, for the moving frame reference in this case, if I identify them now the frame reference itself is moving.

So, we will be moving along with this way. So, if I take a control section, if I take a control section now what happens? So, I am taking a control section like this now. And now this control section will also be moving with a velocity V_w . So, this entire control section it is moving with a velocity V_w . And you can now easily apply the principles of this control volume Reynolds transport theorem you have already studied the same principles you can use them and try to analyze the things.

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The continuity equation

- When we consider the equivalent steady state, we suggest the frame of reference is moving with the same velocity of the surge (i.e. V_w).
- We can define a term called **wave celerity** C_w where the wave velocity is measured relative with respect to the water.

$$C_w = V_w - \bar{v} ; C_w = V_w - \bar{v}_1$$
- Applying the continuity equation now for the equivalent steady state:

$$(V_w - \bar{v}_2) A_2 = (V_w - \bar{v}_1) A_1 = Q_r$$
- This discharge Q_r is in negative direction. It is called **Overrun**

So, the continuity equation in the moving frame analysis, let us apply them before that it is better, let me give you the control frame now in this case a wave is moving with velocity V_w . So, actual velocity of water in this section it is say v_2 bar average velocity, in this portion it is v_1 bar. Let us suggest this depth as y_1 , let us suggest this depth as y_2 . Now, why I am using y_1 on the right side, y_2 as in the left side? For this cases, that you recall the hydraulic jump in the steady state condition. So, the y_1 usually you used it for the super critical condition, or for example, lower depth condition, and y_2 for the higher depth conditions it is corresponding to its sequent depth or like that.

So, in this case also, we are now suggesting in that same way the lower magnitude of depth is generally given as y_1 , higher magnitude of depth of flow is given as y_2 . So, the frame of reference, now this frame of reference is also moving with a velocity same as the surge velocity V_w . You can define a term called waves celerity I can now define as term called waves celerity C_w , such that the velocity of the wave is now measured

relative to the velocity of water. So, instead of measuring the velocity of wave to a stationary object on the banks or anything I am now measuring the velocity of wave with respect the velocity of water. So, what happens? C_w I can just give it as velocity of wave, relative velocity of wave \bar{v} so C_w it can be V_w minus v_1 bar or C_w can be V_w minus v_2 bar like this if you are able to visualize, if you are able to define this wave celerity term as defined here the wave celerity.

Now I can proceed for the next quantities to analyze the thing. So, what happens to the wave here? If V_w , if it is greater than v_2 then, you have a direction in this case, if V_w is greater than v_1 , you have a quantity in this direction, or if V_w is less than v_1 you have a quantity in this direction; you have quantity in this direction. So, we do not know the actual situation so V_w , we do not know whether it is greater than v_2 or it is less than it is greater than v_2 or it is his greater than v_1 . So, in that way you cannot try to analyze the quantities I can apply the continuity equation for this entire control volume, the continuity equation is now applied it is applied for the equivalent steady state condition. So I am going to use the relative velocity term so the relative velocity term. So, we do not know so we will just write it this quantity V_w minus v_1 bar, we are assuming it in this direction. Similarly, V_w minus v_2 bar we are assuming it in this direction.

So, in this entire moving frame method now the continuity equation if I apply for the relative terms for the using the relative terms, we can have the continuity equation in the following form the, you we are using the relative velocities now. So V_w minus v_2 bar as we have already taken the steady state approach. So, V_w minus v_2 bar into A_2 , the cross section in the upstream portion here. This is equal to V_w minus v_1 bar into A_1 that is there is some discharge and that using the relative terms and that discharge. Let us define it as Q_r so this discharge Q_r is now in the negative direction it is also called the overrun it is called the overrun due to the wave propagation.

So, here what we have studied that due to the moving frame difference the relative terms were taken into account. And due to this relative term as we suggested we are not quite clear if V_w is greater than, if V_w is less than or greater than v_1 bar and v_2 bar. So, we are not quite clear. So, we are assuming that let us let V_w be less than v_1 and v_2 in that condition we will be having V_w minus v_1 bar in the negative direction, V_w minus v_2 bar also in the negative direction and that is that has been taken into care here. So, the

equivalence in the equivalence steady state the continuity equation will be given in the following form.

