Advanced Hydraulics
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Module - 2
Uniform Flow
Lecture - 3
Uniform Flow Computations (Part 1)

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Good morning to everyone, welcome back to our lecture series on advanced hydraulics. So, for last few weeks, we are dealing with the module on uniform flows. And in the last class, we have started the portion on uniform flow computations. We had elaborated various methods on that we were discussing on the various methodologies of for computing uniform flow and all. First let us start with the quiz for the last class. In the last class, whatever we have taught based on that we will take a brief quiz. After that we will just go through that briefly again those portions, and today we will start the next portion of the module. So, please begin your quiz, you will be given five minutes to answer the quiz.

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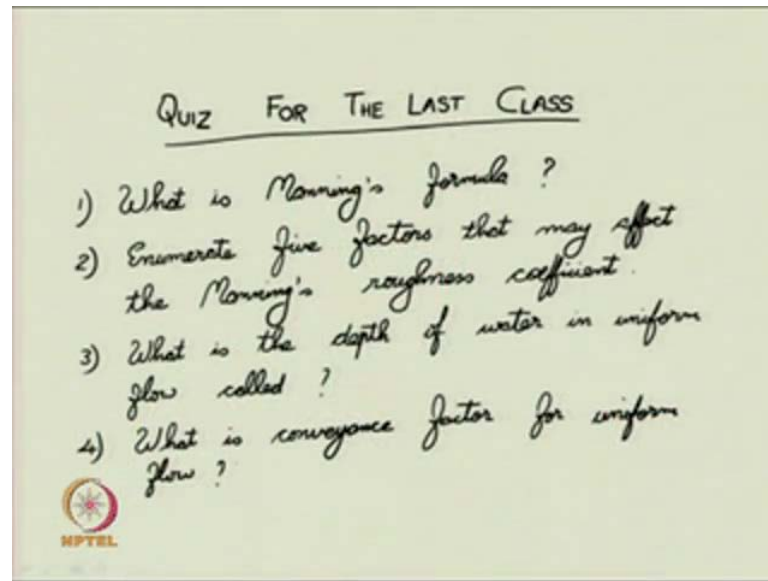
First question for the quiz is what is Manning's formula? So, will you please write down what is manning's formula, it is a very easy one. The second question for you is enumerate five factors that may affect the Manning's roughness coefficient. You know the Manning's formula in that there is a coefficient called Manning's roughness coefficient. So, what are the five factors some just enumerate some five factors; there are many factors that affect roughness manning's roughness coefficient. You enumerate any five factors that may affect the Manning's roughness coefficient. I hope you have answered that.

The next one is, what is the depth of water in uniform flow called? What is the depth of water in uniform flow called? It is just a one word answer; it should these answer should be given in one or two words. Now the last question for you is, what is conveyance factor for uniform flow? What is conveyance factor for uniform flow? I hope you have answered that. So, this quiz was a just a brief quiz, just to test your updation of the course, how you are going in proper time and proper methodology. So, we hope that you have answered all the questions.

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The solutions for this quiz is Manning's formula, I will just write it. So, the solutions first one, the average velocity in any cross section of the channel for uniform flow, it is nothing but 1 by n R to the power of 2 by 3 S naught to the power of half; where n is called manning's coefficient - roughness coefficient, r is your hydraulic radius, s naught is your bed slope.

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The solution for your second question, your second question was enumerate five factors that may affect the manning's roughness coefficient. So, you can enumerate any one of

them. You can tell that say surface roughness, for example, you can tell channel irregularity. You can also tell channel alignment, silting, scouring, off course one important point, vegetation. Vegetation also considerably affects your roughness coefficient. If any obstruction is there in the channel - seasonal changes. So, you can enumerate any one of any five of this or even if you know more than these things that can also be that is also most welcome from your side.

