Scheduling Techniques in Projects
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Lecture - 03
Critical Path Method

Welcome you all to lecture number 3, in the week 1 series; topic of today's lecture is Critical Path Method. So, in the previous two lectures I have primarily covered on the general aspects of project management, construction management and so on and we have also discussed on what are the basic inputs required for scheduling, and all the pros and cons with all the basic inputs different methods available we have seen all those.

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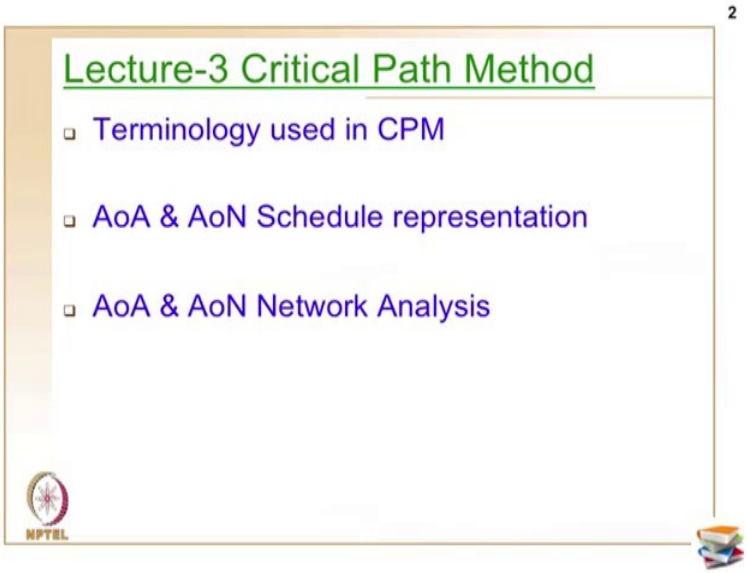

So, today's lecture we will be covering on the first logic driven scheduling method, which is very famously called as Critical Path Method or CPM.

So, in this class we are going to see the terminology used in CPM and the two forms of CPM representation; one is Activity on Arrow diagram and the other one is Activity on Node diagram. So, both the methods we are going to see in today's class, along with their scheduling presentation and the network analysis.

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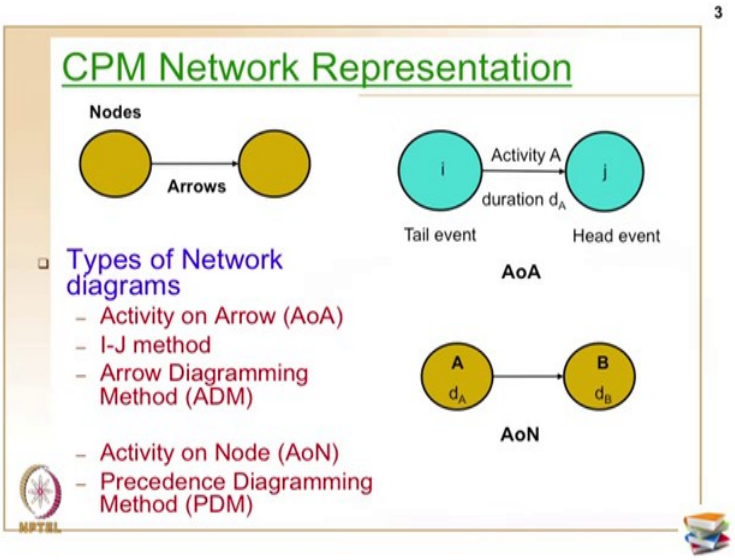

So, CPM network representation generally has two terminology: one is called nodes as you can see here; the nodes are generally in circle or in rectangles and the other one is called arrows. So, in the two types the way you represent the activities along the nodes or arrows we have the two methods.

So, if you see here the types of network diagram, there are two broad categories: one is called Activity on Arrow diagram very famously called AoA and the other one is Activity on Node diagram and which is a AoN method. The other names for AoA are I-J method Arrow Diagramming Method and in your AoN method we also called Precedence Diagramming Method, but PDM is very generally referred for an advanced version of an AoN. So, this is all on the background.

Now if you see the network representation, as I told you where you represent the activities there in only you have the two methods. So in the first method activity on arrow diagram. Now activities are represented along the arrows as how the name also implies. So if you see here, I am representing the events along the nodes and I am representing the activities along the arrows.

So, activity A is represented here and i is the tail event and j is the head event. So, primarily any activity I do not represent an activity, I primarily use the events to talk about the activity and that is a real reason why we also called the AoA method as an I-J method.

In the AoN as the name implies activities are on the nodes. So, these are the nodes and activity representations are generally done on the nodes. So, here if you see; so this is the nodes and activities are represented here and the arrow marks generally show the relationships between the two activities. So that is how we have the two methods and these are the two representations for the same.

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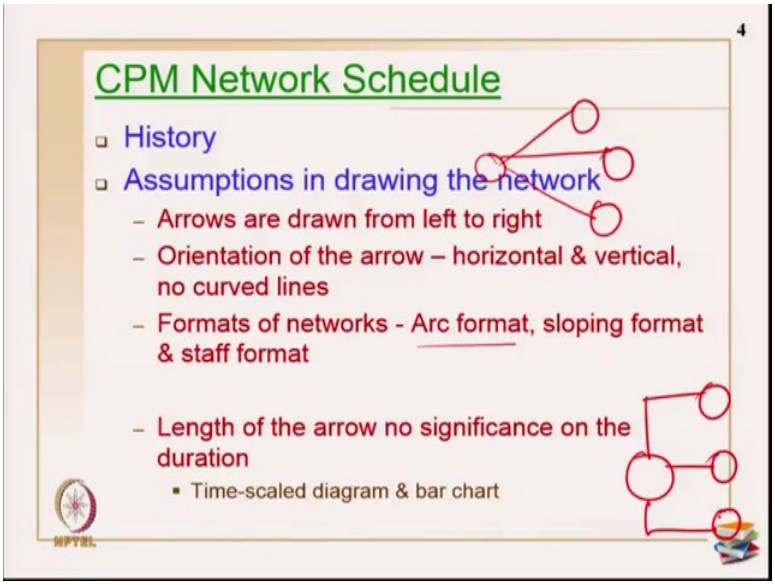

Now, little more on CPM network schedule. So, let us look a little on the history. CPM network is an very old method almost like 1950s or so it started and still going on. The first method which came in existence was a AoA method, because of little problems with the dummies and etc, so AoN came into existence and people are very comfortable in using both the methods as their choice.

Now what are the assumptions in drawing the CPM networks? There are few assumptions: the arrows whether it is AoA or AoN are generally drawn from left to right. And the orientation either horizontal or vertical there is no backward links or something and no curve lines are also available. And there are fewer formats for network representation.

For example, arc format which is not that predominantly used in practice. The other one is called sloping format. Sloping format generally has horizontal lines and also sloping lines which we call it as an angular lines for representing the activities. So, it is represented like this. In staff format the same thing what we do is we represent in terms

of horizontal and vertical lines only. There are no sloping lines and as such there are two ways of presenting your diagrams.

The length of the arrow generally has no significance on duration. Apart from methods like timescale diagram, bar charts and so on, but they are not called CPM network schedules. Generally, any network schedule the length of the arrow is insignificant of the duration.

Now Activity on Arrow diagram.

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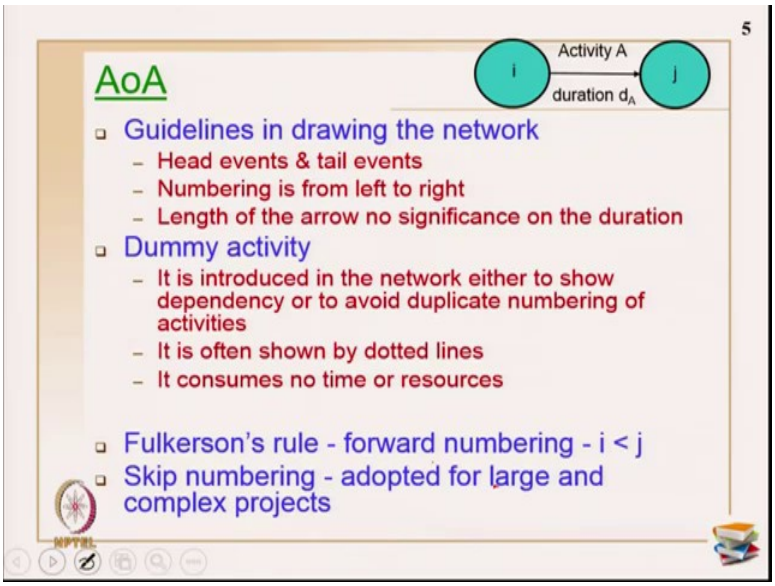

So, we will see little more on that. So, the guidelines in drawing. So, primarily you have head events and tail events, as I tool i is primarily the tail event and j is the head event. Numbering is generally done from left to right which implies it just starts from i and proceeds to j-th event and so on. And the length of the arrow as I have told already its there is no significance on duration, even though you are representing the duration along the arrows.

Then there is something called dummy activity and as I said it is a drawback in AoA and for which only you have an AoN diagram into existence. So, what is this dummy activity; dummy activity is generally introduced in a CPM network either to show a dependency relationships or to avoid duplicate numbering of activities. For any activity I should have an unique i and a j and this i-j cannot be shared for another activity, so for

that reason. And the other reason is just to show the logical dependency between the relationships there may be a requirement of usage on dummies. These are the two reasons in the examples which I am covering I will show both the examples, and how to use for and where are the dummies coming in and what is the purpose of the same. Dummies are often shown by dotted lines. It is easy to identify. It consumes no time or resources.

There is something called Fulkerson's rule, so that is talks about the forward numbering. So, i always has a number which is less than j and there is also something called skip numbering adapted for large complex projects. Every project whether it is medium or large project generally where changes are unavoidable and if there is a change of scope of the work and few activities are added up what happens is, you may have to redo the numbering if you are planning in consecutive 1, 2, 3, 4, 5, 6 and so on.

So, general advice is to leave some numbers or gaps in the middle and start like 10, 20, 30 or 10, 15, 30, 50 and so on, so that you can add activities with the change in scope of the project. That is the main reason on skip numbering.

