Advanced Hydraulics

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Module - 1
Open Channel Flow
Lecture - 3
Flow Classifications & Velocity Distribution

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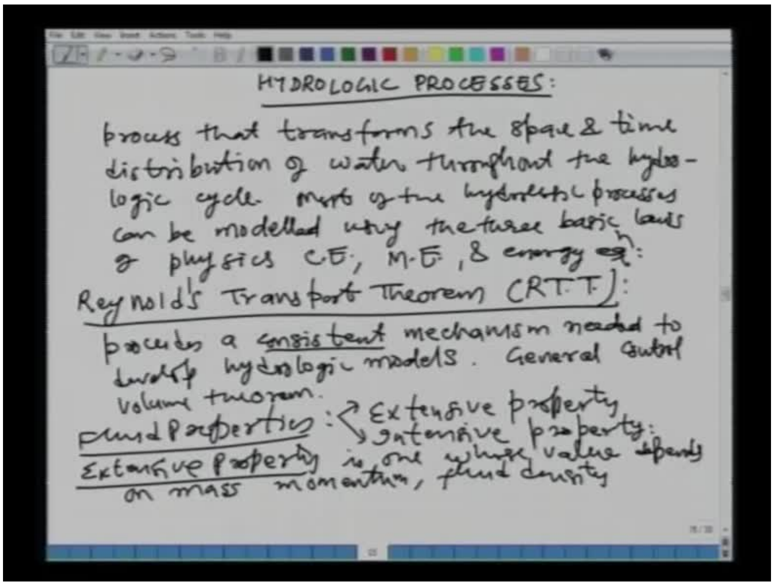

Very good morning to everyone. So, today we will be continuing our lecture series on the course Advanced Hydraulics. Today, the third lecture of the series starts. We are going through the first module of this course that is open channel flows. In the earlier class, we had described on the various classifications of open channel flow. Also, we have described what is meant by open channel flow? We have suggested that the classification is predominantly based on channel properties and flow properties. Based on channel properties, we have suggested some of the classifications of open channel flow; say you had prismatic and non prismatic channels. We have been discussing on natural and artificial channels and we have also discussed on rigid boundary and mobile boundary channels.

Based on the flow properties, we had defined channels or the flowing channels as, say steady and unsteady, uniform and non uniform. In non uniform flow, further we have just briefly mentioned about gradually varied flow and rapidly varied flow. We have also suggested the other parameter randomness. We are not going to take in our hydraulics course here. Also we discussed on some of the channel parameters, say if you have any cross section of a channel and say if its longitudinal profile, if it is been drawn, this is called channel bed. And if this is the level of water or water level or level of the surface of water and if that profile is being given like this, if the channel bed is often following form, we can suggest for any cross section parameters called hydraulic radius, hydraulic depth.

If you recall in the last class, we had briefly mentioned on hydraulic radius. It was given as the ratio of area of the cross section divided by the wetted perimeter. Now, let us go into detail, what are the property and what are the channel parameters available. Say, if you have a longitudinal profile of a channel, say a channel bed, if its slope is, let us give the slope of the channel bed as s 0. We can define say at the cross section, the depth of water at any cross section. Let us define it as y. That is, we are just following the following coordinates system. That is x y z, y is the now taken in the vertical direction and z will be into the, it will be going into the plane of the board. Or if you are taking any longitudinal profile, in the longitudinal profile, here y will be in this direction; this will be the z direction and into the plane of the board, that will be considered as the x direction and we are mainly considering the flow in the x direction.

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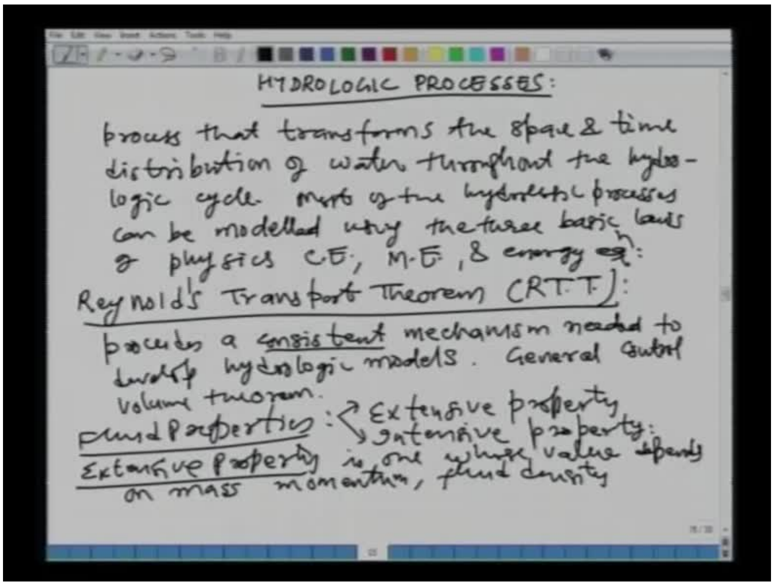

