Advanced Hydraulics
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Module - 4
Hydraulic Jumps
Lecture - 3
Characteristics of Jumps in
Rectangular Channel

Welcome back to our lecture series on advanced hydraulics. We are in the 4th module on hydraulic jumps.

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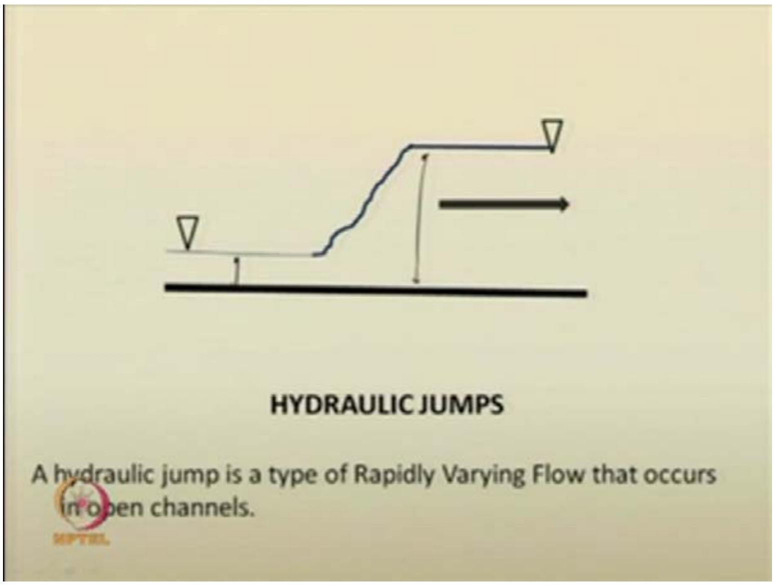

If you recall hydraulic jump means, it is a type of rapidly varying flow, where the depth of flow suddenly increases from a low depth to a greater depth. These things we are discussed it in the last class.

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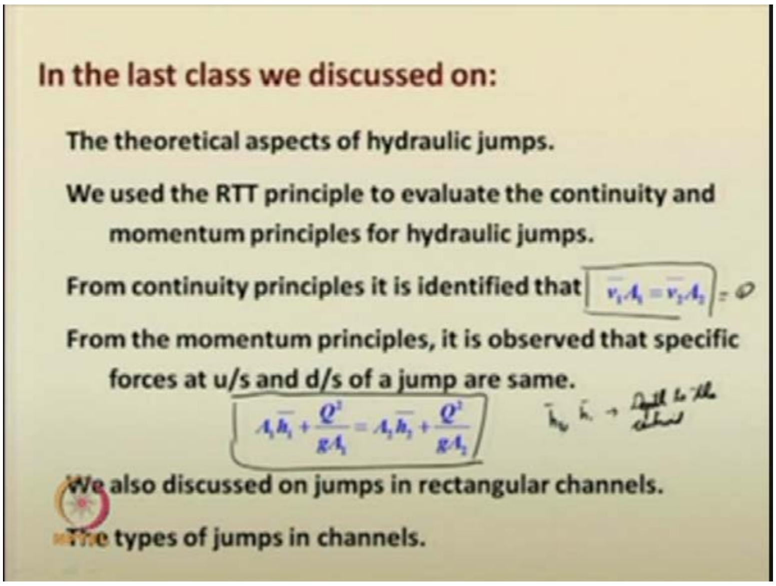

In the last class, we had generally discussed on the theoretical aspects of the hydraulic jumps. What are the various theories coming into picture? For example, we used the Reynolds transport theorem principle, to evaluate the continuity and momentum principles for hydraulic jump. So, if you take the location of hydraulic jump in a control volume, how you can use the principles continuity principle as well as momentum principles using the Reynolds transport theorem? If you recall them in the from the continuity principle, we had easily arrived at this particular relationship that suggest that the discharge is same at upstream, that is pre-jump section as well as the post jump section, the discharge will be same.

From the momentum principle, you had observed that the specific forces at the upstream and downstream of a jump are same that is in the jump. This is the upstream section; this is the downstream section, y 1 y 2. So, the specific forces at these upstream sections as well as at this downstream section both are same. So, the relationship of the specific force was given as such A 1 h 1 bar plus Q square by g A 1 is equal to A 2 h 2 bar plus Q square by g A 2 where h 1 bar is the depth to the centroid in section 1, that is in the upstream section, depth to the centroid. Similarly, h 2 bar is also the depth to the centroid in section 2. We have discussed briefly on the rectangular channels, how the jumps, how you can analyze jumps in rectangular channels? In fact we will be continuing that portion today also. We have also studied the U S reclamation bureaus mentioning about the types of jumps according to the Froude number. So, today we will be going through the characteristics of jumps in rectangular channels.

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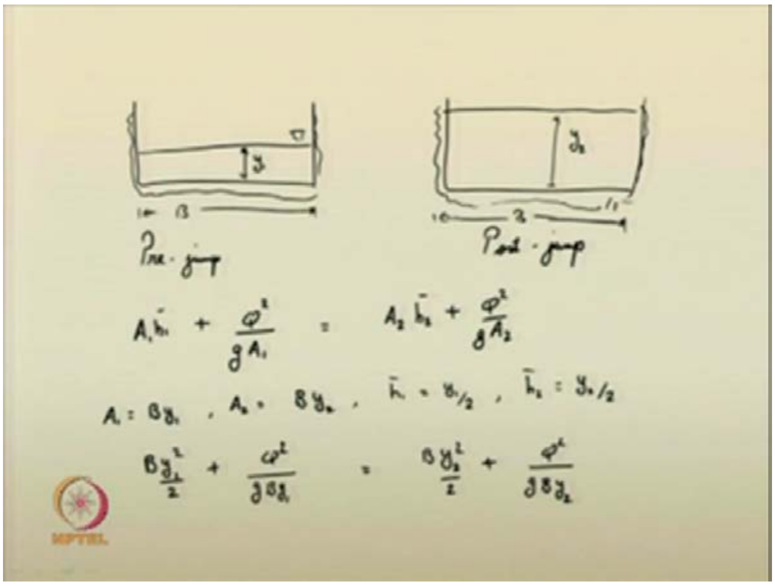

So, the rectangular channel whichever section you take, you can draw the 2 sections that is one pre jump section and this is post jump section. So, jump means rise in water. Here let us assume that the depth water is y 1. Here let us assume that the depth water it is y 2 y 2 greater than y 1, we have to analyze the characteristics now for the rectangular channel. So what are the properties? You know the specific force at the per jump section as well as the post jump section both are same, that is A 1 h 1 bar plus Q square by g A 1 is equal to A 2 h 2 bar plus Q square by g A 2.

Now for the rectangular channel section, you know A 1 is equal to B y 1, A 2 is equal to B y 2 where B is the width of the channel that will be same at the upstream and downstream sections of the jump. Then h 1 bar for the rectangular channel is nothing but, y 1 by 2 h 2 bar is equal to y 2 by 2. So, incorporating all these relationship, what will we get? We will get the following relationship now B y 1 square by 2 plus Q square by g B y 1 this is equal to B y 2 square by 2 plus q square by g B y 2, let us continue with the simplifications.

