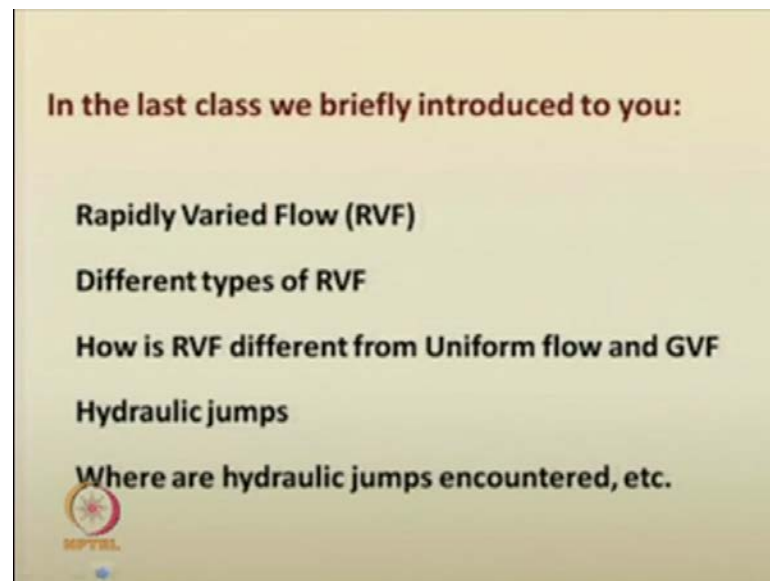


Advanced Hydraulics
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Module - 4
Hydraulics Jumps
Lecture - 2
Theoretical aspects of Hydraulic Jump

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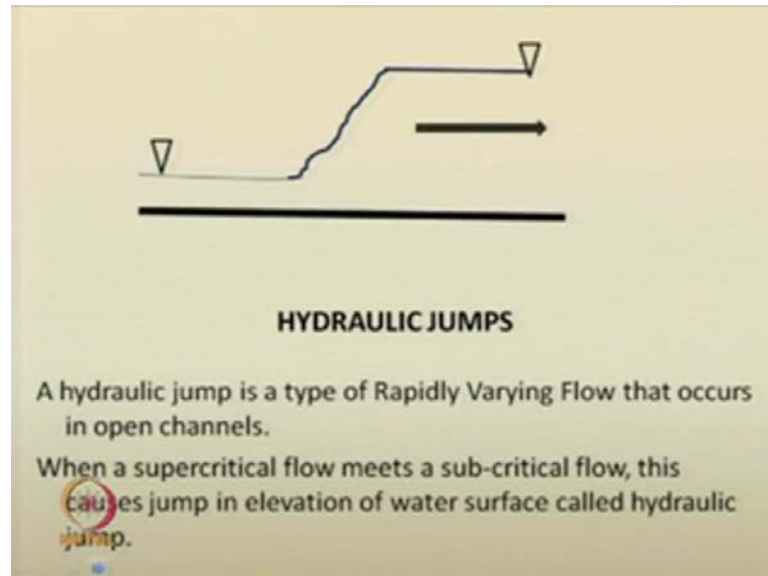


Welcome back to our lecture series on advanced hydraulics. Last class we started the module 4 on hydraulic jumps, if you recall them. So, especially in the last class, we introduced to you the concepts on rapidly varied flow. What are the different types of rapidly varied flow? We also discussed on what is the difference between rapidly varied flow, uniform flow and gradually varied flow. For example, in uniform flow and gradually varied flow, you were able to take the parallel flow assumption whereas in rapidly varied flow it is not possible and you have also seen that the streamlines have quite a good curvature.

You were also introduced to you, you were also introduced on hydraulic jumps. We also gone through in which all locations or in which all situations hydraulic jumps are formed. Today as we discussed in the last class, we will go through the theoretical aspects on the hydraulic jumps. Some of the theoretical aspects that are required, some of

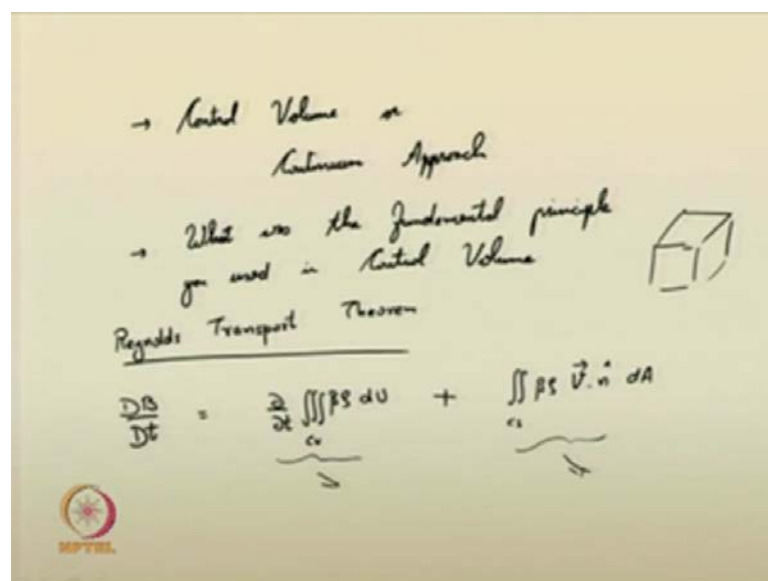
the fundamental principles, how we can employ them, even for analysing hydraulic jump this is we will see them.

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So, just recalling back hydraulic jumps means, it is a type of rapidly varying flow that occurs in open channels or the definition it was given like that, when a super critical flow meets a sub critical flow, this causes a jump in elevation of water surface and it is called hydraulic jump. So, throughout our open channel hydraulics course.

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We have used the control volume or continuum approach for analysing the fluid flow in open channels. So, the same approach will be again maintained here for analysing the hydraulic jump as well. What was the, what was the fundamental principle for any phenomenon, what was the fundamental principle you used in control volume approach. If you recall them for any control volume, we have suggested that the Reynolds transport theorem, Reynolds transport theorem can be used for analysing any extensive property in a control volume. If you are taking the hydraulic jump as a control volume, the entire jump portion if you are taking it as a part of the control volume and if you are trying to analyse that, the same Reynolds transport theorem should hold good while analysing the flow phenomenon in hydraulic jump.

So, what was the Reynolds transport theorem? If you recall that, the material derivative of any extensive property B , this is equal to the rate of change of that extensive property stored inside the control volume plus the net out flux of this extensive property across the control surfaces of the control volume. So, if you have a cubical shape as a control volume, you know that it has now 6 surfaces that is forming this control volume. 6 surfaces are there. So, what is the net out flux from these surfaces that is being given by this particular term? The change in the extensive of the rate of change of extensive property, extensive properties stored inside this control volume that is given as the first term in the Reynolds transport equation if you recall them. So, we have seen that the some of the extensive properties are mass, momentum, force, etcetera, and all similarly, again we are going to use the Reynolds transport theorem for analysing the hydraulic jump.

