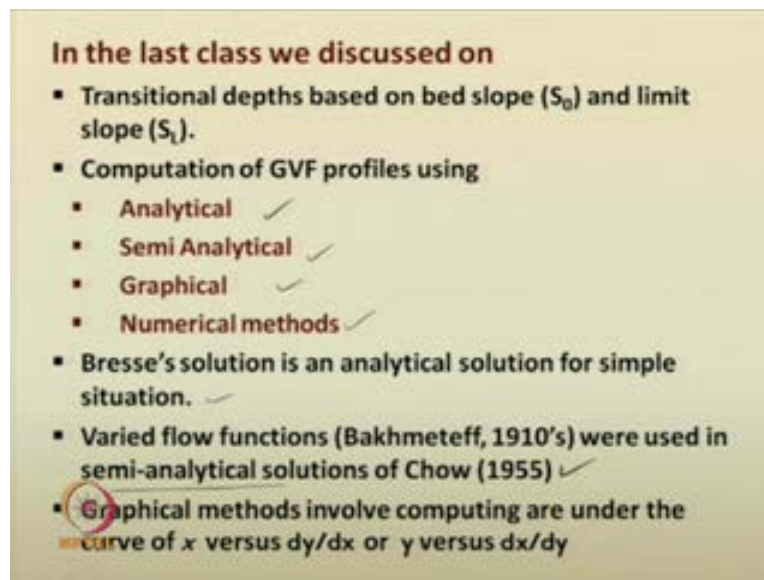


Advanced Hydraulics
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Module - 3
Varied Flows
Lecture - 8
Gradually Varied Flow Computations:
Part 2 R K Method

Welcome back to our lecture series on advanced hydraulics. This is a part of the NPTEL program for the post graduate courses in civil engineering. We are in the module three at present; that deals on varied flows.

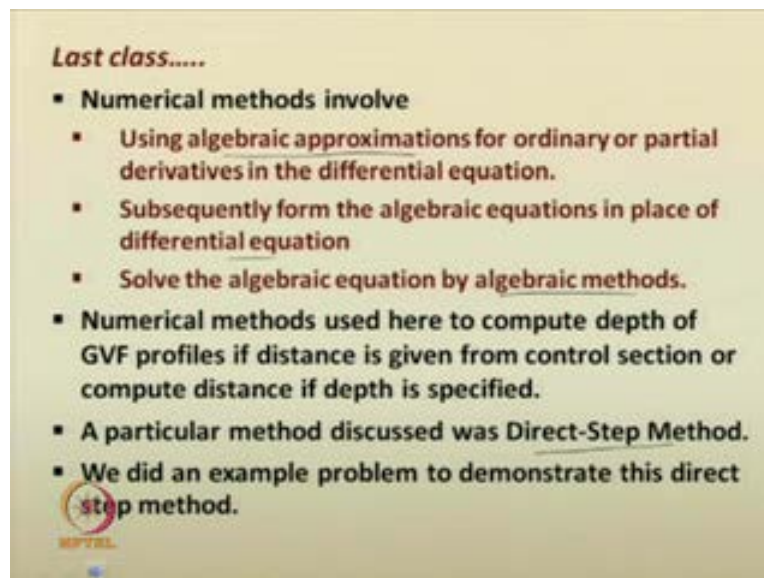
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Last class, if you recall them, we have discussed on the concepts of transitional depths based on the bed slope given to you, and the limit slope that was computed; from that one can easily avail the transitional depth for the gradually varied flow. Also, the computation of gradually varied flow profiles were discussed- using analytical methods, using semi analytical methods, graphical methods, and numerical methods. We have suggested that these are the methods that can be used for computing gradually varied flow.

If you recall, in the analytical methods, Bresse obtained a very simple form of solution for the gradually varied flow profiles; that was discussed in the class. In 1910's Bakhmeteff obtained a varied flow functions; that were subsequently used in the semi analytical solutions by Chow in 1955. You had also seen the graphical method that can be used for computing the profiles, gradually varied flow profiles, using the curves, that is, if it is the distance x versus dy by dx , or the depth of flow y versus dx by dy . If you have these graph, one can integrate; that is, you can compute the area under the curve of these graph, and that can be used for the gradually varied flow profile computations.

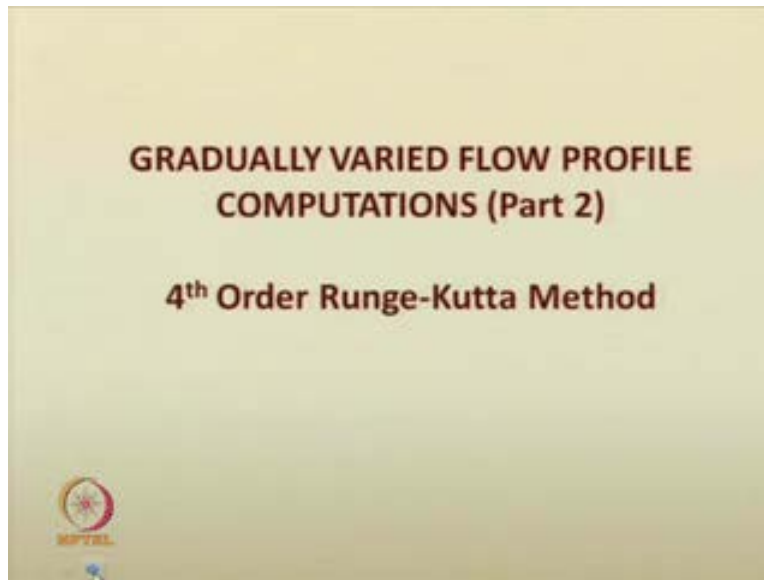
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Also, in the numerical method, we have suggested that the numerical methods use algebraic approximations; that is, they use algebraic approximations for ordinary or partial derivatives in the differential equation. So, once you use these algebraic approximations to the derivatives, you will subsequently obtain algebraic equation instead of the differential equation. So, this algebraic equation, they are very easy to solve using the various available algebraic methods. Now, that is the objective in numerical methods.

