Advanced Hydraulics
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Module - 3
Varied Flows
Lecture - 6
GVF Profile Properties and
Transitional depths

Welcome back to our lecture series on advanced hydraulics. We are still in the module three, where we are dealing with the varied flows in open channels.

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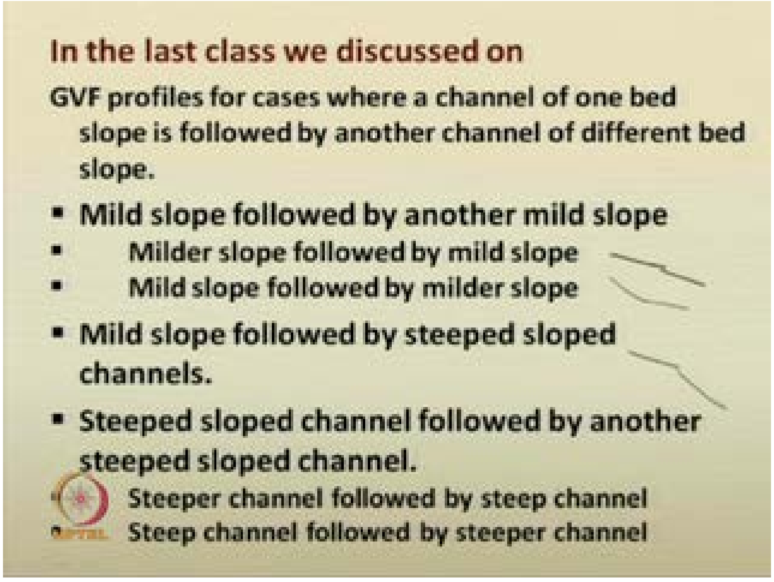

If you recall the last class; we had discussed on gradually varied profile cases, where a channel of one particular type of bed slope is followed by another channel with a different bed slope. Some of the cases, where mild slope followed by another mild slope; in this itself, there were two different cases: milder slope followed by mild slope. Say I am just giving rough idea again. A milder slope followed by a mild slope; that is, this one is having less slope compared to the followed one. Similarly, a mild slope followed by a milder slope. So, mild slope followed by a milder slope; like that. We have discussed the gradually varied flow profiles in these cases. Similarly, another situation is where mild slope is followed by a steep sloped channel; that is, the mild slopes are followed by steep sloped channels. See you can see a mild slope followed by steep slope;

how the profile slopes. Steep sloped channel followed by another steep sloped channel – here also, there are two different cases: first, a steeper channel followed by a steep channel; and second one is steep channel followed by the steeper channel.

Today, we are now going to discuss on the gradually varied flow profile properties and the concept of transitional depths. Before starting the computations of gradually varied profiles and all, it is better to have some more theoretical back ground – what are the properties of the slope, water surface profile slope and all. For that, we will be going in a theoretical way in this class particular class.

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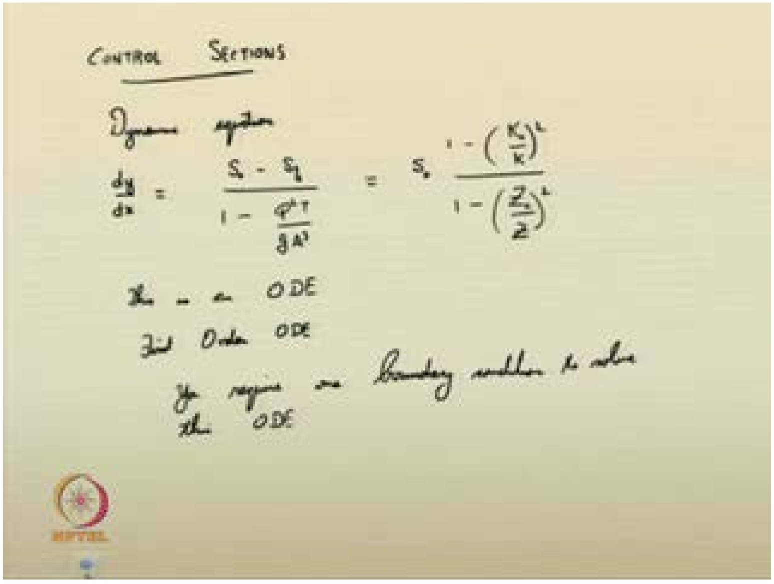

We will start with the concept called control sections. Can any one of you here tell me what is meant by control section or where the control section is huge, and what is meant by control section? If you recall the dynamic equation for water surface profile, we had given that as... Of course, with various assumptions, we have discarded the correction factors and all. This was the expression for the dynamic equation. This was again represented in terms of section factor and conveyance factors – 1 minus k n by k whole square by 1 minus z c by z whole square. These expressions – what do you understand from this? This is an ordinary differential equation in terms y. This is a first order ordinary differential equation. And of course, considering the section factors and conveyance factors and all, it is almost true that, it is non-linear; these expressions are non-linear in terms of y. So, this is a non-linear first order ordinary differential equation. And you can solve them using any solving procedures for the differential equations.

As this a first order this thing, you require one boundary condition to solve this ordinary differential equation. That is the mathematical basis. That is, mathematically, require one boundary condition to solve this particular differential equation. Now, according to the field situation, how will you determine this boundary condition? To begin or for solving this differential equation, we can start from the boundary condition in such a way that, you know all the type of the properties of that particular channel or the section.

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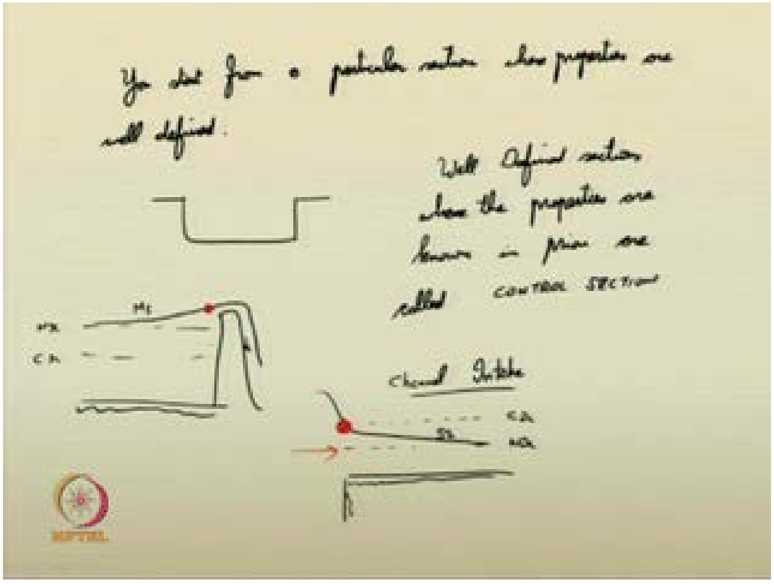

Or, you start... That is, you start from a particular section, the computation in such a way that, you know all the properties of that section; that is, what is the discharge, what is the various channel properties – all those things. And it is well-defined; where, properties are well-defined. For example, if you have rectangular weir; the section of the rectangular – the rectangular cross section – it is well-defined. And you know, if it is a steady discharge coming out from there, you know various properties in that section. So, you can use the rectangular weir as a control section. If you have spill ways or if you have (( )) gates; those sections are also usually used as control section. So, such well-defined sections, where the properties are known in prior are called control sections.