So, these are the general issues with AoA diagram.

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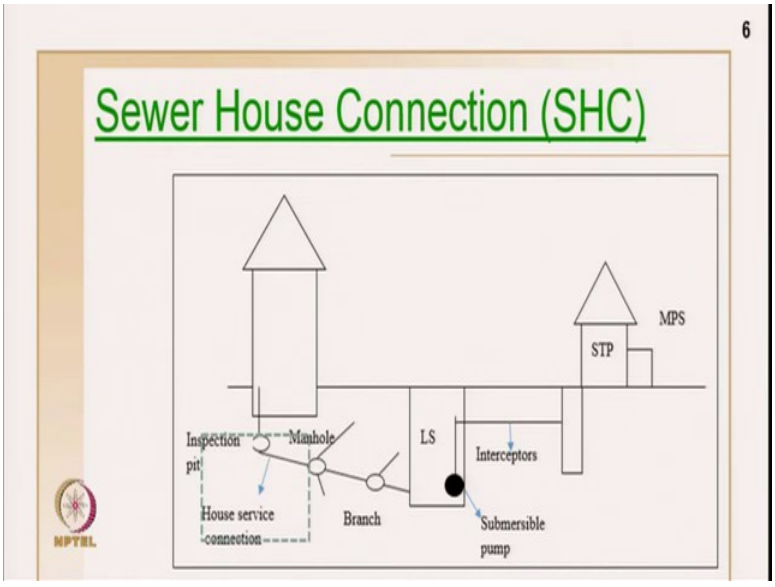

Now, let us take an example in order to really understand how to represent a AoA diagram and the analysis later on. So, first example primarily is a sewer house

connection for collecting sewer from households and how it links to the public or streets sewer.

So, if you see here this is a schematic diagram, I have a main pumping station and there is a sewer treatment plant; STP stands for Sewer Treatment Plant and MPS is for the Main Pumping Station. I have interceptors as shown here and there are lifting stations also, there are pumps for pumping up the sewer collected also and there are branches from the house it just moves on and so on. And inspection pits from the pits connections are all happening. Now.

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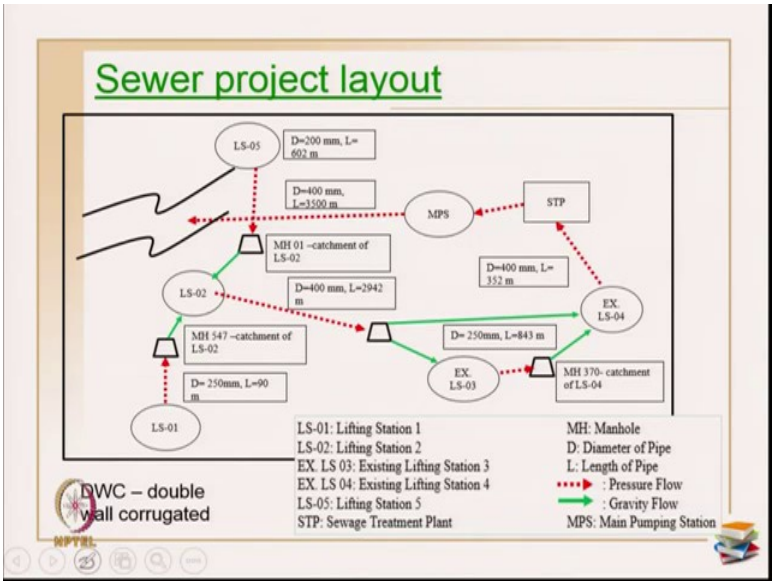

So, a little more on the layout. So, there are pressure flows, there are gravity flows in the area as well. So, you will have location of manholes like this, lifting stations are all here located at station 1, station 2, station 3, and there are manholes also based on the slope and gradients and the pressure on the flow of the sewer.

You may have to do either gravity flow or there can be pressure flow and accordingly there are lines marked here on green and red and there are several existing lifting stations also, new stations are also planned for connected with the with the existing stations in order to do the pumping action and collecting of the sewers to a line, right.

Now the scope of the present project is only on DWC pipes; double wall corrugated pipes just for one manhole I have and the pipeline which I have as a connection. So, what

are the activities, how was the duration calculated on, how was the construction really done?

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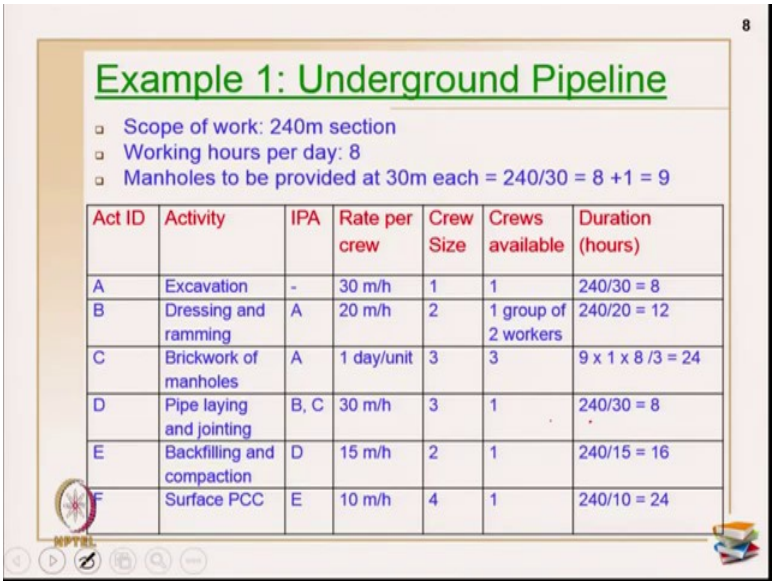

So, let us talk only about the underground pipeline including the manholes. So, how do you plan for the whole section of the work? So, scope of the work is planned is 240 meter sections. So, covering the entire flows I have 240 meter of a stretch I am planning to take, so how much time it generally takes for finishing of the entire tasks? Working hours per days by default is 8 hours per day which is also taken. Manholes general norms in this particular project was to be provided at 30 meters interval. So, approximately there is 8, starting with the connection or the starting points you may have to do 9 manholes in the whole sections, ok.

So, now the activities for this particular construction, there were six activities we have taken it was not completely exclusive, but we just took out only the major activities for the whole project. So, excavation, dressing and ramming, brickwork for the manholes, pipe laying and joints, then backfilling and compaction and the surface PCC; these were the 6 activities that were taken.

In order to have an easy way of writing IPA's and going ahead with the analysis we also introduced an activity ID, so A B C D E F. IPA I have assigned based on the activity IDs of these activities only. If you see here and all activities are done in sequence except the activities B and C which have shared the same IPA, so they can be done in parallel ok.

So now, what is the crew productivity? So, this is not rate per crew this is a crew productivity, the productivity of the following crew that was planned for the project was; for example, excavation 30 meters per hour and the crew size available was 1 crew size and the crews available were 1 crew; which implies one group of only one worker was available for the entire excavation activity.

So, duration taken is a simple formula which you always know; what is the quantity of work to be done? Which is 240 meters divided by the productivity constant which is 30, so this will be like 8 hours is the duration for excavation. Now for the second activity dressing and ramming. So, 20 meters per hour and there were two workers were available in one group and one group was only available at that time. So, one group of two workers, so it is primarily 240 by 20 which is actually equal to 12 hours is a duration calculation.

So, similarly all the durations for all the activities were calculated in that way. This is only to show how to represent, how to define the scope of the work, how to list out the activities and how to get the duration. In this case the resources were primarily driving out the duration and the duration was not fixed as a constant and then the resources were are same. In this case the duration came out naturally as a result of the crew size which were available, ok.

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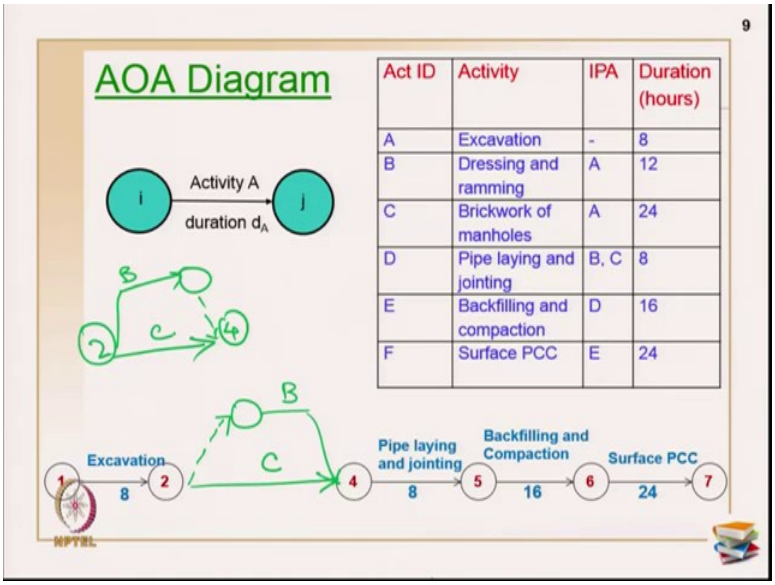

Now, this project is represented here. So, I have the list, so from the previous table I have taken up. I need only these information, so I am just showing up only the IPAs and the duration for all these activities, ok. As I told the activity B and C are in parallel.

So, I have not shown that activity, but I have shown all the other sequential activities in the network. And the duration and the activities are represented along the arrow marks. Generally, activities represented above the arrow mark as shown here and duration is generally written below the arrow mark as seen here and numbering is also done.

Now, in this particular case. So, this is primarily I am going to introduce a dummy, because if I connect the two activities like this what will happen is; I am going to share the same i j for both the activities. So, I wanted to avoid that. So, what I have to do is there are several options for; there are several options for showing the dummies primarily I have to introduce the dummy.

So, one option is I can introduce a dummy here and then I can connect the activity, this is one way. The second way is, so from 2 to 4, I can introduce the activity and then I can connect the dummy. This is activity B, this is activity C, this is activity B, this is activity C.

Other option is; so two options are already shown here.

