**Advanced Hydraulics**
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**Module - 3**
**Varied Flows**
**Lecture - 3**
**Classifications of Gradually Varied Flow**

So, we are back into our lecture series on advance hydraulics. So, this we are going through the third module of the lecture series. The third module, this title has varied flows if you have gone through the syllabus, we will know that. So, today is a third lecture of this module, and we can give the title of today's lecture as gradually varied flows, and its classifications. So, definitely the classification, the entire classification of the gradually varied flow may not be able to be completed in today's lecture alone, so there will be a next part in the following lectures as well. In the last class we discussed on varied flows in open channels.

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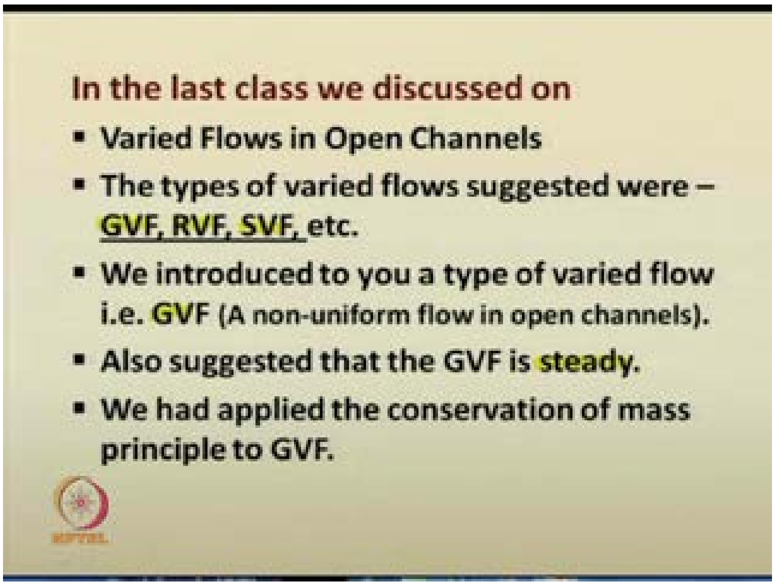


The various types of varied flows that we have suggested; the gradually varied flow, rapidly varied flow, specially varied flow, etcetera. These things we have just briefly discussed, we have not gone through the technical discussion on those aspects, definite, but however we have introduced to you a type a particular type of varied flow; that is the gradually varied flow. This we suggested that it is a non uniform flow in open channel, and it is also having a steady state condition; that is the flow conditions in the channel are not changing with respect to time, even though the flow is non uniform, but that non uniformity is maintained, that particular non uniformity, it is maintained, completely throughout the time. So, we are we also had applied the conservation of mass principle to gradually varied flow, and we have seen how the continuity equation will look in the, gradually varied flow.

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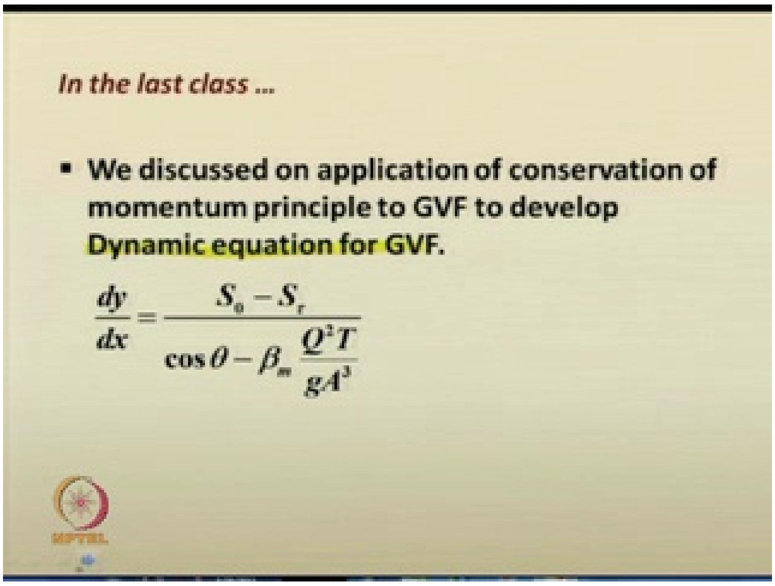


Also we have discussed on the application of, conservation of momentum principle to gradually varied flow; that is these principles were used to develop, the dynamic equation for gradually varied flow, if you have gone through that. See the slope of the water surface d y by d x is equal to S 0 minus S 2 by cos theta minus beta m Q square T by g A cube. The terms S 0 is the bed slope, S 2 is the shear slope, Q is the study discharge in the gradually flow are varied flow, T is your channel top width, A is the area if cross section, theta is the angle, the channel bed makes with the horizontal. So, all this things we have discussed in the last class.

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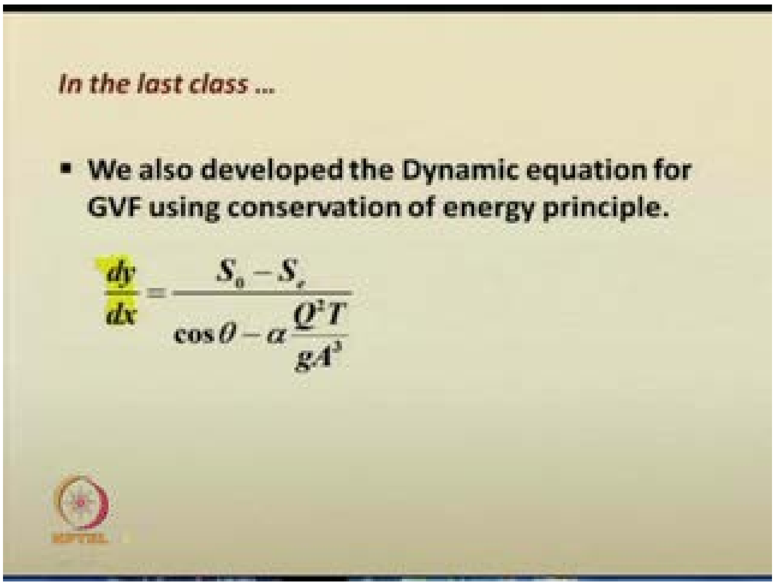


We had also developed the dynamic equation for gradually varied flow, using conservation of energy principle. There also you had derived it the, this particular portion, slope of the water surface d y by d x is equal to S 0 minus S e. So, here the S e term is being used that is the energy slope is being used, and also you have use a correction factor as the kinetic energy correction factor alpha.

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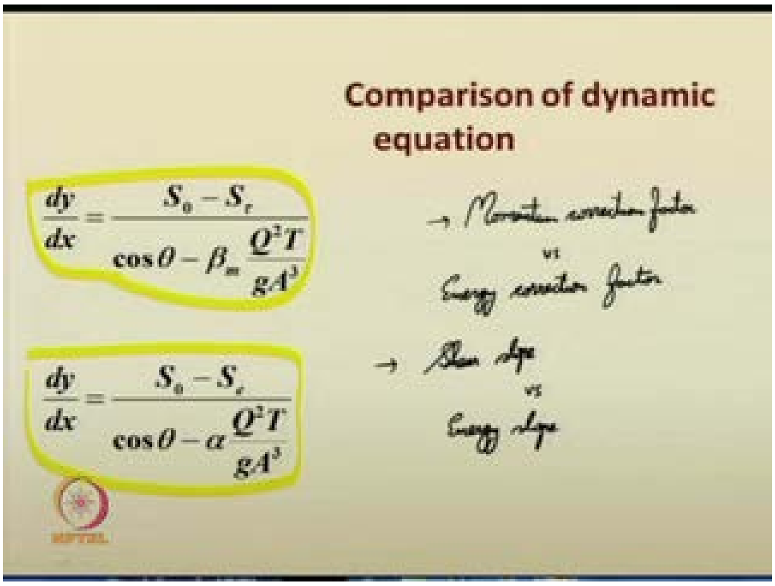


So, these some of the differences is, these means that, how the dynamic equation differs in energy as well as momentum equations. If you apply it in energy equation, it will look like this, if you apply it in momentum equation it will look like this. So, there are quite difference, some differences; that is the differences quite obvious are; momentum correction factor verses kinetic energy correction factor. So, whatever differences are there in this correction factor, that will be reflected, when you compute your slope of the water surface. Another obvious change you can observe is, shear slope verses energy slope.

