Advanced Hydraulics
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Module - 3
Varied Flows
Lecture - 10
Spatially Varied Flow

We are on the lecture series on advanced hydraulics, part of the post graduate courses in civil engineering for the NPTEL program development. So, we are going through the third module on varied flows, for last few classes.

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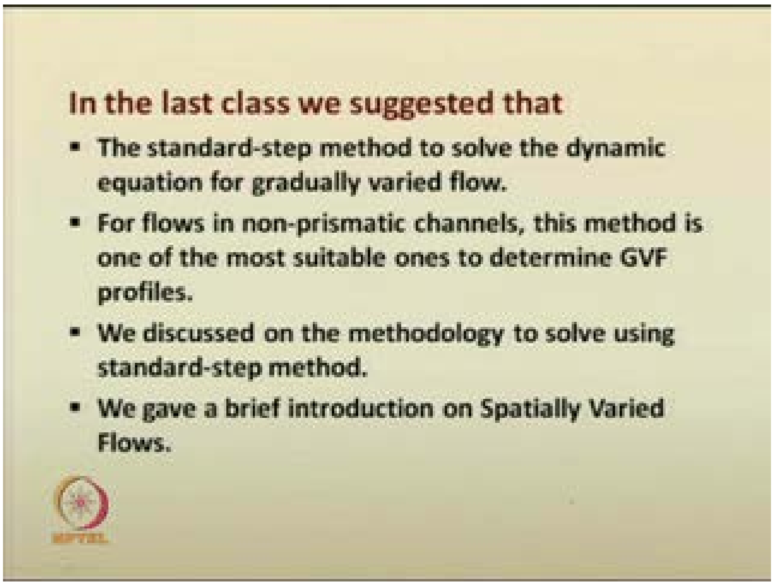

In the last class, we had discussed on the standard step method to solve the dynamic equation for gradually varied flow. We are, we have seen that this method is very much suitable for non prismatic channels; for prismatic channels, we have already seen the other methods of solving dynamic equation. We have also discussed on the methodology to solve using the standard step method. We have discussed it elaborately; although, we have not solved that problem but we have discussed the methodology on how to solve that problem and all. After that, we had given a brief introduction on spatially varied flows. So today, we will be elaborately discussing on spatially varied flow.

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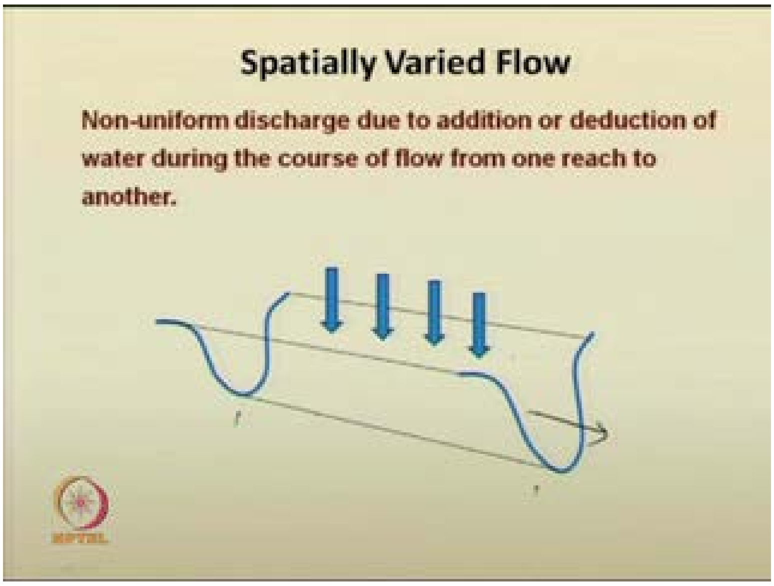

So, as discussed in the last class, spatially varied flow means, it is a non uniform discharge due to the addition or deduction of water during the course of flow from one reach to another reach. So like this, when it flows in this direction, some more quantities of water are getting added; and the flow entirely becomes non uniform; so that, these such type of flows are called spatially varied flow.

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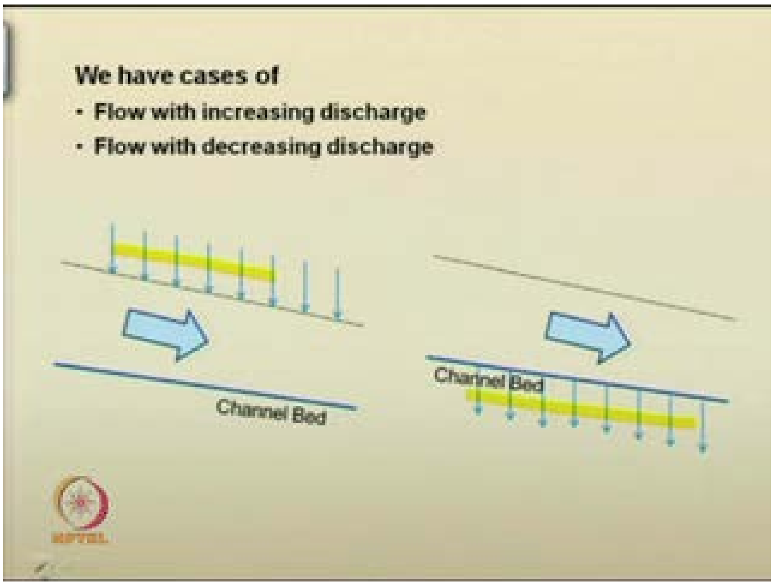

We have seen that, there are two types of spatially varied flow; that is, flow with increasing discharge; in the reach, in the flow, if the channel flow is in this direction and if some quantity gets added into the channel in that reach, this is called increasing flow with increasing discharge; this is flow with increasing discharge. And, flow with decreasing discharge, when the flow occurs from this direction to, from the upstream to downstream, some quantity gets deducted as seen here; this is flow with decreasing discharge.

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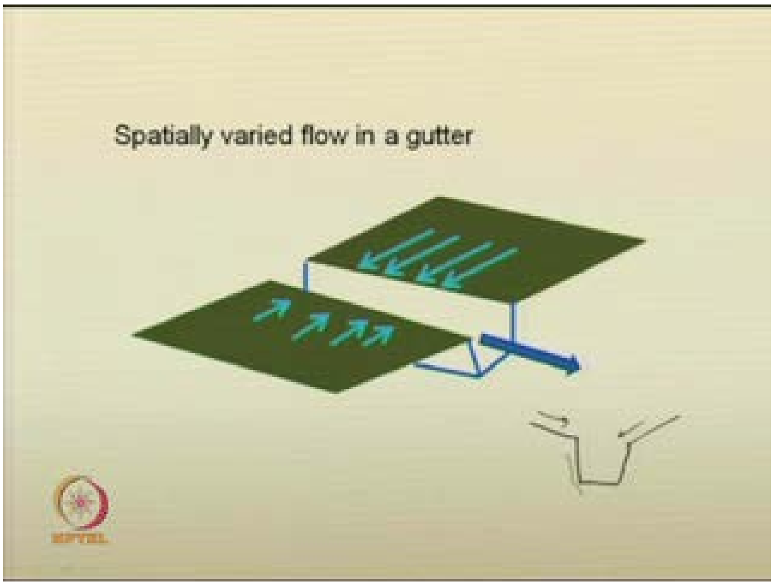

You can also see some more examples of spatially varied flow, especially if there is a gutter, road gutter, or a roof gutter, water from the roofs and all, they flow in these directions and gets collected in this channel, in this gutter; and the predominant direction of flow in the gutter is this. So, here, this, in this entire roof is collecting water and draining into this channel; so, the flow occurs in this main channel, like that; it is also a spatially varied flow; you can imagine several types of flow. Here, the cross section may be like this; if you look into that, flow occurs in this directions and the channel cross section is of the flowing form.

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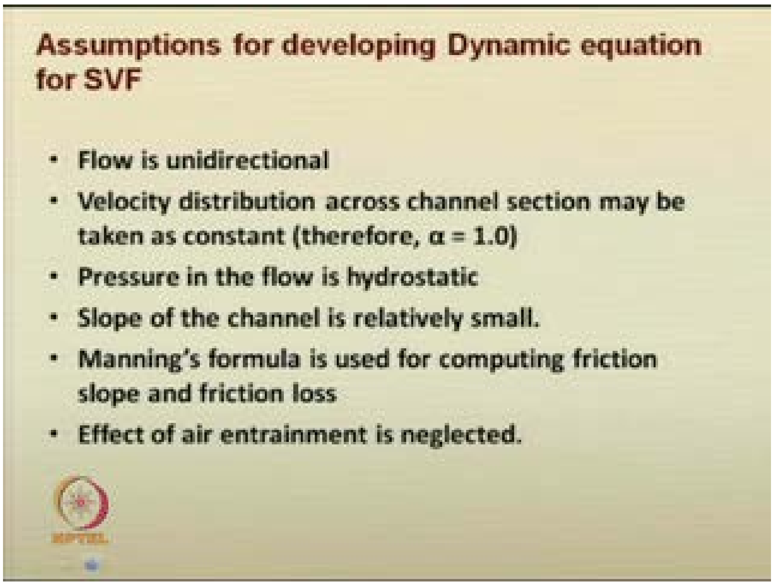

The assumptions involved for developing dynamic equation for spatially varied flow, we have discussed that in the last class. We suggested that the flow is unidirectional; means, velocity distribution across the channel section may be taken as constant, so that we may approximate various places, alpha and beta as 1 and all; pressure in the flow is hydrostatic; slope of the channel is relatively small; Manning's formula is used for computing friction slope and friction loss; effect of air entrainment is neglected. These things we discussed in the last class.

