Advanced Hydraulics
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Module - 3
Varied Flows
Lecture - 2
Gradually Varied Flow Equations

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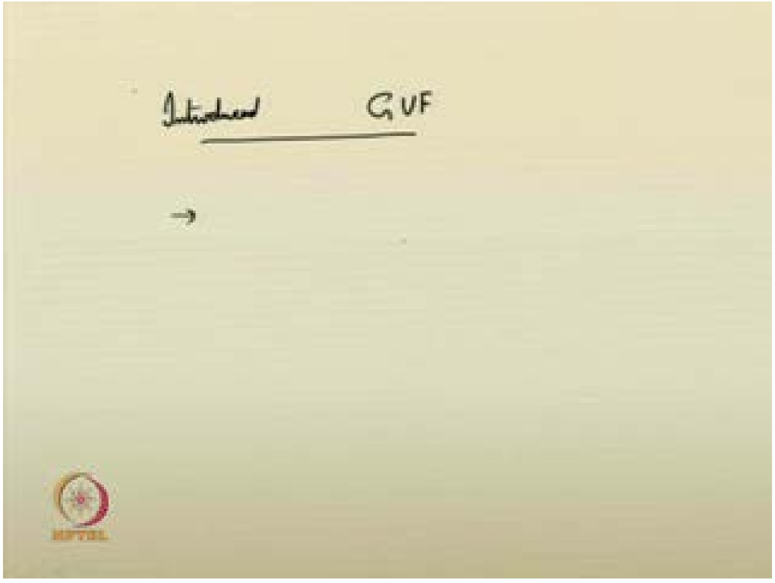

We are back into our lecture series of on advance hydraulics, we are in the third module; varied flows. In the last class we discussed on introducing or we introduced you gradually varied flow. So, we also suggested you, what is meant by gradually varied flow, where they are seen. We also discussed on developing the conservation equation, especially the conservation of mass equation was already developed to you. Conservation of momentum equation we started, and we suggest it will be completed today the conservation of momentum equation. Then, we will see that we will continue on those aspects.

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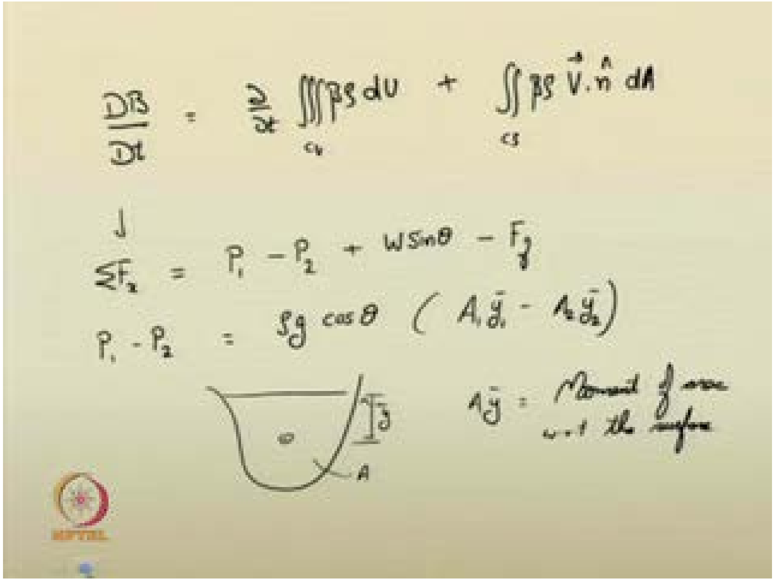

If you recall the conservation of momentum equation, we developed it from the general Reynolds transport theorem. So, we had discussed all the terminologies of these equations earlier also, and as we suggested that the flow is predominantly one-dimensional, we are just going to consider only the momentum. You know that there is net change of momentum with respect to time is equal to, the net force acting in the control volume. So, therefore, this quantity was given as sigma F x, and if you recall them, the net forces acting on the control volume where, pressure forces on the left side, pressure forces on the right side, the component of gravity adding the flow, then the frictional forces, these things we have suggested.

If you have we have also seen that, the component p 1 minus p 2, it was derived in the last class, it is rho g cos theta into A 1 y 1 bar minus A 2 y 2 bar; that is for any cross section, if this is s centroidal area; that is the depth to the centroid from the surface if it is y bar, and if this area is A, then A y bar is equal to moment of area, with respect to the surface, this we have discussed earlier. So, we have two pro sectional areas on the downstream; that is, on upstream; that is section 1 1, downstream section 2 2. So, therefore, p 1 minus p 2 are derived on this way.

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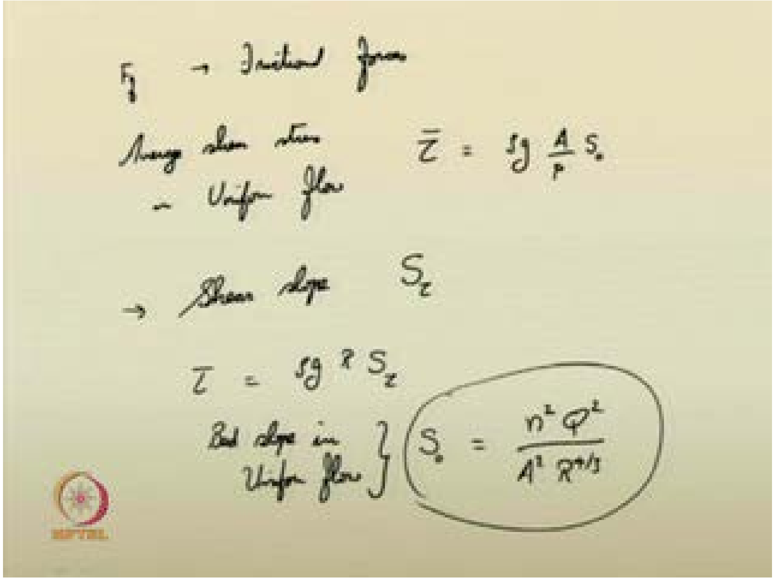

We suggested that for the frictional forces F F, to obtain the frictional forces F F, one need to understand the shear stresses acting on the boundary surfaces of the control volume. So, if you recall in the uniform flow also, if you recall in the uniform flow, we had seen that, average shear stress in uniform flow, while deriving the Chezy's equation, it was given as; rho g A by p into S naught, if you recall them, the average shear stress was given in of the following form. Same quantity we will now applied here. Now instead of the bed slope, here in the non uniform flow, or in the gradually varied flow, we will be using a new term call shear flow, so we will be using a new term call shear flow, and we will be cooperating them in the average here stress equations. So, the average shear stress in the gradually varied flow, can be given as rho g R S 2.

So, this average shear slope, how do you how do you compute shear slope. So, in the uniform flow, your bed slope, your water surface slope, energy slope all were same, all were equal. Means there were no changes in them, therefore you were able to use the bed slope in computation in the manning's equation and all, you could directly incorporate bed slope. And even the bed slope of was computed, if you recall them. I hope it is that bed slope in shears bed slope in uniform flow. It was given as manning's roughness square of manning's roughness coefficient, A square R to the power of 4 by 3, if you recall this equation, the same equation now we are going to incorporate to calculate shear slope, in gradually varied flow.

