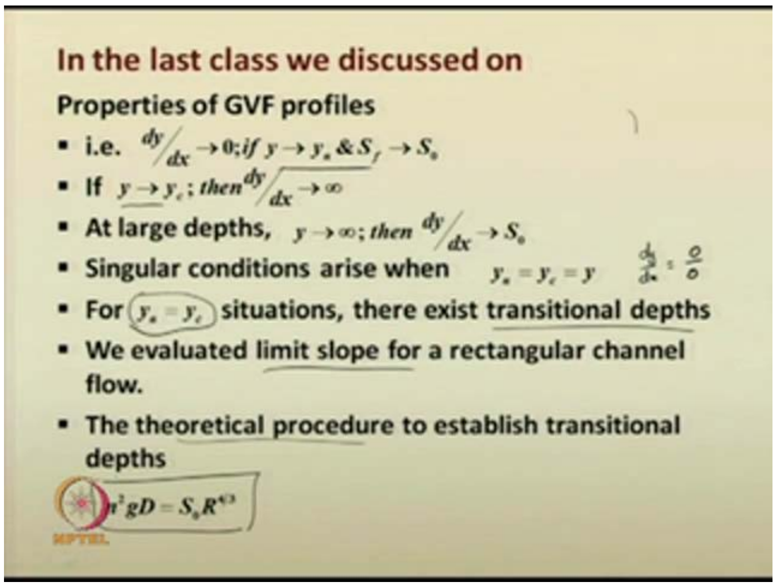
Advanced Hydraulics
Dr. Suresh A. Kartha
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module - 3
Varied Flows
Lecture - 7
Gradually Varied Flow
Computations Part 1

(Refer Slide Time: 00:25)



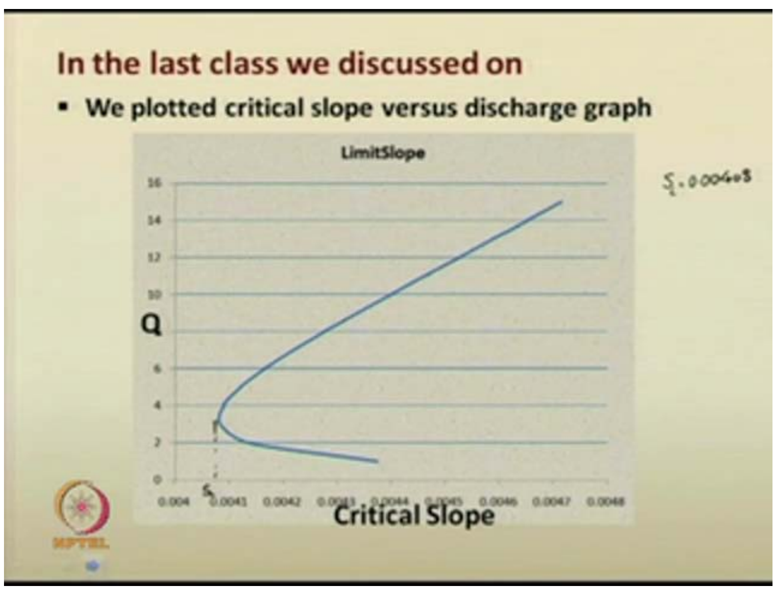
We are back into our lecture series on advance hydraulics. We are in the third module on varied flows. If you recall the last class, in the last class we had discussed on the various properties of the gradually varied flow, flow profiles. So, if you recall them; you have seen that the slope of the water surface that is d y by d x, it tends to 0, when the depth of the water tends to the normal depth or the energy slope tends to the bed slope. If these two situations are present, then you have seen that the d y by d x, means, the water surface is almost horizontal. If the depth of the water tends to the critical depth, then d y by d x is almost infinite; that is the slope at the critical depth, you will see the water surface is almost perpendicular. So, it is a theoretical concept.

You have also seen that at large depths, that is when y tends to infinite and all, the water surface slope, it is almost equal to the bed slope. You have also seen certain singular conditions arise; singular conditions means when d y by d x is equal to 0 by 0 form; if these things arise that is called singular conditions. So, singular condition arises when the normal depth is same as the critical depth, and this is same as the depth of flow.

You have also, you have gone through the situation where y, when y n, that is a normal depth is equal to critical depth. In such situations, there exist transitional depths or transitional profiles. For that, first we have evaluated limit slope; as a demonstrative example we had evaluated limit slope for a rectangular channel. And we also had given a theoretical procedure, how to establish the transitional depth thereafter; say, this was the equation derived if you recall them, n square g D is equal to S 0 R to the power of 4 by 3.

So, you see the transitional depths and all, they have some characteristic profiles. It is not dependent on the certain means, seen that. This theoretical procedure, it depends, means, it depends only on the channel geometry and the roughness of the channel. So, that property and all, if you see, means, transitional depth is also, theoretically one can establish them.

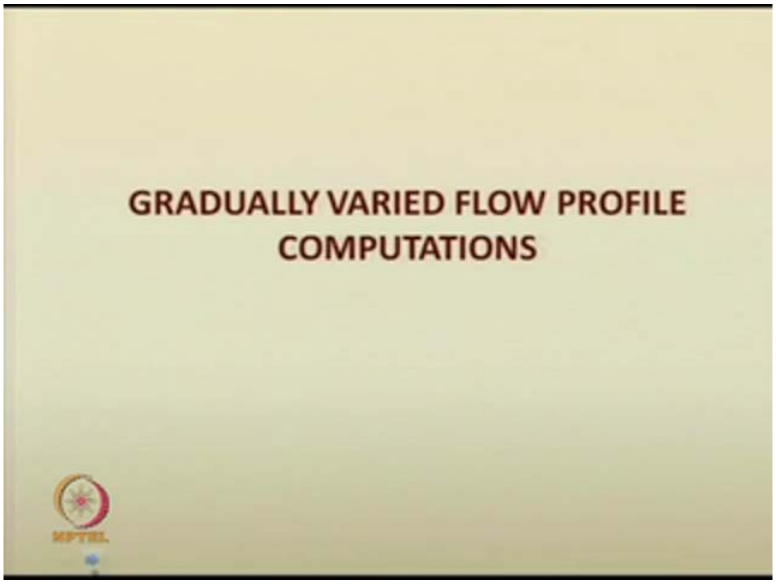
(Refer Slide Time: 03:09)



In the last class, when we suggested about the discussing on the transitional depth and all, we have suggested or we have evaluated critical slope curve versus the discharge for the rectangular channel problem, whichever we had discussed in the last class. So, there, you had obtained such a type of curve, and we have suggested that the minimum critical slope that can be possible in this case, that is given as the limit slope S L, if you have recall, if you are able to recall them. So, this is the limit slope; this value.

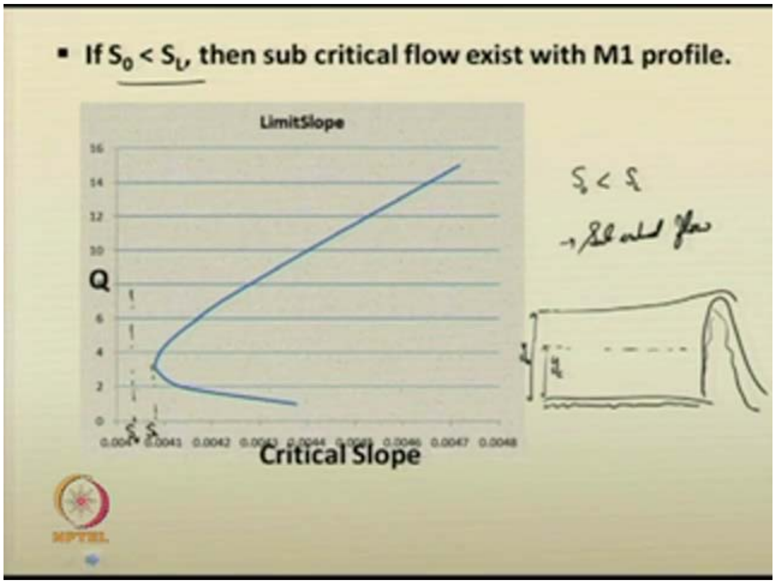
So, it was for that particular problem, S L was almost around 0.00408, something, like that. Now, your actual bed slope in the problem, it can be either less than the limit slope or it can be greater than the limit slope; what are the different conditions for transitional depths? Means, suppose if you want to compute, or if you want to observe the transitional depths or transitional profile, how will you proceed further?