So, if these things are quite different in a particular channel, then obviously, your results will the water surface slope, that will also differ when you compute the water surface slope, or if you compute the water surface, these things and all, you are seeing this dynamic equation. The results will be quite different in both, if you use the two different equations. So, what we have to do is that, we can make certain assumptions; that is if you incorporate certain assumptions in the dynamic equation, just for a simple case that we are normally, we are considering prismatic channels. Suppose if you are a rectangular channel or if you are taking a trap trapezoidal channel or if you are taking a well designed prismatic channel. The entire reach of the channel, if it is prismatic then some of the assumptions that can hold good for such situations are, you can have.

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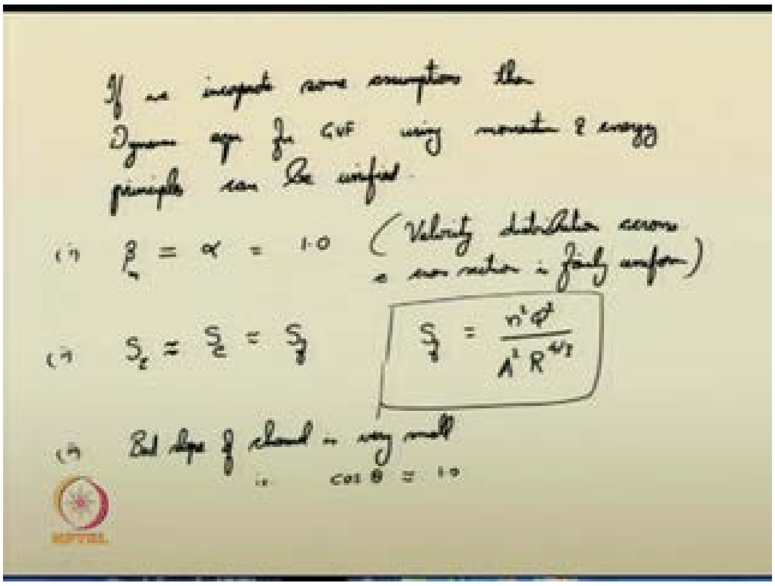

Say, if you incorporate some assumptions, then dynamic equation for gradually varied flow, using momentum and energy principles can be unified, so what are those assumptions. Say if you assume that, the momentum correction factor beta m is almost equal to you kinetic energy correction factor alpha, and if they are almost equal to one, same we are approximating it as one, if there is no significant correction factors involved, so this suggest that the velocity distribution. So, that is a velocity distribution across a cross section is fairly uniform. So, if you can incorporate this assumption, also another assumption we can suggest is that, if the shears slope S 2, if it can be approximated as energy slope, both of them can be approximated as a friction slope S f if you can do that, then further you will commence, it will help us in further unifying the dynamic equations of gradually varied flow.

So, you can see that; that is friction slope is the one, which is used in manning's equation, if you use the manning's equation and all, you can suggest that the friction slope S f is nothing but equal to n squared Q squared by A square R to the power of 4 by 3. One can also think of using Chezy's equation, or I am not going to discuss it here. So, by this way, one can use the friction slope, one can compute the friction slope, and if you can approximate your shear slope, and energy slope with this friction slope. Well good you can directly substitute them in the dynamic equation. Also we are suggesting that, the bed slope of the channel is very small; that is, this approximation is used such a way, that we are suggesting cos theta is approximately equal to 1.

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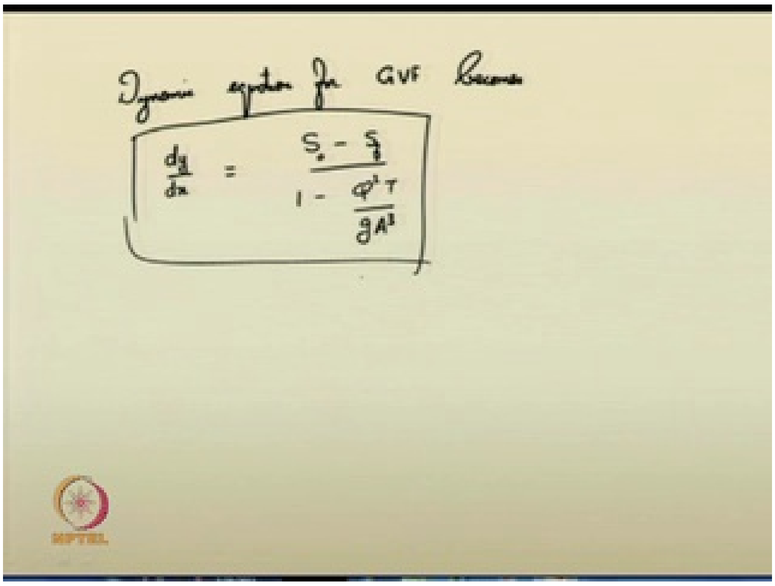

So, if we incorporate these three assumptions, your dynamic equation for gradually varied flow becomes d y by d x is equal to S 0 minus S f by 1 minus Q square T by g A cube, so you have the dynamic equation. Now unified both using the conservation of energy, as well as conservation of momentum principle, by applying those assumptions, we are able to unify the dynamic equation, and you are getting it in a much simplified as well as better form. How will you tackle if you have given this dynamic equation. Now, what is a purpose, how will you try to understand equation now in a physical way, what does this mean.

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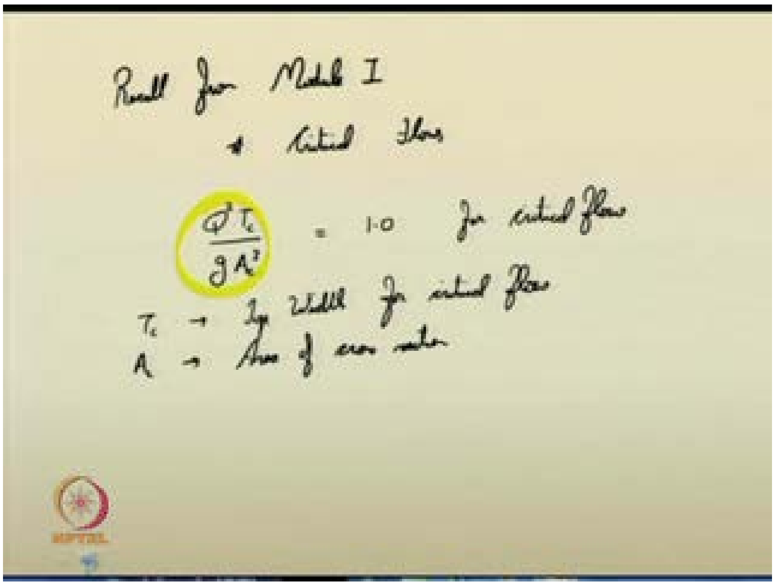

You just recall, you recall from the module one, recall from module one we discussed on critical flows, critical flows in open channel, you had discuss them if you recall them. At that time, we had suggested that the following quantity Q square T c by g A c cube. This will be equal to 1, for critical flow this will be equal to 1, for critical flow if you recall them right. So, where T c is the top width for critical flow A c is a area of cross section for critical flow. So, entire quantity if you recall them, this was almost related with the Froude number also these entire quantity, and this for critical flow this quantity is equal to one. This particular quantity, we suggested that this is equal to one. So, if we have these things, if you recall them. Now it is quite easier, if you incorporate the similar principles now in the dynamic equation for gradually varied flow and all, it will be quite easier now.