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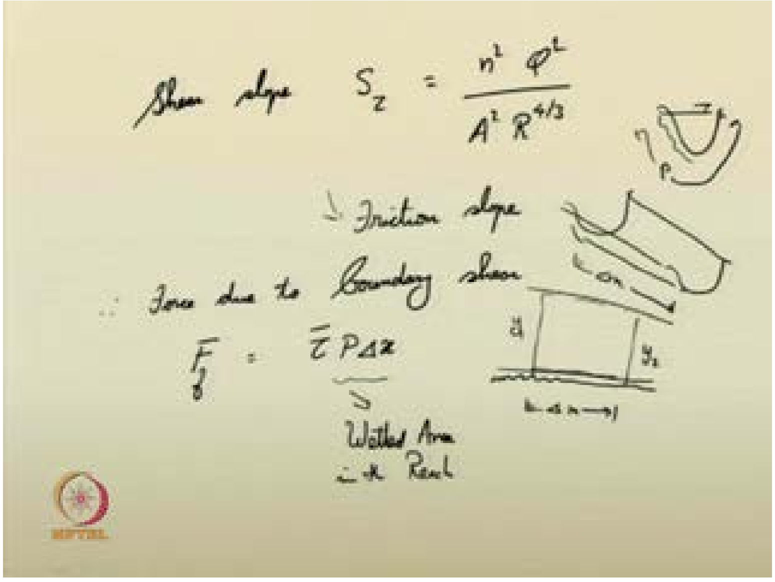

So, shear slope is seen only in non uniform flow. So, in uniform flow we do not have concept called shear slope. So, here the shear slope S 2, you will be using the same manning's equation, and you will be computing the following thing; n square Q square A square R to the power of 4 by 3, so this also it is also called friction slope. So, in some of the literatures you may see this terms friction slope. Therefore, force due to friction or boundary shear, I can give this as F F, this is equal to shear stress into the wetted perimeter, into the length. Just recall the figure, or channel bed is like this, two sections, control volume. If they are, the two sections are separated by A distance del x, this is having depth y 1 y 2 then the wetted perimeter P into del x, that will give you the area, where the water is interacting with the channel boundary.

So, wherever water is interacting the channel boundary, the friction force will be encountered, and that friction forces n by shear stress into the wetted area, so this is nothing but your wetted area in the reach. So, I can now suggest that, or if let me for your benefit again, you cannot visualize the thing, just consider the two sections of the channel. Now these are the wetted areas, and if this length is del x, and if the for any of the cross section, the wetted perimeter is given by the following form. So, P into del x; that will give you the wetted area, so the same thing we have incorporated.

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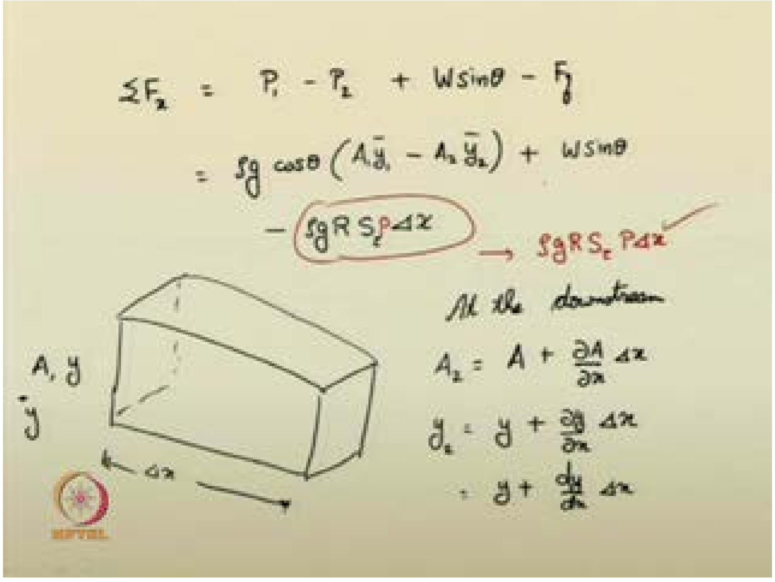

So the net force, in the x direction. Please recall that the flow is one dimensional, so it is given as P 1 minus P 2 plus w sin theta minus F F. So, p 1 minus p 2 you know what is that, it is rho g cos theta; theta is the angle of the bed of the channel, with respect to horizontal, A 1 y 1 bar minus A 2 y 2 bar plus w sin theta minus rho g R into S 2 del x. So, let me draw the channel reach again. So, in this channel reach the control volume whichever you are taking into account. If the upstream area, is separated by a distance del x, they are separated by a distance del x; say if you are suggesting that, the upstream area is A, the depth of flow in the upstream if it is y, the depth of the centroid in the upstream if it is y bar, then you can also suggest now the following quantities; at the downstream you may give A 2 is equal to nothing but the upstream area plus dou A by dou x into del x, because this area, it is varying with respect to x only, that you know; that means, the change in area this is, the property of x in flow direction. Similarly, y y 2 can be given as y plus dou by dou x into del x, or we know that y is varying only with respect x, you can write this as y plus d y by d x into del x.

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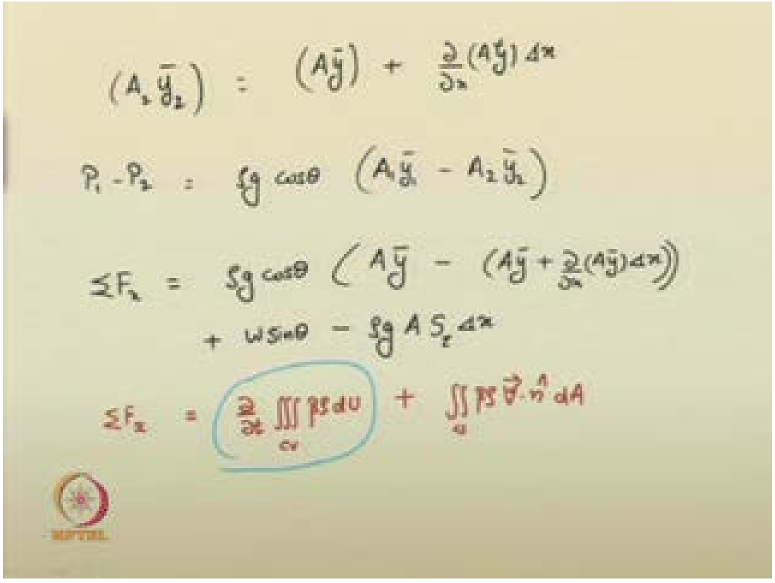

Similarly, one can also suggest the quantity A y bar, if you take into account, the moment of the area. So, A y bar this can also be given as; say in the section two is nothing but A y bar in the section one, plus dou by dou x of del A y bar into del x. Like that also one can easily give isn't it, because you have seen in the sigma f x equation, this is rho g cos theta A 1 y 1 bar minus A 2 y 2 bar, or I can say p 1 minus p 2 similar. So, this quantity now can be easily substituted A 2 y 2 bar, can be easily substituted here, so I can write it like this now, A 2 y 2 bar is equal to A y bar plus dou by dou x of A y bar into del x. So, on using this relationships, you will see that, the net force in the x direction, it can be now written as rho g cos theta A y bar minus A y bar plus dou by dou x of A y bar into del x plus w sin theta minus rho g A S 2 del x.

