

Advanced Hydraulics
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Module - 3
Varied Flows
Lecture - 11
Features on Spatially Varied Flow

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In the last class we discussed on

- Spatially varied flow.
- Derived dynamic equation for SVF for increasing discharge
 - From continuity equation $\dot{q} = \bar{v} \frac{\partial A}{\partial x} + A \frac{\partial \bar{v}}{\partial x}$
 - From momentum equation $\frac{dy}{dx} = \frac{(S_0 - S_f) - 2 \frac{Q \dot{q}}{g A^2}}{1 - \frac{Q^2 T}{g A^3}}$

NPTEL

Welcome back to our lecture series on advanced hydraulics, part of the NPTEL program, phase two program. We are in the third module on varied flows. We have finished ten lectures in this module. Today is the eleventh lecture. And, by today's lecture, we hope to wind up this module.

In the last class, we had discussed on the concepts of spatially varied flow. We had derived the dynamic equation for spatially varied flow for increasing discharge. That is, first using the continuity equation, we identified what is the rate that is the rate of increasing discharge per unit length that was identified; it was given by the following expression.

Then, subsequently, using the momentum equation, we had derived the dynamic equation for spatially varied flow. I hope you recall all the terms here, y is the depth of flow, S_0 is the bed slope, S_f is the friction slope, capital Q is the discharge, $\dot{q}$ star is the rate of increase in discharge per unit length, g acceleration due to gravity, area A , A is the area of cross section, again T is the top width of the channel, all these things are self explanatory.

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Similarly we discussed on

- **Dynamic equation for SVF for decreasing discharge**
From energy principles

$$\frac{dy}{dx} = \frac{(S_0 - S_f) - \frac{Q q'}{g A^2}}{1 - \frac{Q^2 T}{g A^3}}$$

- Also suggested that numerical methods can be employed to obtain solutions of the dynamic equation for SVF



Similarly, we had discussed on the dynamic equation for spatially varied flow for decreasing discharge. If there is a continuous decrease in the discharge through a channel, then using the energy principles we had derived the corresponding, we had derived them, the corresponding equation, dynamic equation. Also, we have suggested how to use numerical methods; that is, numerical methods can be applied to solve these two types of dynamic equations. From solving these dynamic equations, either we can get the depth of flow at various cross sections, or you can get, say, for a specific depth what, which, at which cross section it is available, that depth of flow is maintain and all, that can be readily obtained.

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Features of Spatially Varied Flow Profiles

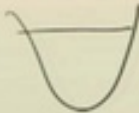


Today, we will be dealing with the concepts on, or some of the features on spatially varied flow and its profiles.

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As discussed earlier

- The Dynamic equation for SVF for increasing discharge using momentum principles

$$\frac{dy}{dx} = \frac{(S_0 - S_f) - 2 \frac{Q q^*}{g A^2}}{1 - \frac{Q^2 T}{g A^3}}$$


- This is without momentum corrections

For SVF, **momentum corrections** are significantly required and therefore the dynamic equation becomes

$$\frac{dy}{dx} = \frac{(S_0 - S_f) - 2 \frac{\beta Q q^*}{g A^2}}{1 - \frac{\beta Q^2 T}{g A^3}}$$

β → Momentum Correction Factor



So, as discussed earlier, the dynamic equation for spatially varied flow for the, in the case of increasing discharge, we had derived it using momentum principles. It was given in the following form. So, if you look closely into these things, this has been given without the momentum corrections applied to the cross section. So, if any cross section is there, then we have taken the average velocity across this cross section as a parameter, and based on that average velocity, whatever correction for the momentum is there, that was not incorporated; we had taken momentum correction factor beta is equal to 1 and we had developed this equation.

So, usually the momentum corrections are very much significant in spatially varied flow. Therefore, we have to apply the momentum corrections factor, momentum corrections factor; and the improved or modified dynamic equation will look like this; that is $\frac{dy}{dx}$ is equal to S_0 minus S_f minus, twice $\beta Q q^*$ by $g A^2$, the whole thing divided by 1 minus $\beta Q^2 T$ by $g A^3$. So, beta, the momentum correction factor is incorporated in the dynamic equation.

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Similarly

The Dynamic equation for SVF for decreasing discharge using energy principles

$$\frac{dy}{dx} = \frac{(S_0 - S_f) - \frac{Q q^3}{g A^3}}{1 - \frac{Q^2 T}{g A^3}}$$

This is without energy corrections

Energy corrections are significant in SVF and therefore the dynamic equation is modified as:

$$\frac{dy}{dx} = \frac{(S_0 - S_f) - \frac{\alpha Q q^3}{g A^3}}{1 - \frac{\alpha Q^2 T}{g A^3}} \quad \alpha = \text{Energy correction factor}$$


Subsequently, a similar concept, similar principle can be applied for the dynamic equation for decreasing discharge. We had derived it using energy principles earlier, in the following form. So, as we have mentioned here; here also, in the cross section, the corrections for energy are not incorporated. So, we need the energy corrections for spatially varied flow, because there is a rapid change or rapid decrease in energy due to the change in flow and all; so, rapid decrease, in the sense that, some changes in the energy is occurring due to the change in quantity of water and all. So, you need energy corrections also.

Therefore, with energy correction factor, the dynamic equation can be specified as given here- $\frac{dy}{dx}$ is equal to S_0 minus S_f minus, $\alpha Q q^3$ by $g A^3$ the whole thing divided by 1 minus $\alpha Q^2 T$ by $g A^3$ where your α is the energy correction factor. I hope you recall these factors; we had studied them in detail in module one and all.