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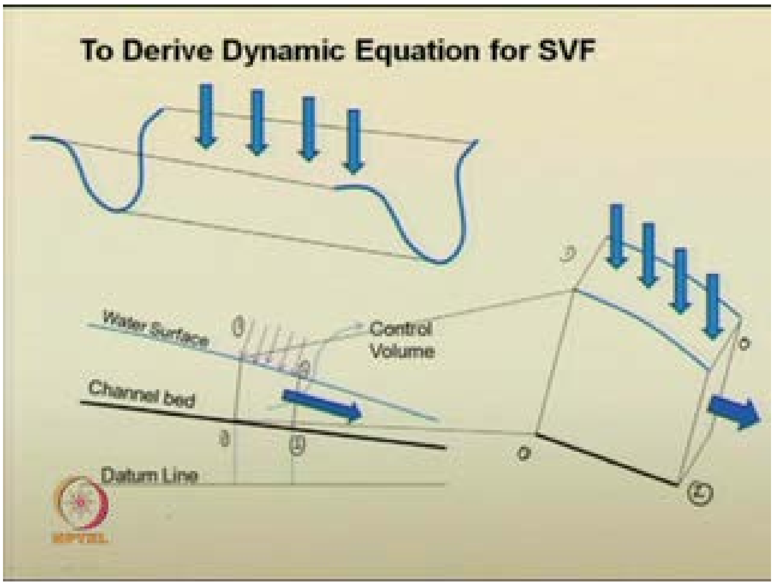

Now, to derive the dynamic equation for spatially varied flow: as shown here, this is a 3 dimensional view of the channel. Some quantity is getting added here. If I take a longitudinal view from this thing, a datum line is there, this is the channel bed, water surface line. I am taking a control volume between 2 sections, 1 1 and section 2 2. So, that entire control volume is elaborately shown here, in the magnified form here. So, this is section 1 1, section 2 2. So, you can see some quantities of water is getting added along the reach of that control volume.

So, we will derive the continuity; we will derive the equations, governing equations that are required to obtain the dynamic equation; or, the governing equations that are called dynamic equations for spatially varied flow, so, as seen in the channel. So, the quantity increases, the quantity is increasing along the reach of this channel; so, let me suggest that this is q star, it is increase; the quantity is increased at a rate of q star; that is, q star is the volume, or it is the discharge per unit length of the reach, that is getting added into the channel.

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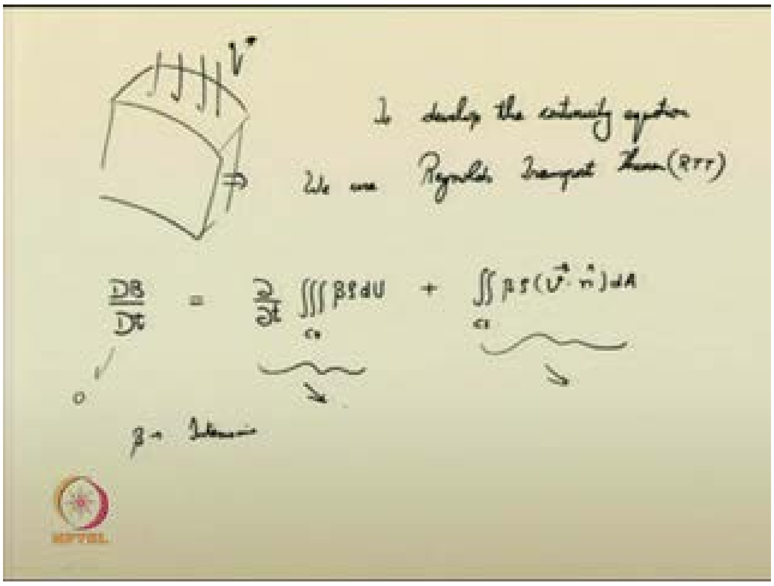

So if you take it like that, q star quantity is getting added here. Now, as the phenomenon here, we have suggested that this quantity it is increasing; q star, it is a uniform rate; this discharge per unit length, it is a uniform rate; and it is adding equally. So, we are assuming that the entire process is now in a steady state condition, whatever inflow is coming into the control section, whatever is going out, and whatever is adding into the control volume, all those quantities, they are now in a steady state.

So, we are suggesting that the flow is predominantly in one, in this direction; the flow direction is unidimensional or one dimensional, to develop the continuity equation, for such a situation we use Reynold's transport theorem. We have already discussed about R T T in earlier lectures also, I am not going to discuss further on that. So, if you recall the Reynold's transport theorem, the equation for Reynold's transport theorem suggest that the material derivative of any extensive property capital B, that is D B by B t, this is equal to dou by dou t of that control volume, whichever control volume you are taking into account, beta rho d u plus, integral area, integral area beta rho, V dot n, d A, where this quantity, the first term we suggested that, it is the rate of change of extensive properties stored inside the control volume, and this is the net outflow of the extensive property across the control surfaces of the control volume; we suggested that beta is intensive property.

So, let us just take B is equal to mass of that water in the control volume. So, when we took this mass as the control volume, so you know that in this control volume, B is equal to mass. I can suggest that, mass can neither be created nor be destroyed; ,so this quantity, this quantity will be 0; now, as the entire process is steady state condition, this also becomes 0; we have the net outflow from the control surfaces, which has to match.

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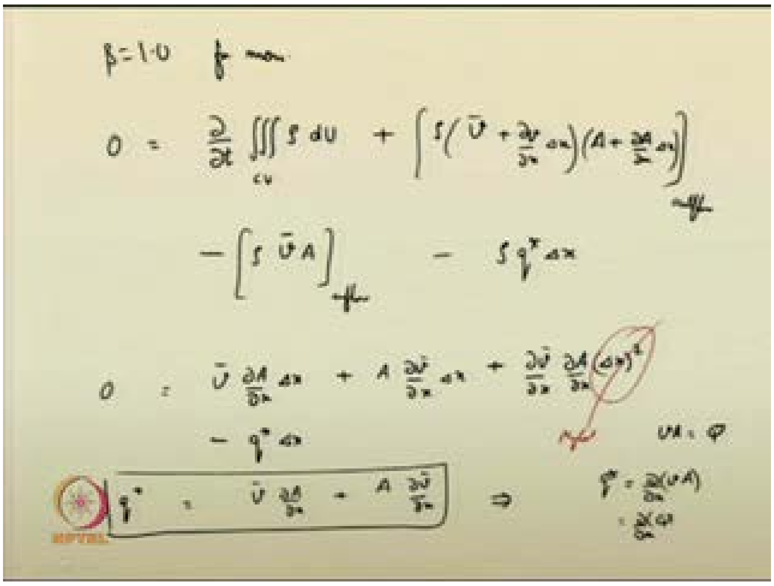

So, I will write it in the following form. 0 is equal to dou by dou t of, you know small beta is equal to 1 for mass; so, I am writing it, rho d u plus, rho V, at this section, on this left hand side section one, section 1 1, this is section 2 2. Here, the properties are V, the average velocity is V; area is A; discharge is q. Here, we are suggesting that this is select, we are assuming the control volume is having a reach length of del x. So, V plus, dou v by dou x into, del x; then area, we are considering, A plus, dou A by dou x, into del x; and q, we are suggesting, q plus, dou q by dou x, into del x; like that, we are suggesting the properties at each sections.

So, here, I can write now, dou, rho V, plus dou v by dou x into del x, A plus dou A by dou x into del x, so, this is the entire outflow; then, as v dot n, the vector means the v dot n in the expression, v dot n is negative for the inflow conditions; so, we are suggesting that, minus sign here, so, rho v A, in the inflow; then, what are the quantities, we have rho q star into del x; some quantity is added through the top also, that is also incorporated here.

So, we have 0 is equal to, you just rearrange the terms here, 0 is equal to v bar dou A by dou x into del x plus, A dou v bar by dou x into del x plus, some higher order terms, higher differential terms, this is del x square minus, q star del x. Here, what we are going to do is that, we are going to neglect higher order terms; wherever these things are coming, we are going to neglect them. So, this expression now changes into q star is equal to v bar dou A by dou x plus, A dou v bar by dou x. So, this, from the continuity equation, we are going to get the expression q star; that is, the discharge per unit length, that is getting added or deducted in the spatially varied flow, along the reach, that is small q star; from the continuity equation, we are getting this particular expression.

If you closely watch that, you know v A is equal to the discharge at any section q, so this quantity is, is nothing but, q star is actually, dou by dou x of v A, that is dou by dou x of q.

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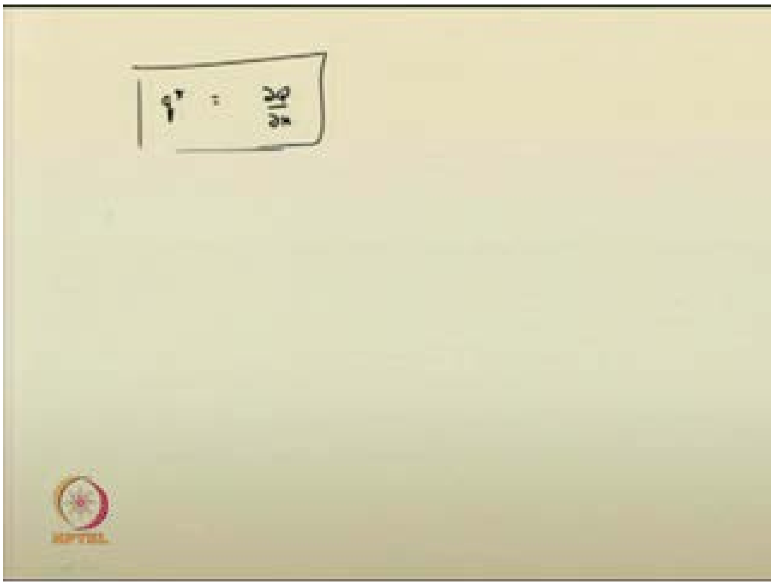

So, I can suggest that q star is nothing but, dou q by dou x. So, this will be an useful expression. Later on, we will be using it extensively.

