Advanced Hydraulics
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Module - 3
Varied Flows
Lecture - 4
Classification of Gradually Varied Flow Part 2

Welcome back to our lecture series on advanced hydraulics. We are in the third module of varied flows. Last class, we had briefly mentioned on the gradually varied flow and how to classify it. We have not completed the portion on classification of gradually varied flow. So, we will continue that classification of gradually varied flow in today's lecture.

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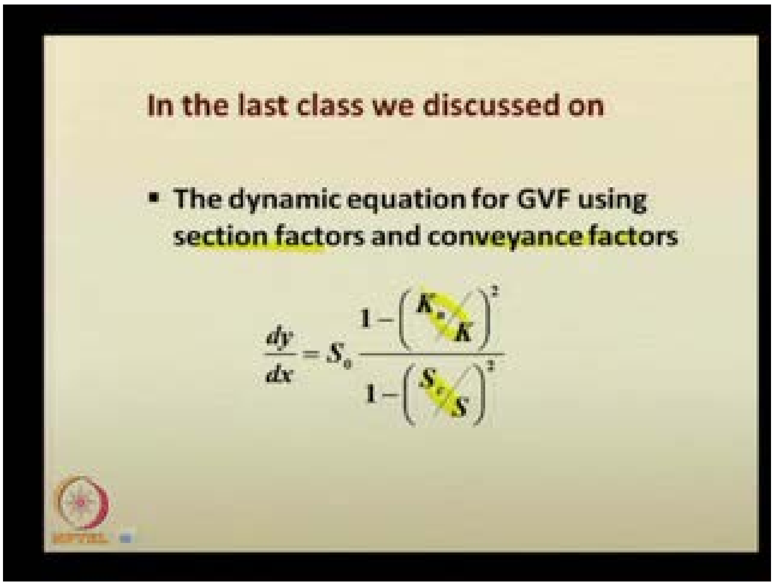

If you recall in the last class; in the last class, we suggested about the dynamic equation for gradually varied flow using section factors and conveyance factor. So, I hope you recall them – the section factors and the conveyance factors. This is your conveyance factor – this; and these are the sections factor for critical flow as well as for non-critical flow. All those things we had described it in the last class. In this way, you can describe the dynamic equation for gradually varied flow. So, the dynamic equation – it gives you the slope of the water surface profile.

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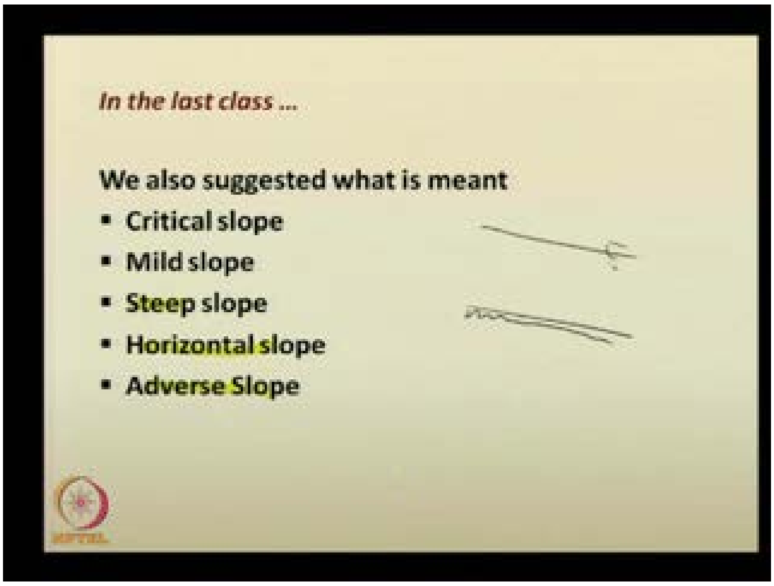

We also suggested you, what is meant by critical slope; that is regarding the bed slope of the channel. If you recall the bed slope of the channel; for any channel, the bed slope – it can be considered as a critical slope; it can be considered as a mild slope; also, it can be considered as a steep slope, horizontal slope, adverse slope. All this things we have described it in the last class.

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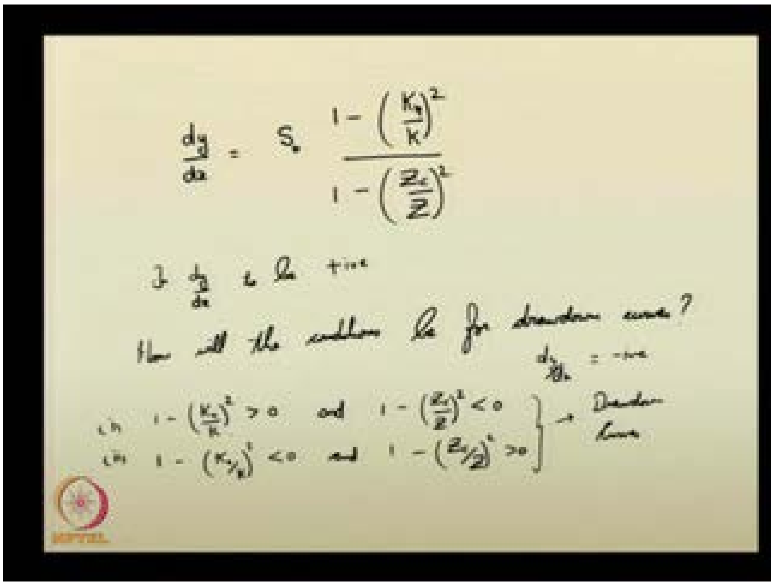

Today, we will see what are the other things. Especially in the last class, we suggested you, in the dynamic equation, S naught into 1 minus k n by k whole square divided by 1 minus z c by z whole square. So, these things in this equation, for dy by dx to be positive; the criteria for that we had discussed in the last class, that is, for the back water curves, how the new… If the numerator is positive, then the denominator also has to be positive; or, if the numerator is negative, then the denominator also has to be negative. Like that those things we had described it in the last class.

Now, in today's lecture, we will see how will the conditions be for the drawdown curves. You can simply answer them. Just recall the last class thing, how we described it for the back water curves. Similarly, for drawdown curves, what are the conditions? That is, you require dy by dx is equal to negative. Therefore, the conditions are 1 minus k n by k whole square – this should be greater than 0; as well as 1 minus z c by z whole square – this should be less than 0. This is one criterion. Similarly, another criterion is 1 minus k n by k whole square – this should be less than 0; and 1 minus z c by z whole square – this should be greater than 0. If these two conditions are there, then you will get drawdown curves – the gradually varied flow. For the gradually varied flow, the water surface profiles will give you drawdown curves. So, these… Then what are the physical meanings of these conditions?

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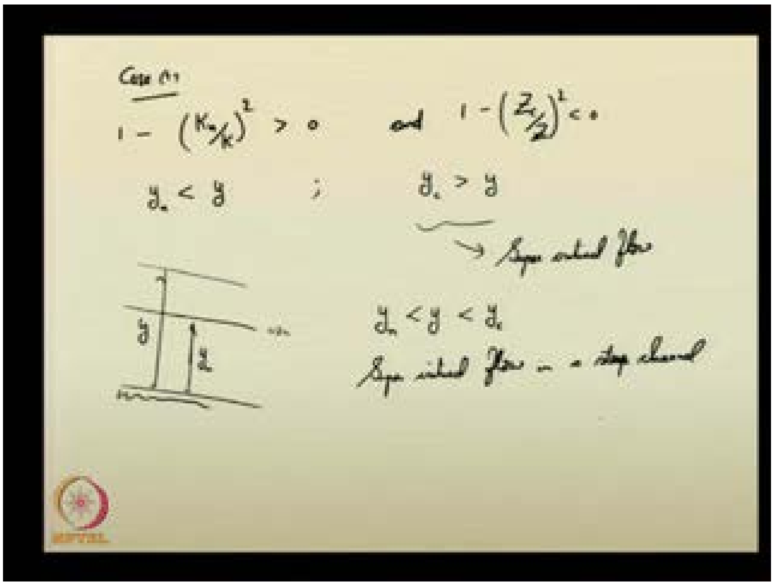

Let us consider the case 1. The case 1 is say, 1 minus k n by k whole square – how did I write the in the last slide? That is, 1 minus k n by k whole square greater than 0; and 1 minus z c by z whole square should be less than 0. What does that this mean? So, k n by k whole square; that is, 1 minus k n by k whole square – this quantity has to be greater than 0. That implies that, the normal depth of flow will be less than the actual depth of flow, y. See in the channel, see if this is the normal depth line; then the flow may be somewhere here; this y may be greater than… This is y; this is y n. Not only that; what does this mean here? 1 minus z c by z whole square should be less than 0; that is, this quantity z c by z – it should be greater than 1.