So, why we need to write it like this, in the previous slide if you recall them, I think I made a mistake in the earlier slide, if you see here, this is rho g S 2 R S 2 into p into del x; that is the, so please correct it this quantity rho g R S 2 p into del x, p into del x is your wetted perimeter, so you have to incorporate this quantity. So, I had made mistake there, so you correct them, so that P into R; that gives you the area of the cross section A, so I have directly incorporated A here. In the equation sigma f x is equal to beta rho d u plus V dot n d A, if you recall this equation now one can write that, as the flow is steady state these component in the equation, it will vanish of…

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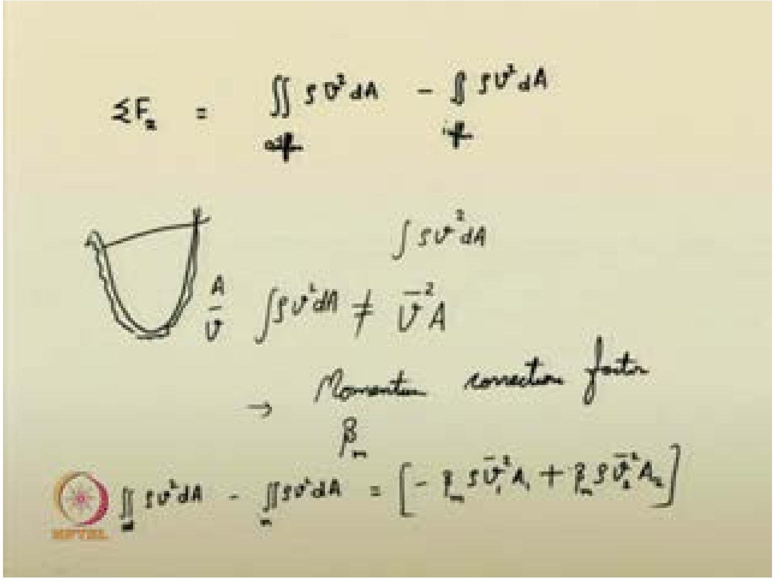

Therefore, you can write now, out flux rho V square d A minus influx rho V square d A, so what does this mean. So, what does this mean to you, as if you take any arbitrage cross section; say this is section, it is having area A. If the average velocity is V, the quantity, this quantity; that is a aerial integral, that is why we are giving aerial integral in the that form. So, the aerial integral, one can now write this quantity because rho V square d A, you know that it can. This is not the average velocity; this is only the velocity at the any at the any point in the section. So, this quantity now can be represented in the form of average velocity V square into A, but they are not same, because you know the momentum, this quantity signifies the momentum flux, across this control surface, or across this cross section, what is the momentum flux transmitted.

So, the momentum flux integral rho V square d A is not equal to the average quantity. If you take the average velocity into A that momentum flux, or whatever momentum flux you obtain through the average velocity; that will not be same. So, you need to means you have already seen them you have incorporated momentum correction factor earlier. So, that momentum correction factor, beta m need to be incorporated here, in subsequently we can address the issue here. So, this is, the above quantity now will become outflow rho V square d A minus inflow rho V square d A. This can be now easily written as, minus beta m rho V 1; that is the average velocity in the section one; that is why I put the bar here, V 1 bar square into A 1 plus beta m rho V 2 square A 2.

So, we are taking the momentum correction factor through almost same. So, beta m and rho can be taken out now in this expression, so you will get.

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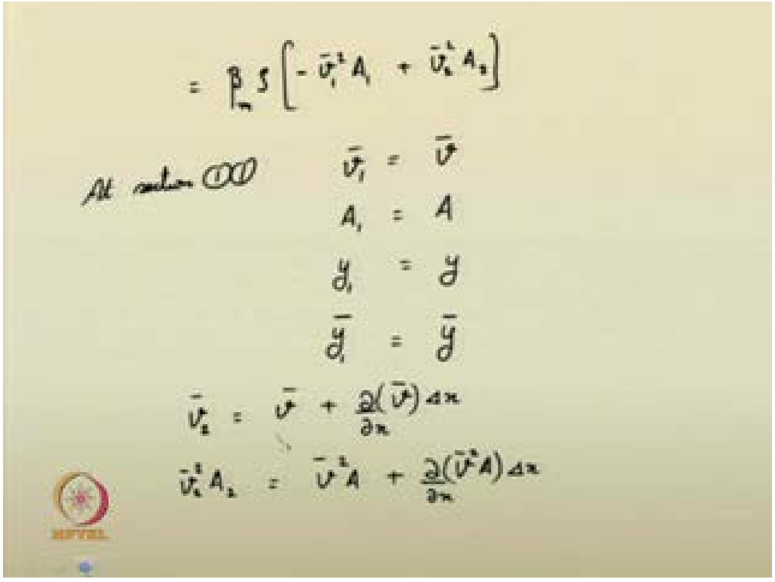

This is equal to beta m rho V 1 squared A 1 plus V 2 square A 2. So, as again mentioned earlier, at section 1 1 V 1 bar, we are taking it as v bar A 1, we are taking it as A, depth of flow y 1, y 1 we are talking it as y, the centroid depth y 1 bar, this was been taken as y bar. We have done it earlier also, same quantity, we can same relationship we can subsequently incorporated it here. You will see that V 2 bar is nothing but V 1 bar plus dou by dou x of v bar into del x; that is the separation the change in velocity, as it move in the x direction is given by this thing. This is the velocity in the section 1 1, and this is the change in velocity, as it travels additions del x, so that relationship (( )) given here in this following form. Now you can also give the following thing, the following product v 2 square A 2 is nothing but v square a plus dou by dou x. All these quantities we are using the same first principles, so there is no much complexity involve, we are suggesting that this entire quantity, it is being changed using this relationship, as it goes from upstream to downstream. So, this relationship also we can incorporate.

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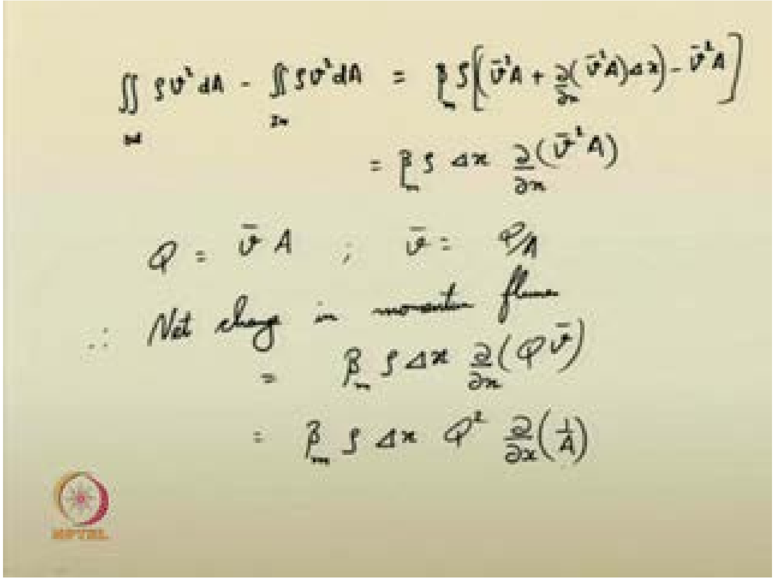

So, we will get the following equation now; rho v square d A, this is equal to beta m rho v square A plus dou by dou x of v square A del x minus v square A equal to beta m rho del x dou by dou x of v square A. So, you know beta A is the momentum correction factor, also Q is a constant quantity, v bar A. Therefore, net change in momentum fluxes, across the control volume, across the control surfaces of the volume, this is equal to beta m rho del x dou by dou x of Q v bar. I hope you agree, and we writing this thing, or again one can infer v is equal to Q by A, like this also you can infer. Then I can change this quantity as, beta m rho del x, Q is a constant value, so Q square I can take it out, and this is dou by dou x of 1 by A. Well fine we can easily write it, so you can write this quantity.