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$$\frac{DB}{Dt} = \frac{\partial}{\partial t} \iiint_{cv} \beta s dV + \iint_{cs} \beta s \vec{V} \cdot \hat{n} dA$$

B : Extensive Property
 β : Intensive Property

Conservation of MASS

$B = m, \beta = 1$

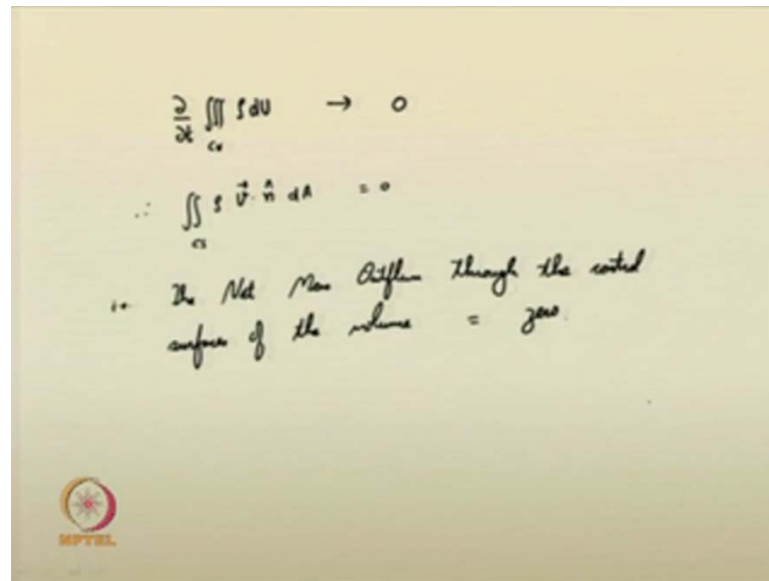
$$0 = \frac{\partial}{\partial t} \iiint_{cv} s dV + \iint_{cs} s \vec{V} \cdot \hat{n} dA$$

Diagram: A rectangular control volume (dashed red line) is shown in an open channel flow. The flow is represented by a wavy line (hydraulic jump) moving through the channel. The channel boundaries are solid lines.

So just let me write it again $\frac{DB}{Dt}$ is equal to the rate of change of this first term is the rate of change of extensive properties stored inside the control volume plus the second term is, the net out flux of the this extensive property through the control surfaces. You know B is equal to the extensive property β was called as the intensive property. Now what happens if you are taking say conservation of mass, if you are taking the conservation of mass property, then the extensive property B becomes the mass of the liquid. Then you can see that, say the hydraulic jump, say in this hydraulic jump, if I am going to consider this, if I am going to consider the control volume in the following form say, this control volume is now enclosing the hydraulic jump. So, in our open channel flow predominantly we consider the flow as A 1 dimensional in the flow direction.

Therefore, this particular control volume is having only 2 surfaces which allow flow through it that is on the left side and on the right side. So, that phenomenon we will be taking into taking into care here. So, when the in the Reynolds transport theorem, if you have taken the extensive property B as a mass then I hope you remember the intensive property β is 1. Then subsequently the conservation of mass equation as per the Reynolds transport theorem was given as 0 means the net change, the rate of change of their extensive property that will be 0 mass can either be created not destroyed, so based on that same principle, now you will be getting $\rho \frac{D}{Dt} U$ plus control surface $\rho \vec{V} \cdot \hat{n} dA$. So, this equation we have to apply it for the following control volume.

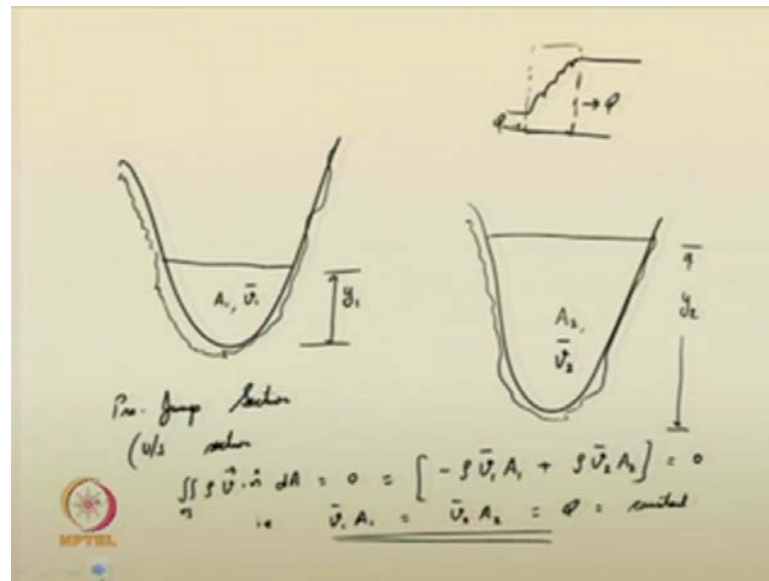
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$$\frac{\partial}{\partial t} \iiint_V \rho dU \rightarrow 0$$
$$\therefore \iint_V \rho \vec{V} \cdot \hat{n} dA = 0$$

\therefore The Net Mass Outflow through the control surface of the volume = zero

How we are going to what is the first term? The first term due to out of control volume ρdU as inside the control volume, you have that control volume. Now inside that control volume, you may see that here inside this control volume, you are not adding further water or you are not deducting among certain water within this control volume. So therefore, the first term will be 0, so it becomes a 0 quantity. Therefore, your control volume equation Reynolds transport equation becomes for the conservation of mass becomes $\rho \vec{V} \cdot \hat{n} dA$ is equal to 0, that is the net out flux of mass through the control surfaces of the hydraulic jump that will be 0. That is, the meaning of this term that is the net mass out flux through the control surfaces of the volume is equal to 0. That is the meaning of the entire equation now. So, as we mentioned earlier, there are only 2 control surfaces that allow flow through the control volume, so which are they.

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So, we have in the pre jump section or the upstream section, that is the pre jump section or the upstream section of the jump. So, again just for your benefit, I am just drawing the hydraulic jump. It shows that, so your control volume is like this. So, on the left hand side section will be may be in this form. So, the depth of flow let me give this as y_1 , the area of flow is A_1 . And on the right hand side, this control volume this surface allows flow and as you knows depth of flow increases in the downstream section. So, we are suggesting a higher elevation and it is being given as y_2 and the area of cross section is given as A_2 . So, what does this equation mean then? Therefore, your equation $\rho \vec{V} \cdot \vec{n} dA$, this integration is equal to 0 it is nothing but you have now 2 sections left hand side and right hand side. So, what is the net out flux to these 2 surfaces that you have to combine now?

So, as we know that we use to take the average velocities in the sections. So, this average velocity into area can be combined for this entire integral term. As in the left hand side the outward normal is opposite to the flow velocity direction therefore, this is a negative quantity. So, I can write this quantity, now $\rho \bar{V}_1 A_1$ plus $\rho \bar{V}_2 A_2$ is equal to 0, that is $\bar{V}_1 A_1$ is equal to $\bar{V}_2 A_2$, this is equal to discharge Q . This assumption based on the incompressible flow assumption, the density of the liquid is not changing. Based on that assumption, we are able to write it in the following form. So, we are, you are a getting constant discharge that is the discharge on the down upstream side of the jump as well as on the downstream side of the jump. Both are same, both are

having same discharge. Here also it is Q here also it is Q, that is the meaning of this equation. So, from the conservation of mass equation you are now getting the following relationship.