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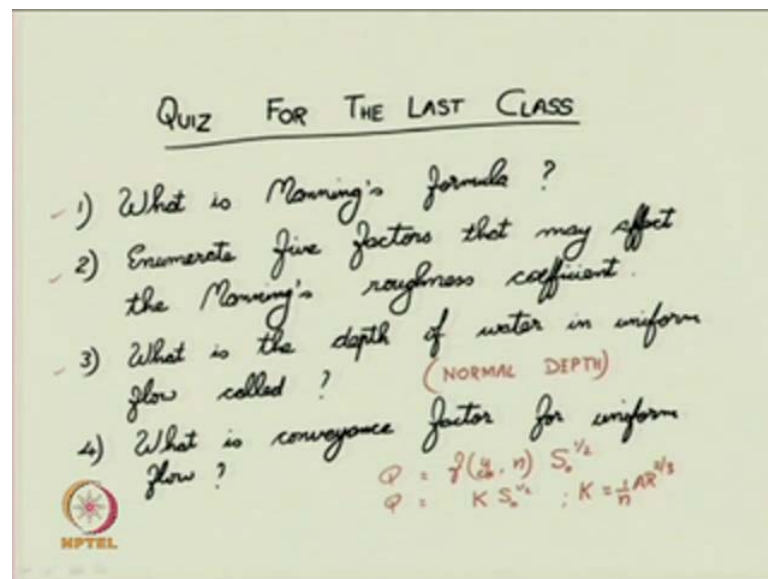
So, what was your third question? Your third question is what is the depth of water in uniform flow called? I will just mention it here itself; it is called normal depth. No need to further elaborate that expression. Then the next question is what is conveyance factor for uniform flow? If you recall the discharge the uniform flow discharge, it was given as a function of normal depth y n and manning's roughness this thing into s naught to the power of half. This function of normal depth y n and manning's roughness coefficient n was subsequently given as conveyance factor k into s naught to the power of half. So, your conveyance factor k is a function of, your conveyance factor k is a function of normal depth y n and roughness coefficient n. It is given as 1 by n a r to the power of 2 by 3.
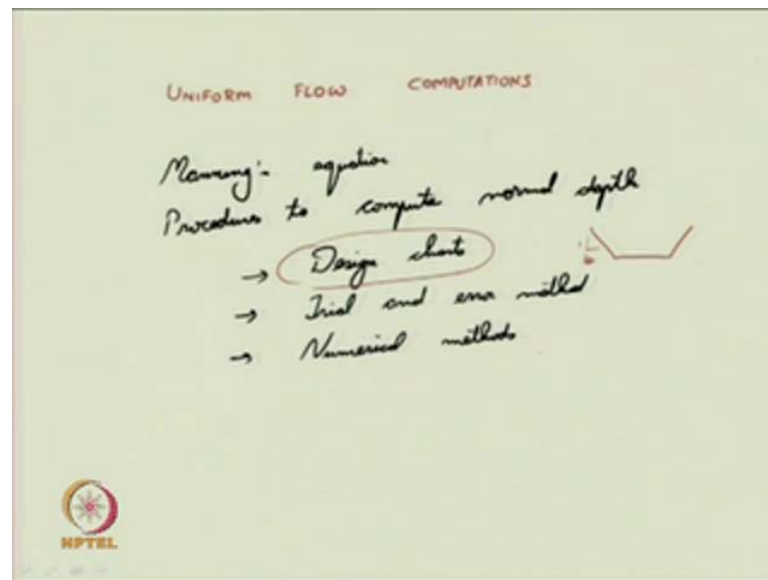
Then let us start today's lecture now. So, today we will be continuing with uniform flow computations. So, in the last class, we had studied on manning's equation. We have studied normal depth conveyance factors, section factor; we have also briefly discussed that what are the procedures to compute normal depth. So, in that we mentioned that you can use design charts, you can imply trial and error method, you can use numerical methods. So, how to develop your design charts, this was explained in the last class. We had, in fact, developed some design charts for trapezoidal channel or how the table how to compute design charts and all that we have done it, we have derived. In fact, derived it for trapezoidal channel for any sides slope one is to v, you recall them. Any slope one is to v, and v is equal to one, v when we substitute that the corresponding table was also developed.

You can see how the trial and error method is also employed to compute normal depth, and how to use numerical methods also to compute normal depth. We will just see it in the following portion now. So, the trial and error method. So, you recall your manning situation for discharge - v 1 by n r to the power of 2 by 3 s naught to the power of half or your discharge Q - this is one by n AR to the power of 2 by 3 s naught to the power of half. In this relationship, if you remember then you had obtained the following relationship, AR to the power of 2 by 3, this was clubbed together this is nothing but, n times the discharge Q by s naught to the power of half. You had obtained such a relationship in the last class, subsequently we suggested this quantity as a section factor and all if you recall them.

Now, the point here is to use your trial and error method, in the left hand side, this is your left hand side, this is your right hand side, for a given channel usually n is a given quantity; q is also specified for you, bed slope is also given, or it is a channel property. You are now requested to find the normal depth y n, that is the objective here, that is you are using trial and error method to compute your normal depth. So, how will you compute normal depth using trial and error procedure?

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Again just as an illustration, let us go with the trapezoidal channel; you are following the same norms as we are doing in our earlier classes. The depth of uniform flow is called normal depth, the bottom width is B 0, side slope 1 is to b. If you recall the area of cross section for such a trapezoidal channel is y n into B 0 plus small b into y n, your wetted perimeter p is equal to B 0 plus 2 y n into root of 1 plus b square. Therefore, your hydraulics radius R is equal to a by p y n y B 0 plus twice y n into root of 1 plus b square.