(Refer Slide Time: 04:24)



So, today, we will see some of the gradually varied flow profile computational methods.

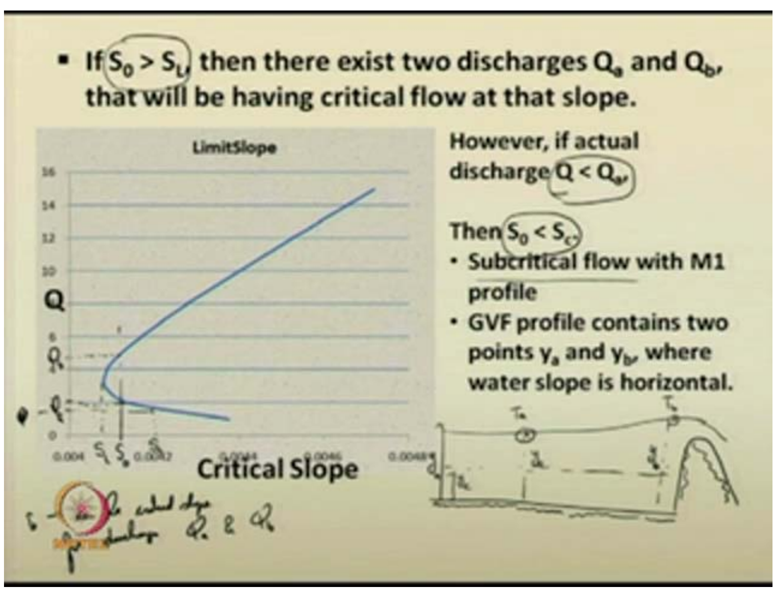
(Refer Slide Time: 04:35)



So, before starting the computational methods, we will quickly glance through, how the transitional profiles, especially in the transitional cases and all, how one can evaluate the transitional depths, or how you can observe it in the channel cross section and all? So, recall the same graph; so as we mentioned earlier, this is your limit slope S L. Now, if the actual bed slope of the channel for the given discharge, if the actual bed slope of the channel, if it is less than the limit slope; that is somewhere here, if your bed slope is somewhere here, then what happens?

This is the critical slope line. So, your slope S 0 is less than S L, means, subcritical flow exist, right. So, it is a subcritical flow. And in subcritical flow, what is a gradually varied flow profile? That you can observe in such situation, yes. So, it is the same M 1 profile that you can observe. You see here, this is a channel cross section. Suppose, if you are creating a bent, then this is your critical depth; it is your normal depth; then the gradually varied profile will be something of this form, fine. This will be the gradually varied flow profile. So, here, in such situations there will not be any transitional depths and all, because your bed slope is already less than the critical limit slope; so it is a subcritical flow existing there. What happens to the next situation?

(Refer Slide Time: 06:43)



If your bed slope is greater than the limit slope, then; let us see that, how you can demonstrate? This is your limit slope. So, if your bed slope is greater than, say, let this be your bed slope S 0, if this is greater than the limit slope, then you can observe easily 2 discharges; say, this is discharge, I can write it as Q a, and this is discharge Q b. That is, for the same bed slope S 0, for

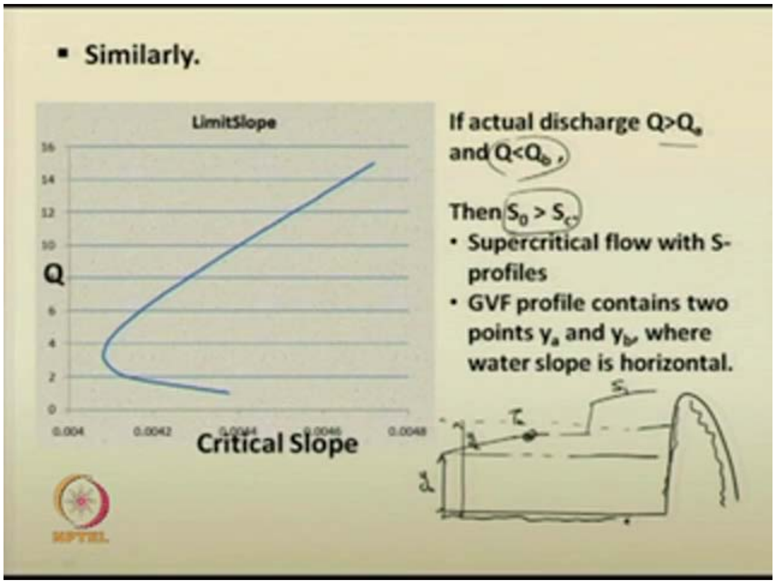
S 0 can become critical slope for discharges Q a and Q b. So, from this profile you can see that. So, S 0, at this location, for almost magnitude of 2, it is the Q a; that is Q a is equal to 2 meter cube per second, in this particular from, in this particular graph. So, it is almost, the bed slope is almost a critical slope. Similarly, just above 4, 4, between 4 and 5, here Q b, the magnet of Q b, at that situation also S 0 is acting as a critical slope. Like that, you can easily observe that.

So, how can you observe the transitional profiles in such situations? So, it depends on, what is your actual discharge Q? Say, if your actual discharge Q, if it is less than Q a; even though your bed slope is S 0; if Q is less than Q a; from this graph, when do you think that Q can become less than Q a? It can become less than Q a only if the corresponding slope is greater than S 0, right. Say, somewhere here, this is Q, some actual Q value. So, this is some S, some, particular slope S. So, the, for that Q, the critical slope S c is at this location. But, the actual bed slope is S 0. So, S 0, in such situation is less than the critical slope.

Therefore, again, the subcritical flow with M 1 profile will exist, fine. Because, the gradually varied flow profile contains 2 points. You know that, for Q a there is one particular depth y a, for Q b there is another particular depth y b. If I want to plot them, say, your bed profile, initially the normal depth y n, you have the critical depth y c. So, in this normal depth, this is going to follow an M 1 profile. So, you know that this will gradually increase like this, then right. So, this is a gradually varied flow profile.

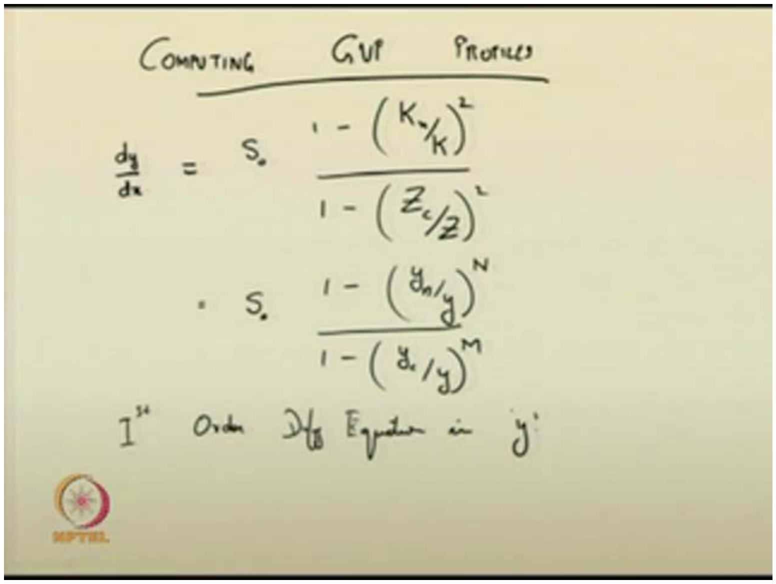
In that case, there exists 2 points, say T a; on the profile there exist 2 points, T a and T b, ok. There exists 2 points. Why we are mentioning such 2 points are? So, the corresponding depths will be y a and y b. So, these points y a and y b represents the normal discharge Q a and Q b. So, T a; at these locations, the gradually varied flow, flow profile will be horizontal. So, they are the points of infraction; that is, the slope changes from that location onward, the slope of the curve changes from that location onwards. So, such horizontal; So these T a points, T a and T b are the transitional locations. Transitional depths are, then the corresponding y a and y b, fine.