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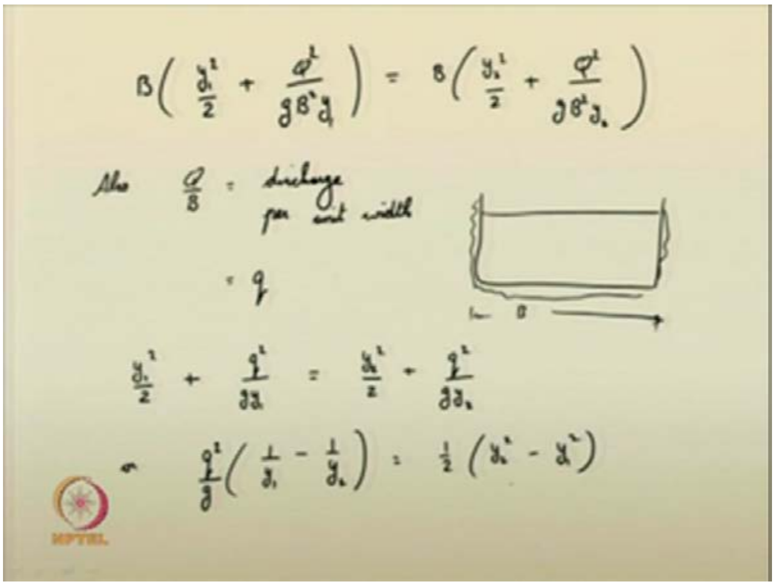

I am just taking B out now, that is B times y 1 square by 2 plus Q square by g B square, it is Q square g y 1. This quantity is equal to in the right hand side B times y 2 square by 2 plus Q square by g B square y 2. So, you can cancel off B now in these portions appropriately, you can cancel off. Also Q by B the quantity Q by B in a rectangular channel section say rectangular channel section, it is carrying some discharge. So, this Q by B quantity it is a property of the channel now. Q by B is equal to discharge per unit width of the rectangular channel. So, for a rectangular channel it is a property you can just define this quantity as a small q. So, that can we use it in our equations and all.

So, this entire equation will now become y 1 square by 2 plus q square by g y 1 is equal to y 2 square by 2 plus q square by g y 2. Or, you will get a relationship for q small q as follows; q square by g into 1 by y 1 minus 1 by y 2. That is, we have taking it into the left hand side, these quantities you are taking it into the right hand side, y 2 square minus y 1 square continue the real you can see that there are some common terms and all here.

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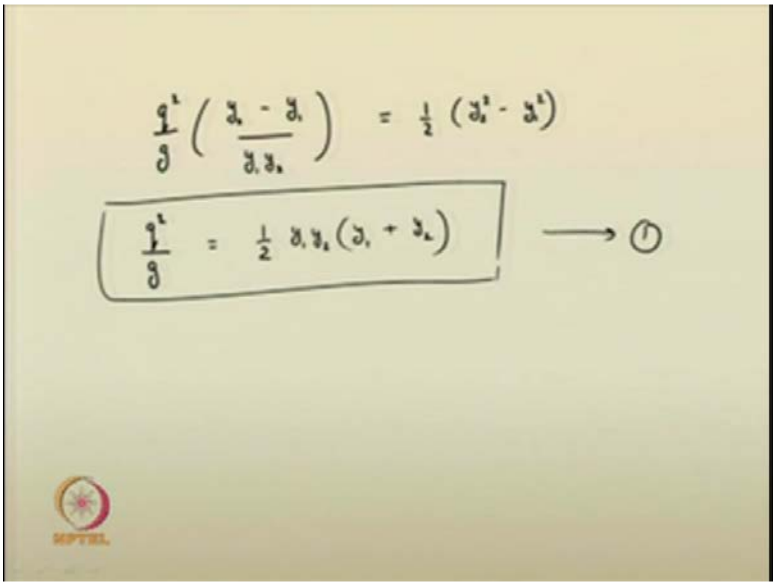

You will see this quantity becomes q square by g into y 2 minus y 1 by y 1 y 2 is equal to half of y 2 square minus y 1 square. So, you know from your mathematics A square any 2 variables A and B, A square minus B square is equal to A minus B into A plus B. The same principles you are applying logic you are applying here for the simplifying the things. You will get q square by g; this is equal to half of y 1 y 2 into y 1 plus y 2. Like this I will get a relationship. So, let me call this as equation 1. So, in you may be using them for many analyses today. So, this relationship that is the discharge per unit width of the channel, you are now relating it with the pre and post jump post jump depths of water. Like that you are able to now correlate it just for your benefit. We can just if you recall in the rectangular channel section, we had come up with a representation according to the upstream Froude number. We have come up with a relationship for the rectangular channel; we will see how it is arriving at now.

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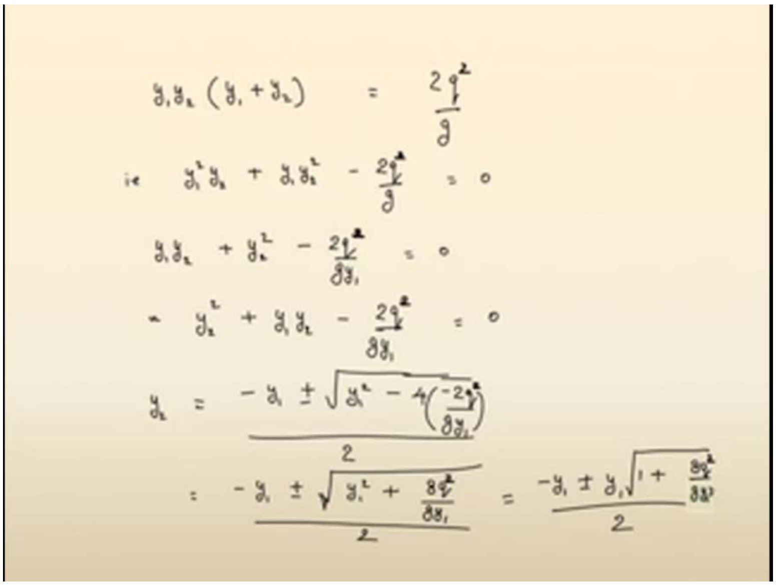

The same equation now we are going to rearrange it that is y 1 into y 2 plus y 1 plus y 2. This is nothing but, equal to twice q square by g, that is y 1 square y 2 plus y 1 y 2 square minus twice q square by g equal to 0. It is a quadratic form, you can represent the quadratic form either in terms of y 1 or y 2 as y 2 is the unknown term. In most of the cases, we do not know what is the post jump depth? Because pre jump depth, we already have it is, it is we who is releasing the water either from the sluice gate or it is we who is allowing the spill way flow and all in the over grass spill ways and all.

So, we know the pre jump depth mostly. So, we want to identify with the post jump depth. So, based on those manipulations, let me suggest this as now quadratic form means a quadratic equation in terms of y 2 or the post jump depth. So, this can be now rearranged as y 1 y 2 plus y 2 square minus 2 q square by g y 1 equal to 0. That is I just divide it by y 1 or y 2 square plus y 1 times y 2 minus 2 q square by g y 1 equal to 0. You use your mathematics principles and all. So, you will get the solution for this quadratic equation y 2 is nothing but equal to minus y 1 plus or minus y 1 times root of say 1 plus 8 q square by g y 1 cube by 2. Like this, this portion can be written from that.