Usually, the control section... You start or you give the boundary condition as any of these control sections and you start the gradually varied flow computations. This is the normal procedure. If you... For example, if I construct a dam and a spill way is there; if this is the normal depth line, critical depth line; then the M 1 profile – it will go like this. This is the way discharge. This is the M 1 gradually varied flow profile. And the control section for of this gradually varied flow profile – to obtain this gradually varied flow profile, you start from the control section; and the control section is normally given – it is given as... In this particular case, it is given at this location, where the discharge and the section details are well-defined. And like that you compute.

Similarly, if there is a channel intake; for example, channel intake is there from a reservoir. And if the critical depth line is being shown like this; normal depth line; let us assume that this is having a super critical flow in the channel intake and the profile will be something of S 2. So, this is the S 2 gradually varied flow profile. And this S 2 gradually varied flow profile starts from a control section as shown here. So, this will be the control section now. This will be the control section. So, from there, you begin the computations or to determine the gradually varied flow profile in that intake; like that you can (( )) This is the flow direction.

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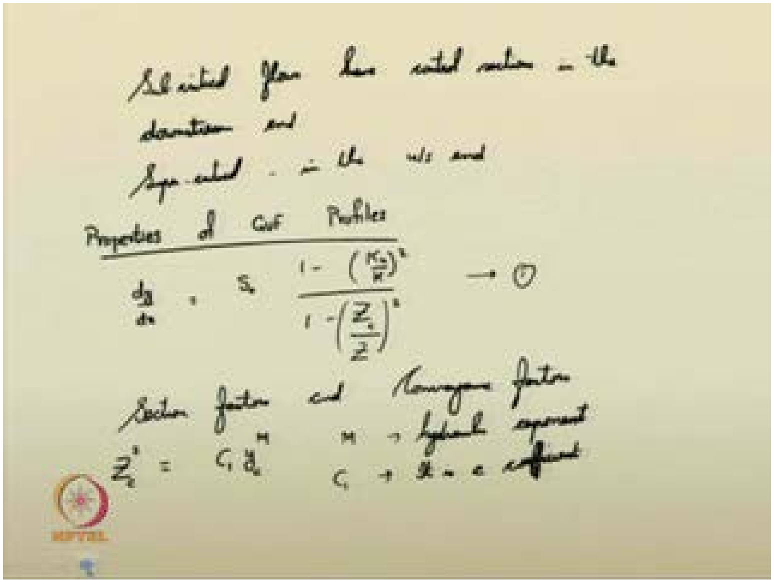

We have to note that, usually, the subcritical flows have control sections at the downstream of the profile; that is, the subcritical flows have control points in the downstream end as you have seen in the spill way example, say in the previous slide for the... Say this is the M 1 profile; it starts somewhere here and it goes like this. But the control section is at the downstream of this profile. Similarly, here this is the supercritical flow. And here the profile is shown like this and the control section is in the upstream of the beginning of the... So, subcritical flows have control sections in the downstream; and supercritical have control sections in the upstream end. This you can note it down.

Now, let us see what are the properties of these gradually varied flow profiles. It is the simple mathematical analysis or it is not a higher this thing; whatever you have studied in your high school level and... Also, that will help you in understanding the properties of the gradually varied flow profiles. Again, I am just writing down the dynamic equation – S 0 1 minus k n by k whole square – please note that this is k suffix n – by 1 minus z c by z whole square. In this equation, you can determine the various properties of the slope dy by dx. For that, just recall from module one and module two. Module one was based on the critical flows; module two was based on uniform flows. So, in those lectures and all, you have already dealt with section factors and conveyance factors. So, from module two or... Let us go back into the first module. From module one, you had studied that, at the critical section, the section factor z c – it can be given as some coefficient C 1 in to the critical depth raised to M. This was dealt at that time. It was clearly explained to them; where, you studied that M is the hydraulic exponent. It is a hydraulic exponent. z c is the section factor at critical flow; y c is the critical depth; C 1 – it is a coefficient.

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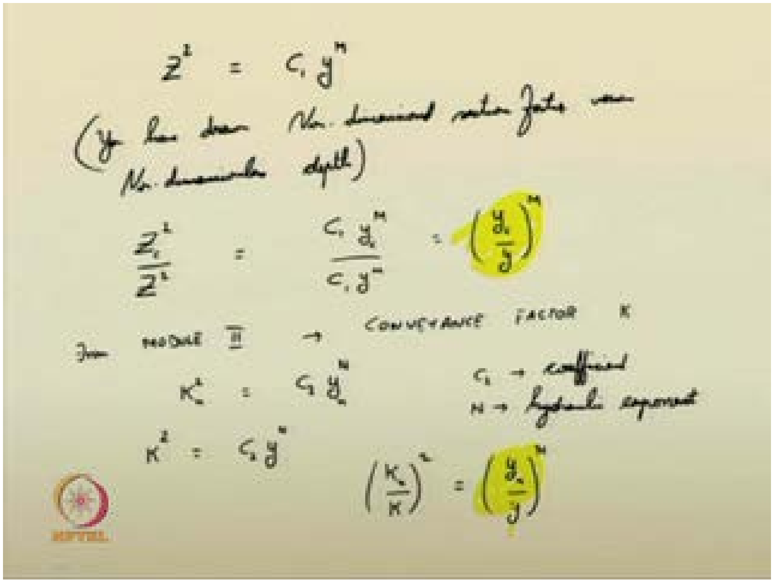

Again, for any flow, if it is not a critical flow or that (( )) still you can suggest the section factor z square – this is equal to again C 1 into y to the power of m. If you have drawn non-dimensional section factor versus non-dimensional depth; these graphs and all were drawn; you have been taught also how to draw these graphs and all. Just recall those portions again. Now, we can see that, from the two relationships: z c square by z square – this can be given as C 1 y c to the power of M by C 1 y to the power of M. This is nothing but y c by y to the power of M. From module two, you had been taught about the conveyance factor, which was defined as k n square; that is, for the normal flow, k n square is equal to C 2 y n to the power of capital N; C 2 is a coefficient; and N is another hydraulic exponent. So, this is hydraulic exponent for conveyance factor. For any flow, even if it is not a normal flow, you can write the conveyance factor as k square is equal to C 2 y to the power of n. So, from these two relationships, you have the following ratio: k n by k whole square is nothing but equal to y n by y to the power of N. So, based on these two relationships, there is this one as well as this one.

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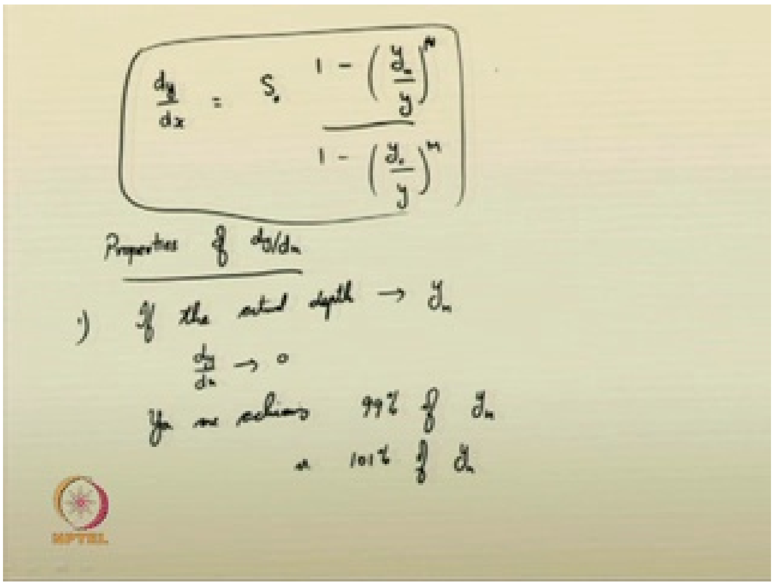

Substituting them, you will get in the dynamic equation; dy by dx is equal to S 0 into 1 minus y n by y whole to the power of N by 1 minus y c by y whole to the power of M. So, this gives you means... Now, it is clearly understood by... I hope you are able to understand that. This equation clearly represents the first order differential equation in y and the terms of y are non-linear. So, it is a non-linear differential equation – non-linear first order differential equation. So, you can solve to obtain the gradually varied flow profile. So, let us describe the properties of dy by dx. Let me ask you. Just recall this equation from this particular equation, what is inferred or what can you infer from this thing. If the actual depth of the flow – if it is approximately equal to the normal depth of the flow, then what happens? dy by dx; or, if the actual depth tends to normal depth, then dy by dx tends to 0; that is, the water surface profile becomes horizontal there. It becomes parallel to the bed.

