Advanced Hydraulics
Prof. Dr. Suresh A. Kartha
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module - 2
Uniform Flow
Lecture - 2
Manning's Equation and Normal Depth

Good afternoon everyone, we are back into our lecture series on advance hydraulics, we are dealing with the second module on uniform flows.

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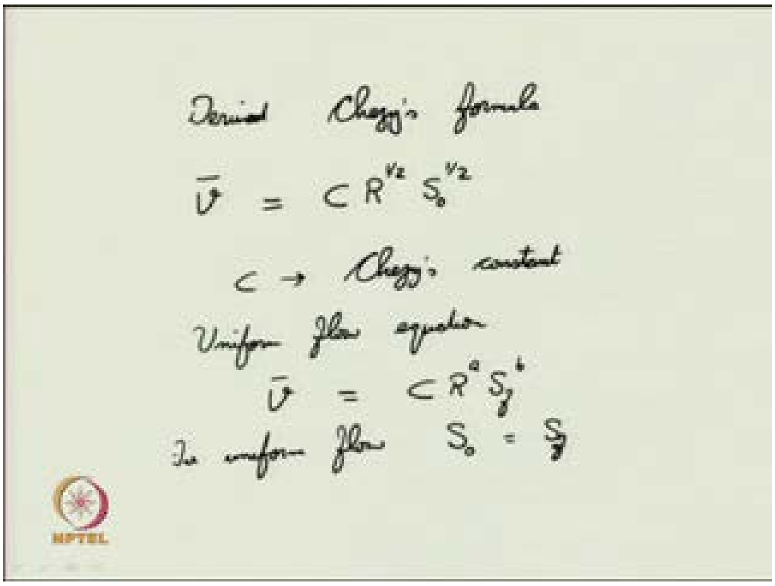

In last class we gave you the introduction to uniform flows; we also derived the Chezy's formula. If you recall the Chezy's formula, we suggested that the average velocity through a cross section v is equal to some coefficient C into R to the power of half S naught to the power of half, where C was suggested as Chezy's constant. We also have seen that this Chezy's equation, it is very much compiling with the uniform flow equation, which was given as v is equal to some coefficient C R to the power of a S f to the power of b. So, there is a quite analogy between these two equations; the uniform flow equation, and the Chezy's equation. Here as you know for uniform flow, your bed slope and energy slope are same. So, there is a quite matching, between uniform flow equation and Chezy's equation.

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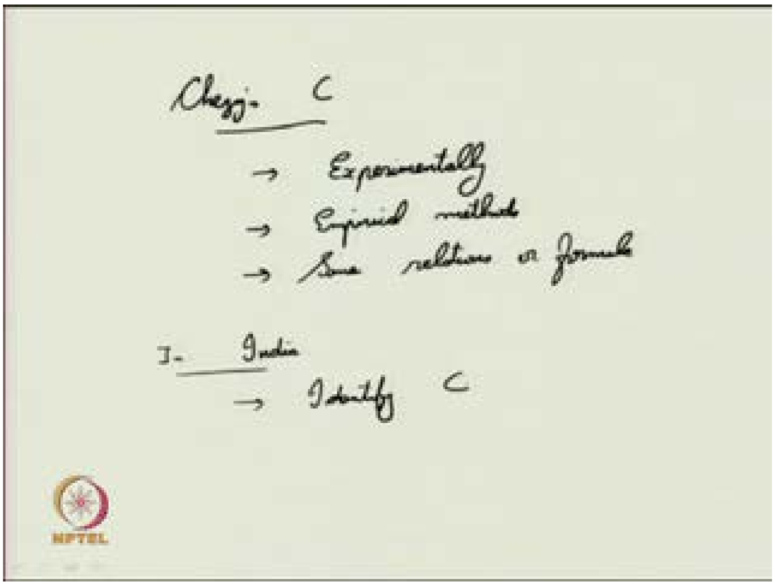

Now you are Chezy's C, this can be determined either experimentally or you can use empirical methods, you can also some relations or formulas. Especially in India, there are very less experimentally works done, regarded to determination of Chezy's coefficient C and all. If you are interested you can just have a go through this or you can work on such items, invents or you can just do research and identify C for various streams in our country; that will be a quite good job, and it will be a quite benefit for our country also.

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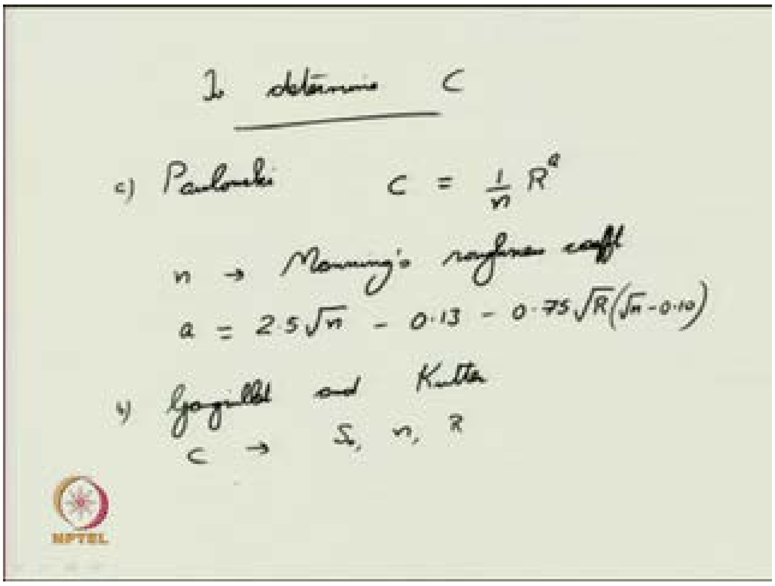

The some of the formulas that are being used to determine C to determine C some of the formulas that are very much use are; first one Pavalovski, the scientist Pavalovski. He suggested a relation that, your Chezy's C is nothing but 1 by n into R to the power of a, where n is your Manning's roughness coefficient. We will coming to that what is Manning's roughness coefficient, a is a term that is define by the following mathematical equation, 0.13 minus 0.75 into root n minus 0.10. This is one particular equation, another scientist, two scientist Ganguly and Kutter, they had develop the relationship with respect to, means C they had related with respect to your bed slope, your roughness coefficient, your hydraulic radius, based on that they suggested that your C is nothing but C is equal to 23 plus 1 by n plus 0.00155 by S naught divided by the whole thing divided by 1 plus 23 plus 0.00155 by S naught n divided by root R.