Then only, this can be become negative. So, what does that mean? y c is greater than the depth of flow. So, this implies… What does this imply? It implies super critical flow; that is, the flow is now super critical. Also, let us suggest that, y n is less than y; y c is greater than y. So, y n less than y less than y c – this will be the condition of the flow and this gives you super critical flow in a steep channel. It gives you super critical flow in a steep channel, why? This is a super critical flow; y n is less than y c. Therefore, this is a steep channel.

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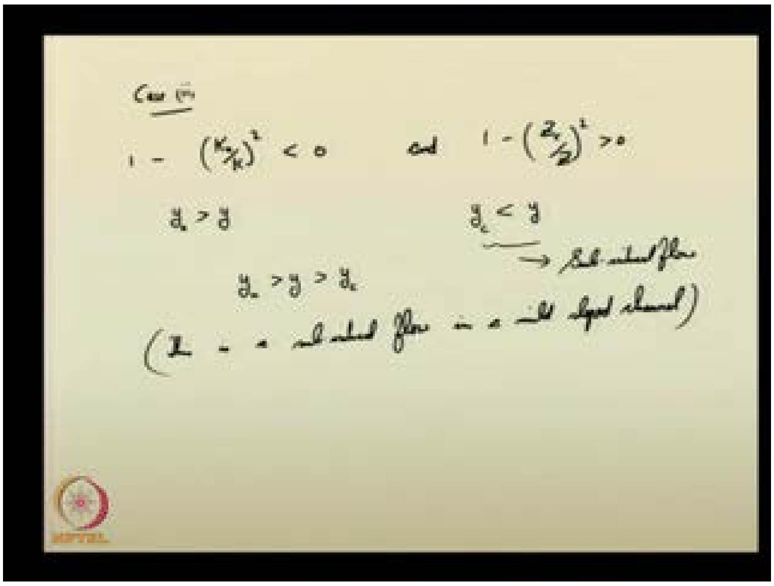

Similarly, in the case 2, one can easily elaborate it. So, 1 minus k n by k whole square – this should be less than 0; and 1 minus z c by z whole square – this should be greater than 0. So, the condition arrive whatever is coming into the picture here is that, the normal depth is now greater than the actual depth of flow. Then only, this quantity k n by k can become greater than 1. And from here, 1 minus z c by z whole square greater than 0, the condition is that, the critical flow should be less than the actual flow depth. So, what does this imply? This gives you that, y n is greater than y, which itself is greater than the critical flow. So, this is, if y c is less than y, it automatically gives you a subcritical flow; and also, y n is greater than y c. So, this is a subcritical flow in a mild sloped channel. Like this, you can describe the flow conditions. This is how you give the… means you can infer from the relationships given in the dynamic equations. So, just recall the various bed slopes. We mentioned in the last class about the critical slope, mild slope, steep slope and all. So, we will just…

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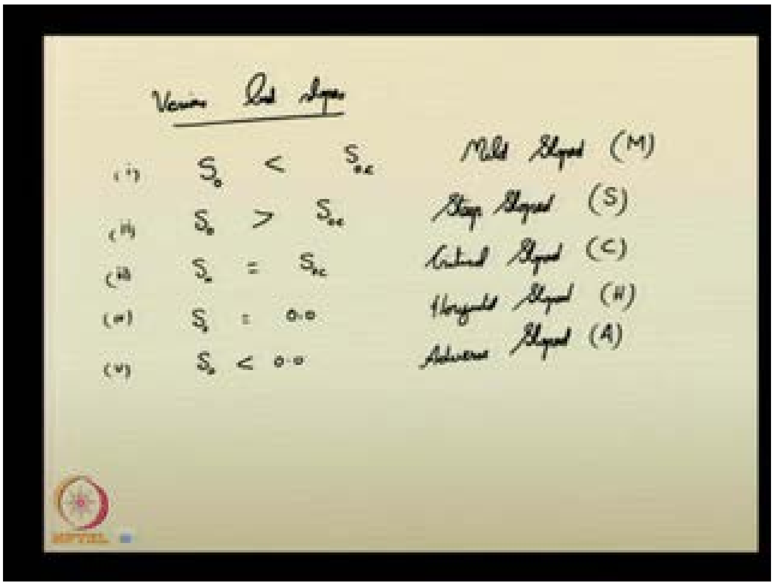

I will again enumerate it the various bed slopes. If the actual bed slope S 0 – it is less than the critical slope; then this is called mild sloped channel. And we give the abbreviation later – M for mild sloped conditions. So, if S 0 – it is greater than the critical slope, this is called steep sloped channel and we give the abbreviated representation S. If the actual bed slope is equal to the critical slope; then such bed sloped channels are called critically sloped channels; we give the symbolic letter C. If the actual bed slope is 0; that is, there is no slope at all; they are called horizontally sloped channels, represented by H. And if the bed slope is less than 0, they are given as adverse sloped channels as A. So, now, these bed conditions are given.

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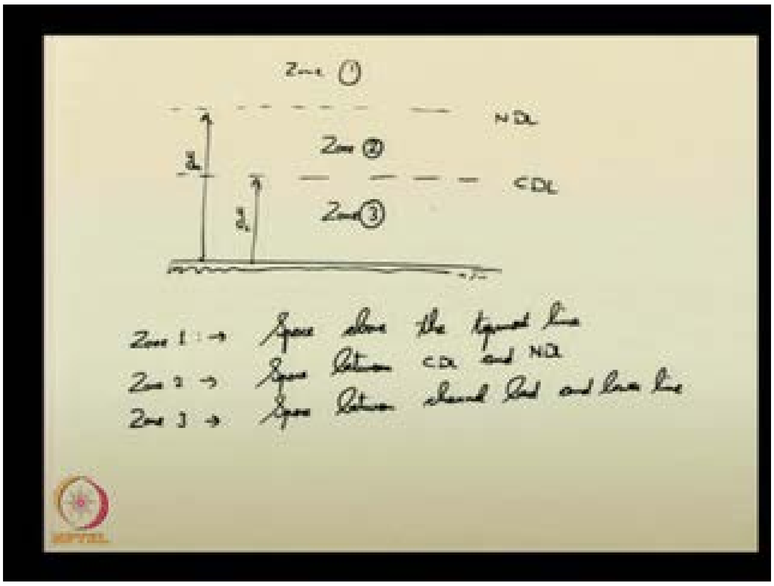

And, for any given flow situations, one can easily give three zones. Say if this is the channel bed; say for the given discharge, if this is the critical depth; for a considerable reach, you can draw the critical and depth line. So, that critical depth line is the… And if the normal depth line is given like this; then this depth is the normal depth; this depth is the critical depth. And if you see in such channels, there are now three zones. Zone 1 – this is zone 1; this is zone 2; and this is zone 3. So, for all the channels we have mentioned in the previous slide…

In the previous slide, we mentioned here the mild slope, steep slope, critical slope, adverse slope. One can easily identify three zones of flow there. Zone 1 represents the region above the top line. So, the top line – it can be either the critical depth line or the normal depth line, whichever case it may be; if it is a super critical flow, you know that critical depth line is above the normal depth line. Therefore, whichever region is above the upper line, that region is considered as zone 1. Zone 2 is the region in between the normal depth line and critical depth line. The zone 3 is given as the region between the bed and the lower line. So, like that, you can easily identify three zones.