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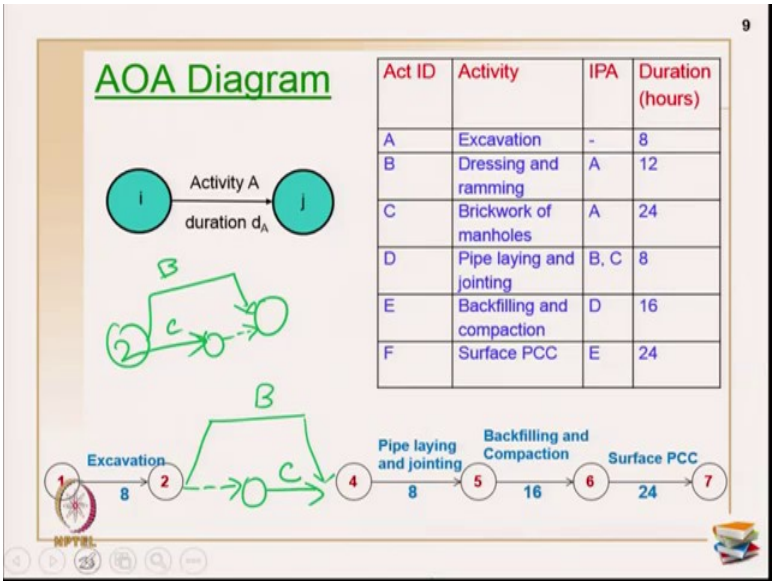

The next option is I am going to connect my activity B as such. So, this is my activity B. Introduce a dummy here and then connect the activity which is C or else I am going to introduce my activity here and then connect my dummy, ok. In this way how many options I can have 4 options of putting the dummies and all 4 options are equally good. It is primarily based on the convenience with which you are going to choose the options. So now, what I am going to do is, I am just going to choose one of them here.

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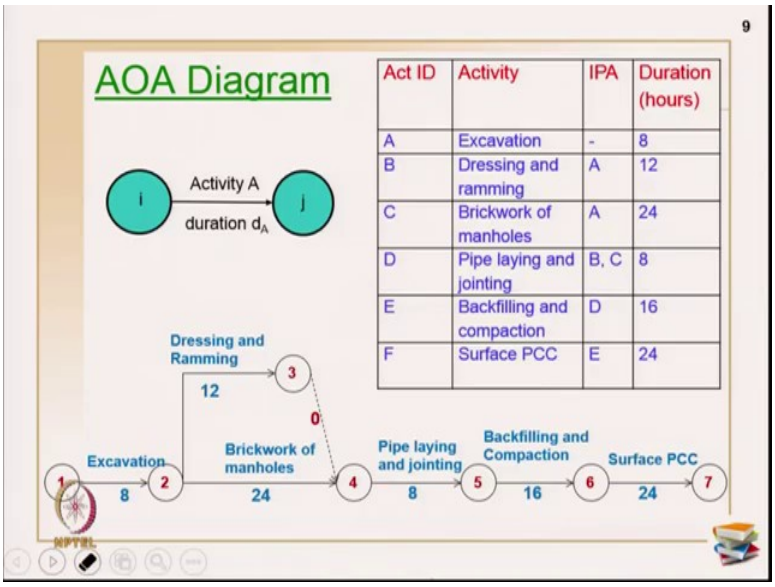

So, this is primarily the option which I have chosen and I have just represented the network, but there are other options also available and this dummy is introduced primarily to avoid the unique numbering of activities between the activity B and activity C. So, that is all on the AoA diagram.

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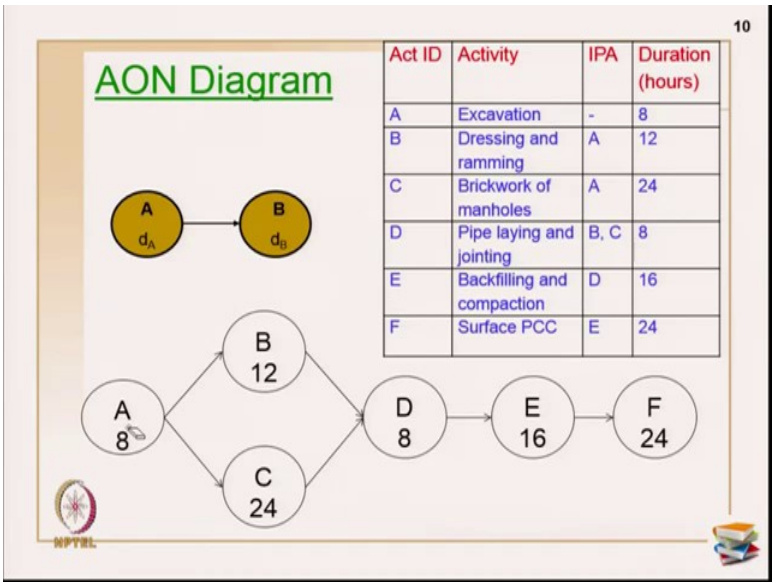

Now let us move on to the other form of CPM representation which is a AoN diagram. So, the same example I am going to take. So, the same example I have taken: duration everything I have taken from the previous example which I was listing out right now.

So now, how do I represent this example; as I have told earlier, activities are generally written above the inside the circle but above, and the durations of those activities are written below, but inside the same circle. And generally, we use circle representation for a for a CPM network and we use a rectangle representation for a PDM network, but the choice is among the users you can use anything.

So, this is how the activities are represented. So, after A, I have B and C in parallel and after B and C, I have to start my D. After D is completed E will start, and after E is done F starts and finishes.

This is a whole network. As you can see here this is really an easy network to draw compared to a AoA because of the absence of dummy activities, ok. So, that is where this is most preferred compared to the AoA method.

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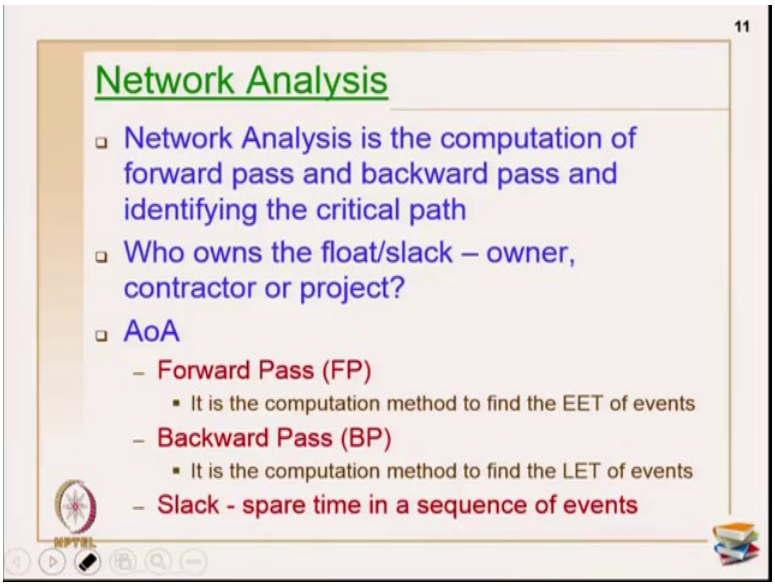

Now moving onto network analysis. So far we have seen network schedule, the terminology when you say network schedule is primarily representing your networks on to the with either in the AoA format or in the AoN format and the minute you say analysis, primarily you are going to do forward pass, backward pass, and critically you are going to find the critical activities in the path. So, network analysis is computation of forward pass and backward pass and identify the critical path.

Now, why we do this calculation and who wants the float or the slack; now there are two terminologies which are coming in: one is called float other one is called slack. Float is a word which we generally use in the AoN format and slack is a word which we generally use in a AoA format, after few slides you will understand that. Primarily what is it? It tells you how much is the delay in time, how much you can actually delay any activity without affecting the final project duration of the whole project ok.

Now, you know that there is a buffer time existing for a few paths or for a few activities and so on. Now who has the responsibility or who owns that free paths in the luxury of starting and activity or a particular stretch, late or delaying and slowly progressing and so on. Is it owner or contractor? It is primarily definitely not the owner and definitely not the contractor, it should be a project related float, ok. So, that should be a good. if you If there should be a successful projects to happen then the project should own the float and neither the owner and not the contractor.

Now, let us see the first method which is a AoA method and the forward pass, backward pass, and the slack calculation. So, forward pass it is a computation method to find the early event of early event time of events, that what is the EET; Early Event Time. And backward pass it is a computation method to find the late event time of event, so which is LET and slack is nothing but the spare time in the entire stretch of sequence of events, and generally slack is given as LET minus EET, ok.

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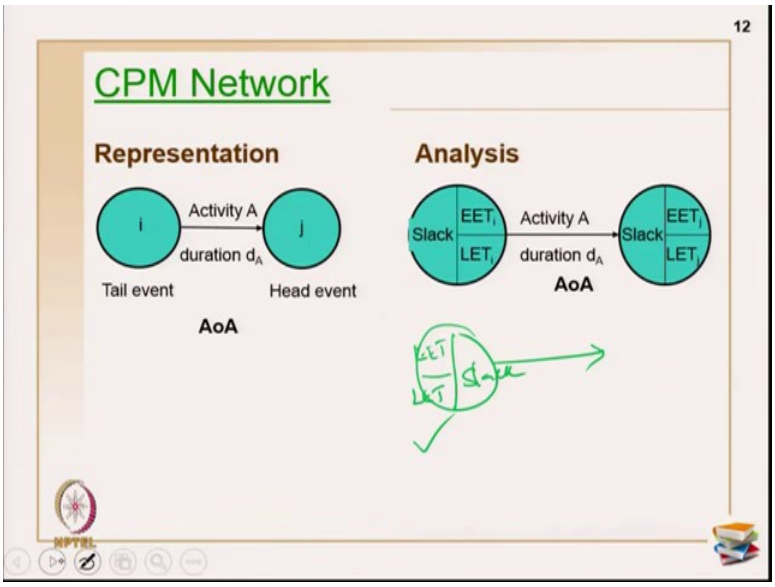

Now, how do we represent? For representation AoA, this has been discussed earlier. For analysis how do we represent? This numbering is not generally used when we are doing analysis. So, the late event time or early event time is generally represented inside the events and then slack actually comes on one side, ok. So now, this can be also altered. So, LET and EET.