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Both the Dynamic equations for SVF - for increasing discharge and decreasing can be written in a common form

$$\frac{dy}{dx} = \frac{(S_0 - S_f) - c_1 \frac{Q q^*}{g A^2}}{1 - c_2 \frac{Q^2 T}{g A^3}}$$

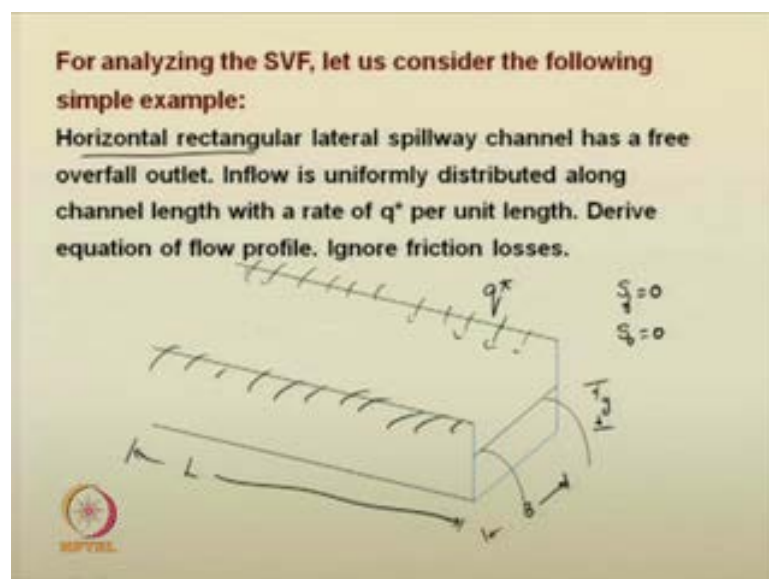
$c_1 = 2\beta, \quad c_2 = \beta \rightarrow$ When we use Dyn. eqn for increasing discharge

$c_1 = c_2 = \alpha \rightarrow$ When we use Dyn. eqn for decreasing discharge



So, if you look into both the equations, say dy by the momentum means, dynamic equation derived through the momentum principles as well as the dynamic equation derived through the energy principles; both of them can be written in a common form. So, I have just expressed this thing in a common form here, dy by dx is equal to S_0 minus S_f minus, a quantity c_1 into $Q q^*$ by $g A^2$ by the whole thing divided by 1 minus $c_2 Q^2 T$ by $g A^3$; where c_1 is equal to twice beta and c_2 is equal to beta, when we use dynamic equation for increasing discharge. Similarly, c_1 and c_2 , both are equal to alpha, when we use dynamic equation for decreasing discharge. So, it is upto you; just for the convenience we have written it in a common form.

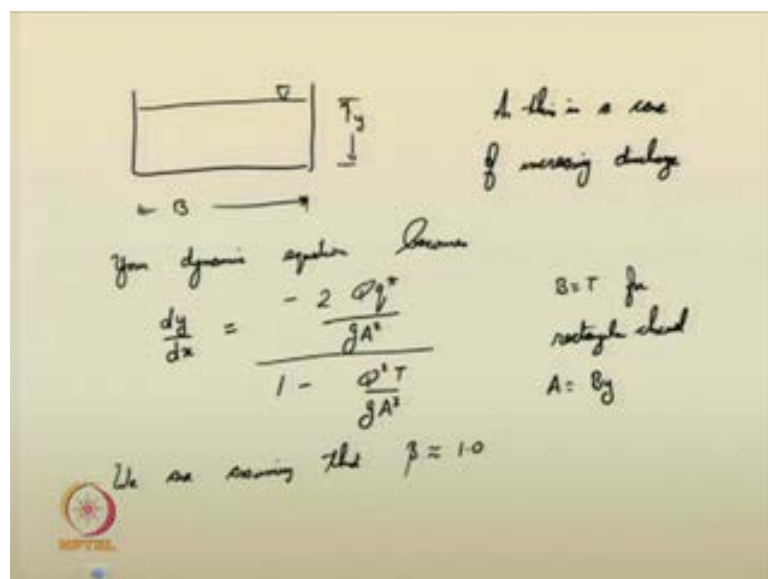
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So, for analyzing the flow profiles in a spatially varied flow, let us first consider through a simple example. So, let me state the example. There is a horizontal rectangular channel that collects water laterally through the over flow spill way. That is, water is entering the rectangular channel through over flow spill way, like this. It is entering like this, say, may be from both sides. This rectangular channel has a free fall, over fall outlet, like this. It is the, at the end of this rectangular channel, the water is falling freely. Inflow is uniformly distributed along the channel length. So, the inflow to this channel, it is distributed uniformly, at a rate of q star per unit length. Now, derive the equation of flow profile. You can ignore the friction losses. How will you try to obtain the flow profile or the equation for the flow profile?

So, this is the very interesting problem. You can see that it is a horizontal channel; spill, means water is coming into this channel; the length of the channel, let us assume it as capital L ; then, the width of the rectangular channel is capital B , according to our normal convention; depth of flow at any location, it is y , right. So, based on this, how will you obtain the flow profile?

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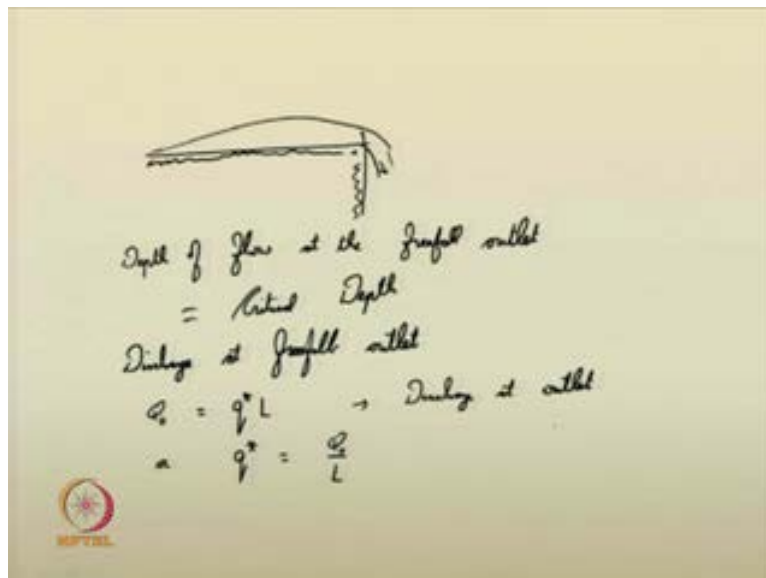


So, again the cross section; we have our rectangular channel, B is the width, y is the depth of flow. So, B and T , both are same for the rectangular channel. Now, read the question carefully. What are the assumptions or what are the things given here? There is an assumption that you can ignore friction losses, so what does that mean? While ignoring friction, to ignore friction losses means, you are considering S_f is equal to 0. Another criteria given is, this is the horizontal rectangular channel, so that means your bed slope is also equal to 0.