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Applying the MOMENTUM PRINCIPLE

$B = m\vec{V} \quad , \quad \beta = \vec{V}$

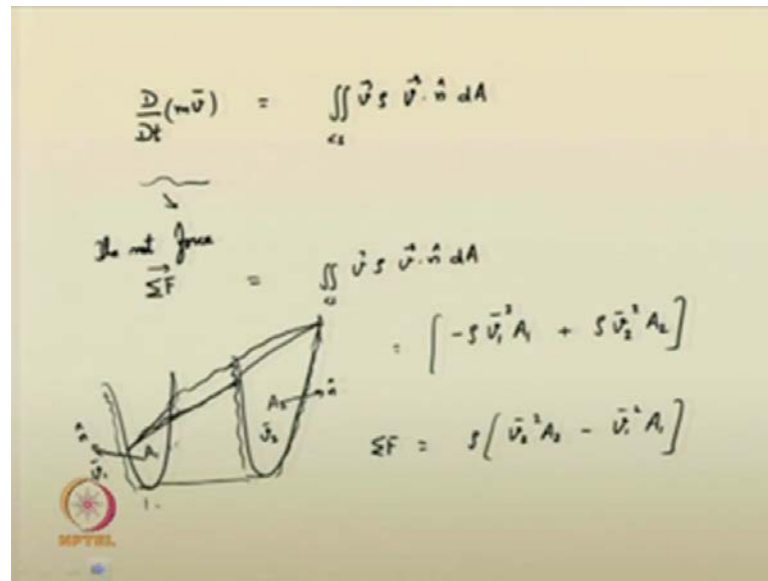
$$\frac{D(m\vec{V})}{Dt} = \underbrace{\frac{d}{dt} \int_V \vec{V} \rho dU}_{\text{Rate of change of momentum stored inside the control volume}} + \underbrace{\int_{CS} \vec{V} \rho \vec{V} \cdot \vec{n} dA}_{\text{The net out flux of momentum across control surface of the volume}}$$

$\frac{d}{dt} \int_V \vec{V} \rho dU = 0 \quad (\because \text{We are not adding or deducting momentum inside the control volume})$

So, what happen I mean applying the momentum principle, so if you apply the momentum principle to the control volume, same control volume if you apply the momentum principle, you know that the extensive property B is now equal to mass into velocity. Therefore, the intensive property is taken as V. Again the Reynolds transport theorem the material derivative of B that is m V this is nothing but equal to dou by dou t of volumetric integral V rho d U plus the surface integral, that gives you the net momentum flux that passes through the control surfaces of the control volume.

So, this is the first term shows the rate of change of momentum stored inside the control volume, and this term shows the net out flux of momentum, net out flux of momentum across the control surfaces of the volume. So, again let me suggest to you, you are not adding any momentum inside the control volume or you are not deducting any momentum from the control volume inside the control volume you are neither adding it nor deducting. So, we can avoid the first term that is we can suggest the first term as 0 dou by dou t of because we are not adding or deducting momentum inside the control volume. So, inside the control volume, we are not neither adding nor deducting it. So, that first term will be 0.

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$$\frac{D}{Dt}(m\vec{v}) = \iint_{CS} \rho \vec{v} \cdot \vec{n} dA$$

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$$\text{The net force } \Sigma \vec{F} = \iint_{CS} \rho \vec{v} \cdot \vec{n} dA$$

$$= \left[-\rho \bar{v}_1^2 A_1 + \rho \bar{v}_2^2 A_2 \right]$$

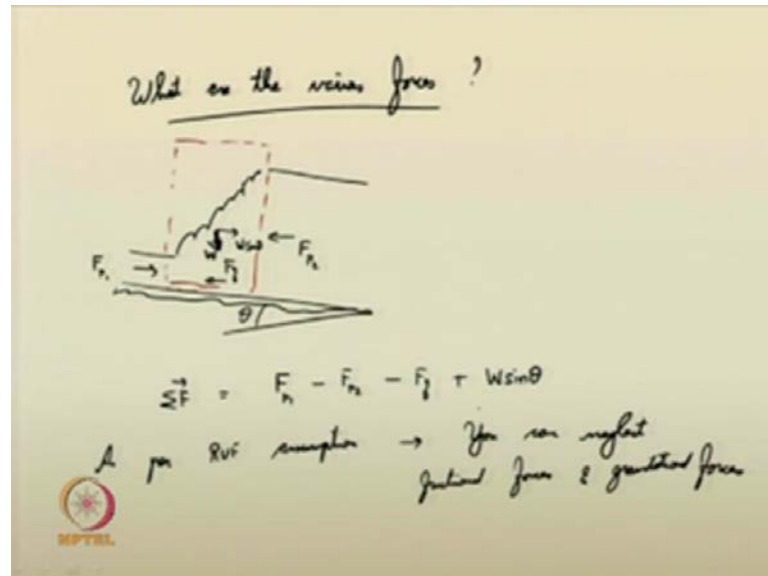
$$\Sigma \vec{F} = \rho \left[\bar{v}_2^2 A_2 - \bar{v}_1^2 A_1 \right]$$

So this means that your momentum equation now becomes $\frac{D}{Dt}$ of mV is equal to the net out flux of momentum through the control surfaces of the control volume. What is this quantity? The material derivative of momentum, that means, the rate of change of momentum in a control volume that is that will give you the net force acting in the control volume. This gives you the net force ΣF and the second term, so I can write this entire quantity as ΣF is now equal to $\rho \int \vec{v} \cdot \vec{n} dA$. So, you have now 2 surfaces. Again only 2 surfaces allow the momentum flux to pass through it. So, you have the upstream section where the flow depth of water is like this you have the downstream section where the depth of flow may be considered at a higher elevation compared to the first one and you have the jump occurring in the channel. So, there is a jump it is, it may not be a realistic figure which we have drawn here. So, I can just draw it very close this entire section now upstream.

So, the jump between the, between that short reach it can be given here, here the area is A_2 , here the area is A_1 , V_1 is the velocity, here V_2 means we are averaging it. I hope you remember the $V_1 \bar{V}_2$, the discharge is same in both the sections. So, the net momentum out flux thorough the control surfaces; this can be given as here the outward normal vector is in this direction of that surface whereas, here it is in this direction of the flow itself. Therefore, the left hand quantity will be a negative quantity. So, I can now write it as $\rho \int V_1^2 A_1$ plus $\rho \int V_2 V_1 V_2 \bar{V}_2^2 A_2$. So, this

will give you the momentum equation that is $\sum F$ is equal to $\rho V_2^2 A_2 - \rho V_1^2 A_1$. Like this, I can easily write the quantity now.