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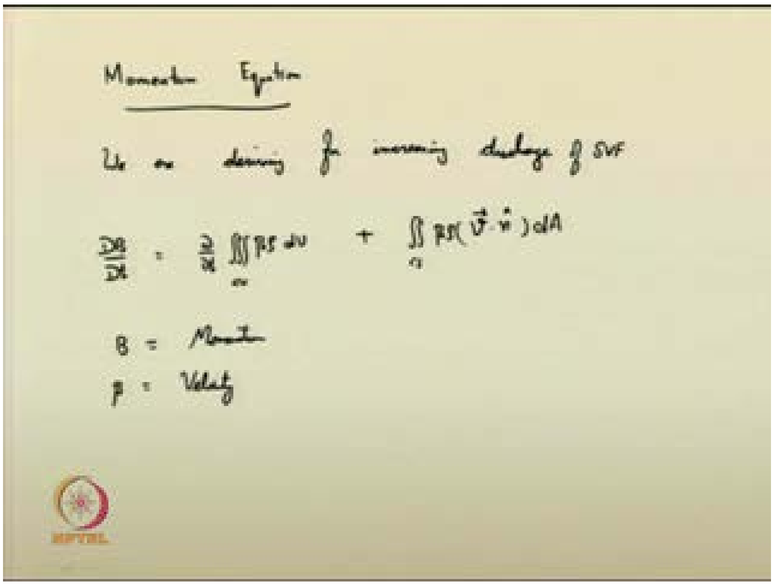

Now, let us go for the momentum equation. So, we are deriving for increasing flow. So, in the earlier class we mentioned that, for increasing discharge we will be using momentum equation. So, we are doing, for increasing discharge. We are deriving for increasing discharge of spatially varied flow. So, again, the Reynold's transport theorem, we will be going to use them; beta rho d u plus, control section beta rho v dot n d A.

Now, in this case, the extensive property B is equal to momentum, and intensive property beta is equal to, it will come out to be velocity, because momentum divided by mass will give you velocity, so that you have to note it.

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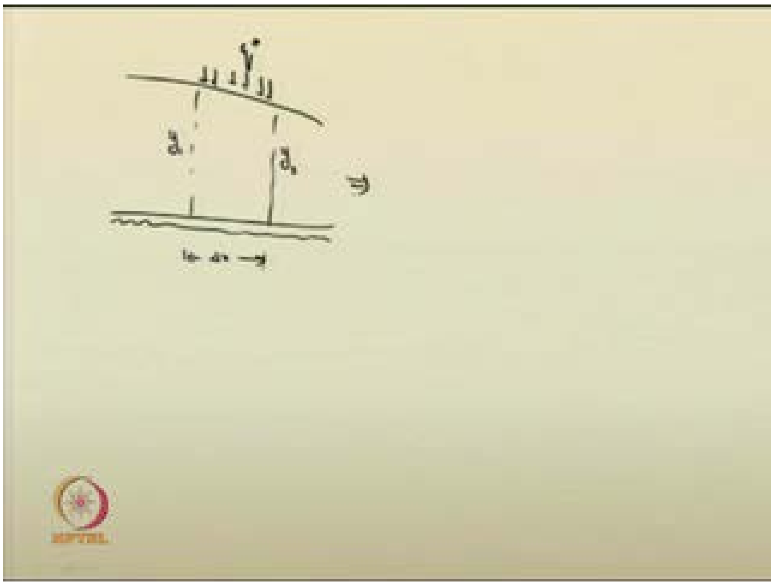

So based on that thing, I can just suggest now, the following thing. This is the bed slope; let this be the depth y 1 at section 1 1; y 2 at section 2 2; they are separated by a del x; this is the control volume, earlier we suggested; we are having an increasing discharge at rate q star per unit length, having flow.

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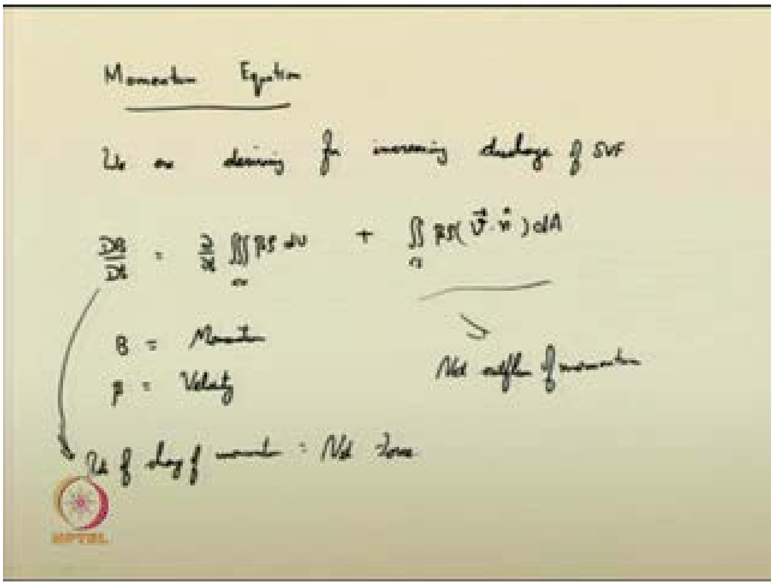

So, the net outflow, according to this equation; so, this is the net out flux of momentum across the control surfaces; what are they? That we have to analyse. This is the rate of change of momentum, that will give you rate of change of momentum; so, that will give you the net force, net force in the system or in the control volume. So, based on these principles, we are going to give the momentum equation here.

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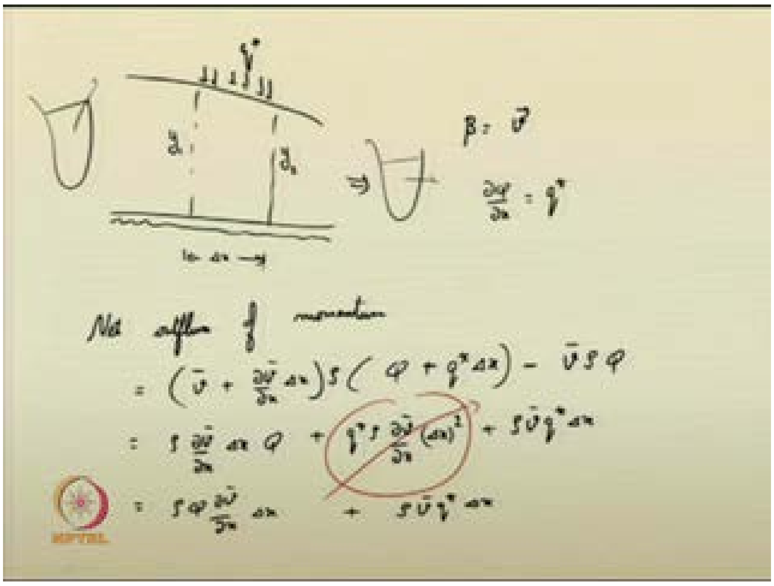

So, small beta is equal to v. Now, the net out flux of momentum, I can write it; just recall in the continuity equation in a similar form, so we will be using that thing, net out flux of momentum across the control surfaces, this is equal to, first we are writing the out flux, dou v by dou x into del x, into rho, Q plus, dou q by dou x into del x, that I can replace it; that is, if you recall dou q by dou x is nothing but q star. So, I have just replaced it by q star; then, minus, in the inflow section v rho q; so, in the whichever section, here the section will be like this, here the section will be like this, whatever area is there, we are taking the average velocity in these sections; so, that is why I am putting the bar here. So, v rho q is the inflow momentum, influx of momentum and this quantity is the outflux of momentum.

So, I can now easily write the quantity as, rho dou v by dou x del x into Q plus, q star rho dou v by dou x del x square plus, rho v q star del x. So, as we mentioned earlier, we are going to neglect the higher order terms; higher order terms, these quantities we are going to neglect; so, we will get the expression for net out flux of momentum as rho Q dou v bar by dou x into del x plus, rho v bar q star del x .

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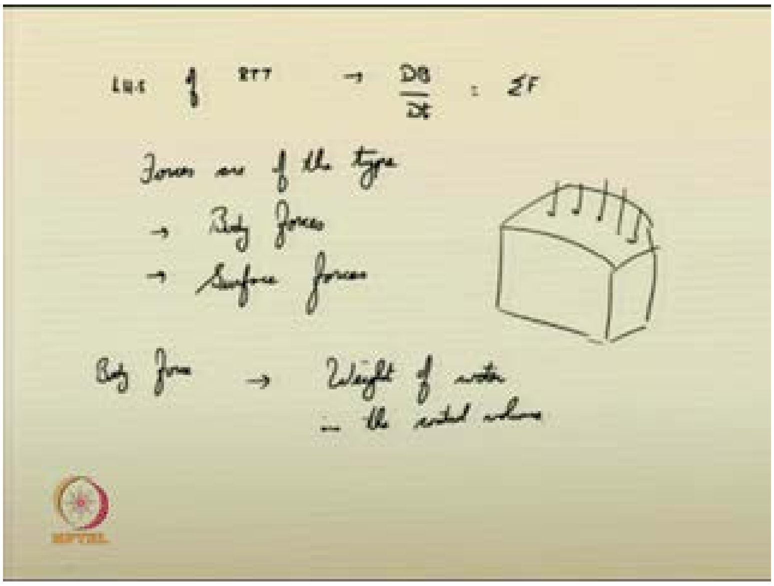

So the left hand side of Reynold’s transport theorem equation, it consist of the material derivative D B by D t. So, this is nothing but, the net force in the system. So, in the the net forces for this control volume, whichever control volume you are going to take into picture, so, in this control volume the net forces are, means, the types of forces are body forces and surface forces. So, forces are of the type body forces and surface forces, surface forces.