What happens now? In the gradually varied flow computations and all, you have to terminate the computations of gradually varied flow; that is, you have to terminate the gradually varied flow computations in such a way that you are achieving either 99 percentage of normal depth if you are starting from a supercritical condition or 101 percentage of y n when you start from subcritical conditions. So, like this you have to think on that. Why? Because now, if the actual depth – they are tending to normal depth, there dy by dx – it is zero; and normal depth phenomenon – it is only theoretically possible conditions. So, you have to compute...

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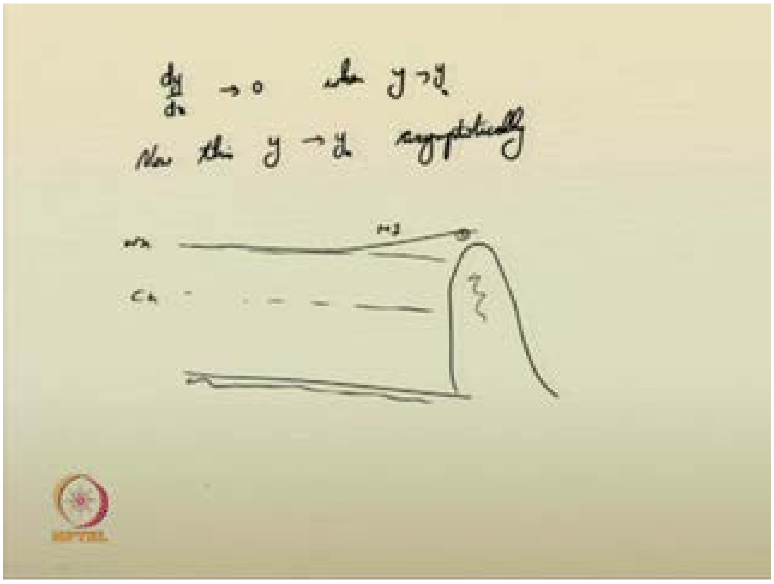

As we mentioned earlier, dy by dx tends to 0 when y tends to y n. Now, this y tends to y n is possible or it reaches or becomes y n asymptotically. So, you will require a very large length. Say if you are starting say in this particular case, you have such an obstruction dam, critical depth line, normal depth line. And if you want to compute the M 1 profile, this M 1 – this gradually varied flow profile reaches the normal depth asymptotically. So, theoretically, according to the mathematical case, it will reach only at an infinite distance. So, that is not feasible for us. We cannot spend that much time and it is not at all possible. So, we stop the computation whenever you have reached, say in this particular case from the control section, if you have reached 101 percentage of y n, then we stop; we suggest that the depth has reached the normal depth and the gradually varied flow profile is completed, like that. Similarly, in the supercritical flow situations, one can achieve up to 99 percentage of normal depth and suggest that the normal depth has been achieved; like that you can solve.

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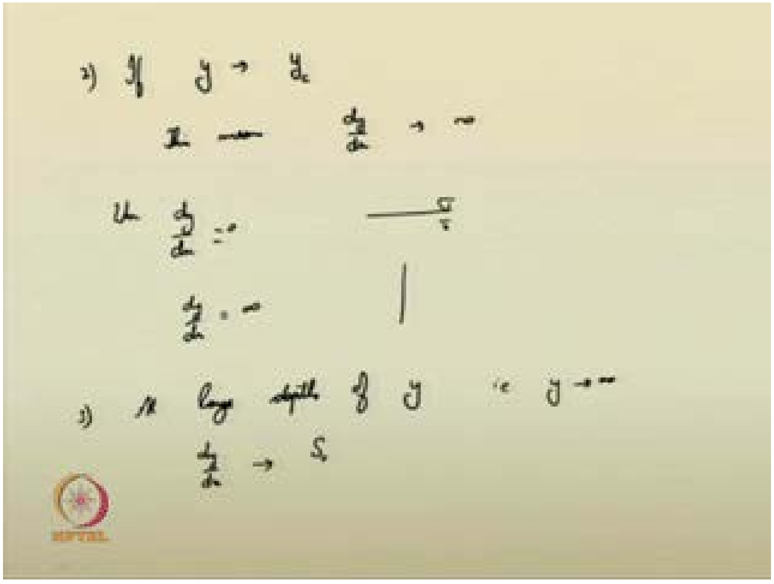

Another situation – if the actual depth tends to the critical depth of the channel, what happens then? This means dy by dx tends to infinity; or, dy by dx tends to infinity – what is the physical inference of this particular term? When dy by dx is equal to 0, the water surface is almost horizontal. When dy by dx is equal to infinity, the water surface – this becomes almost vertical. And the vertical state of water surface – it is not at all possible in the nature. So, it is only... Whenever it reaches critical state and all, water surface becomes unstable and tries to achieve either the supercritical or subcritical condition as soon as possible. Now, what happens at large depths of y? At large depths of y, that is, y tends to infinity, dy by dx tends to become equal to the bed slope, S 0; and also, it is approximately horizontal. So, at last depth (( )) you can note that, the dy by dx is almost same as the bed slope.

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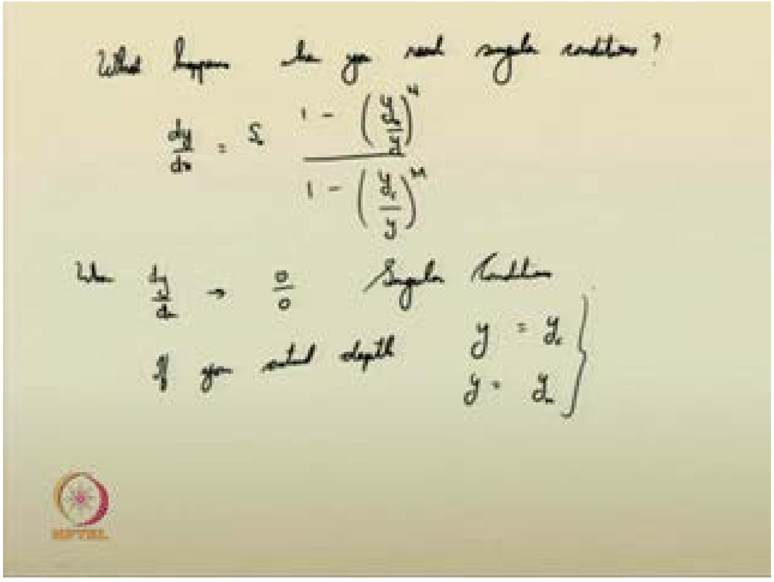

What happens when you reach singular conditions? Anyhow I am not going to explain in detail various singular conditions just for your benefit; that is, dy by dx is equal to S 0 into 1 minus y n by y whole to the power of N by 1 minus y c by y whole to the power of M. In this particular equation, when dy by dx have the form 0 by 0, this is called singular conditions. When do you think that it becomes singular? This condition becomes singular if the actual depth y is equal to the critical depth as well as y is equal to the normal depth. So, when the flow depth is in this situation, then the singular condition arrives and dy by dx cannot exist in those situations. So, we have to use alternate theory or we have to try to infer subsequent things from this thing.