So, inside the circle I can also have EET and LET, and I can also write my slacks this is also possible and both the methods are equally fine, ok. So, whatever you want you can choose here, but generally this is a way to represent the analysis.

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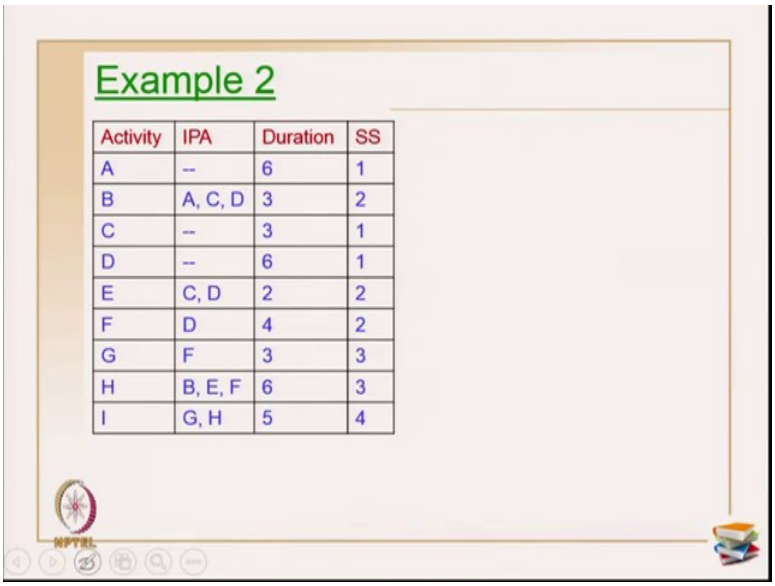

Now, let us move on I am just going to take a hypothetical example right now in order to show the analysis in detail, ok. So, I am just having a few activities here represented alphabetically A, B, C, D, E, F, G, H, I. And the immediately preceding activities (IPA) for these activities are given as like this ok. And I can also have table which can also equally have an ISA as well ok.

So, it is primarily again based on convenience, but the default relationship procedure is to show it on IPAs only. And then I have duration for all these activities given here, ok.

Now what do I wanted to do? First step last example was very simple so it was easy for you to just represent the activities. This example it is little big, as and when you move on in take a real construction project you may have 50 activities are even more. So, how do you start and represent the activities? This is especially good for beginners that is something called sequence steps.

So, first step is to identify the sequence step for the entire table. So, that is easy for you to represent the activities and then connect the activities so that you have a neat diagram in a very few round, ok. So, sequence steps how do you do? So, those activities which does not have an IPA we will come in the sequence step number 1. For example, A, C, and D that is not have an immediately preceding activity.

So, they all will form the first activity to start with, ok. All those activities which had SS 1 we will all be become as I predecessors we will have the sequence steps number 2. For example, A, C, D had SS 1s, so those activities where all its coming as a predecessor. So, primarily activity B, activity E, activity F we will all will form as a sequence steps number 2, ok.

Now, what all the activities which have sequence steps number 2 as an IPA will all become the sequence steps number 3. For example, so it should be E F or B. So, primarily the activity G and H is from is under sequence step number 3 and G and H is IPA for I and hence that becomes a sequence steps number 4. In this way if you progress it is easy for you to finish up the entire arrangement of the activities.

Now what do you do after arrangement? So, first is AoA diagram.

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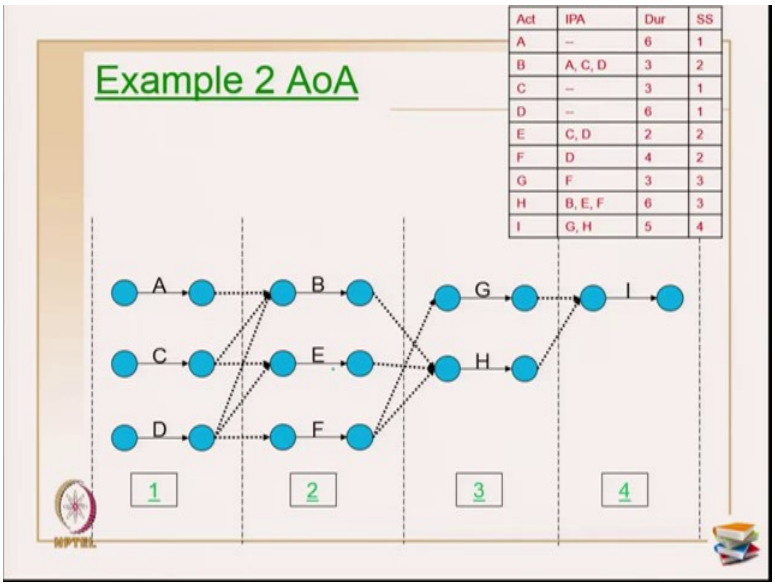

So, the same table I have shown it on the top for you to really correlate and understand. So, primarily the first sequence step, second sequence step, third sequence step and fourth sequence steps are given here. So, I have arranged my activities accordingly. This is activity on arrow diagram.

So, I am representing all the activities along the arrows. So, A, C, D is written here B, E, F is the second sequence step; G, H is the third sequence step; and I is coming in the fourth sequence step, ok.

Next is connect all these activities using a dummy relationship ok. So, seeing these relationship which I have along the IPAs. So, A has no IPA and B has A, C, D as an IPAs, so I am having A, C and D as IPA. Then C does not have any IPA, D does not have any IPA. For E activity I have C and D as my IPA, for F it is activity D is my IPA. Then for my activity G, F alone is my IPA; for activity H, I have B, E and F and then I have for activity I, G and H are my predecessors.

So, connect all these with the help a dummies; that will be the second step which you have to do, ok.

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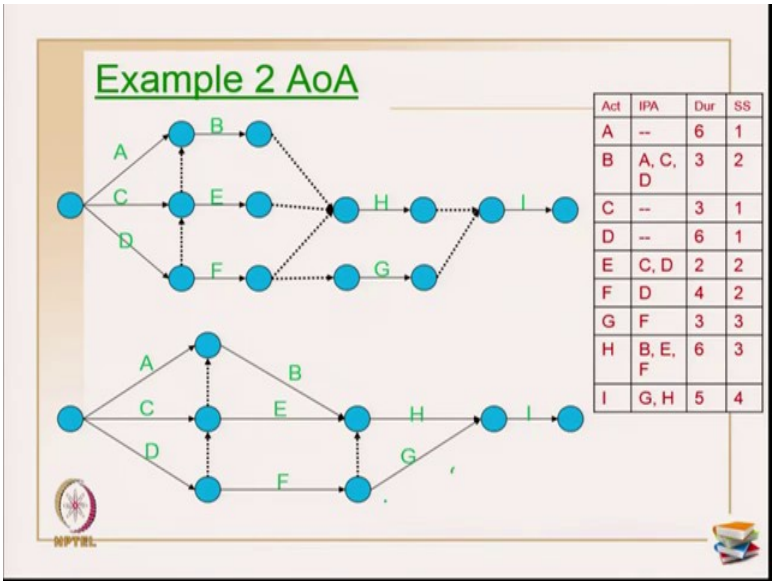

So, the diagram is something like this. Now once you have connected all the relationships, then see whether you can remove some of the dummies so that your relationship is still maintained, but unnecessarily you are putting all the dummies can be really avoided ok.

So, in this particular case for activity E, C and D are the predecessors. For activity B; A, C and D are the predecessors. So, keeping only these two dummies in order to represent the logical flow. The first example I showed a dummy representation to primarily showcase the unique representation of activities. In this example the dummies are used for showing the logical relationship between the activities.

So, the next step will be like; now check for these relationships. Again these 3 dummies can be easily removed, so I have removed all the dummies. And to maintain the relationship, so B, E and F is primarily for H and for F it is for G and then I have I which is H and G. So, when you are removing the dummies see to that and correct all your IPAs whether it is right or wrong and then confirm for the figure before you are removing it.

Now check for each and every activity, I think all my activities are checked for all the dummies. So, this looks like a final network, ok.

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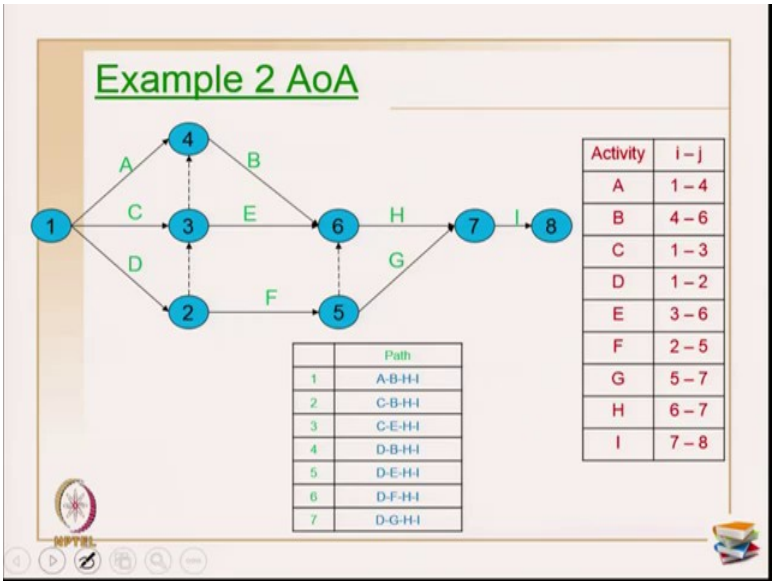

Once the final network is arrived at then the numbering generally happens. Numbering here I have given in sequence only. So, 1, 2, 3, then 4, and then after 5, 6 because I should be always less than G and then i have 7 and 8. So, I have a done my numbering also, ok. And now paths: I have so many paths in this entire network, ok. For example, there are seven paths in this network A, B, H, I C; so this is primarily A, B, H, I ; then I have C a dummy B, H, I. Then I have C, E, H, I; then I have D, B, H, I; D, E, H, I; D, F, H, I; and I have D, F, G, I also. So, these are the different paths I have in this particular network, ok.

So, path explanation also I am just showing it along with this case. Now the main issue behind us now which is a critical path for me and what is the duration for all these paths, ok.