Therefore, in your relationship A R to the power of 2 by 3 is equal to n q by s naught to the power of half; substitute the quantities whatever relationship here with respect to y n, you substitute it here. Now what about the r h s term your r h s terms here all the quantities are known, all the quantities here are known to you. So, you can just specify it as a known quantity J fine. So, you we you were now going to do your equating A R to the power of 2 by 3 is equal to a known quantity J.

$$\frac{(y_n B_0 + by_n^2)(B_0 y_n + by_n^2)^{2/3}}{(B_0 + 2y_n\sqrt{1+b^2})^{2/3}} = J$$

$$\frac{(B_0 + by_n)^{5/3} \, y_n^{5/3}}{(B_0 + 2y_n\sqrt{1+b^2})^{2/3}} = J \quad \text{①}$$

Try with various values of y_n
Which value of y_n satisfies eq ①
→ Trial and error method

Further proceed, substitute the value of A and R, you will see that you are getting the following relationship y n into B 0 plus b y n square b 0 plus 2 y n into root of 1 plus b square whole to the power of 2 by 3. This is equal to a known quantity J. You can do one thing you can just bring all the terms here. So, B 0 plus b into y n whole to the power of 5 by 3 into y n whole to the power of 5 by 3 by B 0 plus twice y n root of 1 plus b square whole to the power of 2 by 3. This is equal to a known quantity j. So, once you have such a relationship, now you are having such a relationship.

Now substitute for various value of y, you try with various values of y n, you see which value of, which value of y n satisfy your equation one, satisfies equation one. You have to find that which value of y n satisfies that equation one; you have to select that this is called the trial and error method. So, you have to try it for several values of y, may be some of you, if you are lucky you may get it in two or three trials itself. Some of you may have to go for some nearly hundred trials then only they may arrive at the solution, depends on luck also. And also through intuition based on your problem and all, what could be the initial depth means or guess value, how you try initial guess and all how you do that depends on your intuition as well. So, you can try it many problems for various this is only for a trapezoidal cross section. You can try it for various.

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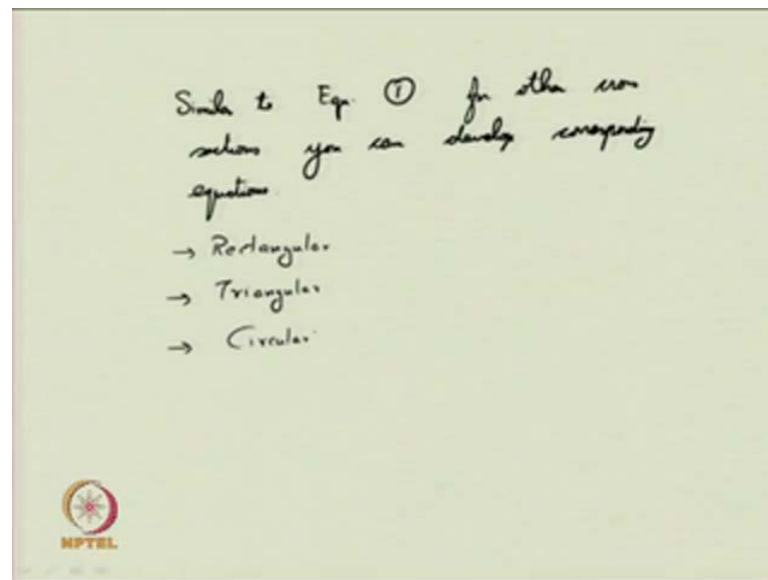
So, similar to, similar to equation one for other cross sections, you can develop corresponding equations. Now it is off to you, say whether it is for rectangular, triangular, even circular, it is up to you to do work it out and just find it. What are those corresponding equation try the trial, I mean use the trial and error method to find the normal depth.

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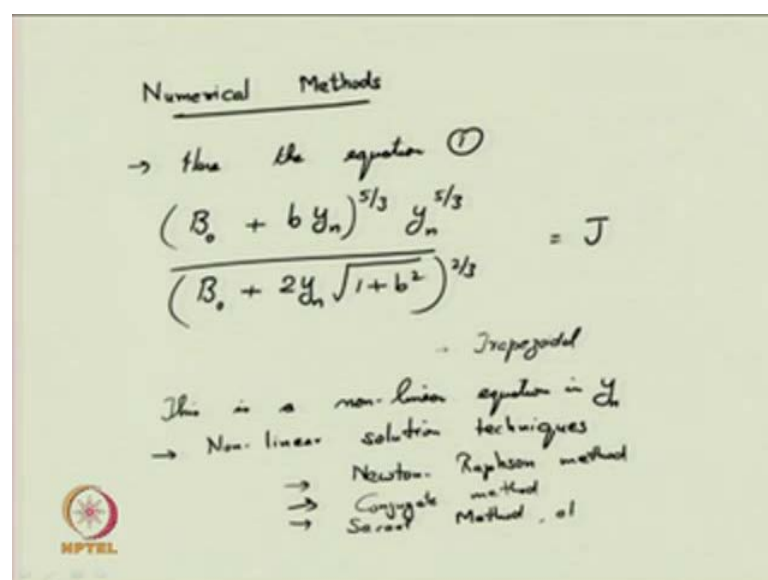
Now let us go with the next method. So, now, we next method, we are going to do is the how to apply numerical methods, to obtain the normal depth for uniform flow. Here, if

you recall the equation one previously develop that is B 0 plus b y n whole to the power of five by three y n whole to the power of five by three by B 0 plus two y n one plus b square. This equation, it was of course developed for the trapezoidal channel, you can obtain similar equations for other cross sectional channels also. In this equation, this is a non-linear in y n.

