Advanced Hydraulics
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Module - 2
Uniform Flow
Lecture - 1
Introduction to Uniform Flow

Good morning everyone, we are back into the lecture series on advanced hydraulic. This as you know it is the postgraduate course, related to civil engineering, and most of you have applied for this course.

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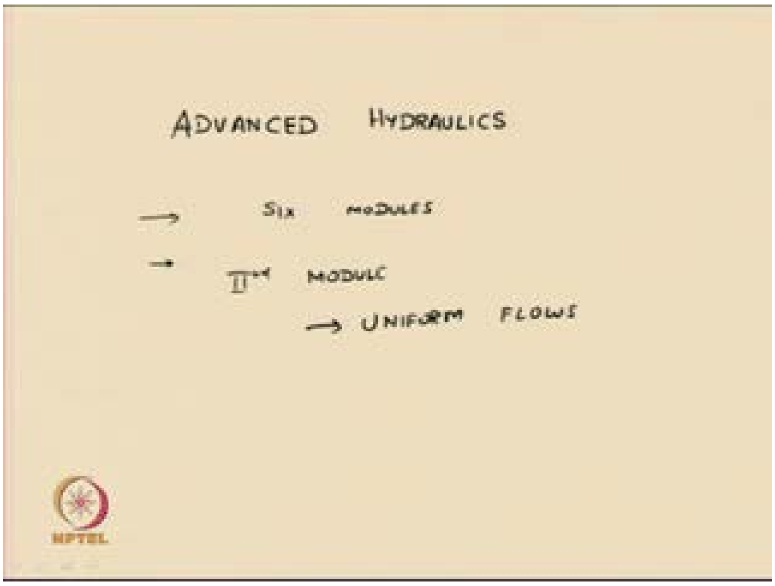

Till now we were (( )), as you recall from our beginning lecture, there are six modules in this particular course; say till the last lecture, we were dealing with the first module; that deals with the open channel flows, in general the open channel flows and all. Today, we will come into the second module of the course; that is the second module is uniform flows. So, if you recall those lecture series in the first class, we had given an overall view of all the topics, related to this course. So, in the uniform flow, we had at the time briefly mentioned, what is meant by uniform flow, what are the qualification criteria for uniform flow, what do you mean by velocity measurement in uniform flow, Manning and Chezy's formula, determination of the roughness coefficient, determination of normal depth and velocity, determination of most economical section, non erodible channels, flow in channels, section with composite roughness, etcetera. So, we will slowly deal with all these topics, as we proceed in this lecture. Let us start with the simple case of uniform flow.

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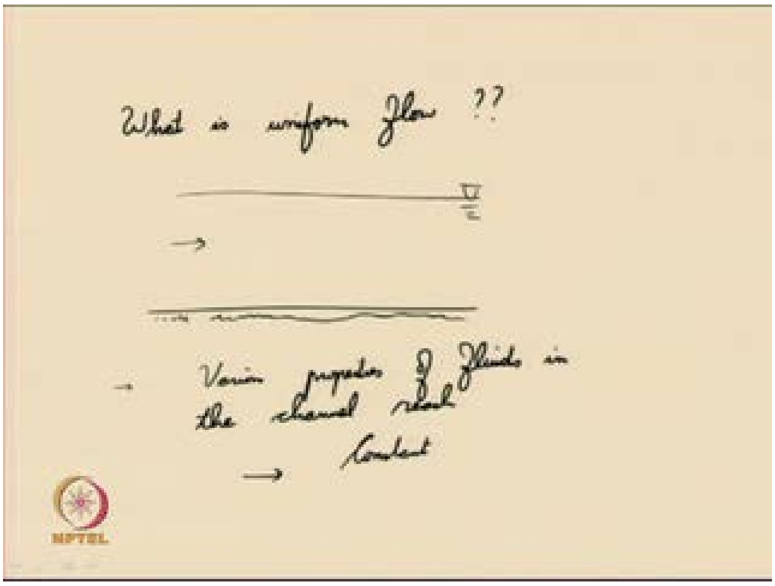

Let me pose you a question, let me pose you a simple question; what is uniform flow, what do you understand by uniform flow. If I draw the a particular reach of channel, with depth of water, having flow direction in this. So, what do you mean by uniform flow. So, uniform flow, it suggests that whatever properties of fluids are there, whatever properties of fluids are there in the channel for the fluids. Various properties of fluid in the channel reach, they become constant, if such a flow arises, then that flow is called uniform flow.

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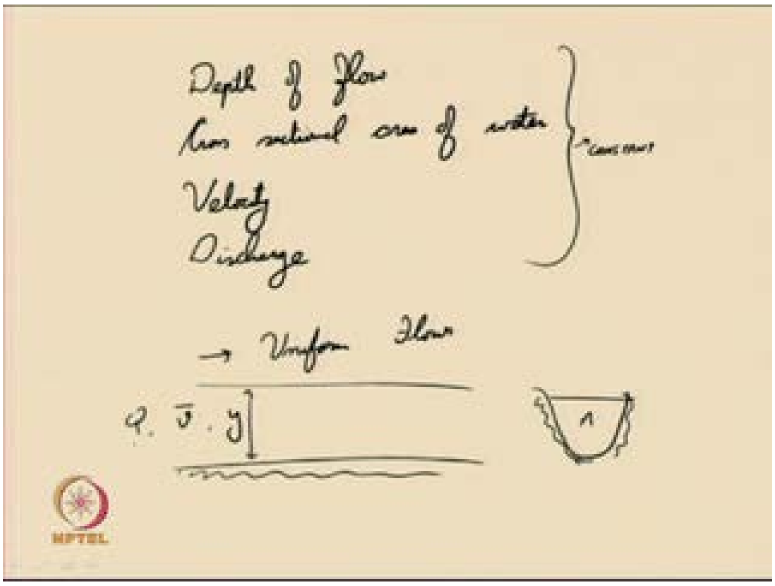

What are the various properties that you are generally aware of; depth of flow, cross sectional area of water, velocity of flow, then discharge. You can even enumerate some other properties, let me just stop here. So, for example, if these properties, if they are constant, then such flows are called uniform flows. So, what do you encounter in such type of flows, if the depth of flow in a. Let me just again draw the channel reach, depth of flow, cross sectional area. This is your cross sectional area, velocity, discharge, Q.