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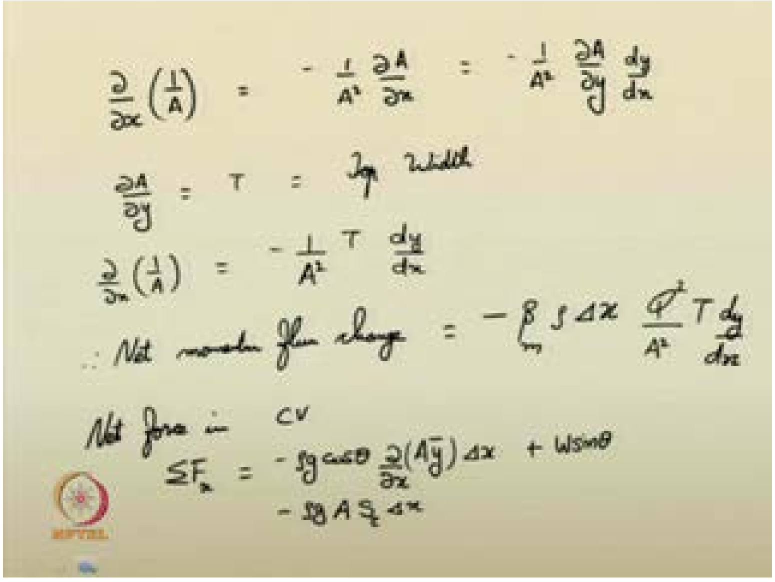

Now, you can see their dou by dou x of 1 by A, this is nothing but 1 minus A square dou A by dou x, or you can also see that 1 minus 1 by A square dou A by dou y into d y by d x, like this also you can write. Recall our earlier lectures dou A by dou y throughout we have given this as top width of the channel, top width of the flow of the channel. We have been dealing it in almost all of the classes, so that same thing I am going to adopt it here. So, dou by dou x of 1 by A, this is nothing but minus 1 by A square T d y by d x. Therefore, net momentum flux change is equal to, see the negative quantity is appeared here, so I have to incorporate that here, minus beta m rho del x q square by A square T d y by d x. So, come back into our net force in the control volume net force in control volume sigma F x, this is equal to minus rho g cos theta, or let me show you that equation. So, sigma F x is equal to rho g cos theta A y bar minus A y bar plus this entire quantity. So, these things get cancelled off, so I can use the remaining terms minus rho g cos theta dou by dou x of A y bar into del x plus W sin theta minus rho g A into the shear slope S 2 into del x.

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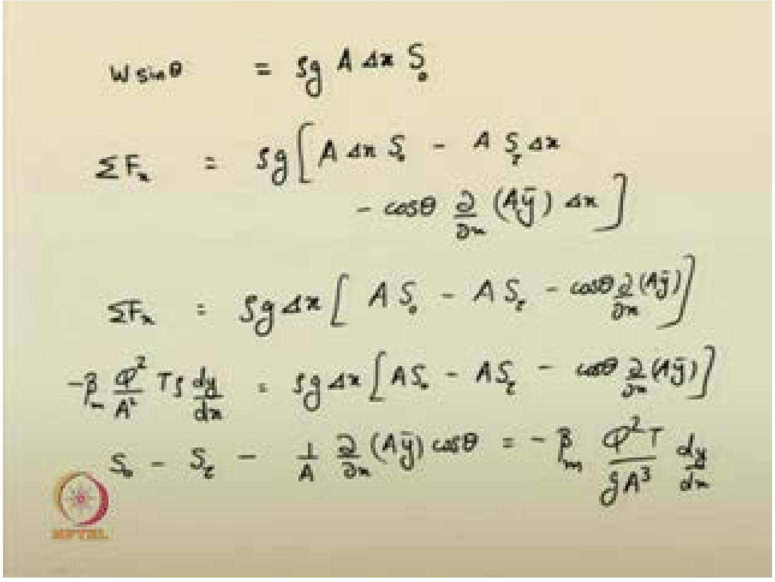

So, you know W sin theta is nothing but rho g into area into del x, and the slope sin theta, what is sign theta, it is nothing but your bed slope S 0 directly incorporate. So, therefore, your sigma F x is nothing but rho g into A del x S naught minus A into S 2 del x minus cos theta dou by dou x of A y bar into del x. So, again rearrange the terms here, you will see, what can you get here now; sigma f x simplify the terms all the things, you will get very good expression now; rho g del x A into S 0 minus A into S 2 minus cos theta dou by dou x of A y bar. So, dou by dou x of A y bar, we have already seen that earlier, substitute those quantity now, what will you get. You see here, I can take the quantity now as sigma F x is equal to minus beta m Q squared by A squared T d y by d x is equal to rho g del x A S 0 A S 2 minus cos theta dou by dou x of A y bar.

So, you can cancel rho, as it is a incompressible liquid and all, you see that rho as come out, you can cancel them, you can also take A out from here, you can incorporate it here, you will get means very good relationship now. I can easily write now this as S 0 minus; that is the bed slope minus the shear slope, this quantity minus 1 by A of dou by dou x of A y bar cos theta, this is nothing but equal to minus of beta m Q square T by g A cube. So, I hope you remember this particular term Q square T by g A cube, you have seen in some of other portions also. So, how the terms they are all coinciding, how they have, how the physical significance are coming into picture, I hope you are getting it clear now. So, like this, this equation is obtained.