So to solve this equation, you have to use non-linear solutions techniques. This in the numerical methods, there are methods or in the methods or techniques to solve non-linear equations. So, you have to employ them here to solve (( )) and get the normal depth. Some of them, I will just briefly mentioned the you might of heard about Newton-Raphson method, conjugate method - by conjugate method, sequence method, there are various method to solve non-linear equations in the numerical methods portion. So I am not going to describe all of them, just briefly we will see through the Newton-Raphson method.

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So you have equation, I can now obtain the equation as from this B 0 b y n, again I am iterating here, iterating that this equation was developed for the trapezoidal channel for other cross section it is quite different. So in this equation, L H S is unknown; R H S is the known quantity. So the Newton Raphson iteration or Newton Raphson method, it suggest that if you can develop a function of y n and if that function is differentiable, so you can obtain the first differentiation f dash y n. You can iterate and obtain the value of

y as y n new is equal to y n old minus f of y n by f dash y n. This is the standard Newton-Raphson iteration or Newton Raphson method to find non-linear solution. So here what does y n new and y n old mean? This is the previous value or guessed value; y n new is it is the modified value or improved value. So the steps of that if you have any non-linear equation in y n, you can give an initial guess of y n some value. And start using this relationship you first obtain the function of y n which is also possible to be differentiated then using your initial guess or old guess, you can get a new value of y n or improved value of y n using the given equation mentioned here, so that we can employ here.

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In our case for open channel for a computation of uniform flow, we can suggest that your f of y n is equal to B 0 plus b y n whole to the power of five by three y n to the power of five by three minus J. Let us consider that your function of y n is equal to the is in the following form. In this following form now, if you obtain this function, this is also quite possible to be differentiated. This was for trapezoidal for a general case, you can suggest f of y n for any cross sectional method you can suggest f of y n is equal to A R to the power of two by three, minus J. And for exact value of y n, this relationship A R two to the power of three minus J should be equal to zero. Now your initial guess, y n initial, it may not equal to be your y n actual.

In that case, then f of y n initial not equal to zero. So, this is the principle behind that. Suppose if you are guessing initially and if you are getting a function f of y n then that suggest that for the exact value of y n, if your initial guess was an exact value of y n then

f of y n would have directly yield you zero. Then there is no need of further solving the equation. If your initial guess is not the actual normal depth y n, what you can do is that you are now going to evaluate the function f of y n, you are now going to evaluate this function f of y n, subsequently you are going to evaluate f dash y n also. You are going to evaluate f dash y n.

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Now what is f dash y n? This is d f by d y n, so as taken from Anil Chaudhry, two thousand eight on his book in flow through open channels. So, d f by d y n, you can give it as d by d y of A to the power of five by three P to the power of two by three minus J. So this is nothing but five by three A to the power of two by three P to the power of two by three d A by d y n minus two by three P to the power of minus five by three A five by three d P by d y n. So you rearrange the terms, recall that d A by d y n in any cross section of the channel, any channel we had suggested that the top width T is nothing but equal to d A by d y. So therefore, the uniform flow d A by d y n will give you the top width of that uniform flow in the channel. If you recall them, we had discussed these things in the earlier classes.

So, just substitute those quantities here, if you substitute them appropriately you will get the corresponding relationship for the trapezoidal channel. Your f dash y n is nothing but equal to one by three R to the power of two by three into five times the top width minus twice R d p by d y n. And you also know that f of y n is equal to A R to the power of two by three minus J. So what you can do is the f of y n is equal to A R to the power of two

by three minus J. So you have now two function f of y n and its corresponding differentiated function f dash y n.

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Start with initial guess value for y n that is y n 0 is equal to some value, some value you can specify, we start with the initial guess value. Using your, so using your y n value what you have to do is evaluate f of y n, using your initial guess; evaluate f dash y n using your initial guess. Now your improved value for your normal depth y n, so I am just giving it as super fixed one the improved value which is nothing but a old value minus f of y n that has been obtained using your initial guess and f dash y n that has been obtained using your initial guess. So this will give you an improved value for your normal depth y n, so this is given as y n one. Once you obtain y n one, you can check whether f of y n one whether it is equal to zero.
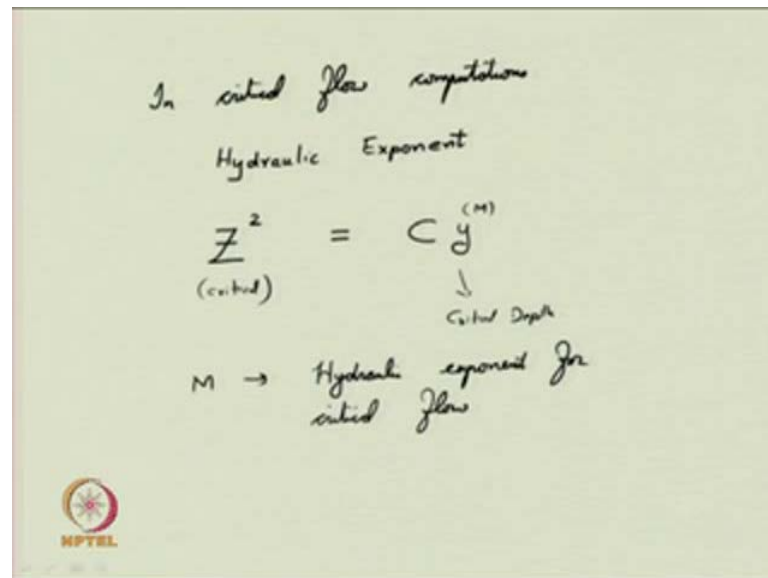
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$$y_n^{(2)} = y_n^{(1)} - \frac{f(y_n^{(1)})}{f'(y_n^{(1)})}$$