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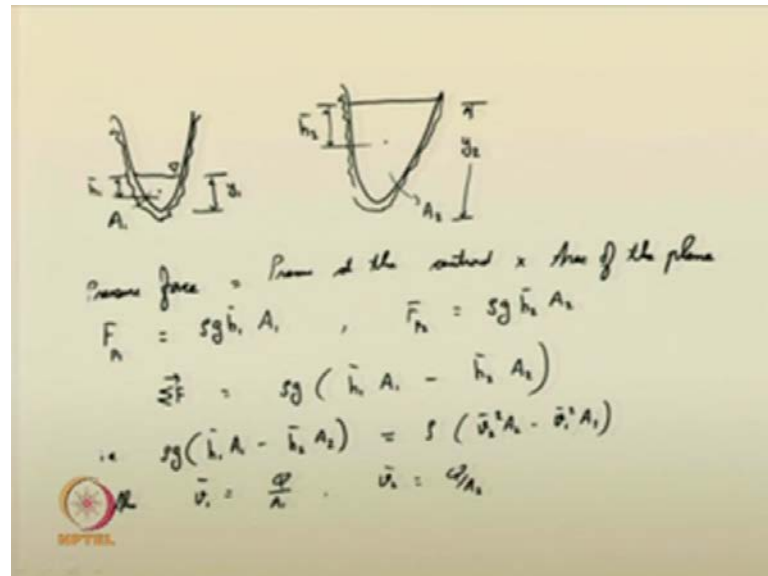


What are the various forces acting? What are the various forces acting in the control volume for the hydraulic jump for your benefit? I am again repeatedly drawing the same figure. So, the control volume like this you have a pressure force on this surface acting in the flow direction $F_P 1$, we can give it as $F_P 1$. You have pressure force in this direction opposite to the flow direction here on the right control surface. All other forces on the control surfaces they are all other pressure forces they are neglected means they gets cancelled off because there is no flow occurring. So, these 2 here will be remaining there, you have the friction force F_f acting in the opposite direction of the flow you have the weight of liquid acting down perpendicularly down.

Let us assume that the slope is theta, and then this weight will be having component $W \sin \theta$. So, the net forces acting in the control volume, this can be given as $F_P 1$ minus $F_P 2$ minus F_f that is friction force plus $W \sin \theta$. So, like this you can write the left hand side of that equation. So, the net force will be given in the following form as per our hydraulic jump definition or as per rapidly varying flow assumption you can neglect because of the short reach frictional forces. You can neglect frictional forces and gravitational forces in the flow direction. Please note that frictional forces and

gravitational forces in the flow direction, we can neglect them. Therefore, your sigma F becomes F P 1 minus F P 2.

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I hope you recall what are the pressure forces, how you compute them the pressure forces. You have now 2 cross sections; the pre jump cross section with lower elevation depth of flow is y_1 , the depth of flow is y_1 here. You have the post jump section where the depth of flow is higher we are giving this as y_2 . So, this area of cross section here it is A_1 here, it is A_2 . Now if you recall that if you observe for this particular area A_1 , if from the top of the water surface if h_1 bar is the height to the centroid of this area similarly, if h_2 bar is the height or distance from the water surface to the centroid, then you can use the following relationship. That is the pressure force is equal to what is the pressure force equal to, that is pressure at the centroid into area of the plane. This we had clearly explained it or we have done it many times.

So, again we are repeating same thing, we are following them. Therefore, F P 1, I can write this as the pressure there is $\rho g h_1$ bar, now into A_1 . Similarly F P 2 this will be equal to $\rho g h_2$ bar is the distance to the centroid from the water surface. There in the second section or in the post jump section and A_2 is the area of cross section at the post jump section. So, you have sigma F is equal to ρg times h_1 bar A_1 minus h_2 bar A_2 . So, you combine both the equations, this ρg this expression as well as the previous expression sigma F is equal to ρ times V^2 square A_2 minus A_1 square A_1 . So,

combine them both of them you will get $\rho g h_1 A_1 - \rho g h_2 A_2$. This is equal to $\rho V_2^2 A_2 - \rho V_1^2 A_1$. So, rearrange the terms ρ , ρ will get cancelled off g you can take it to the denominator here. Also from your continuity equation relationship $V_1 A_1$ is nothing but equal to Q , $V_2 A_2$ is equal to Q by A_2 . So you substitute these terms also here, now in the terms.

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$$g(h_1 A_1 - h_2 A_2) = Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$$

$$\Rightarrow \boxed{A_1 h_1 + \frac{Q^2}{g A_1} = A_2 h_2 + \frac{Q^2}{g A_2}}$$

Eqn 1 Eqn 2

Specific force $F_s = \frac{Q^2}{g A} + h A$

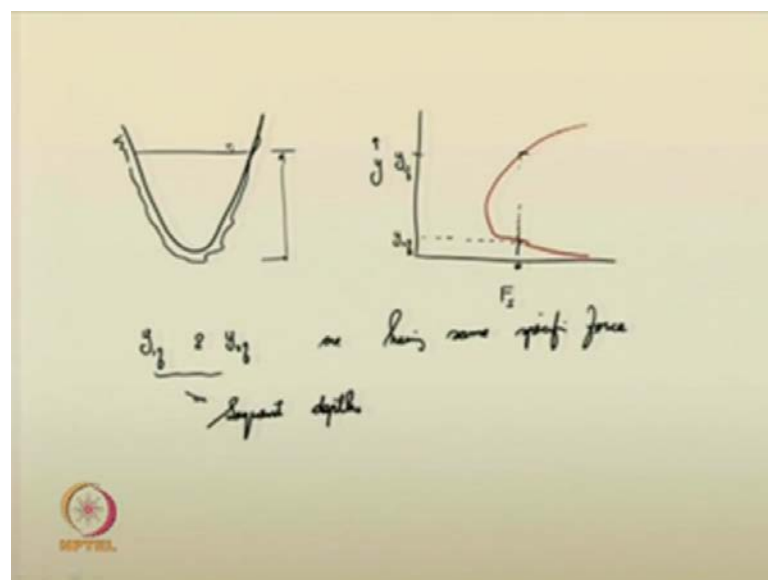
At section 1 & 2

So, I can write the quantities now $g h_1 A_1 - g h_2 A_2$ this is equal to $Q^2/A_2 - Q^2/A_1$ or you will get $A_1 h_1 + Q^2/g A_1 = A_2 h_2 + Q^2/g A_2$. So, this like, this expression you will get relationship. You can say relationship, you will get I can just box it for your benefit. Can you tell me what are the terms this is nothing but I can say some quantity in section 1 and some quantity in section 2 they are same that is being specified recall our first module especially the lecture 7 and lecture 8, where we had given you the concepts on specific force.

In the first module, we had described you what is meant by specific force and all. So, specific force was defined as F_s is equal to $Q^2/g A + h A$. At any given section, the specific force can be computed using these relationships. So, this is the first term is the momentum of flow passing through the channel section per unit time per unit weight of the water. Second quantity that also we had described at that time it is force per unit weight of water. So, these quantities if you analyse it, you will see that from the

momentum equation for the hydraulic jump you are getting that at section 1 and section 2, at section 1 and section 2 of the hydraulic jump you are having the same specific forces. So, the specific force is same at the upstream section and at the downstream section of the hydraulic jump. So, that is the peculiarity or that is the principle now from which we are going to analyse the hydraulic jump. That is the most important principle here for analysing the hydraulic jump that is the specific forces on the upstream as well as on the downstream they are the same. So, if you recall that at that time also we had mentioned that the specific forces between 2 adjacent cross sections they will be same for such steady flow conditions and all.