(Refer Slide Time: 11:55)



Similarly, you can explain for situation, if Q is slightly greater than Q a, but but however less than Q b, then in that situation your bed slope will be greater than the critical slope, and your supercritical flow will be having S profiles. So, that S profile, one can think of in the following form. Say, initially, if this is your normal depth, if this is the critical depth line, then your profile, initially, it will be in the following form; up to a certain location T a. There, the slope will be horizontal, water surface profile will be horizontal; then it will make a jump and will have S 1 profile. So, it will go through S; it will be having S 3 and S 1 curves, right; it will be having S 3 and S 1 curves.

(Refer Slide Time: 13:41)

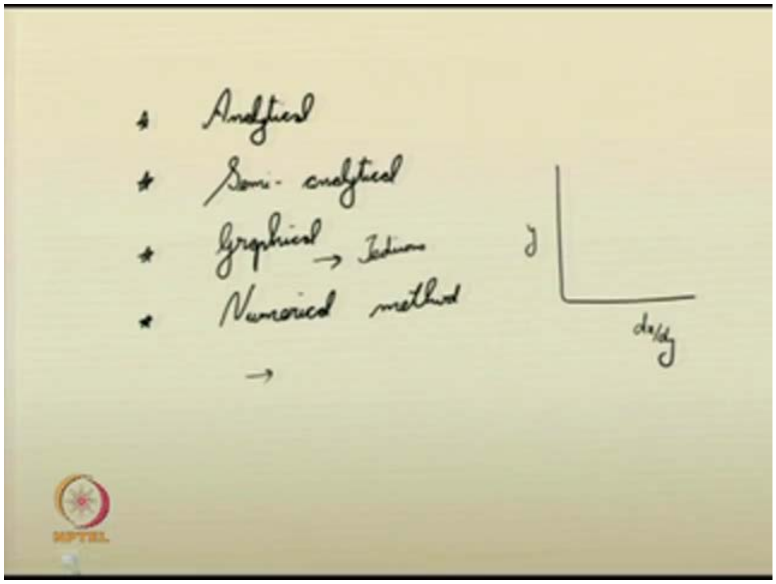


So, using these theoretical knowledge and all, now we will see, what are the various methods of computing gradually varied flow profiles? Fine. So, let me recall the dynamic equation for gradually varied flow; d y by d x, this is equal to S 0, 1 minus K n by K whole square by, 1 minus; may I beg pardon; Z c by Z whole square; this you can write in terms of y also, we have derived it in the last class. 1 minus y, beg pardon, y n by y, whole to the power of hydraulic exponent N, divided by 1 minus by y c by y, whole to the power of M. So, you can see that, this is a first order differential equation in y, and we have, it is a first order differential equation in y.

So, we have seen, also seen that it requires one boundary condition to solve. And also to solve that we have described the control sections in the open channel flows. One can begin with the control sections, where the properties are known initially. And from there one can start computing the profile. Like that we have discussed in the last class, about the control sections. So, based on one control section, one can easily start the profile computations.

Solving this differential equation, means, you can identify the distance for a particular depth; for a particular depth of flow, how much it is, how much distance it is from the control section? Whichever referred control section is there, how much distance it is from that control section? That you can identify. Or for a given distance from the control section, what could be the depth of the water profile? That can also be identified from solving these equations.

(Refer Slide Time: 16:15)



In general, to solve a differential equation, the various methods- you have analytical methods, you can also use semi analytical methods, you can also use graphical methods, and the most common and widely used method is the numerical methods. So, you can use any of these methods to solve the differential equation, that is the dynamic equation for gradually varied flow.

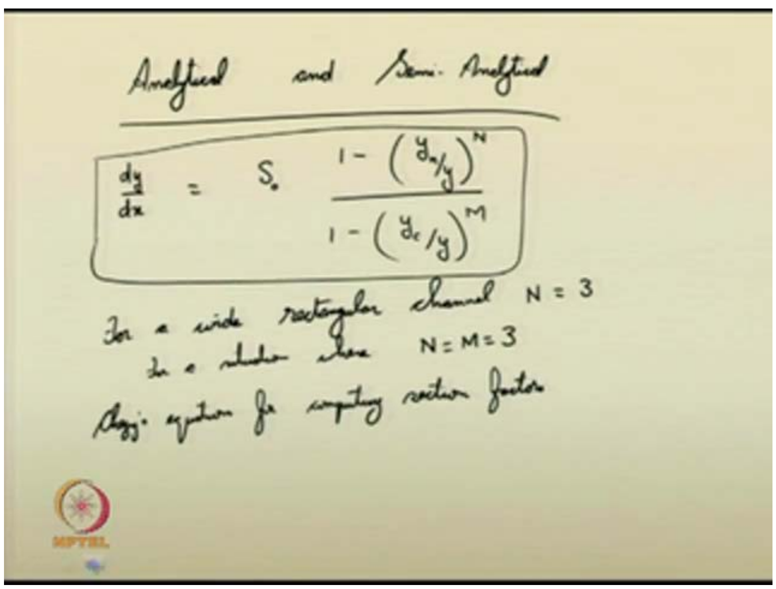
So, to solve these equations; analytical methods, it is, it is possible only for a very few cases; very few simple cases only, one can solve the gradually varied flow problem using analytical method. Semi analytical methods, it involves, where; as you have seen this differential equations, to solve this differential equations, means, you have to integrate upon y and x, right, the equation. That is the meaning of solving the differential equation. So, that means, in semi analytical form, what you are doing is that, you are clubbing the actual integration with certain numerical integration approximations that are available for some functions.

For example, if you have cosine function, there can be a tabular values of cosine function, numerically integrated values for the cosine function, and you can directly substitute them; and like that, you can develop the semi analytical solutions. If you have sine function, or if you have exponential function, based on that one can; in a tabular form, if it is this already available the tabular data; or tabular things are already available, the integrated values for those functions and you can directly substitute; that is the approach used in semi analytical form.

Graphical form, what you are doing is that, you are just directly plotting the slope of the curve verses a depth of the curve, directly. And from that you just compute the area; say, d x by d y, inverse of the slope, one can just plot like that; and then try to club it; that is also possible. d y by d x with y that can also be plotted, and you can compute the area graphically; graphically the area has to be computed. So, this is a very tedious. Although, the approach is correct and it may give very good result, it is quite tedious. So, the graphical method is quite tedious.

The most common method is the numerical method. Due to the advance, even in the computational technology and all, one can use, readily use the numerical methods to solve such type of differential equation. Not only such differential equation, even the higher order differential equations are also solved using numerical methods. Very common, very common methods like finite difference method, finite element methods and all, you might have already seen them in some of your other courses, there are, they can be used here in the open channel flow also.