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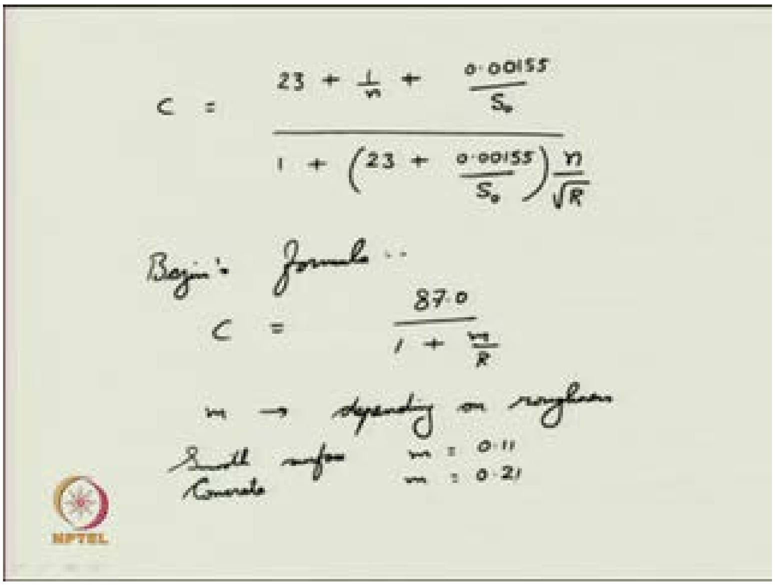

So, these are some of the mathematical relationships, or they might have been empirically obtained these relations, you can just use them, we are not going to derive this relationships and all. Another scientist who developed relationship for determining Chezy's coefficient C is Basin's formula, where the Chezy's coefficient C is given as 87 by 1 plus a coefficient m by R. So m, it is a coefficient depending on roughness. Please note that this is not your Manning's roughness coefficient, this is some coefficient m depending on roughness. In fact, there are some tables related to identify m also, using Basins formula, what are the m values. For example, for smooth surface, you have m is equal to 0.11. For concrete, you have m is equal to 0.21. For earth channels, you have the values ranging from 1.5 to 3.1; you can just note it down.

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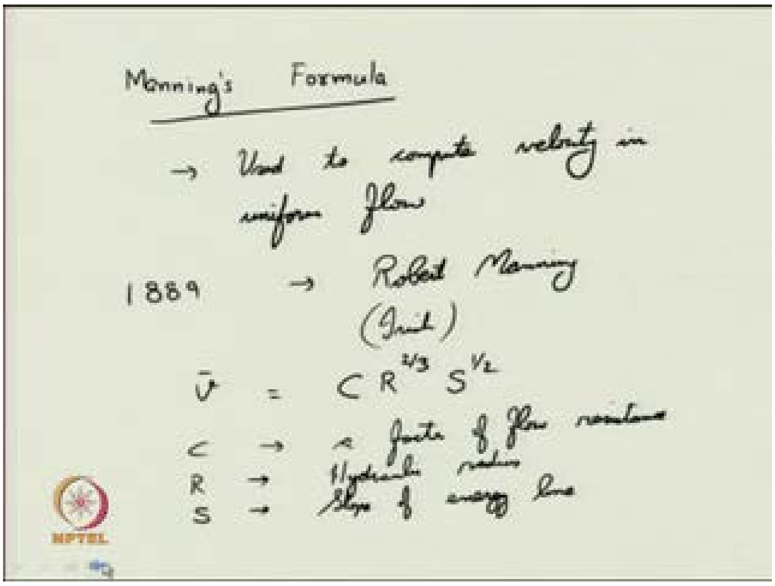

Now, the next relationship which we are going to study is Manning's formula. So, we had derived the Chezy's equation, similarly Manning's equation is also there, to compute your or velocity for a uniform flow. So used, this relationship is used to compute velocity in uniform flow. So, this relationship was developed long back in 1889, the scientist Robert Manning, he had developed this relationship. In 1889 Robert Manning, he is an Irish scientist, he presented it in front of the institute of Irish engineers. Some of (( )) some of his theories related to computation of velocity in channels, he presented that your velocity, the average velocity in a channel, can be computed as some coefficient C into R to the power of 2 by 3 into S to the power of half. Pleases note that we are just representing like this, where C here stresses consider it as factor of flow resistance. R as you know is your hydraulic radius. S here, he suggested that, this is slope of energy line.

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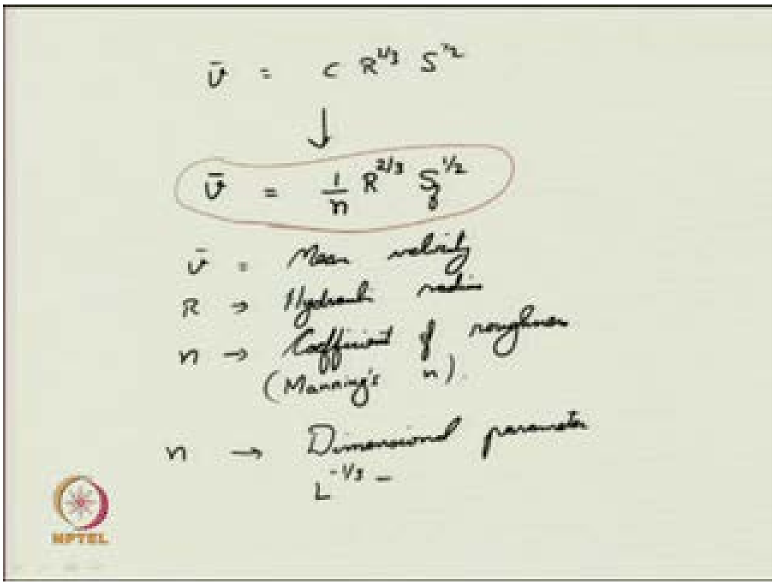

Based on this, such some other scientist worked on the, some other scientist they worked on these Manning's equation; v is equal to C R to the power of 2 by 3, S to the power of half, they worked on. Subsequently, they develop the final forms of Manning's equation as v is equal to 1 by n R to the power of 2 by 3 S f to the power of half, this is according to our symbolic notations and all, so here this 1 by n, your v. As we are dealing with what is mean velocity, R the hydraulic radius, n is a new coefficient called coefficient of roughness, or it is also called Manning's n, S f is your energy slope. So, like that after several studies, the scientists have arrived at this particular form of…

So, they have arrived at this particular final form of your Manning's equation, so v is equal to 1 by and R to the power of 2 by 3 S f to the power of half. This if you observed that, your n is now, it is a dimensional parameter, it is not non dimensional, it has the units of, or it has the dimensions of l to the power of 1 by 3 into t. So, it is something observed that, or something that your roughness coefficient is depending on time and all, but of course, if you later on further del into the subject, you may appreciate those phenomenon.