$$y_n^{(3)}$$

$$y_n^{(i)} = y_n^{(i-1)} - \frac{f(y_n^{(i-1)})}{f'(y_n^{(i-1)})}$$

$\rightarrow$ Once y_n converges

$$\left| \frac{y_n^{(i)} - y_n^{(i-1)}}{y_n^{(i-1)}} \right| \le \varepsilon_{\text{tolerance}}$$

You can check that. If not again go for the next iteration, that is in the next iteration y n two is equal y n one minus f of y n. So again check if your y n two satisfies that condition f of y n or using this y value, if it satisfies that if we get equated to 0 then find the that is the solution. If not, then go again for the next iteration y n three. Like this any general iteration can be given as i minus one minus f of y n using the i minus one by f dash, so this you go on. So you can stop your iterations once y n converges. What do you mean by converging? Convergence means, you can put some convergence criteria, such that between two iterations y n in the ith iteration minus y n in the i minus one eth iteration by y n in the i minus one eth iteration mod of this thing if this difference is some less than some tolerance value.

Tolerance, you can specify according to your the some of them some people may desired one into ten to the power minus three, one into ten to the power of minus four, one into ten to the power of minus five, what are we according to the requirement, you can specify any convergence criteria. Use them, and see whether your y n is getting converge. Once it converges, you can use that value of y as the normal depth, so that is the method of using numerical methods to compute normal depth.

If you recall in the critical flow, in critical flow computation we had we had use the concept called hydraulic exponent, so the corresponding section factor for critical flow, please note that this section factor is for critical flow. This was given as zeta square is equal to some coefficient C into y to the power of M, where y is your critical depth; M was given as hydraulic exponent for critical flow. As similar analogous is there for uniform flow computations also.
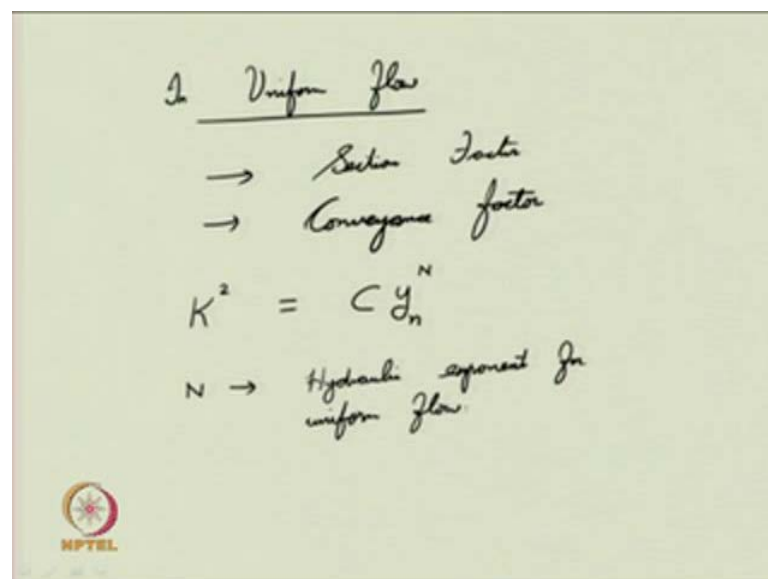
In uniform flow, you have already seen section factor, conveyance factor. So in uniform flow also, the conveyance factor now can be given as K square is equal to some

coefficient C into the normal depth raise to and exponent capital N. So this capital N, this is called the hydraulic exponent, it is called the hydraulic exponent for uniform flow. So these you can use, why this is been used, it gives some certain characteristic of the channel. You can, it will be of a if one know the hydraulic exponent of the particular channel section (( )), it will be quite useful to compute the normal depth.

There is no need to further go and measure the depth or no need to go and measure the various other features; using simple discharge and roughness coefficient, you will be able to evaluate the normal depth. So now the hydraulic exponent is also use, therefore hydraulic exponent is also used in computation of uniform flow. How we are arrival that, so in this thing I hope you every one know the K is your conveyance factor, N is your hydraulic exponent for uniform flow. In this case, I can just derive the following quantity.