So, this discharge Q_r is now in the, this it is due to this thing the additional discharge of whatever discharge is occurring it is called the overrun due to the moving frame of reference.

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The momentum equation

- We can apply the momentum conservation equation as we are doing in this entire course for various flow analyses.
- The same Reynold's Transport theorem can be employed. (We are not going to write them again here).
- As the overrun is in negative direction that implies that the net force is in the negative direction.
- Therefore, momentum equation applied appropriately in this direction.

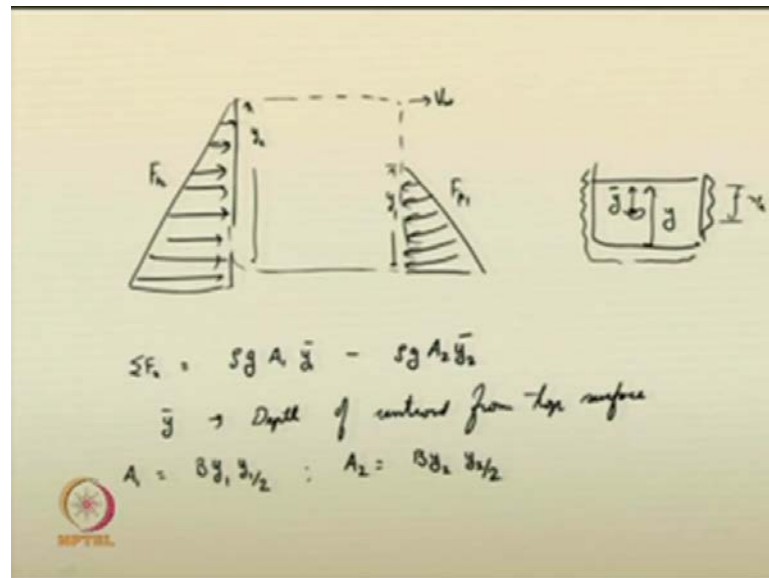
Q_r is in -ve dir
 \rightarrow Assume hydrostatic condition
 \rightarrow Assume unit weight factor = 1

You can similarly use the momentum equation also to analyze the fluid flow due to search. So, same control volume method, if I apply now I can consider, say consider the following control section now or the control volume. So, this is the portion so the wave is moving with a velocity V_w . So, we are suggesting this frame is also moving with a velocity V_w . So, you have a relative velocity in this direction V_w minus v_1 bar here, you have a relative velocity in this direction V_w minus v_2 bar in this direction.

So, the one dimension as this is the dimensional one dimensional case, the we are using the same Reynolds transport theorem as mentioned earlier here I am not going to write it means we have written it for many cases. So, the overrun Q_r is in negative direction so your Q_r is in negative direction. So, what does that mean? So, if the overrun is in the negative direction the net force is also in the negative direction? Therefore, I am taking the momentum equation I am applying the momentum equation for the net force in the negative direction, based on that we can write the momentum equation. For that I am just assuming hydrostatic pressure condition.

So, we have to assume for the momentum analysis, assume hydrostatic condition now also assume momentum correction factors as 1, momentum correction factors equal to 1. So, that way if I consider these things I can write the momentum equation as follows.

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So, the net force say in this control volume now will be, such that here you have the hydrostatic force on these walls. You have the hydrostatic force on these walls like this. So, this is $F_p 1$, the force will be in this direction I will just rub it again. So, here the hydrostatic force is in this direction. So, this is $F_p 2$ $F_p 1$ the weight as we are considering horizontal channels, the weight effect of weight and all that is being neglected here.

You have the depth of flow here as y_2 , here this is depth of flow is as y_1 , the frame is moving with A velocity V . So, the net force acting here the net force acting here is, it can be given by $\rho g A_1 \bar{y}_1$ minus $\rho g A_2 \bar{y}_2$ where \bar{y} , this terms \bar{y} suggest depth of centroid, any section you take say if I am taking a rectangular section say if this is the depth of flow y . Then if the centroid is at this depth this particular quantity is called depth of centroid from top surface, from top surface of water so that is this quantity. So, for a rectangular channel you know the centroid is same as $y/2$.