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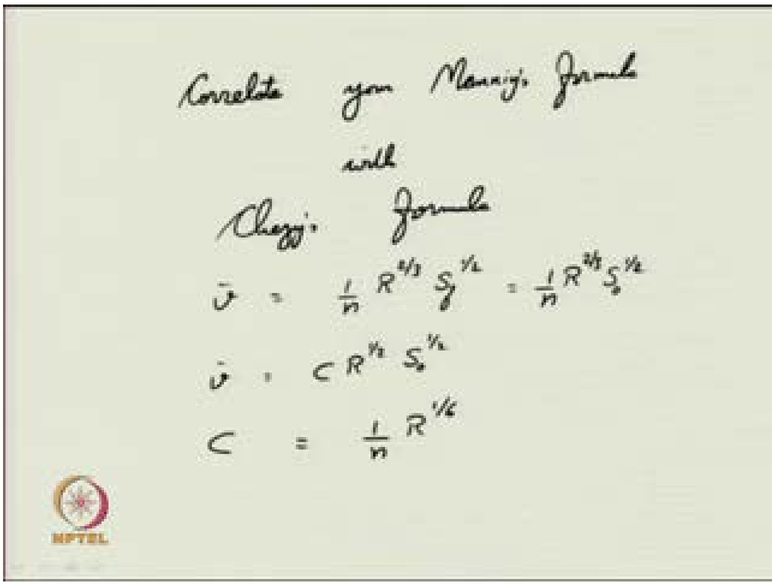

Now correlate your Manning's formula with Chezy's formula. One's equation suggest that 1 by n R to the power of 2 by 3 S f to the power of half, or equal to 1 by n R to the power of 2 by 3 S naught to the power of half for the uniform flow. Chezy's equation suggested you, C R to the power of half S naught to the power of half. From this thing your Chezy's co efficiency, it is nothing but 1 by n R to the power of 1 by 6, this you can easily see that, is not it.

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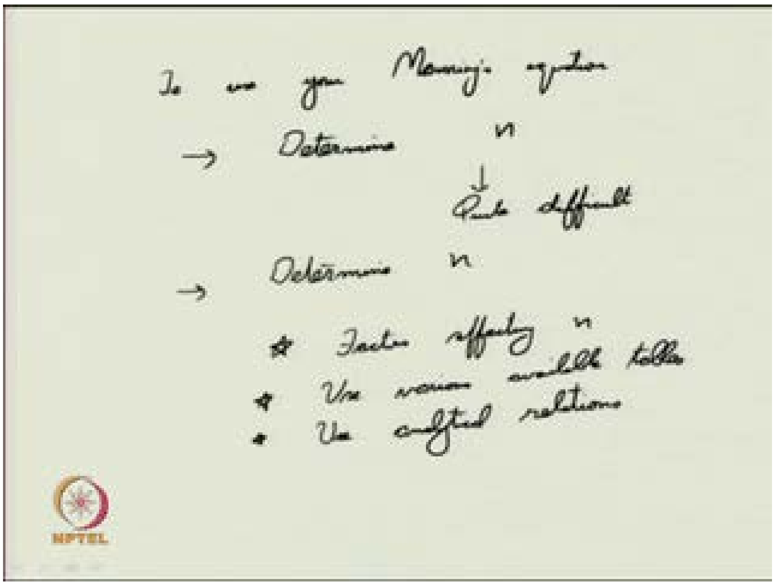

To use your Manning's equation, because Manning's equation is one of the most highly use equation for open channel flow computation. For uniform flow computation Manning's equation is one of the most widely used, and it is also one of the most reliably used equations. So, when to use your Manning's equation, you need to determine n; the roughness coefficient n, and to determine the roughness coefficient n it's quite difficult. You have to do many experiments to finally at m, finally arrive at particular value of n for any particular surface. So, people have done lot of experiments related to these phenomenon and all. You will also require lots of experience on seeing the objects, what are the different types of; means how the roughness co-efficience can affect, or what are the different types of roughness co-efficience for various surfaces.

So, let us see what are the factors means, what do you understand, means how you can compute n, for how you can determine n. To determine n, you need to understand factors affecting n, this is quite essential what are the various factors affecting n. Then you can also use various available tables, in various engineering offices, in various academics institutes various places related to open channel flow and all. You will find printed tables for the roughness coefficient of various surfaces, you can use them, no need to go, means there is no need for you to individual compute the value of n. You can use already available data there, use available tables, or you can also use analytical relations to compute n. There are some analytical relationships, as well as empirical relationships, that relate n with various parameters of the surface, you can use them.

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What are the factors affecting n, just enumerate them, what are the various affecting your Manning's n, definitely the first thing which you will coming to the mind, is your surface roughness, which ever surface you are taking into the account, whether it is a clay bed channel, or whether it is a channel of sand bed, whether it is a channel of concrete bed, wooden, whether the bed is made of wood, depends on the surface. So, the surface roughness plays an important role, what are the thing that governs the surface roughness; the size of the particles, the shape of the particles.

For example; clay have fine grains, therefore its roughness coefficient is much less compare to sand, or compare to gravel, whose grain sizes are much closer. Next factor that can determine your, or that may affect your roughness coefficient Manning's n is vegetations. In a region the if the vegetation if it is dens, if it is densely fact, or if it is distributed at a certain location, type of crop, the height of the crop, various thing affects the discharge in the channel. So, vegetation also plays quite an important role in determining your roughness coefficient n.

Next parameter that can, you can think of is the channel irregularity. If your channel if it is irregular, if your cross section of the channel, if it is getting affected, or it is being changed abruptly, or means all of us are suddenly width get reduce, or if it gets enlarge, and now, these factors will affect the roughness coefficient considerably. If the transition is smooth, you will observes that your roughness coefficient will not get affected that much, but if there is a sudden hump, or sudden restriction in the width, that may affect your roughness coefficient in a high way. For example if there is sand bars in coming into picture, or sand waves that may obstruct your channel flow and all, your roughness coefficient is going to increase, so that you need to take into account.

Next factor which can determine, or which can affect your roughness coefficient is, channel alignment. So, you have seen various types of channel alignment; there are smooth transition curves, there are abrupt transition curves, there are curves or sorry perpendicular intersections in channels certain regions, and certain regions you have seen small smooth curves, or if it is a bent, depending on this two type, where the channel is smooth, there transition is smooth, there the factors on the roughness coefficient will be much less.