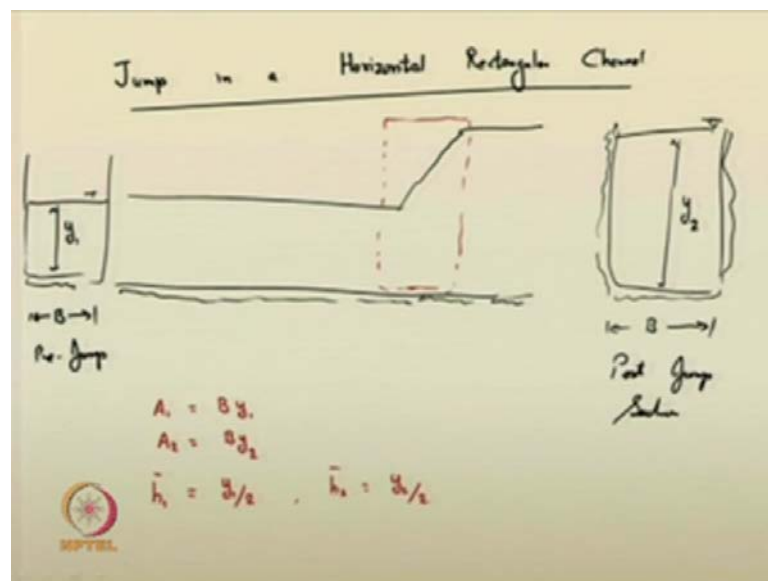
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And also for any given cross section for any given cross section if you have a depth of flow, one can easily draw the corresponding specific force versus depth of flow for any given section. I hope you recall them, so you were getting a curve of this particular nature like this. So, same thing now at this case here is the depth of flow. So, this can be given as y_1 and y_2 for the same specific force F_x . Now for the same depth you know this is the specific force here. This is the for that same specific force you have another depth of flow also y_2 and we have suggested that y_1 and y_2 are having same specific force. So, that is for a given section you can have 2 depths that can have same specific force. So, these 2 depths are called sequent depths that also we had described at that time.

So, what we want to suggest here is that as the hydraulic jump, in the, in the hydraulic jump the pre jump and the post jump sections they are having the same specific forces. The elevation there in the downstream and the upstream sections they are not sequent depths. So, if you can compute the sequent depths at any elevation, any cross section, your height after the jump that can easily be computed. Just let us just go through the situation.

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Where you have jump in a horizontal rectangular channel, so you have a horizontal rectangular channel. A horizontal rectangular channel, a jump occurs on that and you want to analyze how you will use the same theory. Here the section is like this, depth of flow here the section is like this; depth of flow. So, let us assume the width of the rectangular channel is B , so in the upstream section you have depth y_1 at the downstream section you have depth y_2 in the rectangular channel. So, that is this is pre jump section, this is post jump section. You are taking the portions like this within this region; within this region you are taking the portion, now the 2 sections.

So, what are the, how will you analyse now using the same specific force criteria specific force on the upstream side as well as on the downstream side. Both should be same. So, what can you do means, how you can further simplify for a horizontal rectangular channel section? This is a simple mathematical rearrangement of the things you know just for this thing. I can suggest that A_1 that is in the upstream section A_1 is equal to B

into y_1 , A_2 is equal to B into y_2 . Also the distance to the centroid from the water surface in a rectangular channel it is nothing but half of the depth of flow. So, h_1 bar is equal to y_1 by 2 similarly, h_2 bar is equal to y_2 by 2 if you have all these quantities.

(Refer Slide Time: 35:20)

Specific force

$$F_s = \frac{By_1^2}{2} + \frac{Q^2}{gBy_1} = \frac{By_2^2}{2} + \frac{Q^2}{gBy_2}$$

Result Inertial Number

$$F_r = \frac{\text{Inertial force}}{\text{Gravitational force}}$$

$$= \frac{\bar{V}}{\sqrt{gy}}$$

For rectangular channel, $F_r^2 = \frac{\bar{V}^2}{gy} = \frac{Q^2}{gB^3y^3}$

Then you can easily write specific force. Now specific force F_s this is equal to $B y_1$ square by 2 plus Q square by $g B y_1$, this will be same for the downstream section $B y_2$ square by 2 plus Q squared by $g B y_2$. So, the some of the, there are some quantities now you can rearrange the things. That is the objective here is, I want to find a relation between the sequent depths you have the sequent depths y_2 and y_1 for the rectangular channel hydraulic jump. In a rectangular channel you have the sequent depths y_1 and y_2 how you can find the relation for them. That is our objective here, how I can just do it.

Now in the following, rearrange the things quantities here. Now, just recall the Froude number. We have written Froude number F_r this is equal to inertial forces ratio of inertial forces by gravitational forces. So, in our module 1 we had explained these quantities for the open channels we have given this as V by root of $g y$. Now for rectangular channels, for rectangular channels I can write F_r square is equal to V bar square by $g y$ this is nothing but you know V is equal to Q by A . So, Q squared by $g B$ square y cube for rectangular channels, the Froude square of the Froude number will be in the following form.

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∴ In the force equation

$$\frac{B y_1^3}{2} + F_{r1} B y_1^2 = \frac{B y_2^3}{2} + F_{r2} B y_2^2$$

$$\therefore y_1^2 \left(\frac{1}{2} + F_{r1} \right) = y_2^2 \left(\frac{1}{2} + F_{r2} \right)$$

$$\left(\frac{y_2}{y_1} \right)^2 = \frac{\frac{1}{2} + F_{r1}}{\frac{1}{2} + F_{r2}}$$

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

$$\bar{V}_1 B y_1 = \bar{V}_2 B y_2 \Rightarrow \frac{\bar{V}_1}{\bar{V}_2} = \frac{y_2}{y_1}$$

$$F_{r1} = \frac{\bar{V}_1^2}{g y_1} \quad F_{r2} = \frac{\bar{V}_2^2}{g y_2}$$

Therefore, in the specific force equation I can write $B y_1^3$ square by 2 plus, just recall it. This is the specific force equation, so these are the specific force quantities. So, Q^2 square by $g B y_1$, so Q^2 square by $g B y_1$ that quantity will be equal to $F r$ square into $B y$, $B y$ square. So, the same thing I will be just substituting it here. So, I will just put the pen now, $F r_1$ square B times y_1 square. So, on the right hand side also similarly, I can write plus $F r_2$ square $B y_2$ square, that is B is getting cancelled off in all the both left hand side as well as the right hand side.

So, I will get y_1 square into half of $F r_1$ square this is equal to y_2 square into half of $F r_2$ square, it is a good relationship or you can suggest y_2 by y_1 . One relationship is between the sequent depths of the channel; this y_2 by y_1 whole square is nothing but equal to half of $F r_1$ square plus half of plus divided by half plus $F r_2$ square. Like this, you can easily get one relationship. To get the Froude number of both usually, you have or you will be you may be evaluating Froude number at the upstream section and your objective is based on the upstream section condition, what could be the elevation after the hydraulic jump and all that is your objective.