Mostly in open channel flow, we will consider in the longitudinal direction of the flow, the proper fluid flow. So, let us define the parameters for the various aspects. As we mentioned, the depth of flow can be given as y and the bed slope can be given as s 0. We can provide the width at the bottom of the channel as b width. At the width at the top of the channel, that is at the top surface, water surface, the width there, let us give it as capital T. We can define wetted perimeter as the perimeter along the boundary, where the boundary is wetted. So, this is your wetted perimeter p. You can define now the cross sectional area. This entire cross sectional area, let us give it as a. So, let me recall that you have defined y, you have defined b, you have defined t, you have defined area of cross section a, you have defined channel bed slope as s 0, and slope of this water surface, that can also be given as a parameter. So, let us give it as s w. That is also defined.

Based on these following properties and all, your hydraulic radius R, this is defined as the area of cross section divided by the wetted perimeter p. Hydraulic depth, another parameter, we can give it as capital D symbolically. This is nothing but the area of cross section divided by the top width. So, these parameters are some of the hydraulic parameters that can be defined for any channel cross section and we will be using these parameters in most of our analysis for various classifications as well as various analysis.

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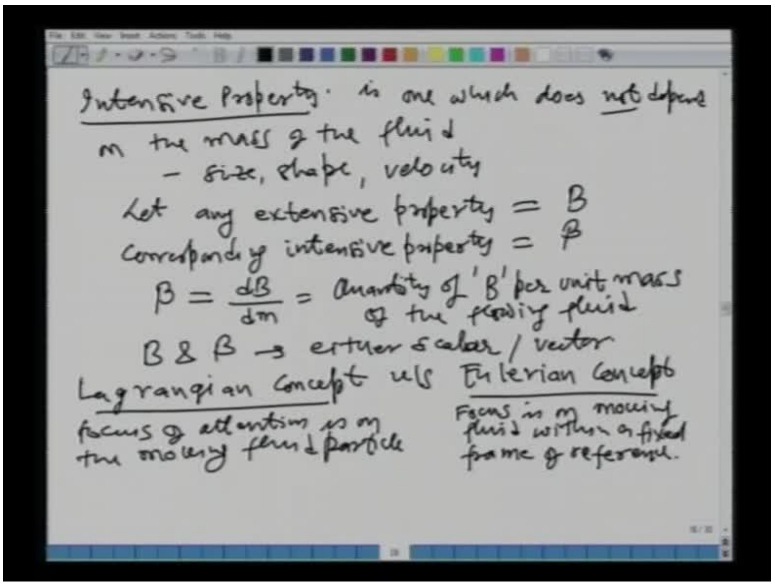

So, let us again look back into the thing. If you recall your fluid mechanics principles and all, energy can be described using energy head. Energy at any location in fluid flow, it can be described using energy head and energy head, you might have studied that it is the summation of pressure head plus velocity head plus datum head. You might have given it like this, p by rho g plus v square by 2 g plus z, like this and all you might have studied in your under graduate courses and all.

So, we are suggesting the same thing here. If you are referring it to any, if the channel is, if it is being referred with any datum line, so any location, at any section, you can easily describe the energy gradient line or energy line also. That is the datum head plus the pressure head plus velocity head, that will give you the energy head at that location. At any particular cross section, you will get your energy head. That can also be described. If you profile the energy at various sections and if you connect them, that will give you energy gradient line.

If you are taking only pressure head and datum head, if you are summating them, you are getting the profile of this water surface. In the free surface flow, this summation of pressure head plus datum head, that is given as hydraulic gradient line. The hydraulic grading line is the summation of pressure head plus datum head and in the free surface flow, it appears, that it is as good as the profile or profile of the water surface. Now, based on these channel parameters and all, we can further classify the open channel flow. Some more classifications are there. You might have heard about laminar flow, turbulent flow, especially in five flows and all. In your under graduate fluid mechanic courses, you might have studied them. So, similar laminar flows and turbulent flow occur in your open channel flows as well.

So, how are they classified as laminar flow and turbulent flow? Again, this is based on your Reynold's number. Your definition for laminar flow is that, if any flow, if it is flowing in a uniform manner without mixing, if it is flowing uniformly, then such type of flow is called laminar flow. Turbulent flow means, wherever the water gets mixed with the surface or from one location to another location, it just mixes in a random way and that flows on. Those are turbulent flows. They are classified or they are defined as using Reynold's number. Reynold's number is nothing but the ratio of initial forces to viscous forces.