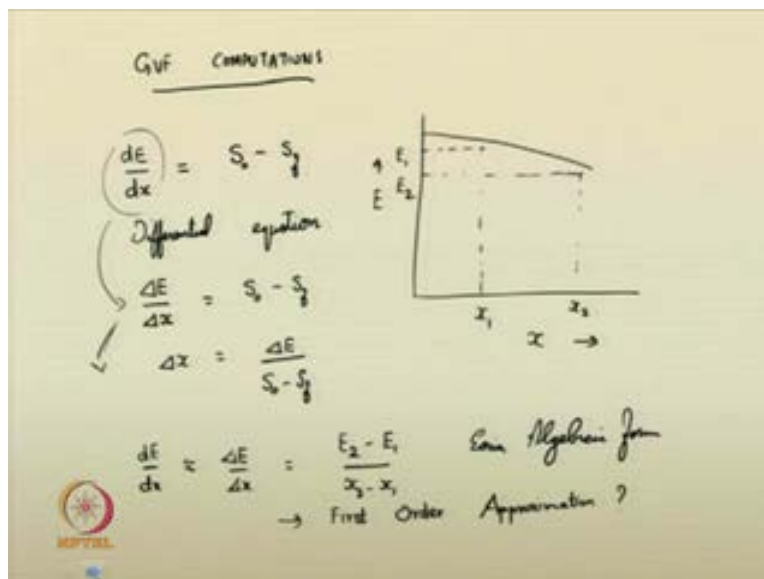
So, numerical methods, we have used here to compute the depth of gradually varied flow profiles, if the distance is given from the control section; or, you can compute the distance if the depth is specified. A one particular method which we discussed was the direct step method. We also did an example problem on this particular method.

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So, today, in our lecture, we will be again continuing with the gradually varied flow profile computations. Today, we will see on the fourth order Runge Kutta method used to compute gradually varied flow profile. We will also see a demonstrative example on that.

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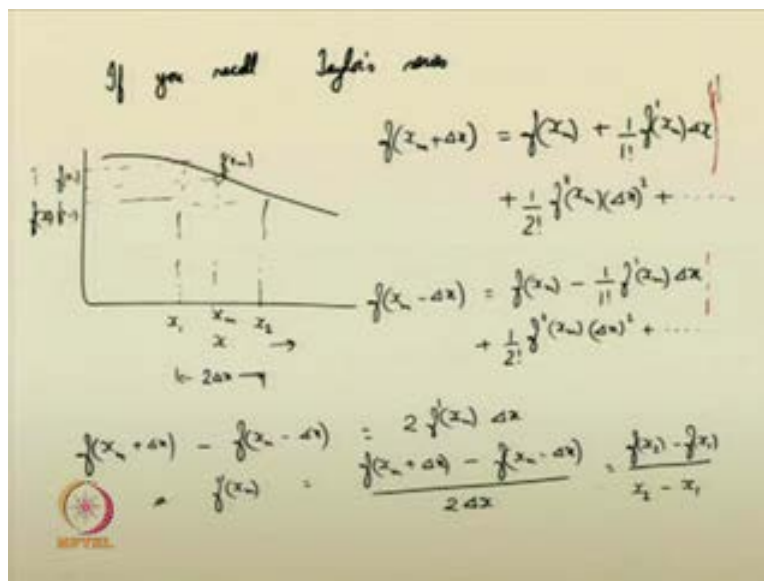
So, if you have closely observed the processes, the, in the last class also, direct step method, for the gradually varied flow computations, you can see that we had derived or we have suggested that the slope of the energy equation is S_0 minus S_f . Then, this is a differential equation. Now, this differential equation, the derivative is now substituted by an algebraic expression, $\frac{\Delta E}{\Delta x}$

Δx is equal to $S_0 - S_f$, like this we computed, or we suggested. So, here the partial derivative, this derivative is being approximated by the algebraic expression. From this, you had subsequently obtained the direct step method, for computing gradually varied flow profile, Δx is equal to ΔE by $S_0 - S_f$.

What is the principle behind this? See, if you have E versus distance curve; suppose, if the curve is of this particular form, any particular form of like this and all; if this is your expression for E . Then, at any location, x_1 , you have the corresponding energy E_1 ; and any position x_2 , we have the corresponding energy E_2 . So, what this particular approximation do, is that, the slope of the curve dE by dx is now approximated by ΔE by Δx , which can be subsequently given as $E_2 - E_1$ by $x_2 - x_1$. Now, this is an easy, easier algebraic form, right.

So, what type of approximation was incorporated for this particular derivative? We suggested that, the direct step method involves linear or the first order; it involve first order approximation for the partial derivative, sorry, for the derivative. So, what do you mean by first order approximation?

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Just going back into our basic mathematics, if you recall, if you recall Taylor series. Now, just recall your Taylor series. So, what can be done? See, again, for any function, if I have a curve, I have a function f of x with respect to x , if you have such a curve and all. So, we do not know the pattern of the curve. Then, between any two points say, say this is x_1 , this is x_2 , let the

midpoint of this distance; so, let this x_1 and x_2 be separated by a distance of $2\Delta x$, and this midpoint let it be x_m . Let the corresponding values be, this is $f(x_1)$, this is $f(x_2)$, and, let this be $f(x_n)$, let these be the situations.

According to your Taylor series, I can now find this particular function f of x_m , based on the location f of x_m plus Δx ; this is nothing but f of x_m , plus 1 by 1 factorial, the derivative of this function at x_m , into Δx , plus 1 by 2 factorial, the second derivative of this function at x_m , into Δx whole square, and the series goes on; you can go for a higher order derivatives also in this thing, the series goes on.

Similarly, according to the Taylor series, f of x_m minus Δx can be easily written in terms of the known functional value f of x_m as such; f of x_m , now minus 1 by 1 factorial, f' at x_m , into Δx , plus 1 by 2 factorial, f'' at x_m Δx whole square, like this, plus going on; the series goes on like that. So, at the location, at which you truncate your series, based on that if the approximations you obtained for the derivatives are termed as such; whether it is a first order approximation, whether it is second order approximation and all. In the direct step method, why did we call this as a first order approximation?

There, the both the series were truncated after the first derivative; so that is, if I truncate at this location; now, and if I again start using algebraic manipulation, f of x_m minus, f of x_m minus Δx ; after truncating the series, soon after the first derivative, and then if you compute this algebraic expression, what will you get from this thing? You will get; these term gets canceled of; and you will get twice f' at x_m , into Δx , right. Or, f' at x_m , is equal to x_m plus Δx , minus f of x_m minus Δx , by twice Δx ; like this you got the expression for the first derivative, f' at x_m .

What is f of x_m plus Δx , Δx according to this graph? It is, $f(x_1)$, $f(x_m - \Delta x)$; this is, sorry, $f(x_2)$, this is $f(x_1)$. Like that, I can now write it this in the following form, $f(x_2)$, minus $f(x_1)$, and $2\Delta x$ is nothing but $x_2 - x_1$. So, this was the same expression you had obtained for the direct step method.