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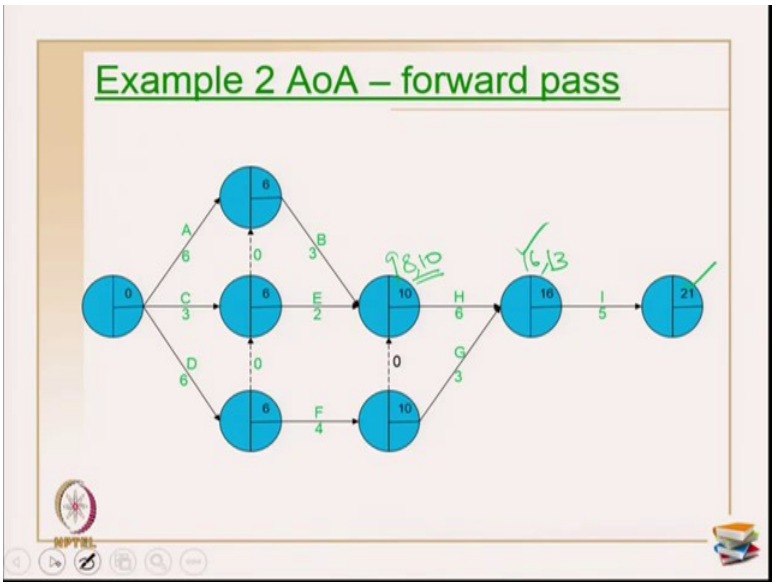

So, now let us start the analysis segments. So far I showed you the simple steps in order to arrive at representing a network, now we will start with the analysis. The same diagram which I have taken up earlier. As I told you numbering will not be done when you are doing the analysis part ok.

So, I am just taking the example and as is shown you earlier the representation for analysis I have divided the circle into 3 segments: one for EET, one for LET, and one for the slack. Forward pass is primarily to calculate the early event time of the events ok.

So now, how do we start? As usual, so you have to start the first event at 0, and again this also I forgot to say earlier. So, these three, and every time you should see to that there is a one starting event and one end event as here, instead of having multiple stars and multiple lanes to avoid dangling of activities. So now, here I am starting with 0 ok. So, D has a duration of 6.

So, for this particular event the early event time is 6 here and for activity C; so 0 plus 3 is 3, this is 6 plus 0 is 0, so maximum is 6. So, I am going to keep this as 6. Now coming back to A, so this is 0 plus 6 is 6, this is 6 plus 0 is 6. Again there is no minimum maximum, but generally you are supposed to take maximum.

So, this becomes 6. Now for this particular activity I have 6 plus 3; sorry I have to do this because this is in the successor list. So, 6 plus 4 is 10, so this early event time of this event is 10.

Now, coming here I have three arrows connecting to this event 6 plus 3 is 9, 6 plus 2 is 8. As a result of 6 plus 3 this is 9, 6 plus 2 this is 8. And this is 10, 10 plus 0 is 10 here. So, I have 3 event numberings here, maximum only you should take when you are doing the forward pass, so this becomes 10. Now if you look at here 10 plus 6 is 16, 10 plus 3 is 13 so maximum is 16 so this becomes 16, and 16 plus 5 is 21. This is primarily the forward pass.

Forward pass what you do? Early start for the first event generally we take 0 and for all other events is maximum of the preceding events plus the duration, ok. So, that is what we have to take here. Next is backward pass: in the backboard pass we have to generally start from the last late event and then we have to proceed backwards and late evens are generally for the successor activities is primarily minus you have to subtract the duration of the activity and go ahead. So, backward pass is generally done in this way and your forward passes are generally done from left to right.

Now we are seeing the backward pass so this is 21, 21 minus 5 is 16 here. Now there are two lines which are two preceding activities after this event. So, 16 minus 6 is 10 here and this I have on more arrow so I cannot do this, so 16 minus 6 is 10.

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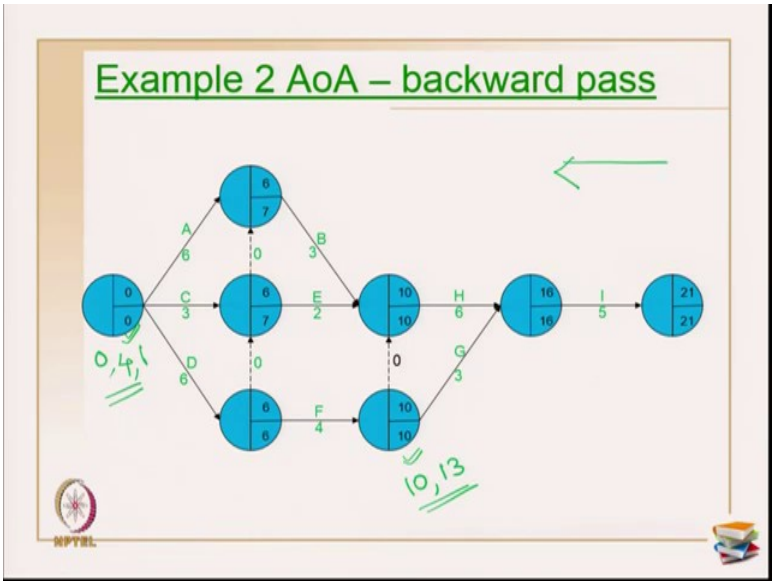

And if you look at here I have 10 minus 0 is 10 and 16 minus 3 is 13. And when you are doing the backward pass we have to take the minimum of the two numbers. So, this will be 10 here I have written 10 here. So, 10 minus 3 is 7 here, then 7 minus 0 is again 7, 7 minus 0 is again 6 here. So, this will be 6; now 6 minus 6 to 0 here 7 minus 3 is 4, 7 minus 6 is 1. So, I have 3 event times and the minimum I should take which is 0 ok. So, that is how the backward passes are done.

So, forward pass is generally calculation of early event times taking into the activities which are all preceding, and you should take the maximum of the value and then go ahead until completion. When you are doing the backward pass you have to start from the last event and then proceed till the first event and every time you have to subtract from the activities and go backwards with the arrows. So, you have to start from the head event and then proceed to the tail event every time, ok. That is primarily the backward pass. So, I have got LETs and EETs.

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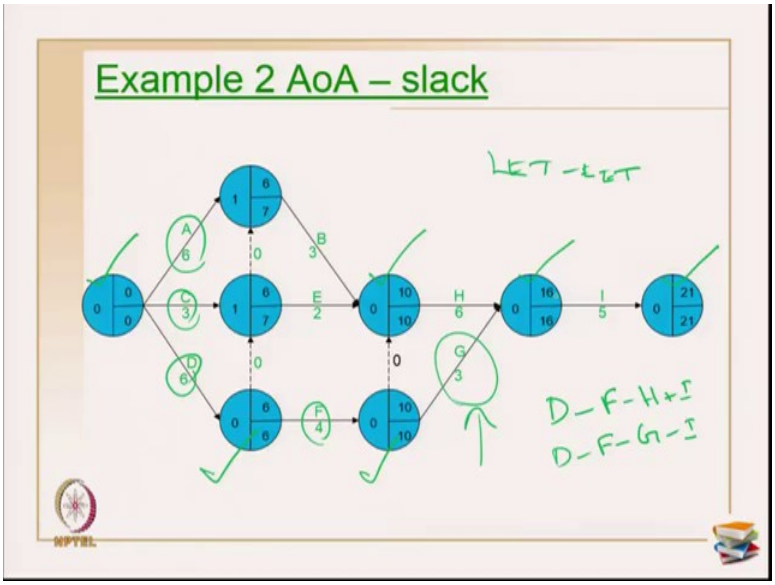

Now, the next procedure is to find the slack. As I told you the slack formula is LET minus EET ok. So, 0 minus 0 is 0 here, 7 minus 6 is 1, 7 minus 6 is 1, 6 minus 6 is 0, 10 minus 10 is 0, 10 minus 10 is 0. So, you have to show for all the events, ok. Once you have done all these calculations primarily the evens which have 0 as a slack are all on the critical path ok. So, primarily what happens; this event is having a 0 slack, this event is

having a 0 slack, this is having a 0 slack, this is also having a 0 slack this and this, so all these are having a 0 slacks, ok.

Now if you see the path, so how many paths I am able to build? I can build D, F, H, I ok, on the other path is D, F, G, I ok. Blindly if you go only with the slack values and then go ahead and fixing up your critical path I may land up in the assessment that there are two critical paths in the entire network, ok. But is the logic really true. If you see here 10 plus 3 is 13, 10 plus 6 is 16, you really know that G is not on the critical path, ok.

Now coming back to the drawbacks in AoA method, one of the drawback we have already discussed what is it; the presence of so many dummies in the network only to do unique representation of activities or to represent the logical relationships. In this network I had 3 dummies, in the last example I had 1 dummy. So, dummies are one biggest disadvantage in a AoA diagram.

The other disadvantage is: sometimes you may be misguided with the values on the slack calculations, ok. What to do in these cases? You may have to again work out on all these activities and cross check for your values whether they are on the critical path or not and then you have to go ahead with the calculations. This is another biggest disadvantage in a AoA network, ok. That is where people started preferring a AoN method.

Now let us show the AoA representation again. So now, the critical path as I told you G is not on the critical path, so the critical path is D, F, H and I along with the dummy in the middle and the duration for this particular project has taken 21 units of time. That is how the critical path calculations are done, ok.

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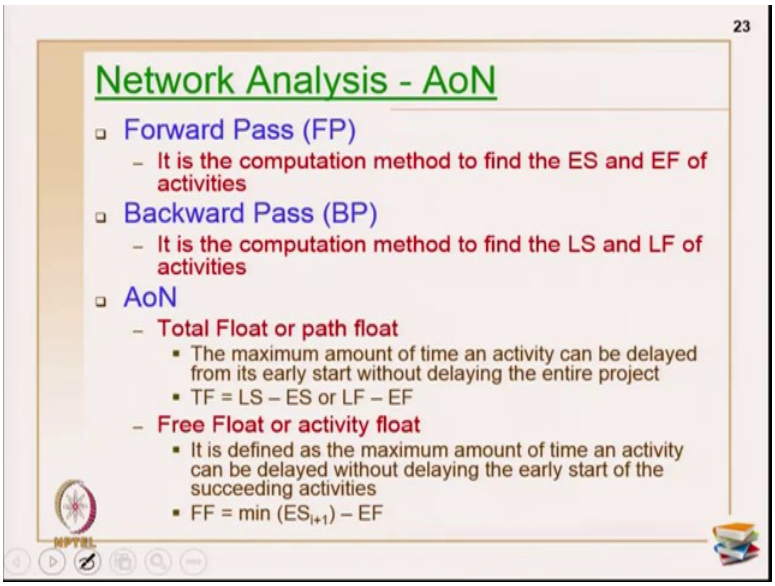

Now in the AoA method whether you are using or the AoN method the procedure for computation on analysis is same. First you have to do forward pass next backward pass and the flow calculations, in AoA we have slack calculations ok. That you should keep in mind.