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K square is equal to C into y to the power of capital N; taking logarithm both sides, this is twice log k is equal to log c plus N log of normal depth. If you differentiate the quantity with respect to your normal depth, this is nothing but N by twice y n. Just keep this as equation number two.

Manning's equation, from the Manning's equation, you remember the conveyance factor K is equal to one by n A R to the power of two by three. Use the logarithm here. Differentiate that differentiating this thing, you will get d by d y of log K is equal to this

quantity, differentiating this quantity in zero. So you will see that this is nothing but one by A d A by d y n plus twice by three R d R by d y n. So you know the quantity d R by d y n, because R is equal to A by P, this is we are quite aware. So therefore, your d R by d y n term this is nothing but equal to one by P d A by d y n minus A by P square d p by d y n. Again, if you recall that d A by d y n is equal to top width T, use the following relationship.

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$$\frac{d}{dy}(\ell k) = \frac{T}{A} + \frac{2}{3}\frac{P}{A}\left(\frac{T}{P} - \frac{A}{P^2}\frac{dP}{dy_n}\right)$$

$$= \frac{1}{3A}\left(5T - 2R\frac{dP}{dy_n}\right) \longrightarrow ③$$

comparing eqns ② and ③

$$N = \frac{2y_n}{3A}\left(5T - 2R\frac{dP}{dy_n}\right)$$

→ Based on cross section

→ Trapezoidal
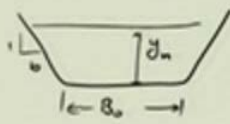→ Rectangular   one can find N
→ Irregular   for each type
                        of cross section

Go ahead, your d by d y n of log K is now equal to T by A plus two by three P by A into T by P minus A by P square d P by d y n. Or this is equal to one by thrice area into 5 times that the top width minus 2 R d P by d y n, from this becomes your equation three. Comparing equations two and three, you will get the hydraulic exponent for uniform flow N is equal to twice y n by 3 A into 5 times T minus twice R d P by d y n. Based on say, based on cross section, whether it is trapezoidal, whether it is rectangular, triangular, parabolic, circular what are the you can identified as you can find, you can find N for each type of cross section.

Now for example, a trapezoidal channel with bottom width B zero, side slope one is to be normal depth of flow y n. What can you expect now? In this thing, again your area of cross section, I can write it in the following form one plus b into y n by B naught. R is equal to one plus b into y n by B naught whole thing into y n by one plus two y n by B naught root of one plus b square. Top width T is equal to B naught into one plus two b y n by B naught, is not it? This quantities are already aware, you can evaluate it on your own also, using these things on substituting them in the equation for N hydraulic exponent N, I am not going to do it for here, you can do it as homework.

You will see that N becomes ten by three into one plus twice b y n by B naught by one plus b into y n by B naught minus eight by three into root of one plus b square into y n by B naught one plus twice root of one plus b square into y n by B naught. Like this you will get expression for N for trapezoidal, channel. So why I wrote N in such a form is that, you can have a relationship of N versus non-dimensional depth y n by B naught. You can plot them, your plots may be, you know how to plot by this time now.