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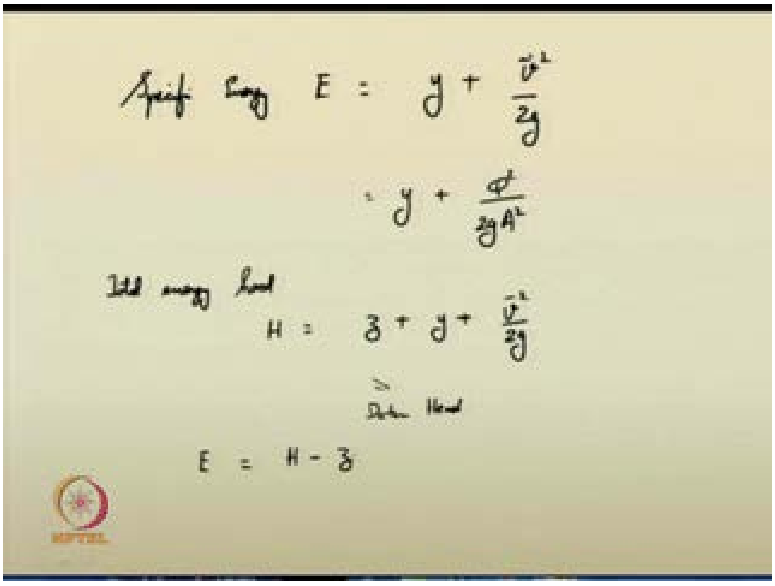


So, for the specific energy in module one, you discussed that this is nothing but the pressure head plus velocity head. This is y plus Q square by 2 g A square. So, before deriving specific energy in that class, you had also suggested that total energy head; H is equal to datum head plus pressure head plus 2 g, where z is your datum head, so what does this mean. So, from these things it is quite obvious that, your energy, specific energy E is nothing but total energy minus datum head; that is E is equal to H minus z.

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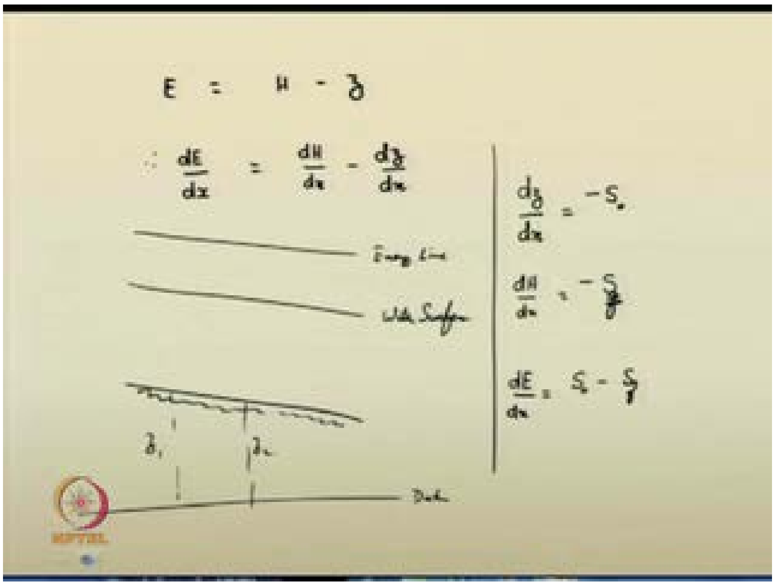

Therefore, d E by d x, if you go along the reach of the channel; that is in the flow direction d E by d x, this means that this is equal to d H by d x minus d z by d x. So, if you have the channel bed like this, this is your datum line, so one, if you proceed in the x direction, this is z 1, this is z 2 datum head at two locations. So, this is the energy line, water surface line. So, what does this mean, from this things; d z by d x, this quantity d z by d x, this gives you the slope of the bed, it gives a, or say what does this mean. Here the height is z 1, here the height is z 2. So, the height or the datum head it decreasing in the x direction. So, d z by d x gives you the slope. So, we have already discussed this thing, this is the negative minus S 0. We have to give it minus S 0, because the datum head it is decreasing in the flow direction, d H by d x, this is your energy slope, that can also be given here S e, or S f, if you recall that we have approximated energy slope with the friction slope and all, so d H, so it is S f. Therefore, you get d E by d x quantity as S 0 minus S f. So, why we suggested this thing here is that, in your dynamic equation, the numerator consist of S 0 minus S f term. So, the numerated term is actually suggesting the slope of the specific energy. So, you can think in that direction, you can further evolve these equations, further try to proceed them.

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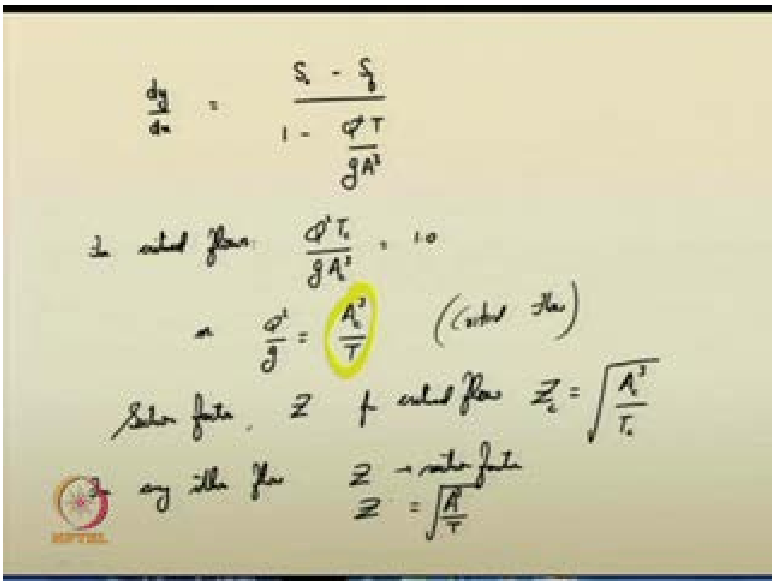

So, the dynamic equation for gradually varied flow, S 0 minus S f by 1 minus Q square T by g A cube. So, as suggested earlier, for critical flow Q square T c by g A c cube is equal to 1 or Q squared by g is equal to A c cube by T, this is for critical flow, for critical flow Q square g by is equal to A c cube by T. In that class, at that time we had suggested a quantity call section factor z. In module one, we had discuss this things section factor z, this is nothing but for section factor z for critical flow. Let us give it as z c, this was given as A c cube by T c root. So, in the critical flow equation, the right hands, this right, this particular term, this is nothing but the square of your section factor. So, z c this was defined in those portions, so here again you can use them. So, for any other flow, not only critical flow, you can define section factor this is nothing but equal to the corresponding cube of area by top with the whole thing, you have to square root it this, z is equal to root of A cube by T, like this one can define that.

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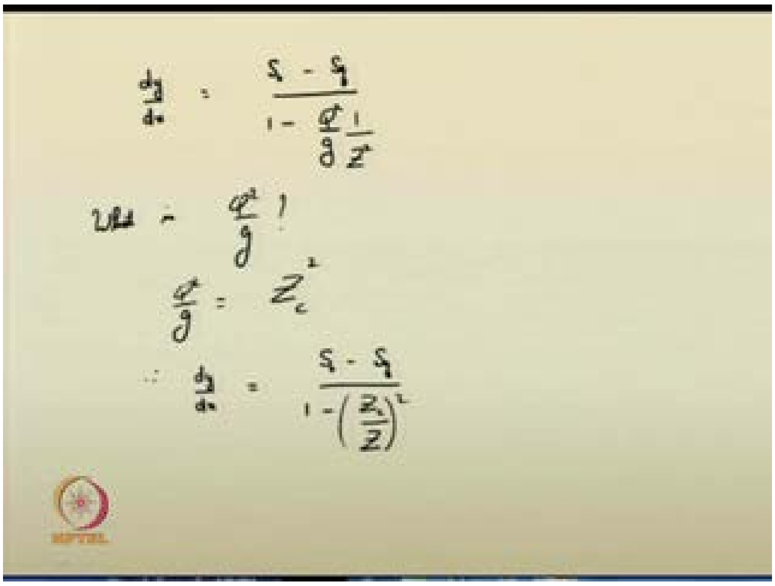

So, again I am going back to the dynamic equation now, S 0 minus S f by 1 minus Q squared by g, and just look into the equation. This particular portion T by A cube, this is nothing but your section factor for the channel at that, for that particular flow, what is the section factor for the channel; that is being given by T by A cube, is not it. So, I will just substitute those things now in the equation, so this is Q squared by g into 1 by z square. Now what is Q squared by g, go back to the previous slide Q squared by g, this is nothing but equal to the square of the section factor for critical flow; that is Q squared by g is equal to z c square. Just go back again here, Q square by g is equal to A c cube by T c, and z c this is z c is given by A c cube by T c the square root of that. So, here substitute this thing now this is Q squared by g, you can substitute those thing, so you will get the following relationship. Therefore, your dynamic equation becomes d y by d x is equal to S 0 minus S f by 1 minus z c by z whole square right. So, now we will see, we are just trying to further simplify the dynamic equation, in terms of certain non dimension, some certain parameters like conveyance factor, section factor and all. So, here we use the section factor.