(Refer Slide Time: 19:54)



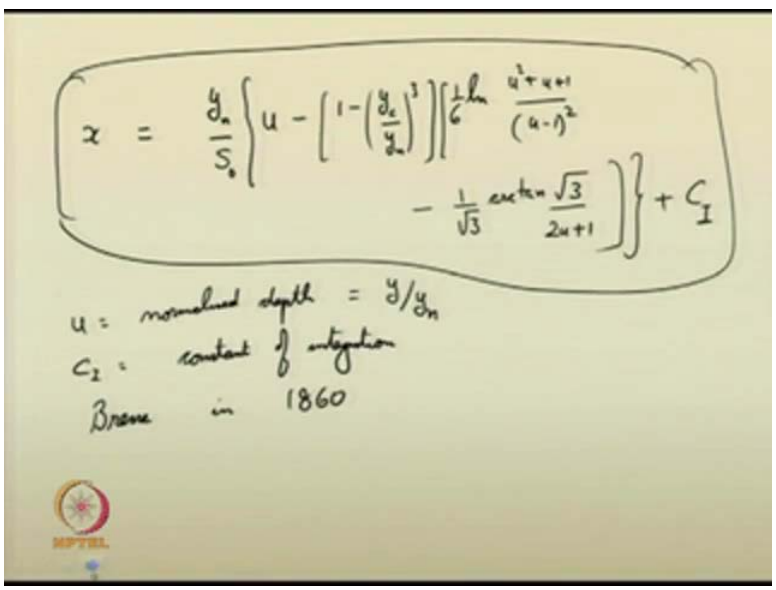
So, some of the, in the analytical and semi analytical method, I would just briefly describe one or two methods here. We are, definitely these methods are not going to help you solve the practical problem. They can be only used in a very limited situation, where, especially in the laboratory cases where you have all the constrains established. In such situations only, one will be able to solve the gradually varied flow equation, dynamic equation, using analytical form.

So, here, the analytical methods, they were developed quite earlier itself; in the early, in the 20th, earlier part of the 20th century, or in the later part of the 19th century and all, their mathematicians have developed analytical forms for these equations. We will just briefly go through them. I am not going to, for discuss elaborately on those things.

So, your governing dynamic equation, if I write it in the terms of depth of flow, y c by y whole to the power of N; so where N and M are hydraulic exponents, that you are quite aware, right. Now, y n is the normal depth, y c is the critical depth. If you recall in the module two, in module two we had discussed on the hydraulic exponents, N and M, for various type of channels- for rectangular channels, for wide rectangular channels and all, our trapezoidal channels; we had in fact derived the expressions for N and M also. You just refer back them.

For a simple case, for a wide rectangular channel, using those relationships, for a wide rectangular channels it is observed that your hydraulic exponent N is equal to 3. Now, for a situation, where N is equal to M is equal to 3, if you take this thing; and if you had used Chazy's equation for computing section factors, instead of Manning's equation if you have used Chazy's equation for computing section factors in critical flow and all, then you will observe that this particular dynamic equation can be solved with the following expression.

(Refer Slide Time: 23:17)

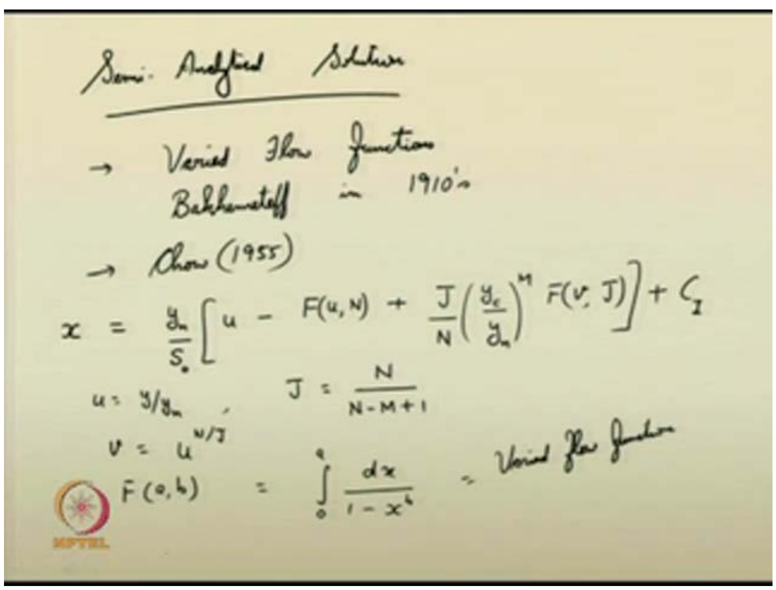


If the solution can be given in the following form: x, that is the distance; horizontal distance x, this is equal to y n by S 0 into, u minus, 1 minus, y c by y n whole cube into, 1 by 6th of natural log of u square plus, u plus 1 by, u minus 1 whole square minus, 1 by root 3 arc, tan of root 3 by, 2 u plus 1; this square bracket is closed, plus some constant of integration C I. Like this one can give a solution for the dynamic equation, where x is the horizontal distance, u is the normalized depth that is given as y by y n, C I is equal to constant of integration your C I, then all other terms are explained earlier also.

So, using this relationship, you can obtain the solution for the gradually; or, this the solution for the gradually varied flow profile. From this you can easily determine the horizontal length of the profile and all; so or horizontal location of the, for a, for a particular depth, what is the corresponding horizontal distance? That can also be identified from this equation.

So, this solution was obtained by Bresse in 1860. So, you see, it was long back; in the 19th century itself such a solution was identified. So, this is called the famous Bresse solution for gradually varied flow profile. Although, this solution, it is obsolete in the present, present generation, but still, you can appreciate, long back, even in almost 200 years before itself, more than nearly, more than 200 years before itself, or nearly 200 years before, scientists had almost reached for a approximate solutions, or a nearby solutions to the flow problems. So, that way, the science has to be quite appreciated.

(Refer Slide Time: 26:27)

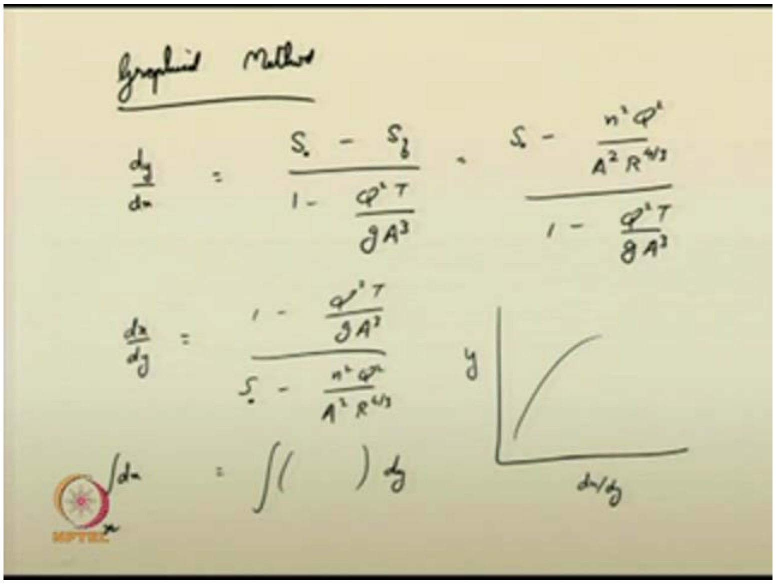


The semi analytical form, which I would like to briefly introduce to you, is the following form, semi analytical solution. So, the semi analytical solutions, they are usually obtained; means, in the open channel profile, gradually varied flow, they are usually done by generating varied flow functions. Such varied flow functions, different type of varied flow functions, they were developed by Bakhmeteff in 1910s, between 1910 1920 and all, varied flow functions were derived, mathematical varied flow functions were derived by Bakhmeteff. Such functions were later used by Venti Chow in, Chow in 1955, in his book on open channel hydraulics and all, he has used the varied flow functions and obtained flow profiles, or obtained solutions for the gradually varied flow profile equation.

So, one such solution is, your horizontal distance x is equal to y n by S naught into, u minus, the varied flow function F of u and hydraulic exponent n plus, a parameter J divided by N, into y c by y n raise to M into, the varied flow function of a variable v and parameter J, plus constant of integration C I; where u is your normalized depth y by y n; J is a parameter obtained as N by N minus M plus 1, if you have recalled, in the hydraulic exponent literature and all, whichever we have dealt in the module two, there also we had encountered with similar relationship, so it is the same thing. So, J, you have just substituted the quantity J here; the variable v is nothing but it is u to the power of N by J, like this you can obtain the variable v; and your varied flow function F of any 2 parameters a and b, it is given as integral of 0 to a, d x by 1 minus x to the power of b, like this; and this is called varied flow function. So, you can use these semi analytical forms.