For an abrupt change in the cross alignment, the roughness coefficient is going to increase. It is quite intuitional, you can just think on your own also, related to this thing. If there is silting and scouring, these factors are also going to affect your roughness coefficient. For example due to silting of fine grains and all, your channel bed may become smooth, and the roughness coefficient may get decreased, and due to scouring the channel bed may become rough, and your roughness coefficient may increase. So, that factors and all one has to see, other factor that may affect the roughness coefficient, if there is some abstraction like bridge, or dam, any other parameter or any other abstraction peers, well foundation and all. They may affect your affect the flow by changing the roughness coefficient of the section, so you need to take into account those parameter as well.

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Your roughness coefficient is also depended on the stage and discharge. Normally you may see that, for large depth of flow, the roughness, means whatever roughness is there in the bed, that may not affect much of the flow in the channel, but if their flow is shallow, you may see whatever roughness is there in the channel, where it may affect your flow considerably, so that also need to take into account while determining the roughness coefficient. Seasonal changes; for example, especially in vegetation or in other in rainy season, you may see lot of erosion, lot of landslides occurring and the they may carried lot of sediments along with the flow, though all this parameters may cause wide ranges, means the seasonal affects are also playing important role in determining the coefficient of roughness.

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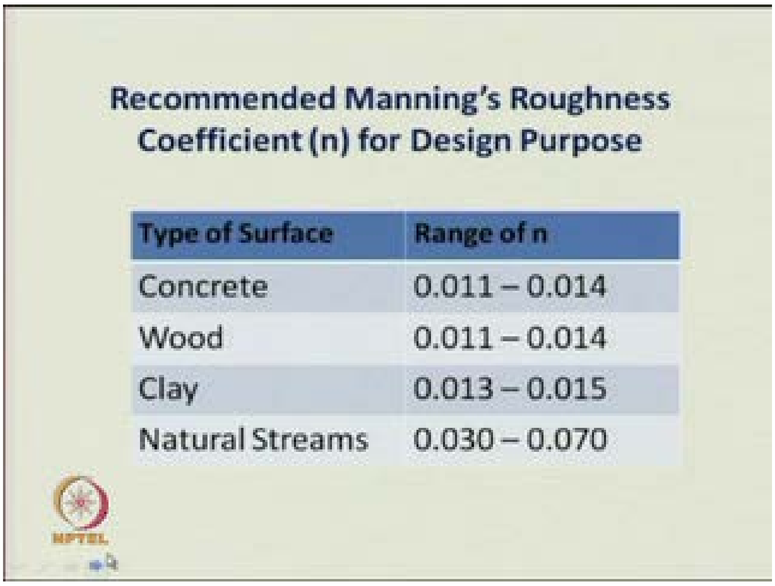

Recommended Manning's Roughness Coefficient (n) for Design Purpose

Type of Surface	Range of n
Concrete	0.011 – 0.014
Wood	0.011 – 0.014
Clay	0.013 – 0.015
Natural Streams	0.030 – 0.070

We will just see, say some of the recommended Manning's roughness coefficient, for various design purposes, we can just show it here. For example, if this a concrete surface, you can take the range of n from 0.011 to 0.014, for wood, for clay, for natural streams. Of course, in this parameters of also there are further deviations, for concrete, whether it is smooth concrete, whether if it is a stratified concrete, whether it is a rough concrete wood, whether it is plane wood, whether it is a stratified wood, clay naturals streams, wide streams or narrow streams. Based on the all things there are further elaborations, which I have not discussed. We are just suggesting that, for designing purpose you may use the range of Manning's abruptly. So, we will come back we will see an example.

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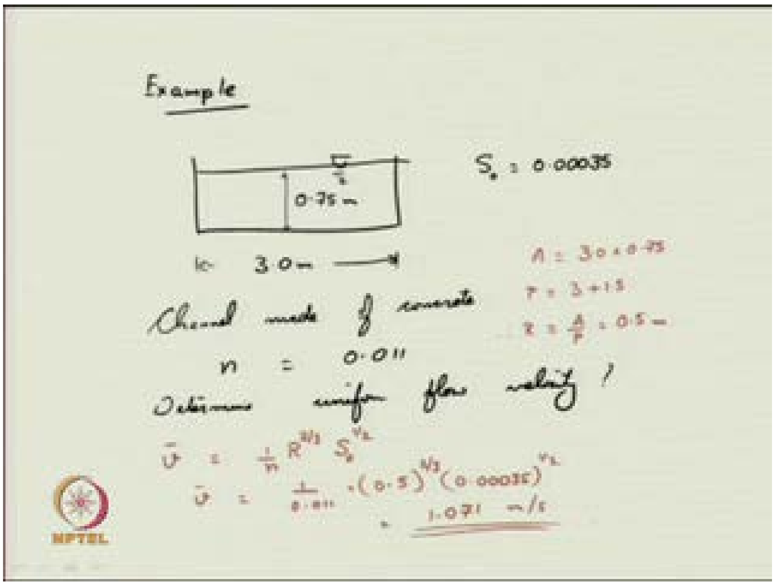

Suppose if there is a rectangular channel rectangular cross section channel, the width of the channel is 3 meters, depth of the flow is 0.75 meters, your beds slope is given as 0.00035, channel is made of concrete. So, for design purpose you may take the Manning's roughness coefficient as 0.011 for concrete. Then determine uniform flow velocity. Now it is up to you to determine in the uniform flow velocity, how will you compute the uniform flow velocity now. Let me give you the solution you know, your Manning's formula is the average velocity is equal to 1 by n R to the power of 2 by 3 S naught to the power of half, substitute the formulas appropriately, use the appropriate terms for rectangular channel, you have the 8 here. This is nothing but 3 into 0.75, your wetted perimeter p is equal to 3 plus 1.5. So, therefore, your hydraulic radius R is equal to a by p is nothing but 0.5 meter.

Therefore, your Manning's velocity v is equal to 1 by 0.011 into 0.5 to the power of 2 by 3 into 0.00035 to the power of half, and this will give you roughly a velocity 1.071 meter per second. This is how you use your Manning's formula to compute velocity. If you want to compute discharge, then just multiply this velocity term with your area of the cross section, you will get the discharge quantity.