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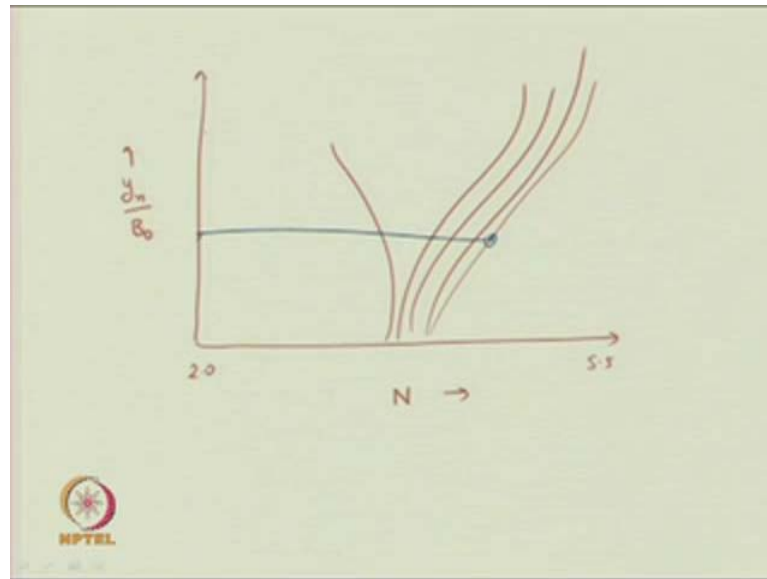
So these figures, you see it may range from two to five point five and your N value may be something it is going like this for various cross sections. So if you have any particular value, say if you happened to observe N value for any particular channel, say it is here then you can just check the corresponding depth of flow, you can interpret that and from that you can get your normal depth of flow. Similarly, if it is here corresponding thing, you can easily interpret them. So it is up to you to determine that, so that is you can use the normal, you can use your hydraulic exponent for uniform flow, you can compare it with the non dimensional normal depth of flow. Use those graphs and interpret the normal depth of flow.
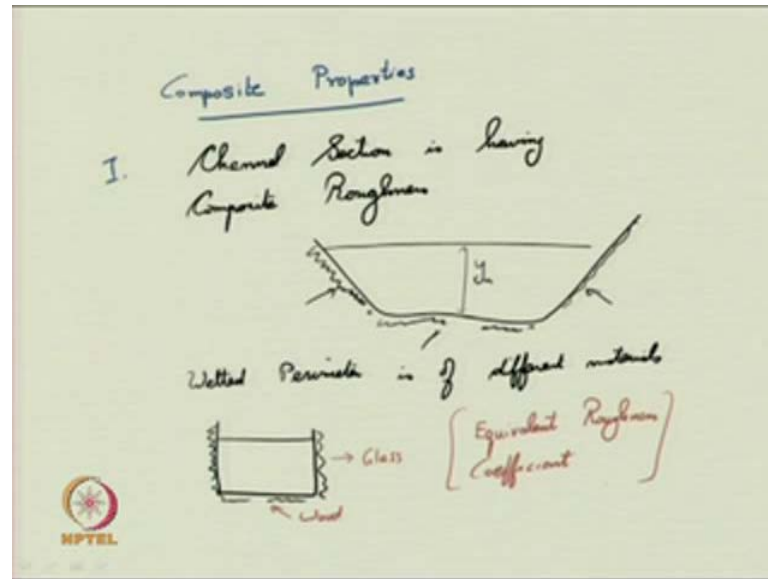
So next we are going to deal with is what happens to channel flow or uniform flow if composite properties are there. If composite properties are there, what happens to your channel flow. First one, if your channel section is having composite roughness, if it is having composite roughness what happens to your uniform flow. See it can be, say for example, in the trapezoidal channel, the sidewalls may be of one particular material, the bottom bed may be of some material, this may be of another material, still the uniform flow is there in the channel. How will you compute uniform flow? It is quite difficult or it is deviates, if you then take the corresponding areas allotted to this perimeter and try to evaluate it independently to be quite different fondly to get your normal flow, normal or normal depth.
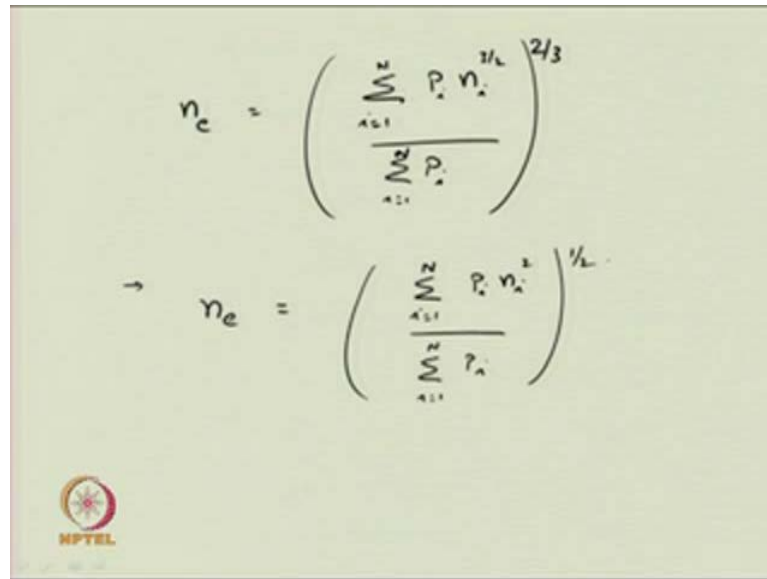
So how will you evaluate the uniform flow in such situation? This is different, this is different, this is different, so you can just suggest now that the wetted perimeter is of different material; or for example, another simple case is in a rectangular channel the sidewalls are made up of glass and the bottom is made up of wood, what happens to your uniform flow. How will you compute them? So in these situation, you require the concept called equivalent roughness coefficient. So, we can use the same (( )) equations to compute the uniform flow, but using your equivalent roughness coefficient. So how will you evaluate equivalent roughness coefficient? What is the procedure?
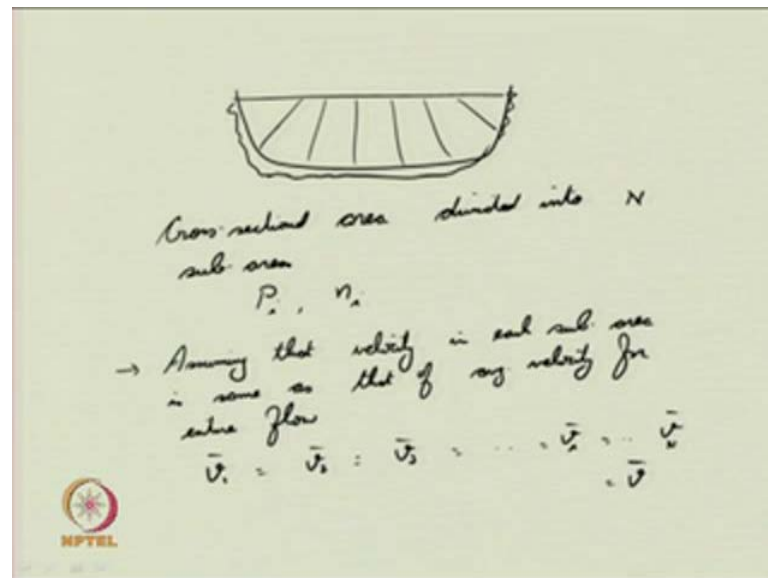
You may or may not divide the areas wherever the properties are changing, corresponding perimeters say cross sectional area divided into sub areas, into N sub areas, each having wetted perimeter P i, its roughness coefficient n i. Now, we are assuming that here the channel cross section it is not that much, the roughness things are the roughness is the thing that is getting differed here. As so we are assuming that the velocity the average velocity in each section, each sub area is same as that of average velocity for entire flow, if you assume that. So that means that say v one bar is equal to v two bar is equal to v three bar equal to v i bar equal to v n bar. And these are all equal to the entire average velocity of the cross section of the channel. Then you can evaluate the equivalent roughness coefficient in the following form.