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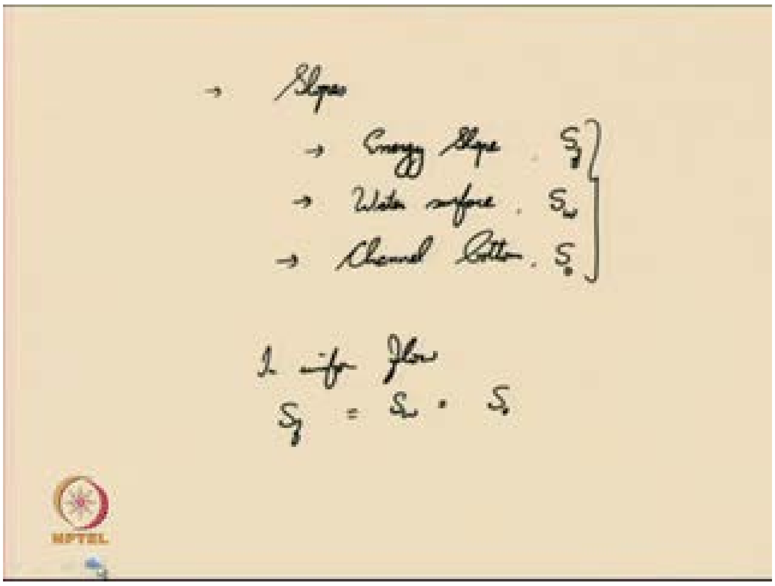

So, if all these properties, if they are same in a particular reach of channel. Then that means, you can suggest that, the following slopes which you are aware of, slopes like energy slope, water surface slope, channel bottom slope. So, symbolically I think we have mentioned it as S f S w S 0 if you recall them from earlier lecture, what happens to these slopes. In uniform flow, your S f will become equal to S w; this will become equal to your channel bed slope. This you can see very easily from the graph, or from here. You just draw the corresponding energy line; energy line, you are bed slope, your water surface slope, all of them become parallel. So, you will get a same slope for energy, water surface, as well as channel bottom. So, this is one peculiarity of uniform flow.

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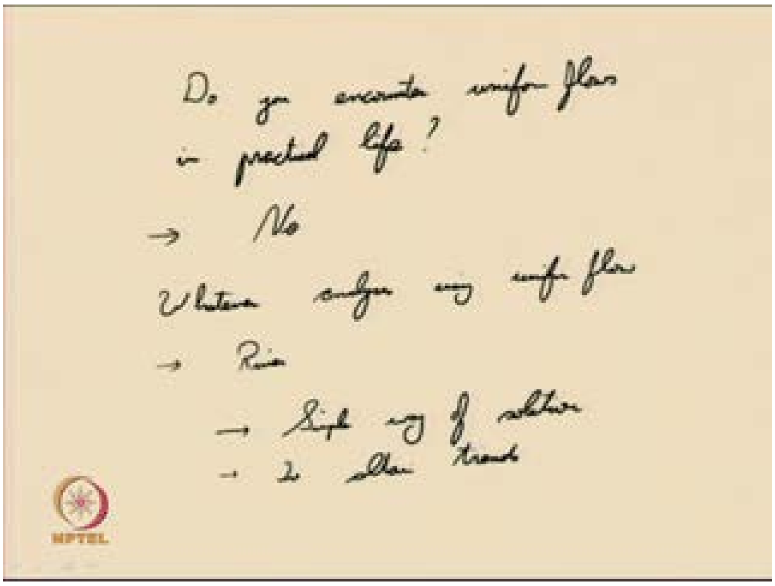

So, do you encounter uniform flows in practical life, this is a simple question, you encounter uniform flows in your practical life. A very simple answer is no, but whatever assumptions you are taking, for whatever analysis using uniform flow you do, that can be used for interpreting in general, the non uniform flow or the flows that occurs in the nature. So, that is how you have to approach the case, do you think. So, for example, river, in a wide river and all, if there is no flood situation. In most of the cases we can approximate for a considerable large reach of the channel, or large reach of the river that the flow is uniform. It is just a simple approximation, they have any complex situation; of course, in the river there are many secondary issues, where higher issues and all are present, but still, just to solve your engineering problem, you can try it in a simple way first, so that is the way we approach. First you solve it in a simple way, simple way of solution, that can help us to get better idea of the situation there. You can then subsequently incorporate many other parameters, and try to solve them with other non uniform conditions, but this is for the simple to obtain trends.

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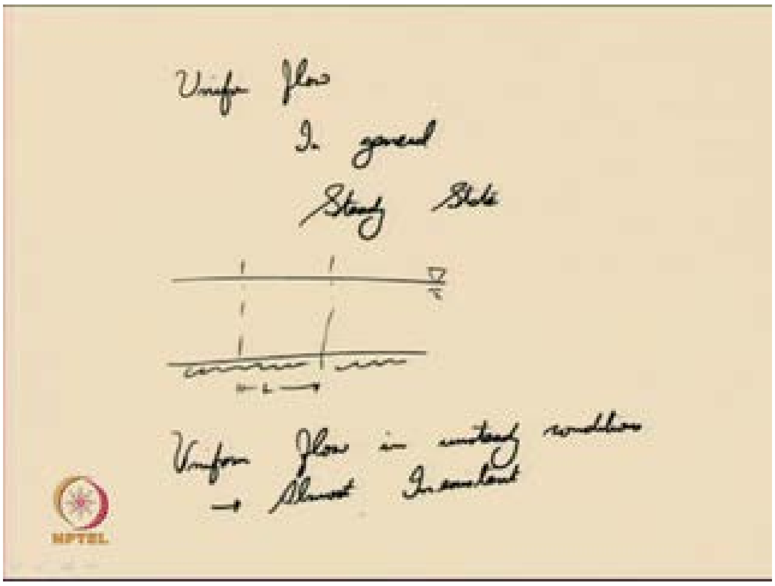

So, in the natural rivers, very rare that you will be having uniform flow; uniform flow in general, they are steady state. Again in the same channel each if I draw that, as we mentioned the depth of flow, the velocity of the flow, the discharge cross sectional area, everything will be uniform in the particular stretch, whichever stretch you are going to take. If it is uniform, then that means, that the flow is steady, your parameters now are also not going to vary with respect to time. If it varies with respect to time, it is very difficult to obtain uniform flow. So, uniform flow in unsteady conditions, I can say that it is almost inexistent, may be in future if someone analysis it properly, and find that you can even have uniform flow in unsteady conditions, it can be proved or it may be proved, but we are not sure at present, till the. From the present knowledge, I can only suggest that uniform flow in unsteady conditions, are almost nonexistent.