Now, based on these three zones… I will just write it for your benefit here. Zone 1 – space above the topmost line; zone 2 – space between critical depth line and normal depth line; zone 3 – space between channel bed and lower line. So, why I am classifying is that, the water surface – it can exist in any of these three regions. Now, based on the channel bed slope and the existence of the water surface in any of this region, you can easily classify the gradually varied flow profiles. So, we are now going to classify the gradually varied flow profiles based on these three zones as well as the type of bed slope of the channel. Like that we are now going to categorize them.

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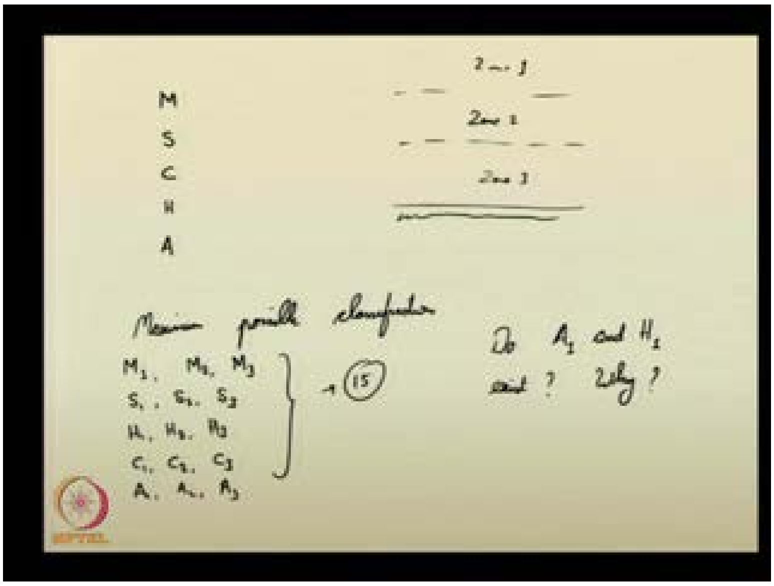

We have already suggested the mild sloped channels, steep sloped channels, critically sloped channels, horizontally sloped channels, adversely sloped channels. So, in each of these things, in the mild channel also, the water surface may exist in any of three zones. So, we have given you the three zones: zone 3, zone 2, zone 1. So, we have given you these three zones. So, the water surface can exist in any of these zones even in the mild sloped channel or even in the steep sloped channel, even in critical flow. And we will ask you, what about in the horizontally sloped channel and adversely sloped channels. So, maximum possible classifications one can make is that, if I give the zone number as suffix to the abbreviated letter; the maximum possible classifications one can infer is M 1, M 2, M 3; S 1, S 2, S 3; H 1, H 2, H 3; C 1, C 2, C 3; A 1, A 2, A 3. Like that fifteen possible classifications can be theoretically now you can visualize them; or, not theoretically; we can say suggest that, based on the combinations, one can infer fifteen possible classifications for the gradually varied flow.

Now, let me ask you from these classifications; do the classification A 1 and H 1 – whether they exist? whether such classifications are possible? H 1 exists, why? Now, the question is, why? So, what is your response for that? You now casually think on this thing.

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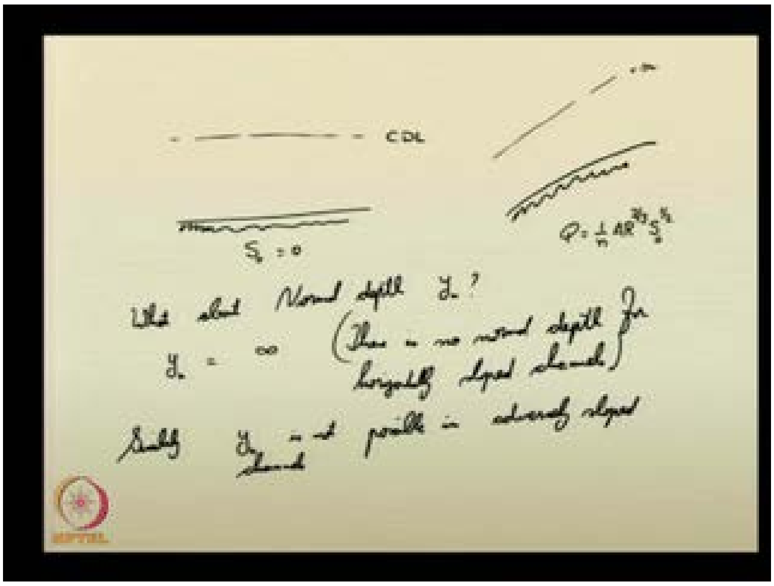

We will see that. Say if consider the horizontally sloped channel; for the horizontally sloped channel, the bed slope is equal to 0. Definitely, you will be having critical depth line – CDL; that is there, available for the given discharge. What about normal depth? So, whether normally depth will be there in the computation? What is your influence in this? You will see that, y n is equal to some infinite value. So, there is no normal depth for horizontally sloped channels; it is not possible. So, theoretically, it is not possible to get normal depth for the horizontally sloped channels.

Similarly, for adverse sloped channels; if this is the adverse sloped channel, you can infer the critical depth line; but y n is not possible in adversely sloped channels. So, just recall the Manning's equation. The Manning's equation suggested that, Q is equal to… I will just change the color for your benefit. So, Q is equal to 1 by n AR to the power of 2 by 3 S naught to the power of half. This is was the Manning's equation for uniform flow; if you recall them. As S naught is equal to 0 in horizontally sloped channels, y n cannot be inferred. Similarly, as S naught is negative, you cannot have the root of a negative value and subsequently compute y n. That is also not possible. So, y n is not possible in adversely sloped channels as well. Like that you can infer the things.

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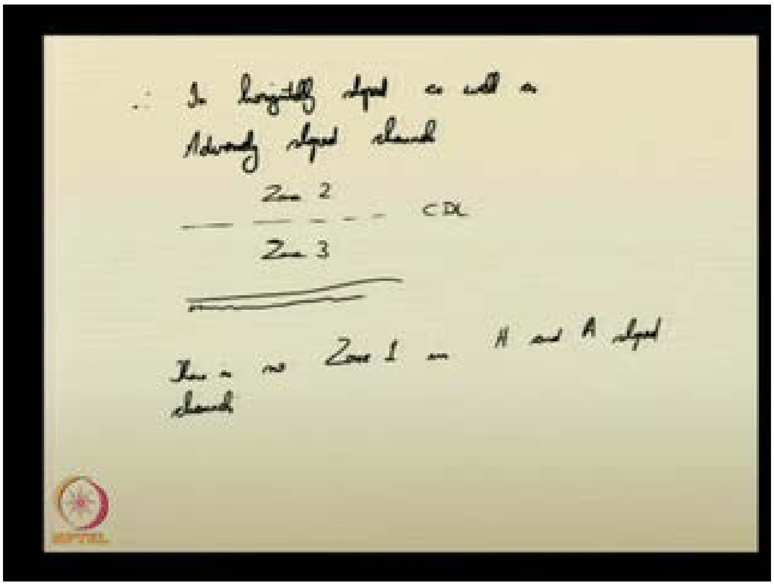

Therefore, in horizontally sloped as well as adversely sloped channels, we mention that, there do not exists any normal depth line or there is no normal depth for the adversely sloped channels as well as horizontally sloped channel. So, you can easily suggest now, there are only two regions or only two zones for flow in such type of channels. Now, they are classified as zone 2 and zone 3. Therefore, in these two channels, there exists only zone 2 and zone 3. So, there is no zone 1 in horizontally sloped and adversely sloped channels. So, when you devise or when you consider the classifications of the gradually varied flow, you have to keep these things in mind.