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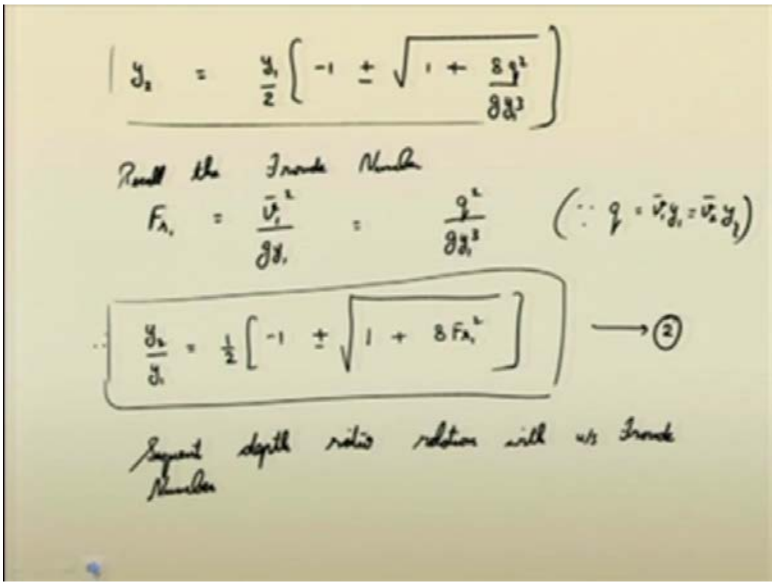

I will be getting it in the following form; I can write it easily in this form. Recall the Froude number, so Froude number in the upstream F r 1 we gave it as V 1 square by g y 1 is not it. This is nothing but, now q square by g y 1 cube, because you know that is the flow per unit width discharge per unit width q is nothing but equal to V 1 into y 1. This is again is nothing but equal to V 2 into y 2. So, the same quantities we will be using it. So, I am witting the Froude numbers in terms of discharge per unit width q square by g y 1 cube. Therefore, the above equation, this particular equation, this particular equation this becomes y 2 by y 1 is equal to half times minus 1 plus or minus root of 1 plus say q square by g y 1 cube. What is this thing? This is Froude number, so 8 F r 1 square.

So, we have already seen this relationship that is we had at that time we have just briefly mentioned that you can relate the sequent depth ratio with respect to the Froude number. And this how it is derived at, let me give this as equation number 2. So, that is the sequent depth ratio relationship upstream Froude number. Now let me go through some of the characteristics of jumps.

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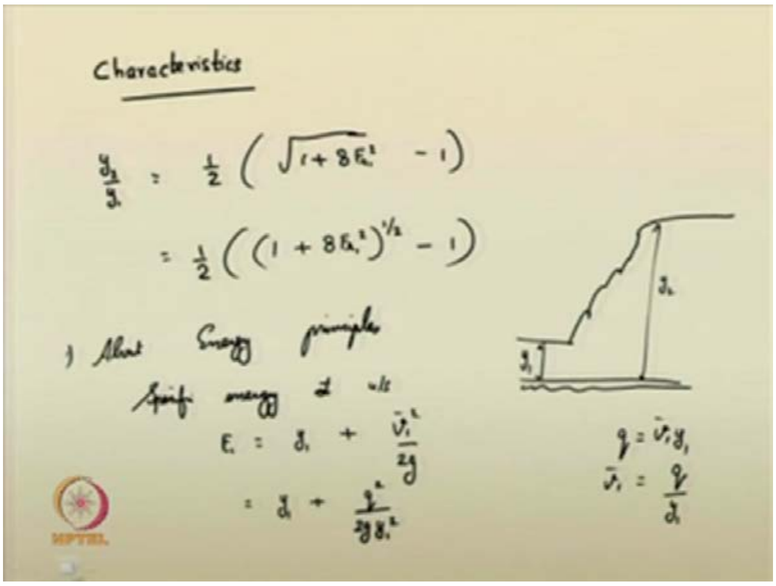

Some of the characteristics in the jumps of course, we are dealing with the rectangular channels as the heading means; it is today's class is mainly on the rectangular channel jumps in rectangular channels itself. So, the sequent depth relationship as we are aware now y 2 by y 1 is equal to half into root of 1 plus 8 F r 1 square minus 1. This I can represent it in another form; 8 F r 1 square raise to half minus 1. Like this also I can just write it. So from this equation you can, if one depth, if the per jump depth, if it is given to you, you can easily identify what is the post jump depth there is no need to go and measure it in the means if you aware of the type of flow and all you can easily now measure the quantities first one for the among the characteristics is about the energy principles.

When you apply the energy principles, let me suggest you here in the flow, in flow, in rectangular channels while dealing with hydraulic. I mean hydraulic jumps in rectangular channels and all hydraulic jump you know that it dissipates energy. Is not it? It dissipates energy means, from the upstream whatever amount of energy is there, some quantity of energy gets lost. So, whatever change in energy is there at present we are assuming that, that change in energy due to the change in specific energies at the upstream and downstream section. The other type of energy losses and all we are not taking into account here. So, we can compare the specific energies at the upstream and downstream first. So, specific energy at the upstream of the jump, we can give this as E 1 is equal to the depth of flow plus V 1 square by 2 g. So, you know what is meant by specific energy is? So, that specific energy at the per jump section, we can write it as follows; just recall the hydraulic jump. So, at this section what is this specific energy that is being measured? Now at this is the post jump section, what is the specific energy there?

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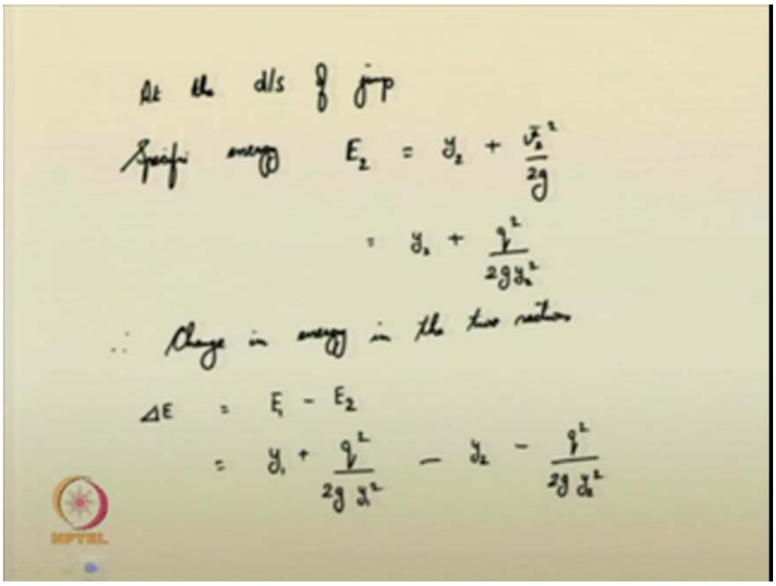

That also can be suggested at the downstream of jump specific energy E 2 is equal to y 2 plus V 2 square by 2 g. Just going back into the previous slide, if you recall V 1 the discharge per unit width, it was given as q is equal to V 1 into y 1. Therefore, V 1 is nothing but, q by y 1. So, that thing will be substituted here. I can write this as y 1 plus q square by 2 g y 1 square. Similarly, for the upstream portion also, we can write this specific energy E 2. Now this is equal to y 2 plus q square by 2 g y 2 square. Therefore, change in energy in 2 sections, the change in energy in 2 sections, again I am reiterating that the hydraulic jump, say the 2 sections whichever we are taking into account, the reach of that section it is small.

So, we are considering that whatever change in energies at those 2 sections are there, it is due to the change in the specific energies at those sections. The other type of losses, we are not taking into account again because the short the reach of that hydraulic jump is too small. So, coming back into this thing I can write the change in energy del E. This is equal to E 1 minus E 2. Naturally E 1 is greater than E 2, you know that. Otherwise the flow means you cannot accumulate the quantities or hydraulic jump itself means it is dissipating the energy. So, I am just writing the quantities again here, q square by 2 g y 1 square minus y 2 minus q square by 2 g y 2 square. Like this I can easily write it rearrange the terms.