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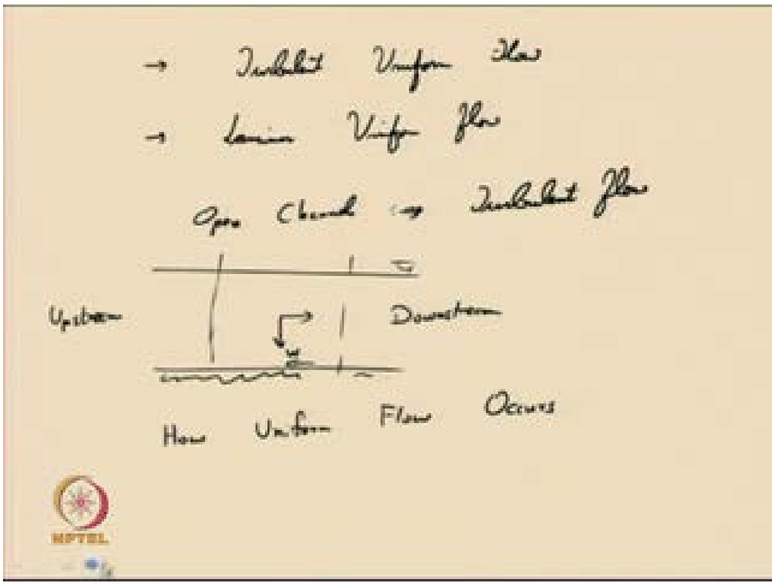

We will be mainly dealing with turbulent uniform flow; of course, there are laminar uniform flows, but as a part of civil engineering, we are mostly dealing with the open channel, we are dealing with open channel, and there you see turbulent flow in general. Laminar flow in nature is very rare, and this is difficult, it may be used some other applications, may be in some chemical engineering or mechanical engineering technology and all. We may be using laminar flow methods and all; however in our case we are generally using the turbulent uniform flow techniques. So, in the open channel, I draw in the same reach, you have the upstream location, you have the downstream location.

As the flow starts from the upstream location to the downstream location, in general we are talking about how the uniform flow occurs, or how generally the flow occurs from upstream to downstream, that you are quite aware, so how uniform flow occurs in a channel flow. So, when the flow occurs from upstream to downstream, you have to balance many forces. There are many type of forces which you have already dealt in the momentum equation and all, we may again go through them.

So, we will see that, once you try to balance the forces, what are the driving forces for the flow from upstream to downstream, certainly it is the gravitational force. The component of the gravitation force, is the gravitational force, and the component in the in this flow direction, that will help the liquid particle from the upstream to move to the downstream side. There are certain forces that oppose the fluid motion, so there is resistance of the flow. So, if both of them get balance at certain situation, then generally your flow become uniform, that is the simple approach, it has been done.

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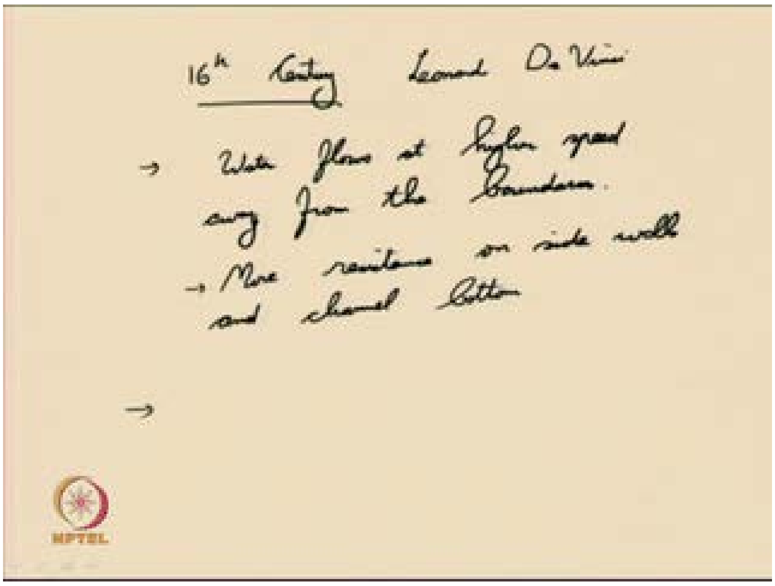

Even this approach has been done in the 16th century, Leonardo Da Vinci himself has suggested about such flow, means how the uniform flow occurs. So, there he suggested that, at that time he suggested that simply that water flows at higher speed; water flows at higher sped away from the boundaries, and there will be more resistance on sidewalls, and channel bottom. He used the simple theory; he suggested that water surface, means surface of the water is open to air. Air is offering less resistance, compare to the channel bottom which is solid. So, he used such a simple method, and analyse that, water flows at higher speed away from the boundaries, both the side boundaries as well as the channel bottom, and side walls and channel bottom they give more resistance to channel flow, and fluid flow in the channels and all. So, this, in the 16 century itself it was been identified, so that means this science is pretty old, it is not that new as many may be thinking of it.

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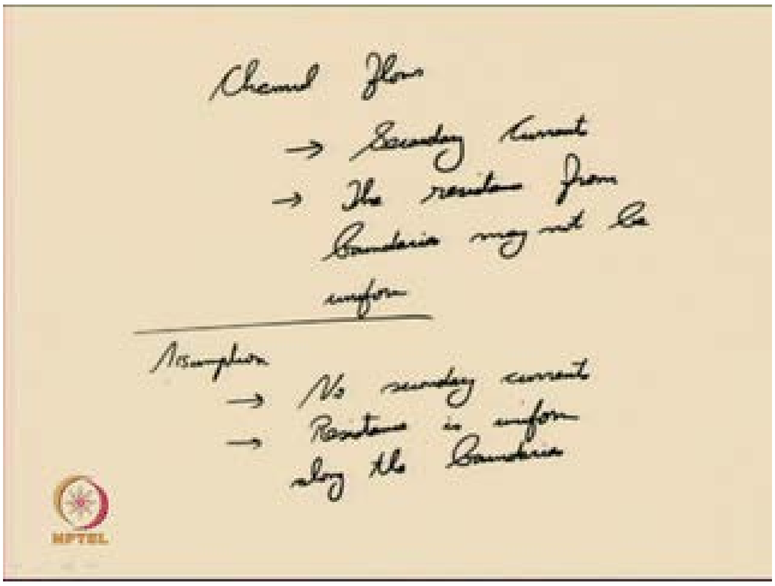

In the true sense if one goes through channel flows, you may see that the channel flows; there may be lot of secondary currents. The resistance from boundaries may not be uniform. So, these are some of the practical situations, do we need to take into those conditions here, just to analyse the uniform flow. Well as an assumption here, we are suggesting that no secondary currents. As I mentioned earlier, this is to simplify your results, just to understand the phenomenon, it is not that you have to solve the natural problem in it is (( )). You can simplify the things, you can obtain the trend. Suppose, if I avoid this secondary currents, and try to interpret that there is no secondary current, and the flow is uniform, and also I am suggesting that the resistance offered by the channel bed as well as the side wall, they are uniform in nature, if they are not changing it is not non-linear or not like that. So, such assumptions if we can incorporate, then we can go in a better way, or we can simplify the results and let us see what happens, resistance is uniform along the boundaries.