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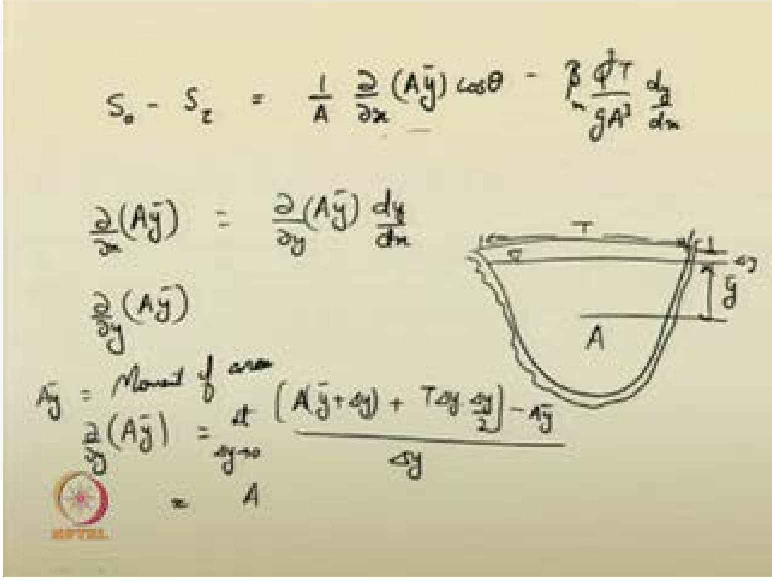

Again I can rearrange them; S 0 minus S 2 nothing but equal to 1 by A dou by dou x of A y bar cos theta minus beta m Q square T by g A cube d y by d x. Let us see what is the term dou by dou x of A y bar, this particular term. This I can now write it as A y bar into d y by d x; the differentiation rule, dou by dou y of A y bar what it could be. As we are mentioning it earlier also, this is moment of area with respect to the water surface. So, if you have the section like this, you have your cross section, and if this is the area A, and the centroid of this area is y bar. So, A y bar is the moment of this area, with respect to the surface of water. If the surface of the water changes; say if it increase by height del y, then how that change is incorporated in this thing. So, that can be used for getting this particular derivative dou by dou y of A y bar.

So, I can write this as now dou by dou y of A y bar is nothing but as limit del y tends to 0 A into y bar plus del y plus, this is the top width T, this width is T. So, T into del y into del y by 2 minus A into y bar; that was the previous moment of the area if you recall them divided by del y. So, as limits del y tends to 0 what happens to this quantity A y bar A y bar gets cancelled of, and A del y term remains T del y square by 2 also remains 1 del y is already cancelled out, and del y is instating to 0, you will get this quantity as a area A itself.

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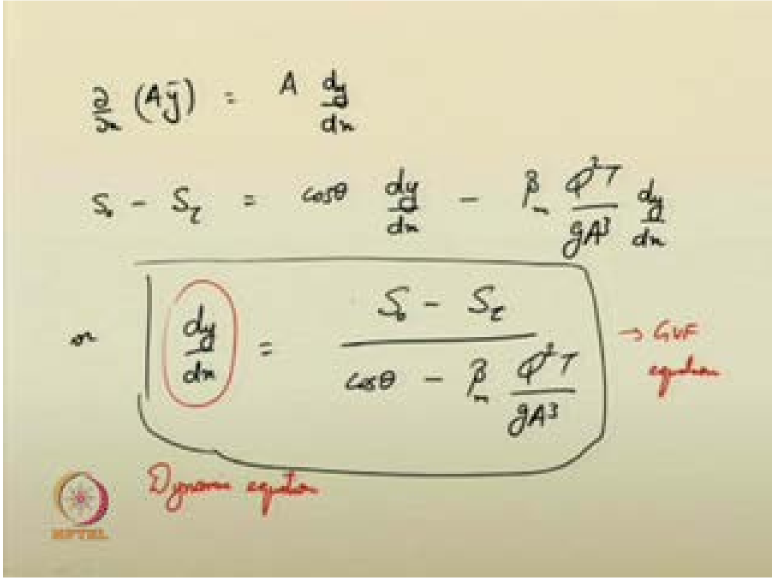

So, wonderful, we can now directly write this dou by dou x of A y bar as; A into d y by d x, now you write this directly. So, what happens to our momentum question is 0 minus S 2, S 2 is equal to cos theta into d y by d x now minus beta m Q square T by g A cube d y by d x or your d y by d x this is nothing but S 0 minus S 2 by cos theta minus beta m Q square T by g A cube. So, this is the most fundamental equation in the gradually varied flow. What is the physical significance, what did I obtain after all these derivative, of the all these derivations whatever we have done here, what is the thinner obtaining. You are obtaining the quantity, the slope of the water surface. So, please recall gradually varied flow, what is meant by gradually varied flow. Gradually varied flow, we suggested that the surface the depth of the water, decreases or increases gradually, as it flows from upstream to downstream.

So, that change in the depth of the water surface, the depth of the water; that is signified by this particular quantity d y by d x. So, it also signifies the change in slope of the water surface. So, that quantity is obtained by the following relationship. This is the most fundamental equation in the gradually varied flow. So, if you have to solve any gradually varied flow problem, you have to solve this equation. It is also call the dynamic equation for gradually varied flow. This is also called dynamic equation, why it is call dynamic; that is this is a gradually varied flow equation a gradually varied flow equation as described above, it is also called dynamic equation for the only reason that, this equation was derive using momentum conservation, you have developed it the using the equation of forces; therefore, this is call dynamic equation. You can also suggest it as a differential equation for gradually varied flow, we will see how, what happens if you use the conservation of energy.

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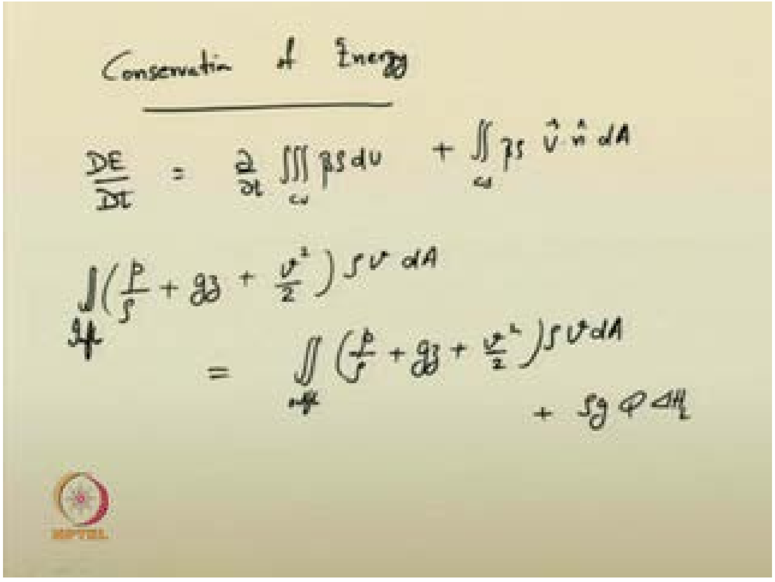

So, I am not going to go deep into this portion, conservation of energy if you use in the, if you use control volume, conservation of energy. So, I can write the equation now D E by D T as dou by dou t of beta rho V dot n d A. So, they are extensive property, B is now equal to energy of the system, or energy in control volume; small beta the corresponding intensive property, all those things club. And as the flow, you know that the material derivative of energy then it will be 0; that is energy can be neither created nor be destroyed.

And another case as this is steady flow condition, this quantity will also the first, the left hand side term will also is also 0, and first among the right hand side that is also 0, you will get the following relationship now; that is this particular quantity, it can be given as, in the inflow section, the intensive property this thing given as p by rho plus g z plus v square by 2 rho v d A. This is equal to in the out flow section p by rho plus g z plus v square by 2 rho v d A plus rho g Q into, the head loss between the two sections; section one and section 1 1 and sections 2 2. So, Q is the steady discharge, del H L is the head loss, it is also called the mechanical, loss of mechanical energy per unit weight.