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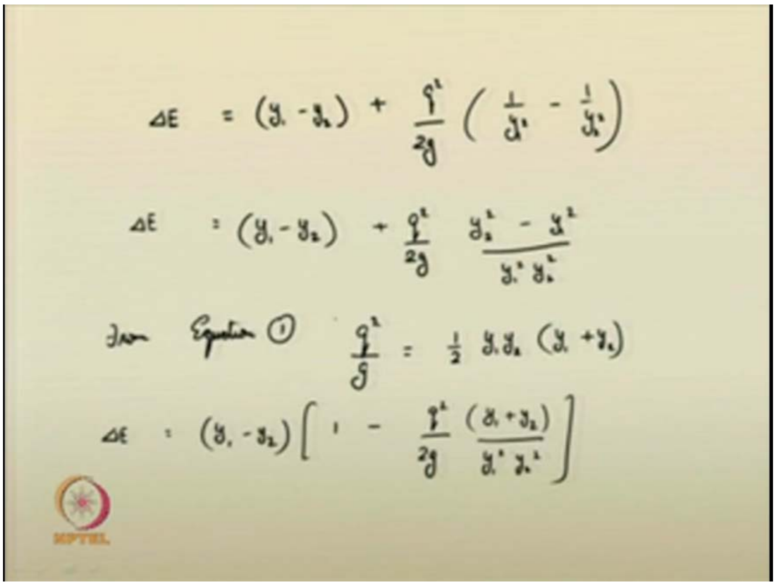

So, this becomes the change in energy del E. Now this becomes y 1 minus y 2 plus q square by 2 g into 1 by y 1 square minus y 2 square. Again y 1 minus y 2 plus q square by 2 g y 2 square minus y 1 square by y 1 square y 2 square. Like this I can write the thing from the equation 1, we have derived today q square by g is equal to half of y 1 y 2 into y 1 plus y 2. You recall them and you just substitute these quantities here. So, you will get an expression for del E now. So, I can write the quantity now del E this is equal to y 1 minus y 2 from here, this is y 1 minus y 2 into 1 minus q square by 2 g, y 1 plus y 2 by y 1 square y 2 square. So, you can write it in the other form also. That is instead of writing y 1 minus y 2 you can write it in terms of y 2 minus y 1.

Because, you naturally know that the hydraulic jump y 2 is the larger depth, y 1 is the smaller depth, so to avoid the negative things and all and also to give a more physical relevance, you write it in terms of y 2 minus y 1. Also you substitute in the expression, this expression whichever is given to you. Substitute them here, so I can write the quantity del E, del E this is nothing but equal to y 2 minus y 1 into y 1 plus y 2 whole square by 4 y 1 y 2 minus 1. Do I need to explain it? You are substituting these entire quantity in q square by g. So, the q square by g is equal to half of this quantity. So, here y 2 minus y 1 time is there, y 1 plus y 2 y 1 plus y 2 y 1 those things gets multiplied and.

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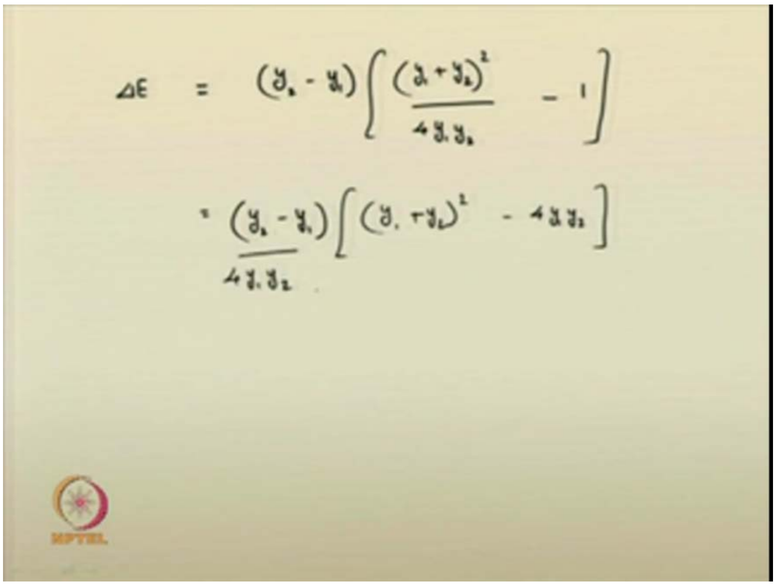

You are getting this square of those quantities that is the result it is coming into picture in this way. So I can rearrange the quantities again, y 2 minus y 1 into y 1 plus y 2 whole square minus 4 y 1 y 2 by 4 y 1 y 2, like this also I can write.

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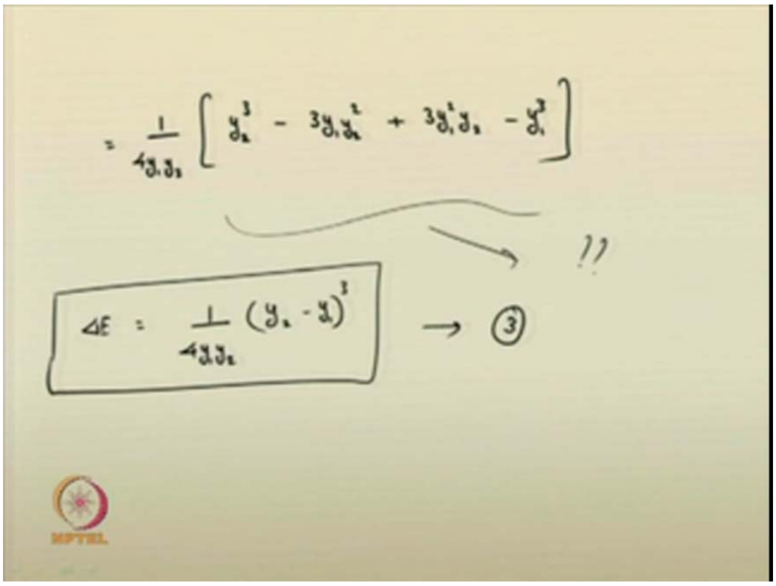

That is your simple cubic expansion of y 2 minus y 1 quantity so and y 4 y 1 y 2 into y 2 minus y 1 whole cube. So, this is the expression for change in energy. So, let me assign this as equation number 3, this is how you derive the change in energy term. If you want to express energy in terms of relative terms you can explain.

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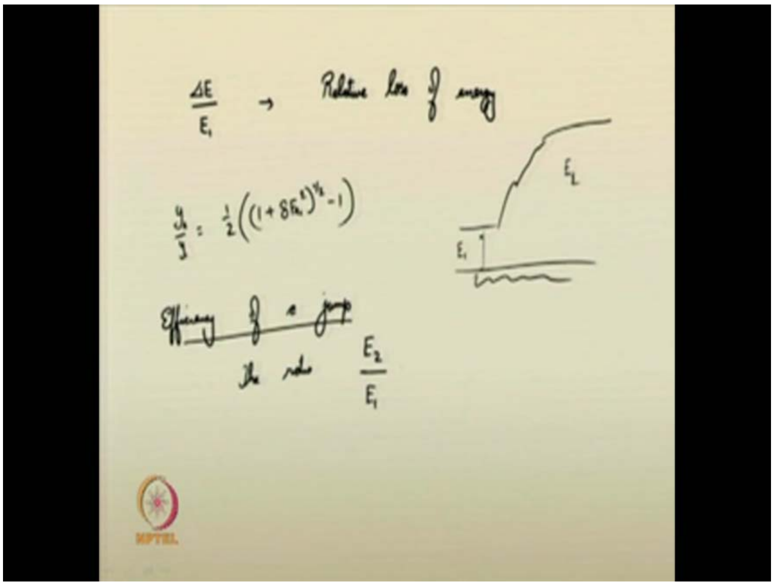

Say for example if I express this quantity, now you can define a ratio del E by E 1. So, this is called relative loss of energy. Because, you know the upstream section energy, specific energy and what is the energy loss? So, like that the non-dimensional form of loss in energy can be expressed. So, it is a relative loss in energy term. If you want to find it in terms of Froude number, that is also quite possible. You know that del E, del E is now in terms of y 1 and y 2. Similarly, E 1 it is in terms of y 1, when you if you find their ratios, corresponding ratios if you incorporate the corresponding relationship y 2 by y 1 is equal to half of 1 plus 8 F r 1 square to the power of half minus 1. If this relationship if it is incorporated in those ratios and all you will get the relative loss of energy in terms of Froude number, that is also quite useful thing.