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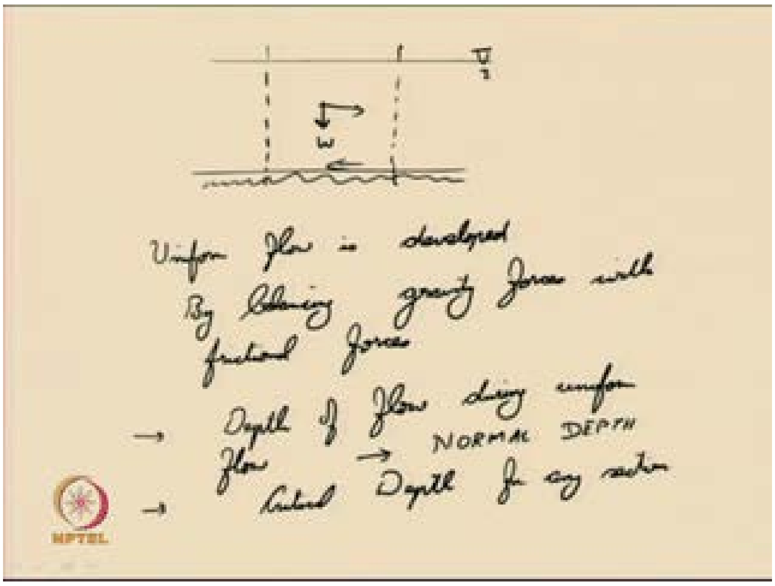

So, such situations are there I can suggest that; for the same channel reach, whatever force due to gravity is there, whatever force; that is the uniform flow, is developed by balancing gravity forces with frictional forces; that is as I mentioned earlier, if there are no secondary currents, and if the fictional or resistance from the channel boundaries if they are uniform. You will be able to see that, the component of the gravity force say; this is component of the gravity force along the flow direction, this is the frictional force. They have to balance then only flow becomes uniform, how do you see that. You can even describe them mathematically also, let us see. Before that, let me define the following terms for the uniform flow.

The depth of flow during uniform flow, it is called normal depth. So, here after whenever we say normal depth, it means that depth of flow, for the uniform flow conditions for that channel. So, you may even able to draw for a particular cross section, and particular reach of channel. The normal depth line, the various normal depth line and all that, one can easily obtain them. You have also seen the critical depth for any flow section. Using the critical depth, one can see whether the flow is uniform in subcritical condition, or whether the flow is formed in supercritical condition, and how the flow changes form subcritical to supercritical and all, that one can analyse from the previous chapters and all as you have seen.

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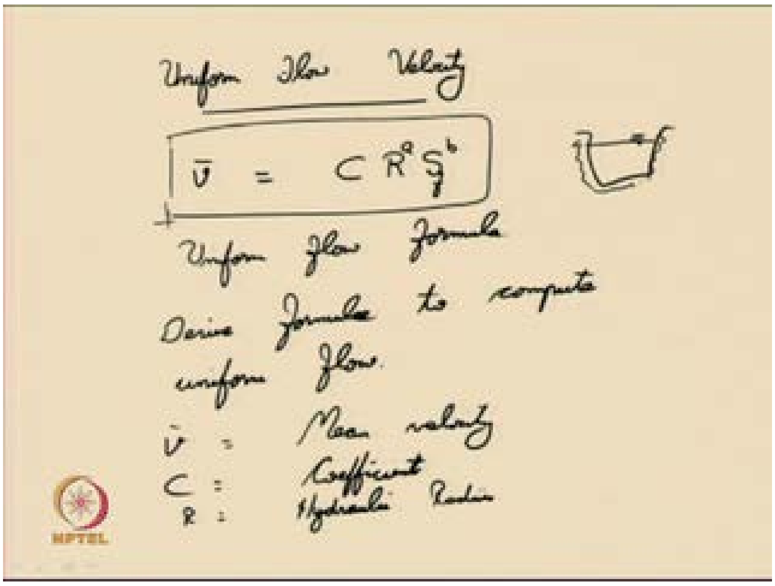

The scientist, various scientists have done several experiments. They have through the dimensionless analysis and all. They have suggested that, the velocity for the uniform flow in general, one can express the velocity for uniform flow in general, as v is equal to some coefficient C then R to the power of a S f to the power of B, this is called uniform flow formula. You can use this; they have obtained these, using scientific study and non dimensional analysis. In general from this method, you can use the various formulas, or you can derive formulas to compute uniform flow. In this you know that v is equal to the mean or average velocity in the cross section, C coefficient.

So, you do not know, what is that coefficient? This coefficient in general, it depends on type of the boundaries, what are the materials on the boundaries, the cross sectional. It may depend on the cross sectional area, it may depend on perimeter. Various factors it may depend, we are not aware of that; that is the objective now. It is objective of us to obtain what are the values of C, and how it is how it has to be computed. R you know this is the hydraulic radius. S f as you are aware from the previous slide, this is the energy slope.

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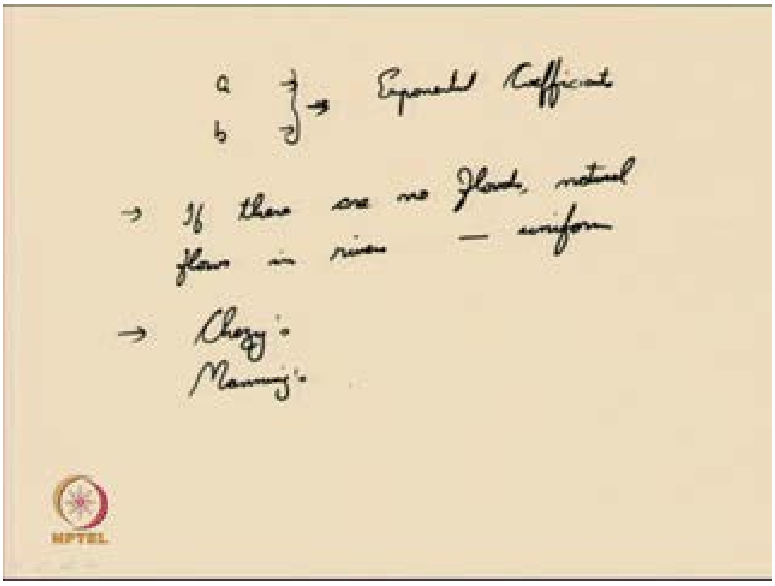

Your coefficients a and b, they are simple exponential coefficients, that has to be determine for various methods being used, either experimentally or analytically, one can determine this coefficients a and b. At present the equation which we are showing here; v is equal to C R to the power of a S f to the power of B, it is a general uniform flow equation. If in general or most of the cases, if there are no floods, the natural flows in river are approximated as uniform flow. So, once this approximation is done, you can use the uniform flow formula, and compute the velocity as well as subsequently the discharge. Some of the formulas; uniform flow formulas, you might have earlier heard about Chezy's formula, Manning's formula or may be some other equations which you may be aware, they are also. They are all derived using the uniform flow formula.