Therefore, your dynamic equation, now this becomes $\frac{dy}{dx}$ is equal to, both S_0 and S_f are 0, so minus twice; ok, I am just taking the common form, ok; is it sufficient, common form of the thing; so, minus; or, let me say, as this is a case of increasing discharge; so, let us see; so, this becomes $\frac{-2Q}{gA^3} \frac{dQ}{dx}$, the whole quantity divided by, $1 - \frac{Q^2}{gA^3}$. We are assuming that the momentum correction factor, it is approximately 1; for, only for this particular problem, we are assuming that situation.

So, just go through here; so, from here, starting on from here, the water is collected in this channel, it, and it is flowing like this. So, at a rate of q , it is being collected, the entire thing. So, we can suggest now, for the rectangular channel, B is equal to T for rectangular channel; A is equal to $B y$, right.

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What is, let me ask you the simple question now. Ok, I will just redraw the thing here. So, this is the horizontal channel. We are looking the longitudinal profile; so, the flow, the channel, if it is having a free out fall, ok. Just I will just redraw the thing again, like this. So, in this case, what will be the depth, depth of the flow at the free fall out, outlet; this should be equal to the critical depth, this should be equal to the critical depth in the flow.

So, if that, if that is clear to you, then discharge at free fall outlet; what is the discharge at free fall outlet? If you recall the thing, we had suggested that water is slowly increasing, water is slowly increasing from this section to this section as it goes downstream. So, the rate of increase is q star per unit length. So, we give the quantity Q_0 ; this is nothing but q star into

the length of channel. So, this Q_0 is the discharge at outlet; or you can suggest now that q_{star} is nothing but equal to Q_0 by L .

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Also for any small elemental length in the channel dx
 → the change in discharge
 $dQ = q^* dx$
 $\int dQ = \int q^* dx$
 $Q = q^* x$
 We have the by velocity $\bar{v} = \frac{Q}{A}$
 $Q = \bar{v} A$

Also, for any small elemental length in the channel, any small elemental length in the channel, say, dx , you can easily identify the change in discharge dQ ; this is nothing but equal to q_{star} into dx . So, this quantity now, it can be easily suggested. You can integrate this thing, dQ is equal to, into $q_{star} dx$. or, you will find that discharge at any section along, in the channel, at any section along channel Q , this will be nothing but equal to q_{star} into x , because q_{star} is independent of length x . So therefore, you can take it out, an integral dx will give you x .

So, we will use the same relationship. We know that average velocity at any section, this is nothing but discharge divided by the area of flow; or Q is equal to $\bar{v} A$. These things, you have been dealing it earlier onwards.

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Now $Q = q^* x$

$\therefore \bar{v} = \frac{q^* x}{A}$

In the rectangular channel, $\bar{v} = \frac{q^* x}{By}$

Substitute these q^* in the eqn

$$\frac{dy}{dx} = \frac{-2 \frac{q^{*2} x}{g B^3 y^3}}{1 - \frac{q^{*2} x^2 B}{g B^3 y^3}}$$

Now, Q is equal to q star into x . Therefore, your average velocity becomes, q star into x divided by area. For this rectangular channel, $\bar{v}$ is nothing but, q star x by $B y$. So, what do you get from this thing? Substitute these quantities in dynamic equation; so, dy by dx is equal to, minus twice, capital Q is q star x , so this becomes q star square x by $B y$; may I beg pardon, this become q star x by, $g B$ square y square, the whole thing divided by 1 minus q star square into, x square B , by $g B$ cube y cube. So, we have just substituted it, all the quantities.

You know what is Q ; so, Q is equal to q star x ; that is substituted here; instead of the top width T , we have substituted the width B , and we got this. So, you can cancel out some of the terms and all, here; you will get the following equation.

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$$\frac{dy}{dx} = \frac{-2q^2xy}{gB^2y^3 - q^2x^2}$$

$$\frac{dx}{dy} = \frac{q^2x^2 - gB^2y^3}{2q^2xy}$$

Again

$$\frac{dx}{dy} = \frac{x}{2y} - \frac{gB^2y^2}{2q^2x}$$

$$\left(2x \frac{dx}{dy}\right) = \frac{x^2}{y} - \frac{gB^2y^2}{q^2}$$

I am just working out for your benefit, showing the working, working procedure here. So, your dy by dx will now become minus twice q star square, sorry; I am just rubbing and writing it again, this is not clear in the screen here; so, minus twice q star square x y by, g B square y cube minus, q star square x square. You can just inverse this relationship, dx by dy is equal to, q star square x square, because minus is already there, so I am just writing as per convenience, twice q star square x y .

So, again, how can you write these things? I am just splitting the entire portion now; say, dx by dy is nothing but equal to, this is x by $2y$ minus, g B square y cube by, sorry; this is g B square y square by, twice q star square x . So, when you get such an expression, you can just again look into the thing. You can just take $2x$ quantity into the numerator of the left hand side, I am just taking it like that. So, $2x \frac{dx}{dy}$ is equal to; so there was no x in the denominator here, in the first expression, on the RHS, so I have to take into that into account; so, this becomes x square here, x square by y minus, g B square y square by, q star square. So, what is $2x \frac{dx}{dy}$? What is this quantity $2x \frac{dx}{dy}$? Just go, go through your basic mathematics and all.