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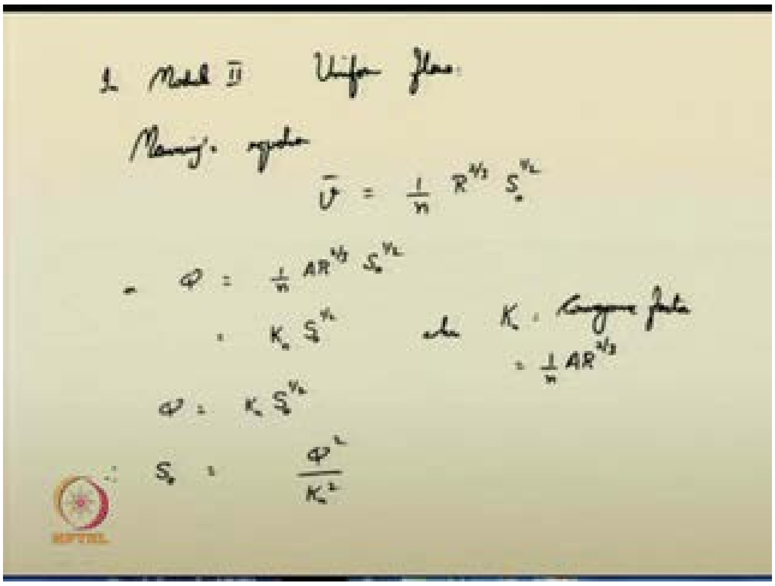

Now, in module two, in module two you had discussed the uniform flow. In uniform flow, uniform flows were discussed manning's equation, it was given as 1 by n R to the power of 2 S naught to the power of half or Q is equal to 1 by n A R to the power of 2 by 3 S naught to the power of half; that means, this is equal to some conveyance factor K n into S naught to the power of half, where conveyance factor is equal to 1 by n A R to the power of 2 by 3, if you know this thing.

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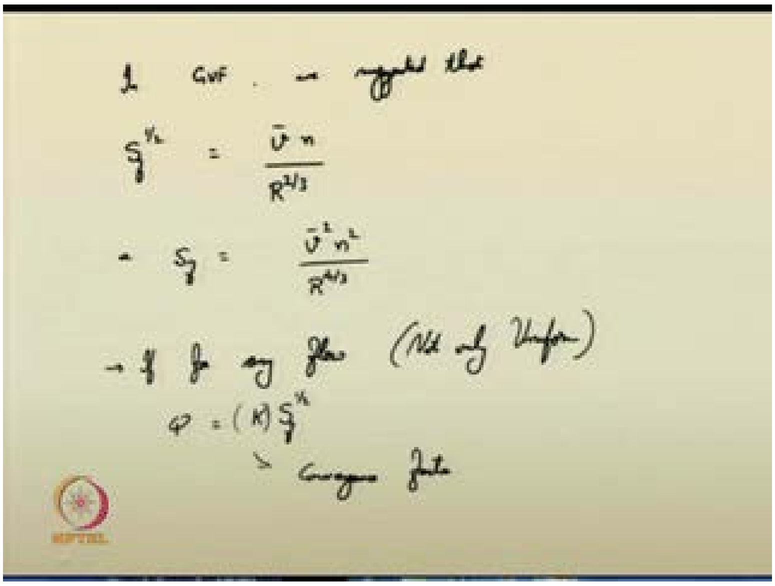

So, now in the uniform flow, if your discharge Q, if it is following a uniform flow pattern, it can be easily written in the following form. In gradually varied flow, we suggested, that the friction slope S f, it has to be computed using manning's equation, and you have to use the parameters that is the hydraulic radius as well as the area of the cross section, at that particular point, at that particular section what is the corresponding value of A and R, you have to substitute them, and find identify the this thing, so you identified the friction slope. So, this was given the following form; R to the power of 2 by 3, or S f is equal to v square n squared by R to the power of 4 by 3. So, now you have expressions; that is, S f, expressions for S f, you have expressions for S naught also here. S naught you can easily think now in this following term; therefore, S naught is equal to Q squared by K n square, so that thing you can think in that terms now.

If for any flow; that is not only uniform, if we apply the manning's equation, and if you see that, your discharge Q, it can be computed using a conveyance factor K into the friction slope, the square root of the friction slope, if you can device like this. Now this is the conveyance factor, this is your conveyance factor, so what does the if. This is just a empirical relationship, which we suggested formulated, means as we stated it here. Here you have suggested that, this entire quantity can now we call conveyance factor in the uniform flow. Similarly, in the other types of flow, you can suggest the discharge Q is equal to some conveyance factor K, into the square root of the friction slope S f like that if you can do that.

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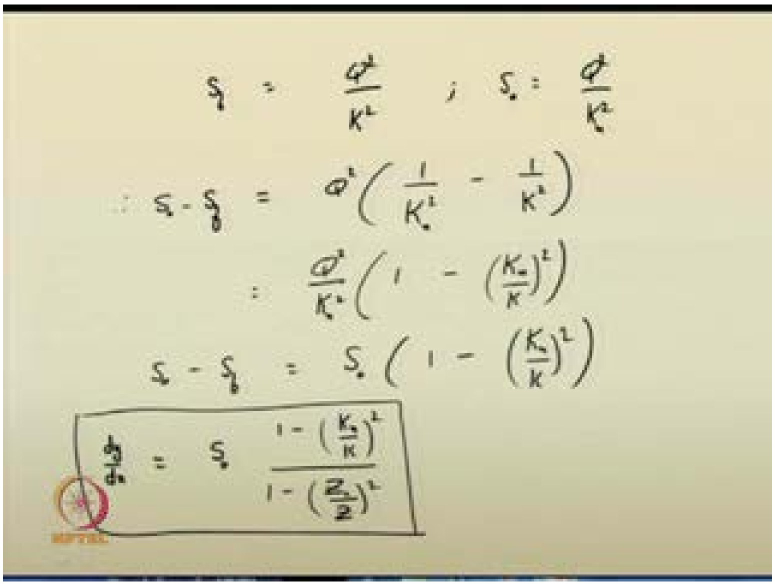

One can easily now derive, your S f or your energy slope friction slope, this is nothing but Q squared by K square. So, therefore, you also have S 0 is equal to Q squared by K n square; that is only in the case of uniform flow you have bed slope equal to friction slope. So, S 0 minus S f, this is now equal to Q square into 1 by K n square minus 1 by K square. So, what is Q square by K n square; that is nothing but your bed slope right Q square by K n square is nothing but . So, if I can take that out, Q square by K n square if I take it as a common thing, so what will be remaining here 1 minus K n by K whole square, like this you will get. So, S 0 minus S f is nothing but S 0 into 1 minus K n by K whole square. So, you got the numerator of your dynamic equation in the following form, it is quite easy now.

So, therefore, your dynamic equation becomes d y by d x equal to S 0 into 1 minus K n by K whole square by 1 minus z c by z whole square. So, your dynamic equation, I have now represented it using conveyance factors and section factors. Why I use this thing is that, by knowing the conveyance factor, by knowing the sections factor and all, you will be easily able to identify using the depth of flow, or based on the depth flow and all, what type of gradually varied flow is there. This can, this relation can now be used for classifying the gradually varied flow profiles; that is coming as a part of this lecture, we will see that.