Based on this ratio, you can define or you can obtain Reynold's number. One of the common expressions you might have seen is rho v average velocity v bar l by mu, where l is given as any characteristic length, mu is given as dynamic viscosity of the liquid, rho is density of fluid, and v bar is the average velocity in the cross section. So, based on these parameters, you can obtain the Reynold's number. In open channel flows or free surface flow, it has been identified through various experiments there. Normally, if Reynold's number is less than or equal to 600, it is laminar flow and if it is greater than 600, turbulent flow starts.

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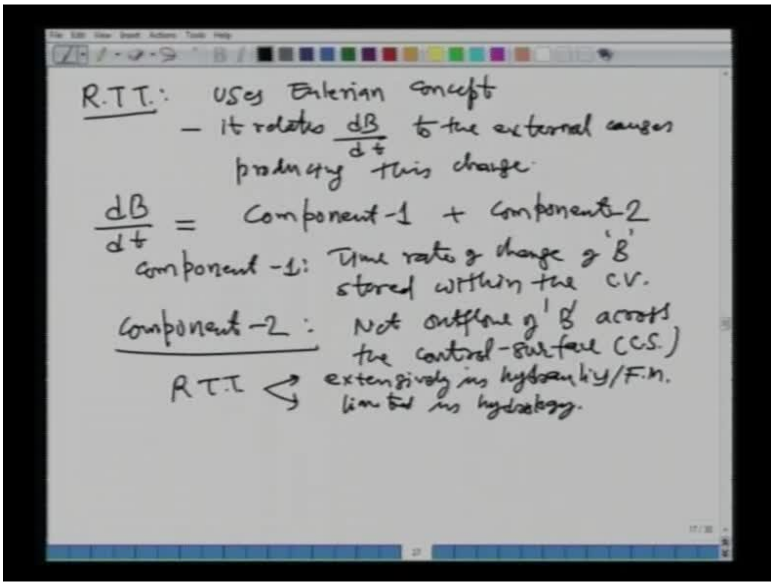

If you try to do, if you try to analyze them in laboratory, sometimes you may visualize that even for Reynold's number greater than 1000 or even for Reynold's number greater than 2000 in the field laboratory flumes, on the laboratory flumes and all, you may still witness laminar flow. This is because due to scaling effects in laboratory flumes, there may be chances of Reynold's number getting a higher value, despite it being a laminar flow. If you want to analyze turbulent flow and all, you may require same other methods and some other type of extrapolations here. So, the characteristic length in your Reynold's number, the characteristic length l in your Reynold's number, so generally this l is taken as either hydraulic radius or hydraulic depth d. So, the earlier criteria, which we mentioned that r e greater than 600 is turbulent. In these cases, the r e is evaluated using hydraulic radius as the characteristic length.

Another type of flows you can mention using a particular constant is called critical flow, subcritical flow and super critical flows. So, just we are briefly touching this here in this module. Just to understand the concept or just to understand it in the first unit of this module, that is different classifications of open channel flow. In this different classification of open channel flow, you will also obtain critical flow, subcritical flow and super critical flow. What is meant by critical flow? If any flow if any flow in a channel, if the velocity of the flow in a channel, if this is equivalent to the velocity of small amplitude wave created in a water of depth y moving with acceleration due to the gravity. If that is equivalent, then such type of flows are called critical flows. That is, you can define velocity v is equal to root of g y. If you can describe any flow in this manner this is called critical flow. That is, any wave of magnitude y or amplitude, this thing y, if it propagates with a gravitational velocity and if that same velocity exists for the fluid flow, then such type of fluid flow are called critical fluid flow.

You can define a parameter called Froude number f r, which is given as, say average velocity by root of g y. If at any value cross section, if v bar is the average velocity obtained for the cross section, this divided by root of g y will give you Froude number and the Froude number is equal to 1. What does that mean? If Froude number is equal to 1, this suggests that the fluid flow is critical.

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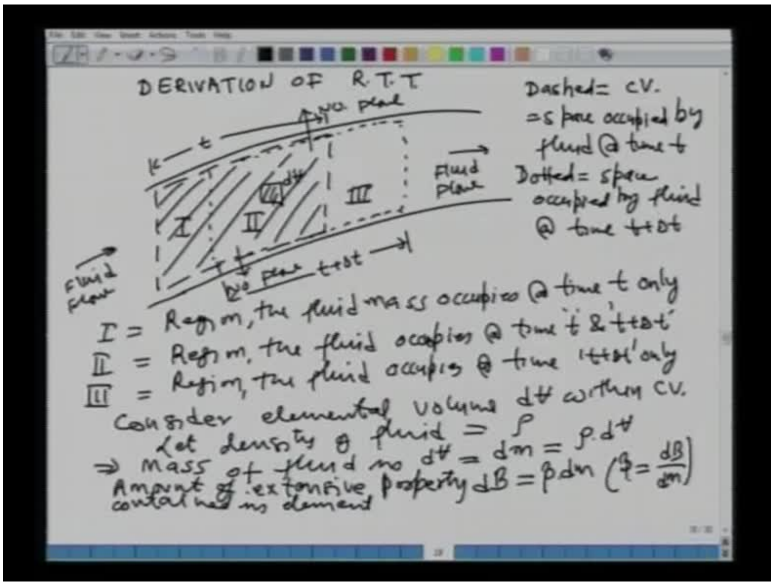