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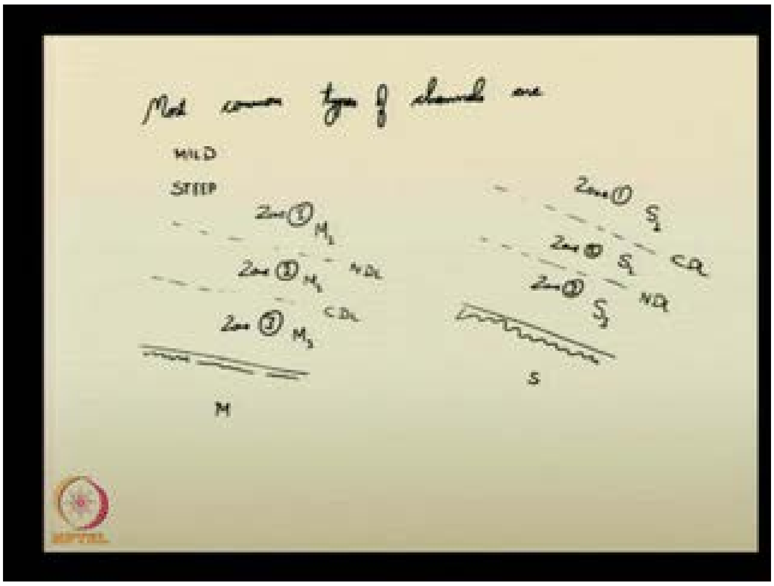

The most common types of channels – they are mild sloped channels as well as steep sloped channels. These two are the most prominent one. You can now suggest that say, the mild sloped channel is the one, where the normal depth line is above the critical depth line; that is, normal depth is greater than the critical depth. Therefore, you can have say… This is mild sloped channel. So, zone 1 of the mild sloped channel; zone 2; zone 3. So, if any of the water surface profiles exist here in these zones, say they can be subsequently considered now – M 1, M 2, M 3. Similarly, in a steep sloped channel, you will see that, the normal depth line is below the critical depth line; that is, the normal depth – it is smaller than the critical depth. You can now easily see or you can easily classify three zones: zone 1, zone 2, zone 3. This is a steep sloped channel. You can also give the corresponding water surface profiles; or, the water surface exists in these zones. Those water surfaces are considered as S 1 profiles; here in this zone, it is considered as S 2 profile; and here in this zone, it is considered as S 3 profile.

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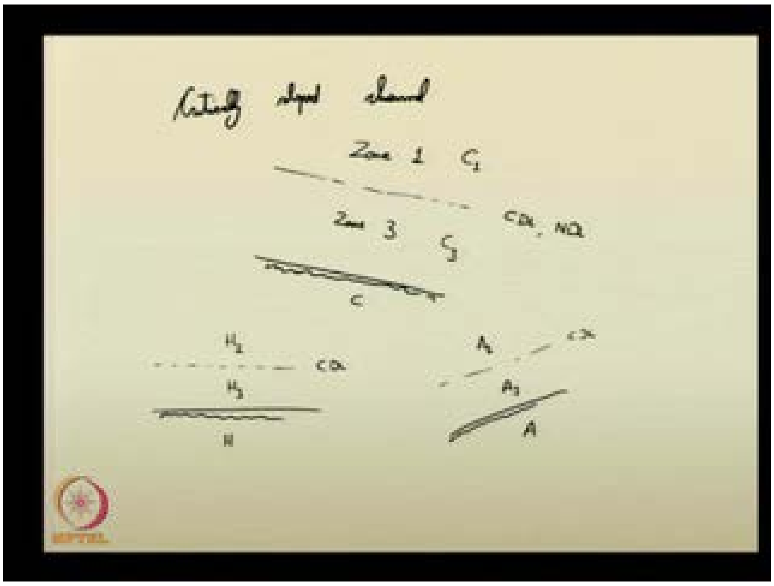

Similarly, in the critical sloped channels or critically sloped channel; what is the peculiarity of critically sloped channel? The critical depth line is same as the normal depth line; that is, the critical depth and normal depth – both are same. So, then you will see that, the region above the critical depth or normal depth line – they are called as zone 1; and the region below that – this is called as zone 3. So, zone 2 is missing in critically sloped channels. Therefore, you cannot give water for the profiles and all. C 1 and C 3; C 2 is not existing in such situations; you will see them. So, adversely sloped channels or horizontally sloped channels – this is C; this is horizontally sloped channels. You will have two regions as mentioned earlier – H 2 and H 3. Adversely sloped channel – A; so A 2 and A 3.

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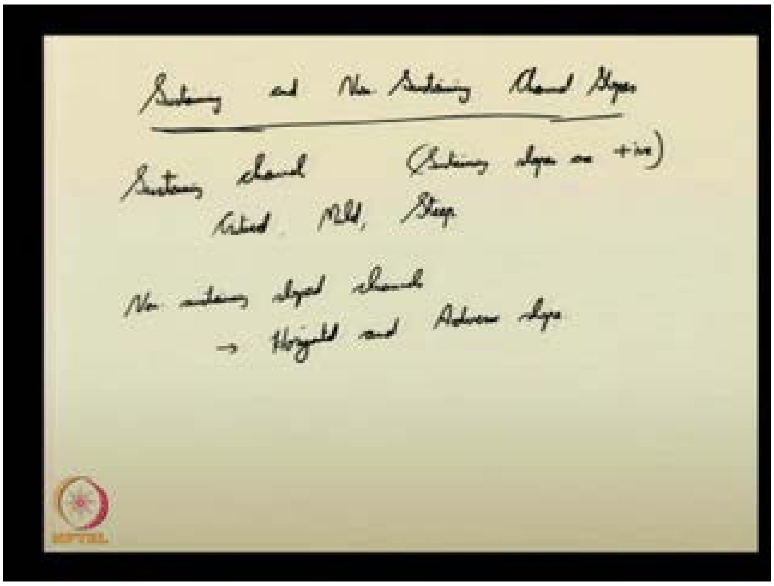

You can also mention… You can also consider channel slopes as sustaining and non-sustaining slopes. These are self-explanatory terms – non-sustaining. So, what does this mean? A channel slope – it is called sustaining if its slope falls or if its channels slope or if the bed of the channel – if it falls, they are called sustaining type of channels. If it is not falling, they are called non-sustaining type of channels. So, let me ask you, where do you encounter sustaining channels out of the five classifications we made for the channel slope in these critical, adverse, horizontal, mild and steep? Out of this five, which are the sustaining sloped channels and which are the non-sustaining sloped channels? Here as it is self-explanatory thing, non-sustaining or sustaining channels or sustaining slopes – they are seen in critical flow, mild sloped channels as well steep sloped channels. So, for the sustaining channels, the sustaining slopes are positive; the non-sustaining sloped channels – their sustaining slopes will be negative and they are found usually in horizontal and adverse slope conditions. Let me give you detail figurative descriptions.