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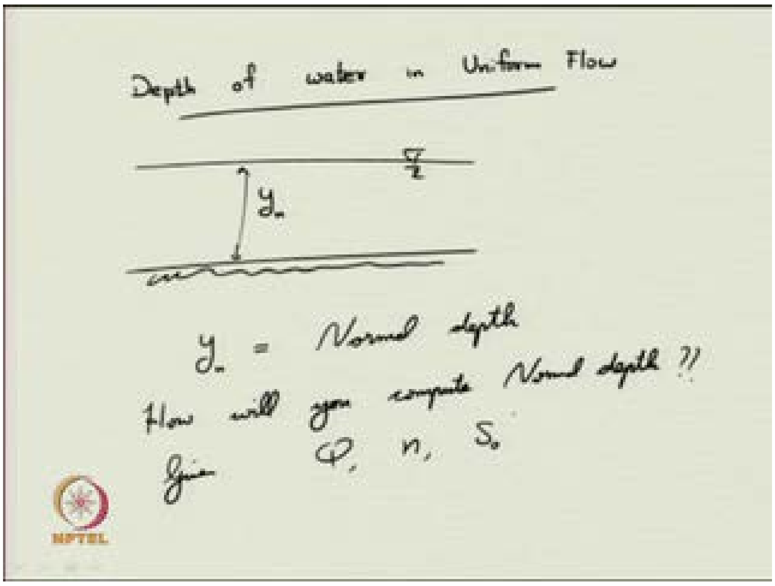

Now, related to the depth of water in uniform flow. So, in a strain, or in a channel, whatever depth is there, this depth is called normal depth. If the flow is uniform, if the stream is having uniform flow, the corresponding depth of flow is called normal depth; this was discussed in the last class, so again I am just mentioning it here. How will you compute normal depth for a uniform flow; say given discharge Q Manning's n beds slope, if all these parameters are given to you, how will compute your normal depth.

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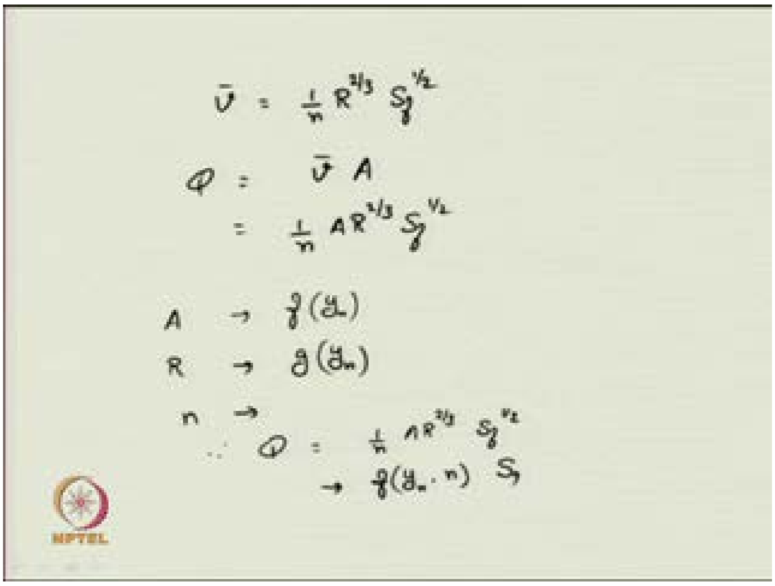

So, you know that velocity is equal to S f to the power of half, your discharge is velocity into area of cross section. So, this is nothing but one by n A R to the power of 2 by 3 S f to the power of half; a is some function of your normal depth y n. Similarly R is also some function of your normal depth y n. So, please note that, the f and g to represent it here, they are just suggesting that function of, it is not related to some gravity terms and please note that. So, R is function of y n, a is also function of y n. You know that n is also may be related to y n or not, we are not exactly sure, but sometimes you may take it them as also constant, for particular surface and all or a for particular term. Therefore Q now, as is equal to one by n A R to the power of 2 by 3 S f to the power of half, can be represented as some function of y n as well as Manning's n into S f to the power of half.

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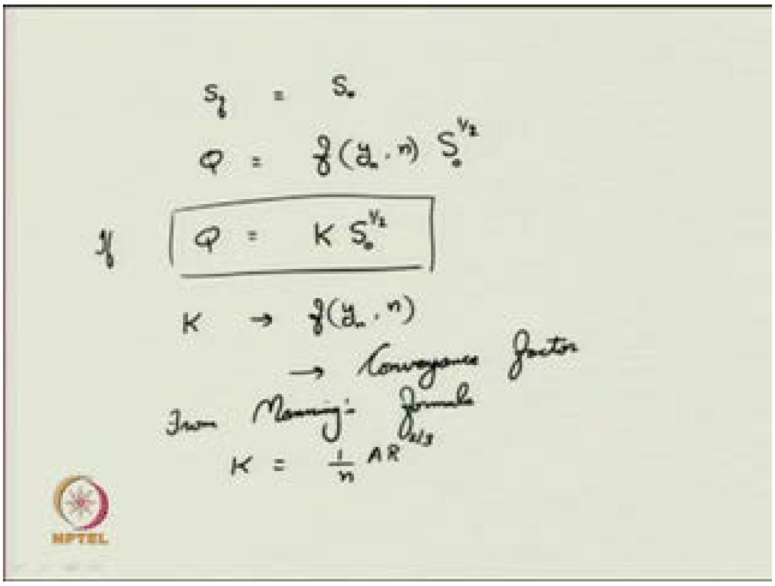

You R S f is equal to S 0; therefore, again function of y n Manning's n into S naught to the power of half. If I suggest this as k into S naught to the power of half, if I obtain such a relationship, now if Q is equal to k into S naught to the power of R, where k is function of y n and n. This term is also called conveyance factor, this term, this k is also called conveyance factor. You can obtain conveyance factor easily through Manning's equation also. From Manning's equation your k is nothing but is 1 by n A R to the power of 2 by 3. Now just think of this, this conveyance factor why we are suggesting is that, to determine your normal depth in the uniform flow. One can use several methods, you can determine your normal depth using trial and error method, or you can determine using design curves, or you can use numerical methods, that use non-linear solution techniques and all, non-linear methods are solving, sorry or solution methods for non-linear equations like, Newton's method, by conjugate method. They all can be employed to determine y n, but how can you arrive at that. So, your conveyance factor k is equal to 1 by n A R to the power of 2 by 3, we can write it in that way