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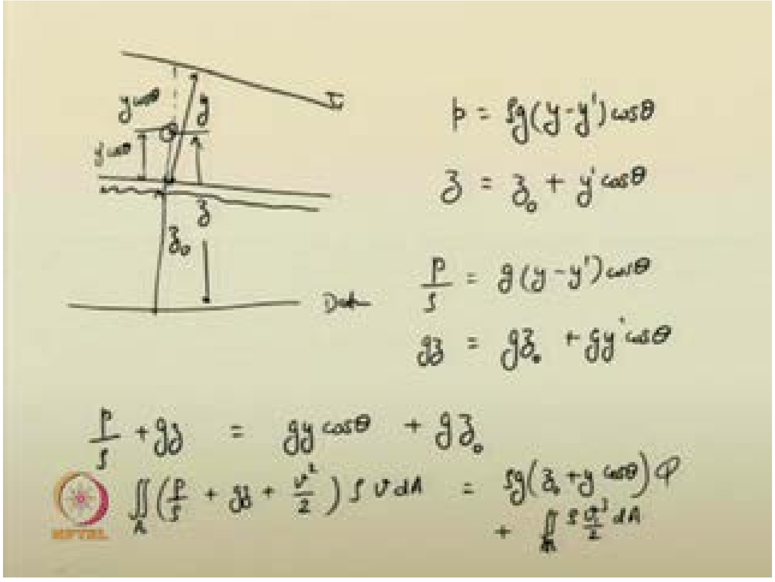

So, considered the any section portion any reach, this is the water surface depth. Then generally we give this as the y depth of flow y, this particular quantity, the vertical portion it is y cos theta; say if you are considering any datum line, then the bed at this location, it is elevated at height z 0 from the datum. And if you want to measure any quantity from the bottom, as we have thin earlier, you can suggest that; say for this point particular point. This is now, say y it is y dash cos theta, let us give it in that form, so y dash cos theta, one can easily suggest that, pressure at that point p, as earlier we are seen rho g into y minus y dash cos theta. Your datum head of that particular point, this particular point, that can be given as z, so that z is equal to z 0 plus y dash cos theta. So, what do you get from this things, you will see that from these two relationships p by rho, this is equal to g y minus y dash cos theta, and g z, this will be equal to g 0 plus g y dash cos theta.

So, therefore, p by rho plus g said quantity, I can easily write, just add those things I can easily write as, g y cos theta plus g z 0. Other terms get cancelled of, so you can get these things, why I have written is that. If you recall in the section A 1 inflow, you had that this particular term p by rho plus; that is the intensive property in the equation, so we got this term, so rho v d A. This can be easily now written as this entire quantity for the section, it is match now. So, rho g, it is rho g, rho is taken out of the integral rho g, z 0 plus y cos theta. If you integrate it aerial v d A, that will give you the discharge Q, so that I have just incorporated it here. Now, there is another integral, integral aerial integral v square by 2 rho v d A, so what is that quantity actually. So, I have to write that here, aerial integral rho v 1 Q by 2 d A.

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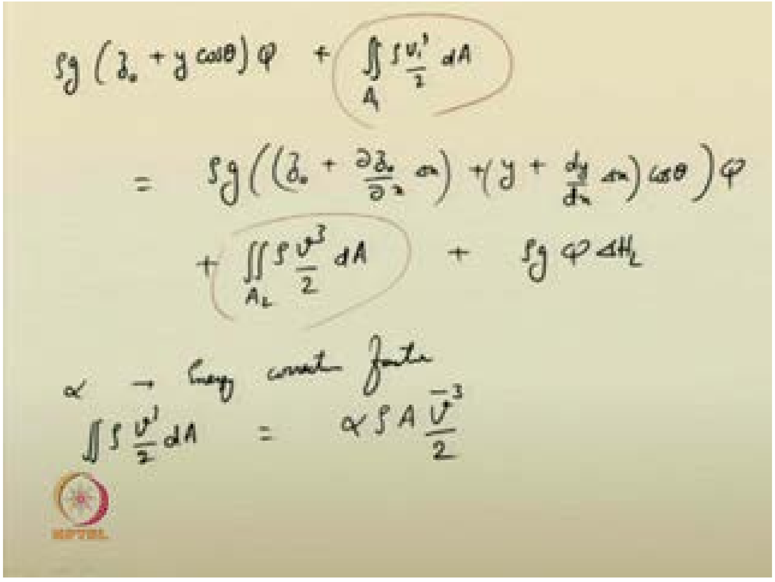

So, the next you can see, rho g z 0 plus y cos theta Q plus integral A 1 rho v 1 by 2 d A, this is will be equal to. In the right hand side z 0 plus, why I am writing is that, just recall the earlier section. So, if this is the right hand side, so this bed slope, this z 0 in the upstream, z 0 in the downstream, if it is changing by this quantity, I can write it like this. So, plus y plus d y by d x into del x cos theta into Q, then you have the aerial integral rho, v cube by 2 d A plus rho g Q del H L. So, this becomes, the energy equation becomes in the following form. So, this particular if you see in these two equations, in this two particular terms. This quantity it is an aerial integral, this quantity is also an aerial integral. So, you need to take into account, the energy correction factor alpha. You were know that energy correction factor, energy correction factor get employee them. So, I can then write the following quantities; any integral rho v Q by 2 d A is equal to the kinetic energy. The energy correction factor rho into the area, into the average velocity, like this you can write them. So, both the places you write there, you will get the following relationship now.

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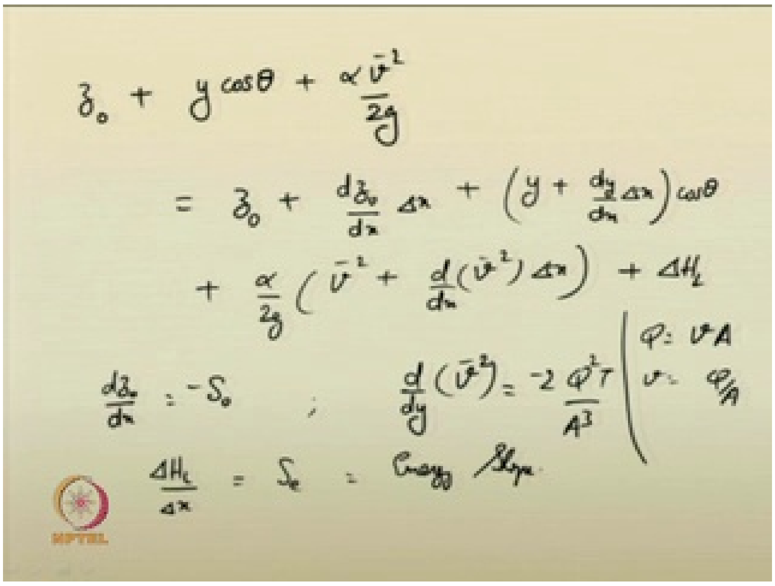

So, I can write the thing; z 0 plus y cos theta plus alpha v square by 2 g, this is equal to z 0 plus d z 0 by d x del x plus y plus d y by d x del x into cos theta plus alpha by 2 g v square plus d by d x of v square into del x plus the head lost. So, all the terms are in units of the length, so these are the energy per unit weight terms. So, you can get it in this following form. You have the elevation head, you have the pressure head, you have the velocity head in these things. We know that d 0 by d x is nothing but the bed slope S 0, I have to give it minus, because it is decreasing, the elevation of the means the value of S 0 is decreasing, also, the quantity d by d x d by d y of v square, what this will be. You know v square; that is Q is equal to v A, or v is equal to Q by A, one can easily identify that. So, substitute that quantity here you will get this quantity as, d by d y v square is equal to minus 2 Q square T by A cube. The quantity del H L by del x, that can also be obtain here, it is given as the energy slope, so one can easily use this thing now.