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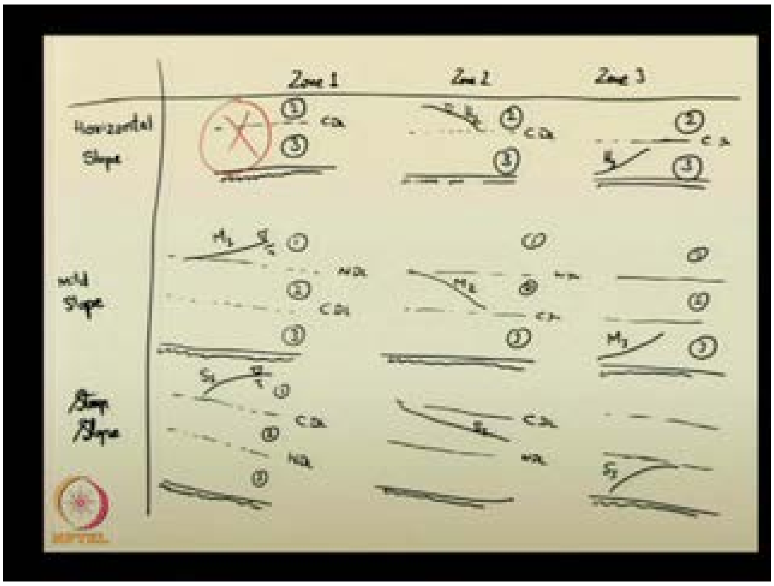

Say if I suggest that, based on the various types of channel flow, how the profiles look. Let us see that. Let me suggest here. First, beginning with the horizontally sloped channels; let us consider the three zones available – zone 1, zone 2, zone 3. So, the horizontally sloped channel – it will look like this. It will have the critical depth line and the bed slope like this. So, there is no water profile. See if I make it in a tabular form like this; just for your benefit, I am just giving it like this. Zone 1 – there will not be any water surface profile in zone 1 for the horizontally sloped channels. So, we can easily suggest now, this portion does not exist. In zone 2, what there is… You can draw the critical depth line. And zone 2 – what does it represent? It suggests you… This is the zone 2 and this is the zone 3 of the horizontally sloped channel.

So, the water surface will be like this; somewhere like this. It should exist in the second zone. So, it should be something like this. So, this is called H 2. That water surface profile is called H 2. Similarly, you will see that, if the water surface profile exists in the zone 3, that profile would be something of this form. And you can name this as H 3. This profile can be named as H 3. So, you can… Say this is zone 2; this is zone 3, zone 2, zone 3, zone 2, zone 3 – this sloped channel. So, a mild sloped channel.

Again, let me draw the bed; say bed may be like this. The critical depth line is below the normal depth line. So, you have three zones: 1, 2, 3. And in the zone 1, if the water surface profile exists in zone 1; that is called M 1 and it may be something of this form like this. So, this water surface profile is called M 1 profile. Similarly, if in the channel again, you draw the same thing – zone 1, 2, 3; and if the water surface profile exists in zone 2; it may be something of this following form. And this water surface profile is called M 2, because it is existing in zone 2 and it is in the following form. So, it is in a mild channel. So, this is M 2. Similarly for the three regions, if the water surface profile is something of this form, it is existing in the third region and it is given – that water surface profile is given as M 3.

One can easily visualize for the steep sloped channels also. For the steep sloped channels, this is the channel bed. And for steep sloped channels, the normal depth line is below the critical depth line. And for such conditions, the flow to existing zone 1; Here the zones are 1, 2, 3. So, the zones are… So, from the critical depth, a profile exists like this. Usually a jump occurs from the normal depth line to the critical depth; then a gradually varied flow profile occurs; and this profile is called S 1. Similarly, if there is a change in water surface in the zone 2 of the steep flow channel, this water surface profile is called S 2. One can easily witness the S 3 curve as well. So, S 3 will be something of this form. So, like that for horizontal mild and steep slope; I have just mentioned it. You can easily think for the horizontal and adverse sloped channels.

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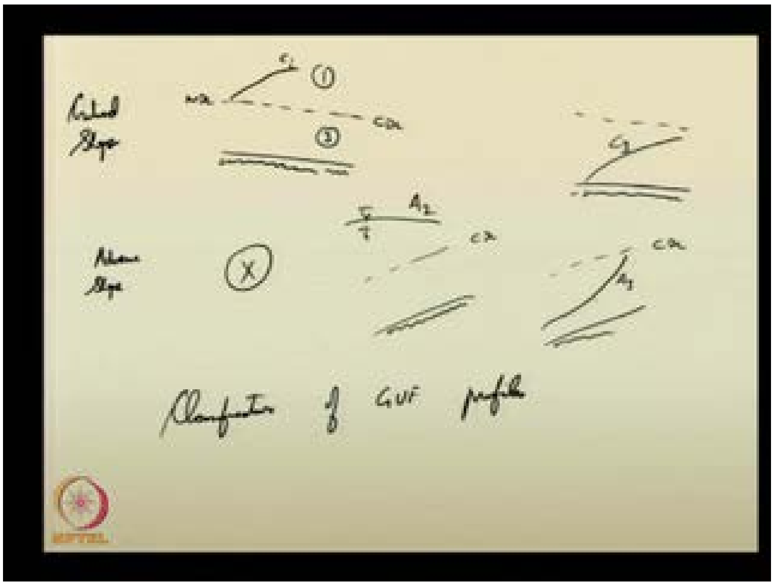

Do I need to describe it here? I will just briefly show it again. For the critically sloped channel, you have both the critical depth line and normal depth line as the same; both are same. This is zone 1; and this is zone 3. And whatever water surface profile is there, this is called C 1 curve. It is in the zone 1. Zone 2 – it is not possible. And in the zone 3, you will see that, this particular curve – it is called C 3. For the adverse sloped channels, definitely, for A 1 is not possible as we have mentioned it earlier. So, I can just omit that portion. In the zone 2, for the adverse loop channels, critical depth line is like this. So, this is the water surface profile that exists in zone 2 and it is called A 2. Similarly, if it exists in the third zone, this water surface profile is called A 3. So, like that, one can easily visualize various water surface profiles. So, these are the classifications of gradually varied flow profiles.

What are the possible things? Gradually, you can have H 2 and H 3 for the horizontally sloped channels. You can have M 1 profile, M 2 profile and M 3 profile for the mild sloped channels. For the steep sloped channels, you can have S 1, S 2 and S 3 profiles. For the critically sloped channels, you can have C 1 and C 3. For the adverse sloped channels, we can have A 2 and A 3 water surface profiles. So, this curve as you have seen in the various profiles, I have just arbitrarily drawn like this. Still in some situations and all, you will see why the curvature of these water surfaces…

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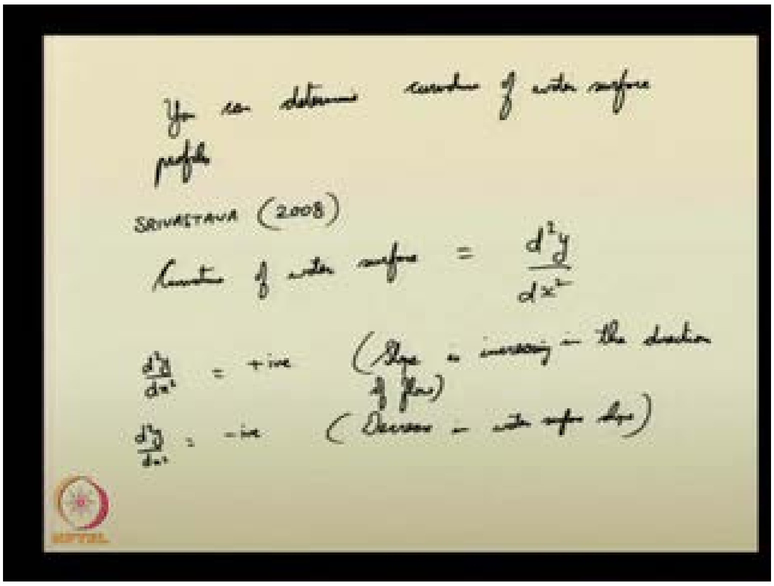

One can easily determine… You can determine curvature of water surface profiles; how the curve should be. This is given by say Prof. Srivastava in his book on flow through open channels. He published the book in 2008. In that, he had suggested that, curvature of water surface – this is nothing but the derivative of the slope of the water surface – d square y by d x square. Now, based on this value; that is, based on the value of d square y by d x square. Suppose if this is a positive quantity; that is, the slope is increasing in the direction of flow. d square y by d x square – if this is a negative value, you are having a decrease in water surface slope. So, there is a decrease in water surface slope in direction of flow.