So, these, what are the things? It has control surfaces; so, whatever forces acts on the control surfaces, they are called surface forces; whatever forces are there inside the control volume, they are suggested as body forces. So, you can easily suggest, means, you can now remember them; that is the body force in this control volume is nothing but, weight of water in the control volume, right; this is the body force, or the component, or whichever, due to the weight of water, what is the force in the flow direction, that we have to take into account.

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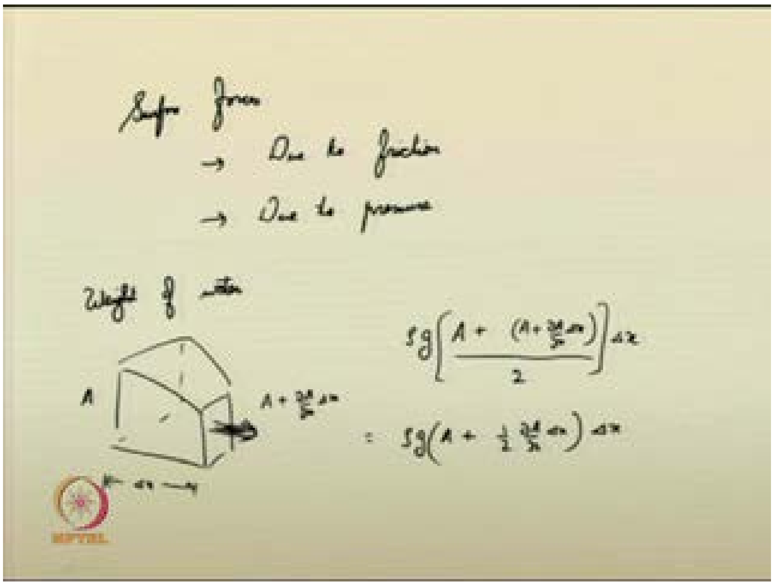

Surface forces in that control volume, it may be developed due to friction, it may also be developed due to pressure exerted by the adjacent liquids on the walls of the control volume or on the control surfaces. So, we have to take into account all these forces and obtain the net force.

So, let me ask you, what is the weight of water? What could be the weight of water? So, if you have such a control volume, here it is A, and here the area is dou A by dou x into del x, then the weight of water is nothing but, density of water into acceleration due to gravity, A plus, A plus dou A by dou x into del x. So, we are taking the average area between this portion and this portion; so, somewhere here, that is being taken into the length del x. So, I can write this as rho g, A plus, half of dou A by dou x into del x, into del x.

So, the component of, as there is a bed slope, and the flow of liquid is in this direction predominantly, we need to find the, actually we are, we have to find the net force in the flow direction.

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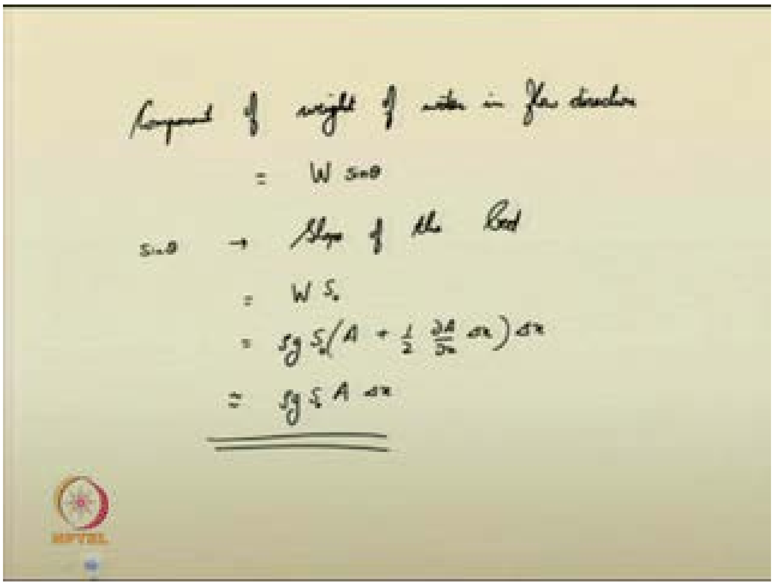

Therefore, the component of weight of water that is available in the flow direction, that can be obtained as; component of weight of water in flow direction, if you are giving the weight as w then this is nothing but, w sin theta. And, sin theta you know, slope of the bed; therefore, this is nothing but, w S 0, is equal to rho g S naught A plus half dou A by dou x into del x, into del x. So, this can be approximated; wherever, del x, higher order of del x terms are coming, we can neglect that; this can be approximated as rho g s naught, A into del x. So, this is your component of weight of water in the flow direction.

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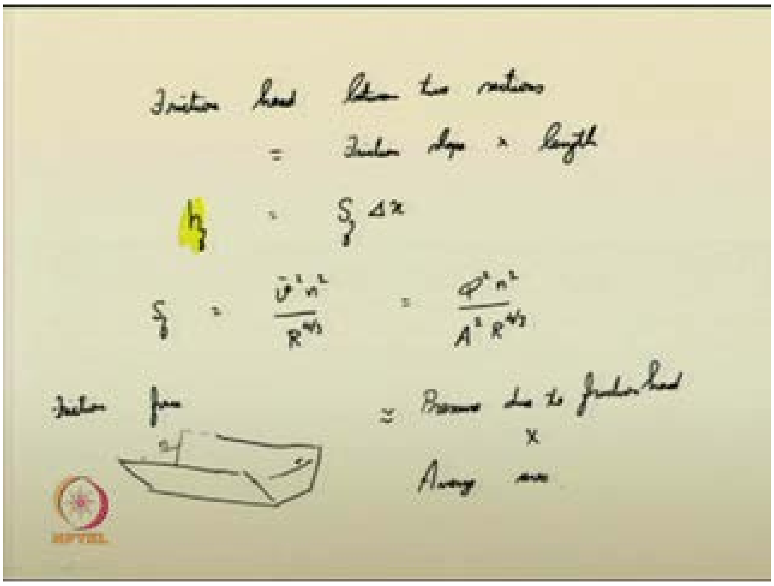

How will you obtain the 2 surface forces- friction and pressure? So, the friction; first consider friction; friction head between 2 sections. According to our theory, this is nothing but, friction slope into length of the reach. So, that is, S f into del x. This is your friction head h f. S f, you know, it is obtained using Manning's equation; it is nothing but, v squared n square by, R to the power of 4 by 3; or, it can be given as Q square n square by, A square R to the power of 4 by 3.

So, how do you compute friction force? Say, you know that friction force, along the, any, along the any reach of the channel, you have the following reach of the channel. So, there are wetted perimeters. So, there are wetted perimeters; and all along that wetted area, the friction force will be acting. And, it has been obtained through Chezy's, earlier in the uniform flow and all, approximations, uniform flow approximations and all, at that time we have suggested that, the entire concept, the entire friction force, it can be approximated, that is friction force can be approximated in such a way that, you can first compute pressure due to friction head, that is friction head whichever we are computing this h f, into the average area of flow section; this was approximated.

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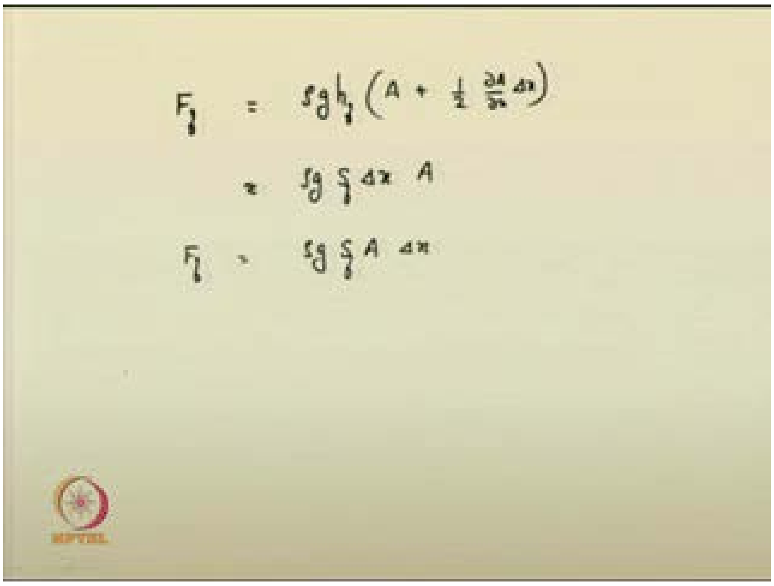

So, based on the same thing, I can now write as, friction force is equal to, F f is equal to; what is the pressure due to friction head? It will be rho g h f; and the average area, you can recall that, it can be given as A plus half dou A by dou x into del x, so, this is your average area. So, friction force, I can write it like this. This you can easily now approximate, S f into del x into A; whatever quantities above that are there, that is del x into del x terms, higher orders of del x terms, we are omitting them; so, this can be approximated. I can write it again. Friction force is equal to rho g S f A into del x. So, we got the weight component, we got the friction force; now, next remaining is the pressure force, force due to pressure.