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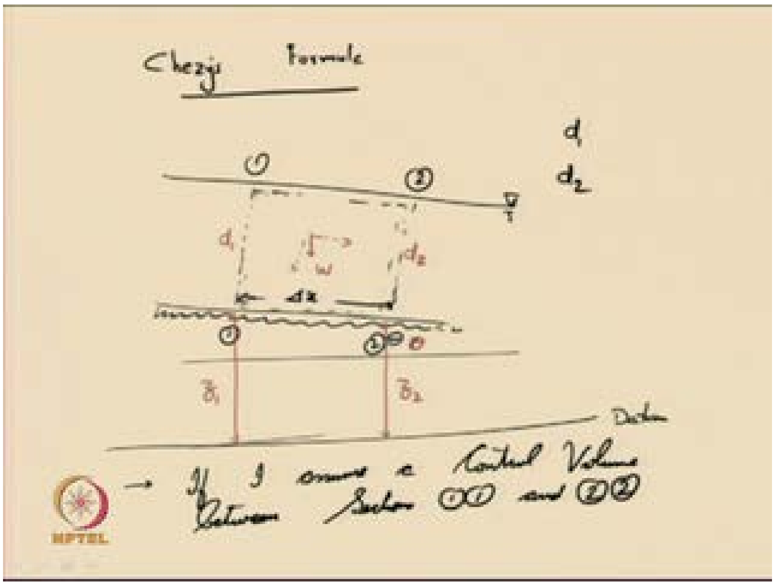

Let us start with the Chezy's formula, how the Chezy's formula is derived. Here again, the same channel cross section I am taking that, so you can think that, this channel slope is theta, if you have the following datum then. This is the inclined depth, and you know the vertical depth, so inclined depth, if you recall from our earlier lectures we had given this as d 1 at section 1 1, d 2 at section 2 2 if you recall them the inclined depth, or the depth that is normal to the water surface, the depth measure normal to the water surface and to the bed; that is called inclined depth, if you recall that. So, d 1 d 2 like this you can have them. If you assume a control volume, a volume inside this, between these two sections. If I assume a control volume between sections 1 1 and 2 2, then you can start thinking, you can derive the Chezy's formula.

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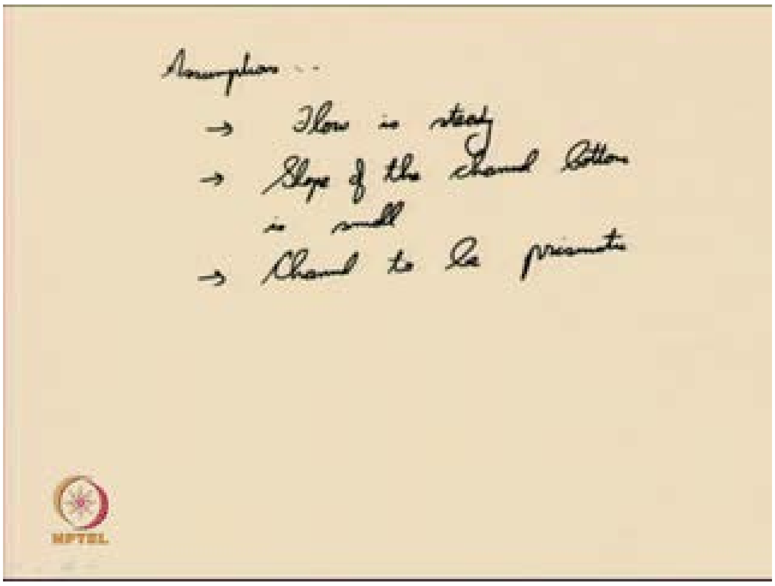

Before that the assumptions used here are; your flow is steady, the slope of the channel bottom is small. Also we are assuming the channel to be prismatic; that is the cross sectional area, the cross sectional shape of the channel is not changing, with along with the length of the channel, so it is the channel that is prismatic. Such is the case, in the previous slide, let me assume this length as; del x, then this depth is given as d 1, the depth here it is given as d 2. The length here is del x, this section is at a height, z 1 from the datum. This one is at the height z 2 from the datum, the angle here is theta, the slope that is the bed slope is having an angle theta. You have the various forces, the gravitational force for this entire volume, it is acting downwards, and it is given as w. So, it is definitely having components in this direction, as well as this direction. Just try to compute the things now.

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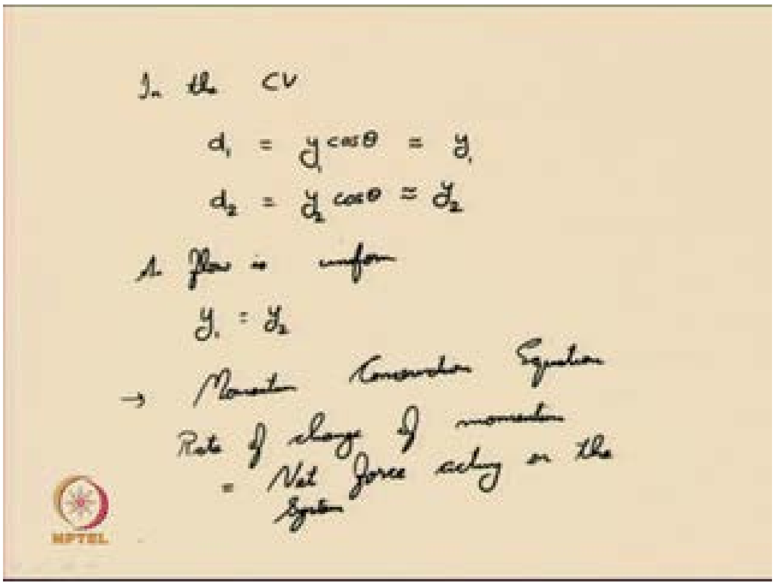

So, now in the control volume whichever you have selected, from our earlier theorem, you can see that, your d 1 is nothing but actually according to the theorem, this y; that is the, y is the depth, the of the vertical depth y cos theta. Now this is approximately equal to, sorry y 1 cos theta, so it is approximately equal to y 1 itself, because we are suggesting that theta is small, so cos theta is approximately 1. So, d 1 is approximately equal to y 1. Similarly, d 2 is equal to y 2 cos theta, so it is same as this thing. As flow is uniform, y 1 is equal to y 2. We shall now apply the momentum conservation equation, what does this theorem suggests if you recall some Newton's theorem. Rate of change of momentum, is equal to the net force acting on the system. The rate of change of momentum in a system is equal to the nets force acting on the system. This you have studied from your high school standard level onwards.