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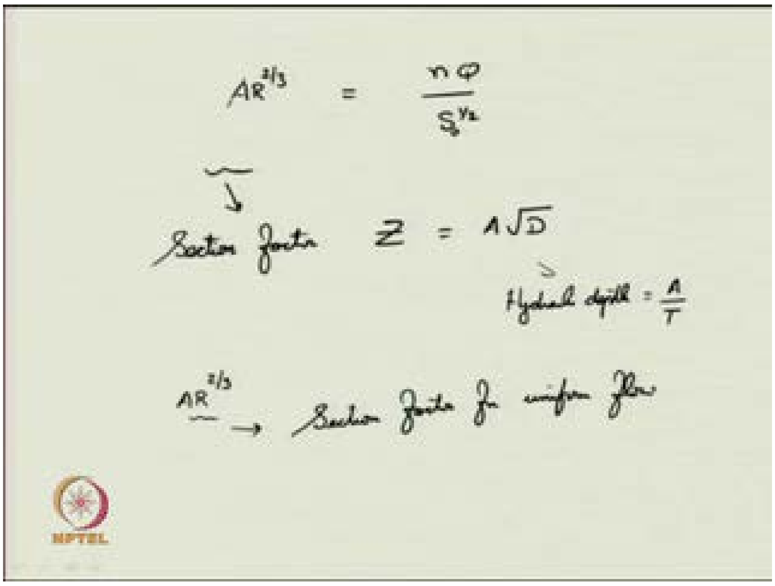

Again proceed it, I can write the same relation now A R to the power of 2 by 3 is nothing but n into Q by S naught to the power of half, recall your flow computations in critical flow and all. You just recall the earlier module, where we computed flow in critical sections and all. There you had define the term call section factor z; that was given as a to the power of a sorry a into root of d, where root of d means, where d is your hydraulic depth, equal to a by top width t, these things we had studied in our earlier module. Now similarly, the quantity A R to the power of 2 by 3, this can also be suggested as a particular type of section factor for uniform flow. So, A R to the, this section factor now can be used for obtaining design curves as well now, how will you obtain them.

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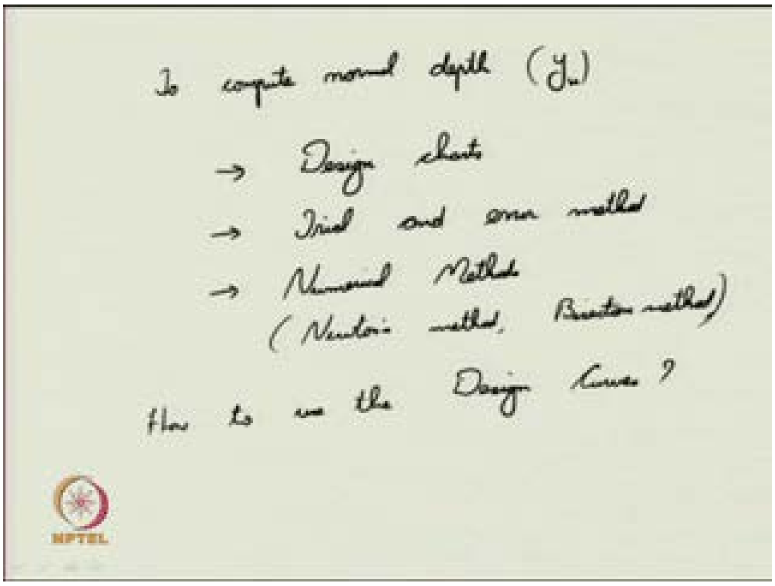

As I mentioned earlier, to compute normal depth y n, you can use design charts, you can also use trial and error method, you can also use numerical methods. So, numerical method like, Newton’s method, bisection method etcetera. You can refer any numerical method books related to this; like that these procedures are there.

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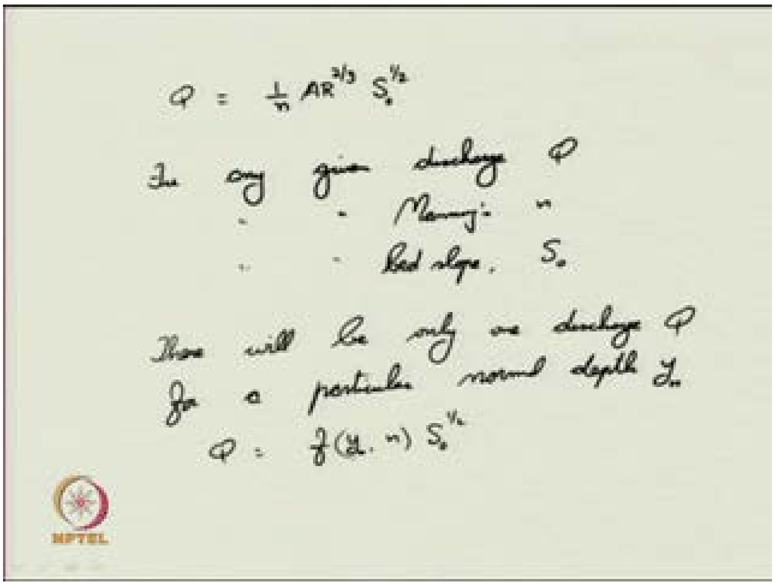

So, how to use the design curves, how do you use the design curves; that is the question post to you now. Q is equal to one by n A R to the power of 2 by 3 S naught to the power of half, how will you develop design curves. If you see the things closely, for any given discharge Q, and any given Manning's n, as well as for any given bed slope S naught. There will be only one discharge Q for a particular normal depth y n, this you have to keep it in mind, as we suggested that, because Q is function of y n and S to the power of half. So, for this thing, for particular depth y n, there can be only 1 possibly, only 1 type of discharge available. So, therefore, for a given discharge vice versa, for a given discharge we can have only one particular normal depth in that particular channel, that you need to take into account base, using that theory, or using that philosophy you have to develop your design curves.