If your Froude number if it is less than 1, your flow is subcritical and if it is greater than 1, it is super critical. We will be going into details on all of these types of flows later. In our later modules and even in the later portions of this module and we will describe them in detail. So, how do you define your velocity? It has been suggested to you that, any particular cross section to obtain your Froude number, you have to use the average velocity. Similarly, in Reynold's number also, you need to use the average velocity, rho v l by mu. In Froude number.

So, here all the cases you have described the term called average velocity. So, what do you mean by average velocity? It is nothing but if you see in a entire cross section, the velocity in the cross sections, they may vary at a particular, say if you take longitudinal profile of the channel at any cross section a a, if your channel section it is drawn, in this entire cross section, the velocity at all the positions, they may not be same. It will differ, especially due to the shear stress components, due to the boundary interactions with the boundary walls and all, there will be shear stress. Definitely, the velocity will be different in different parts of the cross section.

So, you have to compute average velocity from the sections using your mathematical methods. It is nothing but generally v is given as, that is, if you take small small elements in the entire cross section, small small elements of d a area, where v is the velocity at that location. Just get the ratio in following form. You will get your average velocity. So, we will come into that. Just after finishing one more topic, we will just go into the detail of various concepts in the average velocity and how it is utilized in other portions. So, what are the properties of a typical channel cross section? We told that some standard channel sections, if you are given some standard channel sections, you can easily identify its bottom width, its top width, its area, and its wetted perimeter. Subsequently, you can evaluate hydraulic radius a by p and you can also evaluate hydraulic depth a by t.

A question I would like to post you is that, if there is a rectangular cross sectional channel, say depth of flow is y, and width of the channel is b, what could be the hydraulic radius for such rectangular cross sectional channel? Again, a by p. You know what is a. a is b into y. What is p? p is nothing but b plus 2 y. Similarly, your hydraulic depth is nothing but a by t. This is again b y by b. It is same as your depth of flow. If a triangular cross section is given to you, say depth of flow is y, the channel side slope, say if it is given as 1 is to b. That is, every one unit of vertical displacement, it will be displaced horizontally b units. That is, 1 is to b, if the channel side slopes are given in the following form. How will you evaluate the same hydraulic parameters, hydraulic radius, hydraulic depth, and wetted perimeter? How can you obtain them? Your a is nothing but let us start with what is the top width. Your top width is; what is your top width? You see, 1 is to b, 1 is to b is given to you. So here, b y units and here, b y units. So, your top is 2 b y. So, your area is half into 2 b y into y. This is equal to b y square.

Your wetted perimeter, this is nothing but you see the hypotenuse length. So, you can easily get, this is 2 y into root of 1 plus b square. Therefore, your hydraulic radius r is a by p. Try to obtain them. b y square divided by 2 y into root of 1 plus b square, you will get b y into twice of root of 1 plus b square. Again, what is hydraulic depth d? I will write it here. Hydraulic depth d, this is nothing but your area of cross section divided by the top width. That again, you can give it as b y squared by 2 b y and what is that you are getting? y by 2.

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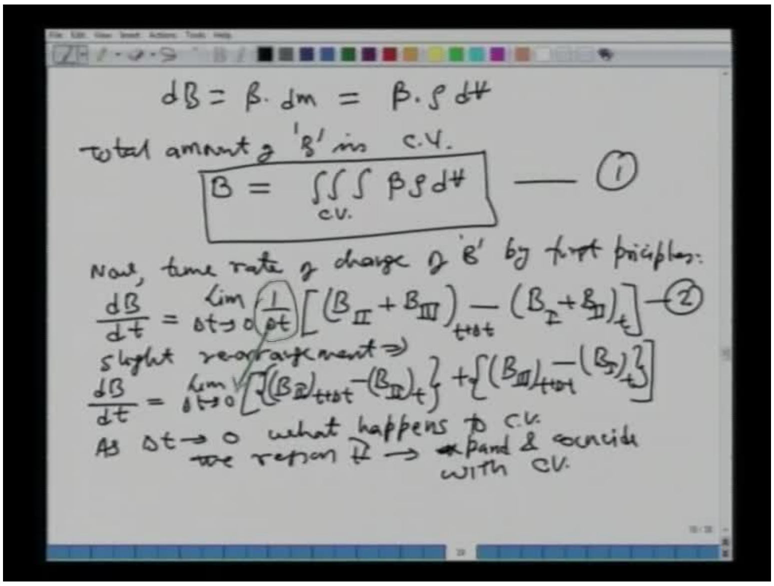

So, you see the difference here. For a rectangular cross section, you are getting the hydraulic depth as y and in a triangular cross section, you are getting it as y by 2. You can evaluate on your own. Say, what happens for a trapezoidal cross section. For a trapezoidal cross section of bottom width b, side slopes 1 is to b. You just evaluate all these hydraulic parameters. First the top width, then the area of cross section, wetted perimeter, hydraulic radius and hydraulic depth.