So now forward pass: it is a computation method to find the early start and early finish of activities. There the focus was events, we were finding out early event time and late event time of events in AoA diagram. Now we are going to see early start and early finish of activities, this is forward pass. Backward pass same way it is a computation to find out the late start and late finish of activities ok. Short forms are ES, EF, LS and LF.

In AoN as I have told you we generally have terminology called floats to show the free time or the delay with which we can have a delay the start of the activities ok. So, primarily the two very famous floats we generally use to find critical paths are total float and the other one is free float. Total float is also called as a path float and free float we also call it as an activity float, you will see when I am explaining with an example ok.

Total float it is a maximum amount of time and activity can be delayed from its early start without delaying the entire project. So, this actually talks about the one stretch of an activity or one particular path, how much is the buffer or delay an activity can do without delaying that particular path in order to finish the in order to without delaying the entire

project duration. So, the formula for calculation is either late start minus the early start or late finish minus the early finish of the particular activity.

Free float as I told you it is also called activity float. It is defined as a maximum amount of time an activity can be delayed without delaying the early start of the successor activities ok. So, the formula is minimum off early start i plus 1 which is the next activity minus the early finish of the current activity. That is the formula we generally use here. In the free float what happens is only tells you what is a buffer time or delay I can have on the activity without effecting the successor activity on the whole network. That is what is the free float meaning behind, ok.

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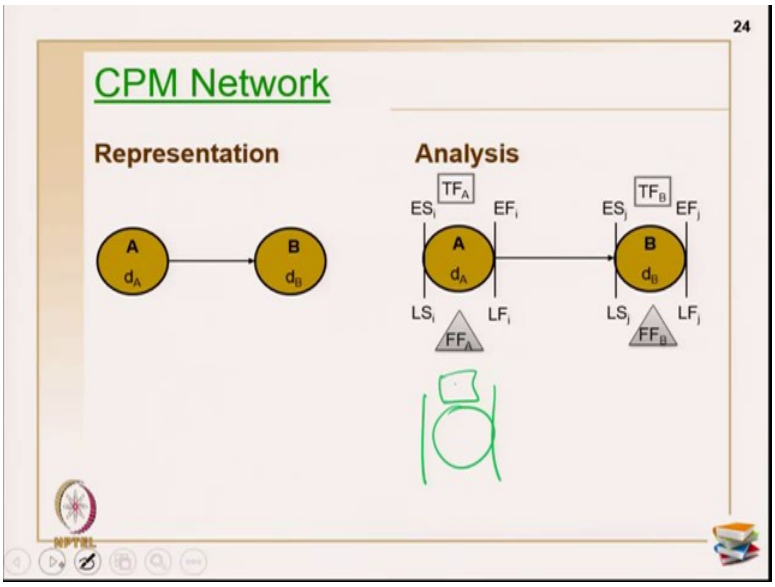

Now, let us see the representation. So, representation for AoN is activity on the die on the node diagram, this is known to you already. Now for analysis how do you represent? So, primarily we have two lines close to the activities. Why these two lines are only to show that that is for the particular activity; if you have a very clumsy network very big network and you keep putting all the 4 values then you will not know for which activity these values are represented for; after the entire calculations are done ok.

So, the early start of the particular activity A, then early finish, this is late start late finish and early start early finish, similarly for B. And total flow generally rectangular blocks on the top of the activity and free float is given by a small triangular below the activity.

Again, these two symbols are only to avoid confusions and to show this is for the total float and this is for the free float. When I am having an activity which is below and I am writing two values, then with the symbol I will know that these are the total float on the other activity and this is free float of this particular activity. Only for that these representations are given.

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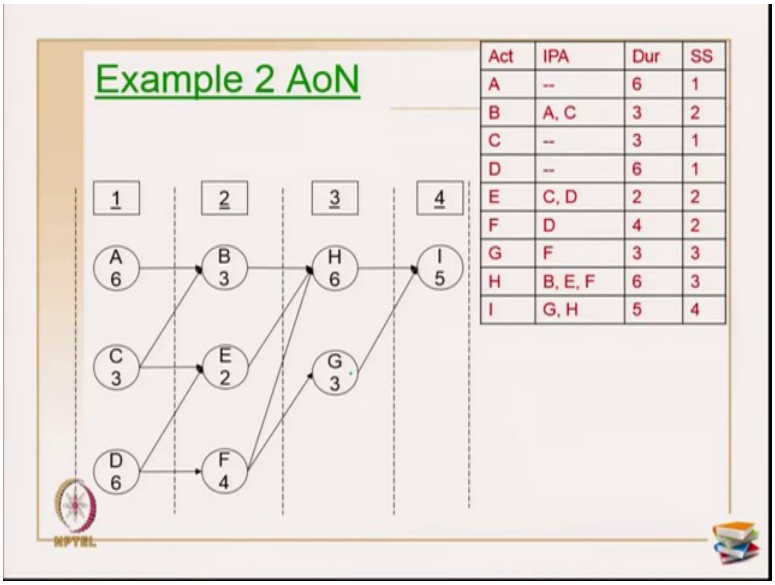

Now, with this we will go back to the same example, not the same I would say this is an other hypothetical example. So, sequence steps as earlier I have given. So, these are the 4 sequence steps and all these activities are listed as per the sequence step as per this figure, I am not expanding the sequence steps again because this is known to you ok.

Now relationships, next step is to connect all the relationships for all these activities. Here I have moved the activity G down, because I know it will be crossing if I am putting it on the other side so I have purposely put the activity G down, ok.

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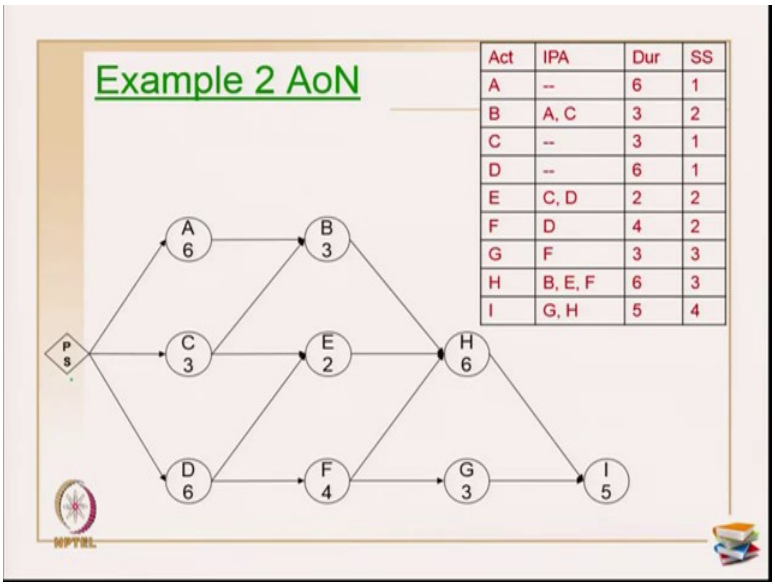

And this, since I have 3 activities in the starting I wanted to avoid the dangling. So, I have introduced a project start, that can be in the same shape as the activity like this and you can introduce as projects start ok; and that has a duration as 0. So, only to avoid the dangling of the activities and this is my entire final network.

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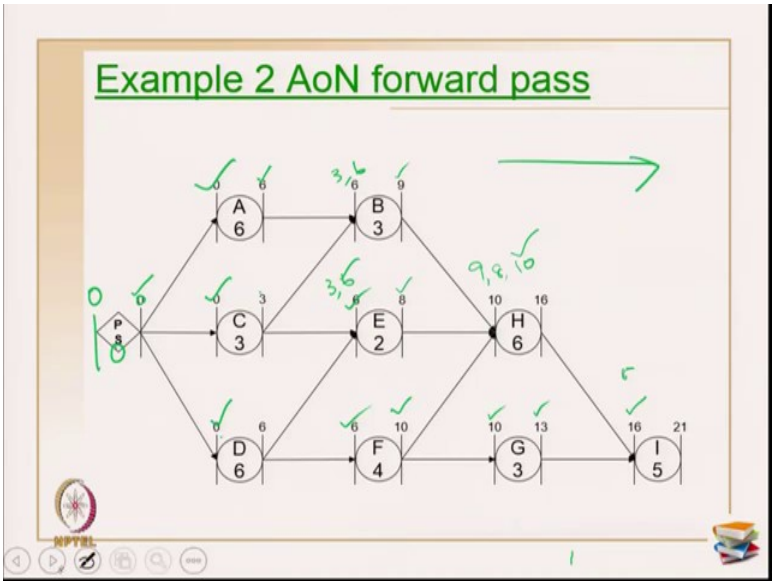

Now, first forward pass, as I told you earlier so primarily you have to put two lines close to each other and forward pass is primarily again from left to right, calculation of early start and early finish of all the activities ok. Project starts, so starting at 0 and the activity

duration is also 0. So, 0s this is also 0, so you generally have 0 ok. So, this 0 0 and 0 all three will have the same 0 values ok. 0 plus 6 is 6, 0 plus 3 is 3, 0 plus 6 is 6.

Now if you see activity B there are two relationships before this activity A and C this is 6 this is 3 maximum time only you should take in case of forward pass. So, this is 3 comma 6 and the maximum is 6 here. Now for this activity again I have C and D as my predecessor and the early finish of C is 3 early finish of D is 6, maximum only I am going to take which is 6 so I am keeping here as 6. And this will be the same number because there is no other relationships ok. Now 6 plus 3 is 9 here, 6 plus 2 is 8 here, 6 plus 4 is 10 here.