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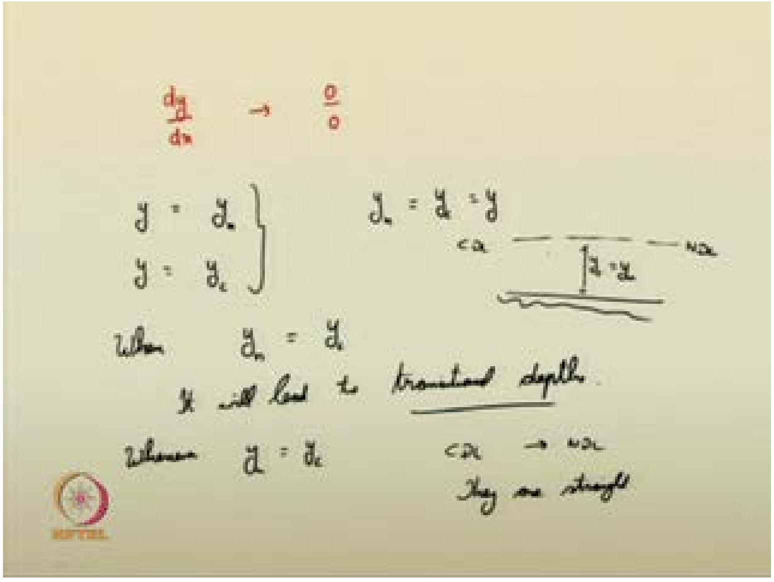

Singular condition occurs when... As mentioned earlier, dy by dx – when it tends to 0 by 0, then the singular condition exists. In that situation, what you have to do is... And this situation arises when the actual depth y is equal to the normal depth; and also, the actual depth y is equal to the critical depth when these two conditions satisfies; that means, the normal depth is also equal to the critical depth and this is also equal to the actual depth. If such situation arises, then it becomes a singular thing and it will give rise to various things. So, during that singular condition, what are the types of depth? How can you compute the gradually varied flow profile and all? We have to understand further phenomenon in that.

Now, just forget about the actual depth of the water when for a given channel or when for the given conditions, if the normal depth is equal to critical depth; that is, in that channel, both the normal depth line and critical depth line – if they are same; y c and y n – if they are same, then what happens? Due to y c and y n becoming same, it will lead to a concept called transitional depths. Due to this, that is, the normal discharge as well as the critical discharge, if it same for a particular channel for a given condition, then that will yield transitional depths. So, what are the transitional depths? That is, whenever y n is equal to y c, the corresponding profile will give you the transitional flow profiles. This is further prismatic channels; the critical depth line as well as normal depth line – they are straight. For the prismatic channels, the normal depth line as well as the critical depth line – they are straight. It is a type of profile. Now, in the transitional flow profiles, there will be a particular... A singular point will pass through these transitional profiles whenever it is required; that is, when the flow – gradually varied flow profile if you are trying to compute and all, it has to pass through this singular point.

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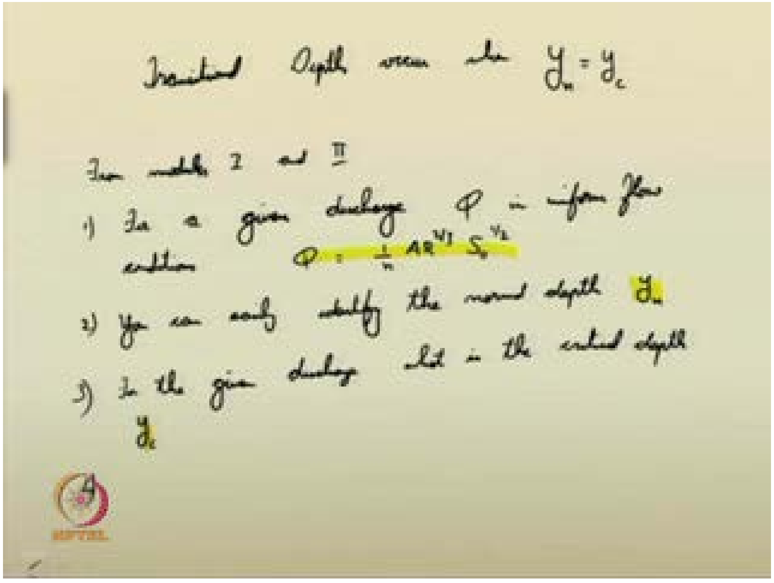

Let me make it clear before that is that transitional... Just I have tried now; just make it that. You just note it down that, the transitional depths occur when normal depth of flow is equal to critical depth. What do you mean by that? From module one and module two – from modules 1 and 2, you just recall that, for a given discharge Q in uniform flow conditions, it can be computed using Manning's equation or Chezy's equation, whatever be; let us use the Manning's equation. So, Q is equal to 1 by n AR to the power of 2 by 3 S naught to the power of half. You know what the terminologies in this equation. Like this you can compute the uniform flow.

You can easily identify the normal depth y n from this equation; say if a steady discharge is given to you – some value of Q; based on the Manning's equation, you can compute the normal depth y n. Once you compute the depth y n for the given discharge, what is the critical depth y c? That can also be computed. You can also compute the critical depth y c. Just recall; in the rectangular channel, what was the critical depth; how the critical depth was measured – y c is equal to... Very easy formula; I am not going to again repeat it here. We will see in another example. For the same discharge Q, if you subsequently... Now, what happens is that, let me state it in a following way.

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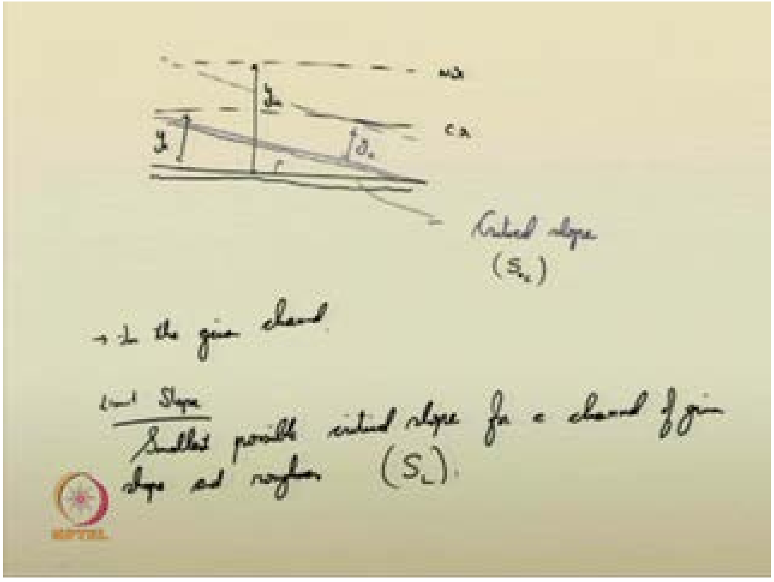

This was the normal depth line; this is the critical depth line; or, that is, this is for the uniform flow. This is the uniform flow with the following normal depth occurring in the channel. If you now increase; that is, I will change the colour and show to you. If you change the bed slope of the channel, what will happen? If you change the bed slope of the channel, you will see that, whatever normal depth was there earlier – it slowly starts... means whatever depth or water depth was there existing – it slowly starts reducing. And it will reach such a stage that, at a particular situation, say if I raise the bed slope from here to here like this; now, the depth of flow will become like this. And whatever depth of flow is there – y c – that will be the actual... Whatever flow depth is there, that will become a critical depth also. And such a slope is called critical slope. This was also explained to you earlier. The critical slope means when you tilted the bed of the channel in a such way that you tilted it further; means you increase the slope of the bed in such way that for the given discharge, whatever discharge is there, that discharge occurs in a critical way. If that happens, then that particular slope is called critical slope. So, I can give the critical slope as S 0 c.