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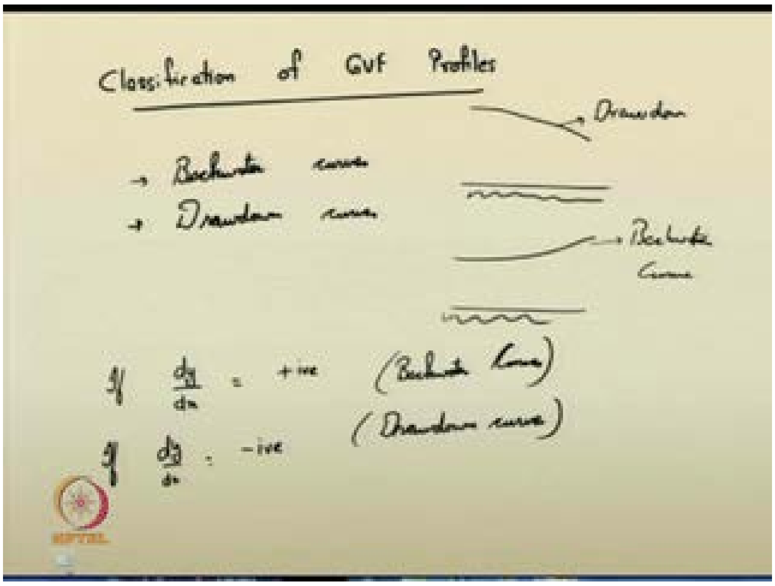


So, I can now write it as say classification of gradually varied flow profiles. So, if you recall the last days lecture, we had suggested that time, gradually varied flow like say, back water curves, draw down curves, if you recall them. We suggested that for any gradually varied flow, if the water surface decreases, if the height of the water surface decreases in the flow direction, then it is called flow down curve. If the height of the water increases, then this is called back water, the water surface curve is called back water curve. So, the same thing, how can you identify now. So, if in the dynamic equation; the d y by d x quantity, if this is a positive term. What do you mean by positive; that is a slope of the water surface it is increasing, we are not talking about the height of the water surface, if the slope of the water surface if it is if it is a positive value, then that will give you, the back water curves. You look at this back water curve, the slope d y by d x, it becomes positive here. Similarly in the draw down curve d y by d x if it is negative, then that will give you draw down curves.

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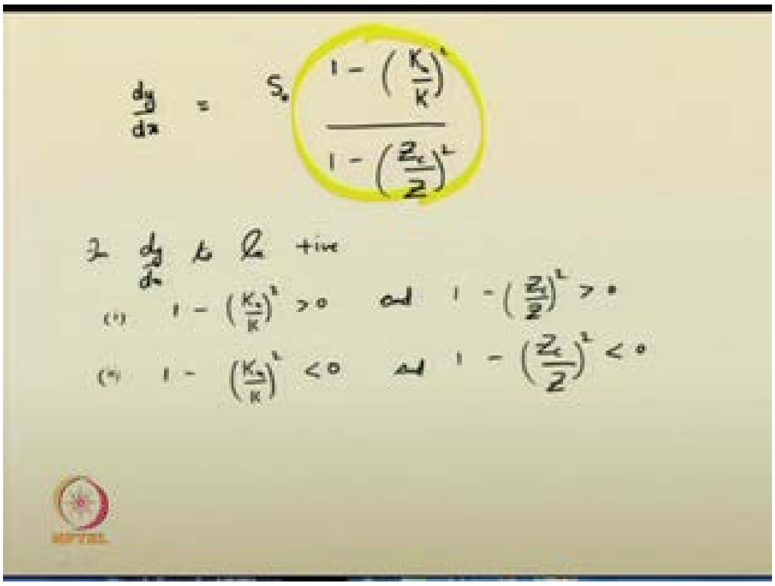

So, what are the conditions now, look in to the dynamic equation d y by d x is equal to S 0 into 1 minus K n by K whole square by 1 minus z c by z whole square. This is your dynamic equation using conveyance factors and section factor. So, what is the relationship, what is the criteria that will suggest that d y by d x has to be positive. For d y by d x to be positive, you can have two cases; that is you can have two cases. Either the numerator in that relationship 1 by K n by K whole squared. This should be greater than 0 1 minus z c by z whole square, this should also be greater than 0. If this is done, then this entire quantity, it becomes positive.

Similarly, the entire quantity can become positive, if 1 minus K n by K whole square, if this is negative, as well as 1 minus z c by z whole square less than 0 if this is also negative. Then also the entire quantity can become positive. So, there are two cases involved here for d y by d x to be positive. So, by giving this relationship what do you mean by this thing, what does this suggest? So, before that, let me introduce you related to the slope of the channel.

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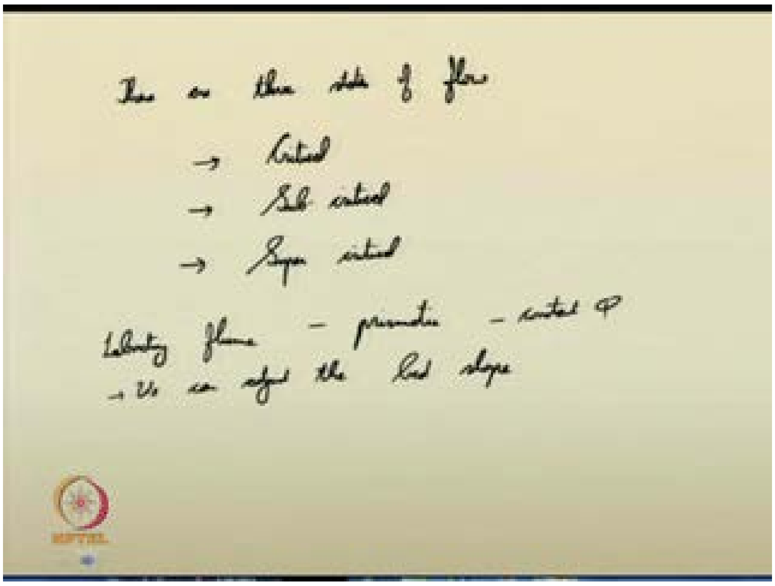

In the last class, we mentioned that, there are three states of flow. You can have critical flow, you can have sub critical flow, you can have super critical flow. One can easily do the laboratic experiments and all, you can see that, there are these three states of flow. Now based on these three states of flow; one can easily define the bed slopes of the various channels, in which category that bed slope of the channel is coming into the picture. For example now, let us consider a laboratory, flume that is prismatic in nature. A prismatic laboratory flume let us consider, it is having constant discharge Q, we can adjust the bed slope of the flume, so what does that mean.

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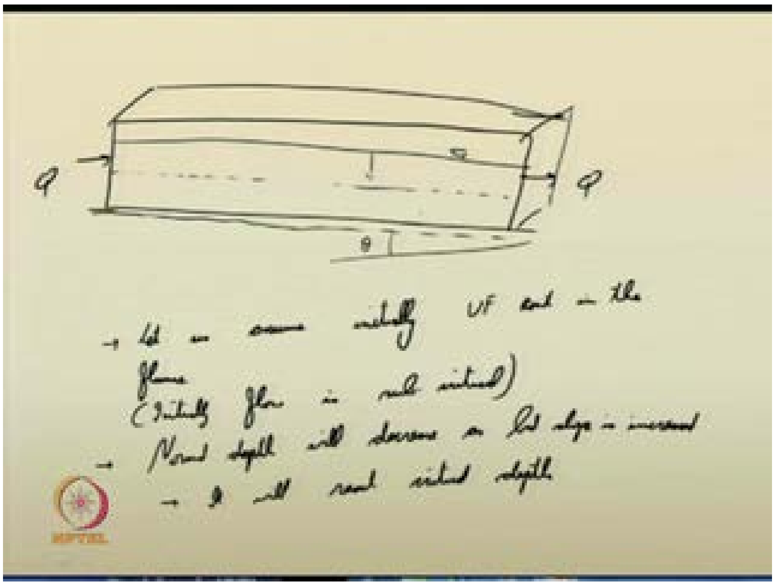

So, you have a laboratory flume like this; say this may be made of glass, and you have the bed like this. So, this slope of the bed, it can be adjusted by tilting it, you can rotate it, this theta it can be adjusted. So, it is giving you a steady discharge Q, Q is coming in, Q is going out of the flume. Say if it is a rectangular shape you can easily draw it like this also, or if it is any other shape you draw the appropriate shape. Now, the reason to show this flume is that, say if we say for this particular discharge, let us assume that, let us assume initially uniform flow exists in the flume, let us assume this thing, let us assume initially uniform flow exists in the flume. Also let us suggest that, the normal depth of the uniform flow is, above the critical depth; therefore, the flow is initially flow is sub critical. So, if we increase the slope of the bed now, if we increase the slope of the bed, you will see that the normal depth of the flow in the channel, it will start decreasing.