Now coming to activity H, I have 3 predecessors one is 9, one is 8, and one is 10 as the result of B E and H F predecessors. The maximum is what I am going to take for the forward pass, so this is 10. So, primarily I have to finish B, E, and F to start my H. There is a logic to that also ok. If I start at 9 then F is not going to complete, so I have to finish for all the 3 activities.

When will all the 3 activities get over that will be on day 10 only. So, I can start H at the earliest starting time of H is day 10 because I want to complete B, E and F before that, ok. Now this becomes 10 ten plus 6 is 16, this is same as 10, 10 plus 3 is 13. Here again I have two relationships 16 and 13. Maximum is 16 so my value is 16 here ok; 16 plus 5 becomes 21. So, this finishes my forward pass calculations.

Now, coming back to the backward pass, backward pass always progresses from right to left from the last event to the first event ok. In the forward pass I finished my early finish at 21, the same value is generally taken for the backward pass late finish ok. The same thing happens in the AoA also. The EET of the last event will be the same as LET of the last event ok.

So, this 21 is here 21 minus 5 is 16, so this become 16, this also become 16. 16 minus 3 is 13, 16 minus 6 is 10 here ok. Now coming here so this is 10, this is 10, and this is 10. And for this particular activity I have two values 10 and 13 ok. As I told you earlier I have to choose a minimum when we are working on the backward pass.

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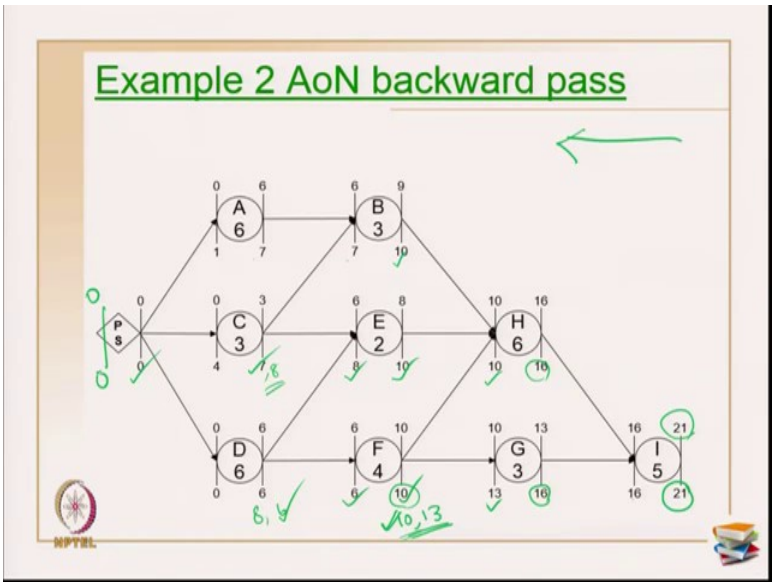

What is a meaning behind this? So, this stops on, this I can start even on 13, but what will happen the latest time with which I can finish should be 10 ok. So, that is the reason why I have to finish the activity F latest by 10th day. So, all these are primarily 10 as a result of this relationship; 10 minus 3 is 7 here, 10 minus 2 is 8 here, 10 minus 4 to 6 here, now this becomes same 7 so this relationship is here.

Now here I have 7, 8 two values are there minimum is 7, so I am going to take 7. Here I am having 8, 6 same way I have two values, and the minimum is 6. I am going to take 6. So, 6 minus 6 is 0, 7 minus 3 is 4, 7 minus 6 is 1. Now here 1, 4 and 0. Minimum value 0, so I am going to do this as 0 ok. So, this finishes my backward pass.

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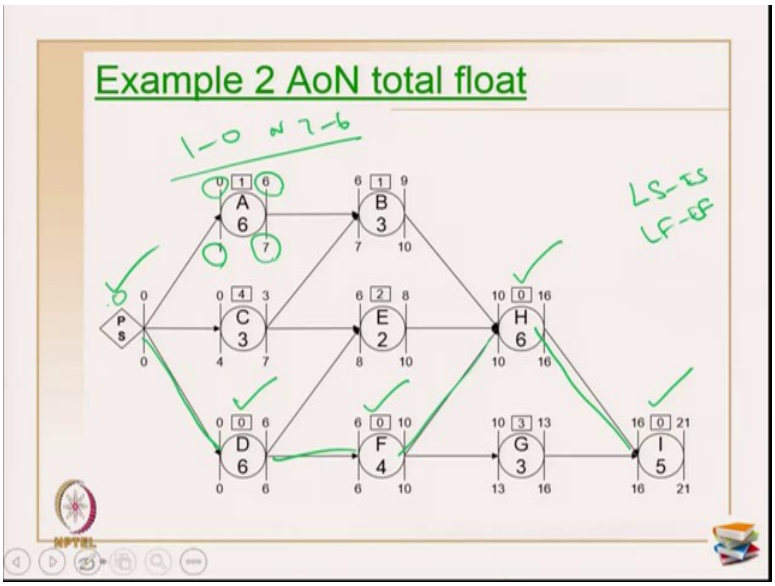

Now if you see here. So, once I have done forward pass. I have done my backward pass, I know the early start, early finish, late start, late finish, of all the activities I have calculated ok. The next step logically is to find the critical path.

To find the critical path we should calculate the float ok. Generally, you can do free float total float. Both the floats you can calculate. But default calculation is by using total float only people find the critical path. Wherever the total float is 0 in an activity those activities will come on the critical path ok. Now I am having the same network. So, how to find the total floats is primarily the late start minus early start or late finish minus early finish is what I said. So, it is primarily the late start minus the early start 1 minus 0 or 7 minus 6. So, both are same here, total float I am getting as 1.

For activity B it is either 7 minus 6 or 10 minus 9 both you can use any one of them, so it is 1. Here 4 minus 0 is 4, 0 minus 0 is 0, 8 minus 6 is 2, 6 minus 6 is 0, 10 minus 10 is 0, 13 minus 10 is 3, 16 minus 16 is 0. So, I have finished my total float calculations completely.

If you look at this network right now where are the total floats 0 values. So, I have at activity I, I have at activity H, I have at activity F and also on D. this logically will be 0 because 0 minus 0 is 0, ok. Now this is also one of the other way to check whether your calculations are right or wrong. Obviously, in any network there will be minimum one critical path in a network and the path in the sense it should be connecting the whole

project and there will be at least one path which is on the critical path ok. If it is not coming then obviously we have done some mistake somewhere in the calculations, ok.

Now let us move on to representing the critical path. So, D, E, F, H, I, E is on the critical path. Now let us go back to the earlier AoA diagram as well.

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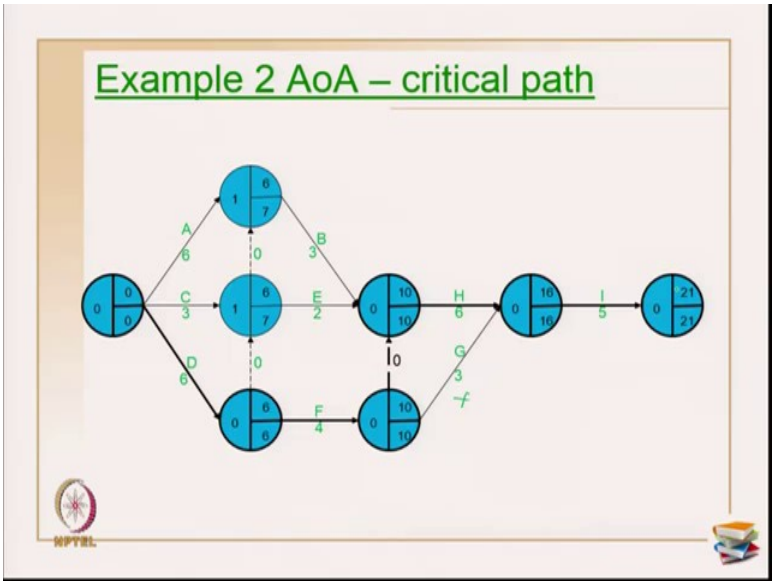

You also had the same network which is D, E, F, H, I, E was on the critical path. So, generally people do not do both the methods for calculations, but I am just showcasing it to you that you will get the same results whatever the method you choose.

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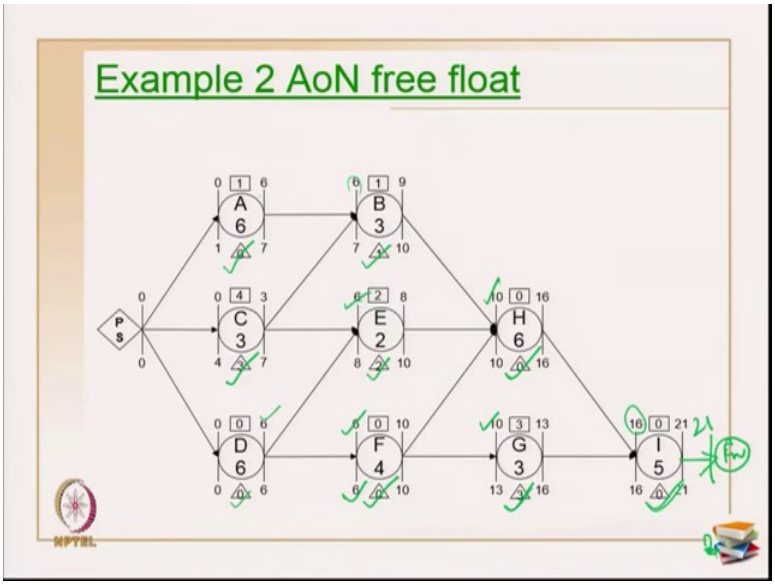

So, the next is free float. So, how do you calculate the free float? Free float for any activity is; so primarily the maximum of the early start of (Refer Time: 44:42) activity minus the early finish of the activity ok. For this particular activity I have only one successor and the early start is 6 minus the early finish is 6, so 6 minus 6 become 0 here. For this particular activity I have two successors and both the values happen to be the same, so 6 minus 3 is 3 for me.