Let me suggest this as quiz number. Today's quiz, in the today's quiz, you have to solve it now itself. I will just give you two minutes to solve this thing. In this today's quiz, you just try to evaluate these following parameters. So, this is one of today's quiz. So, today's quiz also contains three questions. The first question is to evaluate the hydraulic parameters of the following trapezoidal cross section. So, please evaluate it as soon as possible. We will allow you only two minutes to solve this thing. T? P? A? R? So, can you give the solution? I will give you the solution after the end of today's lecture.

Velocity distribution. What do you mean by velocity distribution? Just few minutes ago, we suggested, any cross section, say natural drain cross section or a regular prismoidal channel cross section, the velocity varies at any cross section. It has been suggested to you. How the velocity varies? We suggested that there will be shear stresses along the boundaries; on the side boundaries as well as the bottom boundary. Similarly, in the natural ranges also, there will be shear stresses. Using your shear stress principle and all, it is now quite understood there will be almost negligible velour. There will not be any velocity at the interface here, between the boundary and the liquid. So, there will not be any velocity. The velocity of liquid at those locations will be 0.

How do you find the cross sectional of profiles for the velocity then? As we are dealing with open channel flows, say for large rivers, channels, canals, flumes, aqueducts, whatever type of channels you have, predominantly they have a longitudinal direction of flow of water is there. Compared to the width of the channel and all and compared to its longitudinal distance, the width and all is much much less. Not only that, the variations of any fluid parameter in the transverse directions, they can all be almost neglected. If you are taking velocity as a vector v is equal to v x i plus v y j plus v z k, if you represent it in a vectorial form, you may see that if your longitudinal direction is given as x, then predominantly your velocity vector can be approximated as v x i. This can be almost approximated because v y and v z are almost negligible in most of the cases.

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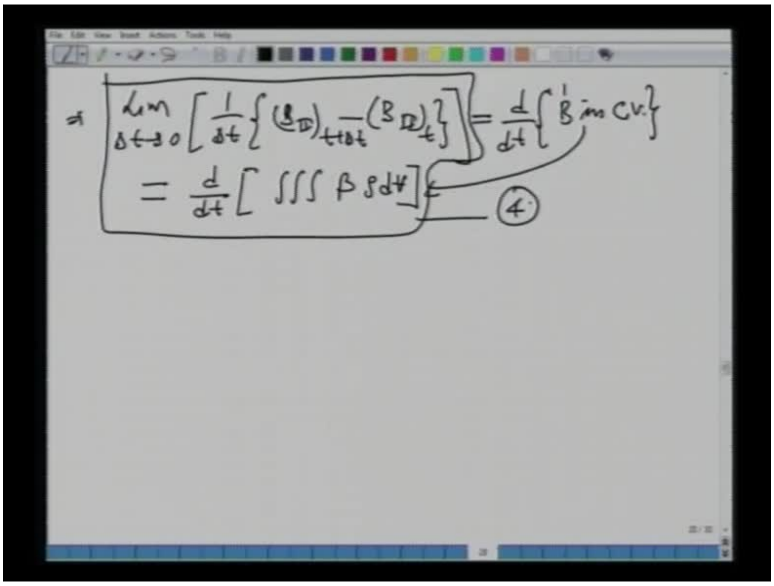

In such a situation, if you take the any cross section, say in this cross section, a a or b b, it does not matter. Say if this is the cross section at a a, you will see, if you plot the velocities contuse and all, you may get several type of isovels. You will get isovels of the following form. So, it is quite experimentally obtained values and all. It looks very nice also such profiles and all. You will see them. The velocity, if it is high, means it is 0 at the boundaries and as it goes far away from the boundaries, the velocities increases. So, this is the portion, where maximum velocity and all it is there. Surprisingly, you may also see that the maximum velocity point normally found from various experimental studies, they are not at the surface, which is the farthest point from the boundary. They are obtained just below the surface level. Those points are also identified through several experimental studies and all. We have to just see how the velocity profiles and all look like.