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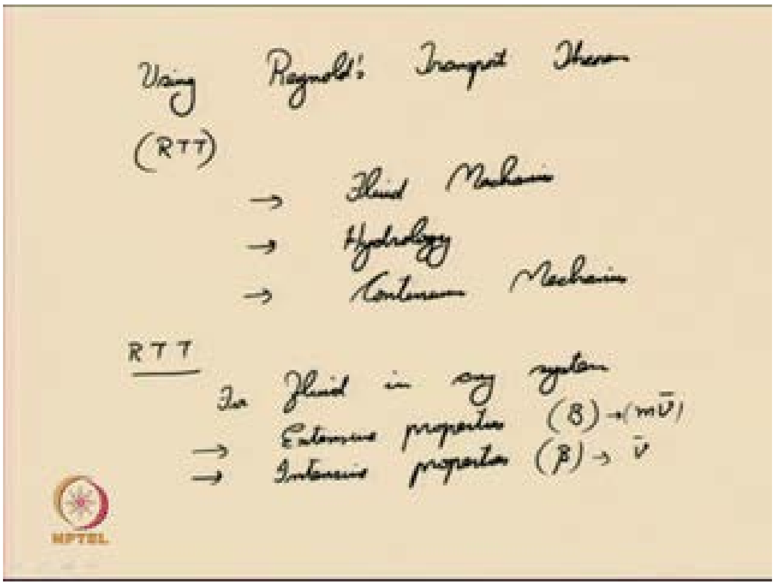

But now at this post graduate level of engineering, we can use, or we can introduce the same concept using Reynolds's transport theorem. Most of you may be familiar with RTT, in short I can call this as RTT. You may have studied them either in your fluid mechanics course or even in your hydrology course, may be at the advanced level or advanced basic level itself, or some of you might have studied in your continuum mechanics course as well. So, we will be using the Reynold's transport theorem, to just simple see that. I am not, definitely I am not going to explain the Reynold's transport theorem, we are just applying your Reynold's transport theorem for momentum conservation, for the particular case of uniform flow, the channel cross section and the reach whichever we have done earlier. So, what does this theorem suggest; your RTT. For fluid in any system you can have extensive properties associated with the fluids. Symbolically we give this as capital B.

Please note that this is not the breadth of the channel and all. This is symbolically to know that, this is an extensive property capital B, and these extensive properties are associated with the mass of the fluid. You can also define intensive properties beta. This is the properties of the fluid that are independent of the mass of the system; for example, velocity. Velocity of a particle is independent of the mass of the particle. There is no role of mass coming to the picture there, whereas if you compute momentum, momentum is a property of mass therefore, it comes as an extensive property, and definitely any extensive property. If you divide it by mass of the fluid system, you will get your corresponding intensive property. For example, in case of momentum, if m is your momentum, and if this is an extensive property, the corresponding intensive property will be momentum divides by mass, so it will give you velocity of their system. So, that way you compute extensive property, you define extensive property, intensive property for the fluid system.

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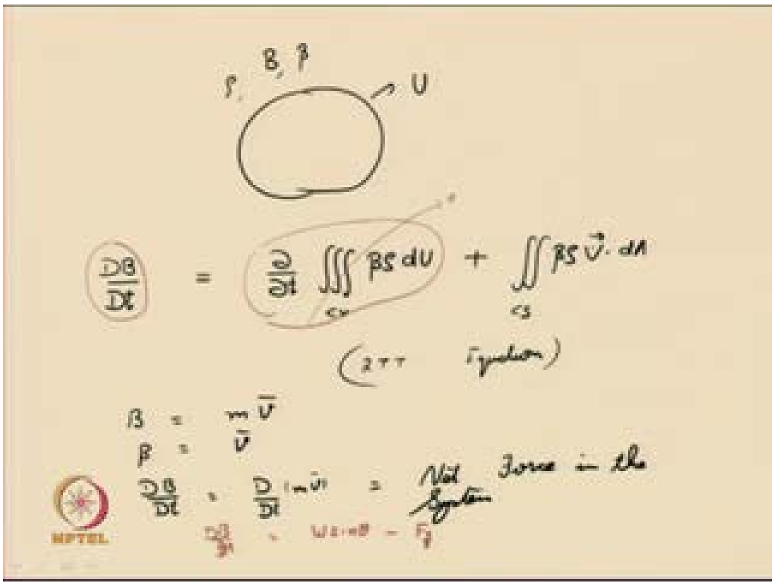

So, the theorem suggests that, or any such volume, any arbitrary volume. I have just drawn a arbitrary volume, the volume, the change of the extensive property with respect to time, this is nothing but the amount of. Say if this is having volume u, if this system is having volume u if the extensive property is B, intensive property is beta. Please note that this beta is not the correction factor, whichever we have introduced in our lecture one or lecture two, I cannot recall them. So, this is independent, it just shows an intensive property. So, if in such a particular volume, having an extensive property B the responding intensive property beta, rho is the density of the fluid. Then the change in, the extensive property with respect to time, this is nothing but equal to, the amount of extensive property created or lost, inside the control volume, or inside this volume, plus the flux of that extensive property, that crosses through the control surfaces of this volume, so that is how we do the thing here; beta rho v dot d a, or you can say v dot n d a like that also one can write it. So, this is the Reynold's transport theorem equation. You can use the same equation here, in our case B is equal to m v, this is the average velocity v bar, your beta is equal to v bar. According to the Newton's law d v by d t is equal to d by d t of your entire this thing, is equal to the net force in the system.

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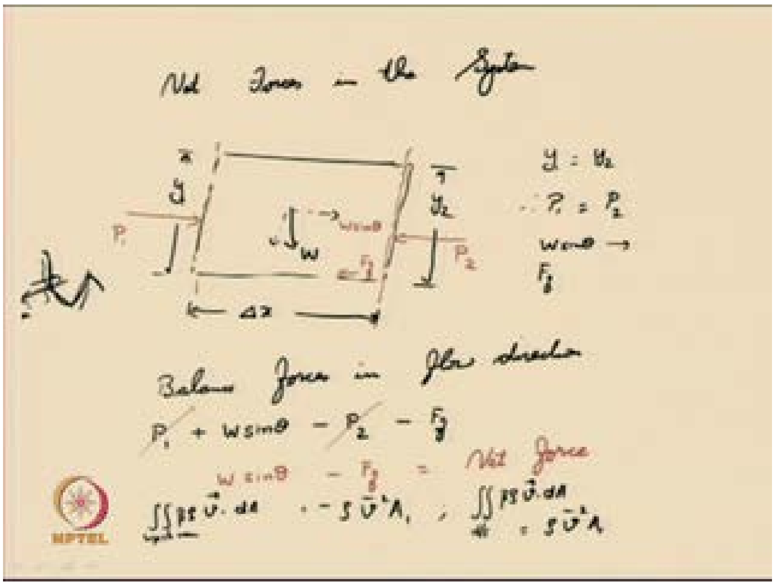

So, what are the net force, net forces in the system. You have your control volume, I just took the control volume like this, the corresponding depth y 1 y 2, weight this length is del x. So, this is a fluid control volume in the channel, you are taking a control volume from the from control volume of the fluid from the open channel, for a case of uniform flow. So, in this particular reach, you can now suggest easily, what are the forces. If I take this control volume, definitely you are going to have pressure force, at this left boundary you will be going to have pressure force in the right boundary, you are also going to have the weight; say w sin theta, there is frictional force S f.