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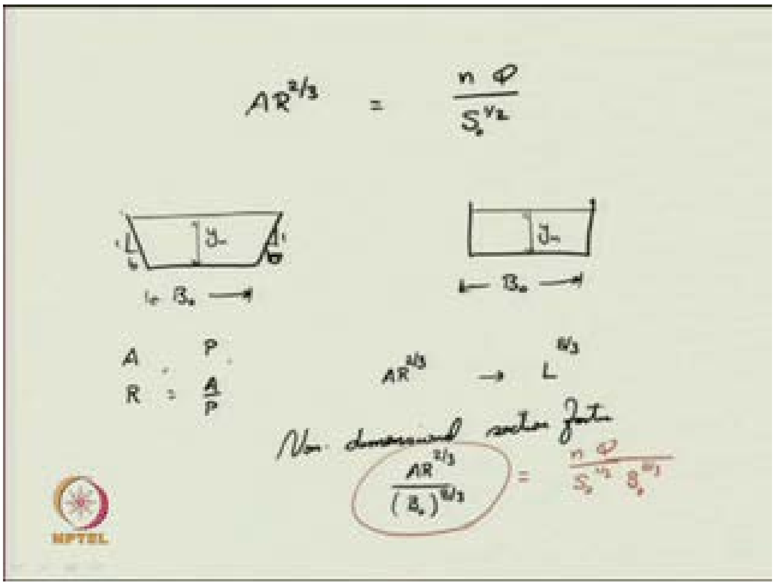

This particular relationship A R to the power of 2 by 3 is equal to n Q, if you use trapezoidal channel, or if you use rectangular channel whatever be; say that trapezoidal channel is having the bottom width B naught as fixed, if the rectangular channel is having B naught as fix width. If this is having sides slope 1 is to b I think, 1 is to b. Then for the normal flow, for the normal depth, or for the uniform flow, you can have. If you recall our earlier classes and all, I think now our earlier class we had suggested, how to compute area of a trapezoidal channel, how to compute the hydraulics radius of a trapezoidal channel, how to compute the wetted perimeter. These things we had given a, some of the relationship also later some of the relationships on that. Similarly for rectangular channels also we had given the corresponding relationships, you employee them. Now the quantity the section factor A R to the power of 2 by 3, this is having a dimension of, it is having a dimension of l to the power of 8 by 3.

Therefore we can now think of a non dimensional section factor, that can be given as A R to the power of 2 by 3 the whole quantity divided by any characteristic length, for the channel the characteristic length is definitely your, fixed width of the channel, because the fix width of the channel B naught; that is not going to change, so you just obtain the following form. So, this quantity is now your non dimensional, so this is your non dimensional section factor. So, this non dimensional section factor, it can be easily computed you know that, from n Q, if n is given, Q is given, S naught is given, and B naught is also given, because it is a fix parameter of the channel B naught is also given. So, b naught 8 to the power of 3 if you compute this right hand side, the left hand side is available now. So, you can now easily identify the non dimensional section factor for a given discharge, in the given channel you can now easily determine the non dimensional section factor.

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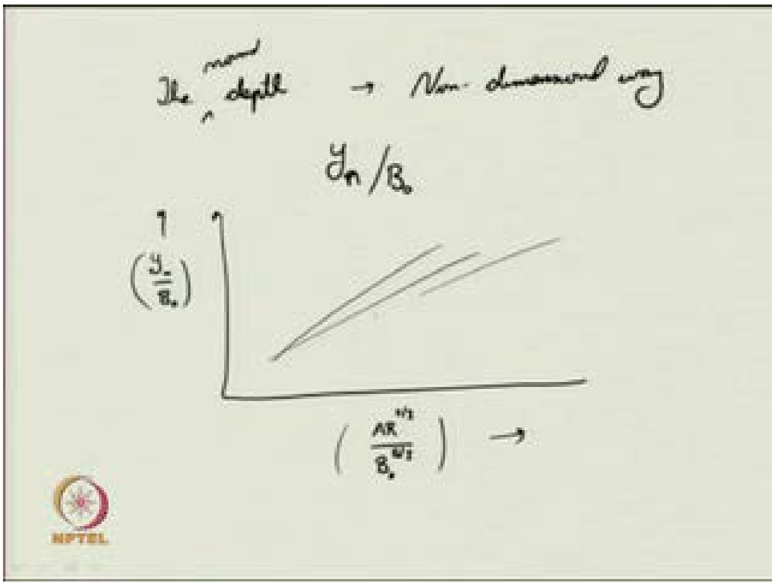

The depth of flow or the normal depth, it can also be given in a non dimensional way; that can be given as y y n by B naught now, so that is your normal depth if it is divided by the width of the channel, you will get a non dimensional form of the depth of the normal depth of the channel. One can now easily plot the non dimensional quantities A R to the power of 2 by 3 by B naught 8 by 3, this in the x axis y n by B naught in the y axis. If you plot them, you may get a curve of such, based on the side slopes and all, you may get various types of curves, so you use the appropriate curves for example; let me suggest you for an example how to develop that.

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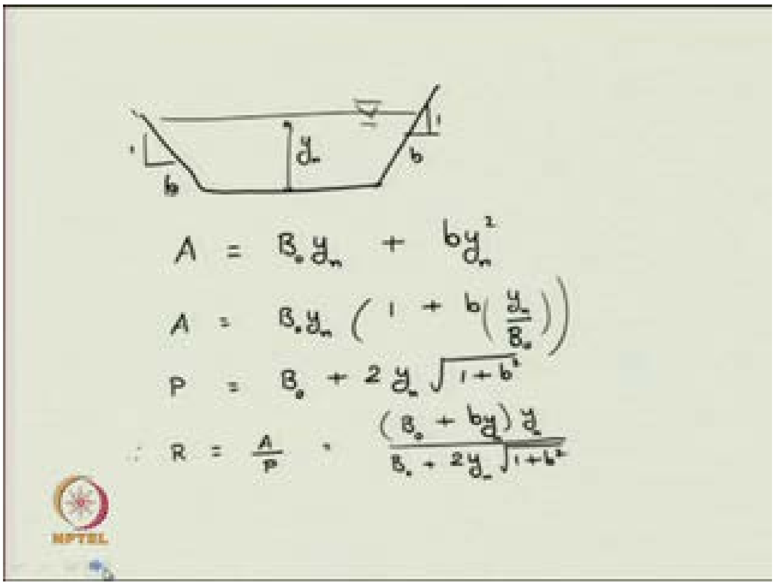

Again, in a trapezoidal channel of side slope 1 is to B, your a is nothing but B naught into y n plus b y n square, or I can write this relationship further as B naught y n into 1 plus, the side slope relationship 1 is to b this is 1 is to b, so that b is coming into picture here, small B into y n by B naught. Like this you can obtain you area, your wetted perimeter p this is also given as, B naught plus twice y n into root of 1 plus B square. Therefore, your hydraulic radius R, is equal to A by p, this can be given as B naught plus b y y n by B naught plus 2 y n into root of 1 plus B square, further proceed the computation formulations, rearrange the terms, I will get the following relationship now.