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$$\frac{d}{dy}(x^2) = \frac{x^2}{y} - \frac{gB^2 y^2}{q^{*2}}$$

$$\Rightarrow \frac{d}{dy}(x^2) - \frac{1}{y}(x^2) = - \frac{gB^2 y^2}{q^{*2}}$$

$x^2 \rightarrow$ Dependent variable in the differential eqn.
 $y \rightarrow$ Independent "
 $x^2 \rightarrow f(y)$

So, I will write this quantity as d by d y of x square by, sorry; d by d y of x square equal to, x square by y minus, g B square y square by, q star square. Or, this can be written according to the standard form of differential equations, so d by d y of x square is, minus 1 by y into x square is equal to, minus g B square y square by q star square.

So, here, x square is the dependent variable in the differential equation; x square is the dependent variable and it is getting differentiated with respect to y , independent variable. So, if you solve this equation, you will get, x square as some function of y. This is how, the objective of solving the differential equation should be. So, the same phenomenon is adopted here.

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$$x^2 = f(y)$$

Based on the

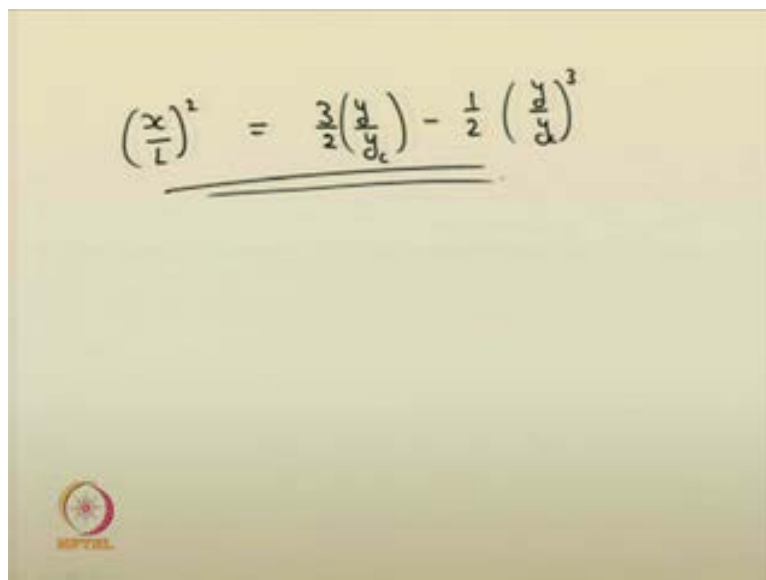


At $x=L$,
depth $\rightarrow$ critical
 y_c
 y_c
You know the
depth $y_c = \frac{q^*}{V}$

We will get, say, an expression x square is equal to some function of y , based on the boundary conditions. So, you know the boundary conditions; for the channel, water is flowing out like this; so, this depth, we suggested that this is at length, at x equal to length; you have depth is critical, your depth is critical, so y_c ; let me write it as y_c .

How do you identify the depth a critical depth, y_c ? Because, you know, the discharge at outlet Q_0 or Q_c also you can write; Q_0 is nothing but equal to q star into L . This you are quite aware. That is the discharge at this outlet. So, based on that, based on these quantities, you can easily identify, what is the critical depth, y_c . So, based on this critical depth, at x is equal to L , y is equal to the critical depth y_c , use, that condition is applied.

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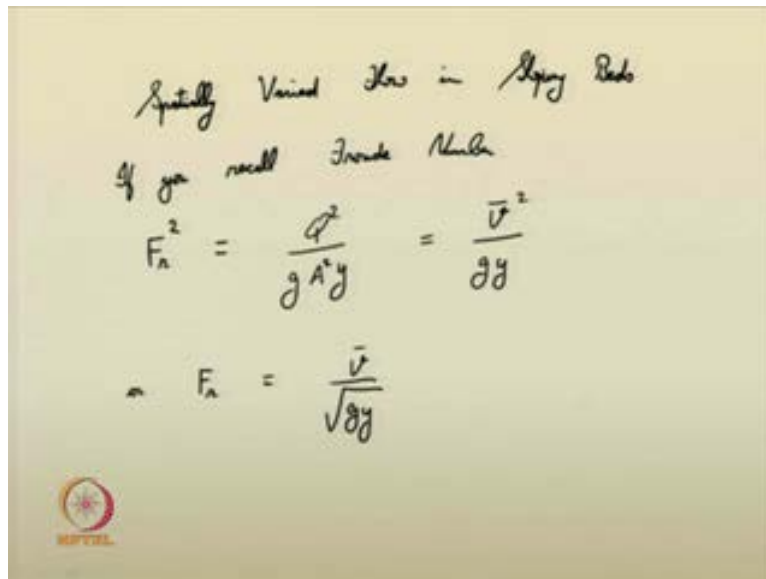


$$\left(\frac{x}{L}\right)^2 = \frac{3}{2} \left(\frac{y}{y_c}\right) - \frac{1}{2} \left(\frac{y}{y_c}\right)^3$$

All these conditions on incorporating, you will get the following expression for x , x by L whole square is equal to 3 by 2 y by y_c minus half of y by y_c whole cube. This expression, we will get, on solving the thing, applying appropriately these conditions.

Some other scientists, some scientist have tried to obtain. If you recall, in the gradually varied flow, we had a well distinguished flow profiles; say, mild slope, steep slope, horizontal, adverse, all those conditions were incorporated and we had various kinds of gradually varied flow profiles. Unfortunately, in the spatially varied flow, a well defined categories are very much limited; only very few scientists have worked on that, and they have come up with certain strategies. For example, the scientist, Lee, in 1955, he performed certain analysis on spatially varied flow. Say, in the last example on the rectangular channel, it was horizontal varied, it sorry; it is a horizontal rectangular channel; so, the bed slope is 0 .