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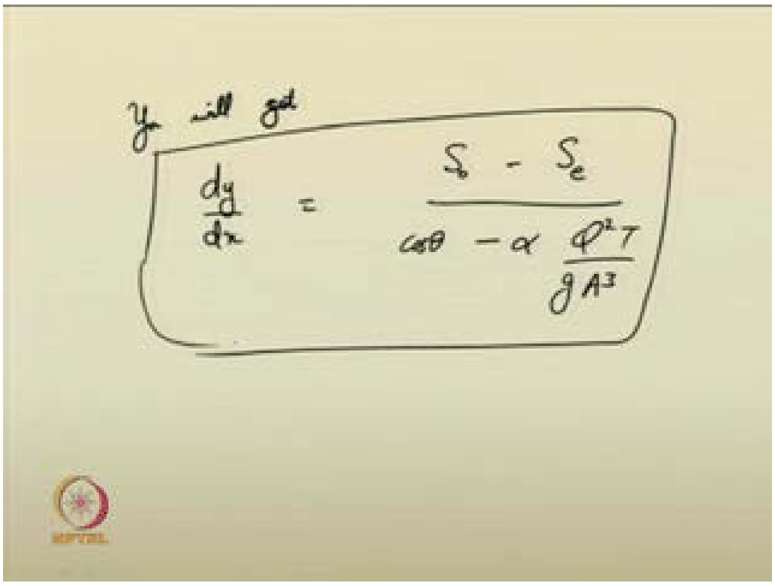

Rearrange the terms, you will get d y by d x is equal to S 0 minus S e by cos theta minus alpha Q square T by g A cube. So, again I got a gradually varied flow equation, now using the energy equation. So, you can use both appropriately, whichever circumstances, according to your circumstances, you can use both the equations.

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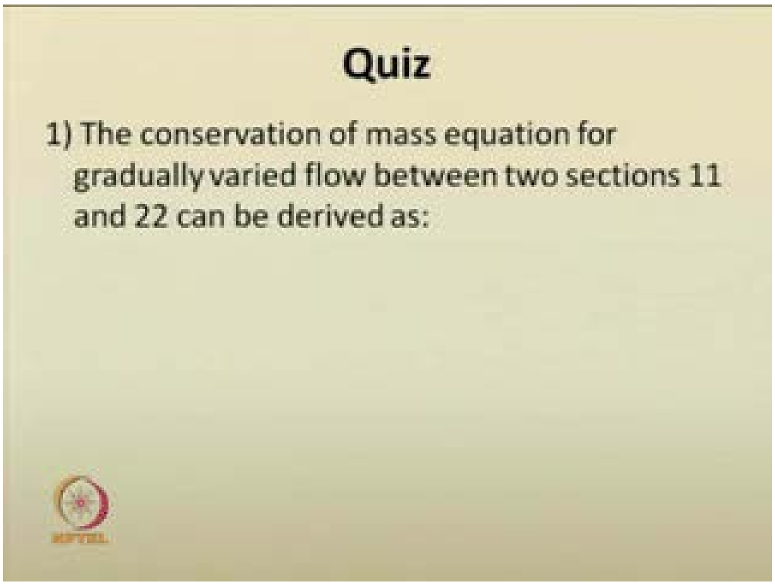

The quiz for this lecture is, the first question to you is, the conservation of mass equation for gradually varied flow between two sections 1 1, and section 2 2, it can be derived. You have derived it in this class today, so you do that derivation again and show it to me.

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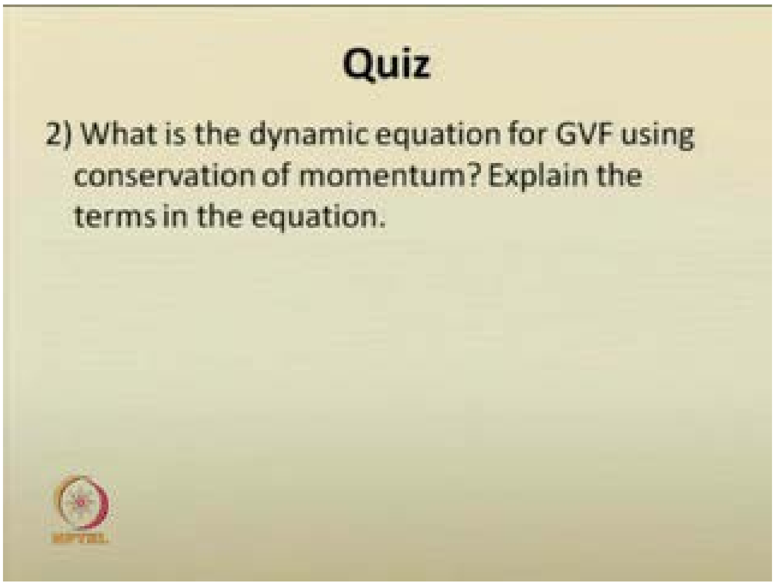

The second equation for the quiz is, what is the dynamic question for gradually varied flow using conservation of momentum principle, explain the terms in the equation. So, I am repeating the question, what is the dynamic equation for gradually varied flow, using conservation of momentum principle, explains the terms in the equation.

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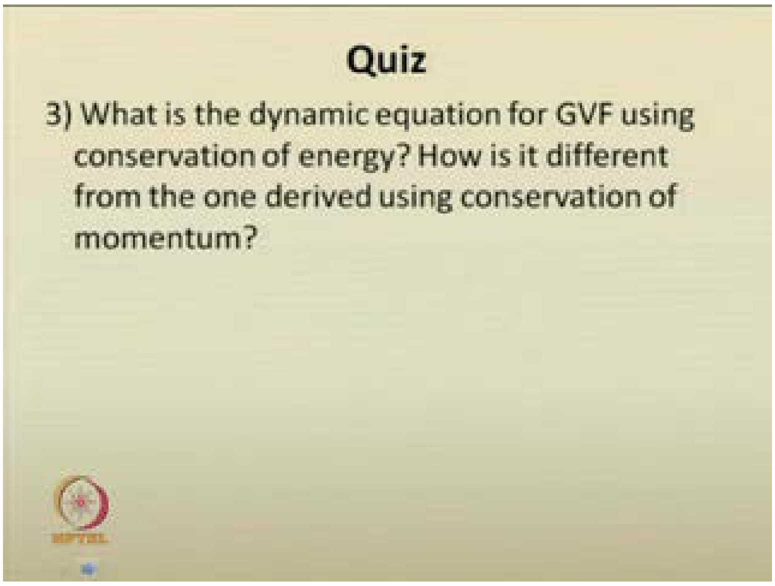

The third question for this quiz is, what is the dynamic equation for gradually varied flow using conservation of energy principle, how is it different from the one derived using conservation of momentum principle. I am repeating the question, what is the dynamic equation for gradually varied flow, using conservation of energy, how is it different from the one derived using conservation of momentum principle. So, the solutions for this quiz are, the first question you were asked to derive the conservation of mass equation for gradually varied flow, between two sections 1 1 and 2 2.