So, the net forces acting are, so the net forces acting will be, you want to balance forces in flow direction. As the depth of flow y 1 is equal to y 2, therefore if you recall how to compute pressure forces, then p 1 is equal to p 2, your w sin theta is equal to the component of gravity force in the flow direction; F f is the force due to friction in the flow oppose; that is opposing the flow. So, we can suggest now, the net force is p 1 plus w sin theta minus p 2 minus F f, this will be your net force in the flow direction.

If you see the net force in the vertical direction, or the direction normal to the P, normal to the bed, they are getting balanced off, so there you need not take them into computation here, definitely p 1 and p 2 they are cancelling of, so I can suggest that, w sin theta minus F f is now your net force. So, net force if you. Now, again go back into your previous equation. Here he suggested that components in the left hand side of this equation d v by d t. So, this components d B by d t, from the earlier shown case, this is now equal to your w sin theta minus F f, so you just substitute it accordingly. There is no creation of momentum inside this control volume, we are assuming, but the momentum is not created inside the volume of the channel. So, this quantity, entire quantity, it can vanish off, you may need not assume that. Now in the channel, along the control surfaces, there are two control surfaces that permit the momentum flux to be transferred; one on the left hand side, one on the right hand side, so that you need to take into account now.

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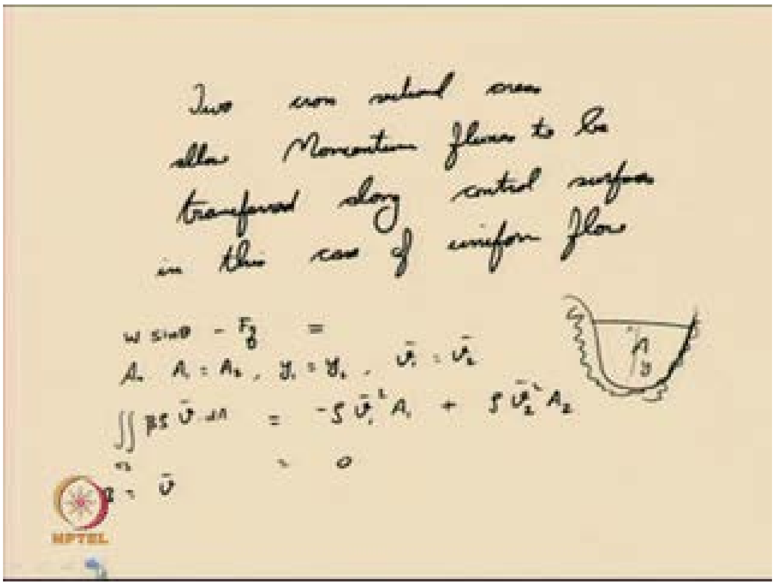

Two cross sectional areas allow momentum fluxes to be transferred, along control surfaces, in this case of uniform flow. So, I can write that now w sin theta minus F f, this will be equal to. What are the two cross sections this thing, you control surface beta rho v dot d a. So, in the channel cross section; a is the cross sectional area, y is the depth of the flow. So, what are the quantities you will get it here, you will get on the left hand side, your beta is equal to v. So, on left hand side the area, here in the left hand side, the corresponding area of the channel; say this is the corresponding. Then outward normal of this area is in this direction, whereas the flow in this direction, so the corresponding integral beta rho v dot d a in the upstream. This can be given as v rho, v rho v square a one and the minus quantity, because n is in the opposite, v dot d a. The v dot d a product in this region, it will be a negative quantity. Similarly, in the downstream section, beta rho v dot d a quantity, this will be equal to rho v square a 2, but you know A 1 and A 2 are same. A 1 equal to A 2, y 1 is equal to y 2 and all, you are getting these quantity; that is beta rho, this is equal to minus rho v 1 square, v 1 equal to v 2 A 1 plus rho v 2 square A 2, this is equal to 0.

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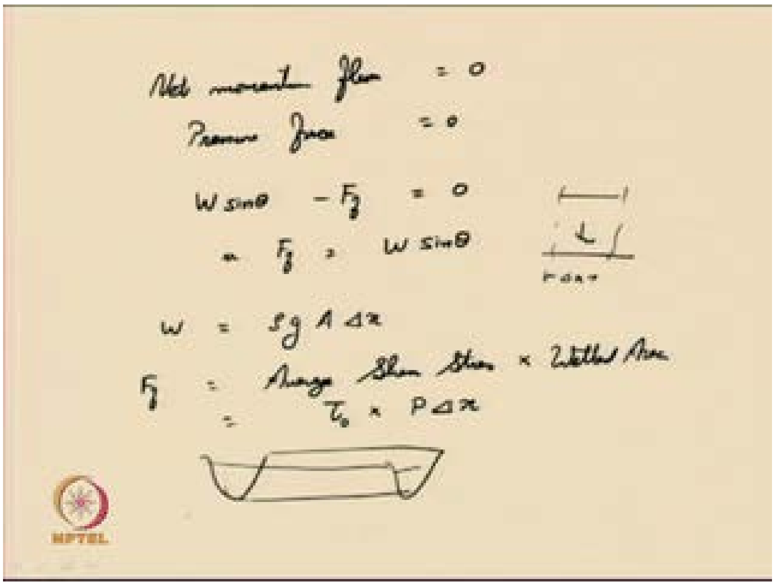

So, your equation will become w sin theta minus F f equal to 0, or F f is equal to w sin theta. Now the weight of the liquid in the control volume, if you recall the control volume stretch del x. So, w, this w is nothing but density in the acceleration due to gravity, into area of cross section into del x. You can also see that the frictional force F f, it can be suggested as, a quantity of average shear stress into, the wetted area of the channel, in the channel reach whichever channel which you have. In the channel reach, the amount of wetted, wherever wetness are there, the entire reach wherever the wetness of channel is there, that area you can compute, wetted area into average shear stress; that will give you the frictional force, so I can write this quantity as tau 0. Now the wetted area, this is nothing but the wetted perimeter of the cross section into the del x quantity.