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Spatially Varied Flow in Sloping Beds

If you recall Froude Number

$$F_r^2 = \frac{Q^2}{g A^3 y} = \frac{\bar{V}^3}{g y}$$
$$\therefore F_r = \frac{\bar{V}}{\sqrt{g y}}$$

Lee had performed experiments for spatially varied flow in sloping beds. So, may be, consider the same type of rectangular channels itself, or the parallel, the walls are parallel and all, if you consider that things into account, let us try to investigate, what are the properties or features that can be seen in spatially varied flow?

If you recall Froude number, if you recall the Froude number, we had suggested that Froude number is equal to Q square by $g A$ square y , or this is equal to v square by $g y$; or, sometimes, we may write this as Froude number F_r is equal to v by root of $g y$. So, this was giving you certain principles or certain quantities on, how, due to change in inertia and gravity; or, how the inertia is affecting; means how the motion of the body is being affected by inertial and gravitational forces and all?

That relationship is being described to the Froude number. We had described it in the critical flow, module one and all, on Froude number. For critical flow, if you recall them, Froude number should be equal to 1. So, this Froude number concept, we are again trying to utilize here.

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$$(F_2)_c^2 = \frac{q^2 L^2}{g A^3 y_c} = 1.0$$

for any depth



So, the Froude number at critical flow in the, in the same type of rectangular channels, we can now suggest that $q^2 L^2 / g A^3 y_c$ is the critical depth; this should be equal to 1. Now, for any depth, along the sloping rectangular channel; say, this is a rectangular, sloping rectangular channel; here lateral increase is there in the flow of water, so we do not know how, whether it will increase like this, or whether it will decrease or whatever be; and it is, say, it is having a outlet, this portion is outlet of the channel. So, this outlet, it may be either having a free outflow like this, or it may be some times jumping to some other; means, if water is already there in the downstream, it may be jumping, or it may be getting stagnated like that, at that level without any jump. So, we do not know the situation.

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$$F_r^2 = \left(\frac{q^2 x^2}{g A^3 y} \right)$$

With another hypothetical parameter

$$J = \frac{S_0 L}{y_c}$$

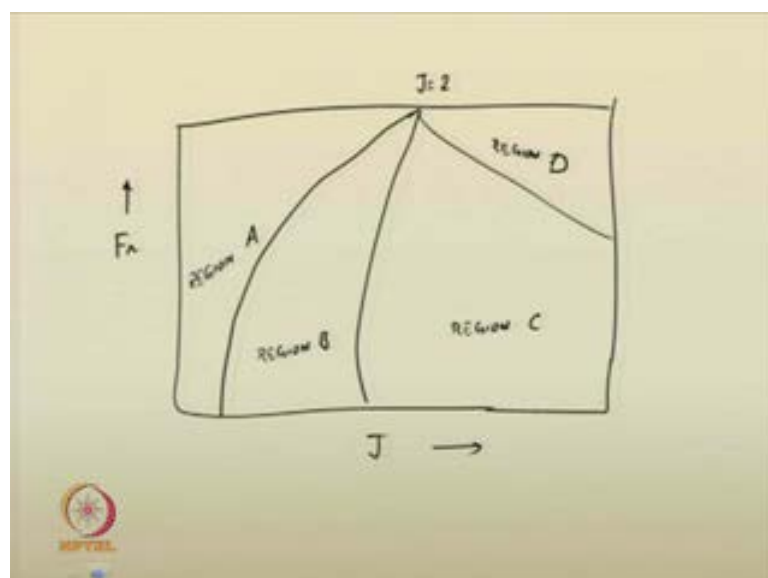
Just to compare F_r with J



Based on this, you can, Lee in his analysis, he suggested that the Froude number at any section, it can be given as, q star square x square by, $g A$ square y . So, when he gave this quantity, Froude number F_r square is equal to q x square by, $g A$ square y . This quantity, subsequently, with this quantity he related with another parameter, another hypothetical parameter, he suggested; he suggested a hypothetical parameter, say, J ; J was defined as the bed slope into length of the channel, divided by the critical flow.

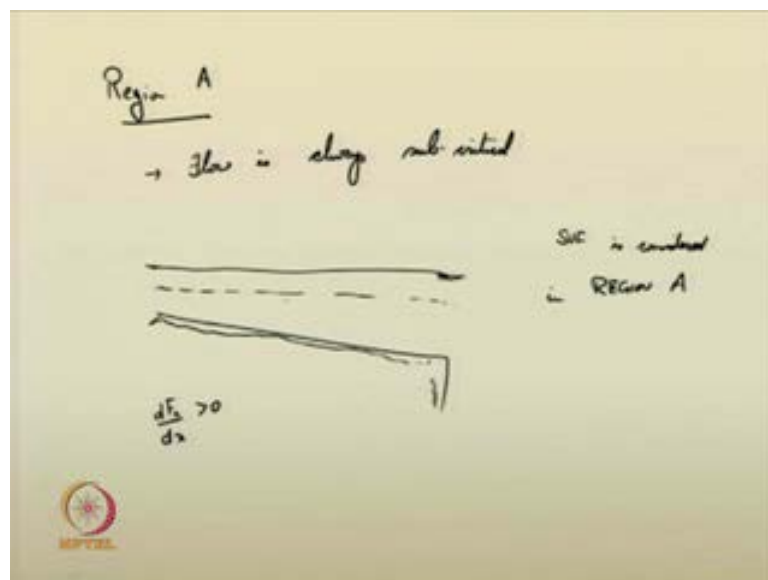
So, we are assuming that, at this brink, the flow is critical; this is the length of the channel; with these things in picture, he defined a parameter J is equal to $S_0 L$ by y_c , and tried, tried to compare F_r with J . He came with amazing profile.

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So, although, we do not have data and all, we have not plotted, plotted these things in scale. We are just simply trying to introduce you the concept. So, with Froude number on the y axis and the hypothetical parameter J on the x axis, they have identified that there are 4 main regions of fluid flow. Say, this is; say, this is a particular region where I can suggest that this is region A; here they have suggested that J is equal to 2, from that point onwards, this is being suggested; another region here identified, and this is region B of fluid flow profile; then, a third region was suggested region C; and this is region D, like this 1 regions were identified for the spatially varied flow. So, your spatially varied flow can occur in any of these regions; it can be in region A, or it can be region B, region C, or region D.