Now, for the given channel, whatever possible – means the limit at which you can raise the bed of the thing; if you raise the bed of this thing – bed of the channel beyond this thing, then supercritical flow occurs. If it is lower further, subcritical itself will be prevailing. Like that you are aware now. The limit for this particular channel at which the critical slope exists, that is called limit slope. The concept of limit slope is given there, is defined in such way that, smallest possible – it is the smallest possible critical slope for a channel of given shape and roughness. For given shape and roughness, the possible critical slope of a channel is called the limit slope. We generally give it as S L and all.

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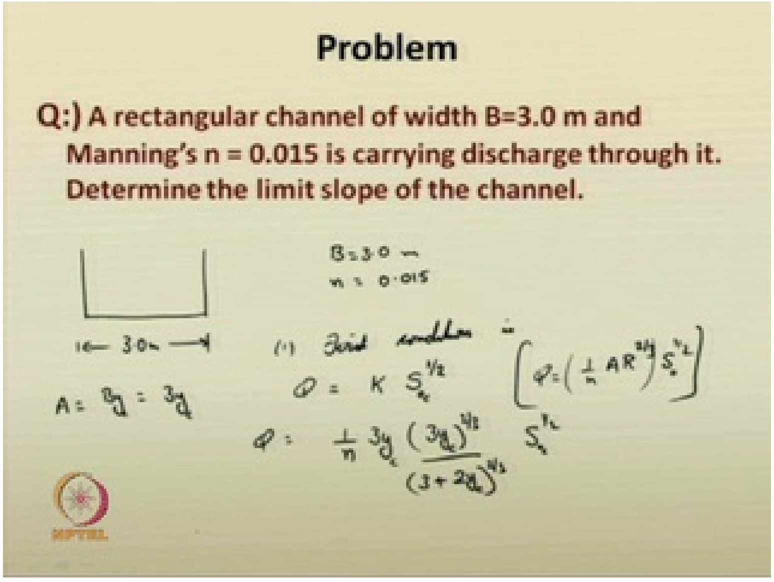

Let us do one example. And with this example, I think it will be more clear for you. Let us do that. I am giving you a problem – a rectangular channel of width B is equal to 3 meters; say a rectangular channel of width 3 meters. It is carrying a steady discharge; its Manning's surface coefficient is 0.015. Determine the limit slope of the channel. So, B is equal to 3.0 meters; manning's n is equal to 0.015. What could be the limit slope? You know that, limit slope – it is a critical slope. Critical slope means both the normal discharge as well as critical discharge are same in such type of conditions. What is the minimum possible slope for this particular rectangular channel in which such a condition exists? That you have to determine. First, you have to...

The first condition we can suggest is, the discharge cube – this is equal to the conveyance factor. And now, the slope – it is a critical slope. We have already suggested that, the limit slope itself is a critical slope. So, the critical slope at which this thing occur – means the minimum possible critical slope is called the limit slope. So, you can measure Q in such a way that this is equal to k S 0 c to the power of half according to the theory. I hope you are well aware of that, Q is equal to 1 by n AR to the power of 2 by 3

S naught to the power of half is the Manning's equation. And now, in this condition, S 0 is a critical slope. So, that is why, I wrote it S 0 c. This particular quantity is k. So, I have written it here. So, I can write Q is equal to 1 by n; and A is equal to B y is equal to 3 y. So, this is a critical depth as well as normal depth; you can also suggest like that. I am just writing it as c – 3 y c to the power of 2 by 3 by 3 plus 2 y c to the power of 2 by 3 S naught c to the power of half. Like this you will get the relationship.

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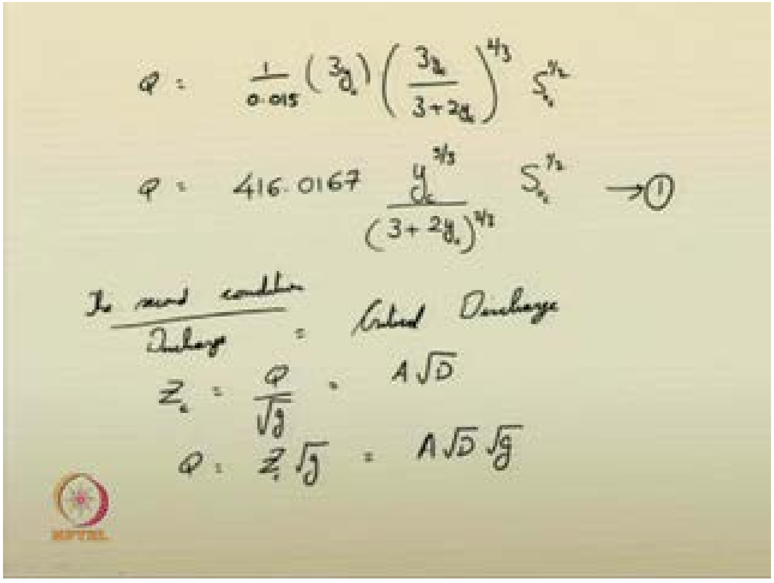

Substitute n is equal to 0.015. You will get Q is equal to 1 by 0.015 3 y c into 3 y c by 3 plus 2 y c whole quantity raised to 2 by 3 S naught c to the power of half. Substitute the terms. You will get Q is equal to... I have just computed everything; 416.0167 y c to the power of 5 by 3 by 3 plus 2 y c raised to 2 by 3 S naught c to the power of half. So, you do not know the Q and you also do not know the slope S 0 c in this equation. So, let me give this as equation number 1. Now, just go to the second condition. The second condition suggests the given discharge is equal to critical discharge. So, from module one, you just recall them. You had defined section factor z c; z c is equal to for critical conditions, Q by root g. This is same as A into root of D. So, from this relationship, you know Q is equal to z c into root g. This is nothing but A root D into root g. Substitute the quantities.

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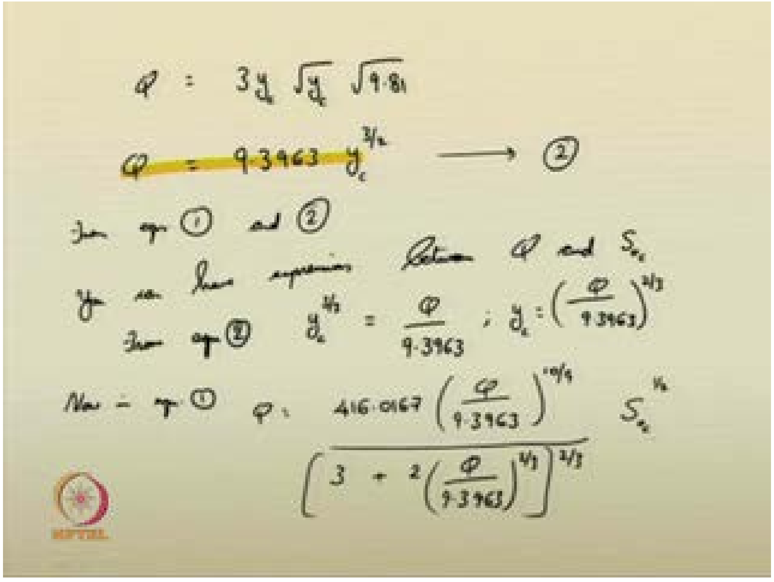