So, efficiency of a jump, you can find the efficiency of a jump by identifying the ratio upstream specific energy by downstream specific energy. So, in the hydraulic jump this is the upstream quantity, whatever specific energy is there, E 1 whatever specific energy is there in the downstream section E 2. So E 2 by E 1 whatever change is there, whatever, what is that ratio, that is called all the efficiency of hydraulic jump. So, you can easily write again as I mentioned it earlier.

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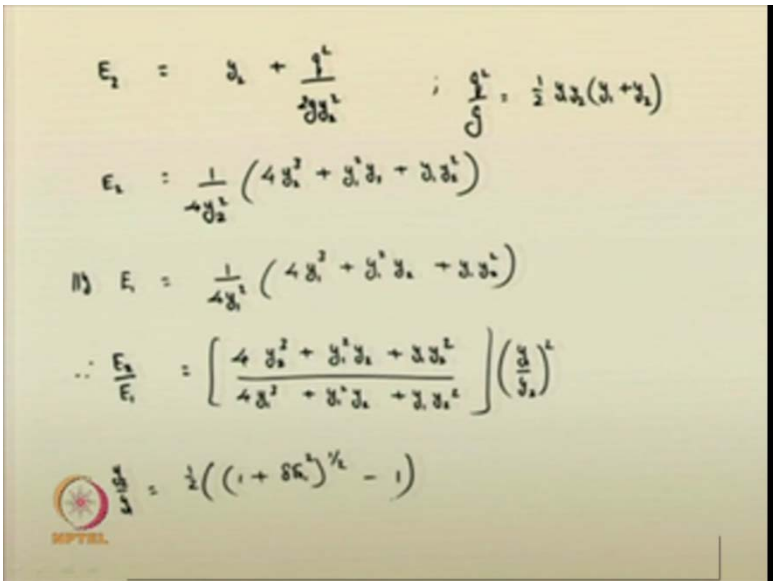

For example, you know E 2 is equal to E 2 is equal to y 2 plus q square by 2 g y 2 square. So, this can be written as you know q square by g is equal to half of y 1 y 2 into y 1 plus y 2. You incorporated, incorporate this relationship here as well. So, I will get a corresponding form 1 by 4 y 2 square into 4 y 2 cube plus y 1 square y 2 plus y 1 y 2 square, like this I will get an expression for E 2 in terms of y 1 and y 2.

Similarly, E 1 is equal to 1 by 4 y 1 square into 4 y 1 cube plus y 1 square y 2 plus y 1 y 2 square, like this you will get. So, their corresponding ratios therefore, E 2 by E 1 you will get the corresponding quantity 4 y 2 cube plus y 1 square y 2 plus y 1 y 2 square by 4 y 1 cube plus y 1 square y 2 plus y 1 y 2 square into y 1 by y 2 whole square like this I will get the corresponding means that. So, efficiency of jump relationship, the efficiency of jump you can obtain it in the particular form. This again, as I mentioned earlier, you can express it in terms of Froude number upstream Froude number. Because, you know that y 2 by y 1 is nothing but, is equal to half of 1 plus 8 F r 1 square, this quantity is minus 1. If this quantity is substituted here, you can easily get the expression in terms F r 1 square.

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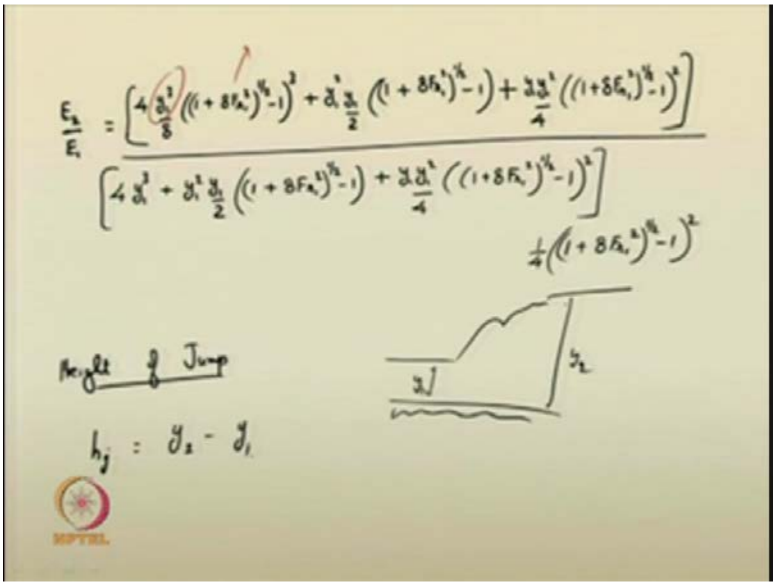

Just for a simple demonstration, I can show it to you now. E 2 by E 1 for the other quantities I expect from you to do the expansion and obtain the corresponding relationship and all. Just here also I will not be deriving it up to the final step, I just, I am just showing you how you incorporate the sequent depth ratio in the above efficiency relationship E 2 by E 1 and correspondingly. You will get all the terms, in that ratio in Froude number quantities. So, the jump efficiency ratio, it can be expressed means as I mentioned earlier now from the previous slide, if you guess have gone through this, the E 2 by E 1 term, I had expressed it in this following relationship.

So, here in a for the y 2 term, we have just substituted from this particular relationship. From this relationship, what is y 2 that has been substituted here? Similarly, wherever y 2 is coming in picture, this relationship is substituted. Similarly, this y 1 by y 2 whole square that is nothing but the inverse of this quantity that is also being substituted there, so like that, we are incorporating the terms here and I am getting the corresponding equation in the expanded form. So, here you will see that even these y 1 cube terms, these y 1 it is uniform everywhere and that will also get cancelled off.

So, you are getting this efficiency practically in terms of Froude number. So, like that you are able to, you are able to derive these efficiency quantities and all you can arrange means for similarly, for all the quantities you can arrange the things. Next we can suggest is that height necks characteristics of the rectangular jump is height of the jump.

You can think of height of the jump also. So, you have jump this is pre jump this is post jump depth. So, height of the jump h f or h j, whatever just to avoid confusion h j, let me write it, this is equal to nothing but, y 2 minus y 1. It is a simple term terminology however, this height of the jump this again can be represented in terms of some standard value, standard in the sense for this particular any rectangular channel, this specific energy of the upstream that will be the maximum depth means the specific energy it is possible.