Say, I can draw, your x direction if it is given and at any location of x, that is, at any location of x and if you plot the velocity distribution with respect to y axis, that is in the vertical direction, you may come upon something of this following form and all. Say, this may be some, if this is the depth of flow y, this may be the velocity at 0.2 times the depth. There can be velocity at 0.8 times the depth and some of the experimental studies have also suggested that, the average velocity in this cross section, they can be given as the velocity obtained at 0.2 plus velocity obtained at 0.8 divided by 2 or it is more suggested that, average velocity is the velocity at 0.6 depth. It is also at the 0.6 depth of the flow.

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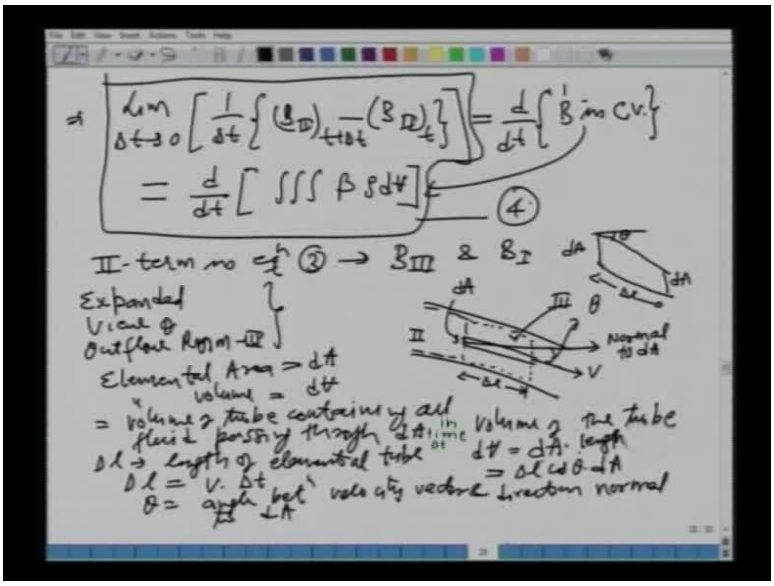

So, these are some experimental analysis. We are not going through these type of methods to study our velocity profiles and all. We will be using some integration methods and all. So, as I have mentioned, so predominately your velocity vector is vector in the x direction only. So, it can be given, this velocity vector can given it in the following form as well. You can now suggest that your average velocity, it can also be computed as any cross section, whatever be the depth. So, this any elemental area, if you take it, d a, any elemental area, if you take d a and if it is having velocity v at that location, so that, you integrate it for, you try to obtain the v d a product at the entire cross section. You know the entire cross sectional area is a. Through that, you will get the, that ratio of the things, you will get the average velocity v. So, in most of the cases, average velocity need to be incorporated.

So, if you recall back your energy head, it was having a component called velocity head. It was the summation of pressure head, velocity head and datum head. So, the velocity head is a part of the energy head. What do you mean by velocity head? It is a quantity related to kinetic energy and it is documented. It is nothing but some velocity, that is velocity at any location v squared by 2 g. It is well documented. If you take any cross section of the channel and all, if you look it, you will see that velocity at any location, at any small elemental area v, v square by 2 g, you will get the velocity head for that particular position. v squared by 2 g at another location, that you will get it. For the entire channel section, if you try to obtain average velocity and try to obtain the velocity head using the average velocity, this term, generally it will not be equal to, if you integrate this position to the entire area. That is, what it is seen in the many experimental such studies.

So, we can suggest now, say if I take v squared by 2 g for the entire cross section, various things it is there and if I take the average of this quantity v squared by 2 g, they are not equal. To correct them, we may have to use a correction factor. So, this can be explained in another way also. If I want to say the same for the same cross sectional details having velocity v and d a, you may see that, the mass of water flowing through that elemental area, it can be given as density into the velocity at that area location into that area. That will give you the mass of water flowing through elemental area per unit time. Please note that. This is per unit time. So, I now can give it this as rho into v into d a. Kinetic energy that gets transferred through this small elemental area, that gaper unit time, that can also be given as half into mass half m v square, right. So, this is nothing but half rho v cubed (( )) a.

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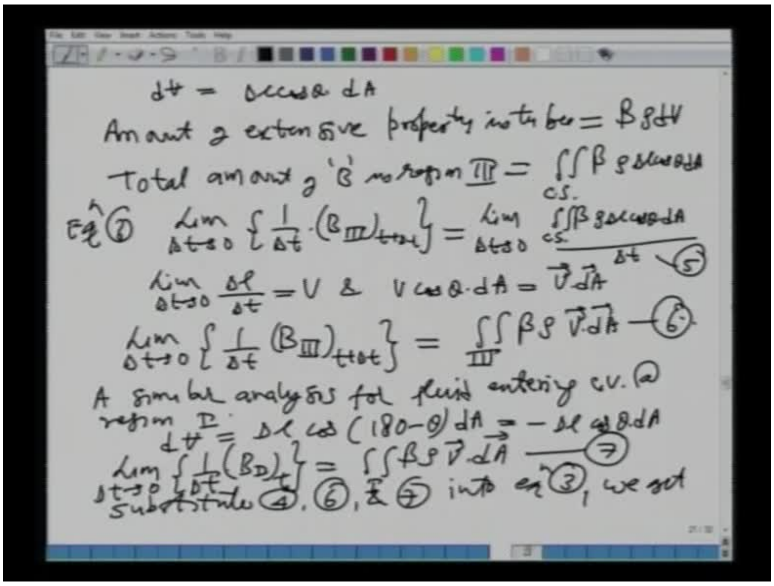