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$$\frac{\Delta E}{\Delta x} = \frac{E_2 - E_1}{x_2 - x_1}$$

First Order Approximation
Similarly you can go for Higher Order Approximations
4th Order Runge-Kutta Method
(R-K)

KPTCL

The $\frac{\Delta E}{\Delta x}$ is equal to $E_2 - E_1$, by $x_2 - x_1$, the similar expression; this was how it was derived, and it was given as first order approximation. Similarly, you can go for higher order approximations, in the Taylor series, you can truncate the series at any location; say here, if I truncate it at the second, after the second derivative, then the derivative expression for the first derivative as well as the second derivative, it can be a different one. Say, if you truncate it after the third, third degree, third order derivative, then the approximation will be corresponding thing. Like that you can do the thing.

So, you can, many places or for the many natural phenomenon, in most of the situation based on the Δx distance, if the Δx value, if it is approximately, if it is small, then you can go for the first order approximation or the derivative. You can go for higher order approximations. And, one such method is, the fourth order Runge Kutta method. So, I am not going to derive this method. This you can refer any book on numerical methods or higher engineering mathematics and all. You can, or it is at, if you are curious you can try to obtain the expressions for the Runge Kutta method and all. I will just show the formulas for the Runge Kutta method here. And then, I will subsequently apply it in our flow computations. It is also called R K method.

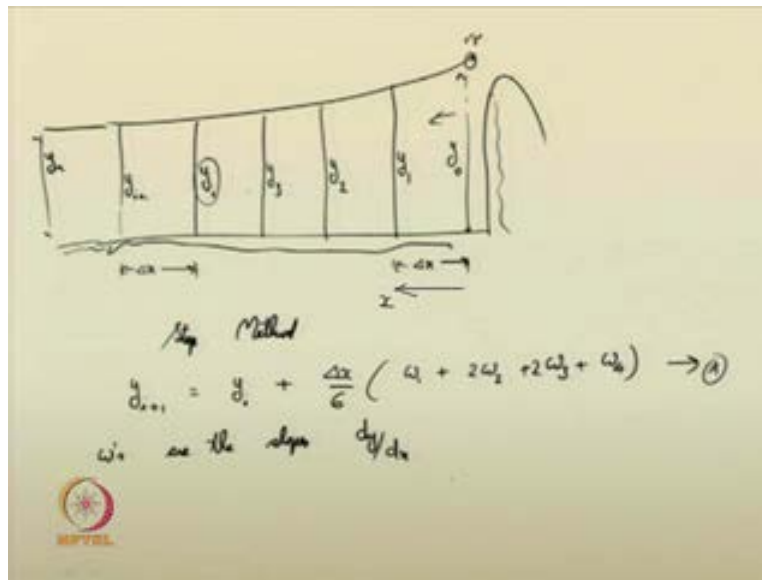
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The image shows a handwritten derivation on a slide. At the top, the equation is $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}$. Below this, the friction slope S_f is defined as $S_f = \frac{n^2 Q^2}{A^3 R^{4/3}}$. This is substituted into the first equation to get $\frac{dy}{dx} = \frac{S_0 - \frac{n^2 Q^2}{A^3 R^{4/3}}}{1 - \frac{Q^2 T}{g A^3}} = f(x, y)$. At the bottom, the final result is boxed: $\frac{dy}{dx} = f(x, y)$. A small logo with the text 'MPTEL' is visible in the bottom left corner of the slide.

So, what is the thing? Just recall your dynamic equation S_0 minus S_f by 1 minus $\frac{Q^2 T}{g A^3}$ by $g A^3$. So, you have seen that the energy slope or the friction slope S_f , it is given as based on the uniform flow $n^2 Q^2$ by $A^3 R^{4/3}$. So, therefore your $\frac{dy}{dx}$ is S_0 minus $\frac{n^2 Q^2}{A^3 R^{4/3}}$ by 1 minus $\frac{Q^2 T}{g A^3}$ by $g A^3$. You know that A , R , T , all these are functions of x and y for a non prismatic channel; off course, for a non prismatic channel.

For prismatic channel, they are functions on, of only y . So, if they are functions of x and y , this $\frac{dy}{dx}$ is, this slope $\frac{dy}{dx}$ is surely a function of x and y . So, if you can relate it in this form, now what do you mean from this thing? This solving, $\frac{dy}{dx}$ is equal to, which is a function of x and y , is the objective involved.

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So, how do you solve them? In the R K method, what you are doing is that, say, the entire reach; if this is the reach of the channel, say, some dam is constructed here, some gradually varied flow profile is occurring. And, if this is your control section; then keeping this as the known value, we are now discretizing the entire stretch or entire reach into small small reaches, say, may be of Δx length; Δx may be uniform or it may be changing.

So, if this is your first valley control section; beginning with the control section, if this is your first value y_0 . Based on that, using the values of y_0 , you now can compute at this; this is the section 1 1, and what is the depth y_1 , that can be computed. After computing y_1 , then you can go to the second section; this is section 2 2 and y_2 ; this is section 3 3, and corresponding depth y_3 . Similarly, go on, y_i y_i plus 1, and this goes on, till you reach your normal depth in the upstream, like this; this is the objective in computing the gradually varied flow profile.

So, if you discretize into such small small reaches, then based on the known value if you are proceeding into the unknown direction; this is called the step method. So, your direct step method was also following the same procedure. Here also, we will be first starting from a known value, and then trying to compute the unknown values, sequentially. So, that way one can proceed; the, in the step, this is called the step method. So, the entire reach as mentioned, it is being discretizing. Then, from the beginning of the control section, which from the known parameters you are computing; means, this depth is known, from that you are computing the

other quantities of the thing, that will be kept in the same form; and, then, proceeding the computation in the left side; for this particular case, it is the left side we are proceeding the computation.

So, if the depth at i , section, section i , if it is known to you, then the depth at section y i plus 1 can be computed with the following algebraic expression, y , the depth at the known section, that is y i plus Δx that is the distance between the 2 section, Δx by 6, into ω_1 plus, 2 ω_2 plus, ω_3 plus, ω_4 . So, this is the fundamental in the Runge Kutta method; fundamental equation to compute the unknown value. So, this is the main equation; I can write it as, or whatever notation you can give. Now, what is ω ? ω s are the slopes $\frac{dy}{dx}$, so you know $\frac{dy}{dx}$, this is function of x and y . So, ω s are nothing but the slopes, $\frac{dy}{dx}$.