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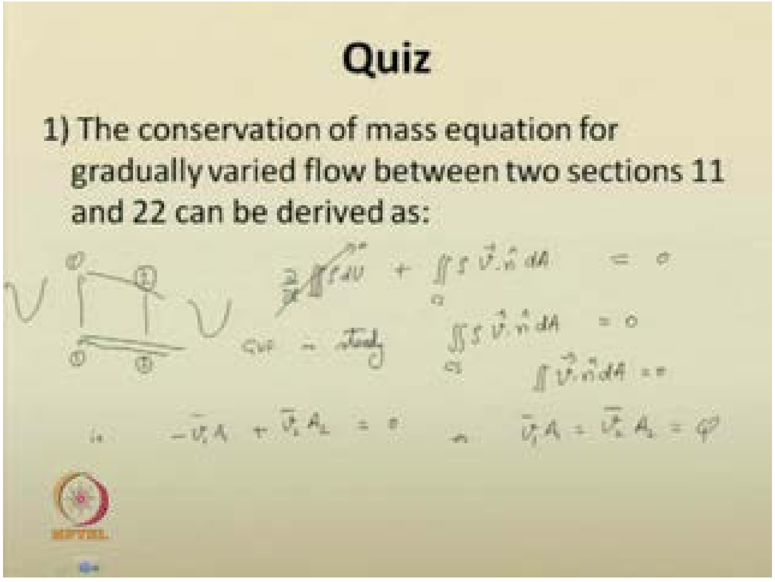

There is a gradually varied flow between two sections; 1 1 and 2 2, you were requested to derive the continuity equation, or the conservation of mass equation. So, from the basic Reynolds transfer theorem, if you recall them, we had suggested that, a change in mass inside the control volume between the two sections, plus the net out flow of mass, across the control surfaces of the control volume, this should be equal to 0, it has been derive, I mean it has already told to you. From this thing, this as, it is suggested that gradually varied flow is steady; therefore, this component vanishes of. So, now you can suggest that equation remains as rho V dot n d A, is equal to 0. From the control volume, two control surfaces along flow across through that, and that is the plane normal to this section 1 1, plane normal to the portion 2 2 here.

Only those two sections allow flow, across through those planes. So, you can now easily write this thing as, as a flow is incompressible you can remove rho also. So, that is you have two sections here, one section is there here, another section is there. So, minus v 1 average velocity into A 1 plus v 2 A 2 is equal to 0 or v 1 A 1 is equal to v 2 A 2, is equal to your discharge Q. So, this is the conservation of mass or the continuity equation for gradually varied flow. So, the solution for the second question is, you were ask what is the dynamic equation for gradually varied flow, using conservation of momentum you had derived those equation, if you recall them.

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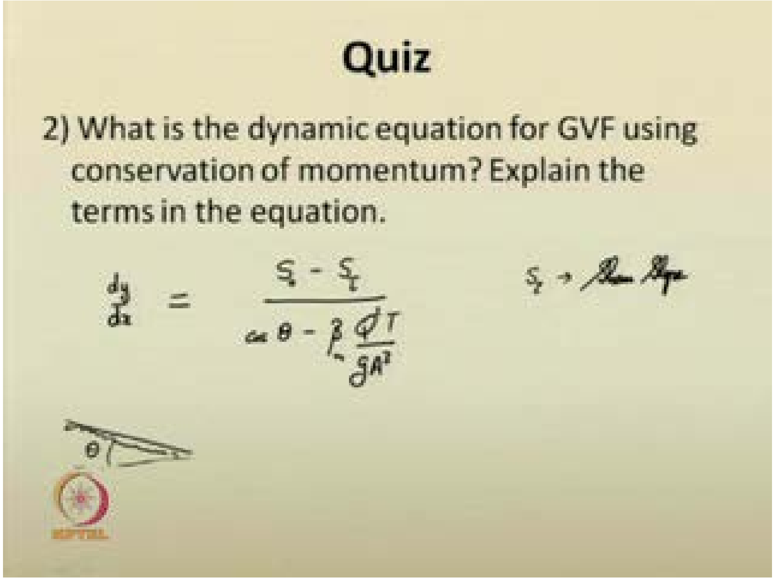

The slope of the water surface d y by d x is equal to S 0 minus S 2 by cos theta minus beta m Q square T by g A cube. So, d y by d x is the slope of the water surface, S 0 is the slope of the bed, S 2 is nothing but shear slope. We had discussed those things in the last class. Q is a steady discharge, T is the top width, A is the area of cross section, theta is the angle, at which the bed, or the channel bed is having with the horizontal; that is if this is your channel bed, how much angle it is making with the horizontal line; that is theta; that is the angle theta, so all the terms are self explanatory. The third question asked to you was, what is the dynamic equation for gradually varied flow using conservation of energy, so this was also derived.

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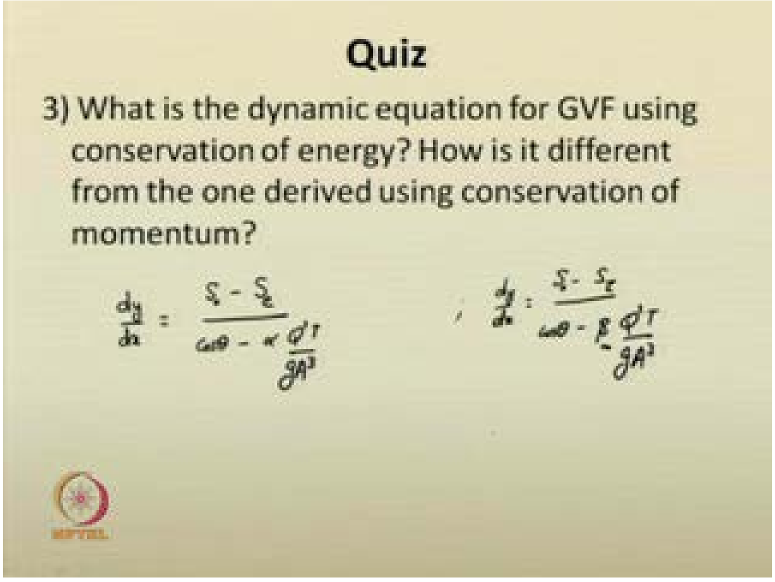

Again, d y by d x is equal to S 0 minus S e, it is not the difference, cos theta minus alpha Q square T by g A cube, this is the dynamic equation for the gradually varied flow, using conservation of energy principle. So, you can see that this is quite different from the one develop using momentum principles; S 0 minus S 2 by cos theta minus beta m Q square T by g A cube. There are similar terms, but there are some differences also. Here in the momentum equation you had used shear slope as S 2, where as in the energy equation you have use the energy slope S e. For the correction factors, you had used the kinetic energy corrections factor in the energy equation, whereas you use the momentum correction factor in the momentum equation. So, there is alpha m and beta m, those differences are there. So, therefore, they are not in the same, in principle they are different, because kinetic energy correction factor, and momentum correction factor, they can differ for any channel section. Similarly the energy slope as well as the shear slope, they can also differ.

Thank you.