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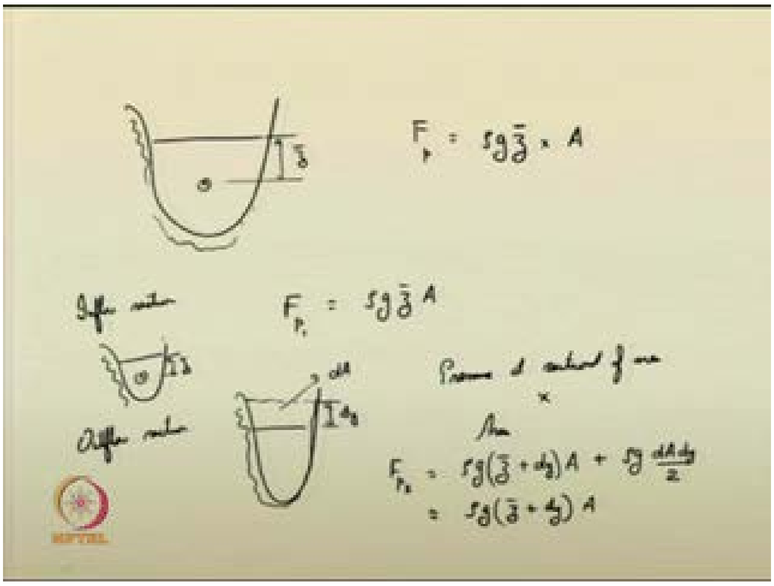

So, for any cross section, if you recall, how did you compute the pressure due to, sorry, how did you compute the force due to pressure? Say, if this is the water level; then, you have suggested that, whatever is the centroid of this area, that is height to the centroid of the area from the top, if we give it as z bar, then the pressure, the force due to pressure in this can be given as the pressure computed at this centroid, into the area of the cross section, so like that. So, it can be suggested that head, or what is the pressure due to this thing, rho g z bar is the pressure, into area; so, this will give you the pressure force in any cross section.

So, similarly, at inflow section, let me suggest that it is having this depth to the centroid is z bar; so, F p 1; I am giving it as rho g z bar into A. Similarly, at outflow section, at outflow section; so, area may increase or may decrease; but, let us assume that it has increased by a quantity d A, the area has increased by a quantity d A; and, in the height or the change in water elevation is d y, let us assume; for this, between the reach del x, that is, we have the 2 sections separated by a distance del x.

So, as we have recalled earlier, here also the pressure at the centroid of this area, at centroid of area, into area will give you the pressure force. So, F p 2, I am computing in the following form; this is nothing but, rho g into, earlier it was z bar, z bar plus d y into, A plus, rho g, whatever change in area means, how much area it is being there, this is elevated by d y, isn’t it; and that increase in area is d; so, d A into d y by 2, that will be the force created there due to pressure in that location. So, I can compute at them. I can approximate now this quantity neglecting higher differential orders; this as rho g z bar plus d y, into A.

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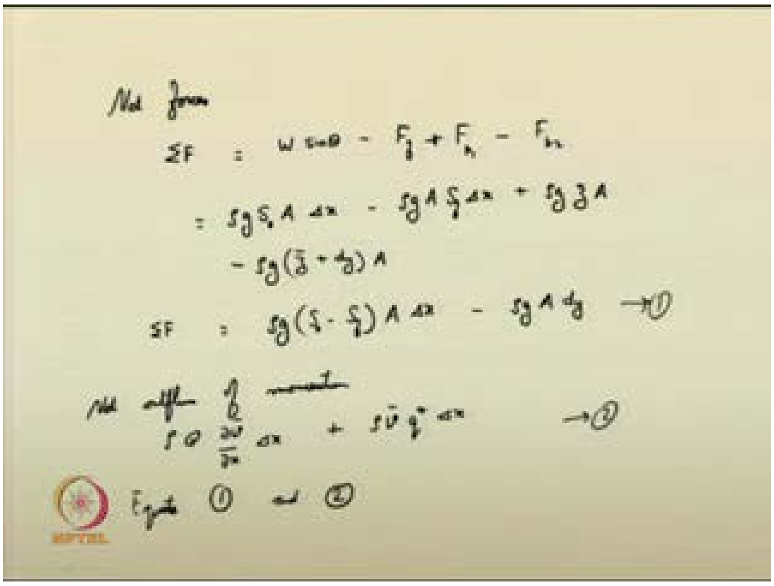

So, you have, now, net forces in the system; net forces in the system, you have sigma F is equal to w sin theta, minus F f plus, F p 1 minus, F p 2. You know the pressure force in the section 2 2, it will be acting oppose; in the opposing direction of the flow, that is the concepts we have already learned earlier also. So, these things can be rearranged, rho g S naught A into del x minus, rho g A S f into del x, plus rho g z bar; if you look into F p 1 minus F p 2, this will be given by the following 2 terms, z bar plus d y, A. So, the net force sigma F is equal to rho g, S naught minus S f into, A into, del x minus, rho g A into d y.

So, according to the Reynold’s transport theorem or the conservation of momentum equation, the net force should be equal to the net, should be equal to the change, rate of change of momentum, that is given as the net out flux of the momentum. So, you equate them both; we will be having net out flux of momentum. If you recall them, we had earlier given that as, rho Q dou v bar by dou x into del x plus, rho v bar q star into del x. So, these 2 quantities, 1 and 2, they have to be equated.

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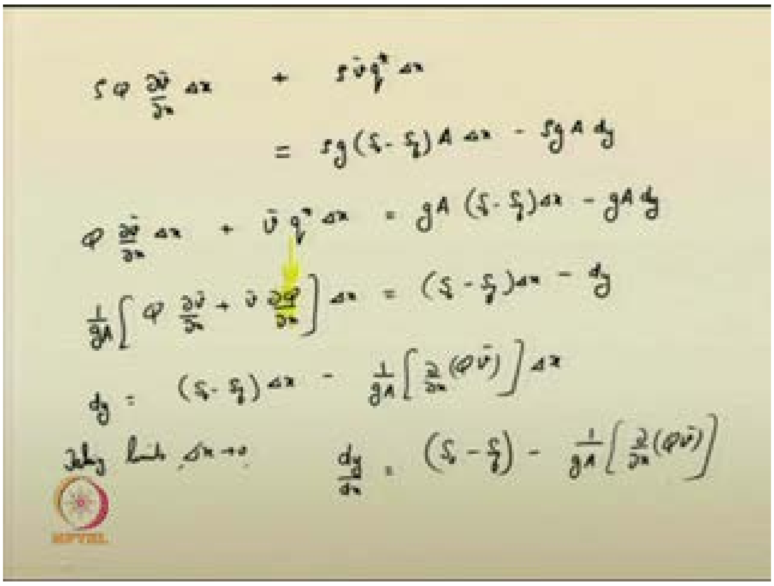

Equate 1 and 2, so we will get the following expression rho Q dou v bar by dou x into del x plus, rho v bar q star del x, this is nothing but, equal to rho g S naught minus S f, A into del x minus, rho g A into d y. So, you can easily take rho terms out, as this is an equation now. You can, and you are dealing with incompressible liquid, so you can take rho easily out of this equation; this equation gets simplified into Q dou v by dou x into del x plus, v q star into del x, equal to g A into, S naught minus S f del x, minus g A d y.

Rearrange the terms; you can, you may do them yourself also, but, however i am just writing for your benefit; dou Q by dou x into del x is equal to S 0 minus S f del x, minus d y. I hope, you understood why I wrote this term as dou Q by dou x; q star I have just converted to dou Q by dou x. So that, this expression we will get in a better way.

So, subsequently, I can now write the following terms; d y is equal to S 0 minus S f into del x minus, 1 by g A; what is this term, Q dou v by dou x plus, v dou q by dou x; this is nothing but, dou by dou x of Q into v, right; so that benefit I am getting here, when I write it in the following form. You see, as del x is the elemental length of the reach, if you are going to, means limit del x, then, and putting it into the denominator, so we will suggest that, taking limits, this can be suggested del x tends to 0 limit; this can be now represented as d y by d x is equal to S 0 minus S f minus, 1 by g A, dou by dou x of Q v bar; like this, we can easily write the expression now.

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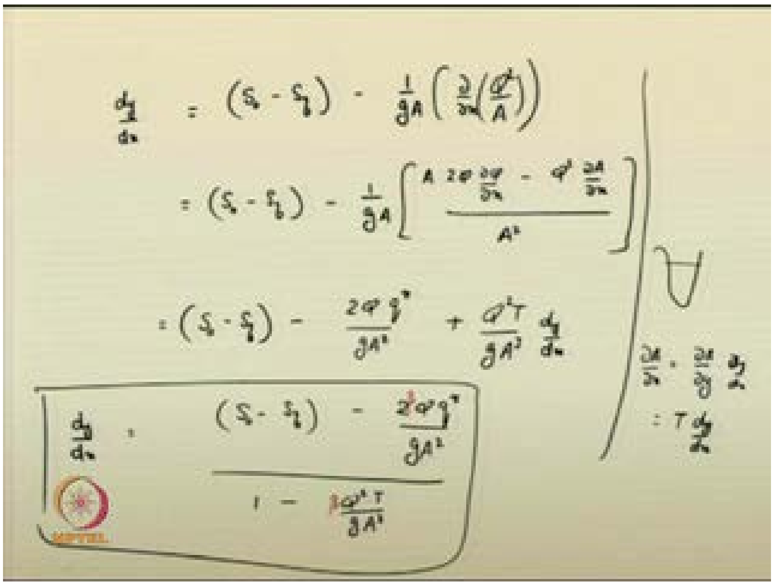

So, what will you get further? That is d y by d x is equal to S 0 minus S f minus, 1 by g A, into dou by dou x of, instead of q v, I can write the things as, Q square by A; so the bracket quantity is just modified into Q square by A. This is S 0 minus S f minus, 1 by g A; you just use the differential rules, A into 2 Q, dou Q by dou x minus, Q square dou A dou x by, A square. This becomes S 0 minus S f minus, 2 Q q star, dou q by dou x is q star, so 2 Q q star by A; so g A, and 1 A is multiplied, so, it becomes 2 Q q star by g A square plus, minus and minus plus, you are going to get; what is the quantity, you are going to get; see dou A by dou x.