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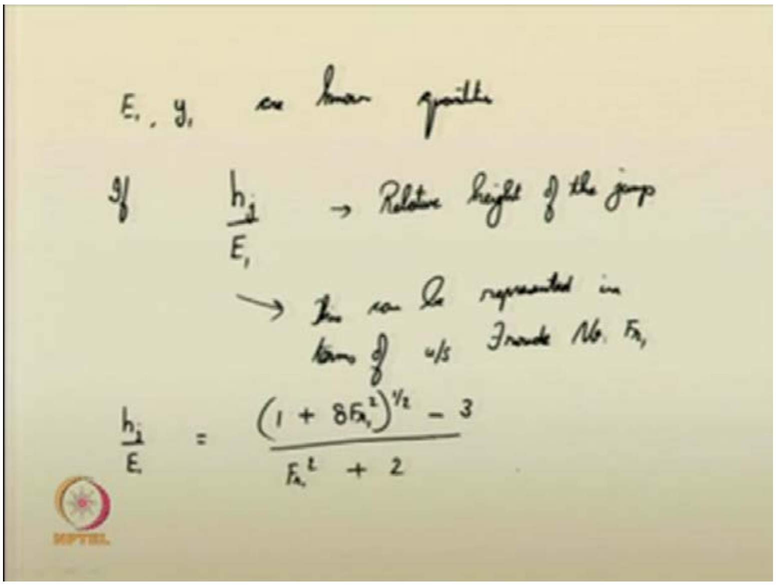

So, E 1 and y 1 are known quantities. So, if the ratio h j by E 1, if this quantity is obtained, then this quantity is called relative height of the jump. This is called relative height of the jump. So, this is a more useful value we will of course, we will see how it is useful means, we can just demonstrate them also. Again, this quantity relative height of the jump, this can be represented in terms of upstream Froude number F r 1. So, I am not deriving that, you have already seen it for the efficiency, how one can derive it. These quantities in a similar approach, you can derive this quantity also. The relative height of the jump it is a non dimensional quantity. This is nothing but equal to 1 plus 8 F r 1 square, the square root of this quantity minus 3 by F r 1 square plus 2, like this you can obtain the relative height of the jump as well. Why you are requiring these quantities? Why such quantities, non dimensional quantities are being identified? We will see that it means it is quite useful.

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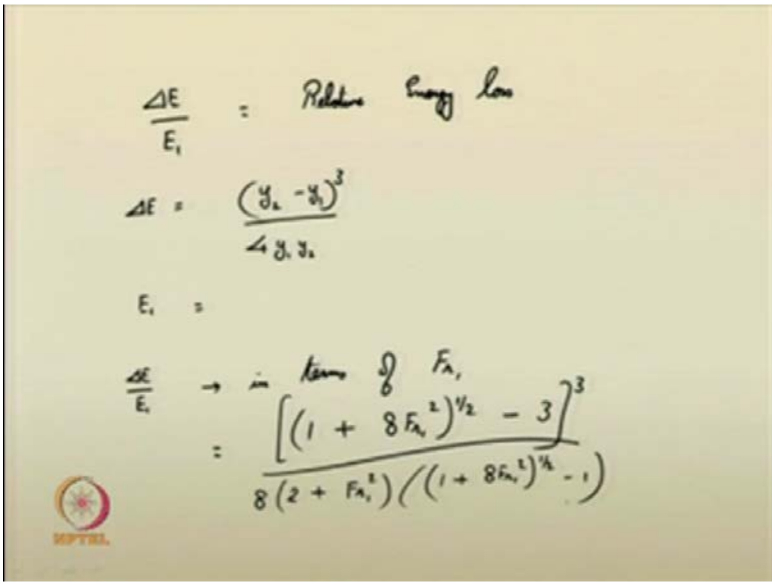

Another quantity non dimensional quantity I can suggest is, say the change in energy was already mentioned. So, this change in energy with respect to the upstream energy, this is called the relative energy loss. So, you know del E is equal to y 2 minus y 1 whole cube by 4 y 1 y 2. Energy relationship you have already derived that. So, and E 1 this can also be expressed in terms of y 1 and y 2, that we have seen in the previous slide. Here E 1 is E 2 is equal to E 1 is equal to in terms of y 1 and y 2, that is also obtained. Substitute those quantities; you will again get this relationship del E by E 1 in terms of Froude number at the upstream section. So, again I am not going to derive it. You have already seen the derivation process.

So, I will just write the final form 1 plus 8 F r 1 square, the square root of this quantity minus 3 whole cube divided by 8 times twice F r 1 square 1 plus 8 F r 1 square raised to half minus 1. So, if we have these ratios, these corresponding ratios in terms of the Froude number, upstream Froude number, then the suggestion is that, the upstream Froude number is the only quantity you require from that. You can easily analyze the jump in the rectangular channel, what are the peculiarities of these quantities? del E by E 1 as well as h j by E 1.

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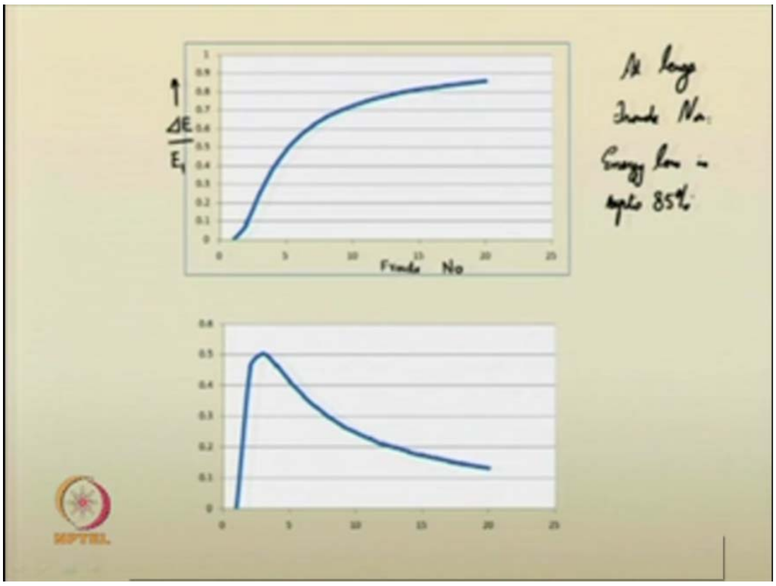

I can just show some 2 plots here. You see here the first one. This corresponds to say the x axis, this is the Froude number and the y axis, and this is the relative energy loss. It is the relative energy loss represented del E by E 1. So, if the Froude number, if it is slowly increased from 0 to 20, you can see the curve following the path in following form right the path is being followed. So, you can see that at very large Froude numbers, that is Froude numbers of 20, and all that will yield a considerable energy loss, that is del E by E 1. It is very high up to 0.9 or means 85 percentage. So, that can be easily inferred from these pictures at large Froude numbers, energy loss is up to 85 percentage, if you recall in the last class in the strong jumps and all you have see that, means if this strong jumps where in the regions of very high Froude number, numbers of the high upstream Froude numbers, there you will see up to 85 percentage of energy loss, you can witness. This is being obtained in the following form the curve we have simply.

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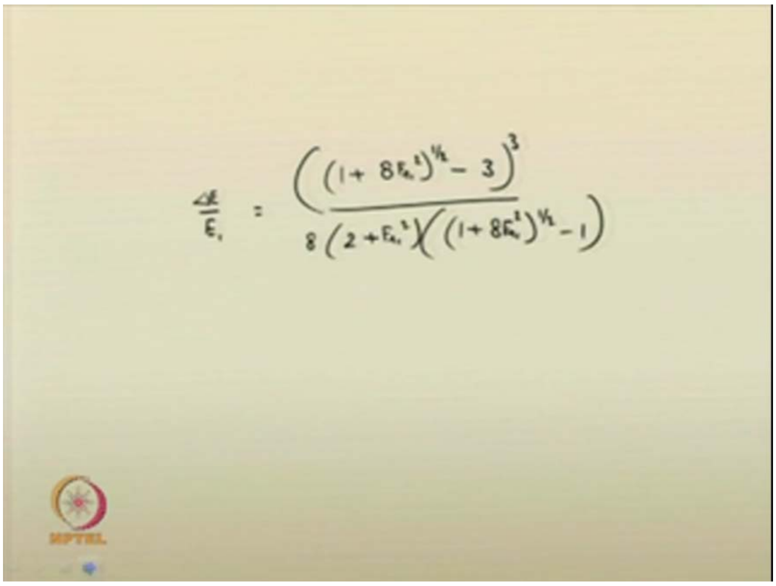

This curve has been derived from the relationship del E by E 1 is equal to 1 plus 8 F r 1 square to the power of half minus 3. The whole quantity raised to 3 by 8 times 2 plus F r 1 square 1 plus 8 F r 1 square the root minus 1.