Now you are required to have the Froude numbers at the upstream as well as downstream section, that may be difficult or that is not Froude. So, what we have to do is that, you use the continuity equation, you have continuity equation $V_1 \bar{A}_1$ is equal to $V_2 \bar{A}_2$. So, from this for the rectangular channel we will get $V_1 B y_1$ is equal to $V_2 B y_2$ or

V_1 by V_2 is equal to y_2 by y_1 . If you have this relationship now, you just incorporate it, what is Froude number? Froude number $F_r 1$ square is equal to V_1 square by $g y_1$. Similarly $F_r 2$ square is equal to V_2 square by $g y_2$. So, you employ this particular relationship here.

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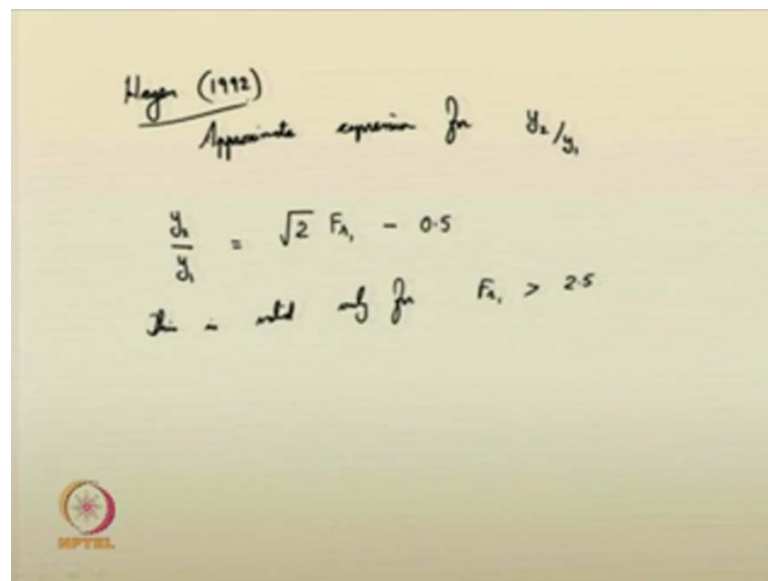
The image shows a handwritten derivation on a yellowed piece of paper. It starts with the ratio of Froude numbers squared: $\frac{F_{r2}^2}{F_{r1}^2} = \left(\frac{y_1}{y_2}\right)^3$. This is rearranged to $F_{r2}^2 = F_{r1}^2 \left(\frac{y_1}{y_2}\right)^3$. A note says "in the eqn for eqn". Then, the specific force equation for F_{r2} is written: $\left(\frac{y_2}{y_1}\right)^2 = \frac{\frac{1}{2} + F_{r1}^2}{\frac{1}{2} + F_{r1}^2 \left(\frac{y_1}{y_2}\right)^2}$. This equation is boxed, and a note says "Implicit eqn". The boxed equation is then rearranged to: $\left(\frac{y_2}{y_1}\right)^3 = \frac{1 + 2 F_{r1}^2}{1 + 2 F_{r1}^2 \left(\frac{y_1}{y_2}\right)^2}$.

We will get $F_r 2$ square by $F_r 1$ square is equal to I will just write the final form. I hope you know how to write it y_1 by y_2 whole cube. Just I substituted say V_1 by V_2 is equal to y_2 by y_1 , that relationship I just incorporated it here, and I am getting the expression y_1 by y_2 whole cube $F_r 2$ by $F_r 1$ whole square is equal to y_1 by y_2 whole cube. Or I can write $F_r 2$ square is equal to $F_r 1$ square into y_1 by y_2 whole cube. Therefore, in the specific force equation, now specific force equation for the rectangular channel I will get an expression y_2 by y_1 whole square is equal to half plus $F_r 1$ square divided by half plus $F_r 2$ square is nothing but $F_r 1$ square into y_1 by y_2 whole cube or just take 2 means unnecessarily putting it half here, I can rearrange the quantity now.

This can be rewritten as 1 plus twice $F_r 1$ square divided by 1 plus twice $F_r 1$ square. This particular quantity divided by y_2 by y_1 whole cube. So, what is the property here? If you just look into this equation, y_2 by y_1 whole cube, this is a ratio the sequent depth the ratios y_2 by y_1 whole cube this is equal to a certain quantity I am getting. So, this is an implicit expression, it is an implicit expression in the sequent depth ratio y_2 by y_1 . So, if you know the ratio or if you want to find this sequent depth ratio, as this is an

implicit quantity, you have to either use the iterative technique or you have to use trial and error method. There are various procedures also from age old means more than 60 70 years itself. Any scientists have given approximate relations especially for the rectangular channels. That is for the hydraulic jumps in rectangular channel scientist have given approximate relationship for this sequent depth ratio, for example as given in Srivatsva's book in open channel hydraulics.

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Hagen (1992)
Approximate expression for y_2/y_1

$$\frac{y_2}{y_1} = \sqrt{2} Fr_1 - 0.5$$

is valid only for $Fr_1 > 2.5$


Scientist Hagen, scientist Hagen in 1992, he has given an expression approximate expression has suggested and approximate expression for sequent depth ratio y_2 by y_1 . He suggested that y_2 by y_1 for rectangular channels can be written as root of 2 into Fr_1 minus 0.5 and this is valid only for cases where Fr_1 is greater than 2.5. So, Hagen has given such an approximation. Many scientists has given different type of approximation even in Ven Te Chow's book on open channel hydraulics, you will find a very good approximation for rectangular channels.

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Chow (1959)
sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8 F_{r1}^2} - 1 \right)$$

Hydraulic jump will form if the equation is satisfied

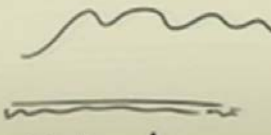


So, in Chow's book in 1959, it gives the approximate relationship of the sequent depth ratio, the sequent depth ratio for rectangular channels y_2 by y_1 can be given as half times 1 plus root of 1 plus 8 times F_{r1} square minus 1. Like this one relationship is given, this is the mean relationship here. So, it suggests that hydraulic jump will form if this equation is satisfied between sections, between 2 sections in the rectangular channel sequent depth ratio. If the sequent depth ratio is being seen in this particular form, then hydraulic jump will be formed.