So, same thing we are going to apply. So, the hydro static force on the left side and on the right side. So, the net force it can be given by the following things. So, $\rho g A_1 y_1$

bar minus $\rho g A_2 y_2$ bar as we have net force in this direction we are taking so $\rho g A_1 y_1$ bar is the first quantity you are taken minus $\rho g A_2 y_2$ bar. So, this is nothing but equal to $\rho g A_1 y_1$ bar and $\rho g A_2 y_2$ bar.

So, you have A_1 is equal to B_1 into B into the width of the channel we are assuming it same. So, B into y_1 into y_1 by 2, A_2 is equal to B into y_2 into y_2 by 2 so these are the areas. So, effect due to these things we can write it now in the effect due to the control surfaces means you remember.

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$$\int_S \beta \rho \vec{v} \cdot \vec{n} dA \rightarrow \text{Net mass outflow through the control surface}$$

$$\int_S \vec{v} \cdot \vec{n} dA \rightarrow SQ_2 [(V_2 - \vec{v}_2) - (V_1 - \vec{v}_1)]$$

$$SQ_1 A_1 \vec{v}_1 - SQ_2 A_2 \vec{v}_2 = SQ_2 [(V_2 - \vec{v}_2) - (V_1 - \vec{v}_1)]$$

$$C_1 = V_1 - \vec{v}_1 \quad ; \quad C_2 = V_2 - \vec{v}_2$$

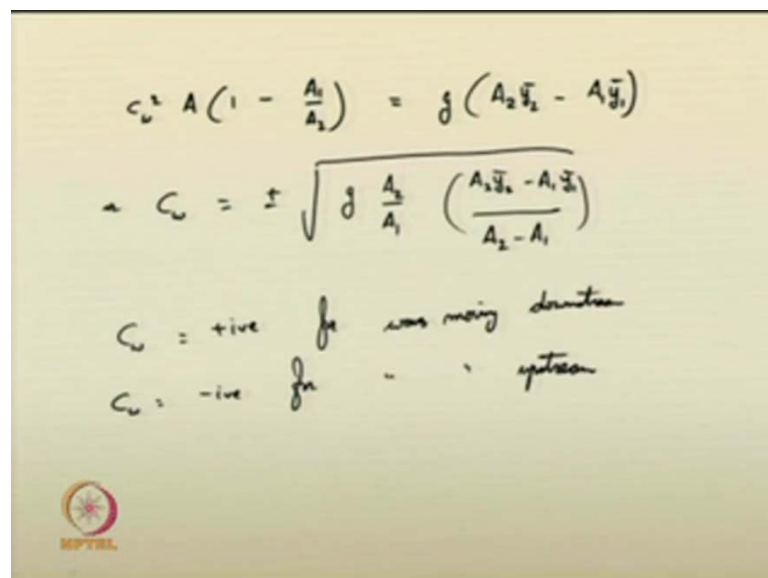
The quantities $\beta \rho \vec{v} \cdot \vec{n} dA$ was the net outflux of the extensive property across the control volume, this you have studied it continuously over this course. So, in the case of momentum, we have β as velocity so $\vec{v} \rho \vec{v} \cdot \vec{n} dA$ for momentum net, momentum out flux through the control surfaces. So, for this case, you have on the left side depth of water as on the left, left side you have the depth of water as y_2 , right side you have the depth of water as y_1 , those two are the control surfaces that allow flow through it. So, we can now suggest this quantity in this is equal to $\vec{v} \rho \vec{v} \cdot \vec{n}$, this is vectorial form. This can be easily represented it I am not writing it as equal to please note that. So, this is represented by the following quantity.

So, $\vec{v} \rho$ into dA in each of the control each of the area surface areas of those control volume, what does that mean? So, the left hand side as well on the right hand side those

quantities denote discharge. So, you have the discharge in this net discharge in this moving frame method as $\rho Q r$ is into $V w$ minus v_2 minus $V w$ minus v_1 bar.

So, we have the following quantities so this will be incorporated there this control quantity we will incorporating our. We will be equating them this is equal to say $\rho g A_1 y_1$ bar minus $\rho g A_2 y_2$ bar is equal to ρ times $Q r$ into $V w$ minus v_2 bar minus $V w$ minus v_1 bar, like this we will be equating them. So, recall the wave celerity as $V w$ minus v_1 bar or $C w$ is equal to $V w$ minus v_2 bar. So, based on those things we will now suggest the following form. Substitute these quantities substitute $C w$ in the expressions here everywhere.