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$$\omega_1 = f(x_i, y_i)$$

$$\omega_2 = f\left(x_i + \frac{\Delta x}{2}, y_i + \omega_1 \frac{\Delta x}{2}\right)$$

$$\omega_3 = f\left(x_i + \frac{\Delta x}{2}, y_i + \omega_2 \frac{\Delta x}{2}\right)$$

$$\omega_4 = f(x_i + \Delta x, y_i + \omega_3 \Delta x)$$

$$y_{i+1} = y_i + \frac{\Delta x}{6} (\omega_1 + 2\omega_2 + 2\omega_3 + \omega_4)$$

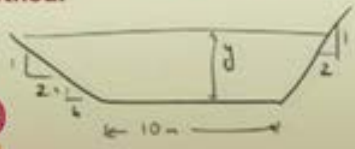
How they are computed? ω_1 , this is nothing but, it is computed as the slope, using the known values x_i and y_i . So, you see the section here; this portion is called x_0 , then x_1 , x_2 , x_3 ; similarly, this is x_i , x_{i+1} , like that you can designate them. So, ω_1 is computed; it is function of x_i and y_i . ω_2 is the slope of the water surface, at the following location, y_i plus Δx by, pardon, y_i plus, whatever slope has been computed at x_i , y_i , ω_1 , that is taken into account here, and Δx by 2.

Similarly, ω_3 is nothing but function of, at the location $x_i + \Delta x$; that is functioned slope, at the location $x_i + \Delta x$, and $y_i + \omega_2 \Delta x$. And, ω_4 is nothing but function at $x_i + \Delta x$; please note, it is $x_i + \Delta x$; that is, $x_i + 1$, $y_i + \omega_3 \Delta x$. So, from the new slope obtained, ω_3 ; that is now applied to the entire small reach. So, this is y_i , $y_i + 1$; this is your Δx length; then, this slope is applied to the entire reach. And, like that, ω_4 is computed. Subsequently, you got the expression, y_{i+1} is equal to $y_i + \Delta x$ into, $\omega_1 + 2\omega_2 + 2\omega_3 + \omega_4$; like that you got the expressions.

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Problem

Q:) A trapezoidal channel of bed width $B=10.0$ m, side slope $2H:1V$, bed slope $= 0.0005$, and Manning's $n = 0.015$ carries water in a region. For the given flow, the uniform flow depth in the channel was observed as 2.0 m. A small dam is constructed across the channel that raises the water height at dam portion to 3.50 m. Whether the GVF will be M-curve or S-curve. For how long does the GVF curve exist. Use 4th order R-K method.

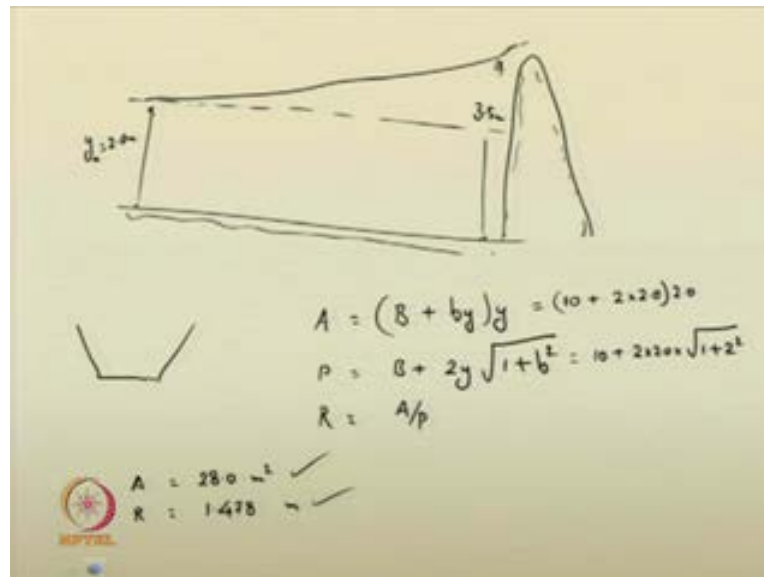


$n =$	0.015
$S_b =$	0.0005
$B =$	10.0
$h =$	2

So, we will see a demonstrative example for, how to solve this method, how to use this method and all. So, the problem, I am just going to dictate it here. It is given in the screen. A trapezoidal channel of bed width B is equal to 10 meter, slide slope 2 horizontal to 1 vertical, bed slope- 0.0005, and Manning's roughness coefficient- 0.015 carries water in a region. For the given flow, the uniform flow depth in the channel was observed as 2 meters. A small dam is constructed across the channel that raises the water height at dam portion to 3.5 meter. Whether the gradually varied flow profile will be a M curve or S curve. By this time, you know what is meant by M curve or S curve in the gradually varied flow curves. So, for how long does this gradually varied flow curve exist. Use the 4th order Runge Kutte method.

So, you have just described the problem. It is a trapezoidal channel section; say, 1 isto 2 is the slide slope; it has a bed width 10 meter; this is the depth of flow y, ok, it's Manning's coefficient, everything is given to you; n is equal to 0.015; S 0 is equal to 0.0005; B is equal to 10 meter, we used to give it this as 1 isto b, if you recall our earlier lectures; so that small b is equal to 2 here; so, a small dam is constructed across the channel.

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Say, if this is the channel, it was supposedly carrying a uniform discharge, as mentioned in the thing. For the given flow, the uniform flow depth in the channel was observed to be 2 meter. It was carrying a, at 2 meters depth uniform discharge. Then, what happen is that, a dam was constructed, and the water level rose like this. So, we do not know whether, what type of profile is this one? So, it has been rose upto 3.5 meters.

What is this length of this gradually varied flow profile? That is the question asked to you now, so, you can compute that. For the trapezoidal channel section, if you recall our earlier lectures, area A is given as B plus b y into y; wetted perimeter is given as B plus twice y into root of 1 plus small b square; R is equal to A by P. So, we do not know whether the flow is critical, subcritical, or supercritical. So, you have to compute the discharge first.