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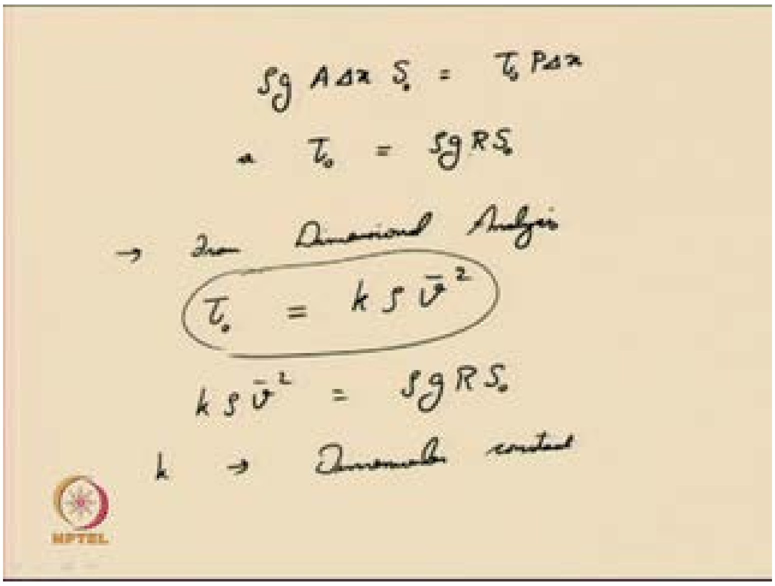

So, I will just go ahead as rho g a del x, S 0 is nothing but tau 0 P del x, or your tau 0 is nothing but rho g R S naught. There is a theory from the dimensional analysis, whoever have studied fluid mechanics, they will be knowing that dimensional analysis. It has been observed, the average shear stress, it can be given as a product of some quantity k, into the density of the liquid, into the square of the average velocity of the cross section. This has been identified through dimensional analysis. We are now just trying to incorporate them, correlate them. We use the same thing here, this is equal to rho g R S naught, your k is a dimensionless constant. So, all other quantities you are aware of.

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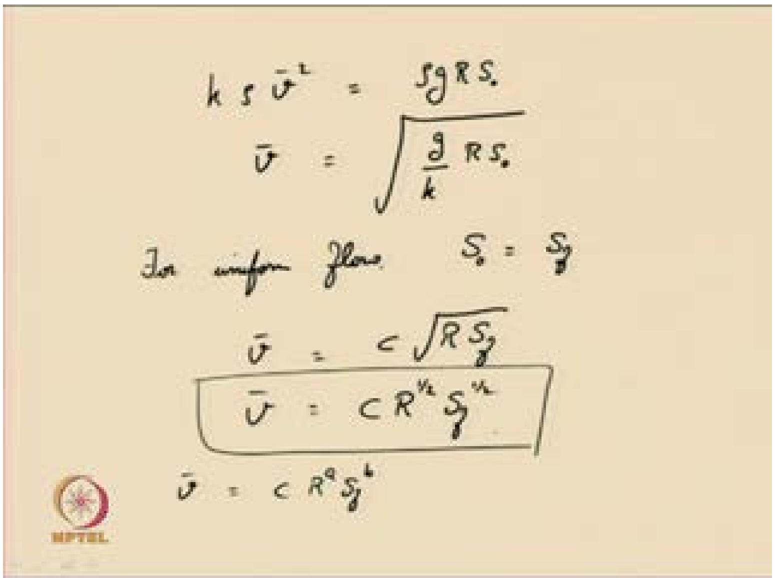

So, I can write the following from now, k dot v square is equal to rho g R S naught, or you R v bar; this is equal to g by k, the square root of that thing. Again as we have mentioned that, for uniform flow your bed slope and energy slope are same. Then we may compute this thing, C R into S f or this is equal to C R to the power of half S f to the power of half, so this is your famous Chezy's formula. Now if you recall your uniform flow formula, I have just few minutes ago mentioned that C R to the power of a S f to the power of B. The same Chezy's formula is complying with your uniform flow formula, and it has been derived using the fundamental momentum conservation of momentum equation. So, this is how you derive Chezy's formula, and this Chezy's formula is used to compute uniform flow in various open channels. There are many applications of uniform flow in open channel; we will see it in the next few classes.

Thank you.

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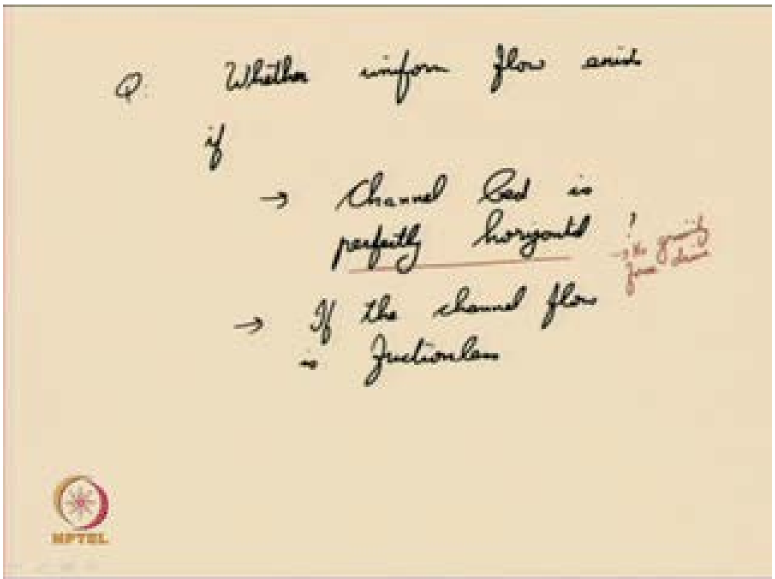

So, let us as part of this today's lecture, let me ask you a simple question, whether uniform flow excess if your channel bed is perfectly horizontal, and whether the uniform flow exists, if the channel flow is frictionless. From the knowledge you have acquired today, or from the concepts you have learnt today, could you answer this simple question, it is just a fundamental thing. What happens if the channel is perfectly horizontal. Yes if you can easily answer that. If your channel bed is perfectly horizontal, then no gravity force drives the fluid downstream. There is no component of drive force that drives the fluid to the downstream right. So, then what happens, if the channel is having friction, your flow is getting reduced, and there would not be any uniform flow existing, so uniform flow will not exist if you channel bed is perfectly horizontal. Similarly, what happens if your channel flow is frictionless. If the quantity is frictionless, then the component or the driving force for the fluid into the downstream; that is not being opposed, or there is no flow resistance to that, and therefore there will be again no uniform flow. The flow of the liquid in such situation gets increased as it goes downstream.

Thank you.