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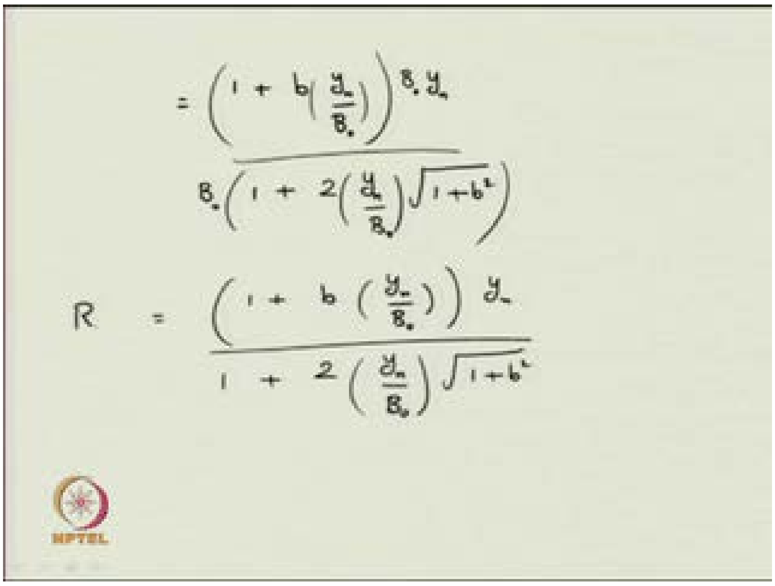

I can write it as 1 plus small bin to y n by B naught whole thing multiplied by B naught into y n divided by, you are taking B naught outside, B naught into 1 plus twice y n by b naught root of 1 plus b square. Again rearrange the terms; you will see that 1 plus b into y n by B naught the whole quantity into y n divided by 1 plus twice y n by B naught into root of 1 plus b square, this is your hydraulic radius R.

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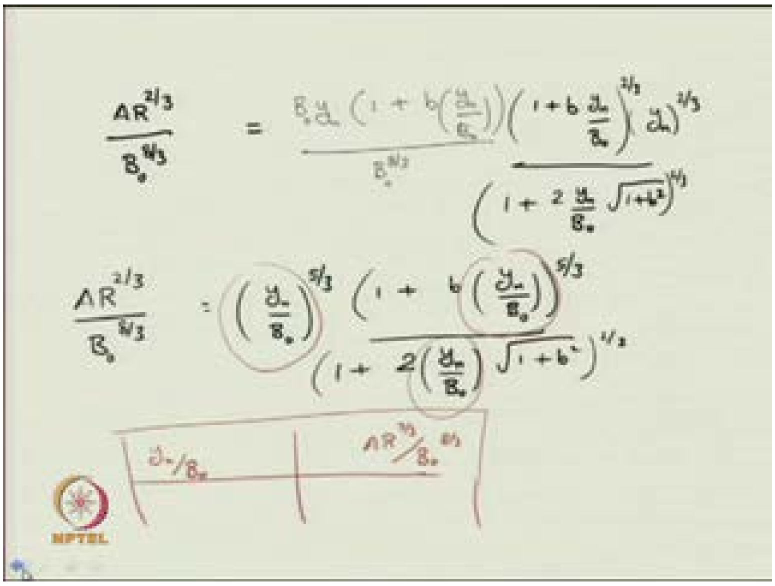

So, if the quantity A R to the power of 2 by 3 by B naught 8 to the power of 3, this can now we easily obtained as, B naught y n, this quantity into 1 plus b into y n by B naught, the whole quantity raise to 2 by 3 by 1 plus 2 y n by B naught root of 1 plus b square whole quantity raise to 2 by 3 into y n whole to the power of 2 by 3. So, this is nothing but y n by B naught whole to the power of 5 by 3 1 plus b into y n by B naught whole to the power of 5 by 3 by 1 plus 2 y n by B naught root of 1 plus b square whole to the power of 2 by 3. So, for a trapezoidal channel, you are getting your non dimensional section factor in the following form. So, here you can easily see that, the right hand side has non dimensional terms y n by B naught at several locations.

So, they have that non dimensional term y n by B naught. So, if you are able to plot now, say if you are able to compute; say for different values of y n by B naught, the corresponding value of A R to the power of 2 by 3 by B naught 8 by 3. You can now easily form a table now, that table can be plotted to obtain your design curves, for different values of b. You can easily obtain a corresponding relationship, I have just made one table, particular table let me show it to you.

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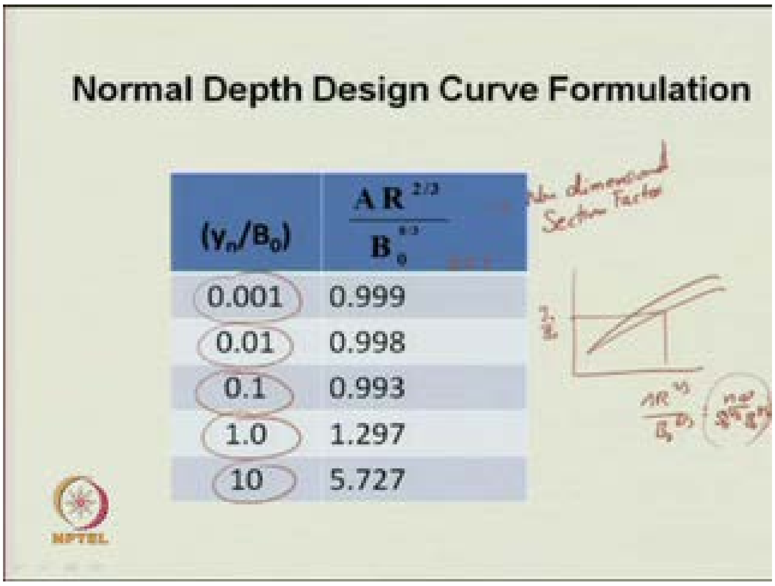

So, here for the normal depth design curve formulation I had taken in the y axis, or in the left column here y n by B naught, for different values of y n by B naught, I computed the corresponding non dimensional section factor. I have computed them, non dimensional section factor, you are getting the following curves, so this you can plot. You will get your corresponding; you can get the corresponding curves. So, here I had taken small b is equal to 1. Please note that you can compute it for small b is equal to 2 similarly, and try to get the updated curves. These curves you can subsequently use in your problem to obtain the corresponding normal depth. Because once you, this you know that it is nothing but. So, if you have this value, this particular value if you have them, try to interpret curve and try to gets your non dimensional normal depth, from that you can interpret your normal depth. So, this way you can compute the normal depth. Let me conclude today's lecture based on this thing, later we will continue in the same topic in the next class also.

Thank you.