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Types of Jumps
U.S. Bureau of Reclamation (1955)

- 1) $F_{r1} = 1 \rightarrow$ No jump
- 2) $1 < F_{r1} < 1.7$

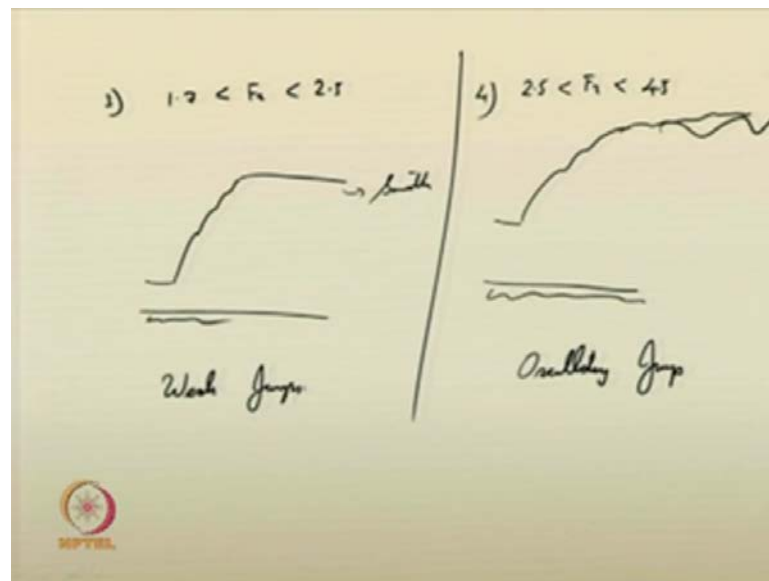


Undular jumps



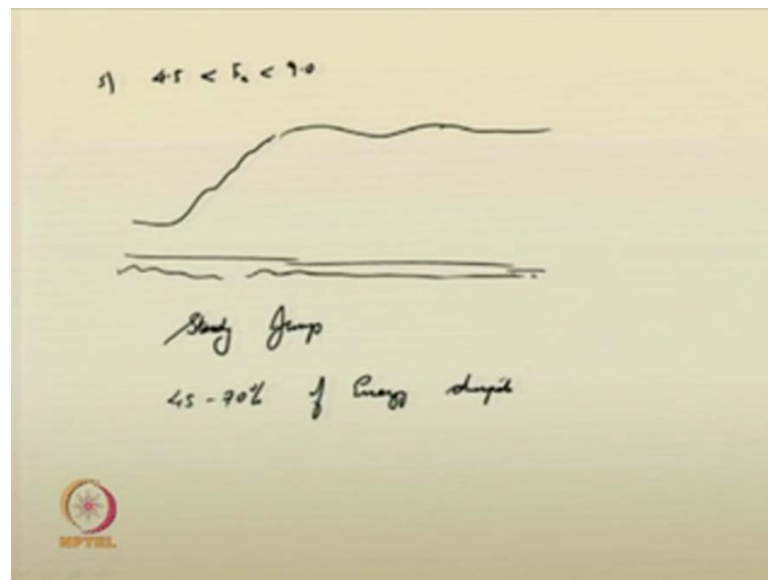
So, from the same U.S types of jumps, we will just quickly cover on what are the types of jumps available? This is given by U.S bureau of reclamation studies in 1955. So, from their studies it is been observed that, so you know that the upstream Froude number that is an important parameter, so for F_r equal to 1 if it is a critical flow, no jump will be formed. If the upstream Froude number ranges between 1 to 1.7, you will get hydraulic jumps of the following form that is undular jump such jumps are called undular jumps.

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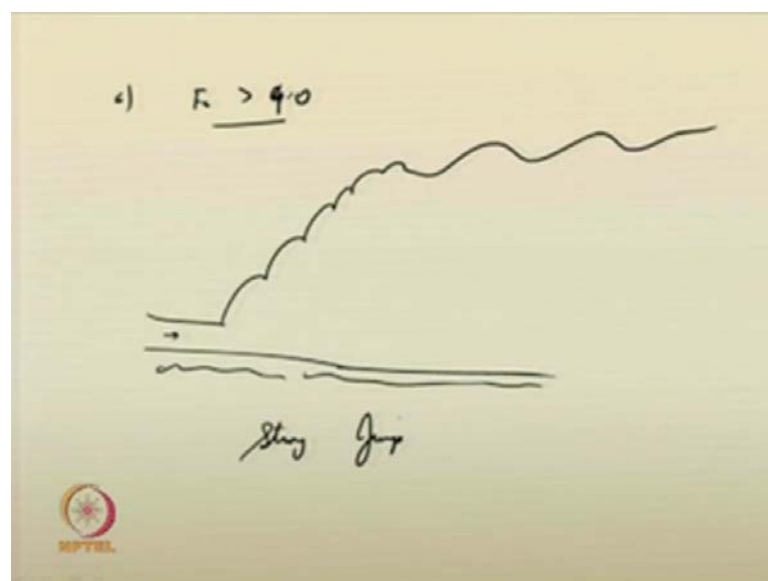
If the Froude number is 1.7 and 2.5, you will see that the downstream section the water surface is still smooth. Although the jump has occurred, smooth water surface is there. Such jumps are called weak jumps. Now the next, I can suggest here is, if your Froude number is between 2.5 and upstream Froude number is between 2.5 and 4.5, you can suggest this quantity now as see oscillations are being formed in the upstream downstream section, several oscillations are formed. This jump is called oscillating jump. So, each oscillation causes waves in the downstream section and that waves can pass on for several kilometres in the canal also. So such oscillations are formed.

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You have if your Froude number is between 4.5 and 9, then you will see that after the jump is formed the undulations are there. But it is almost in a steady state, steady conditions. So, the same discontinuity is maintained, it is not like an oscillating form. So, it is called steady jump. So, the same discontinuity it is maintained with respect to time throughout. So, this is a very efficient type of jump. Such types of jumps are steady jumps are very efficient. It can dissipate almost 45 to 72 percentage of energy, it can dissipate and this is a well balanced jump.

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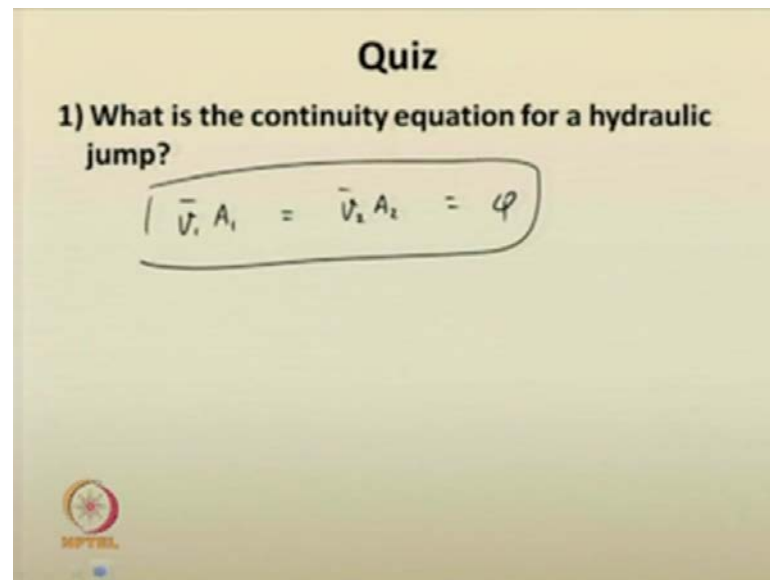


Similarly finally, we can just give that for any Froude number in the upstream section, greater than 9, you have a very rough type of jump. That is water will be gushing into that the upstream water in a supercritical with a high velocity it is gushing in it forms jumps. Then goes on the undulations are also very peculiar, here it is called the strong jump. It will mix with all type of very all type of mixing occurs in such type of strong jumps. So, means there are no water pockets where stagnant water is present or like that situation will not arise in such type of jumps, it is called strong jump. Almost 85 percentage of energy is released in such a type of a jump. So, with that type we have discussed now, the types of jumps also we will continue this portion in the next class.