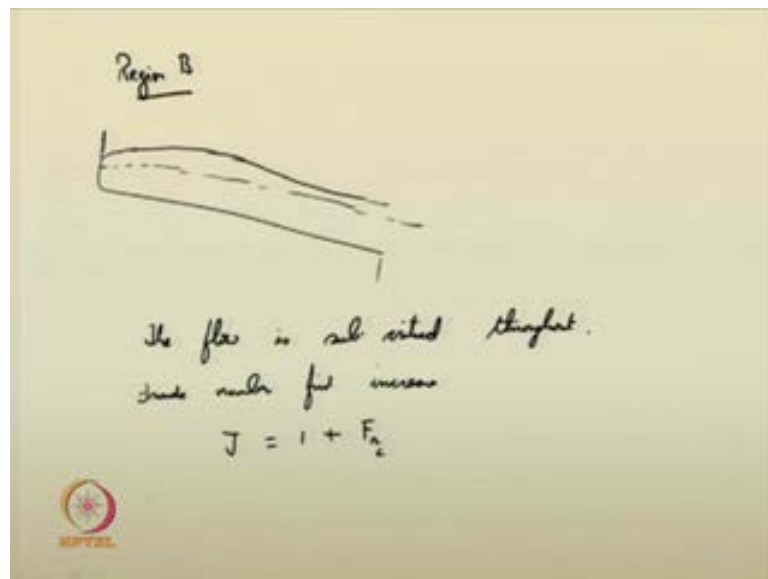
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In the region A, in the region A, it is considered that flow is always sub critical, flow is always sub critical. So, the critical flow depth line, it increases from the starting of the channel; say, if it is starting like this, from here, even the critical depth line increases like this, and your flow is always sub critical, ok. That is the depth of flow is always above the critical flow depth line, like this the flow can occur.

If that is been shown, if the, this type of flow exist, such type of spatially varied flow is considered in region A, in region A, the property is that the slope of the Froude number, $\frac{dF_r}{dx}$, this is always greater than 0. So, the depth of flow is always greater than the critical depth, that is also been suggested.

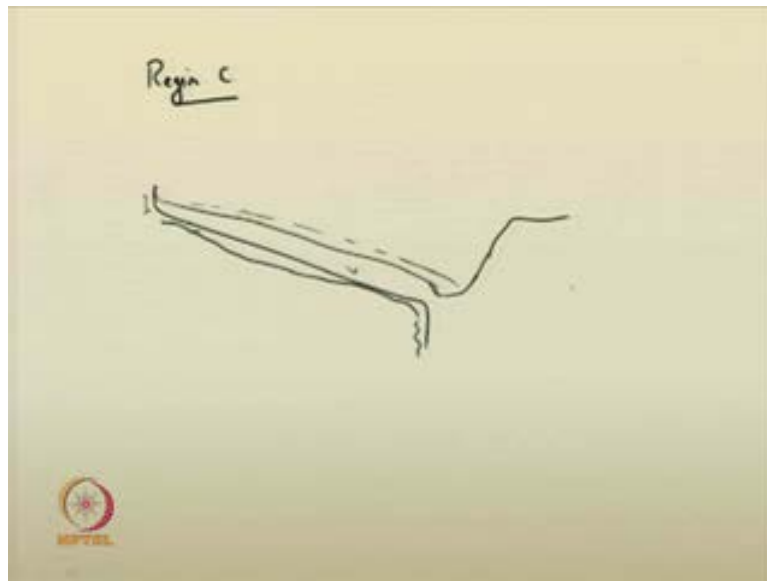
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Region B, in region B what happens is that; say, if this is the critical depth line, again the flow is sub critical throughout; now, the assumption here is that, it will first increase, then it may decrease like this; the depth of flow, it may first increase, then it may decrease. So, Froude number, in this case the Froude number first increase, as the flow proceeds downstream; then it reaches a maximum value, almost near to 80, then it decreases. They have identified that this J is almost equal to 1 plus Froude number at the critical section. So, at this location, this J , you can separate region B with respect to region C.

Similarly, region C and region D, they are also defined. Region C represents; in the graph, this region C, it represents the condition at which there is super critical flow in downstream portion and a hydraulic jump occurs in the channel.

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Region C; that is a super critical flow occurs in the channel; so, this is a channel bed, your flow may be something like this, it is a below the critical depth, and from here it causes a jump and flow like this. So, this is one type of situation that can be encountered in such channels. Super, due to the super critical conditions and all, the it is considered in another region; so the, it is considered as, in a region, defined as region C and all.

Region D also similar, in a similar form; instead of hydraulic jump, it will be purely merging with the water profile, they are existing in the downstream, so, without hydraulic jump. So, that is how region D is also defined. As, we, according to our, these things and all, we are not interested to cover much deeper in these portions and all; and moreover this module has taken more lectures also, we try to minimize the portion here.

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SVF dynamic equation

$$\frac{dy}{dx} = \frac{(S_0 - S_f) - c_1 \frac{Q^2}{gA^3}}{1 - c_2 \frac{Q^2}{gA^3}}$$

$$\frac{dy}{dx} = S_0 \frac{1 - \frac{S_f}{S_0} - c_1 \frac{Q^2}{gA^3 S_0}}{1 - c_2 \frac{Q^2}{gA^3}}$$


So, again, look, come back into the spatially varied flow dynamic equation. So, you have $\frac{dy}{dx}$ by $\frac{dy}{dx}$ this is equal to S_0 minus S_f minus c_1 into $\frac{Q^2}{gA^3}$ by 1 minus $c_2 \frac{Q^2}{gA^3}$. So, this, I can just write it in a convenient form, S_0 , I am taking it out; 1 minus S_f by S_0 minus c_1 into $\frac{Q^2}{gA^3}$ by S_0 the whole thing divided by 1 minus c_2 into $\frac{Q^2}{gA^3}$. So, recall the module one and module two portion, we had defined discharge with respect to conveyance factor, various conveyance factors and all; we had described them.