Let me ask you, if there is a cross section like this, area is, if it is changing, dou A by dou x is nothing but, I can write it as dou A by dou y, into d y by d x; this is nothing but, T d y by d x. So, I will be substituting that term here; plus Q square T by g A, Q d y by d x; or you will see that d y by d x is nothing but, equal to S 0 minus S f minus, 2 Q small q star by, g A square; the whole quantity divided by 1 minus Q square T by, g A cube. So, this is the dynamic equation for spatially varied flow, for increasing discharge using the momentum equation, we have derived that. So, this is how we derive the dynamic equation using momentum principles.

So, if there are, the momentum correction factors are significant, then, please note that, you have to incorporate the corresponding correction factors, may be beta here; the momentum correction factors beta in the numerator and denominators appropriately. So, this is the way you get the dynamic equation; right now, I have assumed beta is equal to 1, and I am getting the following expression.

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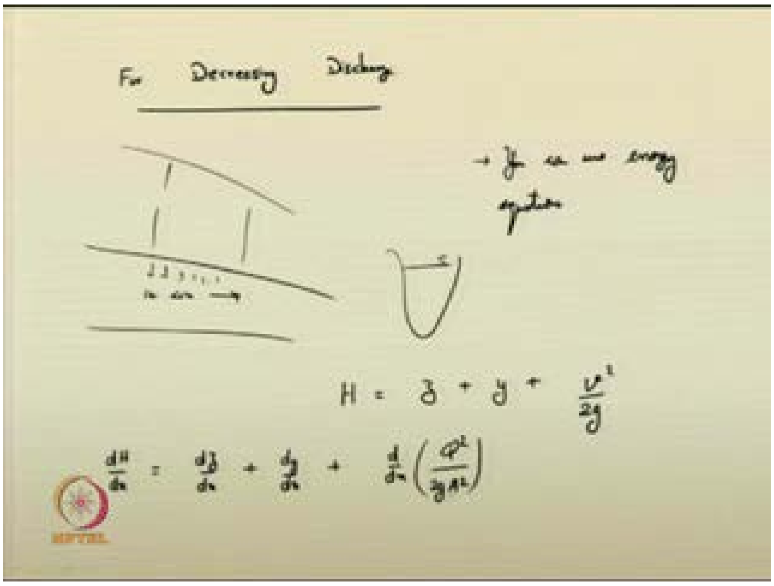

Now, for decreasing discharge; for decreasing discharge, say, along the section; by various process if the flow is decreasing, may be by infiltration, or by side outflow, lateral outflow, whatever be, if it is decreasing, then you can use energy equation. So, what is the total energy at any cross section? At any cross section, how will you compute the total energy? If you recall that, H, total energy head H is equal to, datum head plus, pressure head plus, velocity head, isn't it? So, if I differentiate this with respect to x, if I differentiate this equation with respect to x, I will get d H by d x is equal to d z by d x plus, d y by d x plus, d by d x of Q square by 2 g A square.

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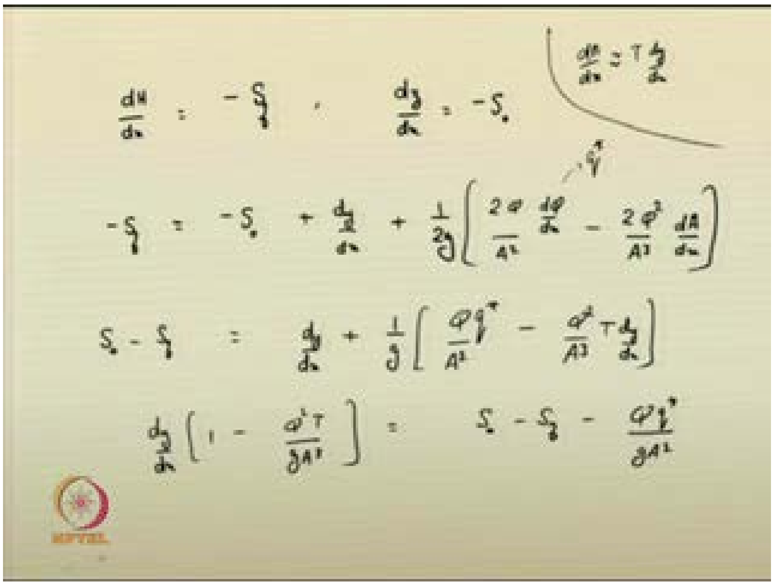

d H by d x is nothing but, the negative of the energy slope or the friction slope; d z by d x, if you recall, z is this datum height; so, d z by d x it is nothing but, the negative of the bed slope; then, you will get the following equation. minus S f is equal to, minus S 0 plus, d y by d x plus, 1 by 2 g; use the differentiation principles, I will be getting the following quantity, 2 Q, d Q by d x, by A square minus, 2 Q square by A cube, d A by d x.

So, I will just, I am just rearranging the terms, S 0 minus S f is equal to, d y by d x plus, 1 by g, Q, and this d Q by d x, it can be given as, for that small elemental area it can be substituted by small q star; so q star by, A square minus, Q square by A cube; d A by d x, if you recall, d A by d x, it can be given as T d y by d x, earlier we have seen that; so, the same quantity I am substituting, T d y by d x. So, rearrange the terms here, d y by d x into, 1 minus Q square T by g A cube, is equal to S 0 minus, S f minus, Q into small q star by, g A square.

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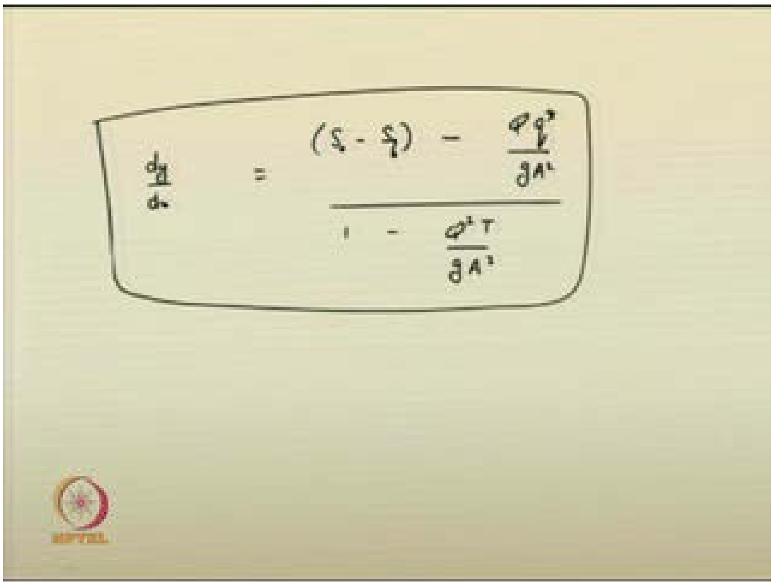

Or, the dynamic equation for spatially varied flow, d y by d x, for decreasing discharge, it can be given as S 0 minus S f minus, Q q star by, g A square by, 1 minus Q square T by g A cube. So, this is the dynamic equation for decreasing discharge spatially varied flow; this you have derived using energy equation. So, you can see the close proximity of these equations with gradually varied flow equations and all; some terms are getting added in this expression; that is all.

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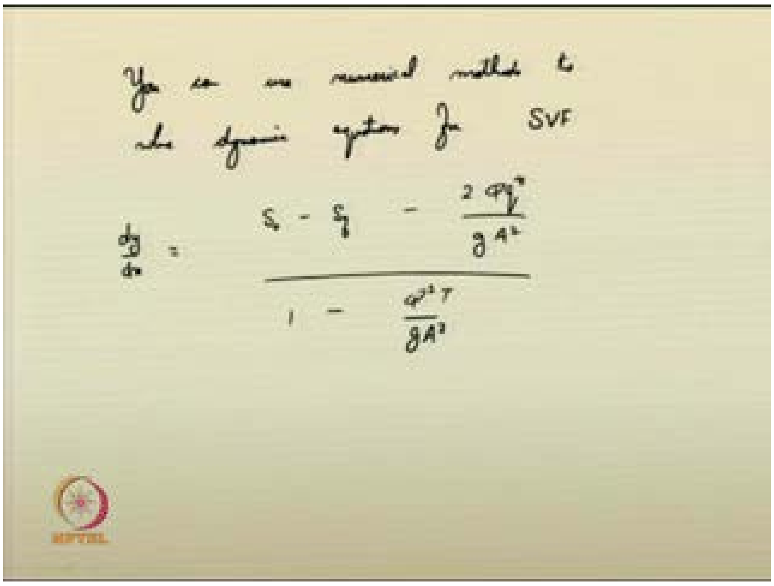

So, you can use numerical methods means; you have seen how to solve dynamic equation for gradually varied flow, in a similar way, we can use the similar techniques; that is, you can use numerical methods, may be first order approximation, second order approximations, fourth order approximations, numerical methods to solve dynamic equations for spatially varied flow. So, you can use these dynamic equations; that is, say, for example, increase in discharge, d y by d x; this is equal to S 0 minus S f minus, 2 Q small q star by, g A square by, 1 minus Q square T by g A cube. To solve these things, you can start from a control point, and you can proceed it in the following form; that is from the control point, you will, you can find the depth of the flow if the distance x is given, like that.