So, the tabular values of this functions, varied flow functions, they are available in the literature; especially, if you go through the Venti Chow's book on Open Channel Hydraulics and all, you will get varied flow functions for different parameters. You can directly substitute them. So, this is a semi analytical approach. One can use the semi analytical approach also for solving the gradually varied flow problem.

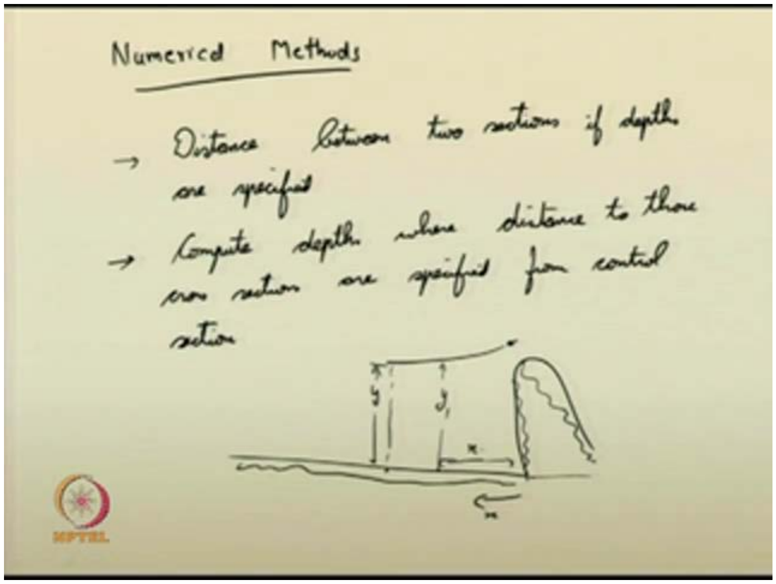
(Refer Slide Time: 30:30)



So, but as we have mentioned earlier, there is graphical method also. Of course, we are not going to do the graphical methods, but this can also be used to solve the gradually varied flow. The fundamental is d y by d x, this is S 0 minus S f by, 1 minus Q square T by g A cube. In the beginning of this module, we have been discussing on this dynamic equation; so please recall them. So, this can be written as S 0 minus, S f is given, an using Manning's equation n square Q square by, A square R to the power of 4 by 3 divided by, 1 minus Q square T by g A cube. So, what do you infer from this? You can write the inverse of this differential equation d x by d y is equal to 1 minus Q square T by g A cube by, S 0 minus, n square Q square by, A square R to the power of 4 by 3.

You can integrate them. So, on integrating them, on the left hand side you integrate it with d x, and this term you integrate it with respect to d y; you are going to solve x in term with respect to y. Like this, you are able to get the term. This can be done in a manual form or in a graphical form also, one can easily plot them y versus d x by d y, and the corresponding plot whichever you are getting, you just compute the area; you just compute it manually that, from that graph, what is the area? And that area will give you the distance x, ok. Or, if suppose, if you want to measure y, go through the reverse process; you plot x with respect to d y by d x, then you plot them, and you will get the corresponding y value.

(Refer Slide Time: 32:53)



So, the most important method, which we hereafter will be always referring is the numerical methods; the idea behind numerical methods. Numerical method, it is an approximate method for solving differential equation. It is not the direct method, whichever you have studied in your high school levels or in your college levels and all, the numerical method does not follow the direct approach. It is an approximate approach.

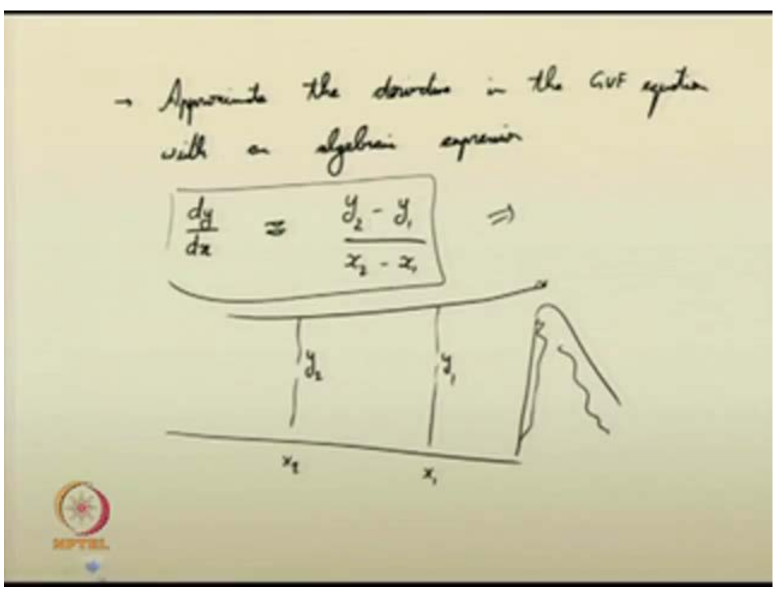
Here, the idea or the crux behind solving the differential equation is, you are going to approximate the various functions or derivates in the differential equation, with certain algebraic expressions. Once you are able to substitute it with algebraic expression, then that corresponding differential equation is converted to an algebraic equation. You can further solve the algebraic equation in, in easier manner. You are already aware how the linear algebra, or the linear algebraic equation is much, much easier to solve, compared to minute of differential equations, right.

So, the numerical methods is just converting the differential form into algebraic form, and then you are solving the corresponding (( )). So, that is the crux behind. So, the computers update, you know that the memory as well as the computational speed, many of the things, it has been increased tremendously. So, there is no limiting computations now. And you can use any number of, in almost you can, say, quote it as infinite number of unknown things and try to inverse them, try to solve them, the computation capability is that high now a days.

So, due to this computer, this thing computation up, one can easily go for the computational approach and solve the open channel flow problem. Why you need to manually go and try to do it analytically, if your computer is able to solve them easily in few seconds; that is the idea behind using numerical methods, as well as the difficulty in the analytical approach.

So, let me tell you. So, in the numerical methods for the open channel flow, we will consider, say, using numerical methods we will, we can find distance between 2 sections, you can find distance between 2 sections if depths are specified. Or you can compute depths where distance to those cross sections are specified from control section, of course. So, that is, say, if I just draw it like this; say, if this is your control section for a particular problem, then you know that this you can consider as x is equal to 0; from this location, you can compute profile in the upstream direction; say, x, this is x. So, for this, at this location, what is the depth of the water y? That can be obtained as per the second statement. Or from this control section, if you are given a certain depth y 1, what is the corresponding distance x? That can also be identified. So, one can easily use the numerical methods.

(Refer Slide Time: 37:31)

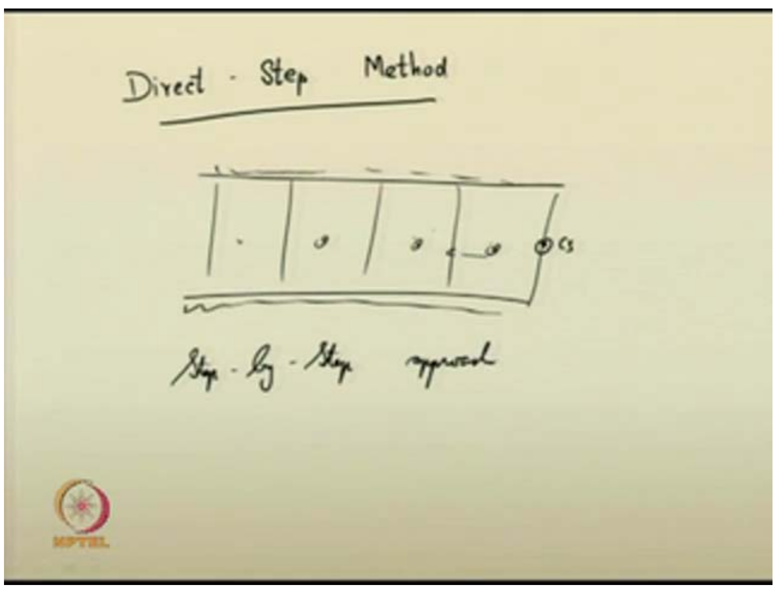


So, here, what we are going to do is that, approximate the derivative in the gradually varied flow equation with an algebraic expression. So, usually, what we are doing is that, this is the derivative available; so d y by d x is now approximated by an expression, algebraic expression in the form of y 2 minus y 1 by, x 2 minus x 1. That is, say, from the control section, if you have 2 locations, this is x 1, this is x 2, and the corresponding depths of y 1 and y 2, if you have them, then the slope of water surface d y by d x between these 2 sections can be approximated by the following algebraic expression: y 2 minus y 1 by, x 2 minus x 1.