Say if this was the normal depth initially, and suppose if this was a critical depth, computed for the particular discharge in this section, if this was the. Then you will see that this normal depth, it will, if you increase the slope it will decrease; that is the normal depth will decrease, as bed slope is increased. So, it will decrease till, if you further go on increasing the thing, if you increase the bed slope, you will see that the normal depth will decrease, and it will reach the critical depth. So, for the given discharge in that uniform flow channel, for the given discharge, when you raise the bed, you have seen the normal depth decreased, it is decreasing, and it will reach the critical depth in that channel. So, that particular critical depth, whichever is there, whatever critical depth, or whatever that depth of the flow that is abiding the critical flow condition, it has been achieved for a particular slope of the bed.

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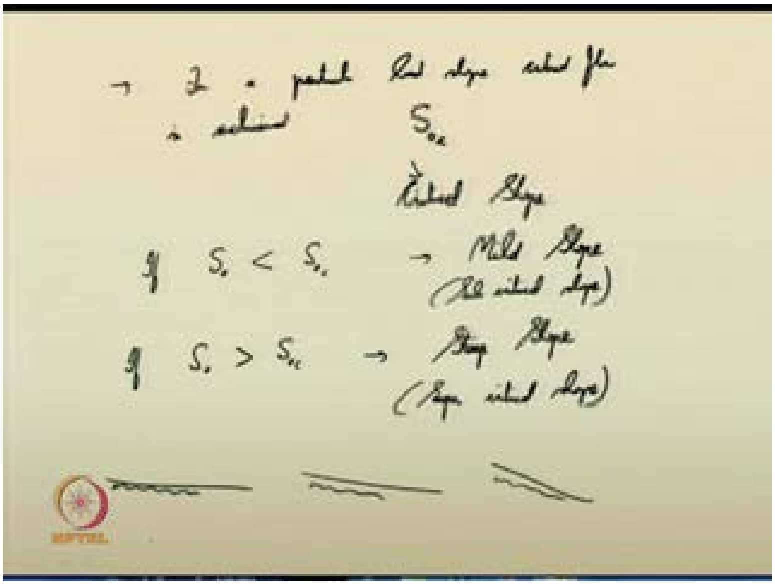

So, for a particular bed slope, critical flow is achieved. So, that bed slope we are now going to say as S 0 c; that is the critical bed slope, so this is called critical slope. So, if the actual bed slope if it is less than critical slope, then that is called mild slope. You have seen the, on further increasing, in this thing on further increase in the slope of the channel, the normal depth of the flow; it will further decrease below the critical depth. Then the flow becomes super critical, and the corresponding channel slope is called super critical flow. It is sub critical slope, it is also called sub critical slope, and if S 0 greater than S 0 c, then the slope is called steep slope or super critical slope. So, one can see in the figure, three different types of slope. I can just show it just like this also say mild slope, critical slope, steep slope; like that one can easily visualize for the discharge, for the same discharge, you can easily visualize three type of slopes, in the channel, so like this one can categories them.

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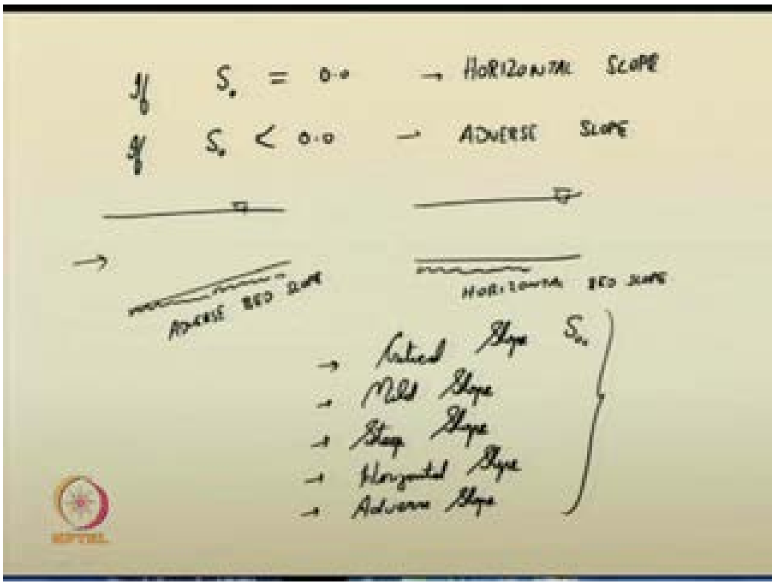


In addition to this, if the bed slopes of the actual flow condition, if it is equal to 0, then those flows are called horizontal slope channels, those flows are called horizontal slope channels. If S 0 what happens if it is less than 0, your bed slope is, although the flow is like this, your bed it is racing in the flow direction, so it is having an adverse slope. So, such conditions are called adverse slope; this is adverse bed slope, and this is horizontal bed slope. So, why we have discussed first the beds condition is, because based on these bed conditions, now, one can identify different types of gradually varied flow in the channel, let us see that. So, I hope you are now able to understand; say there are mainly now, the categories of bed slopes are, critical slope S 0 c, mild slope, steep slope, horizontal slope, and adverse slope, so these are the classifications of bed slope. Based on this bed slope, you can find different combinations of gradually varied flow, how can you identify that.

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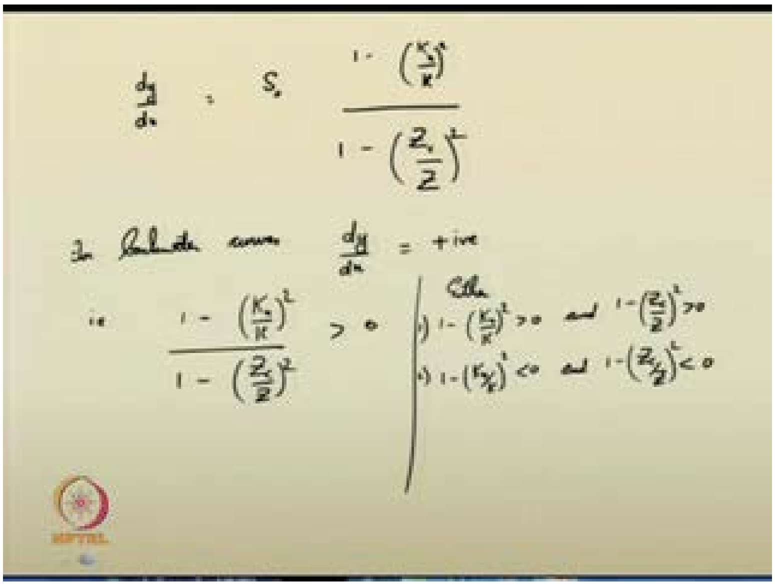

So, let us come back into the equation, our dynamic equation S 0 1 minus K n by K whole square by 1 minus z c by z whole square. So, in this equation, for, we suggested for back water curves; d y by d x is equal to positive; that is you can have 1 minus K n by K whole square by 1 minus z c by z whole square greater than 0. So, the conditions we suggested that 1 minus K n by, if you recall them it can be such that, either 1 minus K n by K whole square should be greater than 0, and 1 minus z c by z whole square, this was one condition. The second condition was 1 minus K n by K whole square less than 0 z c by z less than 0. So, these two conditions has to be satisfy, means either one of this two conditions has to be satisfy, so that this entire quantity should be greater than 0. So, what does this physically mean now.