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In uniform flow is given for $y_n = 2\text{m}$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$
$$= \frac{1}{0.015} \times 28 \times (1.478)^{2/3} \times (0.0005)^{1/2}$$
$$= 54.16 \text{ m}^3/\text{s}$$

In critical flow $\frac{Q^2}{g} = \frac{A_c^3}{T_c}$

The uniform flow is given. So, let us go back into the previous screen, A is equal to B plus b y, it is given to you. So, you know the quantities 10 plus 2 into 2 into 2; this is 10 plus 2 into 2 into 1 plus 2 square. So, you have got A is equal to 28 meter square; R is equal to, just substitute the corresponding values, R is equal to A by P, I am getting it as 1.478 meter, you can verify them. So, therefore, the uniform flow is given for y_n is equal to 2 meters; Q is equal to therefore, 1 by n, A R to the power of 2 by 3, S naught to the power of half; at the normal depth the area A is 28 meter square, hydraulic radius R is equal to 1.478 meter. Substitute them here, 1 by 0.015 into 28, 1.478 to the power of 2 by 3 into 0.0005 to the power of half; this is coming out to be 54.16 meter cube per second. So, this is the normal discharge.

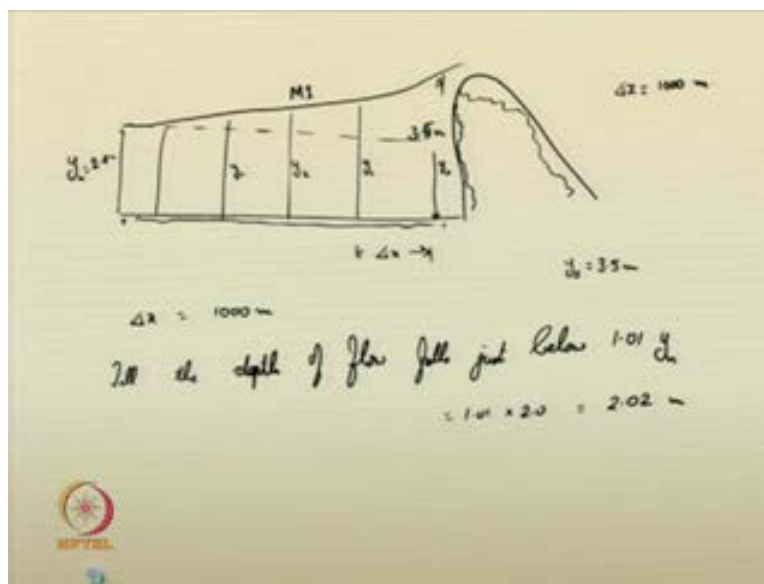
So, this discharge, as we have suggested, gradually varied flow is a study state condition. So, this discharge will be there, even at the upstream as well as in the downstream of the channel, so that same discharge will be following. So, you can keep this discharge now as a constant. You have to identify, whether the flow is critical, subcritical, or supercritical. What is the procedure? For critical flow, you recall from the module one; for critical flow, you had the relationship Q^2/g is equal to the area of cross section, where you have taken the critical depth by T_c , that is the top width at the section where the depth is critical. This relationship is uniform for all type of channels, whether it is rectangular, whether it is trapezoidal triangular, you had seen them. So, you have to use this relationship again here.

I can use the iterative scheme to compute y_c now. Iterative scheme for y_c ; I have just rearranged the equation; this equation can be now rearranged, I can get y_c in this form; an implicit expression for y_c in the following form; 6.687 into, 10 plus $4 y_c$ to the power of 1 by 3 , divided by 10 plus twice y_c . So, this is the LHS of your equation; in this expression this is the RHS of the expression.

You can begin iteration that is computation with a known value. Take, the normal depth y is equal to 2 in the computation start with 2 . And, you can just tablet them, LHS and RHS now. If I tablet it, I am beginning it with 2 meters, and the RHS of this equation; the RHS of this equation is obtaining as 1.252 meter. Again, I am taking this as the LHS, and the corresponding RHS we are getting here; that is, 1.252 is now substituted here, and in this equation I am getting this as 1.319 meter. Again, 1.319 meter, it becomes 1.313 . I substituted in the LHS 1.313 , and in the RHS also 1.313 . Subsequently, I am getting this RHS as 1.313 itself.

Therefore, your critical depth is identified to be this particular quantity. So, I can write this as y_c is equal to 1.313 meter. So, what do you understand from this thing, this particular case? Your y_c is less than your normal depth, ok; your y_c is less than your normal depth. Therefore, flow is subcritical. Therefore, the gradually varied flow profile will be M_1 profile. It will be a back water curve. So, how will you compute the gradually varied flow profile?

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So, this is, as we suggested, it will be an M 1 profile. So, it is, the normal depth was there like this; y_n is equal to 2 meters; due to the dam, the flow profile became M 1 curve; here the depth is 3.5 meters, ok. The depth given is, depth given at the dam site, it is given as 3.5 meters. So, how will you compute the quantities now?

So, I am discretizing the stretch now into small small reaches of Δx length. I am taking Δx is equal to 1000 meter. And, I am just trying to identify, I will be computing, say, the y_0 is given. Now, subsequently, y_1, y_2, y_3 like that, till the depth of flow falls just below 1.01 times of y_n . So, that is; so, at whatever stage, in the flow computation, from 3.5 meters, upto whenever it reaches just below 2.02 meter, you can stop the computation, like that. Then, we are suggesting that the flow, from there onwards it is normal. So, if, gradually varied flow length can be computed upto that location; so, like that, you can do the thing. So, upto this depth, we are now going to compute the flow profile.

So, how will you compute the flow profile? So, start the computation; I will just show from the figure, start from y_0 is equal to 3.5 meter; from there, you start the computation, y_0 is equal to 3.5 meter. You have Δx is equal to 1000 meter. So, in the first iteration, so, on the first computation, what we have to do?