This has been obtained from Taylor series. If you refer further the numerical methods book and all, it will be quite clear. So, so such approximation is done. Then all other terms which involves the quantities, or all other functions that are related to depth, they can be averaged also accordingly, and you can use them in the equation.

So, not only the differential equation; any, if it is partial differential equations, the partial derivatives also substituted by algebraic terms, and the corresponding equation becomes an algebraic equation, so that algebraic equation is solved. So, that is a objective here.

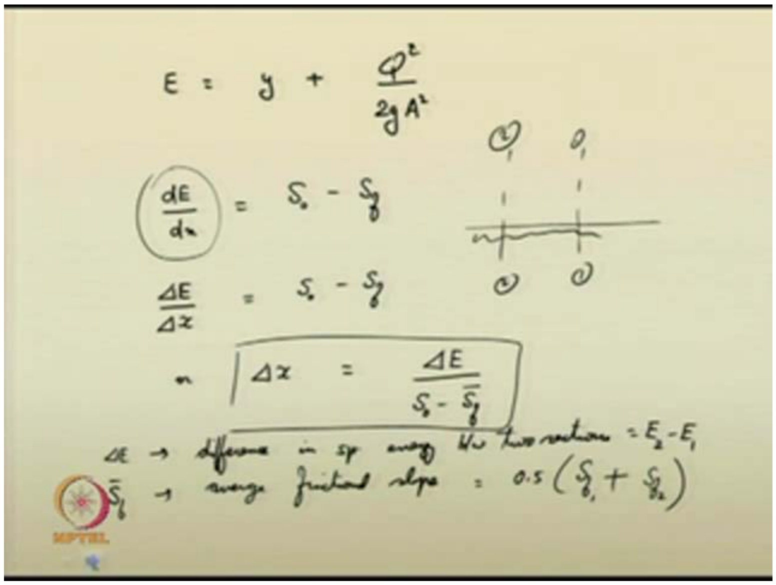
(Refer Slide Time: 39:41)



The first method which we are going to do is, direct step method. The first method is direct step method. What you are going to do is, the channel reach, whichever channel reach is there; say, if there is an obstruction and gradually varied flow profile is there, it is, the channel reach is now divided into short reaches. You will compute the properties now, in each of the small reaches, then you club them.

Or, you say, from this, if this is your control section, then from the control section you begin the thing; when the control section, you know all the properties, then you compute here in this portion, then here, then here; using the values here you compute it here; then using the values here you compute it here, like this, you will go on. So, this is a step by step, step by step approach; so you can use the step method to compute flow or compute the flow properties.

(Refer Slide Time: 41:01)

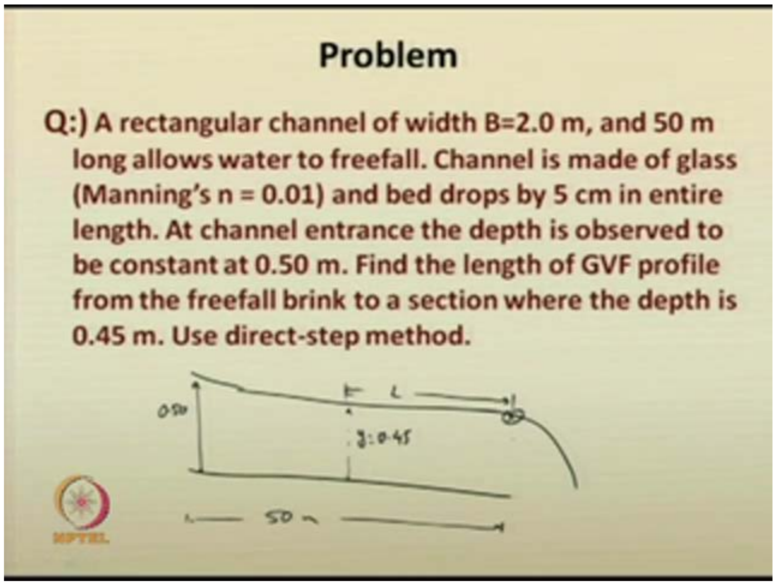


Just recall your specific energy equation, y plus Q square by 2 g A square. So, the direct step method, we can use mostly for the prismatic sections only. For non prismatic sections, this method is not that convenient. So, we will see it later. So, please note that this, how you are going to use them?

You have also seen that d E by d x is equal to S 0 minus S f, in one of our earlier lectures. So, what does this mean? Change in energy. So, this thing you are now going to use the, this derivative. You are going to approximate it using algebraic expression, del E by del x is equal to S 0 minus S f, like this you can write. Or, del x, change in distance or the difference or the distance between 2 points, del x, that can be given as change in specific energy between those 2 points by, S 0 minus S f.

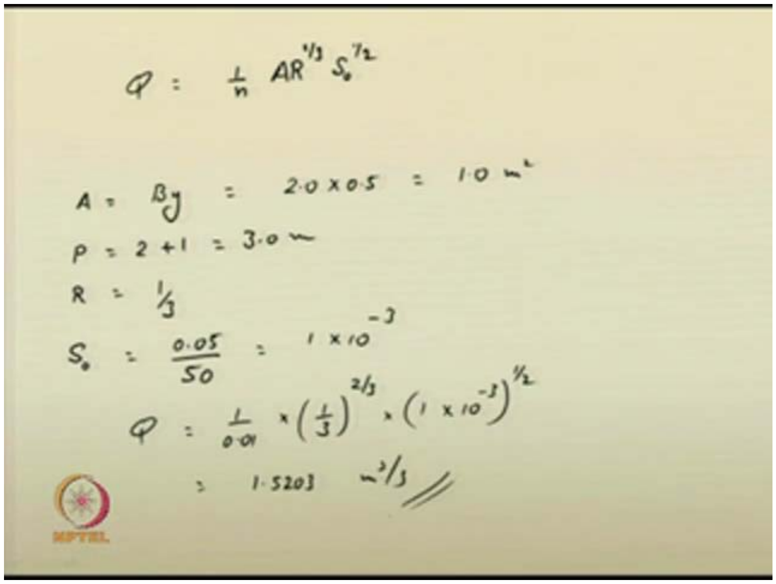
Now, in this case, S f, you have to take as the average one; difference in specific energy between 2 sections, so this is equal to E 2 minus, if you have 2 sections, 1 and 2, say, in the reach 1 1, 2 2; so E 2 minus E 1. S f, you are taking the average value; it is the average frictional slope; it is average frictional slope between the 2 sections 1 1 and 2 2. So, it is generally given as, 0.5 into, S f 1 plus S f 2, like this you can give them. Del is equal to E 2 minus E 1. Using this thing, one can now compute del x. If you are able to compute del E by S 0, you can then further compute the distance between 2 points del x. A demonstrative problem will help you in solving this thing. Let us show that example.