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Using conveyance factor

$$Q = K S_f^{1/2} \rightarrow \text{Actual Discharge}$$

$$Q_c = K S_0^{1/2} \rightarrow \text{Normal discharge}$$

$$\text{The } Q_c = \left(\frac{gA^3}{c_2 T} \right)^{1/2}$$

then we have incorporated the conveyance factor



So, using the conveyance factor, Q is equal to $K S_f$ to the power of half; K is the conveyance factor again, S_f is the friction slope, if you remember them; so, this is the actual discharge.

You can also give normal discharge using the bed slope. So, it is nothing but, $K S_0$ to the power of half; it is for a given depth, what is the normal discharge that can be identified? What is the normal discharge for a given depth? That is that given depth can become a normal depth, using conveyance capital K and bed slope as the parameter.

Also, recall the critical flow; the critical flow Q_c ; it can be given as $g A^3$ by $C^2 T$ to the power of half. Here, we have incorporated the correction factor that is why C^2 is coming into picture here. Earlier C^2 was not there; we have taken correction factors as unity and all.

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$$\frac{dy}{dx} = S_0 \frac{1 - \frac{K^2 Q^2}{K^2 Q_n^2} - \frac{Q^2}{Q_n^2} \cdot \frac{K^2}{g A^3} \frac{dQ}{dx}}{1 - \left(\frac{Q}{Q_n}\right)^2}$$

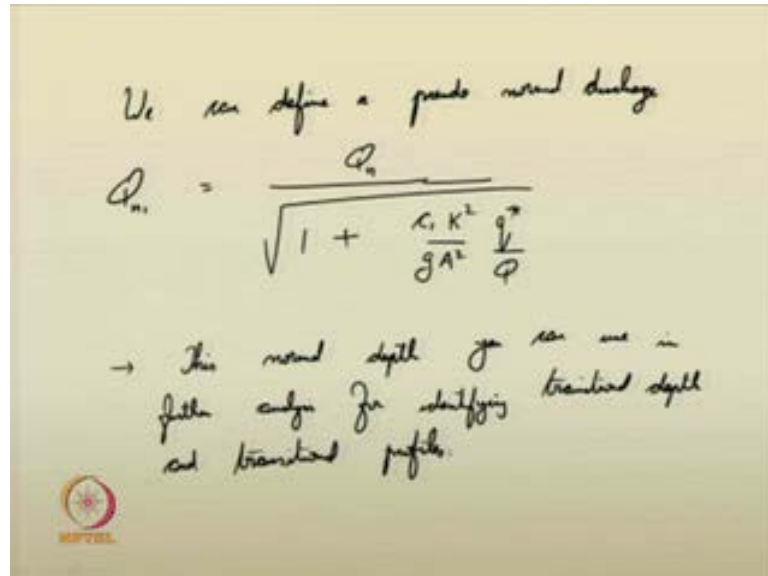
$$= S_0 \left\{ \frac{1 - \left(\frac{Q}{Q_n}\right)^2 \left(1 + \frac{K^2}{g A^3} \frac{dQ}{dx}\right)}{1 - \left(\frac{Q}{Q_n}\right)^2} \right\}$$

So, if the, based on this thing, substitute it in the equation for dy by dx . You will get, dy by dx is equal to, S_0 into 1 minus K square by Q square K square by Q_n square; S_0 is written like this, S_0 square is written like this minus Q square by Q_n square C^2 times K square q star by $g A$ square Q . I hope, you are able to understand. If you recall that, here in the denominator expression S_0 is coming into picture; S_0 is coming into picture, therefore, Q_n term is also coming there. And as S_0 , for obtaining S_0 , you are require Q_n square term, we are multiplying both the numerator and denominator by Q in this expression; that is what, here it is Q square and here it is Q ; this whole quantity divided by 1 minus Q by Q_n whole square. It is in the similar form, as you have done for dynamic equation for gradually varied flow and all.

In a, almost in a similar way, we are writing the expressions. So, now 1 minus Q by Q_n whole square 1 plus $C^2 K$ square by $g A$ square q star by Q ; so, whole thing divided by 1 minus Q by Q_n whole square, like this we get. So, we can, if you just look back the thing,

if you just look back into this expression, and if you recall the gradually varied flow dynamic equation and all, in a similar way, some quantity can be desired.

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$$Q_{n1} = \frac{Q_n}{\sqrt{1 + \frac{C_1 K^2}{g A^3} \frac{q^2}{Q}}}$$

We can define a pseudo normal discharge
 Q_{n1}

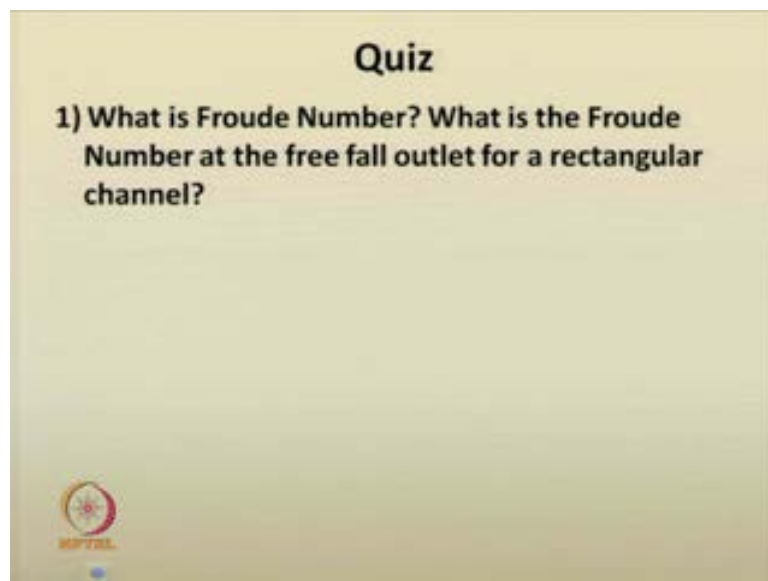
$$Q_{n1} = \frac{Q_n}{\sqrt{1 + \frac{C_1 K^2}{g A^3} \frac{q^2}{Q}}}$$

→ This normal depth Q_{n1} can use in further analysis for identifying transitional depth and transitional profile.