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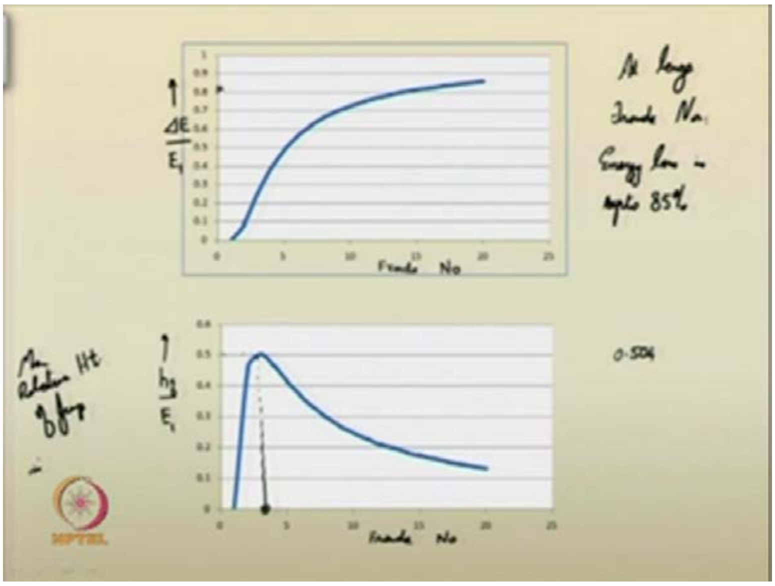

So, using this relationship we had developed this particular curve. The second curve, the second curve is related to again Froude number in the x axis and the relative height. We have already mentioned that the relative height h f by E 1 quantity on the x or y axis. So, this quantity also again we can suggest it in the following form. You will see that the curve increases suddenly, it increases for it reaches up to a maximum of 0.5 around 0.504, I think. So, then it reduces it gradually reduces like this curve. So, the peculiarity of this curve, from this curve also you can identify that the maximum relative energy, that is the maximum relative height of this jump, it is maximum relative height of jump is possible at a Froude number quantity of around.

So, this you can infer it. It may be around means between 0 and 5 that is may be roughly around 3 and all. This can be roughly at around 3, 3 or 3 point something. So, this you can easily derive those curve and check them. So, that way we have just seen some of the characteristics of the jumps. We can have a brief quiz also today unlike the previous quizzes today in the quiz; we have as we have seen the characteristics of the rectangular jumps and all. Like a jumps in the rectangular channels, I am going to ask some problems, numerical problems you just need to apply those formulas whichever are there, even you can just go through the book and just apply those formula and try to get the answer.

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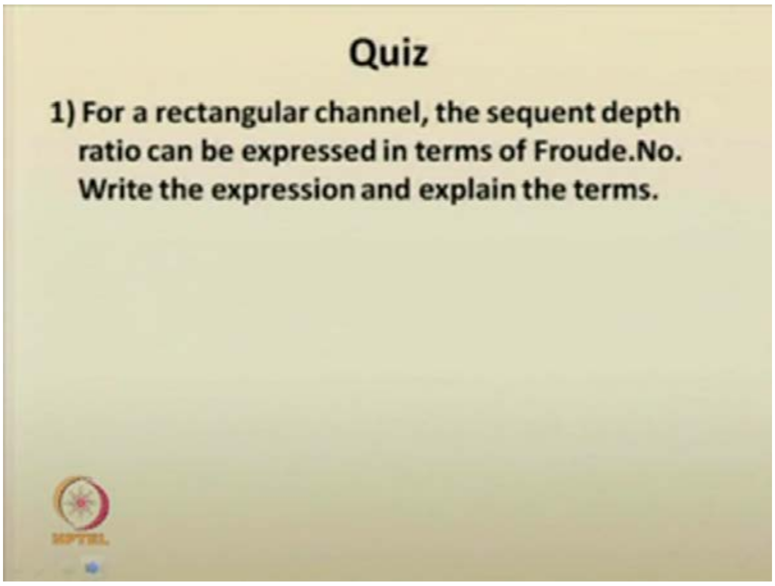

So, the quiz is as follows; the first question for a rectangular channel, the sequent depth ratio can be expressed in terms of Froude number. Write the expression and explain the terms. This is a first question and it is a very simple. You will take hardly 20 seconds I think.

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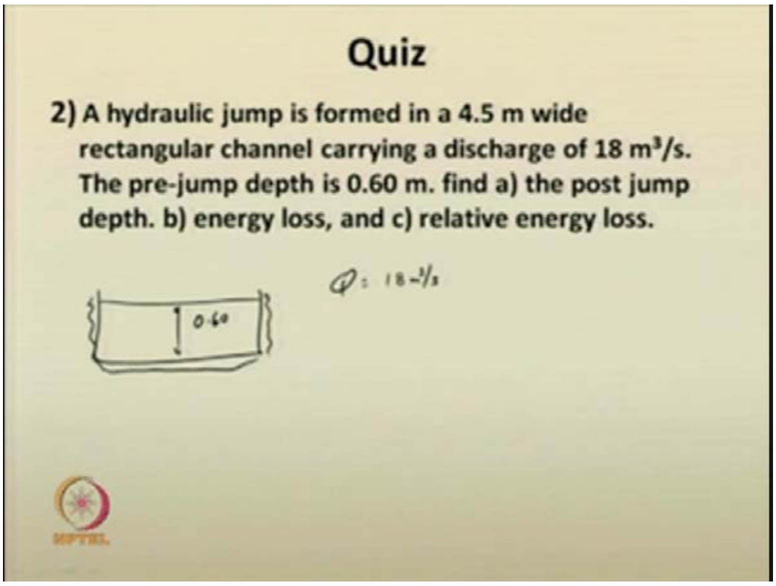

The second question, please note that a hydraulic jump is formed in a 4.5 meter rectangular wide channel, 4.5 meter rectangular channel, the channel is carrying a discharge of 18 meter cube per second. The pre jump depth is 0.6 meter. Find a- the post jump depth, b- energy loss and c- relative energy loss. It is a very simple thing that is we have already seen the relationships. We can just quickly work it out, there is no hurry here, we can just work it out and tell me the answer. Of course, you need to apply it in the field; these are the some field problems; so the solution for the first question.

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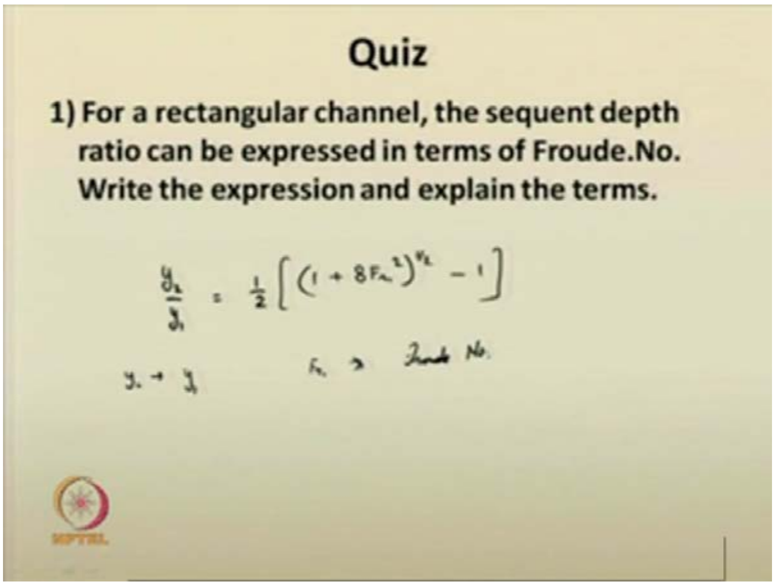

You know the expression y 2 by y 1; this is equal to half of 1 plus 8 F r 1 square. This quantity is square rooted minus 1. So, y 2 is the sequent depth of y 1, F r 1 square, F r 1 is the Froude number, it is a Froude number and all other terms means, as we have asked you to explain the terms, the terms are now explained.