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The image shows a handwritten derivation on a yellowed piece of paper. At the top, the equation $C_w^2 A (1 - \frac{A_1}{A_2}) = g (A_2 \bar{y}_2 - A_1 \bar{y}_1)$ is written. Below it, the equation is rearranged to $C_w = \pm \sqrt{g \frac{A_2}{A_1} \left(\frac{A_2 \bar{y}_2 - A_1 \bar{y}_1}{A_2 - A_1} \right)}$. Further down, two lines of text explain the sign: $C_w = +ve$ for waves moving downstream and $C_w = -ve$ for waves moving upstream. In the bottom left corner, there is a small circular logo with a sun-like symbol and the text 'KPTCL' below it.

So, we will now suggest the quantity as $C w$ square into A into 1 minus A_1 by A_2 is nothing but equal to g times $A_2 y_2$ bar minus $A_1 y_1$ bar, or the wave celerity can be found from this momentum method as square root of g into A_2 by A_1 into $A_2 y_2$ bar minus $A_1 y_1$ bar by A_2 minus A_1 , like this you can find the wave celerity. So, $C w$ will be positive for positive for waves moving downstream. So, whichever waves moving downstream we consider $C w$ as positive, $C w$ is negative for waves moving upstream. So, then using this thing, we will quickly do on numerical problem we will just solve that.

(Refer Slide Time: 49:10)

Example (As reproduced from Srivastava's "Flow through open channels")

A 2 m wide rectangular channel carries discharge = 1 cumec at a flow depth 1 m. The discharge is suddenly increased by opening of a sluice gate in the channel and the depth of flow is 1.4 m. find the increased discharge and the velocity of the surge.

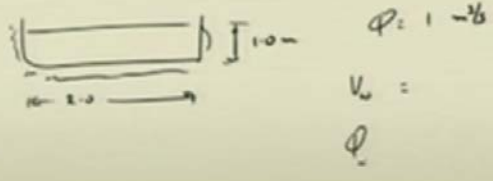


Diagram showing a rectangular channel of width 2.0 m. The initial flow depth is 1.0 m. A sluice gate is shown partially open, increasing the flow depth to 1.4 m. Handwritten notes indicate $Q = 1 \text{ m}^3/\text{s}$ and $V_s = ?$.

And this particular example is taken from Professor Srivastava's book on "Flow through open channels". The question is a 2 meter wide rectangular channel. A 2 meter wide rectangular channel carries discharge Q is equal to 1 meter cube per second cumec at a flow depth 1 meter. So, flow depth is 1 meter the discharge is suddenly increased by opening of a sluice gate in that channel, and the depth of flow has now become 1.4 meter. Find the increased discharge and the velocity of the surge? Find this increased discharge and the velocity of the surge? How will you measure say we need to find the velocity of surge V_w and also we need to find the increased discharge? How do you solve this particular problem?

So, again we will just go through the basics thing we have at the upstream at the as we have suggested the sluice gate in the channel is open. So, the discharge is suddenly increased by opening of a sluice gate in the channel. So, what happens if you open sluice gate in the channel? What type of wave is being propagated when you open the sluice gate? You remember that so it is a positive surge.

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$$\begin{aligned}
 A_1 &= B y_1 = 2 \times 1.0 = 2 \text{ m}^2 \\
 A_2 &= B y_2 = 1.4 \times 2 = 2.8 \text{ m}^2 \\
 \bar{y}_1 &= 0.5 \text{ m}, \quad \bar{y}_2 = 0.7 \text{ m} \\
 C_w &= \pm \sqrt{g \frac{A_2}{A_1} \frac{A_2 \bar{y}_2 - A_1 \bar{y}_1}{A_2 - A_1}} \\
 &= \pm \sqrt{9.81 \times \frac{2.8}{2.0} \frac{2.8 \times 0.7 - 2.0 \times 0.5}{2.8 - 2.0}} \\
 &= \pm 4.06 \text{ m/s}
 \end{aligned}$$