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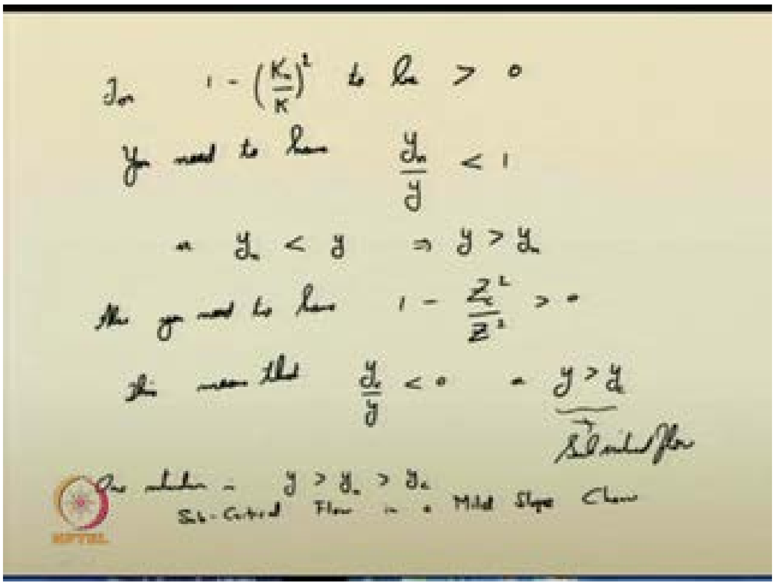

For 1 minus K n by K whole square greater than 0 or for 1 minus this quantity, to be greater than 0, what should be the condition now. You need to have, the normal depth y n by y, it should be less than 1, then only this entire quantity can be greater than 0, or your normal depth should be less than the depth of the flow, this is one criteria, or you can suggest y greater than y n. Also you need to have 1 minus z c square by z square, greater than 0; this means that. So, this mean that your y c by y should be less than 0, or your depth of flow should be greater than your critical depth. So, this means what does. You can now understand the thing. So, this means your depth of flow should be greater than critical flow, this implies subcritical flow.

So, this implies sub critical flow, also your depth of flow should be greater than the normal depth. One situation for this is, on situation is the entire depth of flow is greater than both the normal depth, as well as the normal depth is greater than the critical flow. So, this occurs in a sub critical flow; that is the sub critical flow occurs in a sub mild channel, that can also happen. So, may I beg pardon, I am just rubbing this portion, on situation you can imagine here is, y should be greater than normal depth, y should be greater than critical depth, if I write that y is greater than y n greater than y c. So, this suggests sub critical flow in a mild slope channel.

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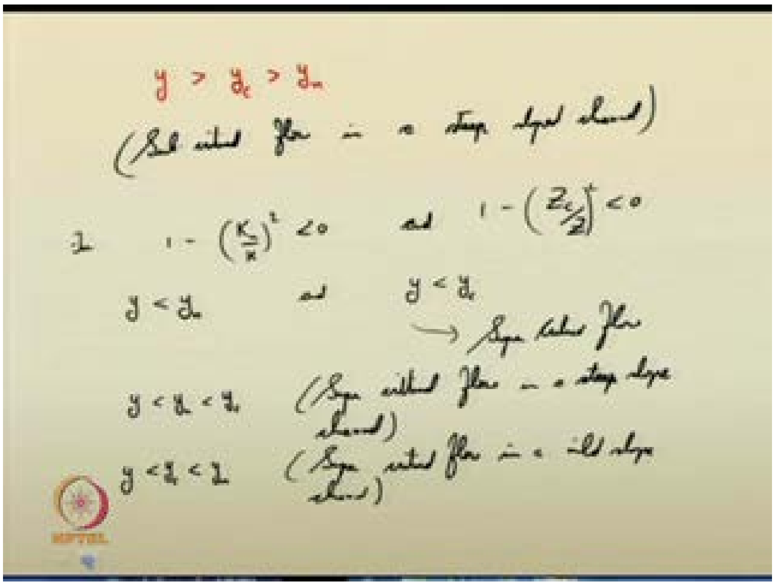

Another situation is, if your y if it is greater than y c, which it is greater than the normal depth y n. Then you can suggest that, if this situation occurs, this suggests sub critical flow in a steep slope channel, that also one can easily witness, so like one can infer the things. Now for the situation; 1 minus K n by K whole square less than 0 and 1 minus z c by z whole square less than 0. For these criteria you can suggest that your depth of flow should be less than of normal depth, as well as your depth of flow should be less than critical flow, so critical flow. So, this means that, super critical flow exists, super critical flow has to be existed, then only you will get the back water curve. So, this means the sup. For example, if y less than y n less than y c, this implies super critical flow in a steep slope channel, for your y less than y c, but less than y n, this implies super critical flow in a mild slope channel. So, similarly for the draw down curves, how the d y by d x has to be negative and all, and subsequently further classification of the gradually varied flow profiles and all, we will discuss it in the next class.

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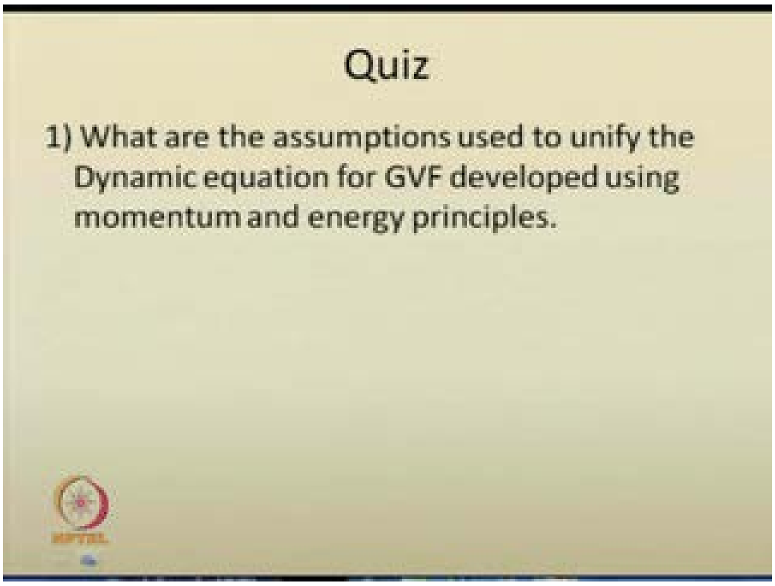


The quiz for today's lecture is as follows; quiz the first question is, what are the assumptions used, to unify the dynamic equation for gradually varied flow, using momentum and energy principle. We had used certain assumptions to unify both the equations, so what are those assumptions, please take them.

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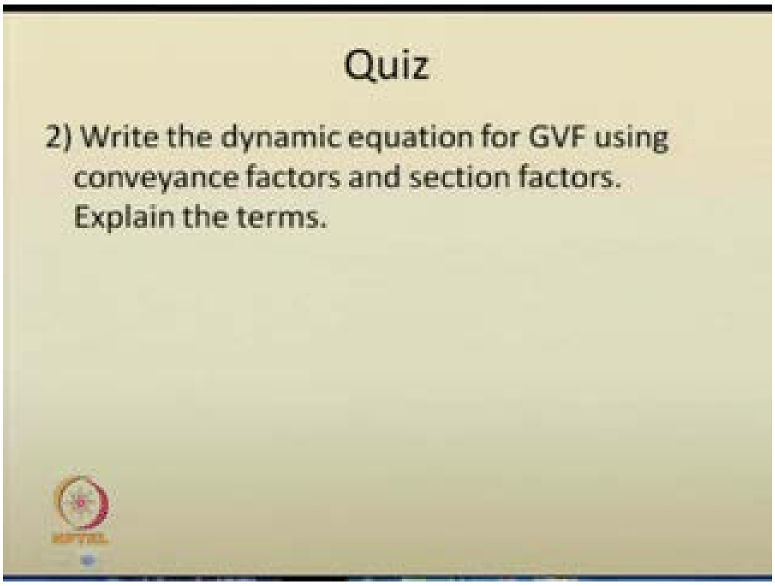


Next question is; write the dynamic equation for gradually varied flow, using the conveyance and section factors, explain the terms in that equation. I m repeating the question write the dynamic equation for gradually varied flow, using conveyance factors and section factors.