You will see Q is equal to 3 y c root into root of y c into root of 9.81. This is equal to 9.3963 y c to the power of 3 by 2. So, I can write Q is equal to 9.3963 y c to the power of 3 by 2. So, this becomes equation 2. So, from equations 1 and 2; that is, you recall equations 1 and 2. This is equation 1 and this is equation 2. So, compare the both equations. From the both the equations, you can eliminate now the critical depth y c and you can have relationship, you can have expressions between discharge Q and critical slope S 0 c. Like this, we can eliminate y c from those expressions. So, when I did that, I got the following thing; that is, from equation 2, I got y c to the power of 3 by 2 is equal to Q by 9.3963; or, this will give you y c is equal to Q by 9.3963 to the power of 2 by 3. Now, in equation 1, you substitute this y c – expression for y c. You will get Q is equal to 416.0167 Q by 9.3963 to the power of 10 by 9 divided by 3 plus twice Q by 9.3963 raised to 2 by 3 this whole quantity raised to 2 by 3 – this multiplied by S naught c to the power of half. Like this, you are now getting an implicit expression for Q and S naught c. You will see them. You can just rearrange them.

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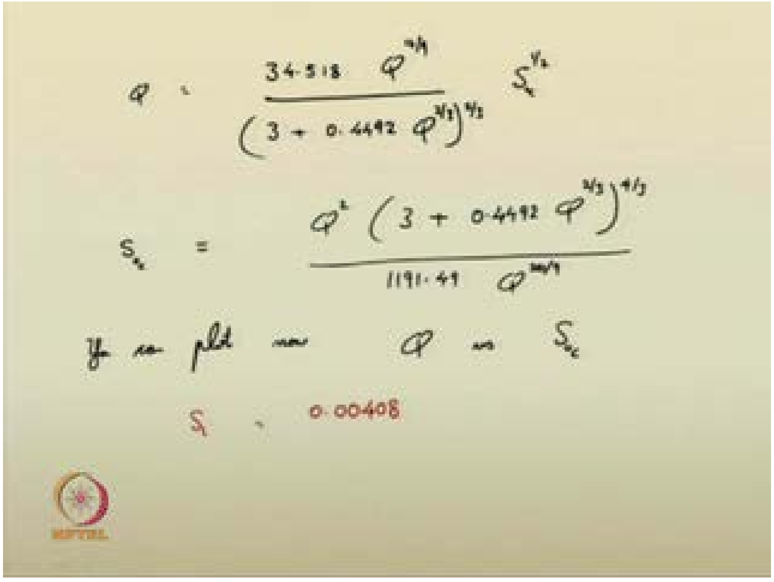

You will see that, either you can write Q is equal to – after computing the terms, 34.518 Q to the power of 10 by 9 by 3 plus 0.4492 Q to the power of 2 by 3 the whole quantity raised to 2 by 3 S naught c to the power of half. You can either write like this or just rearrange the terms; means this equation can be rearranged as S naught c is now equal to Q squared into 3 plus 0.4492 Q to the power of 2 by 3 whole to the power of 4 by 3 – this entire quantity like that; this divided by 1191.49 Q to the power of 20 by 9. Like this you are getting the terms. So, you can plot Q versus S 0 c. So, for various values of Q, what could be the critical slope? Like that you can slot.

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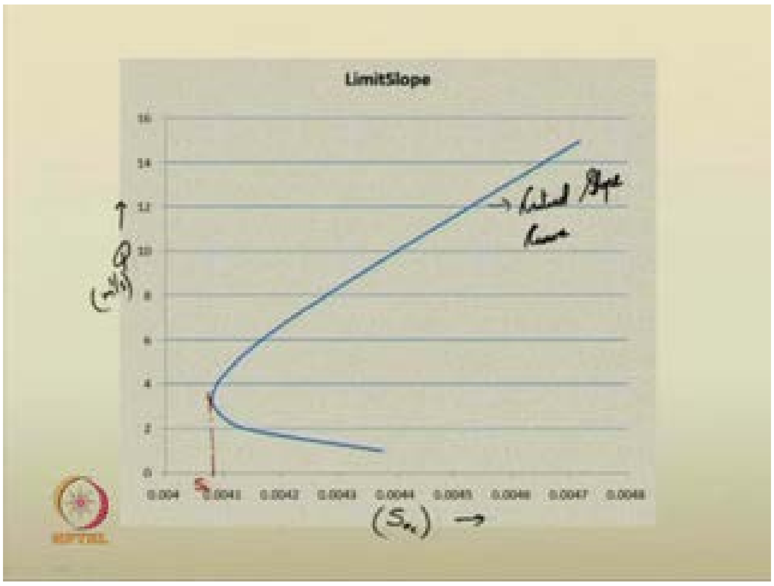

I have plotted them and I am just showing it here. This is the curve I got in this particular form. So, this is the discharge Q in meter cube per second for the given channel conditions for the rectangular. This is the slope – critical slope. This entire curve is now the critical slope curve. So, why I am showing it here? This curve will be useful for when further try to understand the gradually varied flow profile. So, I am repeating in front of you that, you just recall; just remember this particular curve, this particular shape of the curve. We will be dealing with this particular curve later also. So, this is how you compute the critical slope curve. Now, from this curve, it is quite obvious now, what is the... Say this particular portion, whatever is the magnitude, this is giving you the minimum critical slope for the given rectangular channel. So, this is called the limit slope S L. So, the limit slope S L – I got from the curve is 0.00408. So, this is how I got the limit slope.

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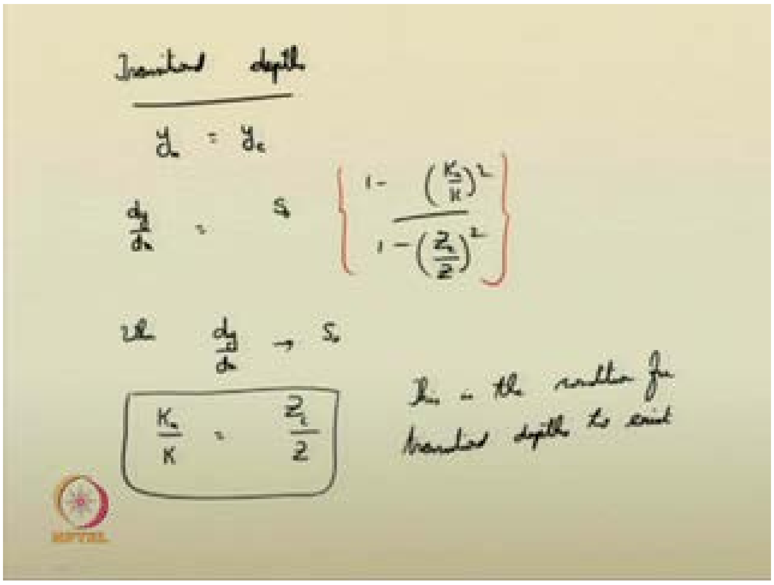

Now, let us come back into the transitional depths. When you are at transitional depth, y n is same as the critical depth y c. Therefore, in the dynamic equation d y by d x – S naught into 1 minus k n by k whole square by 1 minus z c by z whole square. So, in this equation, when dy by dx tends to S 0, what happens? When dy by dx tends to S 0, this entire quantity, that is, this entire quantity – this tends to 1. So, when dy by dx tends to S 0, this entire quantity now will tend to become 1; that means, the k n by k condition will be equal to z c by z condition; that is, the slope will be approximately equal to S 0 in such situation. This is the condition for transitional depths to exist; that is, transitional depth exists when k n by k is equal to z c by z. So, just note it down.