For the same activity D, I have two successors 6 and 6 are the early starts of both the activities and for the current activity I have 6 as my early finish, so this becomes 0 is my free float, ok. Now for activity B again I have only one successor, so this is 10, 10 minus 9 is 1. For activity E, so this is also done. For activity E, I have again only one successor, so this is 10 minus 8 is 2 here so this is my free float.

For activity F, I have two successors. The early start is primarily 10, so this is 10 minus 10 so this become 0, ok. Now when I look at activity H, so the early start on the successor activity 16 minus the finish activity is 16, so 16 minus 16 is 0; so, this become 0.

Now for activity G, the early start of successor activity is 16, 16 minus 13 is 3 so this becomes 3 for me ok and for this particular activity I, what you have to do is I have to assume that there is one more activity which is primarily a finish and what is the early start and the early finish if this activity exist. So, this will be 21 only, this is 21. So, if you have to take that into consideration so 21 minus 21 becomes 0 here, so this become 0 ok. That is how the free floats are calculated.

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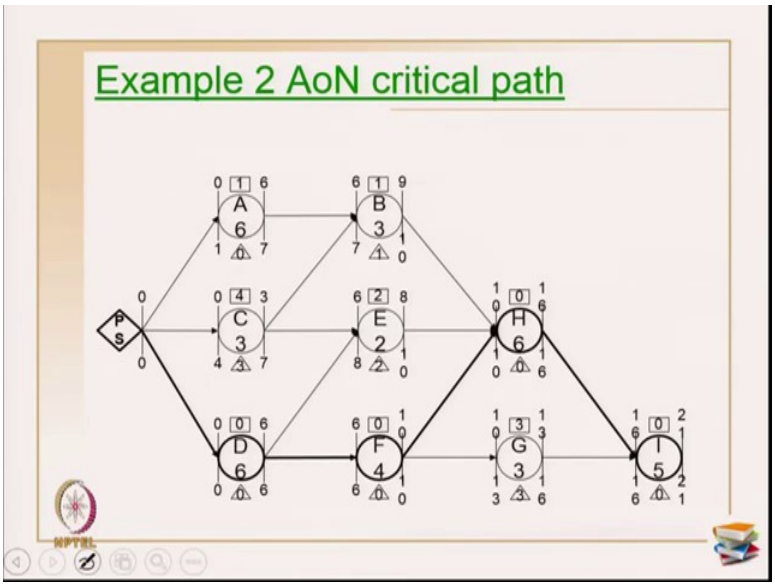

If you look at the forward pass total float and free float, so whatever activity I have for example, are on the critical path the same activities will be on the critical path only for the free float also. So, you will have 0 for activity D, you will have 0 for activity F, you will have 0 for activity H, you will have 0 for activity I. So even with free float also you can still find the critical path, ok.

Now what is the meaning of the free float; that I will explain here. Let us as assume I am taking an activity B, this means I have a 3 day duration for activity B this free float 1 means I can delay the activity B to 1 day and nothing will happen to the delay in the critical path. That is a meaning of the free float I have here, ok.

But if you take in the total float values, the value of this total float which is again 1 this means this value of 1, I can delay my activity B and because of the delay in activity B to 1 day it will not affect my A, B, H, I path to come to delay in the completion. That is why it is called as a path float and in this case it is called as a activity float ok. So, that is all on about the total float and free float.

So, if I wrap up on the AoA diagram and the AoN diagram; AoA diagram came in existence first and then AoN came in. Drawbacks on the AoA I have covered up then and there, primarily the dummies and also sometimes you know miss guidance with the values on the slack values because we are calculating it on events ok. And so, in AoA we

have forward pass, backward pass, primarily with the early event time, late event time, and the slack calculations.

In AoN diagram primarily activities are on the nodes. And we have the early start early finish in the forward pass, late start and late finish in the backward pass. And two types of float we have primarily seen one is total float and the other one is free float. And these two floats are predominantly used in the AoN analysis.

So, that is all about the analysis and the scheduling method in the CPM network. Now what are the other types of floats in the CPM network?

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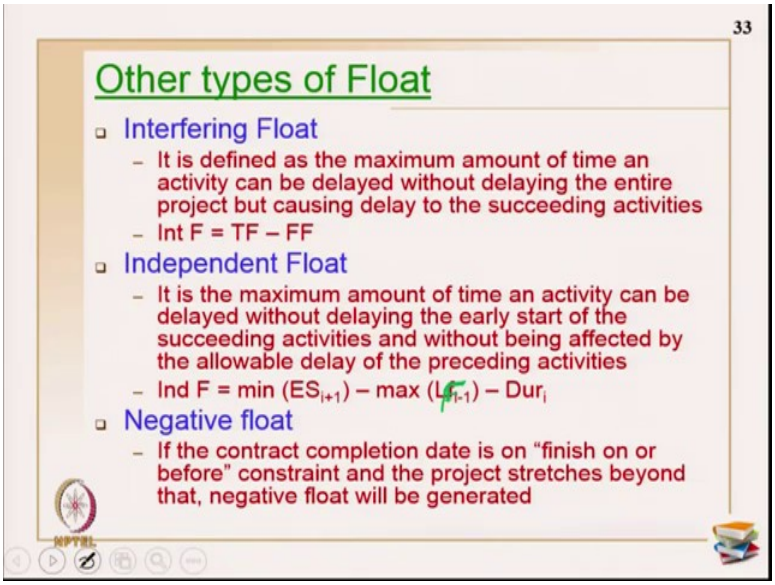

The next one is interfering float. These are all not much used, but there are other types also existing. So, it is defined as the maximum amount of time an activity can be delayed without delaying the entire project, but causing the delay to succeeding activities. So, this interfering float is generally TF minus FF, which implies total float minus free float, ok.

Next is independent float. It is a maximum amount of time and activity can be delayed without delaying the early start of the successor activities all put together, and without being affected by the allowable delay of all the proceeding activities, ok.

So, as the explanation goes the formula also is saying that minimum of all my early start of the successor activities minus maximum of the late finish of all my preceding

activities. There is a mistake here. This should be LF ok. Of all my preceding activities LFs and when I have all my successor activities the early starts, so minimum of the early starts successors minus maximum of late finish predecessors minus duration of the current activity will give my independent float, ok.

Negative float, sometimes what happens in the contract completion date, there may be a definite date called finish on or something there is a definite constraint given there. And sometimes the project expands or delays beyond that when you are putting all your constraints, your resources, schedules and so on.

What happens your project schedule may go on a date which is beyond even the as-planned schedule completion date. Then what happens in that sense you may have a negative load coming into the whole project. That is primarily called negative float.

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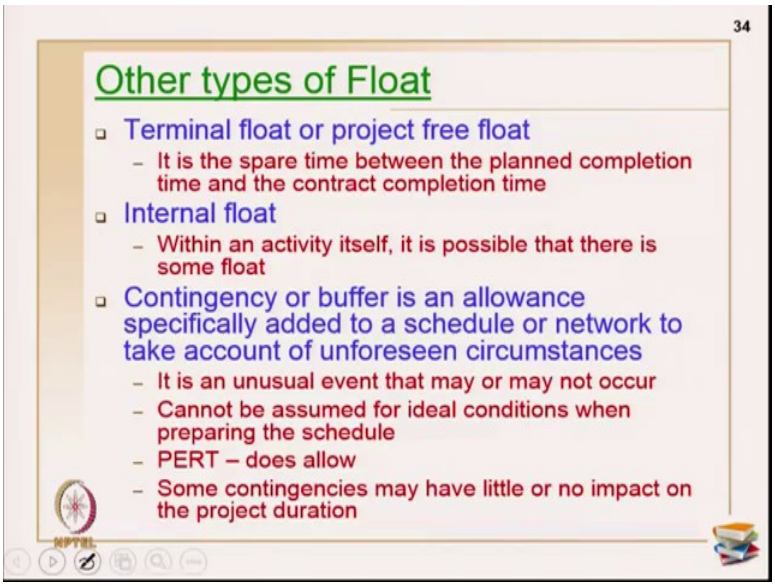

Then next type of floats is terminal float. So, terminal float is nothing but I have a whole project and there is really a different time for my planned completion and also for the contract completion, ok. In the earlier case I said if the contract completion or schedule pressure completion is little early you start getting a negatives on the floats, ok.

So, let us assume I am having a project which is planned to be completed in 30 days as per my TF calculations, forward pass, backward pass. I have done. I have all finished everything, total positive duration for the entire project is coming out as 30. If the

contract completion time is said something as 25 months then what will happen is this 5 months primarily you will get us as a negatives float all along the whole activities.

Suppose if the project completion contract completion time is something around 40 months or something. So, what happens? I can easily finish my entire project in 30 months and the 10 months is primarily a real float which I have on the whole project. This is primarily called the project free float; terminal floats are also called the project free floats.

The next one is internal float. Every activity we are calculating a duration, based on the resources, productivity calculations, crew size available. I am primarily calculating duration for each and every activity. Sometimes what happens is that let us assume I am planning for an 8 day duration for an activity ok. All the 8 days you need not be working on the activity there may be buffer here and there may be a free time you can still take out from the whole activity which is what is primarily called the internal floats. So, within an activity itself it is possible that there is some kind of a float coming out that is primarily called as an internal float.

The next one is contingency or buffer. So, contingency should not be confused with floats. Contingency is purposely you are giving an allowance onto the project and that is primarily called as a contingency. So, contingency or buffer is an allowance specifically added to a schedule or a network to take into account of unforeseen circumstances.

So, it is an unusual event that may or may not occur. That is primarily you cover it up as and contingency and cannot be assumed for ideal conditions when preparing the schedule.

PERT allows contingency because it does contain 3 types of an values like optimistic, pessimistic and most likely.

So, the network itself has uncertainty in the duration calculations and PERT is another network technique which I will cover later. That does allow contingency in some way. And most of the contingencies may have little or no impact on the project duration.

So, that is all on the lecture for today. So, today's class we have seen critical path method. The two types in the critical path method one is the AoA diagram, AoN diagram

the representation of these two networks., the analysis of these two in terms of forward pass, backward pass, slack calculations in AoA, float calculations in AoN and the other types of float also we have seen ok.

So, that is all for today's class and, we will continue on precedence diagramming method in the next class.

Bye.