So, the same thing A_1 is equal to $B y_1$, A_2 is equal to again $B y_2$. This is nothing but 2×1.0 is equal to 2 meter square . A_2 is equal to the new height is 1.4 and the width of the rectangular channel is same. So 2.8 meter square , you have the depth of centroid y_1 bar as 0.5 meter depth of centroid in the downstream section as 0.7 meter . So C_w is given as plus or minus root of g times A_2 by A_1 $A_2 y_2$ bar minus $A_1 y_1$ bar by A_2 minus A_1 , we will be applying the same formula. So, g you know it is nothing but 9.81 , A_2 is 2.8 , A_1 is 2 so 2.8 into 0.7 minus 2 into 0.5 divided by 2.8 minus 2 . So, this quantity it is coming as plus or minus $4.06 \text{ meter per second}$.

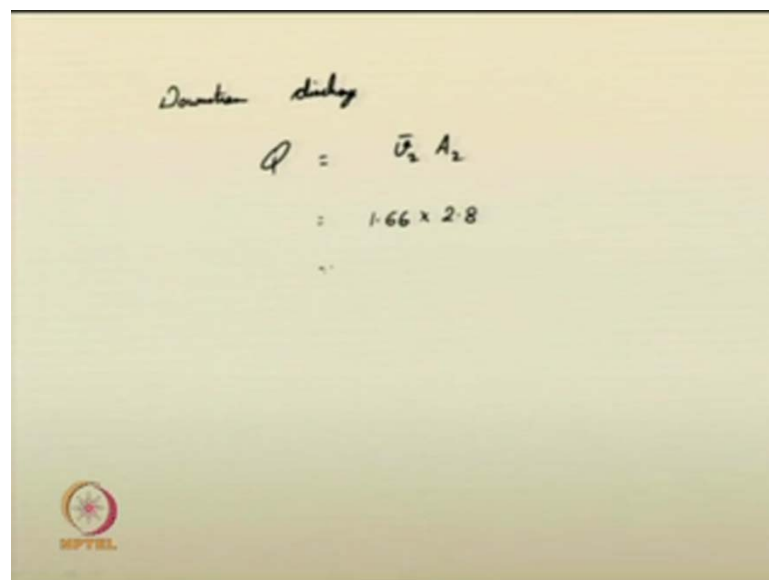
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$$\begin{aligned}
 V_w &= \bar{V}_1 + C_w \\
 &= 0.5 + 4.06 = 4.56 \text{ m/s} \\
 V_1 A_1 &= V_2 A_2 \\
 V_1 &= \frac{Q}{A_1} = \frac{1}{2 \times 1.0} = 0.5 \text{ m/s} \\
 \text{Applying continuity equation} \\
 (V_w - \bar{V}_1) A_1 &= (V_w - \bar{V}_2) A_2 \\
 4.06 \times 2.0 &= (4.56 - \bar{V}_2) \times 2.8 \\
 \bar{V}_2 &= 1.66 \text{ m/s}
 \end{aligned}$$

So, you have now celerity, celerity C_w as 4.06 meter per second. So, the velocity of the surge will be equal to v_1 plus C_w so this is nothing but equal to 0.5 plus 4.06. How did you get this 0.5? You know v_1 is equal to Q by A_1 so 2 into A_1 is 2 into 1 . So the Q , Q is 1 cumec per meter. So, this is 0.5 meter per second that I have incorporated it here so 4.56 meter per second.

So, apply continuity equation now. Apply the continuity equation V_w minus v_1 into A_1 is equal to V_w minus v_2 bar into A_2 . So, you will get V_w minus v_1 bar is nothing but celerity 4.06 into 2, this is equal to V_w is 4.56 minus v_2 bar into 2.8 or v_2 rearranging the terms you will get this as 1.66 meter per second.

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Downstream discharge

$$Q = \bar{v}_2 A_2$$

$$= 1.66 \times 2.8$$

NPTEL

Similarly, downstream discharge increased discharge Q , you will get it as v_2 bar into A_2 , by multiplying this I think I there is no need to for me to do this thing, because you already got this thing. So, two point, A_2 is 2.8, substitute this you will get the modified discharge. So, that way we would like to conclude the following portion. We will continue with this topic in the next class also.

Thank you