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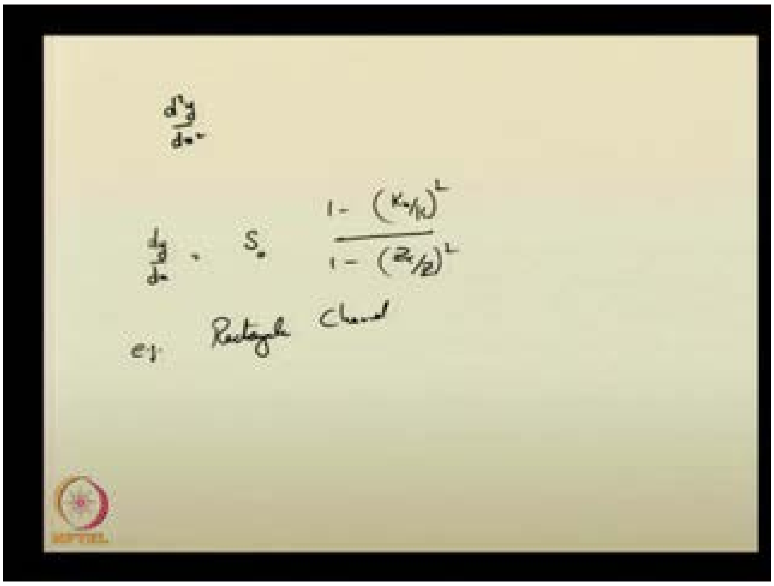

You can easily identify the expression for d square y by d x square. You know d y by d x is equal to S 0 into 1 minus k n by k whole square by 1 minus z c by z whole square – this you are well aware by this time now. So, what could be the second derivative? You take any arbitrarily dimension channels; or, for example, rectangular channels; what happens? You substitute the conveyance factors, section factors for the rectangular channels; substitute that here and again derive it with respect to x. What is the quantity you are going to get? You can do it as a home work and let us see then. Anyhow I am not going to derive them in this particular class. I am just going to write the final expression.

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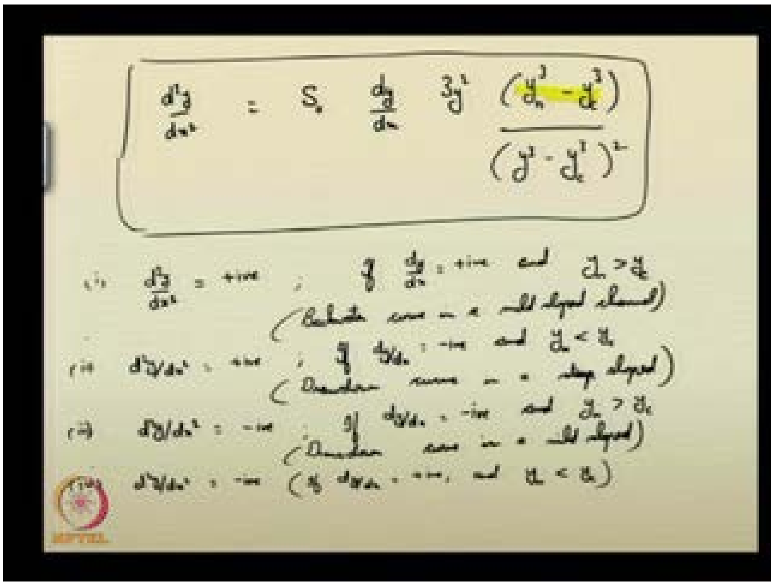

d square y by d x square as given in Prof. Srivastava's book, this is equal to S 0 into dy by dx three times y square into y n cube, that is, normal depth minus y c cube – critical depth by y cube minus y c cube the whole quantity square. So, a relationship for d square y by d x square has been derived in that particular book. I am just showing it here. Why we are showing it here is that, from this, one can easily picture various features of the water surface curve. So, let me ask you. The d square y by dx square – this is will be positive when? At which all times it can be positive? It can be positive if dy by dx is positive and the normal depth – please look into the thing – the normal depth and y n should be greater than the critical depth. If this is true, then d square y by d x square will be a positive value. So, what does that mean? What does that physically mean? It is a back water curve in a mild sloped channel.

Second condition I would like to ask you. d square y by d x square – this is can be positive again if dy by dx is equal to negative and the y n should be less than y c. If this is also true, then also d square y by d x square will be positive. So, it represents a drawdown curve in a steep sloped channel. Another thing – d square by d x square is equal to a negative value if dy by dx is equal to negative and y n is greater than y c. So, this represents a drawdown curve in a mild sloped channel. Similarly, d square y by d x square is equal to negative if dy by dx is positive and y n is less than y c. So, one can easily see these things now.

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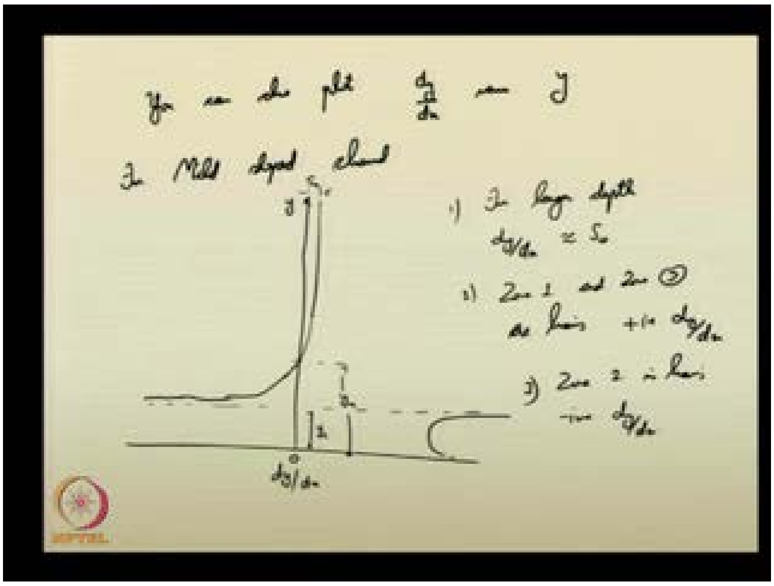

You can also plot dy by dx verses depth of flow using this particular relationship. You can also plot the following things: dy by dx verses y. Say for mild sloped channel, it has been observed that… This is referred from Srivastava book on Open Channel Hydraulics. See if I plot dy by dx with respect to depth; say if this is the quantity of zero depth and zero dy by dx, this is the origin. Then as you… This is the depth of flow. Let us suggest this as the critical depth. Let us suggest this as the normal depth. You will see that, the quantity dy by dx – it will vary with respect to y in a following form. Like this, it may go; then as y decreases, dy by dx goes into the asymptote in the negative direction. So, this quantity is approximately equal to the bed slope as 0.

You can infer from this figure, for larger depth, dy by dx is approximately equal to the bed slope. Then zone 1 – this is zone 1. So, zone 1 and zone 3… Which is zone 3? Here in the zone 3, the dy by dx curve is something of this form. So, like this, you can easily suggest now. Say dy by dx versus y; when you plotted it, it is understood now that, zone 1 and zone 3 are having positive dy by dx. Zone 3 is… Zone 2 is having negative dy by dx. So, this is always true or it is always true in such thing, because from that curvature equation itself, one can easily plot it and see them. So, that is, zone 2 will be always having a negative dy by dx or it means that, it always assists a drawdown curve. You cannot have a back water curve in zone 2. Similarly, zone 1 and zone 3 – they raise in the water surface things.