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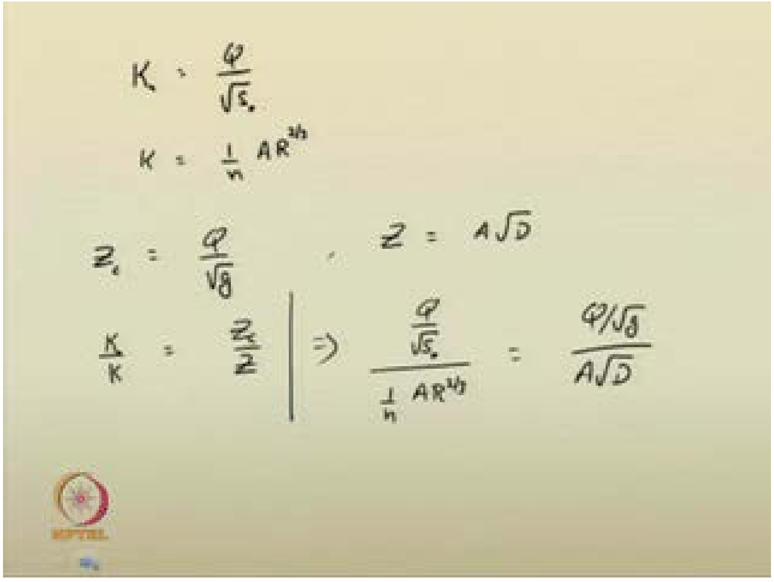

Again, let me repeat it; k n is now equal to Q by root S naught for the normal flow. And also, k is equal to 1 by n AR to the power of 2 by 3. Similarly, the section factor for critical flow is equal to Q by root g. Refer module one. And the simple section factor z is equal to A root D. So, this is the section factor for critical flow; this is the section factor for the actual flow. This is the conveyance factor for normal flow; and this is the conveyance factor for the actual flow – actual depth existing. So, the ratio k n by k is equal to z c by z can be written as... This can be written as Q by root S naught by 1 by n AR to the power of 2 by 3. This is equal to Q by root g by A root D; where, D is the hydraulic depth. I hope you recall what is the term – hydraulic depth.

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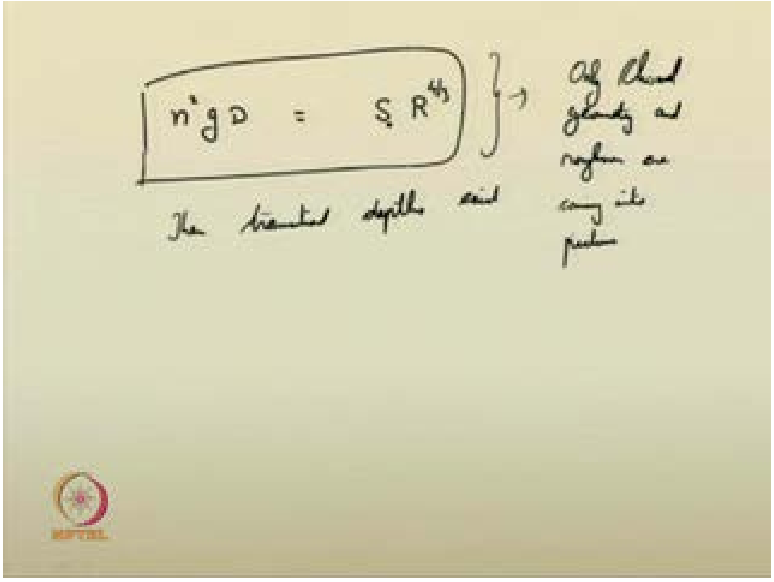

Rearrange the terms here. What will I get? I will get finally, particular relationship n square g D is equal to S naught R to the power of 4 by 3. Please note this relationship. If this relationship exists, then transitional depths exist. Therefore, in such situation, what happens? So, you can compute the transitional depths. Now, what do you infer from this relationship? Only channel geometry and roughness is playing a role for computing the transitional depth for the given slope only. You can see that only channel geometry and roughness are coming into picture. So, that is the discharge. Whatever discharge is being given to you; that is not affecting your transitional depths and all. So, that way, one can clearly infer the things. So, we will continue the portions related to this further in the next class. As we are not able to complete this entire portion in this thing, there are few more minutes of transitional depth theories and all left. So, I will continue it in the next class.

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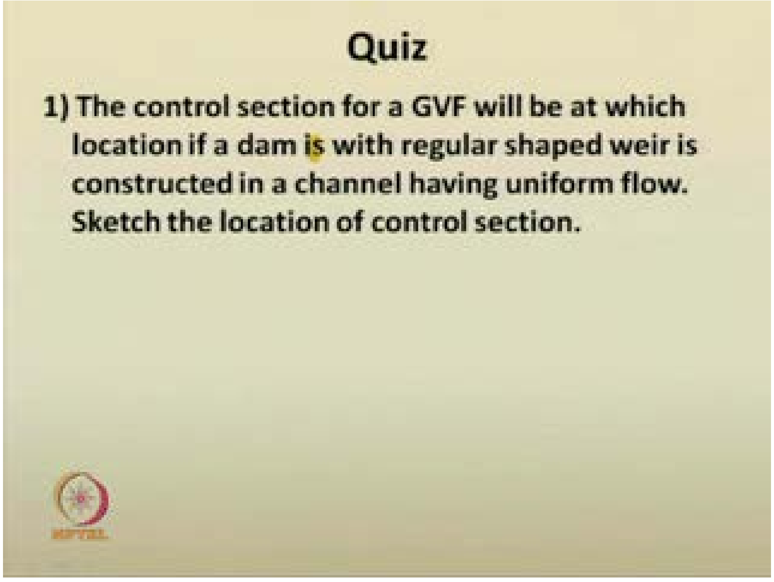

Today's quiz – the first question to you is, the control section for a gradually varied flow will be at which location if a dam with regular shaped weir is constructed in a channel having uniform flow. Sketch the location of the control section. I just want to erase this particular portion – this is not is; if a dam with regular shape weir is constructed in a channel having uniform flow. Sketch the location of the control section.

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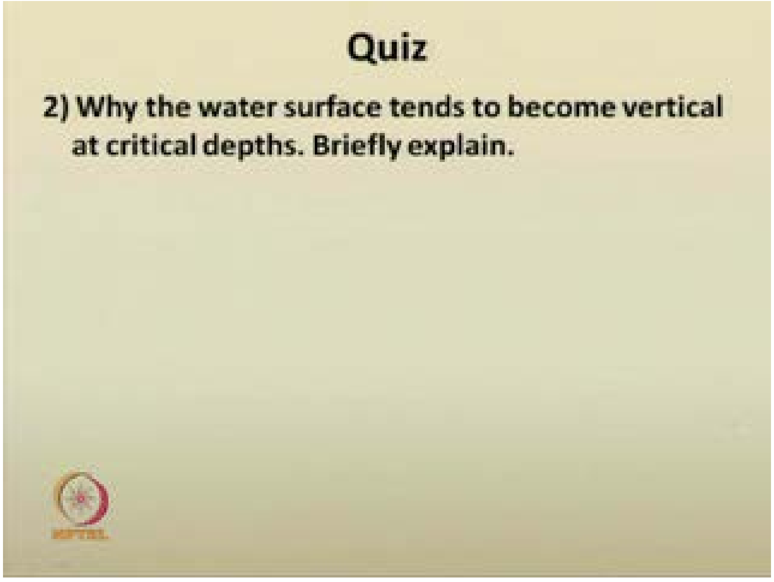

Your second question for the quiz is why the water surface tends to become vertical at critical depths? Briefly explain. Why the water surface tends to become vertical at critical depths? Briefly explain.