If you integrate this kinetic energy transfer for the entire cross sectional area, that kinetic energy transfer per unit time, this will be nothing but you have to integrate that half into rho v cube d a. So, for the entire cross section area, the kinetic energy transfer for unit time, this can be given as integral half of rho v cube d a. Generally, for the open channel flows, we are taking incompressible liquid and in the incompressible liquid assumption, now we can suggest that the kinetic energy transfer or the rate of kinetic energy transfer that is equal to half into row into integral of v cubed a.

So, for the elemental area, we suggested, just for any elemental area d a, the rate of kinetic energy transfer, this was just mentioned previously, half rho v cubed d a. This term, I can just rearrange for our convenience, say rho g into v squared by 2 g, right, into v d a. So, what does this mean? When we started that mass of water flowing through elemental area, elemental cross sectional area d a, this was given as rho v d a. Now, what do you mean by rho g v d a? This is the weight of the locate that gets transferred through this elemental area d a per unit time. So, the weight of water that gets transferred per unit time from that elemental area into velocity head, this is nothing but velocity head; your kinetic energy that is getting transferred through that elemental area per unit time is nothing but the weight of water that gets transferred per unit time into velocity head.

You have already seen how the velocity head, the velocity head term, velocity head obtained by, obtaining the average of all the velocities and subsequently obtaining the corresponding head as well as the average of all the velocity heads from the cross sectional, they are not same. It has to be corrected by some factor. That now we are going to see. As mentioned earlier, that is the velocity head, the average of the velocity head obtained for the entire cross section. This is not equal to the velocity head obtained or the velocity head obtained using the average velocity as they are not equal. We are correcting them using the following theories. That is, v squared by 2 g, this average is equal to alpha times.

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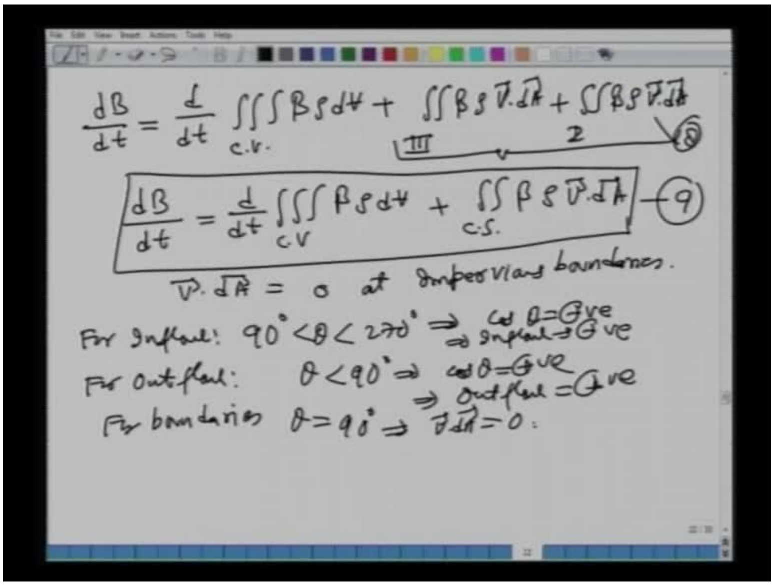

So, why that kinetic energy getting transferred? If you see there, the kinetic energy here, transferring through this particular cross sectional area, similarly many cross sectional areas are there. If you club them, if you add all of them and whatever kinetic energy transfer we are getting and if you try to obtain a mean velocity first for the entire cross section area and then you try to analyze using that mean velocity for the entire cross sectional area and try to obtain the kinetic energy transfer.

They will not be same because velocity it is getting in the third (( )), means third power here. So, they are not same. It is corrected using energy correction factor called alpha and this alpha, you can easily define it in the following form. The kinetic energy obtained after incorporating the alpha term to the mean velocity and the kinetic energy obtained using your general velocity term, they should match or I can write it in the following form as the kinetic energy should match for the both of the cases, kinetic energy transfer for the entire cross section using both the approaches. So, this can be given as rho v bar alpha v bar square 2 using your average velocity. This is how you need to compute the kinetic energy transfer for the entire cross section.