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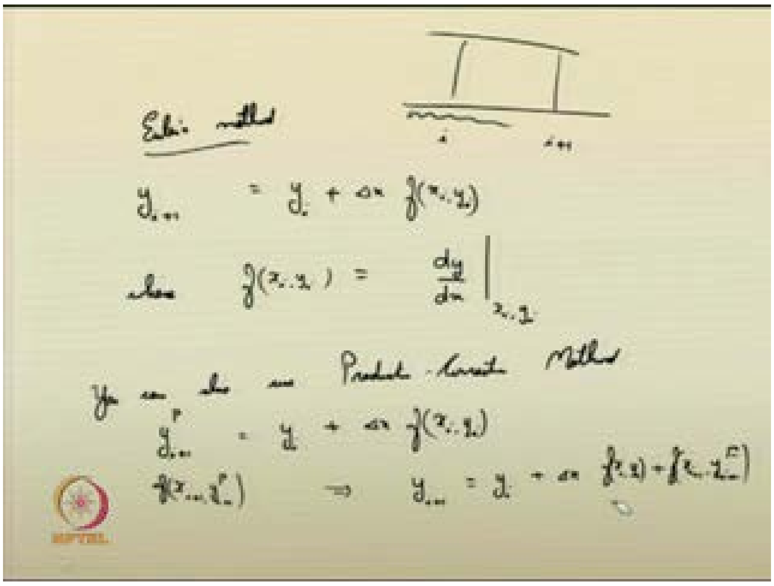

For example, Euler's method; first order Euler's method, numerical approximation, this can be given as y i plus 1, at any section, say, if you have the following reach, say, this is I, this is i plus 1; if you know the values at the section i, and this is at y plus 1; y i plus 1 can be given by y i plus del x into function f of (x i, y i), where f of (x i, y i) is nothing but the slope d y by d x, at x i and y i, ok. You can use such Euler's methods to solve.

Or, you can also use Predictor Corrector method, which is more efficient; in this thing, the Predictor Corrector method uses the same Euler's principle. So, in the first thing, they will predict; a prediction value is given for y i plus 1, using y i plus del x into f (x i, y i). After obtaining a predicted value at y i plus 1, what they are going to do is that, they are going to compute f the slope f x i plus 1, y i plus 1, using the predicted value. Once you compute this slope f (x i plus 1 y i plus 1), then what they are going to do is that, they are further going to improve the depth y; actual depth y i plus 1; this is equal to y i plus, del x f of (x i, y i) plus, f of i plus 1, y i plus 1 obtained using predicted form. Ok, I will just write it in the next page.

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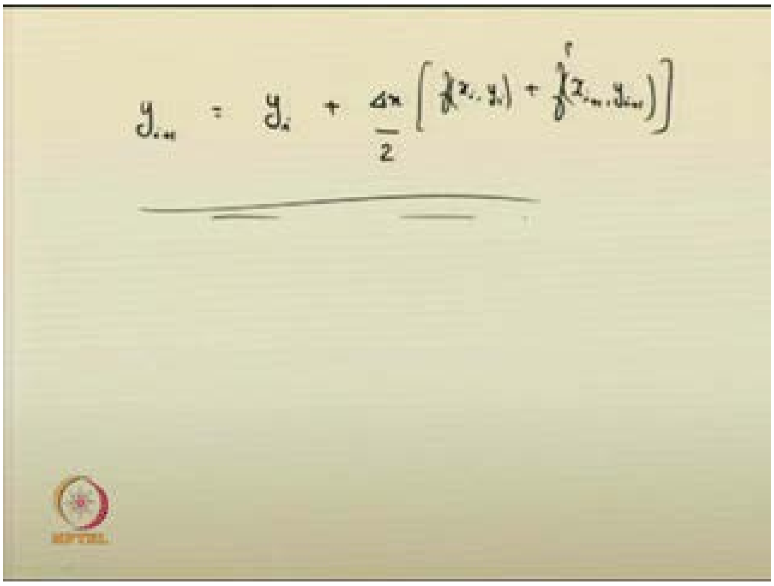

y i plus 1, what, y i plus del x into, f of x i, y i plus, f of x i plus 1, y i plus 1, that is obtained using prediction, this by 2. So, this will give you the Predictor Corrector method. You have seen the R K method, Runge-Kutta method can also be applied for the spatially varied flow, that was the fourth order one, this is a second order one and all; you will see such benefits in many methods, so you can use that.

So, this way we have, we have dealt with the spatially varied flow. If you are curious, you can solve any problem related to the spatially varied flow on your own, and just check that due to several time limitation and all, our portion, we have to finish many of the things; so, I may not demonstrate the problem; may be in the end, if time is permitted, I will just give a demonstrative problem.

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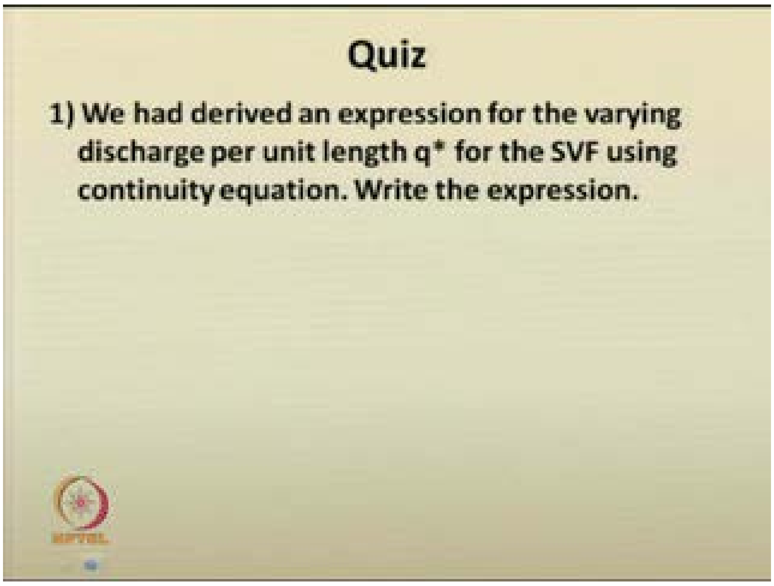

So, for today's quiz, following are the questions. The first question is: we had derived an expression for the varying discharge per unit length that is q star for the spatially varied flow using the continuity equation. Now, I am asking you to write that expression. What was the expression given for q star?

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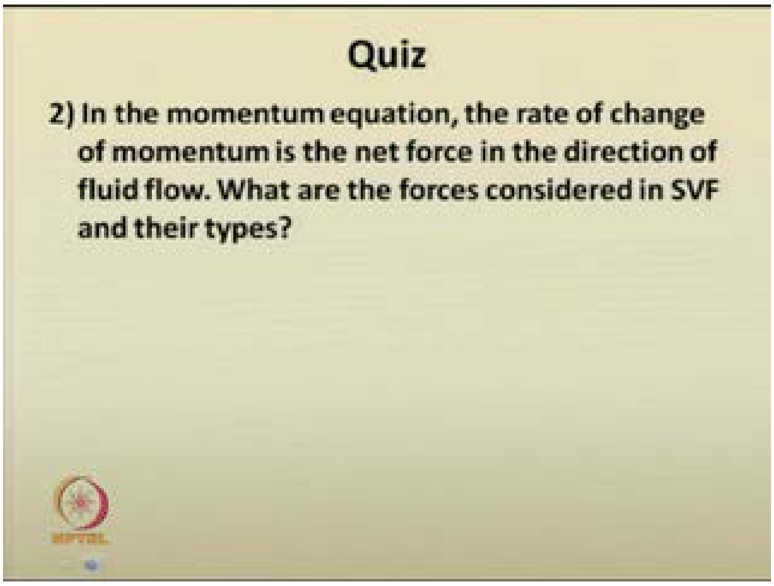

Second question: in the momentum equation, the rate of change of momentum is the net force in the direction of fluid flow. So, what are the forces considered in spatially varied flow control in the control volume, in that control volume, and what are the types of flow?

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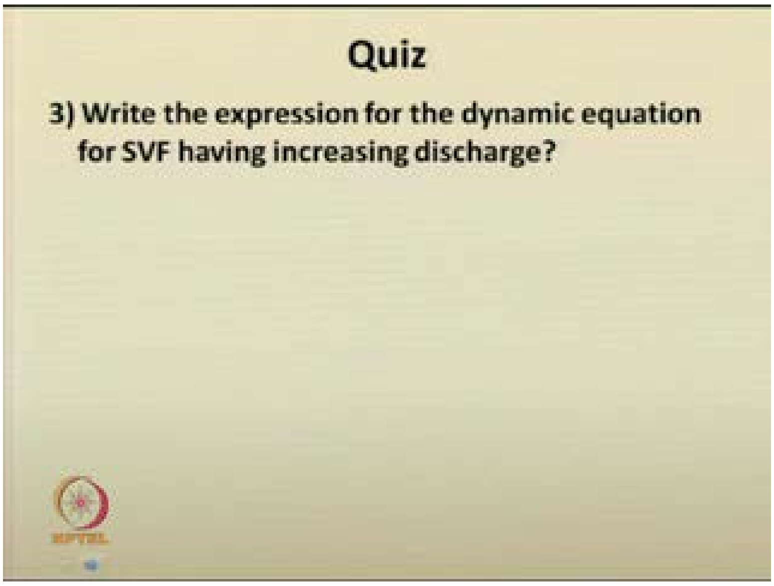

The third question: write the expression for the dynamic equation for spatially varied flow having increasing discharge?