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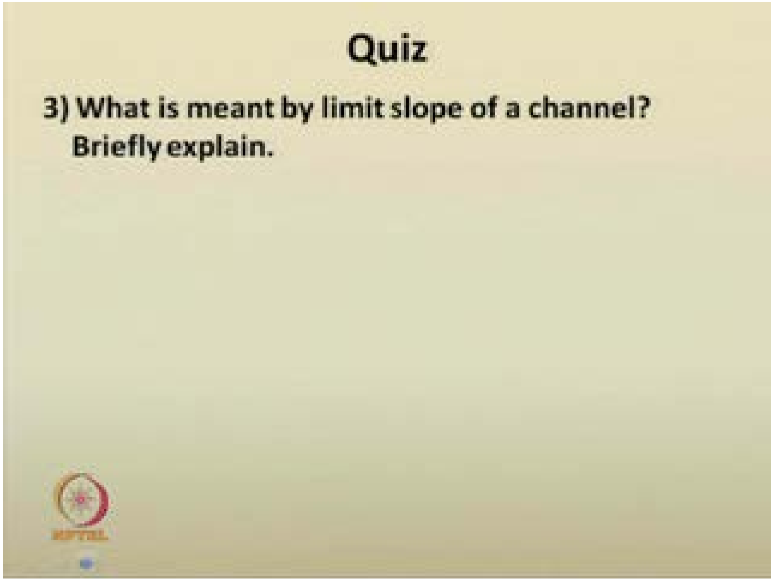

Third question is what is meant by limit slope of a channel? Briefly explain. So, these three are the questions.

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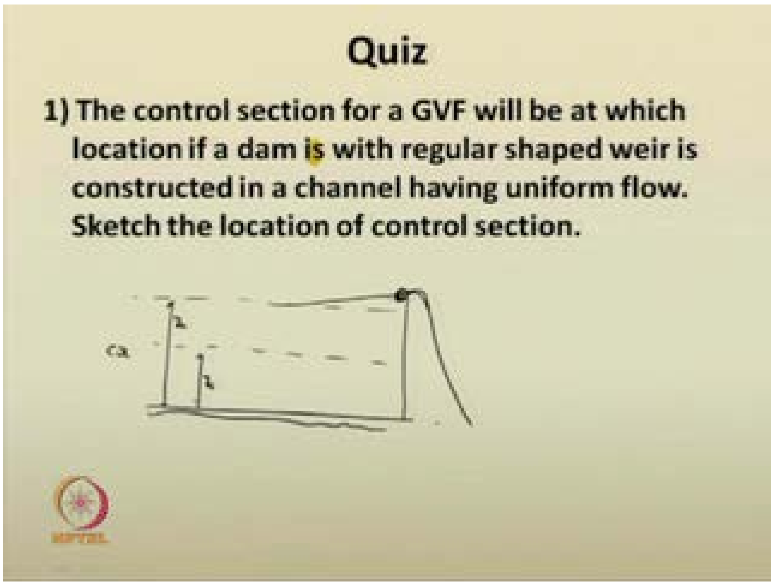

The solutions for today’s quiz – for the first question, you are asked, the control section for a gradually varied flow if a dam is constructed, and we are... Let us assume that, the normal depth is greater than the critical depth. In that condition, you will be having an M 1 profile like this; and the control section is somewhat here. So, that is just above the top of the spill way; where, if you are constructing a weir of regular shape and all, that will give you the control section. So, that is the control section.

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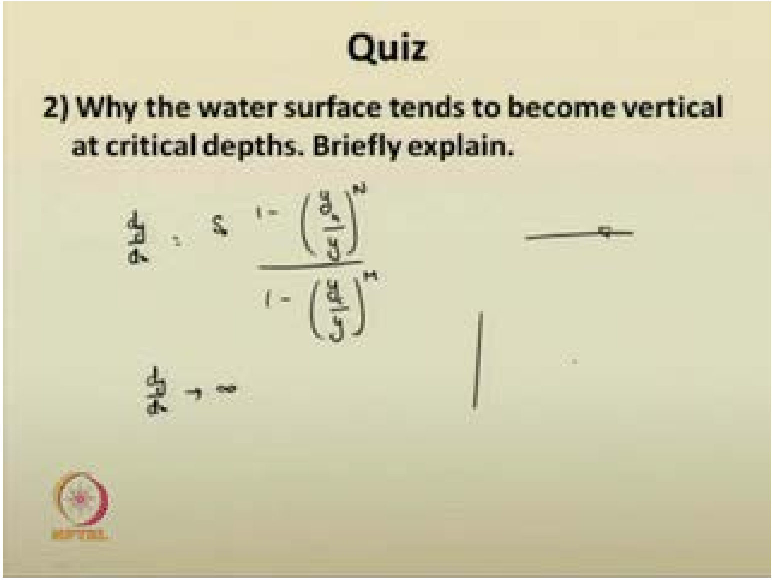

Your second question – why the water surface tends to become vertical at critical depths? Briefly explain. Just recall the dynamic equation – S 0 into 1 minus y n by y whole to the power of N by 1 minus y c by y whole to the power of hydraulic exponent M. You can write the dynamic equation in the following way also. Why the water surface tends to become vertical at critical depths? At critical depth, y is equal to y c. So, this quantity – dy by dx then tends to infinity. So, at infinity, dy by dx is equal to infinity means it is a vertical line. dy by dx is equal to 0 means it is a horizontal line. So, that is why, water surface tends to become vertical at critical depths.

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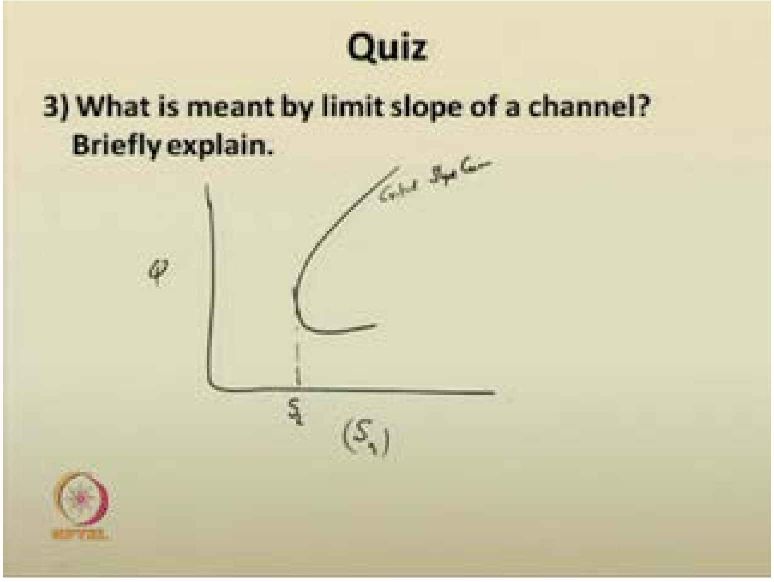

What is meant by limit slope of a channel? We had already drawn that. We have suggested that, for a given condition, for given channels sections, for given channel properties and all, the minimum possible critical slope... You can draw the critical slope curve like this. This is the critical slope curve. So, the minimum possible critical slope for that channel; that is called the limit slope S L. I have already explained that in the class. So, this way we are concluding today's lecture. In the next lecture, we will continue some of the portions left in this topic and then we will start the computations of gradually varied flow profiles.

Thank you.