(Refer Slide Time: 44:04)



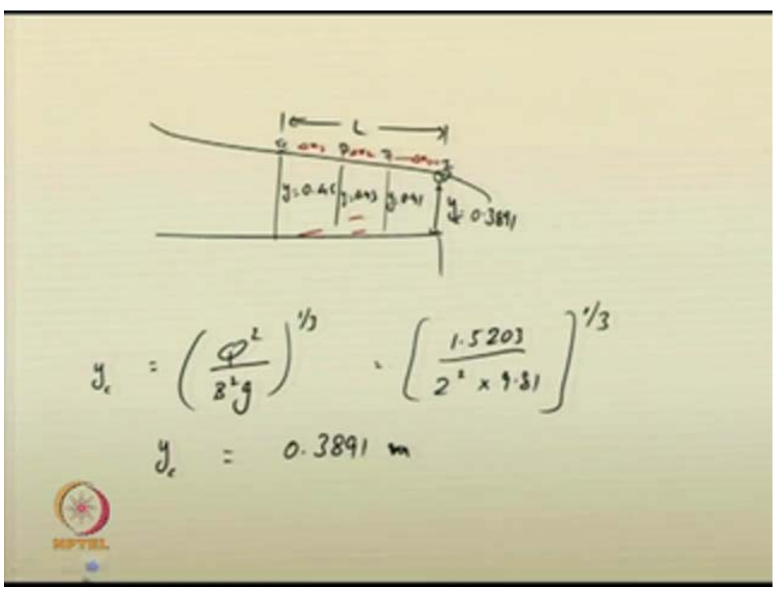
So, please note this particular problem. A rectangular channel of width B is equal to 2 meter, and 50 meter long allows water to freefall. The channel is made of glass; that is Manning's n is equal to 0.01, the bed drops by 5 centimeter in entire length. So, you have the length like this. So, so 50 meter length, channel width is 2 meter, so it is allowing freefall from this edge; so this is the brink; the bed drops by 5 centimeter in the entire length; from here to here it drops 5 centimeter. At channel entrance the depth is observed to be a constant. Here, the depth is almost observed to be a constant of 0.5 meters. Find the length of the gradually varied flow profile from the free fall brink. That is, from here to a particular section here, where the depth y is equal to 0.45. What is the, what is the length of the profile l? That is what, we have to do it now; use the direct step method.

(Refer Slide Time: 45:38)



So, let us see the thing. You know discharge Q is equal to 1 by n, A R to the power of 2 by 3, S naught to the power of half. A is equal to B y. So, in the beginning, in the entrance, channel entrance, you will see that this is nothing but 2 into 0.5 is equal to 1 meter square. Then you have B is equal to 2. Wetted parameter is equal to 2 plus 1 is equal to 3 meter. R is equal to 1 by 3. Your bed slope S 0 is equal to 0.05 by 50; so this is 1 into 10 to the power of minus 3. So, if you have these things, Q, substituting these values, it is observed to be 1.5203 meter cube per second. This is the observed discharge.

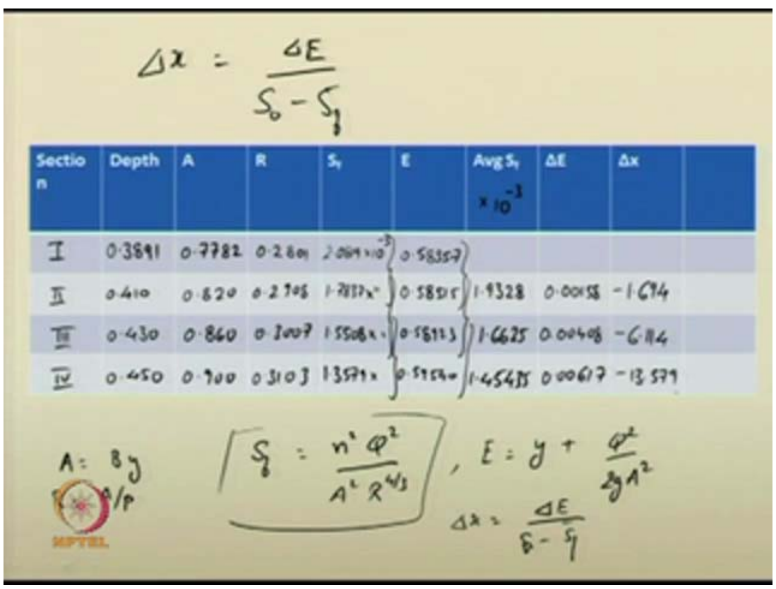
(Refer Slide Time: 47:21)



Now, before that, let me suggest to you, the same discharge at the brink; say, at the brink, here the depth will be critical depth. So, the critical depth, you can easily compute for the rectangular channel as B square g power of 1 by 3. Substitute the values, 1.5203 by 2 square into 9.81 whole to the power of 1 by 3. So, your critical depth is observed to be 0.3891 meters, ok. So, the critical depth for the flow is 0.3891. So, at the brink the depth is 0.3891.

Now, the total length of the profile between this location, 0.3891, and somewhere here y is equal to 0.45. So, what is this length? That is the question asked to you. And we are now going to do it by step, direct step method. So, let me do in a following form, as y c is equal to 0.3891, I am taking another depth y is equal to 0.41, another depth point y is equal to 0.43. Like that, I am having, I am going to take 4, 3 sections, 1, 2, 3. The distance in each section; this is del x 1, this is del x 2, this is del x. The summation of del x 1, del x 2, del x 3, that will give you the total length of the profile. So, that I am just going to do it.

(Refer Slide Time: 47:14)



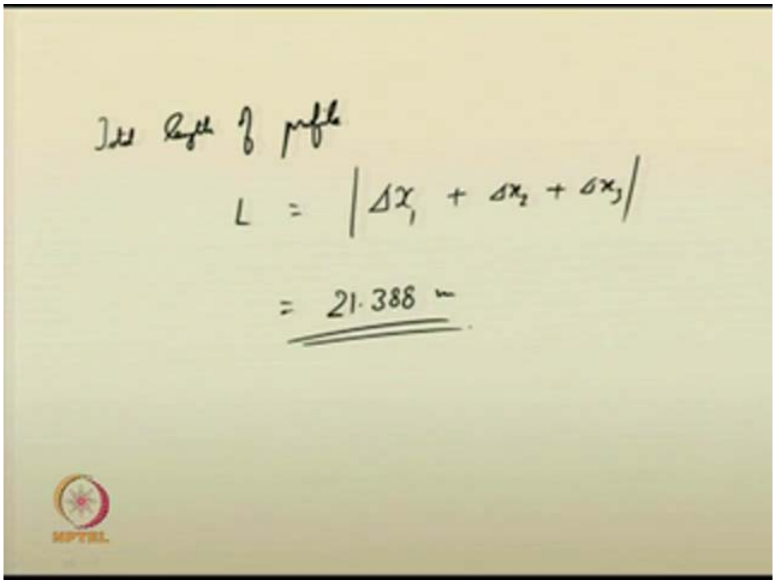
So, now for each of these sections, small, small, small portions, small reaches, I will be completing the corresponding distance and thus computing. So, that can be done in the following table. So, for section this is 1, 2, 3, 4. That is, here is section 1, cross section 2, cross section 3, cross section 4; and these are the reaches 1, 2, 3. So, please do not confuse with the reaches and the sections. I am just rubbing it, for your convenience. So, I have the corresponding section now.

So, at the section 1, the depth is 0.3891, you already know that; we had, it is the critical depth. The corresponding area A, you know it is B y. You compute it, you will get it 0.7782. The corresponding R, this is equal to A by P. I had got it as 0.2801. The corresponding energy slope S f, it is obtained as n square Q square by A square to the, R to the power of 4 by 3. So, this value it is coming around to be 2.0819 into 10 to the power of minus 3. Your energy E, it is nothing but y plus Q square by 2 g A square; substitute this form here, and I am getting 0.58357.

Similarly, for section 2, the corresponding depth is 0.41; at section 3, the depth is 0.43; and at section 4, the depth is 0.45. You can see the corresponding area also, point; the corresponding hydraulic radius, it is coming to be 0.2908 here, 0.3007, 0.3103. S f, you compute it using this relationship for each of the sections; you will see, here it is 1.7837 into 10 to the power of minus 3; here it is 1.5508 into 10 to the power of minus 3; here it is 1.3579 into 10 to the power minus 3. The corresponding energies, specific energies are 0.58515; 0.58923; 0.59540.