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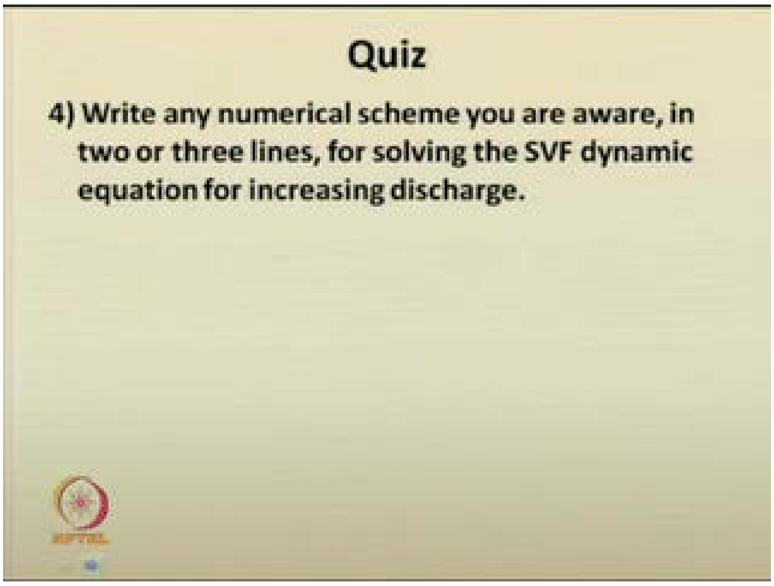

In the fourth: write any numerical scheme you are aware, in two or three lines, or may be three four five lines also, for solving the spatially varied flow dynamic equation for increasing discharge. You write any of the numerical scheme which you are aware, to solve that dynamic equation for spatially varied flow. So, I hope, you have answered the questions.

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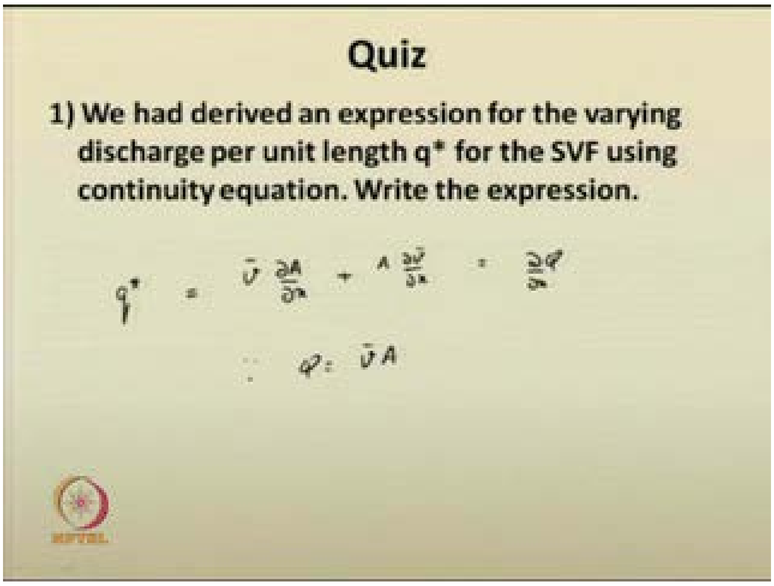

So for the first question, the solution, q star we have derived it using the dynamic equation; it was nothing but, v bar dou A by dou x plus A dou v bar by dou x; this is nothing but dou by dou x of Q, because Q is equal to v A, you know that.

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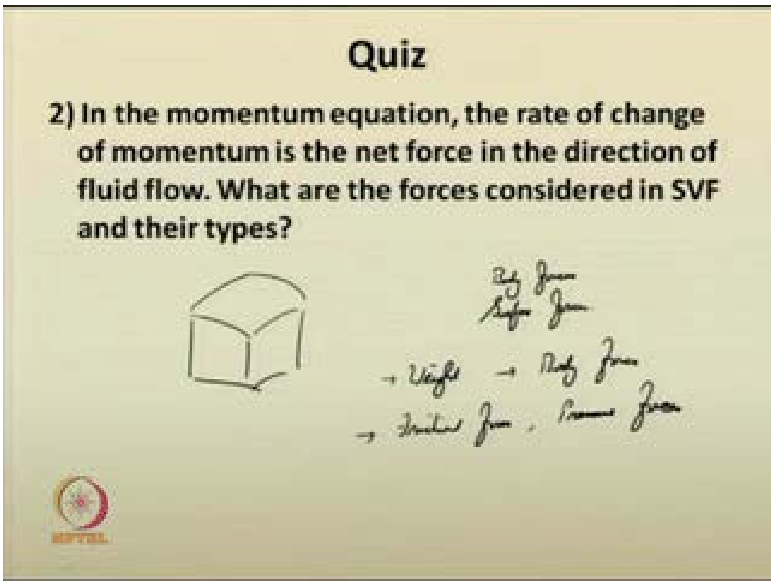

The second question: we have, what are the forces considered in the spatially varied flow and what are the types of forces? So, we have considered the, in any of the control, any type of control volume, there will be body forces and surface forces. So, the body force include the weight, component of the weight, this comes under body forces; this, in the surface forces you have, frictional forces and pressure forces. These are the forces you considered in deriving the dynamic equation.

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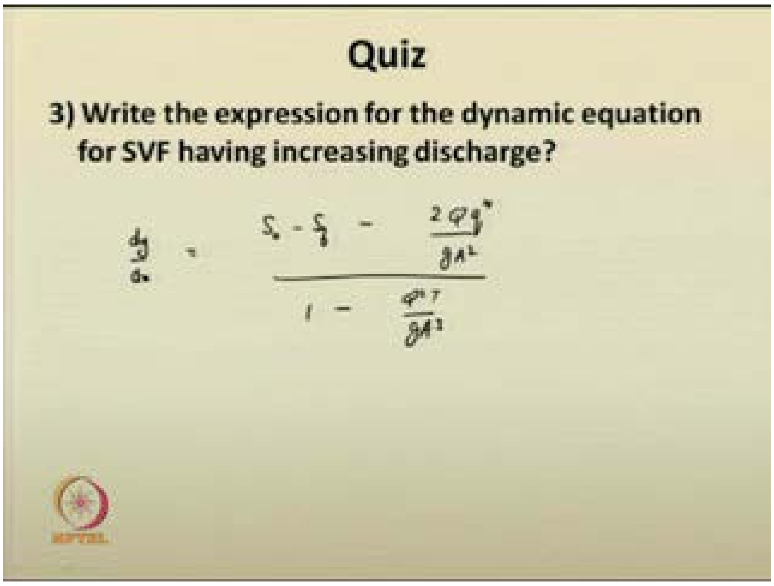

Write the equation for dynamic equation for spatially varied flow having increasing discharge? This we can easily write it; d y by d x is equal to S 0 minus S f 2 times, please note that, this is 2 times Q q star by g A square 1 minus Q square T by g A cube. You can see how it is different from, in the dynamic equation derived using energy equation. So, there you have only 1, means it is not 2 times, only 1 time.

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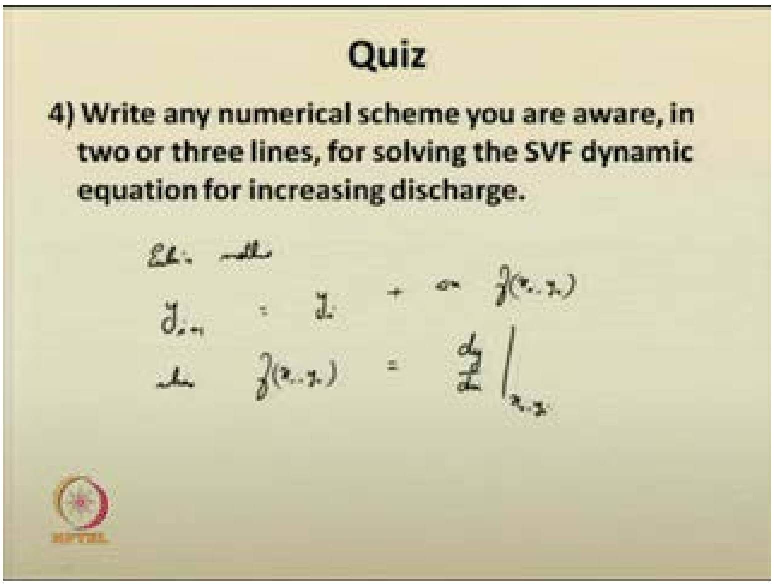

Write any numerical scheme you are aware, in two or three lines. So, I have, as I have told earlier, you can use Euler's method, y i plus 1 is equal to y i plus del x f of (x i, y i), where f of (x i, y i) is nothing but the slope d y by d x at x i y i; this is Euler's method. You can use Predictor Corrector method according to your wish, so that way we are concluding this portion.

Thank you.