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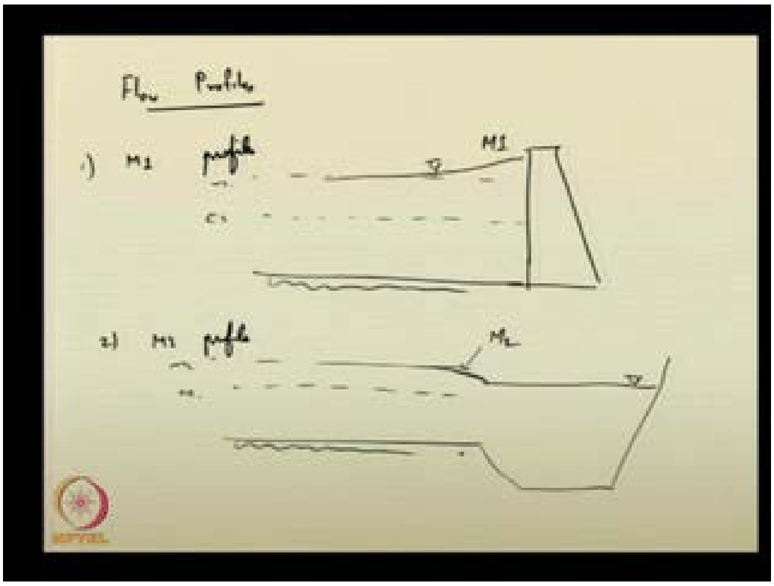

The flow profiles one can easily visualize – say flow profiles – you can see that, the M 1 profiles – it can be seen. Say if there is a uniformly flowing channel and if you construct a dam – uniformly flowing mild sloped channel; say this is the critical depth line; this is the normal depth line; then the M 1 curve is observed in the upstream of this dam. In the upstream of this dam, in the following form, you will see this is the M 1 curve. Similarly, you will see M 2 profiles in the actual field in regions. Say if there is such mild sloped channel, that is, either falling into another tank or where there is water say if the… And if the depth of water in this tank or another channel, if it is below the normal depth line; then the water surface profile will be like this and this is given as the M 2 profile. So, these things are observed in the actual field situations.

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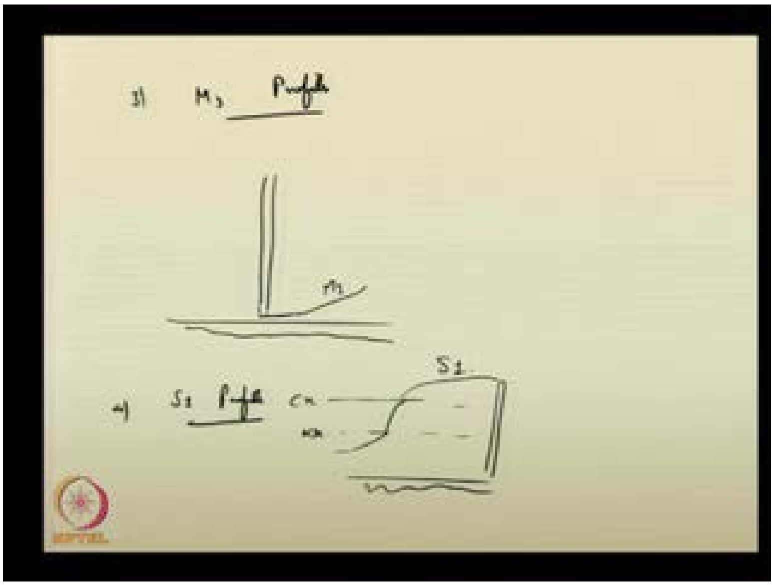

The M 3 profiles – they are witness. Say in the uniformly flowing channel, if you keep a shutter; if you just close the shutter and if you just open it for a small portion, water will gush in like this and this water profile is there; this will be an M 3 – gradually varied flow profile. Similarly, one can visualize S 1 profiles. If there is an obstruction in the gradually varied flow, this is the normal depth line; this is your critical depth line; the water surface – it will cause a jam and it will come like this. This will be the S 1 profile. So, like this, you can see various type of profiles in the actual field condition. Let me now stop here today. Let us conduct the quiz for the today's lecture, the following portion. And we will continue the next lecture next day.

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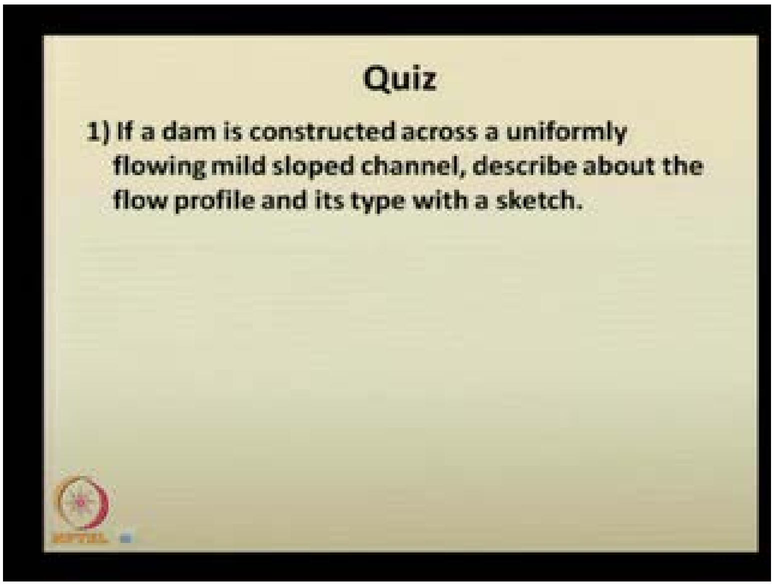

For the quiz – just now only, we discussed on the thing – if a dam is constructed across a uniformly flowing mild sloped channel, describe about the flow profile and its type with a sketch. This is your first question. That is, if a dam is constructed across a uniformly flowing mild sloped channel, describe about the flow profile and its type with a sketch.

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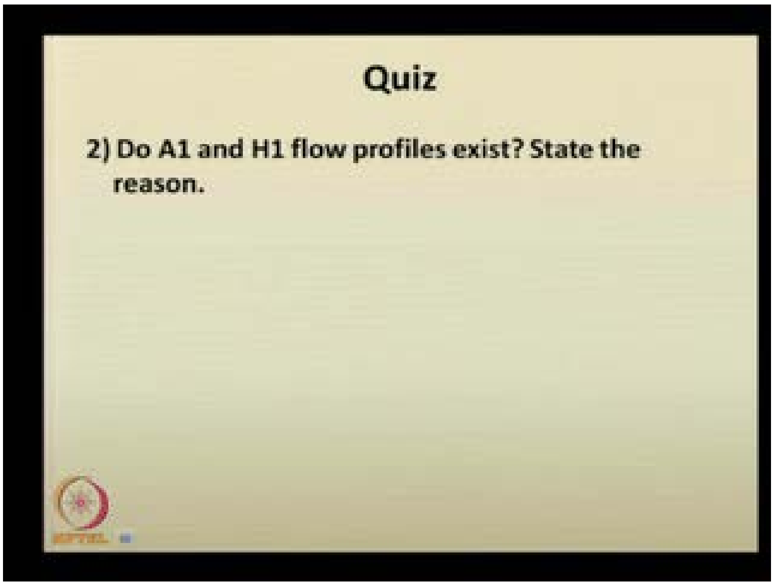

Your second question – do A 1 and H 1 flow profiles exist? Do A 1 and H 1 flow profiles exist in theory? Is it possible? And give the reason. Whatever – whether if it exists, you give the reason; if it does not exist, give the reason specific; means we have to give, substantiate it.