Now, using your simple velocity at each point, the kinetic energy will be measured as half rho. They both should be equal and subsequently, you can get the expression for energy correction factor as alpha is equal to integral v cube d a by or this integral d a can be written as, this can be written as capital a also, if you feel like that, where a is equal to integral d a. Now, you have gone through what is meant by the energy correction factor.

Similarly, if momentum is transferred through any elemental cross sectional area, if any momentum is transferred through any elemental cross sectional area, through elemental area d a, that can also be computed. You know what is meant by momentum. That is, mass. This is mass into momentum transferred through elemental area, mass into velocity rho v d a into v. We can write it in the following form, rho v square d a. So, momentum transfer for the entire cross sectional area per unit time, this is nothing but equal to integral of rho into integral of v square d a.

Now, if you try to compute the momentum transfer using same average velocity, if in the section, instead of analyzing velocity at each points, suppose if you are taking only the average velocity v bar for the entire cross section and then you are trying to compute the momentum transfer per unit time for the entire cross sectional area, you may see that, just by using v bar and computing it in this form, that is rho v bar square into a, this will not be equal to rho into integral v square d a. This again needs to be corrected and we are incorporating momentum correction factor. We are incorporating momentum correction factor using the same energy has you have computed the kinetic energy correction factor. So, beta is nothing but v square d a integral of v square d a divided by average velocity squared into a, where a is nothing but integral of d a.

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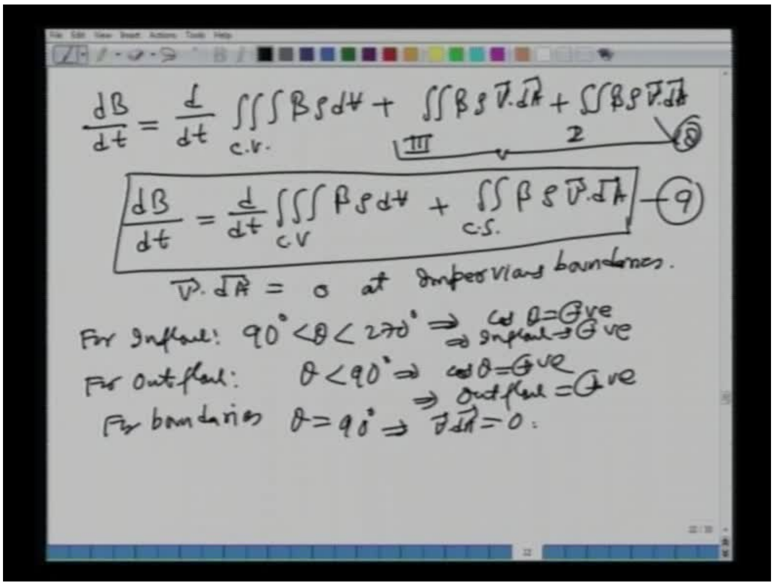

So, whenever you do momentum transfer analysis and all, so in those locations you will, if you are using average velocity, then you need to incorporate momentum correction factor in the average velocity term, while computing the momentum transfer. Similarly, when you are doing kinetic energy transfers or kinetic energy analysis and all and you are using the average velocity terms, then you have to use the corresponding energy correction terms alpha, alpha beta and all. So, the values of alpha beta and all, for regular shaped channels, they are available in the literature. You can go through them.

So, let us start, today we are just going to wind up this lecture. We have seen various topics. You have seen how the velocity distribution is there in the cross section profile. We have seen what is meant by laminar flow, turbulent flow, subcritical, critical and supercritical flows and how to obtain energy correction factor and momentum correction factors. A brief quiz on the topics taken today. So, the first question was already given to you. You were asked for the trapezoidal channel, what are the hydraulic parameters you have evaluated. Now, the second question for, the second question is based on the Froude number, how do you classify flows in open channels? So, we will give you 30 seconds to answer these questions. Based on Froude number, how do you classify flows in open channels? Your third question is, what is energy correction factor alpha and how is it defined. What is energy correction factor alpha and how is it defined. So, these three are the questions for today.

For the trapezoidal channel, you have the following solutions. For the trapezoidal channel, the solutions are the top width is equal to b plus 2 b y, your wetted perimeter is equal to b plus 2 y into root of 1 plus b square, your other quantities are area is equal to b plus b y into y and this r is equal to b plus b y into y 2 y root of 1 plus b square. The second question the solution is, for Froude number equal to 1, the flow is given as critical flow; for Froude number less than 1, the flow is subcritical and for Froude number greater than 1, it is super critical. The next question is, your energy correction factor alpha, this is given as integral of v cubed d a by the cube of the average velocity into a. Then, thank you. So, we will meet in the next lecture, where we will continue in the same module on open channels flow. We will go in detail of some other portions of the course.

Thank you.