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$$y_1 = y_0 + \frac{\Delta x}{6} (\omega_1 + 2\omega_2 + 2\omega_3 + \omega_4)$$

$$\Delta x = R \cdot x \quad \text{m.s.d}$$

$$\omega_1 = -\frac{d}{dx}(y_0) = \frac{d}{dx}(0.0003500)$$

$$\omega_1 = \text{slope of water surface}$$

$$\frac{d}{dx} \left(\frac{Q}{A} \right) = S_0 - \frac{n^2 Q^2}{A^2 R^{4/3}}$$

$$\omega_1 = -0.000454775$$

$$\begin{matrix} A \\ R \\ T \end{matrix}$$

Y 1 we have to compute; and, it can be obtained as y 0 plus, del x by 6 into, omega 1 plus, twice omega 2 plus, twice omega 3 plus, omega 4; this is the our 4th order R K method. And, you have to compute omegas. So, how will you compute omegas? Omega 1 is nothing but; as this is a back water curve, this is a back water curve, right; you have the depth here, and the depth of the flow is reducing to the normal depth. So, this one, omega 1 will be actually, now, minus f, f of (x 0, y 0), ok; omega 1 will be, in the first case, it will be minus f, f of (x 0, y 0); that is, you are taking this as x 0 is equal to 0 meters itself; x 1, this is equal to 1000 meters; this will be y 1. So, f of 0.00, 0.00, this you have to compute. That is, this is slope of water surface; so, you know water surface curve, that is, this is nothing but d y by d x. So, d y by d x at 0, sorry, y 0 is not 0; let me pardon, this is y 0 is equal to 3.5 meters, 3.500 meters, so, 0 3.50.

At this location you have to compute the quantities, S 0 minus, n square Q square by, A square R to the power of 4 by 3, 1 minus, Q square, T by g A cube. So, we will show it in a demonstrative form, in one of the tables. So, what is this value? You know the bed slope; ok, I will show it here.

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$S_0 = 0.005$, $n = 0.015$
 $A = (b + by)y$, $P = b + 2y\sqrt{1+b^2}$
 $R = A/P$

Sect	x	y	A	R	ω
0	0.000	3.5000	59.500	2.31946	0.000454775 ω_1
	500	3.27261267	54.146	2.19788	0.000441167 ω_2
	500	3.05202941	49.150	2.07831	0.000422528 ω_3
	1000	2.62950153	40.124	1.84396	0.000358528 ω_4

$\omega_1 = \frac{f(x_1, y_1) - f(x_0, y_0)}{1 - (x_1 - x_0)^2} = \frac{0.0005 - \frac{(0.015)^2 (54.146)^2}{(59.5)^2 (2.31946)^3}}{1 - \frac{(500-0)^2}{1000^2}}$

So, you know the bed slope, S 0 is equal to 0.005, that is already given to you; n is equal to 0.015. So, you compute now. Based on the given depth, y is equal to 3.5, you compute what is the corresponding area A? So, if you have that corresponding area A, substitute it; corresponding, based on the depth of flow for y is 0 equal to 3.5, the corresponding area, the

corresponding hydraulic radius, this has to be computed; corresponding top width T, that also need to be computed. If you compute them, then you will get the value of omega 1.

So, this I can substitute as in the following form. Just go through this table. Here, I have done that. So, in the first section, where x is equal to 0.0, if it is, it is taken, the depth is 3.500; so, depth is taken as 3.500. The corresponding area, trapezoidal area is 59.500; use that relationship B plus, small b y into, y, that is the relationship of A; similarly, P is equal to B plus, 2 y root of 1 plus b square; R is equal to A by P. All these relationships are substituted. So, in this relationship, b is, small b is equal to 2; capital B is equal to 10; those are all fixed values. So, we get the corresponding value as, corresponding value of R, this as 2.31946.

Then, that is substituted in the relationship; omega 1 is equal to f of (x i, y i). So, in the first section, this is nothing but, S 0 is equal to 0.005 minus, 0.015 whole square into, 54.16 whole square by, 59.5 whole square into, 2.319 divided by, 1 minus 54.16 whole square into, 24 by, 9.181 into, 59.5 to the power of 3; this is the equation, if you recall that. I will just again show it to you, d y by d x is equal to, this equation is substituted at the location 0 and 3.5, so, that will give you omega 1. So, on substituting those values, omega 1 was found to be 0.000454775. So, I am just going back into that table; so, I am getting this. So, this will be your omega 1. Now, using omega 1, you are again going to compute omega 2.

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Handwritten mathematical calculations on a slide:

$$\omega_2 = \int \left(x + \frac{dx}{2}, y + \omega_1 \frac{dx}{2} \right)$$

$$= \int (500, 3.2726)$$

$$\omega_2 = -0.00044167$$

$$y_{min} = y_c - \frac{4x}{6} \left(\omega_1 + 2(\omega_1 + \omega_2) + \omega_2 \right)$$

• **Going to the excel file to show the evaluation.**

Omega 2 is nothing but f of x_i plus, $\frac{\Delta x}{2}$, that is at this location x_i plus $\frac{\Delta x}{2}$; and y_i plus, ω_1 into $\frac{\Delta x}{2}$ at this location. So, that is nothing but, you just find what is y_i plus, ω_1 into, $\frac{\Delta x}{2}$. This comes out to be, at the location, f of (500, 3.2726), ok. So, these things we are getting it. And, ω_2 , on computing in the same equation, I am getting it as 0.000441167. So, just see the table here; so, using the ω_1 value, now at the x location, $\frac{\Delta x}{2}$ location is 500; the corresponding depth is so and so value; so, we are getting the things, 54.146; the corresponding R value is 2.19788; and, I am getting the value of ω_2 .

So, using the ω_2 value, compute ω_3 , at these corresponding locations. So, this is at x_i plus $\frac{\Delta x}{2}$; the corresponding y value, it is observed to be 3.05; that, that has been obtained using, this 3.05, corresponding area is 49.15; 2.07831, corresponding ω_3 is obtained. Similarly, using ω_3 value, you find the depth, or you find this y_i ; y_i plus ω_4 into $\frac{\Delta x}{2}$ that is the quantity, this one, corresponding area, hydraulic radius, then you are getting ω_4 .

So, once you get these values, ω_4 , you can now easily find y_{i+1} is nothing but y_i by, sorry, y_i plus $\frac{\Delta x}{6}$, ω_1 plus, twice ω_2 plus, ω_3 , plus ω_4 . So, please note that this is a back water curve, so this quantity will be minus, you have to reduce it; this is the back water curve in this part. So, we will see the same computations in the following excel file, which I will demonstrate to you, and that shows the computation of the flow profile.