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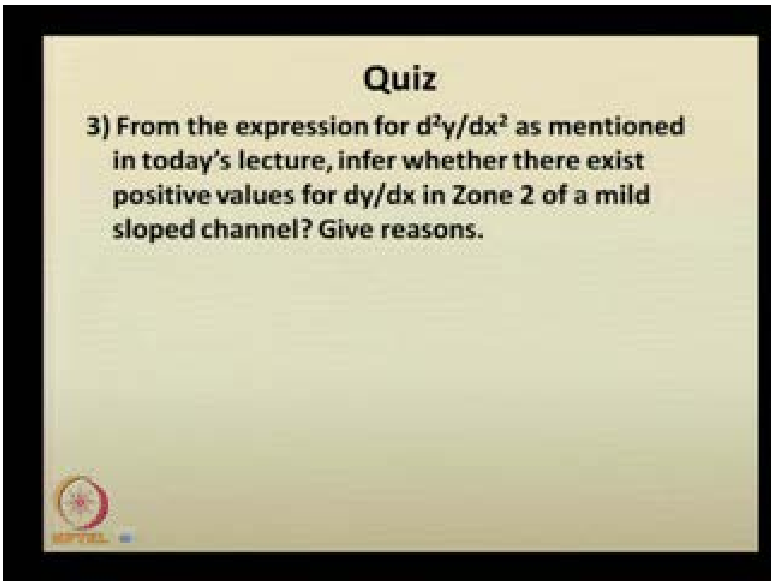

Now, the next question is – from the expression for d square y by d x square as mentioned in today's lecture, infer whether there exist positive values for dy by dx in zone 2 of a mild sloped channel. Give reasons. All are very simple questions today. You can just answer them in 3-4 minutes and just return back the sheets.

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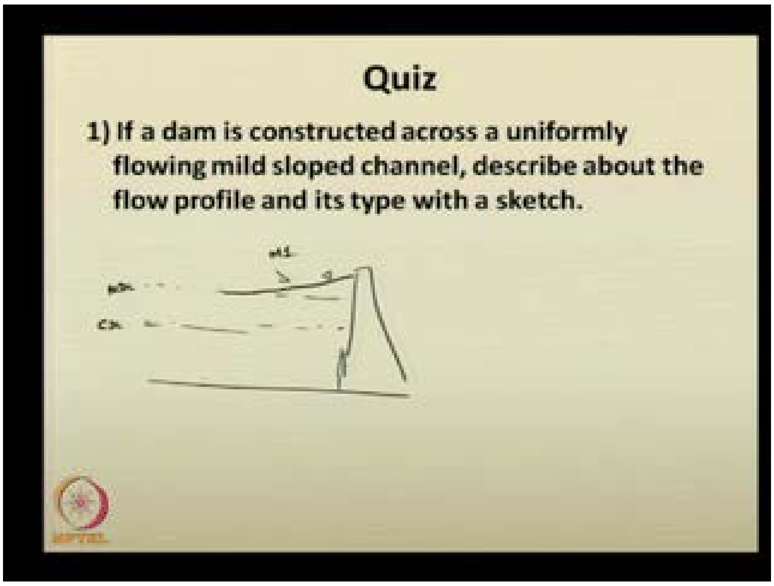

The solutions for today's quiz; you can see that, if a dam is constructed in a mild sloped channel, normal depth… This is your critical depth line. In the upstream of the dam, you will see the water surface profile rising like this. And this water surface profile is called M 1 water surface profile. So, this is in the mild sloped channel.

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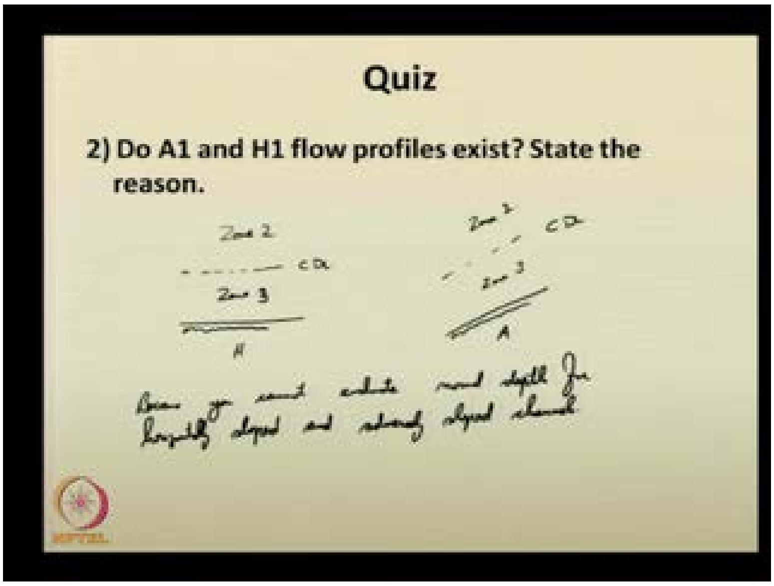

Your next question we asked is – do A 1 and H 1 flow profiles exist? What is your reason? I hope everyone might have answered this thing. For the horizontally sloped channels or for the adversely sloped channels, the critical depth line exists. And we suggested that, there is no zone 1 for horizontally sloped channels as well as adversely sloped channel; there exist only zone 2 and zone 3. So, whatever water surface profile is there above the critical depth line, that comes under zone 2. So, similarly, in both cases: horizontal as well as adversely sloped channels. And you know why zone 1 is not existing. Because you cannot evaluate normal depth for horizontally sloped and adversely sloped channels. So, these are the… This is the solution for the question.

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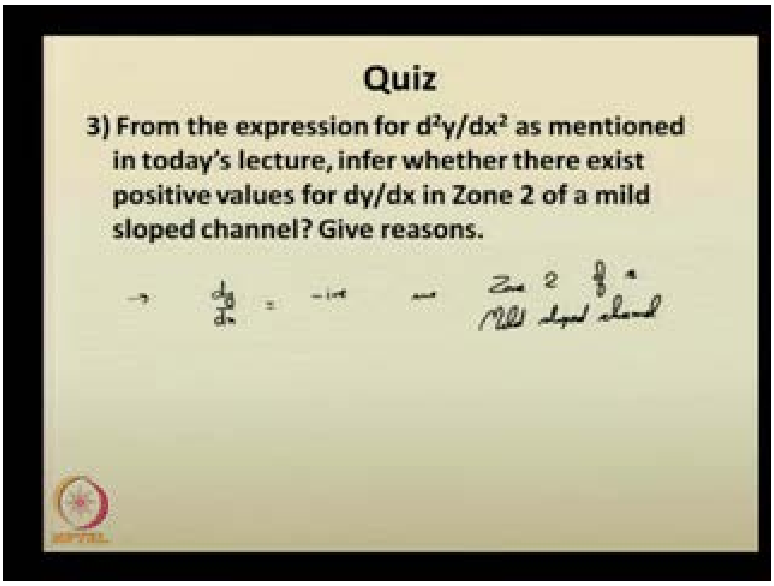

Your third question – from the expression for d square by d x square as mentioned in today's lecture, infer whether there exist positive values for dy by dx in zone 2 of a mild sloped channel. Give the reasons. We had asked you to do that. So, your answer is dy by dx will be always negative in zone 2 of a mild sloped channel. In fact, for steep sloped channel also, dy by dx will be negative in zone 2. That will come… You can infer it on your own also; you can verify it. So, dy by dx will be always negative in such situations. So, there would not be positive dy by dx value. What is the reason? You have already drawn that curve. We can just go back to that lecture and show to you. Say in this curve, dy by dx versus depth; say below normal depth between normal depth and critical depth, the slope – it is going in the negative direction and it goes to minus infinity. Below critical depth, dy by dx profile is being shown like this; it is in the positive side. So, these are today's (( )) We have discussed on the classification of the gradually varied flow. We will see some more other aspects on these things. We will also solve an example problem in the next class.

Thank you.