$$n_c = \left(\frac{\sum\limits_{i=1}^{N} P_i n_i^{3/2}}{\sum\limits_{i=1}^{N} P_i} \right)^{2/3}$$
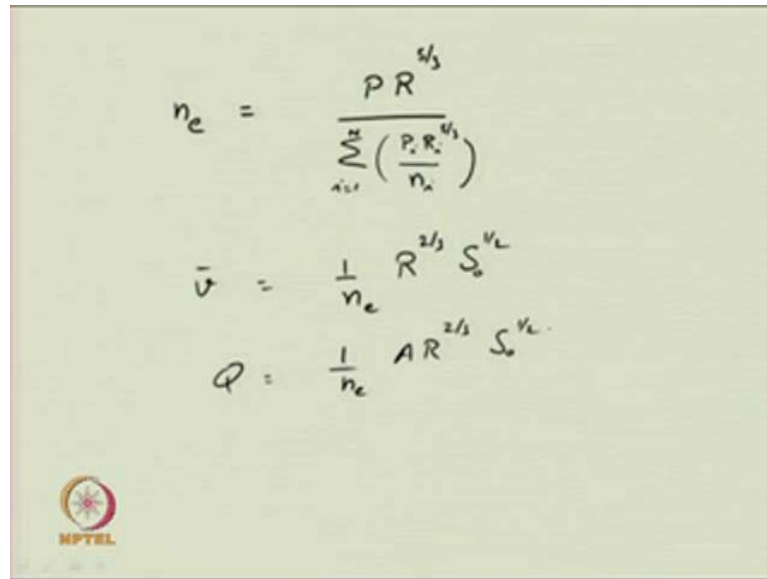
$$\rightarrow \quad n_e = \left(\frac{\sum\limits_{i=1}^{N} P_i n_i^{2}}{\sum\limits_{i=1}^{N} P_i} \right)^{1/2}$$

This is one particular formula, i is equal to one to N p i n i power of three by two by i equal to one to N p I whole to the power of two by three. So students please note that the capital N used here is not your hydraulic exponent, it is just to show the summation. The total number of sub areas divided in the channel cross section, this is just to denote that. We can also evaluate equivalent roughness coefficient, I assuming that the total force resisting the flow in the channel cross section, it is equal to the summation of the forces, individual forces that are opposing the flowing individual sub areas that can also be used. So you can use that method, you can give the equivalent roughness coefficient n e is equal to i is equal to one to N p i n i square p i whole to the power of half. This is one method.

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$$n_e = \frac{P R^{5/3}}{\sum_{i=1}^{N} \left(\frac{P_i R_i^{5/3}}{n_i} \right)}$$

$$\bar{v} = \frac{1}{n_e} R^{2/3} S_o^{1/2}$$

$$Q = \frac{1}{n_e} A R^{2/3} S_o^{1/2}.$$

So was in some other literature you may see, some scientist they might have used that the total discharge in the channel is equal to the summation of the discharges in individual areas. If that concept is used, then your roughness equivalent coefficient n e can be computed as P R to the power of five by three, where P is the total wetted perimeter of the cross section; R is the hydraulic radius for the entire cross section of the channel then i is equal to one to N p i R i to the power of five by three by n i. Like this you can evaluate it. So you can use this particular formula, once get your equivalent roughness coefficient, you can use it to compute your average velocity R to the power of two by three S naught to the power of half. You can also use it to compute your discharge, A R to the power of two by three S naught to the power of half.

So we will conclude here, so we have discussed on the how to compute the composite of effective roughness coefficient or you can tell it as the equivalent coefficient of roughness. To compute the roughness coefficient, if there are different surfaces or there are different surfaces in the channel cross section. You can use them to evaluate the (( )) velocity and discharge. Next time, conclude the portion here, next time we will be dealing with how to evaluate uniform flow, if there is difference in cross sectional areas, if shape of the area and all if it is quite differ how to evaluate the uniform flow.

Thank you.