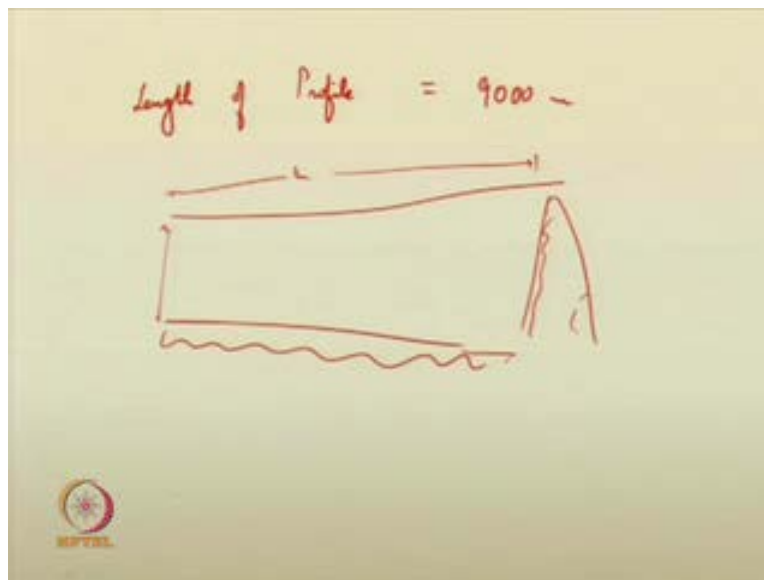
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Depth (m)	x_i	y_i	A	R	ω_1	ω_2	ω_3	ω_4
6000m	500	2.026613	28.480	1.49399	0.000094221			
	1000	1.992391	27.863	1.47344	-0.000010520			
	0	2.078717	29.429	1.52513	0.000091749			
	500	2.032842	28.595	1.49773	0.000041720			
7000m	500	2.011982	28.216	1.48522	0.000015853			
	1000	1.996129	27.930	1.47569	-0.000005323			
	0	2.045121	28.816	1.50507	0.000055960			
	500	2.017131	28.309	1.48831	0.000022441			
8000m	500	2.005911	28.106	1.48157	0.000007906			
	1000	1.998004	27.964	1.47682	-0.000002245			
	0	2.026133	28.472	1.49371	0.000033635			
	500	2.009316	28.168	1.48362	0.000012387			
9000m	500	2.003122	28.056	1.47990	0.000005679			
	1000	1.998933	27.981	1.47738	-0.000003898			
	0	2.015248	28.275	1.48718	0.000020047			
	500	2.005911	28.106	1.48157	0.000007906			

So, here, this excel file, so, I am just starting from this 0th section; so, the first portion, yellowed portion, it has already been shown in the previous slide; so, you are getting at 1000 meters, the corresponding value you are getting this as 3.076551; at 2000 meters, you are getting the value 2.70; at 3000 meters, you are getting 2.4; at 4000 meters, 2.24. So, what do you mean by this?

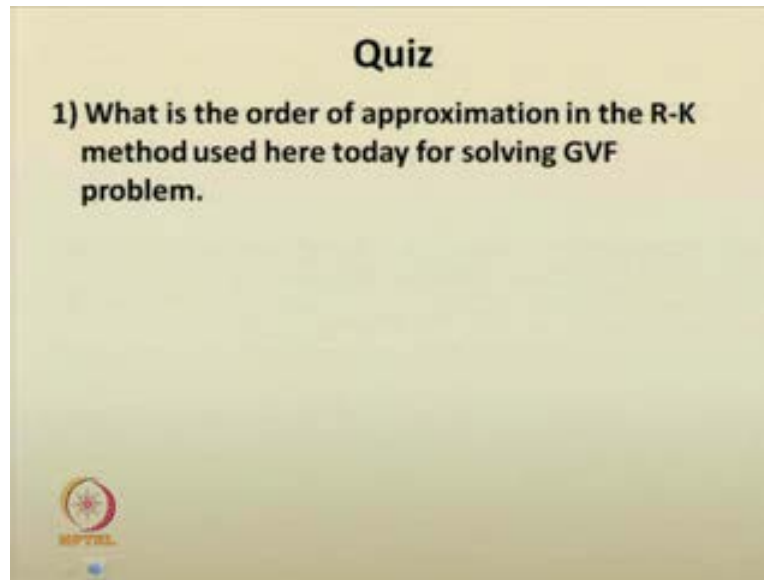
See, I am computing the area, hydraulic radius and the slope subsequently, using the formulas in the excel sheet. See, you can see anywhere, how the computations in excel; how it is easier; how it is to, you can see in the formula bar; how it is easy to show the things, directly substitute their quantities, you will get it, the corresponding slope values, so, everywhere; so, at each sections, at del x to after 2000 meters, after 3000 meters, 4000 meters, like that I have computed that, till I reach, where the depth of flow, where the depth of flow, where the depth of flow is less than 2.02 meters. So, it took a length of 9000 meters. Can you see that? So, you, I have used the Runge R K method, and found that the depth of, reached less than 2.02 meters, at a length of 9000 meters. So, what do you mean by this?

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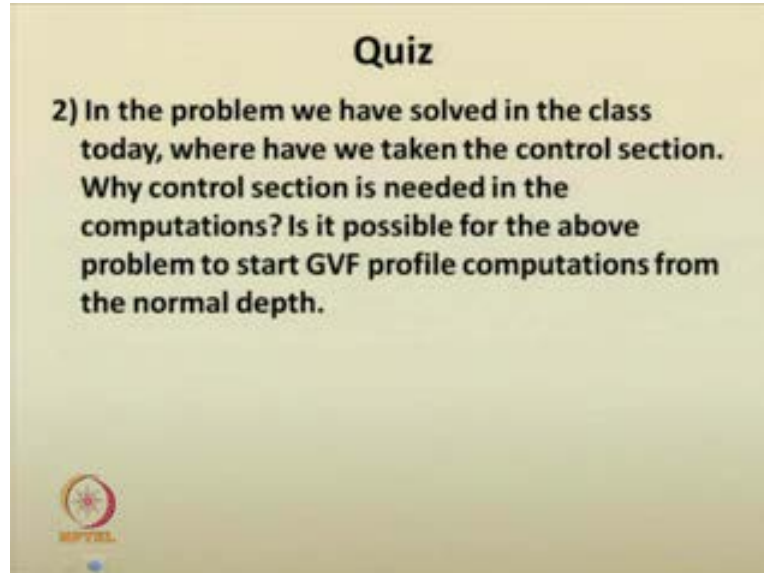
So, this means that, the length of profile, this is equal to 9000 meters. So, in your entire problem, this dam section problem, this corresponding length, length of the profile, this is 9000 meters. So, gradually varied flow profile length is 9000 meter. So, like that you can compute the gradually varied flow profile. So, now, today I am just stopping the lecture here. We will just have a quick quiz today.