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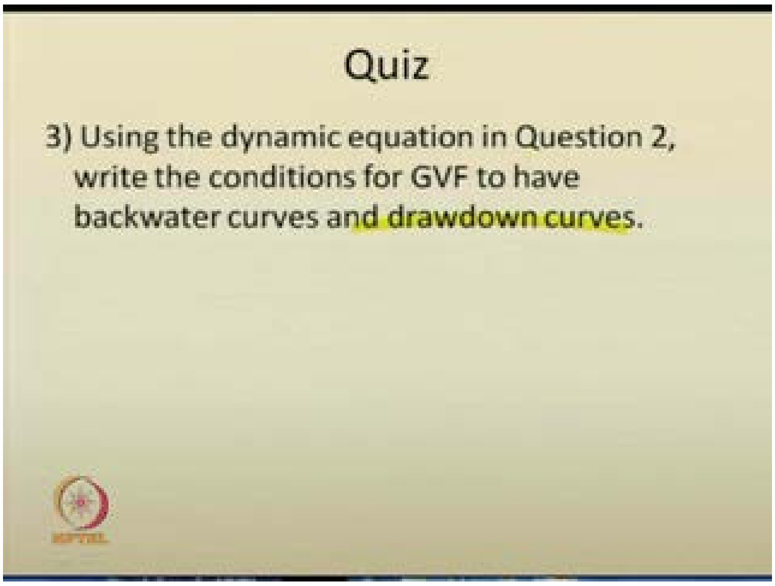


Your third question, now using the dynamic equation in question two, write the conditions for gradually varied flow to have back water curves, and draw down curves. Or especially you write only the back waters, what is a condition for gradually varied flow, so you avoid this portion. Only write gradually varied flow to have back water curves, we have mentioned that conditions you just enumerate them.

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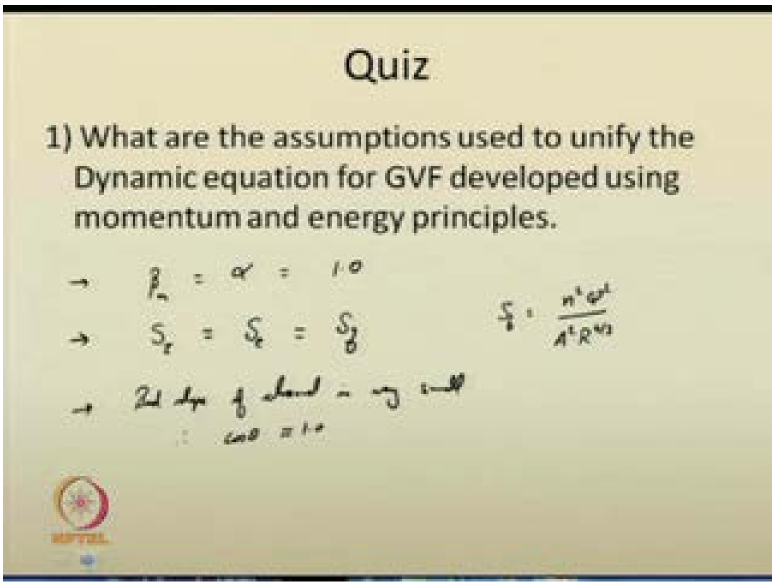

So, the solution for the first question for the, you know that what are the assumptions involved here, it is a straight forward question. You can have the momentum correction factor, as well as the energy correction factor approximately equal to 1 that is the velocity across the cross section is well in uniform, this is one assumption. The second assumption suggested was, the shear slope, is approximately equal to the energy slope, both are equated as friction slope, where friction slope is computed using manning's equation. The third assumption used is, bed slope of channel, is very small. Therefore, cos theta approximately equal to 1, based on these assumptions we had unified the dynamic equations for gradually varied flow.

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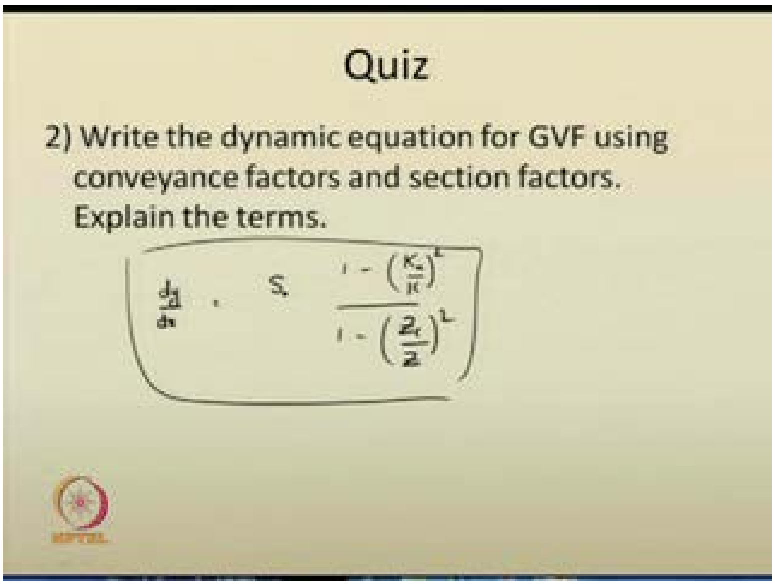


The second question; write the dynamic equation of gradually varied flow, using conveyance factors and section factors. So, I am just writing the equation d y by d x this is equal to bed slope into 1 minus K n by K; that is K n is the conveyance factor for normal flow, K is a conveyance factor for any existing flow by 1 minus z c by z whole square. This is the dynamic equation, using conveyance factors and section factors. So, you K n is the section factor for the normal flow, z c is the sorry cocaine is the conveyance factor, z c is a section factor for critical flow, z is a section factor for any for the existing flow, S 0 is a bed slope. Using the dynamic equation in question two, write the conditions for gradually varied flow to have back water curves.

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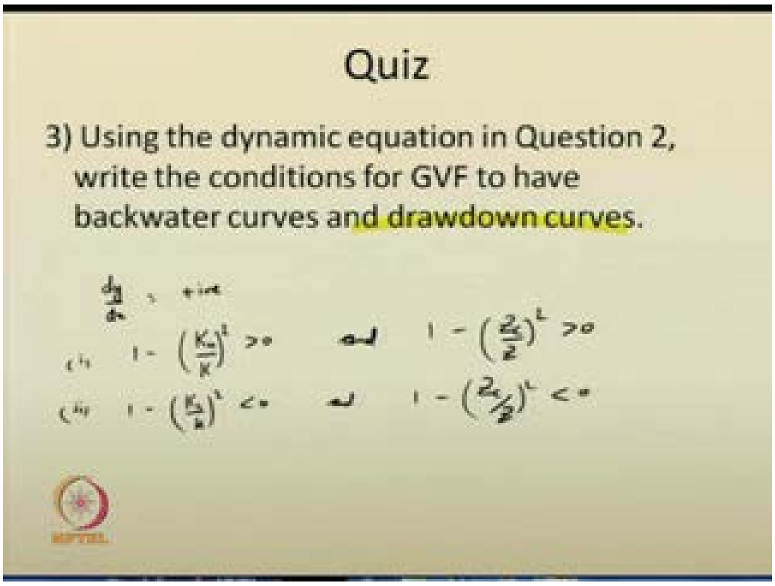


So, in that equation, for back water curves to exists, your d y by d x has to be positive. So, the first condition we suggested was there; 1 minus K n by K whole square should be greater than 0, and 1 minus z c by z whole square should be greater than 0. The second condition was 1 minus K n by K whole square less than 0 and 1 minus z c by z whole square should be less than. So, these two are the criteria for, criteria that has we used in the dynamic equation, for the flow to have back water curves.

Thank you.