This instead of writing Q by Q_n whole square into $1 + C_1$ into Q square K square by $g A$ square q star by Q , we can define a pseudo normal discharge Q_{n1} something like that, where this is nothing but equal to the normal discharge that would have been for that corresponding depth, Q_n divided by root of $1 + C_1 K^2$ by $g A$ square q star by Q , like this you can easily define the thing.

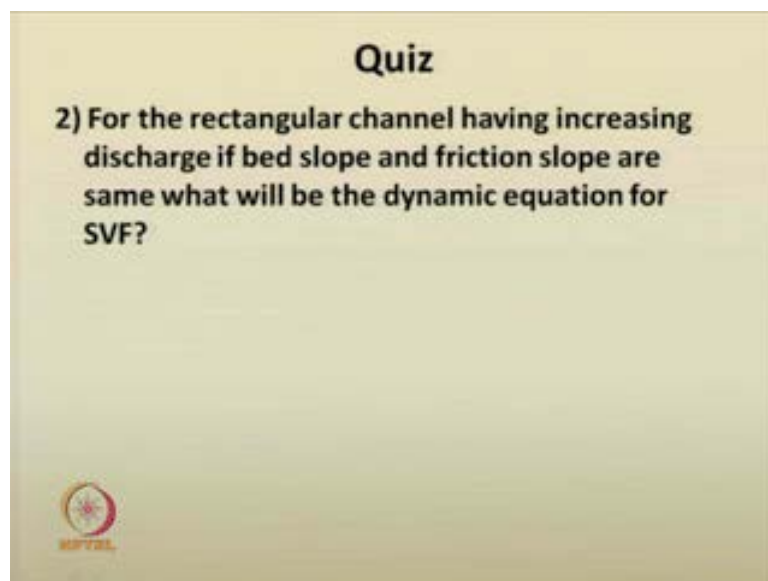
So, this normal depth, if you can identify that; this normal depth, you can use in further analysis for identifying transitional depth or transitional profile and all, transitional depth and transitional profiles. So, this way, the things are described here. As I mentioned earlier, we are, we will not go further into more aspects of the spatially varied flow. If time permits, we may come back again later in the final modules and all, and see that spatially varied flow can be further covered or not. With this way, we would like to wind up this module. We can have a brief quiz also, for today's lecture. The quiz question, I can just dictate it now.

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So, the first question: what is Froude number? What is the Froude number at the free fall outlet for a rectangular channel? What is the Froude number at the free fall outlet for a rectangular channel? This is the first question.

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And, your second question is: For the rectangular channel having increasing discharge, if the bed slope and friction slope are same, what will be the dynamic equation for spatially varied flow? That is, for the rectangular channel having increasing discharge, if your bed slope and friction slope are same, what will be the dynamic equation for spatially varied flow?

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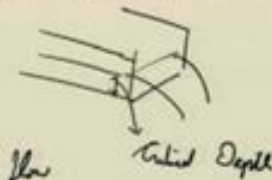
Quiz

1) What is Froude Number? What is the Froude Number at the free fall outlet for a rectangular channel?

$$F_r = \frac{\bar{v}}{\sqrt{gy}}$$

$y \rightarrow$ Depth of flow
 $\bar{v} \rightarrow$ Average velocity

$F_r = 1.0$ at critical depth



So, the solution for the first one; what is Froude number? I hope, you all know that; we have been studying this; we have taken, means, in many of the modules these are coming into picture; in many branches also, Froude number is being taught. Froude number F_r , this is nothing but velocity by root of g and product, root of $g y$; that is y is depth of flow, v is average velocity in a section. So, based on these things, we have derived Froude number.

So, what is the Froude number at the free fall outlet for a rectangular channel? If you have a rectangular channel like this; if you have a rectangular channel and the water is having a free out fall, so the depth of flow here, this will be critical depth, right. So, Froude number will be equal to 1 at critical depth. So, for your question, the Froude number will be 1.

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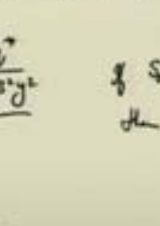
Quiz

2) For the rectangular channel having increasing discharge if bed slope and friction slope are same what will be the dynamic equation for SVF?

$$\frac{dy}{dx} = \frac{S - S_f - \frac{2Q^2}{g^3 y^3}}{1 - \frac{Q^2}{g^3 y^3}}$$

if $S = S_f$

$$\frac{dy}{dx} = \frac{-\frac{2Q^2}{g^3 y^3}}{1 - \frac{Q^2}{g^3 y^3}}$$



Next question: For the rectangular channel having increasing discharge, if bed slope and friction slope are same, what will be the dynamic equation for SVF? Just write, for the increasing discharge, $\frac{dy}{dx}$ is equal to S_0 minus S_f minus, $\frac{2Qq}{gB^3y^3}$ by $1 - Q^2$ by $\frac{T}{A}$ or $\frac{T}{B}$, that also you can write it, like that. Now, you know that this quantity T is same as B , so like that this expression, what happens, now? $\frac{dy}{dx}$, if S_0 is equal to S_f , then $\frac{dy}{dx}$ is equal to minus $\frac{2Qq}{gB^3y^3}$ by $1 - Q^2$. This will be your expression for dynamic equation. So, this way, we are winding up this module. In the next lecture, we will be taking the new module on rapidly varied flow.

Thank you.