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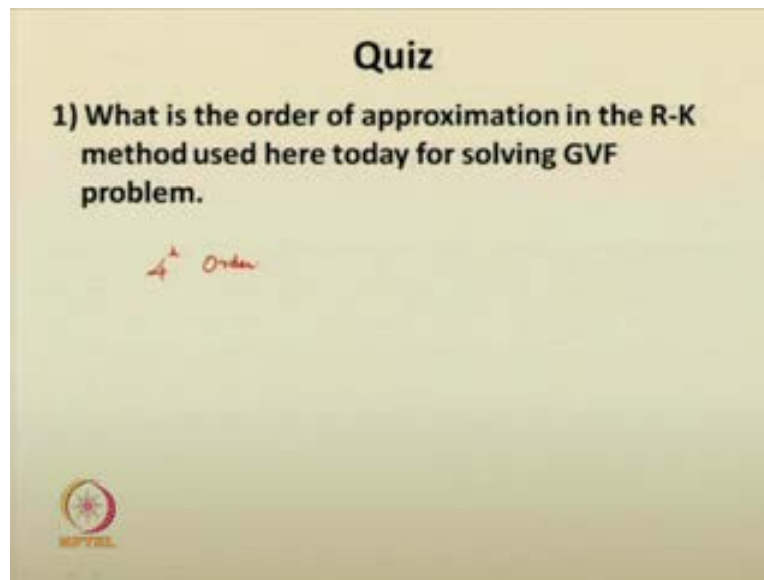
I would like to ask to you, what is the order of approximation in the Runge Kutte method used here, that is used here today for solving gradually varied flow problem.

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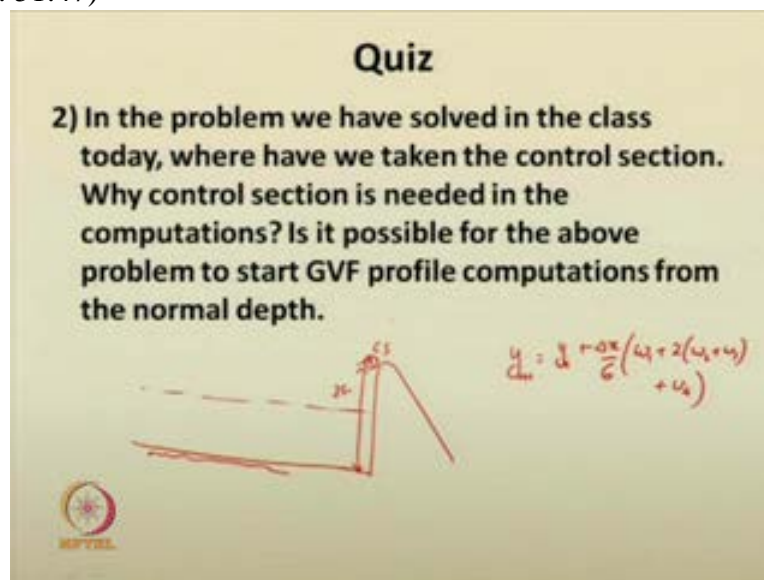
The second question: in the problem we have solved in the class today, where have we taken the control section. And, why do we require the control sections in the computations? Is it possible for the above problem to start gradually varied flow profile computations from the normal depth. Can you compute the gradually varied flow profile computations by starting from the normal depth; is it possible; that is the question asked.

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So, the solutions for the quiz is: the order of approximation in the R K method. As it is in, therein the title itself, it is a forth order approximation.

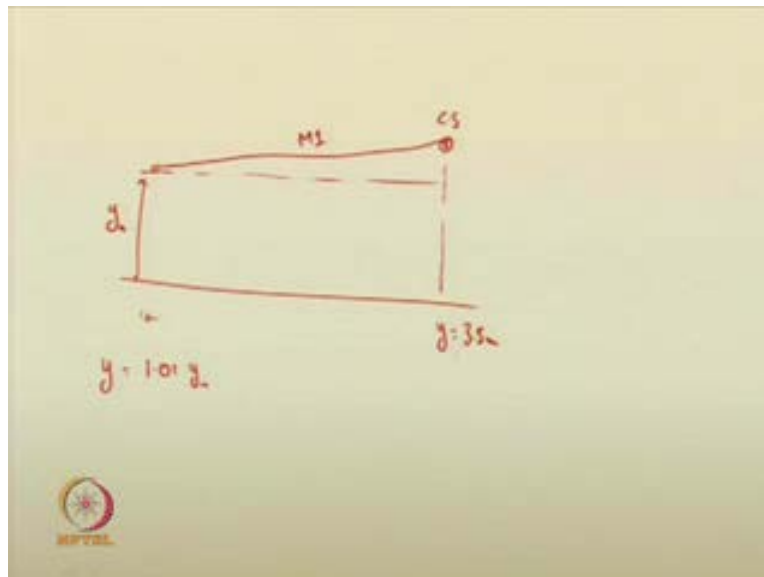
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The second question: in the problem solved in the class, where have we taken the control section? So, if you recall, normal discharge is there, dam was constructed, then just beside that dam, where the depth of flow is 5 meters, sorry, 3.5 meters that is being considered; depth of flow 3.5 meters just beside the dam section, that is taken into consideration. And, it is the control section for the problem; and from there we begin the computation. So, why control section is

needed in the problem? Now, without the control section, it will be difficult for you to compute the gradually varied flow profile; to what extent the gradually varied flow profile is there, what are the depths and all, you require a controlled section, because at the control section you know the value. You remember the equation, y_{i+1} is equal to $y_i + \Delta x \left(\frac{1}{6} \omega_1 + \frac{2}{3} \omega_2 + \frac{1}{6} \omega_3 + \omega_4 \right)$. In this thing, the y_0 value should be the control section; you require a known value initially, so that y_0 value it is given, and it has to be taken from the control section. Next question asked to you: is it possible for the above problem to start with normal depth? Is it possible?

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So, suppose one starts, say, if this is a normal depth, if you want to start; it is good, you can start, say, y is equal to $1.01 y_n$, from that depth onwards you can start computing; and of course, you will get a profile of this form, you will get. What happens is that, at what location y is equal to 3.5 meters is there, that will not be quite distinct, once you start from here. So, it is always better to start from downstream side, for the M_1 profile, ok. If you start from here, this length, as you have seen, you are taking it an approximation, here the length may exceed. So, you do not know the, how much length it will take to reach y is equal to 3.5 meter, that is the situation. So, this way, we are ending the lecture today. Next week, we will go on continuing the topics related to computations in gradually varied flow.

Thank you.