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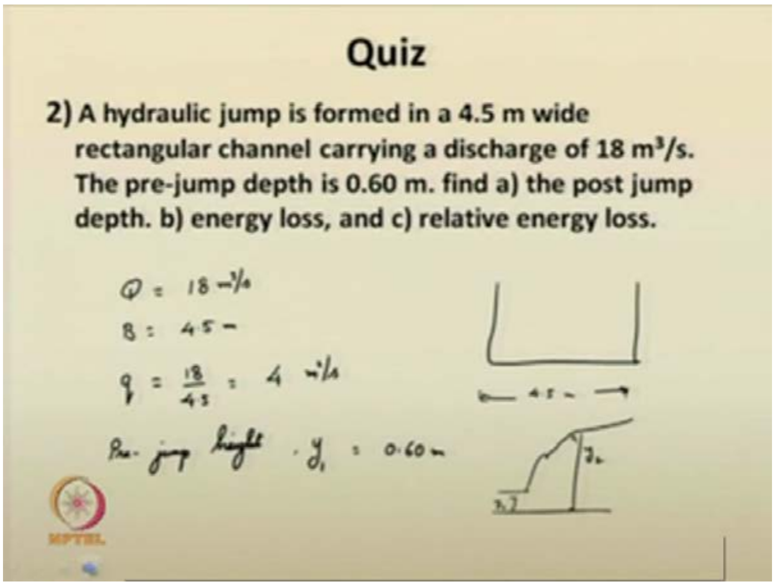

Answer for the second quiz, it can be given in the following form. You have been given q, the discharge q in the rectangular channel as 18 meter cube per second. Let me just draw the cross section of the rectangular channel with width 4.5 meter. So, you have width of the rectangular channel as 4.5 meter. Therefore, you can easily compute discharge per unit width. This can be given as 18 by 4.5. This is nothing but, 4 meter square per second. You are also been provided the data pre jump height pre jump height, that is y 1 is equal to 0.6 meter. This database also given to you, I hope you can recall what is y 1. This is pre jump height, this is post jump height. So, we are requested to find y 2. We are also requested to find energy loss and relative energy loss.

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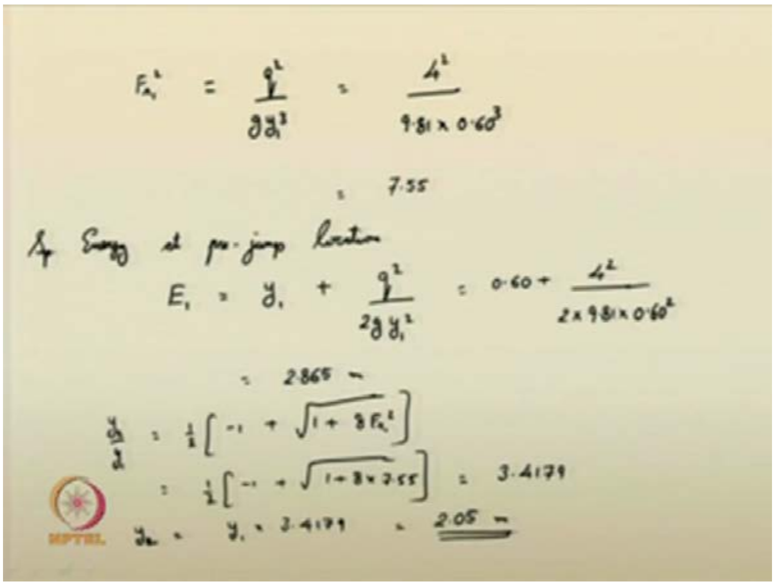

How will you find the quantities? You can easily find the Froude number in the according to the pre jump depth. So, this is can be given as q square g by y 1 cube. So, you know small q is 4. So, 4 square by 9.81 into 0.6 cube. This is equal to, you can compute it, it is coming as 7 roughly 7.55. You can also compute specific energy at pre jump location. So, I can compute this as E 1, E 1 is nothing but equal to depth plus q square by 2 g by 1 square. So, using this relationship, you will get 0.6 plus 4 square by 2 times 9.81 into 0.6 square. So, this is coming as 2.865 meter.

Similarly for the rectangular channel, you had seen the sequent depth ratio y 2 by y 1. This is equal to half of minus 1 plus root of 1 plus 8 F r 1 square. Substitute this relationship F r 1 square is 7.55. So, I will get this quantity as 3.4179. So, you have the pre-jump depth as 0.6 meter. Therefore, post jump depth y 2, this will be equal to y 1 into this particular ratio 3.4179. So, this is coming to be about around 2.05 meter. Therefore, for your post jump the depth is 2.05 meter.

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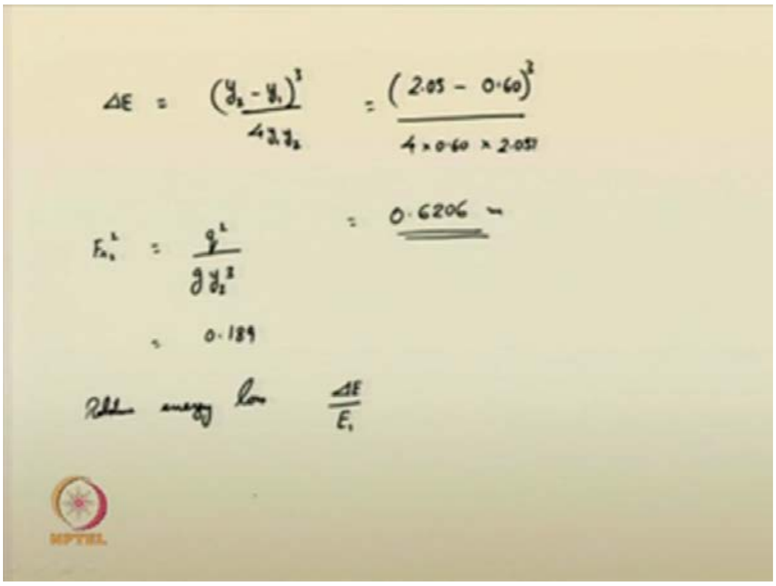

Your energy loss del E, this is given as y 2 minus y 1 whole cube by 4 y 1 y 2. As you have the data of y 1 y 2, substitute them. I am just directly substituting them 2.05 minus 0.6 whole cube by 4 times 0.6 into 2.05. So, you will get roughly 0.6206 meter as the change in energy. You can also compute Froude number in the downstream section. This is given as q square by g y 2 cube. I am not going to further extend the calculations it is coming to be around 0.189, the relative energy loss can also be computed. It is nothing but del E by E 1. You can compute them, you have E 1, you have del E. Just see what could be that value?

That way we are just concluding today's lecture. Next in the next lecture also, we will be continuing with the hydraulics jump. So, it is up to you now to prepare thoroughly on all the aspects of this open channel hydraulics or the hydraulic portions for your benefits and all for your curriculum requirements and all. So, with best wishes we will be continuing in the next class also.

Thank You.