So, today's quiz the first question; what is the continuity equation for a hydraulic jump? Just write the continuity equation for a hydraulic jump, it will take hardly 30 seconds for you. Now the second question it is little bit lengthy, the question is lengthy but the solution is very simple which are the forces present and as well as the forces neglected in the control volume enclosing a hydraulic jump? When you use the momentum equation, you have seen the derivation today. So, which are the forces that are included and which are the forces that are neglected?

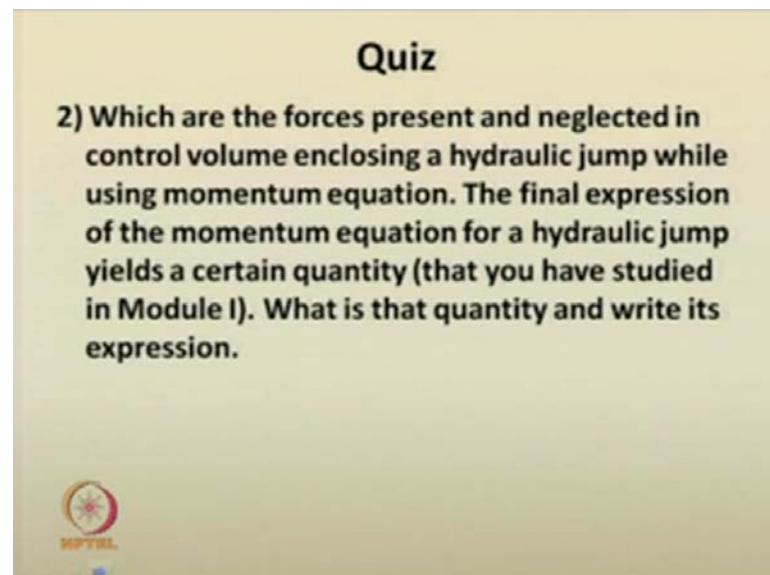
Now the final expression of the momentum equation for the hydraulic jump it yields a certain quantity that you have already studied in module 1 and all, what is that quantity name? What is name that quantity and write its expression, very simple question. The third question the hydraulic jump is classified into 5 types according to the U.S bureau of reclamation. List the types of hydraulic jumps you only give the names, no need to write the Froude number criteria and all just give you give the names.

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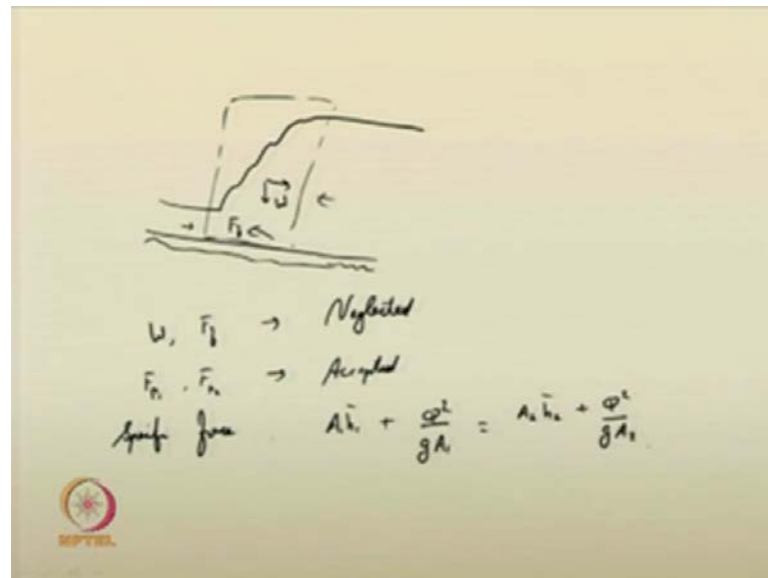
So, the solutions for the today's quiz; what is the continuity equation for a hydraulic jump? As described in many of the courses or subject same thing between 2 sections; it will be having the same discharge. This is the continuity equation we had in fact derived this in our class today.

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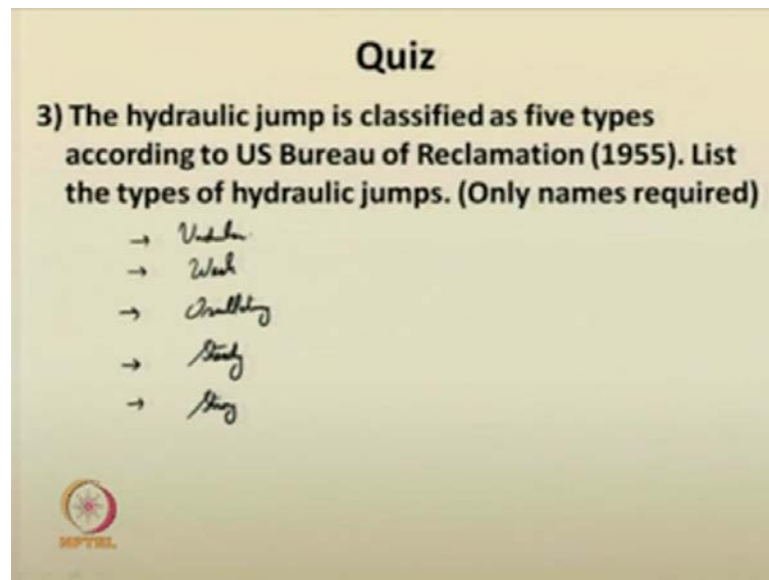
In the second question; which are the forces present and neglected in the control volume or the hydraulic jump?

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You have the hydraulic jump, so in the control volume what are the forces you are incorporating and what are the forces you have neglected? So, the weight component in the direction of flow, the direction of flow $W \sin \theta$ as well as the frictional forces $W F_f$ they are neglected in the hydraulic jump. The pressure forces F_{p1} and F_{p2} they are accepted for the momentum. In the momentum equation, we have incorporated these 2 forces or these 2 forces are accepted. Next part of that question; the final expression of the momentum equation for a hydraulic jump yields a certain quantity, what is that quantity? Do I need to explain it? It is called specific force; we suggested that the specific force at the upstream as well as the downstream section they are same. The expression for specific force, how did we express specific force we suggested that $A_1 h_1 + \frac{Q^2}{g A_1}$ is equal to $A_2 h_2 + \frac{Q^2}{g A_2}$.

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The hydraulic jump we have classified into 5 types according to the US bureau of reclamation list the types, what are the types of hydraulic jump you can just suggest them. If you want I can just briefly write them, you have weak jump, you have oscillating jump, then you have steady jump, you have strong jump, also you have undular jumps. So, these are the 5 classifications for the jumps. So, that way we are concluding today's class. We will continue with the same module in the next class.

Thank You.