You can take the average of these 2 and write it here; average of these 2; similarly, the average of these 2, like this. So, I can write the average values now, this is 1. Average S f into 10 to the power of minus 3. In terms of minus 3, you can write 1.9328, 1.6625, 1.45435, like this you will get. Del E also, you can, difference between these energies, you can write them; here it is 0.00518, 0.00408, 0.00617, so you have got this thing. You know del x is equal to del E by S 0 minus S f bar. Substitute that. I will write it again here, del x is equal to del E by S 0 minus S f bar; substitute these relationship here; you will get the corresponding del x value, here as minus 1.694, this is minus 6.114 and minus 13.579.

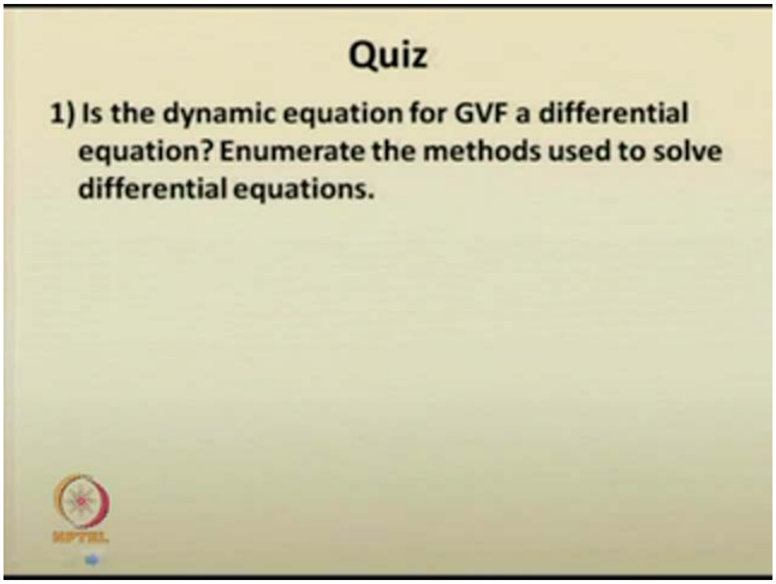
(Refer Slide Time: 54:29)



So, total length of profile, L is equal to the modulus value of del x 1 plus, del x 2 plus, del x 3. Substitute them; I am getting it as 21.388 meters. So, this is the length of the profile. So, the, this way you can do the direct step method. You can approach other methods also. We will see it in the next classes.

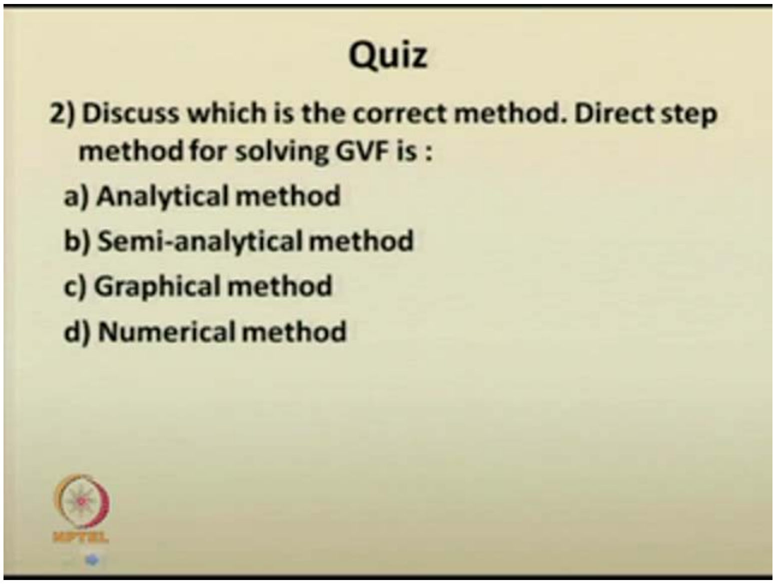
So, today we are going to wind up the lecture. We will just have a very quick quiz session.

(Refer Slide Time: 55:15)



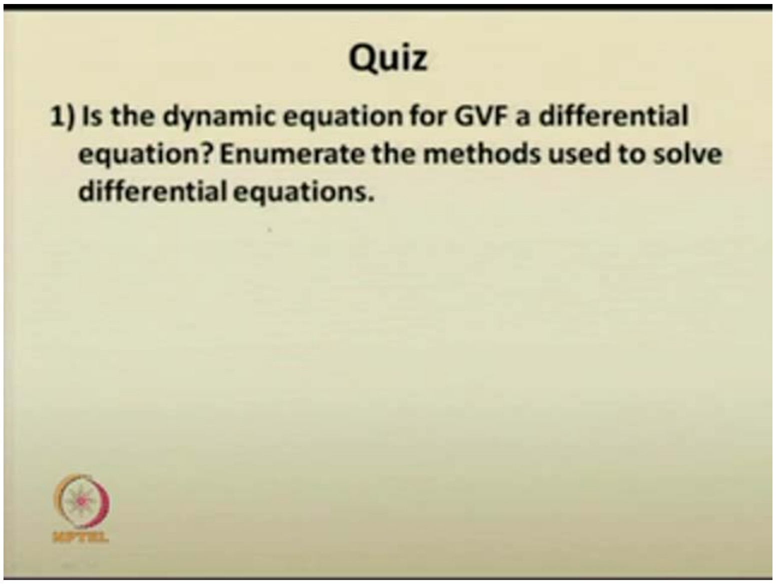
The first question is: Is the dynamic equation for gradually varied flow a differential equation? Now, enumerate the methods used to solve the differential equations. Takes hardly 30 seconds at home.

(Refer Slide Time: 55:29)



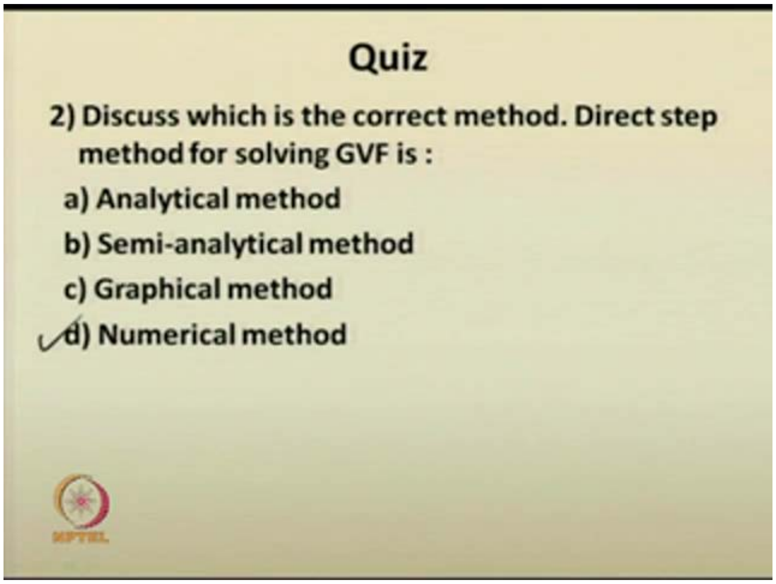
Now, this just here, you just need to tick this thing. Discuss which is the correct method? Direct step method for solving gradually varied flow is: it is an analytical method, or it is a semi analytical method, is it is a graphical method, or it is a numerical method; which is a right one? It is up to you to answer that. So, the solutions for the quiz.

(Refer Slide Time: 55:55)



So, for the first one: of course dynamic equation is a differential equation. You have seen. The various methods are analytical method, semi analytical method, graphical method and numerical method.

(Refer Slide Time: 56:09)



For the second question, you know the correct method. The correct method here, it is the numerical method. So, we will continue with the similar topic in the next class.

Thank you.